Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 3(1): 56–65
DOI:10.3934/Math.2018.1.56
Received: 06 November 2017
Accepted: 23 January 2018
Published: 01 February 2018

Research article

Large time behavior of a bipolar hydrodynamic model with large data and vacuum

Yunlei Zhan*

Department of Mathematics, LiaoCheng infant normal school, LiaoCheng, 252600, China

* **Correspondence:** Email: 1311292966@qq.com; Tel: 17852550178.

Abstract: In this paper, it is considered a hydrodynamic model for the bipolar semiconductor device in the case of a pressure with the exponent $\gamma = 2$. The model has a non-flat doping profile and insulating boundary conditions. Firstly, the existence and uniqueness of the corresponding steady solutions which satisfy some bounded estimates are proved. Then,using a technical energy method and an entropy dissipation estimate,we present a framework for the large time behavior of bounded weak entropy solutions with vacuum. It is shown that the weak solutions converge to the stationary solutions in L^2 norm with exponential decay rate. No smallness and regularity conditions are assumed.

Keywords: Euler-Poisson system; bipolar semiconductor model; entropy solution; stationary solution; large time behavior
Mathematics Subject Classification: 35L20

1. Introduction

Consider the following Euler-Poisson system for the bipolar hydrodynamical model of semiconductor devices:

$$\begin{cases} n_{1t} + j_{1x} = 0, \\ j_{1t} + (\dfrac{j_1^2}{n_1} + p(n_1))_x = n_1 E - j_1, \\ n_{2t} + j_{2x} = 0, \\ j_{2t} + (\dfrac{j_2^2}{n_2} + q(n_2))_x = -n_2 E - j_2, \\ E_x = n_1 - n_2 - D(x), \end{cases} \tag{1}$$

in the region $\Omega = (\mathbf{0}, \mathbf{1}) \times R_+$. In this paper, $n_1(x,t)$, $n_2(x,t)$, $j_1(x,t)$, $j_2(x,t)$ and $E(x,t)$ represent the electron density, the hole density,the electron current density, the hole current density and the electric field, respectively. In this note, we assume that the p and q satisfy the γ-law:$p(n_1) = n_1^2$ and $q(n_2) = n_2^2$

($\gamma = 2$),which denote the pressures of the electrons and the holes. The function $D(x)$, called the doping profile, stands for the density of impurities in semiconductor devices.

For system (1), the initial conditions are

$$n_i(x, 0) = n_{i0}(x) \geq 0, \quad j_i(x, 0) = j_{i0}(x), \quad i = 1, 2, \tag{2}$$

and the boundary conditions at $x = 0$ and $x = 1$ are

$$j_i(0, t) = j_i(1, t) = 0, \quad i = 1, 2, \quad E(0, t) = 0. \tag{3}$$

So,we can get the compatibility condition

$$j_{i0}(0) = j_{i0}(1) = 0, \quad i = 1, 2. \tag{4}$$

Moreover, in this paper, we assume the doping profile $D(x)$ satisfies

$$D(x) \in C[0, 1] \text{ and } D^* = \sup_x D(x) \geq \inf_x D(x) = D_*. \tag{5}$$

Now, the definition of entropy solution to problem (1) − (4) is given. We consider the locally bounded measurable functions $n_1(x, t)$, $j_1(x, t)$, $n_2(x, t)$, $j_2(x, t)$, $E(x, t)$, where $E(x, t)$ is continuous in x, a.e. in t.

Definition 1.1. *The vector function* (n_1, n_2, j_1, j_2, E) *is a weak solution of problem* (1)−(4)*, if it satisfies the equation* (1) *in the distributional sense, verifies the restriction* (2) *and* (3)*. Furthermore, a weak solution of system* (1) − (4) *is called an entropy solution if it satisfies the entropy inequality*

$$\eta_{et} + q_{ex} + \frac{j_1^2}{n_1} + \frac{j_2^2}{n_2} - j_1 E + j_2 E \leq 0, \tag{6}$$

in the sense of distribution. And the (η_e, q_e) *are mechanical entropy-entropy flux pair which satisfy*

$$\begin{cases} \eta_e(n_1, n_2, j_1, j_2) = \dfrac{j_1^2}{2n_1} + n_1^2 + \dfrac{j_2^2}{2n_2} + n_2^2, \\ \\ q_e(n_1, n_2, j_1, j_2) = \dfrac{j_1^3}{2n_1^2} + 2n_1 j_1 + \dfrac{j_2^3}{2n_2^2} + 2n_2 j_2. \end{cases} \tag{7}$$

For bipolar hydrodynamic model, the studies on the existence of solutions and the large time behavior as well as relaxation-time limit have been extensively carried out, for example, see[1][2][3][4][5][6] etc. Now, we make it into a semilinear ODE about the potential and the pressures with the exponent $\gamma = 2$. We can get the existence, uniqueness and some bounded estimates of the steady solution. Then, using a technical energy method and a entropy dissipation estimate, we present a framework for the large time behavior of bounded weak entropy solutions with vacuum. It is shown that the weak solutions converge to the stationary solutions in L^2 norm with exponential decay rate.

The organization of this paper is as follows. In Section 2, the existence, uniqueness and some bounded estimates of stationary solutions are given. we present a framework for the large time behavior of bounded weak entropy solutions with vacuum in Section 3.

2. Steady solutions

In this part, we will prove the existence and uniqueness of steady solution to problem (1) − (4). Moreover, we can obtain some important estimates on the steady solution $(N_1, N_2, \mathcal{E})$.

The steady equation of (1) − (4) is as following

$$\begin{cases} J_1 = J_2 = 0, \\ 2N_1N_{1x} = N_1\mathcal{E}, \\ 2N_2N_{2x} = -N_2\mathcal{E}, \\ \mathcal{E}_x = N_1 - N_2 - D(x), \end{cases} \tag{8}$$

and the boundary condition

$$\mathcal{E}(0) = 0. \tag{9}$$

We only concern the classical solutions in the region where the density

$$\inf_x N_1 > 0 \quad \text{and} \quad \inf_x N_2 > 0. \tag{10}$$

hold.

Now, we introduce a new variation $\Phi(x)$,and make $\Phi'(x):=\mathcal{E}(x)$. To eliminate the additive constants,we set $\int_0^1 \Phi(x)dx = 0$. Then (2.1) turns into

$$\begin{cases} 2N_{1x} = \Phi_x, \\ 2N_{2x} = -\Phi_x, \\ \Phi_{xx} = N_1 - N_2 - D(x). \end{cases} \tag{11}$$

Obviously, $(11)_1$ and $(11)_2$ indicate

$$\begin{cases} N_1(x) = \frac{1}{2}\Phi(x) + C_1, \\ N_2(x) = -\frac{1}{2}\Phi(x) + C_2, \\ \Phi_{xx}(x) = \frac{1}{2}\Phi(x) + C_1 + \frac{1}{2}\Phi(x) - C_2 - D(x). \end{cases} \tag{12}$$

where C_1 and C_2 are two unknown positive constants. To calculate these two constants, we suppose *

$$\int_0^1 \Big(n_i(x,0) - N_i(x)\Big)dx = 0 \ \text{ for } \ i = 1, 2, \tag{13}$$

then

$$\begin{aligned} \bar{n}_1 &:= \int_0^1 n_1(x,0)dx = \int_0^1 N_1(x)dx = \int_0^1 (\frac{\Phi(x)}{2} + C_1)dx = C_1, \\ \bar{n}_2 &:= \int_0^1 n_2(x,0)dx = \int_0^1 N_2(x)dx = \int_0^1 (- \frac{\Phi(x)}{2} + C_2)dx = C_2. \end{aligned} \tag{14}$$

*Using the conservation of the total charge: integrating $(1)_1$ and $(1)_3$ from 0 to 1

$$(\int_0^1 n_i dx)_t = -\int_0^1 j_{ix}dx = 0, \ \text{ for } \ i = 1, 2,$$

we see this assumption is right.

Substituting (14) into $(12)_3$, we have

$$\Phi_{xx} = \Phi(x) + \bar{n}_1 - \bar{n}_2 - D(x). \tag{15}$$

Clearly, we can prove the existence and uniqueness of solutions to (15) with the Neumann boundary condition

$$\Phi_x(0) = \Phi_x(1) = 0. \tag{16}$$

Integrate(15) from $x = 0$ to $x = 1$, we get

$$\bar{n}_1 - \bar{n}_2 = \int_0^1 D(x)dx. \tag{17}$$

Suppose $\Phi(x)$ attains its maximum in $x_0 \in [0, 1]$, then we get $\Phi_{xx}(x_0) \le 0^\dagger$ and

$$\Phi(x_0) + \bar{n}_1 - \bar{n}_2 - D(x_0) \le 0.$$

So we get

$$\Phi(x_0) \le D^* + \bar{n}_2 - \bar{n}_1. \tag{18}$$

Similarly, if Φ attains its minimum in $x_1 \in [0, 1]$, we obtain

$$\Phi(x_1) \ge D_* + \bar{n}_2 - \bar{n}_1. \tag{19}$$

Moreover, from (12), (14), (15), (18), and (19), we have

$$\begin{aligned} \frac{D_* + \bar{n}_2 + \bar{n}_1}{2} &\le N_1(x) \le \frac{D^* + \bar{n}_2 + \bar{n}_1}{2}, \\ \frac{-D^* + \bar{n}_2 + \bar{n}_1}{2} &\le N_2(x) \le \frac{-D_* + \bar{n}_2 + \bar{n}_1}{2}, \end{aligned} \tag{20}$$

$$D_* \le (N_1 - N_2)(x) \le D^* \ \text{ for any } \ x \in [0, 1]. \tag{21}$$

Above that, the theorem of existence and uniqueness of steady equation is given.

Theorem 2.1. *Assume that* (5) *holds, then problem* (8)*,* (9) *has an unique solution* $(N_1, N_2, \mathcal{E})$*, such that for any* $x \in [0, 1]$

$$n_* \le N_1(x) \le n^*, \ \ n_* \le N_2(x) \le n^*, \tag{22}$$

and

$$D_* \le (N_1 - N_2)(x) \le D^*, \tag{23}$$

satisfy, where

$$\begin{aligned} n^* &:= \max\left\{\frac{D^* + \bar{n}_2 + \bar{n}_1}{2}, \frac{-D_* + \bar{n}_2 + \bar{n}_1}{2}\right\}, \\ n_* &:= \min\left\{\frac{D_* + \bar{n}_2 + \bar{n}_1}{2}, \frac{-D^* + \bar{n}_2 + \bar{n}_1}{2}\right\}, \end{aligned} \tag{24}$$

$\bar{n}_1$, $\bar{n}_2$ *are defined in* (14).

---

†If $x_0 \in (0, 1)$, then $\Phi_x(x_0) = 0$, $\Phi_{xx}(x_0) \le 0$ clearly. If $x_0 = 0$ or $x_0 = 1$, the Taylor expansion

$$\Phi(x) = \Phi(x_0) + \Phi'(x_0)(x - x_0) + \frac{\Phi''(x_0)}{2}(x - x_0)^2 + o(x - x_0)^2,$$

the boundary condition (16) indicates $\Phi''(x_0) \le 0$.

3. Large time behavior

Now, our aim is to prove the weak-entropy solution of (1) − (4) convergences to corresponding stationary solution in L^2 norm with exponential decay rate. For this purpose, we introduce the relative entropy-entropy flux pair:

$$\begin{aligned}\eta^*(x,t) &= \sum_{i=1}^{2}\Big(\frac{j_i^2}{2n_i}+n_i^2-N_i^2-2N_i(n_i-N_i)\Big)(x,t)\\ &= \Big(\eta_e-\sum_{i=1}^{2}Q_i\Big)(x,t)\geq 0,\end{aligned}\tag{25}$$

$$\begin{aligned}q^*(x,t) &= \sum_{i=1}^{2}\Big(\frac{j_i^3}{2n_i^2}+2n_ij_i-2N_ij_i\Big)(x,t)\\ &= \Big(q_e-\sum_{i=1}^{2}P_i\Big)(x,t),\end{aligned}\tag{26}$$

where

$$Q_i=N_i^2+2N_i(n_i-N_i),\quad P_i=2N_ij_i,$$

η_e and q_e are the entropy-entropy flux pair defined in (1.7).

The following theorem is our main result in section 3.

Theorem 3.1(Large time behavior) *Suppose* $(n_1,n_2,j_1,j_2,E)(x,t)$ *be any weak entropy solution of problem* (1.1) − (1.4) *satisfying*

$$2(2D^*-\bar{n}_1-\bar{n}_2)<(n_1-n_2)(x,t)<2(2D_*+\bar{n}_1+\bar{n}_2),\tag{27}$$

for a.e. $x\in[0,1]$ *and* $t>0$. $(N_1,N_2,\mathcal{E})(x)$ *is its stationary solution obtained in Theorem 2.1. If*

$$\int_0^1\eta^*(x,0)dx<\infty,\quad \int_0^1\Big(n_i(s,0)-N_i(s)\Big)ds=0,\tag{28}$$

then for any $t>0$*, we have*

$$\begin{aligned}&\int_0^1[j_1^2+j_2^2+(E-\mathcal{E})^2+(n_1-N_1)^2+(n_2-N_2)^2](x,t)dx\\ &\leq C_0e^{-\tilde{C}_0t}\int_0^1\eta^*(x,0)dx.\end{aligned}\tag{29}$$

holds for some positive constant C_0 *and* $\tilde{C}_0$.

Proof. We set

$$y_i(x,t)=-\int_0^x\Big(n_i(s,t)-N_i(s)\Big)ds,\quad i=1,2,\ x\in[0,1],\ t>0.\tag{30}$$

Clearly, $y_i(i=1,2)$ is absolutely continuous in x for a.e. $t>0$. And

$$\begin{aligned} y_{ix} &= -(n_i - N_i), \qquad y_{it} = j_i, \\ y_2 - y_1 &= E - \mathcal{E}, \qquad y_i(0,t) = y_i(1,t) = 0, \end{aligned} \tag{31}$$

following (1.1), (2.1), and (28). From $(1.1)_2$ and $(2.1)_2$, we get y_1 satisfies the equation

$$y_{1tt} + (\frac{y_{1t}^2}{n_1})_x - y_{1xx} + y_{1t} = n_1 E - N_1 \mathcal{E}. \tag{32}$$

Multiplying y_1 with (32) and integrating over $(0,1)$ [‡], we have

$$\begin{aligned} &\frac{d}{dt}\int_0^1 (y_1 y_{1t} + \frac{1}{2}y_1^2)\,dx - \int_0^1 (\frac{y_{1t}^2}{n_1})y_{1x}\,dx - \int_0^1 (n_1^2 - N_1^2)y_{1x}dx - \int_0^1 y_{1t}^2\,dx \\ &= \int_0^1 (N_1(y_2 - y_1)y_1 + \frac{E_x}{2}y_1^2)dx. \end{aligned} \tag{33}$$

In above calculation, we have used the integration by part. Similarly, from $(1.1)_4$ and $(2.1)_3$, we get

$$\begin{aligned} &\frac{d}{dt}\int_0^1 (y_2 y_{2t} + \frac{1}{2}y_2^2)\,dx - \int_0^1 (\frac{y_{2t}^2}{n_2})y_{2x}\,dx - \int_0^1 (n_2^2 - N_2^2)y_{2x}\,dx - \int_0^1 y_{2t}^2\,dx \\ &= -\int_0^1 (N_2(y_2 - y_1)y_2 + \frac{E_x}{2}y_2^2)\,dx. \end{aligned} \tag{34}$$

Add (33) and (34) , we have

$$\begin{aligned} &\frac{d}{dt}\int_0^1 (y_1 y_{1t} + \frac{1}{2}y_1^2 + y_2 y_{2t} + \frac{1}{2}y_2^2)\,dx - \int_0^1 (n_1^2 - N_1^2)y_{1x}dx - \int_0^1 (n_2^2 - N_2^2)y_{2x}\,dx \\ &= \int_0^1 \Big((\frac{y_{1t}^2}{n_1})y_{1x} + (\frac{y_{2t}^2}{n_2})y_{2x}\Big)\,dx + \int_0^1 (y_{1t}^2 + y_{2t}^2)\,dx \\ &+ \int_0^1 \Big(N_1(y_2 - y_1)y_1 + \frac{E_x}{2}y_1^2 - N_2(y_2 - y_1)y_2 - \frac{E_x}{2}y_2^2\Big)\,dx. \end{aligned} \tag{35}$$

Since

$$\begin{aligned} &\int_0^1 \Big(N_1(y_2 - y_1)y_1 + \frac{E_x}{2}y_1^2 - N_2(y_2 - y_1)y_2 - \frac{E_x}{2}y_2^2\Big)\,dx \\ &= \int_0^1 \frac{n_1 - N_1 - n_2 + N_2 - D(x)}{2}y_1^2 dx + \int_0^1 \frac{n_2 - N_2 - n_1 + N_1 + D(x)}{2}y_2^2 dx \\ &- \int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx, \end{aligned} \tag{36}$$

[‡]For weak solutions, (1) satisfies in the sense of distribution. We choose test function $\varphi_n(x,t) \in C_0^\infty\Big((0,1)\times[0,T)\Big)$ and let $\varphi_n(x,t) \to y_i(x,t)$ as $n \to +\infty$ for $i = 1,2$.

then, from $(31)_1$ and (36) we get

$$\begin{aligned}
&\frac{d}{dt}\int_0^1 (y_1 y_{1t} + \frac{1}{2}y_1^2 + y_2 y_{2t} + \frac{1}{2}y_2^2)\,dx + \int_0^1 (N_1 + n_1)y_{1x}^2 \\
&\quad + \int_0^1 (N_2 + n_2)y_{2x}^2 dx + \int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx \\
&= \int_0^1 \Big((\frac{y_{1t}^2}{n_1})y_{1x} + (\frac{y_{2t}^2}{n_2})y_{2x}\Big)\,dx + \int_0^1 (y_{1t}^2 + y_{2t}^2)\,dx \\
&\quad + \int_0^1 \Big(\frac{n_1 - N_1 - n_2 + N_2 - D(x)}{2}y_1^2 + \frac{n_2 - N_2 - n_1 + N_1 + D(x)}{2}y_2^2\Big)dx.
\end{aligned} \tag{37}$$

Moreover, since

$$|y_i(x)| = |\int_0^x y_{is}(s)ds| \le x^{\frac{1}{2}}(\int_0^x y_{is}^2 ds)^{\frac{1}{2}} \le x^{\frac{1}{2}}(\int_0^1 y_{is}^2 ds)^{\frac{1}{2}}, \quad x \in [0,1], \tag{38}$$

we can obtain

$$\|y_i\|_{L^2}^2 = \int_0^1 |y_i|^2 dx \le \frac{1}{2}\|y_{ix}\|_{L^2}^2, \tag{39}$$

verifies for $i = 1, 2$. If the weak solutions $n_1(x,t)$ and $n_2(x,t)$ satisfy (27) then

$$\inf_x\{N_1 + n_1\} > \sup_x\Big\{\frac{n_1 - N_1 - n_2 + N_2 - D(x)}{4}\Big\}, \tag{40}$$

and

$$\inf_x\{N_2 + n_2\} > \sup_x\Big\{\frac{n_2 - N_2 - n_1 + N_1 + D(x)}{4}\Big\}, \tag{41}$$

hold, where we have used the assumption (5) and the estimate (23).
Following (39), (40) and (41), we have

$$\int_0^1 \frac{n_1 - N_1 - n_2 + N_2 - D(x)}{2}y_1^2 dx < \int_0^1 (N_1 + n_1)y_{1x}^2 dx, \tag{42}$$

and

$$\int_0^1 \frac{n_2 - N_2 - n_1 + N_1 + D(x)}{2}y_2^2 dx < \int_0^1 (N_2 + n_2)y_{2x}^2 dx. \tag{43}$$

Thus (36), (42), and (43) indicate there is a positive constant $\beta > 0$, such that

$$\begin{aligned}
&\frac{d}{dt}\int_0^1 (y_1 y_{1t} + \frac{1}{2}y_1^2 + y_2 y_{2t} + \frac{1}{2}y_2^2)\,dx + \beta\int_0^1 (y_{1x}^2 + y_{2x}^2)dx + \int_0^1 \frac{N_1 + N_2}{2}(y_1 - y_2)^2 dx \\
&\le \int_0^1 \Big((\frac{y_{1t}^2}{n_1})y_{1x} + (\frac{y_{2t}^2}{n_2})y_{2x}\Big)\,dx + \int_0^1 (y_{1t}^2 + y_{2t}^2)\,dx \\
&= \int_0^1 \Big(N_1\frac{y_{1t}^2}{n_1} + N_2\frac{y_{2t}^2}{n_2}\Big)\,dx.
\end{aligned} \tag{44}$$

In view of the entropy inequality (6), and the definition of η^* and q^* in (25) and (26), the following inequality holds in the sense of distribution.

$$\begin{aligned}
&\eta_{et} + q_{ex} + \frac{j_1^2}{n_1} + \frac{j_2^2}{n_2} - j_1 E + j_2 E \\
&= \eta_t^* + \sum_{i=1}^{2} Q_{it} + q_x^* + \sum_{i=1}^{2} P_{ix} + \frac{j_1^2}{n_1} + \frac{j_2^2}{n_2} - j_1 E + j_2 E \\
&= \eta_t^* + q_x^* + \frac{j_1^2}{n_1} + \frac{j_2^2}{n_2} - j_1 E + j_2 E + j_1 \mathcal{E} - j_2 \mathcal{E} \\
&\leq 0.
\end{aligned} \tag{45}$$

Since

$$-j_1 E + j_2 E + j_1 \mathcal{E} - j_2 \mathcal{E} = (E - \mathcal{E})(j_2 - j_1) = (y_2 - y_1)(y_{2t} - y_{1t}), \tag{46}$$

then (44) turns into

$$\eta_t^* + q_x^* + \frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2} + (y_2 - y_1)(y_{2t} - y_{1t}) \leq 0. \tag{47}$$

We use the theory of divergence-measure fields , then

$$\frac{d}{dt} \int_0^1 (\eta^* + \frac{1}{2}(y_2 - y_1)^2) dx + \int_0^1 (\frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2})\, dx \ \leq 0, \tag{48}$$

where we use the fact

$$\int_0^1 q_x^* \, dx \ = 0. \tag{49}$$

Let $\lambda > 2 + 2n^* > 0$. Then, we multiply (48) by λ and add the result to (44) to get

$$\begin{aligned}
&\frac{d}{dt} \int_0^1 (\lambda\eta^* + \frac{\lambda}{2}(y_2 - y_1)^2 + y_1 y_{1t} + \frac{1}{2} y_1^2 + y_2 y_{2t} + \frac{1}{2} y_2^2) dx + \beta \int_0^1 (y_{1x}^2 + y_{2x}^2) dx \\
&+ \int_0^1 \frac{N_1 + N_2}{2} (y_1 - y_2)^2 dx + \int_0^1 \left((\lambda - N_1) \frac{y_{1t}^2}{n_1} + (\lambda - N_2) \frac{y_{2t}^2}{n_2} \right) dx \leq 0.
\end{aligned} \tag{50}$$

Using the estimate (22) in Theorem 2.1. and the Poincáre inequality (39), we have

$$\begin{aligned}
&\frac{d}{dt} \int_0^1 (\lambda\eta^* + \frac{\lambda}{2}(y_2 - y_1)^2 + y_1 y_{1t} + \frac{1}{2} y_1^2 + y_2 y_{2t} + \frac{1}{2} y_2^2) dx + \frac{\beta}{2} \int_0^1 (y_{1x}^2 + y_{2x}^2) dx \\
&+ \frac{\beta}{2} \int_0^1 (y_1^2 + y_2^2) dx + n_* \int_0^1 (y_1 - y_2)^2 dx + \int_0^1 \left(\frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2} \right) dx \leq 0.
\end{aligned} \tag{51}$$

Now,we consider η^* in (25). Clearly

$$n_i^2 - N_i^2 - 2N_i(n_i - N_i), \tag{52}$$

is the quadratic remainder of the Taylor expansion of the function n_i^2 around $N_i > n_* > 0$ for $i = 1, 2$. And then, there exist two positive constants C_1 and C_2 such that

$$C_1 y_{ix}^2 \le n_i^2 - N_i^2 - 2N_i(n_i - N_i) \le C_2 y_{ix}^2. \tag{53}$$

Making $C_3 = \min\{C_1, \frac{1}{2}\}$ and $C_4 = \max\{C_2, \frac{1}{2}\}$, then we get

$$C_3(\frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2} + y_{1x}^2 + y_{2x}^2) \le \eta^* \le C_4(\frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2} + y_{1x}^2 + y_{2x}^2). \tag{54}$$

Let

$$F(x,t) = \lambda\eta^* + \frac{\lambda}{2}(y_2 - y_1)^2 + y_1 y_{1t} + \frac{1}{2}y_1^2 + y_2 y_{2t} + \frac{1}{2}y_2^2,$$

then there exist positive constants C_5, C_6, and C_7, depending on λ, n_*, β, such that

$$\begin{aligned}
\int_0^1 F(x,t)dx &= \int_0^1 [\lambda\eta^* + \frac{\lambda}{2}(y_2 - y_1)^2 + y_1 y_{1t} + \frac{1}{2}y_1^2 + y_2 y_{2t} + \frac{1}{2}y_2^2]dx \\
&\le C_5 \int_0^1 [(\frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2}) + n_*(y_2 - y_1)^2 + \frac{\beta}{2}(y_{1x}^2 + y_{2x}^2) \ + \frac{\beta}{2}(y_1^2 + y_2^2)]dx \\
&\le C_6 \int_0^1 \eta^* dx,
\end{aligned} \tag{55}$$

and

$$\begin{aligned}
0 &< C_7 \int_0^1 [(\frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2}) + n_*(y_2 - y_1)^2 + \frac{\beta}{2}(y_{1x}^2 + y_{2x}^2) \ + \frac{\beta}{2}(y_1^2 + y_2^2)]dx \\
&\le \int_0^1 [\lambda\eta^* + \frac{\lambda}{2}(y_2 - y_1)^2 + y_1 y_{1t} + \frac{1}{2}y_1^2 + y_2 y_{2t} + \frac{1}{2}y_2^2]dx = \int_0^1 F(x,t)dx.
\end{aligned} \tag{56}$$

Then

$$\frac{d}{dt}\int_0^1 F(x,t)\, dx + \frac{1}{C_5}\int_0^1 F(x,t)dx \le 0, \tag{57}$$

and

$$\begin{aligned}
&\int_0^1 [(\frac{y_{1t}^2}{n_1} + \frac{y_{2t}^2}{n_2}) + n_*(y_2 - y_1)^2 + \frac{\beta}{2}(y_{1x}^2 + y_{2x}^2) \ + \frac{\beta}{2}(y_1^2 + y_2^2)]dx \\
&\le \frac{1}{C_7}\int_0^1 F(x,t)dx \le \frac{1}{C_7} e^{-\frac{t}{C_5}} \int_0^1 F(x,0)dx \\
&\le C_8 e^{-\frac{t}{C_5}} \int_0^1 \eta^*(x,0)dx.
\end{aligned} \tag{58}$$

are given, following the Growall inequality and the estimates (55) and (56). Up to now, we finish the proof of Theorem 3.1.

Acknowledgments

In the process of the selected topic and write a paper, I get the guidance from my tutor: Huimin Yu. In the teaching process, my tutor helps me develop thinking carefully. The spirit of meticulous and the rigorous attitude of my tutor gives me a lot of help. Gratitude to my tutor is unable to express in words. And this paper supported in part by Shandong Provincial Natural Science Foundation (Grant No. ZR2015AM001).

Conflict of interest

The author declare no conflicts of interest in this paper.

References

1. L. Yeping, Relaxation limit and initial layer analysis of a bipolar hydrodynamic model for semiconductors, Math. Comput. Model., **50** (2009), 470–480.
2. T. Naoki, Existence and uniqueness of stationary solutions to a one-dimensional bipolar hydrodynamic model of semiconductors, Nonlinear Anal-Theor, **73** (2010), 779–787.
3. L. Xing and Y. Yan, Large time behavior of solutions to 1-dimensional bipolar quantum hydrodynamic model for semiconductors, Acta Math. Sci., **37** (2017), 806–835.
4. Y. Huimin, On the stationary solutions of multi-dimensional bipolar hydrodynamic model of semiconductors, Appl. Math. Lett., **64** (2017), 108–112.
5. H. Haifeng, M. Ming and Z. Kaijun, Relaxation limit in bipolar semiconductor hydrodynamic model with non-constant doping profile, J. Math. Anal. Appl., **448** (2017), 1175–1203.
6. L. Jing, Y. Huimin, Large time behavior of solutions to a bipolar hydrodynamic model with big Data and vacuum, Nonlinear Anal-Real., **34** (2017), 446–458.