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Research article

A fractional order alcoholism model via Caputo–Fabrizio derivative

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Abstract: A fractional order mathematical model of the Caputo–Fabrizio type is presented for an alcoholism model. The existence and the uniqueness of the alcoholism model were investigated by using a fixed-point theorem. Numerical solutions for the model were obtained by using special parameter values.

Keywords: Caputo–Fabrizio fractional derivative; alcoholism model; Laplace transform; numerical solution; fixed point theory
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1. Introduction

The many adverse effects of alcohol have been proven. We can divide these effects into two main groups, one of which is social impact and the other of which is self-harm. The negative social impacts of alcohol include antisocial behaviour, criminal behaviour, and violence [6]. On the individual level, alcohol consumption adversely affects all organs of the body and contributes to many diseases such as heart disease, hepatitis, cirrhosis, lupus, and cancer [16, 17]. Some authors have studied models of alcoholism. For instance, Lee [18] introduced a model using two control functions and performed an analysis. Huo [19] worked on a time-delayed model. Huo et al. [20] developed a nonlinear model of alcoholism. This model includes awareness and time delay, with the time delay indicating the time delay of immunity to alcohol use. Mantley developed a new epidemiological model in order to recognize the dynamics of alcohol problems on campus. Twitter is a well-known and important microblogging service that allows Internet users to communicate their thoughts on diverse topics. This service has approximately 500 million users, and these users make an average of 350 million tweets per day. On any topic, users share their thoughts, not exceeding 160 characters. The rapid increase in the popularity of Twitter has attracted the attention of many researchers. It is very easy to reliably learn the thoughts of a user and to keep statistics about tweets.

An alcoholism model was examined by Huo and Zhang [22] with the assumption that the number of susceptible people is not constant. This model includes the positive and negative roles of Twitter. Woo et al. [23] examined the development of influenza outbreaks through social media data, investigating millions of users of social media and blog posts to conduct the research. In fact, this number of studied users corresponded to more people than the population of Korea. Another influenza model was examined by Hou and Zhang [24] using Twitter dynamics. Gomide et al. [25] showed that the spread of dengue could be observed through Twitter and that the data could be used for later situations. Using tweets sent from different regions, Njanku [26] examined the dynamics of Ebola broadcasts in media campaigns. In the present study, the existence and the uniqueness solutions of an alcoholism model are found using Caputo–Fabrizio fractional derivative and integral operators. This model was investigated and studied by Huo and Zhang [22] with integer order. Gómez-Aguilar [27] examined the alcoholism model for existence and a unique solution for the Atangana–Baleanu fractional derivative. The model was also solved numerically. The importance of real-world problems in defining process and memory effects is essential and this article is designed accordingly. The author has published other important works in this regard [10–14].

Modelling and increasing the usability of real-world problems has always played an important role in science. It is important to find the exact solutions of these problems, or the closest solutions. The Caputo–Fabrizio fractional derivative operator provides results very close to the exact solution [7–9, 15].

In the first section, information has been given about the studies on the impacts of alcohol. In the second section, the definitions and theorems used in this paper will be explained. In Section 3, the integration of the classical alcoholism model with the Caputo–Fabrizio fractional derivative will be performed. Section 4 details the existence and uniqueness solutions of the obtained model. Numerical simulations are reported and the results are discussed in Section 5, and concluding remarks are given in Section 6.

2. Preliminaries

In this section, the necessary definitions of fractional derivative and integral operators will be given. More detailed information on the definitions can be found in articles [1–4].

Definition 2.1. *The fractional derivative defined by Caputo is below [3]:*

$$ {}_{a}^{C}D_t^{\nu} f(t)) = \frac{1}{\Gamma(n-\nu)} \int_a^t \frac{f^{(n)}(r)}{(t-r)^{\nu+1-n}} dr, \quad n-1 < \nu < n \in N. $$

Definition 2.2. *Let* $f \in H^1(a,b), b > a, \nu \in (0,1)$ *then, the Caputo–Fabrizio derivative of fractional derivative is defined as [1]:*

$$ {}_{a}^{CF}\mathfrak{D}_t^{\nu}(f(t)) = \frac{\nu M(\nu)}{1-\nu} \int_a^t \frac{df(x)}{dx} exp\Big[-\nu\frac{t-x}{1-\nu}\Big] dx. $$

Here $M(\nu)$ *is a normalization constants. Furthermore,* $M(0) = M(1) = 1$. *The definition is also written as follows,*

$$ {}_{a}^{CF}\mathfrak{D}_t^{\nu}(f(t)) = \frac{\nu M(\nu)}{1-\nu} \int_a^t (f(t)-f(x)) exp\Big[-\nu\frac{t-x}{1-\nu}\Big] dx. $$

Remark 2.3. *If* $\eta = \frac{1-\nu}{\nu} \in (0,\infty), \nu = \frac{1}{1+\eta} =\in [0,1]$, *then the above equation supposes the form*

$$\mathfrak{D}_t^{\eta}(f(t)) = \frac{N(\eta)}{\eta}\int_a^t \frac{df(x)}{dx} exp\Big[-\frac{t-x}{\eta}\Big]dx, \quad N(0) = N(\infty) = 1.$$

Furthermore,

$$\lim_{\nu\to 0}\frac{1}{\nu} exp\Big[-\frac{t-x}{\nu}\Big] = \delta(x-t).$$

Note that, according to the above definition, the fractional integral of Caputo type of function of order $0 < \nu < 1$ is an average between function f and its integral of order one. This therefore imposes

$$M(\nu) = \frac{2}{2-\nu}, \quad 0 \le \nu \le 1.$$

Because of the above, Nieto and Losada proposed that the new Caputo derivative of order $0 < \nu < 1$ can be reformulated as below,

Definition 2.4. *The fractional derivative of order* ν *is defined by [2],*

$${}^{CF}\mathfrak{D}_{\star}^{\nu}(f(t)) = \frac{1}{1-\nu}\int_0^t f'(x) exp\Big[-\nu\frac{t-x}{1-\nu}\Big]dx.$$

At this instant subsequent to the preface of the novel derivative, the connected anti-derivative turns out to be imperative; the connected integral of the derivative was proposed by Nieto and Losada [2],

Theorem 2.5. *For* NFD_t*, if the function* $f(t)$ *is such that*

$$f^{(s)}(a) = 0, \quad s = 1,2,\ldots,n,$$

then, we have

$$\mathfrak{D}_t^{(n)}(\mathfrak{D}_t^{\nu} f(t)) = \mathfrak{D}_t^{(\nu)}(\mathfrak{D}_t^{n} f(t)).$$

Proof. We begin considering $n = 1$, then $\mathfrak{D}_t^{(\nu+1)} f(t)$, we obtain

$$\mathfrak{D}_t^{(\nu)}(\mathfrak{D}_t^{(1)} f(t)) = \frac{M(\nu)}{1-\nu}\int_a^t f'(\tau) exp\Big[-\frac{\nu(t-\tau)}{1-\nu}\Big]d\tau.$$

Hence, after an integration by parts and assuming $f'(a) = 0$, we have

$$\begin{aligned}
\mathfrak{D}_t^{(\nu)}(\mathfrak{D}_t^{(1)} f(t)) &= \frac{M(\nu)}{1-\nu}\int_a^t (\frac{d}{d\tau} f'(\tau)) exp\Big[-\frac{\nu(t-\tau)}{1-\nu}\Big]d\tau \\
&= \frac{M(\nu)}{1-\nu}\Big[\int_a^t \frac{d}{d\tau}(f'(\tau)) exp\Big(-\frac{\nu(t-\tau)}{1-\nu}\Big)d\tau \\
&\quad - \frac{\nu}{1-\nu}\int_a^t f'(\tau) exp\Big(-\frac{\nu(t-\tau)}{1-\nu}\Big)d\tau\Big] \\
&= \frac{M(\nu)}{1-\nu}\Big[f'(t) - \frac{\nu}{1-\nu}\int_a^t f'(\tau) exp\Big(-\frac{\nu(t-\tau)}{1-\nu}\Big)d\tau\Big].
\end{aligned}$$

otherwise

$$\mathfrak{D}_t^{(1)}(\mathfrak{D}_t^{(\nu)} f(t)) = \frac{d}{dt}\Big(\frac{M(\nu)}{1-\nu}\int_a^t f'(\tau))exp\Big[-\frac{\nu(t-\tau)}{1-\nu}\Big]d\tau\Big)$$
$$= \frac{M(\nu)}{1-\nu}\Big[f'(t) - \frac{\nu}{1-\nu}\int_a^t f'(\tau)exp\Big(-\frac{\nu(t-\tau)}{1-\nu}\Big)d\tau\Big]$$

It is easy to generalize the proof for any $n > 1$ [1]. □

It is well known that Laplace Transform plays an important role in the study of ordinary differential equations. In the case of this new fractional definition, it is also known (see [1]) that, for $0 < \nu < 1$,

$$\mathfrak{L}[{}^{CF}\mathfrak{D}_t^{\nu} f(t)](s) = \frac{(2-\nu)M(\nu)}{2(s+\nu(1-s))}(s\mathfrak{L}[f(t)](s) - f(0)), \qquad s > 0.$$

where $\mathfrak{L}[g(t)]$ denotes the Laplace Transform of function g. So, it is clear that if we work with Caputo–Fabrizio derivative, Laplace Transform will also be a very useful tool [2].

After the notion of fractional derivative of order $0 < \nu < 1$, that of fractional integral of order $0 < \nu < 1$ becomes a natural requirement. In this section we obtain the fractional integral associated to the Caputo–Fabrizio fractional derivative previously introduced. Let $0 < \nu < 1$. Consider now the following fractional differential equation,

$${}^{CF}\mathfrak{D}_t^{\nu} f(t) = u(t), \qquad t \geq 0,$$

using Laplace transform, we obtain:

$$\mathfrak{L}[{}^{CF}\mathfrak{D}_t^{\nu} f(t)](s) = \mathfrak{L}[u(t)](s), \qquad s > 0.$$

We have that

$$\frac{(2-\nu)M(\nu)}{2(s+\nu(1-s))}((s\mathfrak{L}[f(t)](s) - f(0)) = \mathfrak{L}[u(t)](s), \qquad s > 0,$$

or equivalently,

$$\mathfrak{L}[f(t)](s) = \frac{1}{s}f(0) + \frac{2\nu}{s(2-\nu)M(\nu)}\mathfrak{L}[u(t)](s) + \frac{2(1-\nu)}{(2-\nu)M(\nu)}\mathfrak{L}[u(t)](s), \qquad s > 0.$$

Hence, using now well known properties of inverse Laplace transform, we deduce that

$$f(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}u(t) + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t u(s)ds + c, \qquad t \geq 0.$$

where $c \in R$ is a constant, is also a solution the above equation

We can also rewrite fractional differential equation as

$$\frac{(2-\nu)M(\nu)}{2(1-\nu)}\int_0^t exp(-\frac{\nu}{1-\nu}(t-s))f'(s)ds = u(t), \qquad t \geq 0.$$

or equivalently,

$$\int_0^t exp(\frac{\nu}{1-\nu}s)f'(s)ds = \frac{2(1-\nu)}{(2-\nu)M(\nu)}exp(\frac{\nu}{1-\nu}t)u(t), \qquad t \geq 0,$$

Differentiating both sides of the latter equation, we obtain that,

$$f'(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Big(u'(t) + \frac{\nu}{1-\nu}u(t)\Big), \qquad t \geq 0.$$

Hence, integrating now from 0 to t, we deduce as in the above equation, that

$$f(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}[u(t) - u(0)] + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t u(s)ds + f(0), \qquad t \geq 0,$$

Definition 2.6. *Let $0 < \nu < 1$. The fractional integral of order ν of a function f is defined by,*

$$^{CF}I^{\nu}f(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}u(t) + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t u(s)ds, \qquad t \geq 0.$$

3. Fractional model

In this section, we expand the alcoholism model [5] to fractional Caputo–Fabrizio derivative of order $\nu \in (0, 1)$. Classic integer order alcoholism model is reformulated in the nonlinear system of differential in Eq. (3.1):

$$\begin{cases} \frac{K(t)}{dt} = \Lambda - \beta K(t)M(t)exp(-\phi T_1(t) + \delta T_2(t)) - \mu K(t) \\ \frac{L(t)}{dt} = \beta K(t)M(t)exp(-\phi T_1(t) + \delta T_2(t)) - (\mu + \rho)K(t) \\ \frac{M(t)}{dt} = \rho L(t) - (\mu + \gamma + d)M(t) \\ \frac{N(t)}{dt} = \gamma M(t) - \mu N(t) \\ \frac{T_1(t)}{dt} = p(\mu_1 K(t) + \mu_2 L(t) + \mu_3 M(t) + \mu_4 N(t)) - \tau T_1(t) \\ \frac{T_2(t)}{dt} = q(\mu_1 K(t) + \mu_2 L(t) + \mu_3 M(t) + \mu_4 N(t)) - \tau T_2(t). \end{cases} \tag{3.1}$$

In the above system (3.1), $K(t), L(t), M(t), N(t)$ represent the moderate drinkers compartment, the light problem drinkers compartment, the heavy problem drinkers compartment and quitting drinkers compartment, respectively. All the parameters are positive constants and μ is the natural death rate of the population, d is the death rate due to heavy alcoholism, Λ is the constant recruitment rate of the population, γ is the permanently quit or removal alcoholism rate of heave problem drinkers, ρ is the transmission coefficient from the light problem drinkers compartment to the heavy problem drinkers compartment, τ is the rate that tweets become outdated as a result of tweets that appeared earlier are less visible and have less effect on the public, β is the basic transmission coefficient. p and q is the ratio that the individual may provide positive and negative information about alcoholism during an alcoholism occasion respectively. The transmission coefficient β is reduced by a factor $exp(-\alpha T_1)$ owing to the behavior change of the public after reading positive tweets about alcoholism, where α determines how effective the positive drinking Twitter information can reduce the transmission coefficient, on the other hand, β is increased by a factor $exp(\delta T_2)$ due to the behavior change of the public after reading negative tweets about alcoholism, where δ determines how effective the negative drinking Twitter information can increase the transmission coefficient. μ_1, μ_2, μ_3, and μ_4 are the rates that the moderate drinkers, light problem drinkers, heavy problem drinkers and quitting drinkers may tweet about alcoholism during an alcoholism occasion, respectively.

The alcoholism model is integrated via Caputo–Fabrizio fractional derivative with alcoholism model and can be written as follows:

$$\begin{cases} {}_0^{CF}\mathfrak{D}_t^{\nu} K(t) = \Lambda - \beta K(t)M(t)exp(-\phi T_1(t) + \delta T_2(t)) - \mu K(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} L(t) = \beta K(t)M(t)exp(-\phi T_1(t) + \delta T_2(t)) - (\mu + \rho)K(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} M(t) = \rho L(t) - (\mu + \gamma + d)M(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} N(t) = \gamma M(t) - \mu N(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} T_1(t) = p(\mu_1 K(t) + \mu_2 L(t) + \mu_3 M(t) + \mu_4 N(t)) - \tau T_1(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} T_2(t) = q(\mu_1 K(t) + \mu_2 L(t) + \mu_3 M(t) + \mu_4 N(t)) - \tau T_2(t). \end{cases} \tag{3.2}$$

where $\nu \in (0, 1)$ is the order of the fractional derivative. Then the following initial values:

$$\begin{cases} K_{(0)}(t) = K(0), \quad L_{(0)}(t) = L(0), \quad M_{(0)}(t) = M(0), \\ N_{(0)}(t) = N(0), \quad T_{1(0)}(t) = T_1(0), \quad T_{2(0)}(t) = T_2(0). \end{cases}$$

4. Existence and uniqueness of alcoholism model

Using fixed point theory, we show the existence of the model in this section. It is used the Caputo–Fabrizio integral operator in [2] on (4.1) to establish

$$\begin{cases} K(t) - K(0) = {}_0^{CF} I_t^{\nu}\{\Lambda - \beta K(t)M(t)exp(-\phi T_1(t) + \delta T_2(t)) - \mu K(t)\}, \\ L(t) - L(0) = {}_0^{CF} I_t^{\nu}\{\beta K(t)M(t)exp(-\phi T_1(t) + \delta T_2(t)) - (\mu + \rho)K(t)\}, \\ M(t) - M(0) = {}_0^{CF} I_t^{\nu}\{\rho L(t) - (\mu + \gamma + d)M(t)\}, \\ N(t) - N(0) = {}_0^{CF} I_t^{\nu}\{\gamma M(t) - \mu N(t)\}, \\ T_1(t) - T_1(0) = {}_0^{CF} I_t^{\nu}\{p(\mu_1 K(t) + \mu_2 L(t) + \mu_3 M(t) + \mu_4 N(t)) - \tau T_1(t)\}, \\ T_2(t) - T_2(0) = {}_0^{CF} I_t^{\nu}\{q(\mu_1 K(t) + \mu_2 L(t) + \mu_3 M(t) + \mu_4 N(t)) - \tau T_2(t)\}. \end{cases} \tag{4.1}$$

Applying the idea used in [2], we obtain

$$\begin{cases} K(t)-K(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\Lambda-\beta K(t)M(t)exp(-\phi T_1(t)+\delta T_2(t))-\mu K(t)\} \\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\Lambda-\beta K(r)M(r)exp(-\phi T_1(r)+\delta T_2(r))-\mu K(r)\}dr, \\ L(t)-L(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\beta K(t)M(t)exp(-\phi T_1(t)+\delta T_2(t))-(\mu+\rho)K(t)\} \\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\beta K(r)M(r)exp(-\phi T_1(r)+\delta T_2(r))-(\mu+\rho)K(r)\}dr, \\ M(t)-M(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\rho L(t)-(\mu+\gamma+d)M(t)\}+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\rho L(r)-(\mu+\gamma+d)M(r)\}dr, \\ N(t)-N(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\gamma M(t)-\mu N(t)\}+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\gamma M(r)-\mu N(r)\}dr, \\ T_1(t)-T_1(0) = \frac{2(1+\nu)}{(2-\nu)M(\nu)}\{p(\mu_1K(t)+\mu_2L(t)+\mu_3M(t)+\mu_4N(t))-\tau T_1(t)\} \\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{p(\mu_1K(r)+\mu_2L(r)+\mu_3M(r)+\mu_4N(r))-\tau T_1(r)\}dr, \\ T_2(t)-T_2(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{q(\mu_1K(t)+\mu_2L(t)+\mu_3M(t)+\mu_4N(t))-\tau T_2(t)\} \\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{q(\mu_1K(r)+\mu_2L(r)+\mu_3M(r)+\mu_4N(r))-\tau T_2(r)\}dr. \end{cases}$$

For simplicity, we replace as follows:

$$\begin{cases} G_1(t,K) = \Lambda-\beta K(t)M(t)exp(-\phi T_1(t)+\delta T_2(t))-\mu K(t), \\ G_2(t,L) = \beta K(t)M(t)exp(-\phi T_1(t)+\delta T_2(t))-(\mu+\rho)K(t), \\ G_3(t,M) = \rho L(t)-(\mu+\gamma+d)M(t), \\ G_4(t,N) = \gamma M(t)-\mu N(t), \\ G_5(t,T_1)T = p(\mu_1K(t)+\mu_2L(t)+\mu_3M(t)+\mu_4N(t))-\tau T_1(t), \\ G_6(t,T_2) = q(\mu_1K(t)+\mu_2L(t)+\mu_3M(t)+\mu_4N(t))-\tau T_2(t). \end{cases}$$

Theorem 4.1. *The kernels G_1, G_2, G_3, G_4, G_5 and G_6 satisfy the Lipschitz condition and contraction if the following inequality holds:*

$$0 \le c_1exp(-\phi c_2+\delta c_3)+\mu < 1$$

Proof. We start with G_1. Suppose that K and K_1 are two functions, then we calculate in below,

$$\|G_1(t,K)-G_1(t,K_1)\| = \|-\beta(K(t)-K_1(t))M(t)exp(-\phi T_1(t)+\delta T_2(t))-\mu(K(t)-K_1(t))\|. \tag{4.2}$$

When using the triangular inequality on Eq. (4.2),

$$\begin{aligned} \|G_1(t,K)-G_1(t,K_1)\| &\le \|-\beta(K(t)-K_1(t))M(t)exp(-\phi T_1(t)+\delta T_2(t))+\mu(K_1(t)-K(t))\| \\ &\le \{M(t)exp(-\phi T_1(t)+\delta T_2(t))+\mu\}\|(K(t)-K_1(t))\| \\ &\le \{\|M(t)\|exp(-\phi\|T_1(t)\|+\delta\|T_2(t)\|)+\mu\}\|(K(t)-K_1(t))\| \\ &\le \{\|M\|exp(-\phi\|T_1\|+\delta\|T_2\|)+\mu\}\|(K(t)-K_1(t))\| \\ &\le \{c_1exp(-\phi c_2+\delta c_3)+\mu\}(K(t)-K_1(t))\| \\ &\le \eta_1\|(K(t)-K_1(t))\|. \end{aligned}$$

Taking $\eta_1 = (c_1exp(-\phi c_2+\delta c_3)+\mu)$, where $\|M(t)\| \le c_1, \|T_1(t)\| \le c_2, \|T_2(t)\| \le c_3$ are a bounded functions, then we get,

$$\|G_1(t,K)-G_1(t,K_1)\| \le \eta_1\|(K(t)-K_1(t))\|. \tag{4.3}$$

Therefore, the Lipshitz condition is satisfyed for G_1, and if also $0 \le (c_1 exp(-\phi c_2 + \delta c_3) + \mu) < 1$, then it is also a contraction. Similiarly, the Lipschitz conditions are given as follows:

$$
\begin{aligned}
&||G_2(t, L) - G_2(t, L_1)|| \le \eta_2||(L(t) - L_1(t))||,\\
&||G_3(t, M) - G_3(t, M_1)|| \le \eta_3||(M(t) - M_1(t))||,\\
&||G_4(t, N) - G_4(t, N_1)|| \le \eta_4||(N(t) - N_1(t))||,\\
&||G_5(t, T_1) - G_5(t, T_{11})|| \le \eta_5||(T_1(t) - T_{11}(t))||,\\
&||G_6(t, T_2) - G_6(t, T_{21})|| \le \eta_6||, (T_2(t) - T_{21}(t))||.
\end{aligned}
$$

Using notations for kernels, Eq. (4.1) becomes

$$
\begin{cases}
K(t) = K(0) + \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_1(t, K) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r, K))dr,\\
L(t) = L(0) + \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_2(t, L) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_2(r, L))dr,\\
M(t) = M(0) + \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_3(t, M) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_3(r, M))dr,\\
N(t) = N(0) + \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_4(t, N) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_4(r, N))dr,\\
T_1(t) = T_1(0) + \frac{2(1+\nu)}{(2-\nu)M(\nu)}G_5(t, T_1) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_5(r, T_1))dr,\\
T_2(t) = T_2(0) + \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_6(t, T_2) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_6(r, T_2))dr.
\end{cases}
$$

The following recursive formula is presented:

$$
\begin{cases}
K_n(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_1(t, K_{n-1}) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r, K_{n-1}))dr,\\
L_n(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_2(t, L_{n-1}) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_2(r, L_{n-1}))dr,\\
M_n(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_3(t, M_{n-1}) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_3(r, M_{n-1}))dr,\\
N_n(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_4(t, N_{n-1}) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_4(r, N_{n-1}))dr,\\
T_{1(n)}(t) = \frac{2(1+\nu)}{(2-\nu)M(\nu)}G_5(t, T_{1(n-1)}) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_5(r, T_{1(n-1)}))dr,\\
T_{2(n)}(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}G_6(t, T_{2(n-1)}) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_6(r, T_{1(n-1)}))dr.
\end{cases}
\tag{4.4}
$$

and

$$
\begin{cases}
K_{(0)}(t) = K(0),\\
L_{(0)}(t) = L(0),\\
M_{(0)}(t) = M(0),\\
N_{(0)}(t) = N(0),\\
T_{1(0)}(t) = T_1(0),\\
T_{2(0)}(t) = T_2(0).
\end{cases}
$$

where $K_{(0)}(t), L_{(0)}(t), M_{(0)}(t), N_{(0)}(t), T_{1(0)}(t)$ and $T_{2(0)}(t)$ are the initial conditions. The difference of the succeeding terms is computed as follows:

$$\begin{cases} \psi_{1n}(t) & = K_n(t) - K_{n-1}(t) \\ & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t, K_{n-1}) - G_1(t, K_{n-2}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r, K_{n-1}) - G_1(r, K_{n-2}) dr, \\ \psi_{2n}(t) & = L_n(t) - L_{n-1}(t) \\ & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_2(t, L_{n-1}) - G_2(t, L_{n-2}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_2(r, L_{n-1}) - G_2(r, L_{n-2}) dr, \\ \psi_{3n}(t) & = M_n(t) - M_{n-1}(t) \\ & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_3(t, M_{n-1}) - G_3(t, M_{n-2}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_3(r, M_{n-1}) - G_3(r, M_{n-2}) dr, \\ \psi_{4n}(t) & = N_n(t) - N_{n-1}(t) \\ & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_4(t, N_{n-1}) - G_4(t, N_{n-2}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_4(r, N_{n-1}) - G_4(r, N_{n-2}) dr, \\ \psi_{5n}(t) & = T_{1(n)}(t) - T_{1(n-1)}(t) \\ & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_5(t, T_{1(n-1)}) - G_5(t, T_{1(n-2)}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_5(r, T_{1(n-1)}) - G_5(r, T_{1(n-2)}) dr, \\ \psi_{6n}(t) & = T_{2(n)}(t) - T_{2(n-1)}(t) \\ & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_6(t, T_{2(n-1)}) - G_6(t, T_{2(n-2)}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_6(r, T_{2(n-1)}) - G_6(r, T_{2(n-2)}) dr. \end{cases}$$

Notive that

$$\begin{cases} K_n(t) = \sum_{i=1}^n \psi_{1i}(t), \\ L_n(t) = \sum_{i=1}^n \psi_{2i}(t), \\ M_n(t) = \sum_{i=1}^n \psi_{3i}(t), \\ N_n(t) = \sum_{i=1}^n \psi_{4i}(t), \\ T_{1(n)}(t) = \sum_{i=1}^n \psi_{5i}(t), \\ T_{2(n)}(t) = \sum_{i=1}^n \psi_{6i}(t). \end{cases}$$

When we want to continue the same process we evaluate the following,

$$\begin{cases} \|\psi_{1n}(t)\| & = \|K_n(t) - K_{n-1}(t)\| \\ & = \|\frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t, K_{n-1}) - G_1(t, K_{n-2}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r, K_{n-1}) - G_1(r, K_{n-2}) dr\|. \end{cases}$$

Using the triangular inequality, Eq. (4.4) is simplified to

$$\begin{cases} \|K_n(t) - K_{n-1}(t)\| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)} \|(G_1(t, K_{n-1}) - G_1(t, K_{n-2})\| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \| \int_0^t (G_1(r, K_{n-1}) - G_1(r, K_{n-2}) dr\|. \end{cases}$$

As the kernel satisfyies the Lipschitz condition, then we have

$$\begin{cases} \|K_n(t) - K_{n-1}(t)\| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1\|K_{n-1} - K_{n-2}\| \\ & \quad +\frac{2\nu}{(2-\nu)M(\nu)}\eta_1\| \int_0^t \|K_{n-1} - K_{n-2}\|dr. \end{cases} \tag{4.5}$$

Then we have

$$\|\psi_{1n}(t)\| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1\|\psi_{1(n-1)}(t)\| + \frac{2\nu}{(2-\nu)M(\nu)}\eta_1 \int_0^t \|\psi_{1(n-1)}(r)\|dr.$$

Similarly, we get the following results:

$$\begin{cases} \|\psi_{2n}(t)\| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_2\|\psi_{2(n-1)}(t)\| + \frac{2\nu}{(2-\nu)M(\nu)}\eta_2 \int_0^t \|\psi_{2(n-1)}(r)\|dr, \\ \|\psi_{3n}(t)\| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_3\|\psi_{3(n-1)}(t)\| + \frac{2\nu}{(2-\nu)M(\nu)}\eta_3 \int_0^t \|\psi_{3(n-1)}(r)\|dr, \\ \|\psi_{4n}(t)\| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_4\|\psi_{4(n-1)}(t)\| + \frac{2\nu}{(2-\nu)M(\nu)}\eta_4 \int_0^t \|\psi_{4(n-1)}(r)\|dr, \\ \|\psi_{5n}(t)\| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_5\|\psi_{5(n-1)}(t)\| + \frac{2\nu}{(2-\nu)M(\nu)}\eta_5 \int_0^t \|\psi_{5(n-1)}(r)\|dr, \\ \|\psi_{6n}(t)\| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_6\|\psi_{6(n-1)}(t)\| + \frac{2\nu}{(2-\nu)M(\nu)}\eta_6 \int_0^t \|\psi_{6(n-1)}(r)\|dr. \end{cases}$$

We shall then state the following theorem. □

Theorem 4.2. *The alcoholism model (4.1) has unique solution if the conditions below hold.*

$$\frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1 - \frac{2\nu}{(2-\nu)M(\nu)}\eta_1 t < 1.$$

Proof. Since all the functions $K(t), L(t), M(t), N(t), T_1(t)$ and $T_2(t)$ are bounded, we have shown that the kernels satisfy the Lipschitz condition, thus by using the recursive method, we obtain the succeeding relation as follows:

$$\begin{cases} \|\psi_{1n}(t)\| \leq \|K_n(0)\|[(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1) + (\frac{2\nu}{(2-\nu)M(\nu)}\eta_1 t)]^n, \\ \|\psi_{2n}(t)\| \leq \|L_n(0)\|[(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_2) + (\frac{2\nu}{(2-\nu)M(\nu)}\eta_2 t)]^n, \\ \|\psi_{3n}(t)\| \leq \|M_n(0)\|[(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_3) + (\frac{2\nu}{(2-\nu)M(\nu)}\eta_3 t)]^n, \\ \|\psi_{4n}(t)\| \leq \|N_n(0)\|[(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_4) + (\frac{2\nu}{(2-\nu)M(\nu)}\eta_4 t)]^n, \\ \|\psi_{5n}(t)\| \leq \|T_{1(n)}(0)\|[(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_5) + (\frac{2\nu}{(2-\nu)M(\nu)}\eta_5 t)]^n, \\ \|\psi_{6n}(t)\| \leq \|T_{2(n)}(0)\|[(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_6) + (\frac{2\nu}{(2-\nu)M(\nu)}\eta_6 t)]^n. \end{cases} \tag{4.6}$$

Hence, the existence and continuity of the said solutions is proved. Furthermore, to ensure that the above function is a solution of Eq. (4.1), we proceed as follows:

$$\begin{cases} K(t) - K(0) = K_n(t) - A_n(t), \\ L(t) - L(0) = L_n(t) - B_n(t), \\ M(t) - M(0) = M_n(t) - C_n(t), \\ N(t) - N(0) = N_n(t) - D_n(t), \\ T_1(t) - T_1(0) = T_{1n}(t) - E_n(t), \\ T_2(t) - T_2(0) = T_{2n}(t) - F_n(t). \end{cases} \tag{4.7}$$

Therefore, we have

$$\begin{cases} \|A_n(t)\| & = \|\frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t,K_n) - G_1(t,K_{n-1}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t (G_1(r,K_n) - G_1(r,K_{n-1}))dr\| \\ & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\|(G_1(t,K_n) - G_1(t,K_{n-1}))\| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t \|(G_1(r,K_n) - G_1(r,K_{n-1}))\|dr \\ & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1\|K - K_{n-1}\| + \frac{2\nu}{(2-\nu)M(\nu)}\eta_1\|K - K_{n-1}\|t. \end{cases}$$

Using the process in a recursive manner gives

$$\|A_n(t)\| \leq \left(\frac{2(1-\nu)}{(2-\nu)M(\nu)} + \frac{2\nu}{(2-\nu)M(\nu)}t\right)^{n-1}\eta_1^{n+1}a. \tag{4.8}$$

By applying the limit on Eq. (4.8) as n tends to infinity, we get

$$\|A_n(t)\| \to 0.$$

Similarly,

$$\|B_n(t)\| \to 0, \quad \|C_n(t)\| \to 0, \quad \|D_n(t)\| \to 0,$$

$$\|E_n(t)\| \to 0, \quad \|F_n(t)\| \to 0.$$

For the uniqueness system (4.1) solution, we take on contrary that there exists another solution of (4.1) given by $K_1(t), L_1(t), M_1(t), N_1(t), T_{11}(t)$ and $T_{12}(t)$. Then

$$\begin{cases} K(t) - K_1(t) & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t,K_n) - G_1(t,K_{n-1}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t (G_1(r,K_n) - G_1(r,K_{n-1}))dr. \end{cases} \tag{4.9}$$

Taking norm on Eq. (4.9), we get

$$\begin{cases} \|K(t) - K_1(t)\| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\|(G_1(t,K_n) - G_1(t,K_{n-1})\| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t \|(G_1(r,K_n) - G_1(r,K_{n-1}))\|dr. \end{cases}$$

By applying the Lipschitz condition of kernel, we have

$$\begin{cases} \|K(t) - K_1(t)\| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1\|K(t) - K_1(t)\| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)}\int_0^t \eta_1 t\|K(t) - K_1(t)\|dr. \end{cases}$$

It gives

$$\|K(t) - K_1(t)\|\left(1 - \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1 - \frac{2\nu}{(2-\nu)M(\nu)}\eta_1 t\right) \leq 0. \tag{4.10}$$

□

Theorem 4.3. *The model (4.1) solution will be unique if*

$$\left(1 - \frac{2(1-\nu)}{(2-\nu)M(\nu)}\eta_1 - \frac{2\nu}{(2-\nu)M(\nu)}\eta_1 t\right) > 0. \tag{4.11}$$

Proof. If condition (4.11) holds, then (4.10) implies that

$$\|K(t) - K_1(t)\| = 0.$$

Hence, we get

$$K(t) = K_1(t).$$

On employing the same procedure, we get

$$\begin{cases} L(t) = L_1(t), \\ M(t) = M_1(t), \\ N(t) = N_1(t), \\ T_1(t) = T_{11}(t), \\ T_2(t) = T_{21}(t). \end{cases}$$

□

5. Numerical solution

The differential transform method was first introduced by Zhou [28], who solved linear and nonlinear initial value problems in electric circuit analysis. This method constructs an analytical solution in the form of a polynomial. It is different from the traditional higher order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally time-consuming for large orders. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of ordinary or partial differential equations. We apply the fractional differential transform method for solving the alcoholism model. By using the basic definitions of the fractional one-dimensional differential transform and taking the corresponding transform of Eq. (3.2), we obtain the following system:

Table 1. The operations for the one-dimensional differential transform method.

Original function	Transformed function
$f(x) = g(x) \pm h(x)$	$F_\nu(k) = G_\nu(k) \pm H_\nu(k)$
$f(x) = ag(x)$	$F_\nu(k) = aG_\nu(k)$
$f(x) = g(x)h(x)$	$F_\nu(k) = \sum_{l=0}^{k} G_\nu(l)H_\nu(k-l)$
$f(x) = g_1(x)g_2(x), \ldots g_n(x)$	$F(k) = \sum_{k_{n-1}=0}^{k} \cdots \sum_{k_1=0}^{k_2} G_1(k_1) \ldots G_n(k - k_{n-1})$
$f(x) = D_{x_0}^{\nu} g(x)$	$F_\nu(k) = \frac{\Gamma(\nu(k+1)+1)}{\Gamma(\nu k+1)} G_\nu(k+1)$
$f(x) = (x - x_0)^\gamma$	$F_\nu(k) = \delta(k - \gamma/\nu)$
$f(x) = D_{x_0}^{\beta} g(x)$	$F_\nu(k) = \frac{\Gamma(\nu k+\beta+1)}{\Gamma(\nu k+1)} G_\nu(k + \beta/\nu)$

We apply the fractional differential transform method for solving the alcoholism model. By using the basic definitions of the fractional one-dimensional differential transform and taking the corresponding transform of the equation (3.2), we obtain the following system:

$$\begin{cases} K(h+1) = \frac{\Gamma(\nu h+1)}{\Gamma(\nu(h+1)+1)}\Big[\Lambda - \beta\sum_{l_3=0}^{h}\sum_{l_2=0}^{l_3}\sum_{l_1=0}^{l_2} K(l_1)M(l_2-l_1)F_1(l_3-l_2)F_2(h-l_3) - \mu K(h)\Big], \\ L(h+1) = \frac{\Gamma(\nu h+1)}{\Gamma(\nu(h+1)+1)}\Big[\beta\sum_{l_3=0}^{h}\sum_{l_2=0}^{l_3}\sum_{l_1=0}^{l_2} K(l_1)M(l_2-l_1)F_1(l_3-l_2)F_2(h-l_3) - (\mu+\rho)K(h)\Big], \\ M(h+1) = \frac{\Gamma(\nu h+1)}{\Gamma(\nu(h+1)+1)}[\rho L(h) - (\mu+\gamma+d)M(h)], \\ N(h+1) = \frac{\Gamma(\nu h+1)}{\Gamma(\nu(h+1)+1)}[\gamma M(h) - \mu N(h)], \\ T_1(h+1) = \frac{\Gamma(\nu h+1)}{\Gamma(\nu(h+1)+1)}[p\mu_1 K(h) + p\mu_2 L(h) + p\mu_3 M(h) - p\mu_4 N(h) - \tau T_1(h)], \\ T_2(h+1) = \frac{\Gamma(\nu h+1)}{\Gamma(\nu(h+1)+1)}[q\mu_1 K(h) + q\mu_2 L(h) + q\mu_3 M(h) - q\mu_4 N(h) - \tau T_2(h)]. \end{cases}$$

where $F_1(k), F_2(k)$ are the T-functions of $e^{-\phi T_1(t)}$ and $e^{\delta T_2(t)}$, respectively.

$$F_1(k) = \begin{cases} e^{-\phi T_1(0)} & h = 0, \\ -\phi\sum_{m=0}^{h-1}\left(\frac{m+1}{h}T_1(m+1)F_1(h-m-1)\right) & h \geq 1, \end{cases}$$

and

$$F_2(k) = \begin{cases} e^{\delta T_2(0)} & h = 0 \\ \delta\sum_{m=0}^{h-1}\left(\frac{m+1}{h}T_2(m+1)F_2(h-m-1)\right) & h \geq 1. \end{cases}$$

By using the following initial conditions and parameters we have

$K(0) = 5, L(0) = 2, M(0) = 3, N(0) = 1, T_1(0) = 6, T_2(0) = 4,$
$\Delta = 0.8, \beta = 0.099, \mu = 0.009, \gamma = 0.8, d = 0.5, \rho = 0.3, \tau = 0.3\mu_1 = 0, \mu_2 = 0.08, \mu_3 = 0.6,$
$\mu_4 = 0.8, \delta = 0.003, p = 0.66, \phi = 0.09.$

For $h = 0, 1, 2, \ldots$ respectively the values of $K(h), M(h), N(h), L(h), T_1(h)$ and $T_2(h)$ are as follows

$K(h)$	$L(h)$	$M(h)$
-0,120828286052481	-2,46917171394752	-3,327
0,49800423437587	-0,457043445404079	1,80714574290787
-0,217706485329673	0,105157528360727	-0,834222270362543
0,100957921788796	-0,0640566725254163	0,280886052603197
-0,0419390616097007	0,0282491674151399	-0,0773793689230419
0,0175455074539311	-0,0128063934920349	0,0182940573574673
-0,00719864713251534	0,00549923941054888	-0,00396983416121931
0,00290951664635863	-0,00229943130187796	0,000855785592525092
-0,00115154658572007	0,000932362998361051	-0,00020111697013097

and

$N(h)$	$T_1(h)$	$T_2(h)$
2,391	0,0216000000000003	-0,2892
-1,3415595	-0,0959481332482144	-0,0029740666241072
0,485930209942099	0,00397961474954874	-0,00251019262522563
-0,167937797044878	-0,0173556087853892	-0,0083403043926946
0,0452440564511922	0,00487684206348898	0,00241817103174449
-0,0103851152744157	-0,00112081081113706	-0,000559392905568532
0,00210410170334908	0,000203024502232462	0,000101468858259088
-0,000399350530538198	-2,8954517283411E-05	-1,44756314095627E-05
7,64691809772131E-05	1,70115521795943E-06	8,50523367908285E-07

By using the differential inverse reduced transform of $K(h), M(h), N(h), L(h), T_1(h)$, and $T_2(h)$, and for special case $\nu = 1$, the solution will be as follows:

$$\begin{aligned}K(t) = \sum_{h=0}^{\infty} K(h)t^{h\nu} =&5 - 0,120828286052481t + 0,49800423437587t^2 - 0,217706485329673t^3\\&+ 0,100957921788796t^4 - 0,0419390616097007t^5 + 0,0175455074539311t^6\\&- 0,00719864713251534t^7 + 0,00290951664635863t^8 - 0,00115154658572007t^9\\&+ \dots,\end{aligned}$$

$$\begin{aligned}L(t) = \sum_{h=0}^{\infty} L(h)t^{h\nu} =&2 - 2,46917171394752t - 0,457043445404079t^2 + 0,105157528360727t^3\\&- 0,0640566725254163t^4 + 0,0282491674151399t^5 - 0,0128063934920349t^6\\&+ 0,00549923941054888t^7 - 0,00229943130187796t^8 + 0,000932362998361051t^9\\&+ \dots,\end{aligned}$$

$$\begin{aligned}M(t) = \sum_{h=0}^{\infty} M(h)t^{h\nu} =&3 - 3,327t + 1,80714574290787t^2 - 0,834222270362543t^3\\&+ 0,280886052603197t^4 - 0,0773793689230419t^5 + 0,0182940573574673t^6\\&- 0,00396983416121931t^7 + 0,000855785592525092t^8 - 0,000201116970130097t^9\\&+ \dots,\end{aligned}$$

$$\begin{aligned}N(t) = \sum_{h=0}^{\infty} N(h)t^{h\nu} =&1 + 2,391t - 1,3415595t^2 + 0,485930209942099t^3\\&- 0,167937797044878t^4 + 0,0452440564511922t^5 - 0,0103851152744157t^6\\&+ 0,00210410170334908t^7 - 0,000399350530538198t^8\\&+ 7,64691809772131 \times 10^{-5}t^9 + \dots,\end{aligned}$$

$$T_1(t) = \sum_{h=0}^{\infty} T_1(h)t^{h\nu} = 6 + 0,0216000000000003t - 0,0959481332482144t^2 + 0,00397961474954874t^3 - 0,0173556087853892t^4 + 0,00487684206348898t^5 - 0,00112081081113706t^6 + 0,000203024502232462t^7 - 2,8954517283411 \times 10^{-5}t^8 + 1,70115521795943 \times 10^{-6}t^9 + \ldots,$$

$$T_2(t) = \sum_{h=0}^{\infty} T_2(h)t^{h\nu} = 4 - 0,2892t - 0,0029740666241072t^2 - 0,00251019262522563t^3 - 0,0083403043926946t^4 + 0,00241817103174449t^5 - 0,000559392905568532t^6 + 0,000101468858259088t^7 - 1,44756314095627 \times 10^{-5}t^8 + 8,50523367908285 \times 10^{-7}t^9 + \ldots.$$

6. Conclusion

In the present study, we have used the new Caputo fractional derivative to bring the alcoholism model into fractional order. The existence and uniqueness of the solution for the CF-derived alcoholism model have been proven in detail. The effect of fractional order was demonstrated through some simulations. The results show that the Caputo–Fabrizio fractional derivative and integral operators are very useful.

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Conflict of interest

The author declares no conflict of interest.

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