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Research article

Mapping algebraic and geometric thinking with the van Hiele model among college students in STEM programs

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Abstract: Mathematical proficiency, grounded in the integration of disciplinary knowledge and fundamental mathematical cognitive skills, is essential for achieving the intended learning outcomes of STEM programs. Despite this curricular emphasis, persistent weaknesses in students' mathematical ability continue to be widely documented. To provide insights on this underperformance, this descriptive–associational study investigated the algebraic and geometric thinking within the van Hiele hierarchy among 167 college students enrolled in STEM programs in a Philippine state university. Data were gathered using the van Hiele geometry test and a researcher-developed algebraic thinking assessment to examine development across three strands of algebraic thinking—generalized arithmetic, functional thinking, and modeling language—within van Hiele hierarchy and analyze variations of algebraic and geometric thinking across programs. Most students remained at van Hiele's visualization and analysis stages and at the transition stage between generalized arithmetic and functional thinking, underscoring a limited progression toward advanced mathematical reasoning. Teacher education students demonstrated stronger geometric and algebraic performance than engineering and computer science students, reflecting differences in disciplinary learning trajectories. The significant and systematic associations between algebraic and geometric thinking confirm the cognitive connections between the two domains, implying that students in higher van Hiele levels are better positioned to achieve higher levels of proficiency in complex algebraic thinking strands. The study recommends an in-depth study on instructional approaches that simultaneously cultivate geometric and algebraic thinking through conceptually rich, developmental, and discipline-aligned learning tasks.

Keywords: algebraic thinking, functional thinking, geometric achievement, van Hiele geometry test, mathematics education

1. Introduction

Mathematical literacy is essential for problem-solving, critical thinking, and informed decision-making in academic and real-world contexts [1]. At its core is mathematical reasoning, an integrated system supported by foundational cognitive abilities across interdependent domains such as algebra, geometry, and linguistic representation [2]. This interconnectedness of mathematical abilities underscores the need to examine how they support one another at both cognitive and pedagogical levels, particularly in higher-education STEM programs, where such foundational abilities may not be taught explicitly yet are essential for performing advanced mathematical and disciplinary tasks. However, large-scale international assessments (ILSAs) (e.g., TIMSS 2019; PISA 2018/2022) and recent literature reviews show that many systems continue to report substantial underperformance among basic education learners in foundational mathematics with persistent low proficiency and frequent item-level failures on algebra and geometry tasks [1,3,4]. These learning deficiencies may have serious implications for higher education, where college students are expected to interpret quantitative information, reason abstractly, and apply mathematical concepts to increasingly complex disciplinary learning tasks and problems. Hence, understanding relationships among mathematical domains is crucial for addressing persistent learning gaps and for designing instruction that strengthens and connects students' mathematical thinking.

The relationship between geometric and algebraic domains has gained renewed attention in mathematics education research. Recent studies demonstrate that geometric contexts play a critical role in supporting higher-level algebraic thinking skills [5,6]. This is because geometry offers rich opportunities to explore structures and relationships, making it a productive setting for developing functional and algebraic modeling competencies [2]. Empirical studies show that two algebraic thinking skills—functional and modeling competencies—strongly predict overall algebraic proficiency [7], thus providing the basis for further studies on their development. Students' algebraic functional thinking improves when tasks are grounded in visual or structural representations such as geometric patterns or transformations [8,9]. However, algebraic modeling ability, which requires learners to translate contextual situations into symbolic forms, interpret, and apply the resulting mathematical representations, remains a persistent challenge for many students [10–12]. Research also highlights that much remains to be desired in the geometric thinking of students from basic education to initial teacher education preparation, where the vast majority reach only the lower levels of geometric thought [13,14].

As scholars examine how students progress from basic to more complex cognitive processes in both algebra and geometry, the need to move beyond treating geometry and algebra as isolated domains becomes increasingly consequential in education and cognitive research—a direction that this study pursued.

2. Literature review

2.1. Algebraic thinking

Algebra has been considered the universal mathematics language, primarily because of its role in promoting students' capacity for generalization, symbolization, and functional thinking, making it the key to learning other mathematics domains [15,16]. Consequently, algebraic thinking has become a central focus in both mathematics pedagogy and mathematics education research.

Algebraic thinking is understood as a sequence of cognitive processes employed when learners summarize and generalize a pattern or structure, along with the mathematical and logical reasoning involved in mathematical representations [7]. Kaput et al. [17] described early algebraic thinking as comprising three strands—generalized arithmetic, functional thinking, and modeling—in which generalization and reasoning are core aspects [7,18]. Generalized arithmetic refers to students' ability to identify patterns, express structure, and extend numerical relationships [7,17]. Functional thinking pertains to students' ability to generalize thinking that focuses on relationships between simultaneously varying quantities (covariation) and rule-based representation of functions [19,20]; thus, it directly fosters the development of other algebraic thinking strands and computational thinking in arithmetic [21]. Modeling involves translating contextual or geometric situations into algebraic forms, requiring symbolic representation, interpretation, and application of algebraic forms or models [5,17,22]. From these definitions, modeling can be deduced to progress from generalized arithmetic and functional thinking skills.

The strands of algebraic thinking have been shown to develop at different rates. Generalized arithmetic typically emerges earlier and more independently before progressing toward symbolic algebraic thinking, characterized by the use of symbols to represent quantities and relationships among them [18,20,23], whereas functional thinking and modeling are late-developing strands that require higher levels of abstraction and reasoning and develop through instructionally mediated experiences, such as constructivist teaching designs [8,9,24]. Some undergraduates, for instance, were noted to demonstrate generalized arithmetic fluency but experience difficulty with generalizing relationships involved in functional thinking [25]; thus, scaffolding is necessary. A review by Sigbatulin et al. [18] emphasized that tasks involving multiple representations, particularly geometric or figural patterns, significantly foster students' algebraic generalization—one of the core components of algebraic thinking. Accordingly, Jamil et al. [26] identified transformative teaching strategies that strengthen algebraic thinking, such as abstraction scaffolding, systematic use of visual representations, and technology-supported modeling across both primary and secondary settings. These reinforce the view that algebraic thinking develops progressively when shaped by both cognitive readiness and instructional experiences [27,28].

While inquiries into the assessment and development of specific algebraic thinking skills have become substantially documented among elementary and secondary learners [7,8,16,19,29], there are limited studies incorporating strands of algebraic thinking to holistically capture the progression of algebraic thought in relation to other mathematical domains, particularly among older learners. Hence, this study looked into how algebraic thinking of college students is associated with their geometric ability.

2.2. Geometric thinking and van Hiele levels

Geometric thinking refers to a cognitive process through which learners understand geometry by progressing through qualitatively different levels of reasoning about shapes, properties, and spatial relationships, as it involves moving from basic recognition of shapes to more abstract reasoning and deduction [30]. Current mathematics education research strongly anchors geometric thinking in the van Hiele model [31,32]. Proposed by Pierre van Hiele and his wife Dina, the model describes geometric thinking as a developmental process in which students' thinking evolves through a sequence of hierarchical levels that include visualization, analysis, informal deduction, deduction, and rigor—each level representing distinct ways of comprehending and reasoning about geometric ideas and progressing primarily through instruction rather than age [33–35].

The van Hiele levels were originally numbered from 0 to 4; however, some subsequent studies adopted a 1–5 numbering scheme [36]. In this alternative scheme, Level 0 is interpreted as a pre-visualization stage, referring to individuals who do not meet the scoring criteria for the visualization level [32,37]. The following descriptions of the original van Hiele levels are derived from the works of Clements and Battista [34] and Hoffer [38]:

Level 0 (Visualization): Learners identify shapes by appearance and struggle to use attributes, requiring experiences that highlight distinguishing features.

Level 1 (Analysis): Learners recognize and use properties of shapes, but cannot relate them based on these properties.

Level 2 (Informal deduction): Learners coordinate properties, understand class inclusions, and form informal justifications, often supported by dynamic or pattern-based tasks.

Level 3 (Deduction): Students engage with formal proofs, though many students struggle to reach this stage without structured instruction.

Level 4 (Rigor): Students reflect an understanding and comparison of axiomatic systems.

van Hieles noted that cases of secondary students reaching Level 4 were rarely seen in schools. Recognizing a consistent, similar empirical evidence, developers of van Hiele tests made corresponding adjustments in the use of their research instruments and in the assignment of level numbers, as previously discussed. The present study utilized the revised Version 1.1 of the van Hiele Geometry Test (VHGT) developed at the University of Chicago [32,37].

Assessment of geometric thinking through the van Hiele model showed persistent gaps relative to the expected student outcomes. Recent work with 9th graders revealed that 56.2% were at the pre-visualization level, 30.75% at visualization, and only 12.5% at Level 2 (analysis), with almost none achieving informal deduction (0.5%) [39]. In a review, Sert-Çelik and Yılmaz [14] revealed that most students across K–12 and even teacher-education programs remained at the first two van Hiele levels, confirming van Hiele's assertion [33] and findings of earlier studies that showed significantly low percentages of learners reaching higher van Hiele levels, with none to very few achieving the highest Level 4 [40,41]. These studies point to systemic learning deficits as contributory factors underlying students' difficulties in navigating geometric tasks across the van Hiele levels [40]. For basic education learners, failing to reach Level 3 and Level 4 is quite understandable, given the scope of learning experience provided in the geometry curriculum [42].

Guided by the assumption that progress within the van Hiele hierarchy is achieved through instructional intervention rather than maturity [33], van Hiele–based instructional designs were used in studies addressing learning deficits in geometric thinking. In a review, Trimurtini et al. [35] reported effect sizes ranging from small to large for interventions using van Hiele–based instruction, dynamic geometry software, and physical manipulatives. This demonstrates that while students often remain at lower levels of visualization and analysis, well-designed instructional sequences can significantly raise the quality of geometric thinking [41].

Although various studies demonstrated improved geometric thinking through van Hiele–based instruction, confirming the model's instructional value, they also pointed out that relatively few studies examined how algebraic or symbolic thinking co-develops with geometric thinking. For instance, the role of the algebraic representational transformation—a core component of functional thinking— from concrete geometric contexts to symbolic remains underexplored [43]. Trimurtini et al. [35] further revealed that interventions using the van Hiele phases, manipulatives, and digital tools often lacked scaffolding capable of pushing learners toward the highest levels of geometric thinking. Research findings further indicate that many students operate at levels where geometric thinking is perceptual rather than relational or symbolic, so that opportunities to integrate algebraic abilities, particularly functional thinking, are limited [13,14,35,43]. Consequently, this study aimed to map the algebraic thinking of college students within the van Hiele model to explicitly show the intersection of mathematical thinking skills between the two domains.

2.3. Intersection of algebraic and geometric domains

Although empirical studies explicitly linking these domains remain limited, recent work provides emerging evidence of their interconnectedness, suggesting progress across levels of geometric thought parallels increasing algebraic thinking complexity [5,7]. Research has shown that geometric, algebraic, and linguistic abilities operate interdependently within a cognitive system, reinforcing the idea that mathematical thinking develops holistically rather than in isolation [2]. This strengthens the assertion that geometric contexts may foster algebraic thinking skills development [44]. Likewise, advancing through geometric thinking involves not only shifts within geometric domains but also includes core algebraic thinking skills associated with functional thinking and modeling such as relational reasoning, abstraction, and symbolic thinking [45].

Across algebra education research, it was found that algebraic thinking abilities do not occur as isolated skills. Sun et al. [7] observed that early algebraic thinking evolves from structural noticing in arithmetic to relational reasoning in functional thinking, a progression that parallels movement from van Hiele Level 2 (analysis) to Level 3 (informal deduction). In modeling, learners need to transition from contextual and visual, which rely on geometric contexts, to algebraic representations, oftentimes demanding processes that many students find challenging [46]. Subsequently, the persistent student difficulties in algebraic thinking processes are often attributed not only to symbolic manipulation alone but also to weaknesses in analyzing relationships and formulating mathematical representations for such relationships [10,11,12], thus providing an empirical basis for the integrative approaches in teaching mathematical competencies within algebraic domains.

On the other hand, geometry offers visual and structural representations that facilitate identification of invariants, relationships, and covariation—core components of functional thinking and modeling [47–49]. Consequently, students working at higher van Hiele levels are theoretically positioned to demonstrate stronger relational and representational aspects of algebraic thinking. Conversely, weaknesses in algebraic thinking can impede learners' ability to engage with analytical aspects of geometry, algebraic proofs of geometric theorems, or coordinate-based transformations, which are features of higher van Hiele levels. In addition, research into creativity [45] found that students who engaged with growing geometric patterns developed stronger abilities to generalize and justify algebraic rules, indicating that creative reasoning is an underutilized but powerful pathway toward advanced algebraic thinking skills such as abstraction. These suggest that coordinated geometric contexts and abilities may serve as an important scaffold for the development of more complex algebraic thinking, particularly in functional thinking and modeling strands.

2.4. Evidence from STEM programs

Current literature on STEM education identifies mathematics teacher education, engineering, and computer science as STEM disciplines [50] in which mathematics takes the foundational role [51–53]. Studies on algebraic and geometric thinking among prospective mathematics teachers have been well-documented, though the explicit link between these domains has been less explored. In the algebraic domain, research on functional thinking has shown that pre-service mathematics teachers often experience difficulty in generalizing relationships involving two variables [54]. In a similar study, Doruk and Gün [55] found that these difficulties may stem from unsuccessful transfers in prior experience with one-variable relationships, suggesting that early emphasis on pattern generalization is crucial for supporting the development of symbolic functional thinking. In the geometric domain, Armah's [13] study of 255 pre-service mathematics teachers in Ghana found that only 12.9% reached Level 5 (rigor), and 15.9% reached Level 4 (deduction), raising concerns about symbolic and deductive readiness among future teachers and calling for stronger emphasis on proof-based geometry in teacher preparation programs. Studies showed that leveraging geometric thinking of prospective mathematics teachers with systematic alignment of content and instructional experiences can promote advancement in the van Hiele levels [41,56].

On the other hand, mathematics also plays an essential role in engineering programs [57], where algebra, geometry, and calculus serve as fundamental resources for engineering education [58]. However, studies in engineering education emphasized that many students entered university with fragile algebraic and analytic-geometric understanding, and that young engineering students often lacked sufficient mathematical foundations and were frequently unaware of the mathematical demands of their programs [59,60]. Ancheta and Subia [61] found widespread misconceptions among engineering students in areas where algebra and geometry interface, such as algebraic manipulation and coordinate geometry. These difficulties suggest that many learners are not yet operating at van Hiele levels where symbolic and geometric reasoning converge. Mahadewsing et al. [62] traced first-year engineering calculus failures to weaknesses in algebra and trigonometry, arguing that success in advanced mathematics requires the ability to coordinate symbolic, graphical, and geometric representations.

In computer science, mathematics provides the foundation by fostering logical and analytical thinking and problem-solving proficiency [63]. Sofowara et al. [64] have shown that there is a strong positive correlation between students' mathematical proficiency and their programming ability. Studies further noted that the assimilation of algebra and computer science requires the development of algorithmic thinking in arithmetic, abstraction, and generalization [65,66]. This is because programming requires working with variables, functions, sequences, and patterns—core elements of algebraic thinking—thus making advanced algebraic and deductive skills necessary for success in discrete mathematics, algorithms, and formal verification [67–69]. Unfortunately, a remarkable number of computer science students often overlook this mathematical applicability, which may hinder engagement and achievement of disciplinary learning outcomes [63].

Despite the close conceptual relationship between algebraic and geometric thinking and the documented students' weaknesses in these domains, it remains unclear whether van Hiele levels or proficiency in algebraic thinking strands can help explain differences in college students' mathematical ability, particularly in STEM programs where algebraic and geometric competencies are essential in the achievement of intended learning outcomes. These limitations underscore the need for empirical work that maps students' geometric thinking levels alongside their algebraic thinking, which forms the basis for the focus of this study.

2.5. Research questions

Anchored on the premise that geometric and algebraic thinking are mutually reinforcing processes that draw on shared cognitive resources, this study aimed to establish the association between algebraic thinking strands and van Hiele levels of geometric thinking among college students in selected STEM programs. Specifically, it sought to address the following research questions:

1. How are the van Hiele levels of geometric thinking demonstrated by college students in STEM programs?
2. What is the extent of algebraic thinking of college students in STEM programs?
3. How does the algebraic thinking of college students in STEM programs vary in relation to their van Hiele levels of geometric thinking?

3. Materials and methods

3.1. Research design

This study aimed to establish an association between algebraic thinking and geometric achievement with the van Hiele model, employing a descriptive–associational design [70]. Specifically, a descriptive approach was employed to determine how van Hiele levels are manifested among college students in STEM programs (Research question 1), to describe the extent of development across the strands of algebraic thinking (Research question 2), and to map the dimensions of algebraic thinking onto the van Hiele levels (Research question 3). An associational approach was utilized to verify the significance of observed differences in algebraic thinking of college students enrolled in initial mathematics teacher education, engineering, and computer science programs to enhance the rigor of analysis in Research question 2 and to test predictive correlation between geometric and algebraic thinking in Research question 3.

3.2. Participants and study setting

As shown in Table 1, the participants of the study were 167 randomly selected second- and third-year college students enrolled in selected four-year STEM degree programs across three campuses of a state university in Capiz, Philippines, representing 72% of their cohort. The programs included pre-service mathematics teacher education (Bachelor of Secondary Education major in

Mathematics), agricultural and biosystems engineering (Bachelor of Science in Agricultural and Biosystems Engineering), and computer science (Bachelor of Science in Computer Science). These programs were selected because they are grounded in foundational mathematical areas such as logic, algebra, and geometry. Consequently, assessing students' algebraic and geometric thinking within these programs may provide insights relevant to curricular implementation and instructional practices. The pre-service mathematics teacher education curriculum prescribed by the Commission on Higher Education includes the study of advanced algebra and Euclidean geometry, emphasizing the development of both content knowledge and pedagogical knowledge [51].

In engineering education, program outcomes emphasize the application of mathematical knowledge in solving engineering problems, which explains the inclusion of courses such as calculus and differential equations in the curriculum [52]. Similarly, the computer science program aims to develop graduates' ability to analyze and solve computing problems, supported by coursework in algorithms and discrete structures within the program of study [53]. For engineering and computer science students, their formal study of algebra and geometry occurred during secondary education approximately two to three years prior to the conduct of this study. As such, the investigation also provided an opportunity to examine the long-term retention of mathematical learning. However, all participants had completed the general education mathematics course Mathematics in the Modern World during their first year of college. This course explores the practical applications of algebra, geometry, and other areas of mathematics in real-world contexts. Thus, it was assumed that students enrolled in these mathematics-intensive STEM programs possessed a working knowledge of the interplay between fundamental algebraic and geometric concepts during the conduct of the study.

Table 1. Distribution of study participants by STEM programs.

STEM programs	*N*	%
Pre-service mathematics teacher education	76	45.51
Agricultural and biosystems engineering	32	19.16
Computer science	59	35.33
Total	167	100

3.3. Research instruments

Two research instruments were utilized in the study: the van Hiele Geometry Test (VHGT) and a researcher-developed algebraic thinking test. The VHGT was used with permission from its lead developer, Zalman Usiskin, from the University of Chicago. The test consists of 25 items organized into 5 components representing the five van Hiele levels: recognition, analysis, order, deduction, and rigor [32,37]. Using the Kuder–Richardson Formula 20 (KR-20), the reliability coefficients for the five components were reported as 0.39, 0.55, 0.56, 0.30, and 0.26. These relatively low values were attributed to the small number of items in each component. The developer noted that by administering an equivalent test containing 25 items per component, the estimated reliability coefficients would have increased to 0.79, 0.88, 0.88, 0.69, and 0.65, respectively [37].

The revised VHGT Version 1.1 [32,71] was adopted in this study, in which the test component on the highest level was excluded. The lowest is Level 0 (pre-visualization), and the highest is Level 4 (deduction). The approach of assigning students to a particular level is guided by the sequential property of the levels [37]: a student is assigned to level *n* if the satisfactory criterion is met in the first *n* levels but not in the *n +1* level. The satisfactory criterion adopted in this study is obtaining at least a score of 3 out of 5 in each level. For example, a student whose scores are 3, 4, 3, 2 is assigned to Level 3. If the scores are, for instance, 2, 1, 2, 2, the student is assigned to Level 0. A special case, No Fit, occurs when the first *n* scores are not satisfactory, but the *n + 1* score is. For example, a student who scores 2, 1, 3, 2 belongs to this category.

The algebraic thinking test was developed based on the identified strands [17,18,19]: generalized arithmetic, functional thinking, and modeling. It consists of three items, each structured to elicit a response for each strand. The test items underwent expert validation by mathematics educators from the university and were pilot-tested with 25 students in the same cohort as the study participants. The test–retest method was used due to the open-ended structure of the test items. The paired sets of scores were correlated, and the obtained reliability coefficient was 0.712, indicating a very satisfactory measure of reliability [72].

3.4. Data collection and analysis

The study instruments were administered in person to the participants in their respective campuses in 90-minute periods: 30 minutes for VHGT and 1 hour for the algebraic thinking test. The accomplished test papers were collated and scored. Data were then processed for analysis. Following the recommended scheme of assigning students to their van Hiele levels, frequency counts and percentages were used to summarize the distribution of students in the van Hiele levels by their STEM program.

In scoring the algebraic thinking test, the following scoring scheme was used: 1 point, correct and complete answer; 0.5 points, correct but incomplete answer; 0 points, incorrect or no attempt made. Mean, standard deviations, and mean percentage scores (MPS) were used to summarize the performance of students. To provide a description of the extent of development in algebraic thinking, this scheme was formulated for the interpretation of MPS levels: *Beginning* (below 50%), *Developing* (50.01%–62.50%), *Approaching proficiency* (62.51%–75.00%), *Proficient* (75.01%–87.50%), and *Highly proficient* (87.51%–100.00%). Content analysis of the responses to the algebraic thinking test was also performed to examine the progression in the algebraic ability of the students across the algebraic thinking strands. Tests of differences were made using the Mann–Whitney test, while tests for correlations and concordance utilized Spearman's rho and Kendall's tau tests, respectively. Simple regression analysis was used to determine the predictive linear relationship between the two skill domains. All inferential analyses were set at a 5% significance level.

4. Results

4.1. van Hiele levels of geometric thinking of college students in STEM programs

Figure 1 shows the overall and program-specific distribution of college students' van Hiele levels, revealing distinct geometric thinking profiles within STEM programs. Pre-service mathematics teachers demonstrated the broadest and most advanced range of van Hiele levels, with approximately 36% (28) reaching Levels 3 and 4, involving axiomatic systems and formal deduction. Engineering students were concentrated primarily at Level 1 (20, 62.50%), with smaller proportions at Levels 2 and 3 (8, 25%). Computer science students exhibited the lowest overall levels, with more than half at

Level 1 (34, 57.63%) and sizable proportions at Level 0 (12, 20.34%) and No Fit (8, 13.56%). Only a small proportion reached Levels 2 and 3 (5, 8.47%), and none attained Level 4.

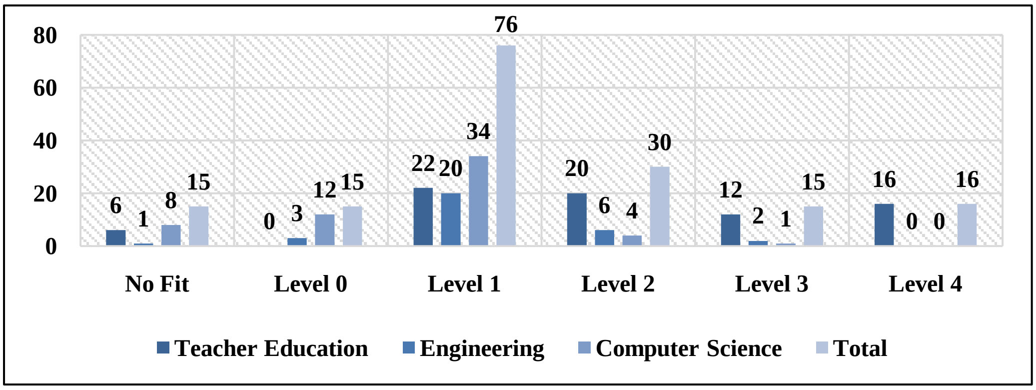


Figure 1. Distribution of college students in terms of van Hiele levels of geometric thinking.

4.2. Algebraic thinking among college students in STEM programs

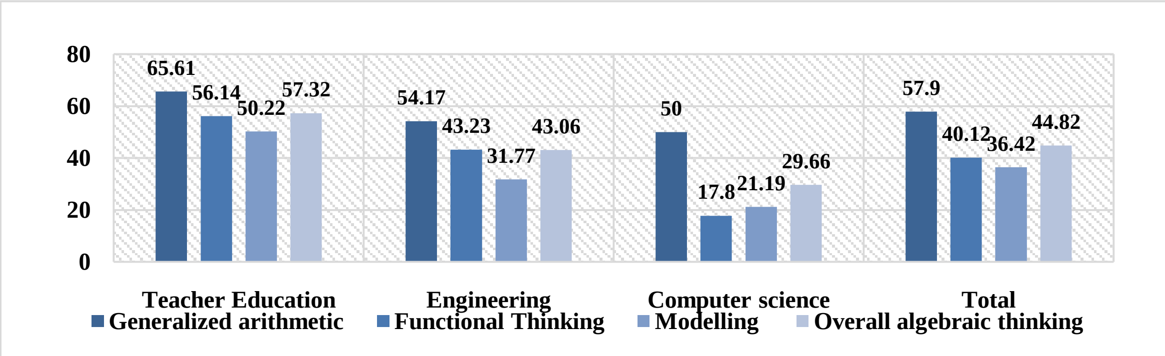
4.2.1. Extent of algebraic thinking of college students

Figure 2 presents the mean percentage scores across three strands of algebraic thinking. The overall performance of college students across programs was at the beginning level (*M* = 4.03, 44.82%). The students achieved the developing level in the generalized arithmetic (*M* = 1.74, 57.90) but the beginning level in functional thinking (*M* = 1.20, 40.12%), and even lower in modeling language (*M* = 1.09, 36.42%).

Among STEM programs, teacher education students obtained the highest scores across all dimensions, reflecting comparatively stronger algebraic thinking. Their highest performance was in generalized arithmetic (*M* = 1.97, 65.61%), followed by functional thinking (*M* = 1.68, 56.14%) and modeling language (*M* = 1.51, 50.22%). Engineering students demonstrated moderate scores in generalized arithmetic at the developing level (*M* = 1.63, 54.17%) but showed lower performance at beginning levels in both functional thinking (*M* = 1.30, 43.23%) and modeling language (*M* = 0.95, 31.77%). Computer science students showed the lowest overall performance, with generalized arithmetic at the threshold of developing level (*M* = 1.50, 50.00%) but much lower scores in functional thinking (*M* = 0.53, 17.80%) and modeling language (*M* = 0.64, 21.19%).

The overall algebraic thinking scores across programs indicate substantial differences in algebraic thinking across programs, with teacher education students showing the strongest grounding in algebra (57.32%), followed by engineering students (43.06%), with computer science students performing the weakest (29.66%).

As shown in Table 2, pairwise comparisons using the Mann–Whitney test were conducted to determine the significance of the observed differences in the strands of algebraic thinking across STEM programs. The results revealed that teacher education students differed significantly from computer science and engineering students in generalized arithmetic, whereas no significant difference was observed between engineering and computer science students in this strand. For functional thinking, modeling, and overall algebraic thinking, significant differences were found across all program groups. These findings indicate that prospective mathematics teachers consistently demonstrate stronger performance in algebraic tasks compared with students enrolled in engineering and computer science programs.



Note: Interpretation of the mean percentage score (MPS) is based on the following scale: below 50: *Beginning*; 50.00–62.50: *Developing*; 62.51–75.00: *Approaching proficiency;* 75.01–87.50: *Proficient*; 87.51–100.00: *Highly proficient.*

Figure 2. Extent of algebraic thinking among college students across STEM programs.

Table 2. Mann–Whitney pairwise comparisons of algebraic thinking of college students.

Pairwise comparison (Mann–Whitney *Z*)		Generalized arithmetic	Functional thinking	Modeling	Overall algebraic thinking
Teacher education	Engineering	-1.793*	-2.011*	-3.207**	-3.261**
	Computer science	-3.170**	-6.805***	-5.683***	-6.517***
Computer science	Engineering	-0.916	-3.938***	-2.248*	-3.299**

*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$

4.2.2. How college students demonstrate algebraic thinking

Content analysis was conducted to examine how college students progressed through the strands of algebraic thinking. Generalizing arithmetic occurs between the initial conditions (0) and the students' formulation of a sequence. In Figure 2, a highly proficient student accurately identified the underlying pattern and applied its rule to generate the rule and determine the next two terms (1). From this point, the student advanced to functional thinking by establishing the relationship between each term and its corresponding term number (*n*), leading to a correct formulation of the general term as a function equation specifying the independent and dependent variables. Finally, the student applied this function equation to verify whether a given value belongs to the sequence, demonstrating effective use of modeling (3).

Figure 3 shows a less successful progression of a student's algebraic thinking. Although the student correctly identified the common ratio as –3, the student was unable to establish the functional relationship between the term number and its corresponding value, leading to incorrectly proposing –3n as the nth term. Nevertheless, by repeatedly multiplying each term by –3, the student managed to verify that 50,049 was not part of the sequence, correctly identifying 59,049 instead. This demonstrates a capacity for arithmetic generalization but points out a gap in functional thinking, particularly in recognizing the covariation between the independent variable (the term number) and the dependent variable (the value of the nth term).

The students' solutions in Figures 4 and 5 illustrate demonstrations of profound algebraic thinking in responding to test item 2. In Figure 4, the student accurately translated the visual pattern by constructing the next two figures and identifying the corresponding numerical sequence. This step shows a sound grasp of pattern recognition as a foundation for generalization.

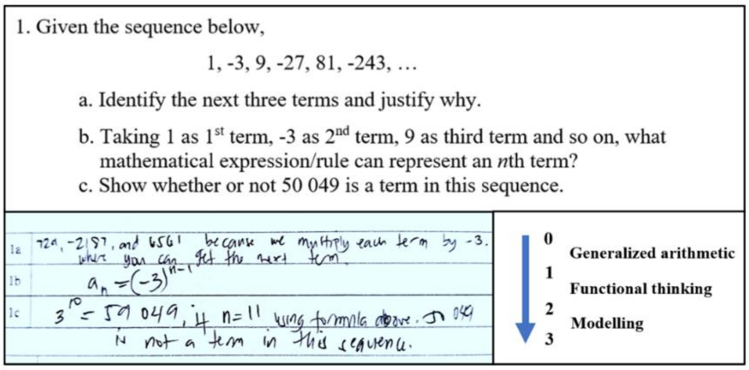


Figure 3. Successful demonstration of algebraic thinking from generalized arithmetic to modeling language.

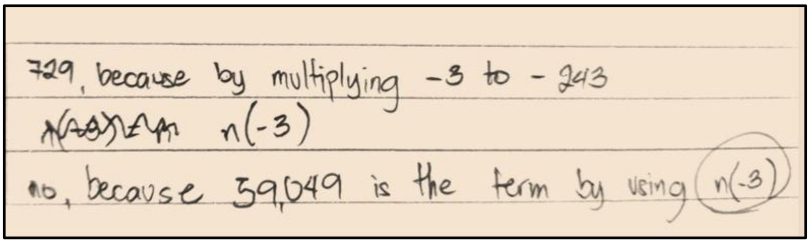


Figure 4. A student's less successful demonstration of algebraic thinking.

The students' solutions in Figures 5 and 6 demonstrate profound algebraic thinking in responding to test item 2. In Figure 5, the student accurately translated the visual pattern by constructing the next two figures and identifying the corresponding numerical sequence. This step shows a sound grasp of pattern recognition as a foundation for generalization. The student in Figure 6 presented a complete and detailed solution process that reflects a high level of algebraic proficiency. The student correctly applied properties of polynomial functions, using successive differences in the dependent variable to determine the function's degree. After establishing the degree, the student employed a system of linear equations to solve for the function's coefficients, an indicator of fluent modeling. With the function explicitly represented, the student was able to compute the required value, demonstrating a thorough understanding of both structure and procedure.

4.3. Mapping algebraic and geometric thinking within van Hiele levels

Table 3 presents the mean percentage scores in generalized arithmetic, functional thinking, modeling, and overall algebraic thinking across van Hiele levels of geometric thinking. Results show an evident progression pattern: algebraic thinking performance increases systematically with higher van Hiele levels.

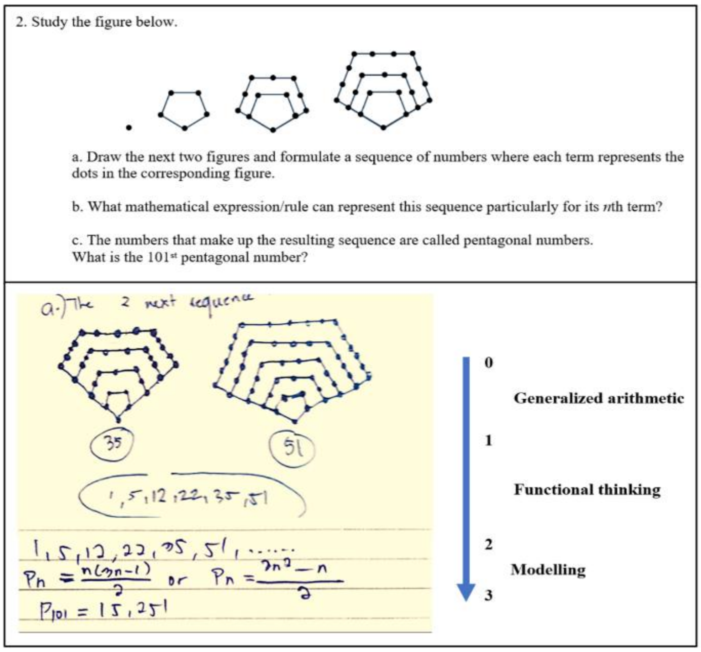


Figure 5. Progression of a student's algebraic thinking from generalized arithmetic to modeling.

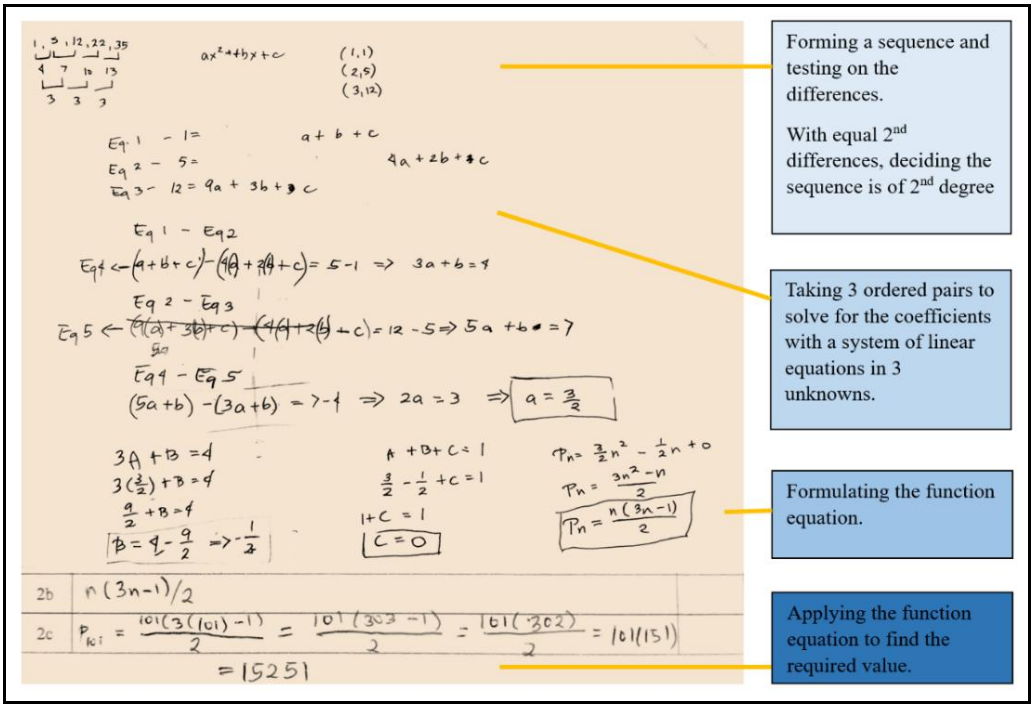


Figure 6. A detailed progression of a student's algebraic thinking from generalized arithmetic to modeling.

Students in the No Fit and Level 0 categories demonstrated the lowest algebraic thinking performance, reaching the *Beginning* level in all strands. At Level 1, students showed improvement in generalized arithmetic at the *Developing* level (57.68%), but functional thinking (33.55%) and modeling (28.95%) remained within the *Beginning* range. There was an observable sharp increase in algebraic thinking from Level 2 onward. Reaching the overall *Developing* level (56.92%), Level 2 students achieved *Approaching proficiency* in generalized arithmetic (65.56%) and moved into the *Developing* range for functional thinking (52.22%). However, their ability in modeling remained at the *Beginning* level (47.78%), indicating that representational fluency may develop more slowly than fundamental generalization skills.

Students at Level 3 consistently achieved *Developing*-level performance across strands, in which functional thinking (57.78%) and modeling (51.11%) showed substantial improvements that correspond to the shift from analysis to a more advanced geometric level of informal deduction. The highest performance across strands was observed at Level 4, where generalized arithmetic (71.04%), functional thinking (69.79%), and modeling (63.54%) were markedly higher than those at any other level. This indicates that students capable of formal geometric deduction also demonstrate higher levels of achievement in numerical generalization, functional thinking, and modeling abilities.

Table 3. Mapping of algebraic and geometric thinking within van Hiele levels among college students.

Modified van Hiele level	*N*	Generalized arithmetic		Functional thinking		Modeling		Total algebraic thinking	
		MPS	Description	MPS	Description	MPS	Description	MPS	Description
No fit	15	42.22	Beginning	32.22	Beginning	34.44	Beginning	36.30	Beginning
Level 0	15	37.78	Beginning	7.78	Beginning	10.00	Beginning	18.52	Beginning
Level 1	76	57.68	Developing	33.55	Beginning	28.95	Beginning	40.06	Beginning
Level 2	30	65.56	Approaching proficiency	52.22	Developing	47.78	Beginning	55.19	Developing
Level 3	15	65.56	Approaching proficiency	57.78	Developing	51.11	Developing	58.15	Developing
Level 4	16	71.04	Approaching proficiency	69.79	Approaching proficiency	63.54	Approaching proficiency	68.13	Approaching proficiency

Note: Interpretation of the mean percentage score (MPS) is based on the following scale: Below 50: *Beginning*; 50.00–62.50: *Developing*; 62.51–75.00: *Approaching proficiency*; 75.01–87.50: *Proficient*; 87.51–100.00: *Highly proficient*.

Figure 7 shows the distribution of college students across van Hiele levels and strands of algebraic thinking. Although the average performance in algebraic thinking reached the *Approaching* level at higher van Hiele levels, individual results reveal that some students achieved a *Highly Proficient* level of algebraic thinking across van Hiele levels with greater concentrations at Levels 3 and 4. Excluding the No Fit group, a clear progression emerges: the concentration of students gradually but steadily shifts from *Beginning* to *Highly Proficient* level of algebraic thinking as van Hiele levels progress from Level 0 to Level 4.

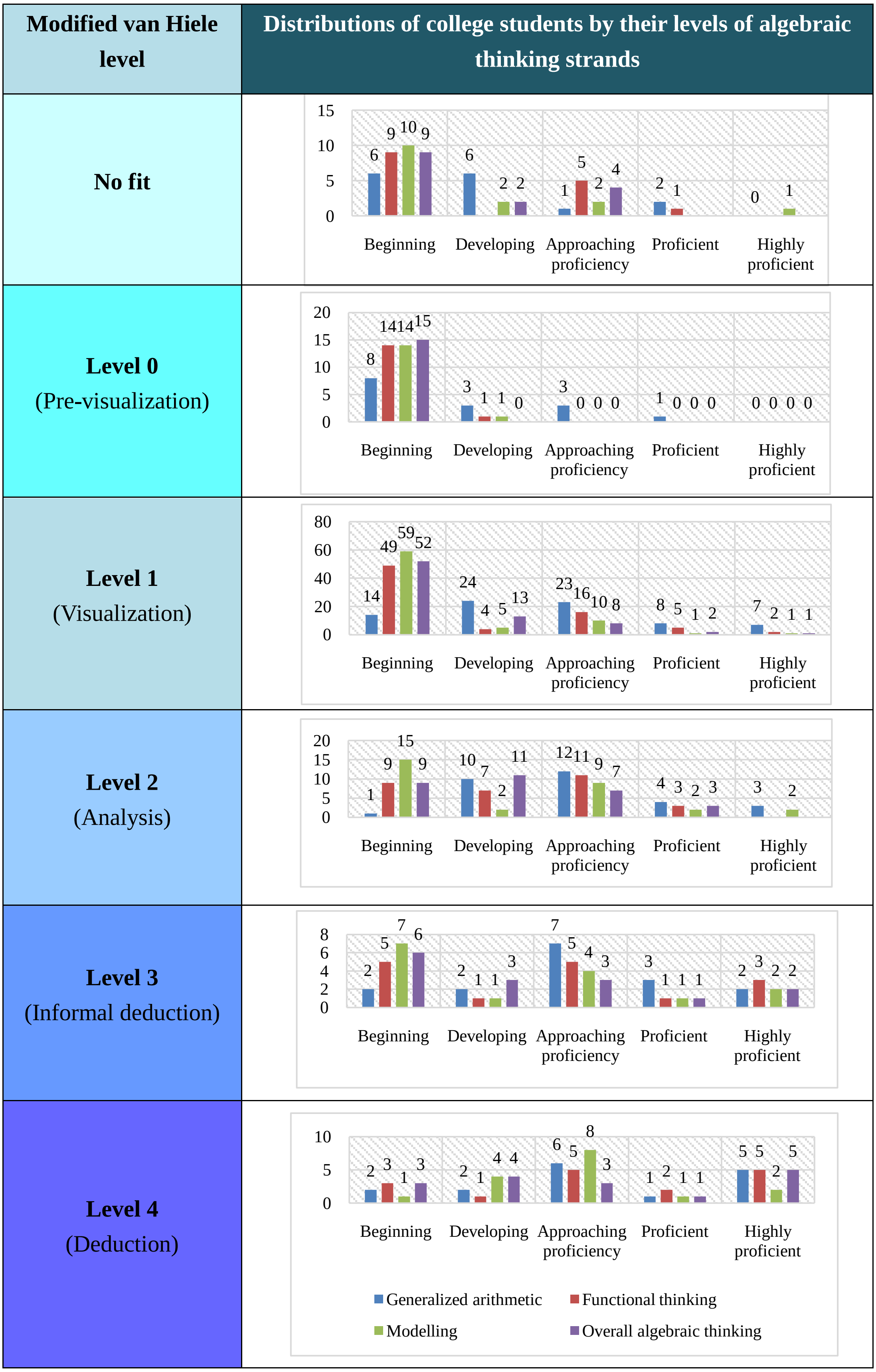


Figure 7. Distribution of college students in terms of proficiency levels in algebraic thinking strands within van Hiele levels.

To verify the evident direct relationship between van Hiele levels and dimensions of algebraic thinking, correlation analysis using Spearman's rho and test of concordance using Kendall's tau were performed. Results in Table 4 show weak-to-moderate measures of highly consistent direct association and an overall moderate and highly significant association between van Hiele levels of geometric thinking and algebraic thinking among college students (ρ = 0.551***, τ = 0.446***). Excluding those in the No Fit category, simple regression analysis was further performed to establish predictive correlation between geometric and algebraic thinking. Results presented in Table 5 show that the linear relationship between geometric and algebraic thinking was highly significant: $F(1, 150) = 62.884, p < 0.001$. Consequently as shown in Table 6, geometric thinking is a significant predictor of algebraic thinking $(B = 11.172, SE = 1.409, t = 7.930,\ p < 0.001)$, explaining 29.50% of the variance in algebraic thinking. This indicates a substantial contribution of geometric thinking abilities of college students to the development of their algebraic thinking.

Table 4. Measures of associations between van Hiele levels of geometric thinking and algebraic thinking.

Aspects of algebraic thinking	ρ	τ
Generalized arithmetic	0.306***	0.258***
Functional thinking	0.511***	0.433***
Modeling	0.518***	0.437***
Overall algebraic thinking	0.551***	0.446***

***p < 0.001

Table 5. Analysis of variance for the linear relationship between geometric and algebraic thinking skills.

ANOVA	Sum of squares	*df*	Mean square	*F*	*p*
Regression	23974.895	1	23974.895		
Residual	57188.747	150	381.258	62.884	<0.001
Total	81163.642	151			

Table 6. Regression result for geometric thinking as a predictor of algebraic thinking.

Variable	*B*	*SE*	*t*	*p*	95.0% confidence interval for *B*	*R*	R^2
Constant	16.480	4.006	4.114	<0.001	8.564–24.395	0.543[a]	0.295
Geometric thinking	11.172	1.409	7.930	<0.001	8.388–13.955		

5. Discussion

5.1. van Hiele levels across STEM programs

The distribution of learners across the van Hiele levels offers a nuanced account of how geometric thinking was manifested by college students in STEM programs in this study. The result showing that most students were clustered at Level 1 suggests that while many can analyze shapes and identify properties, comparatively fewer have transitioned toward the more sophisticated deductive reasoning characteristic of Levels 2–4. This is consistent with earlier studies showing that the majority of students from basic education reached only the first two van Hiele levels [49], and pre-service teacher education students could barely achieve the fourth level [13,73].

The comparatively higher proportion of teacher education students attaining advanced levels within the van Hiele hierarchy suggests that prospective mathematics teachers were positioned to demonstrate more developed abilities beyond recognition and use of properties of geometric figures than their counterparts in engineering and computer science programs. These abilities include proficiency in informal justifications and engagement in formal proofs, though students may require structured instruction in overcoming proving difficulties [34,38] to advance toward understanding of axiomatic systems. Engineering students demonstrated basic descriptive and analytic understanding of geometric properties with limited progression to higher-order deductive reasoning, while computer science students demonstrated weaker geometric foundations and minimal advancement beyond visual or basic descriptive reasoning.

These results should be interpreted cautiously, taking into account the curricular features of the STEM programs involved in the study. According to van Hiele [33], the progression from one level to the next is not a natural developmental process but is achieved through teaching-learning experience. In mathematics teacher education, program curricula emphasize geometric thought development, instructional sequencing, and contextualized pedagogy, which can support advancement through the van Hiele hierarchy [51]. These curricular provisions are not explicitly emphasized in engineering and computer science curricula [52,53]. Consequently, the presence of a substantial number of prospective mathematics teachers reaching Levels 3 and 4 appears to support that instruction aligned with the van Hiele phases allows students to transition toward advanced geometric thinking. This pattern is consistent with contemporary mathematics teacher education discourse emphasizing that professional preparation shapes learners' geometric thinking trajectories through meaningful integration of content, mathematical thinking development, and instructional experiences that support prospective teachers' learning [58,61]. Empirical evidence from other disciplines further suggests that strengthening discipline-specific curricula by embedding geometric thinking tasks, particularly those targeting Level 2 transitions, such as analyzing relationships and constructing informal arguments, may be beneficial [43]. Collectively, the findings imply deeper inquiry into the role of purposeful instructional design where educators can better scaffold learners' movement beyond the visualization level toward higher-order geometric thinking abilities [74].

5.2. Algebraic thinking across STEM programs

The assessment of the three strands of algebraic thinking revealed a general performance that fell short of expected learning outcomes, indicating minimal knowledge and skill acquisition and a continued need for support in transfer and application. Clear disciplinary differences were also evident, despite the shared mathematical foundation of STEM programs involved in the study.

Teacher education students' higher scores reaching an overall *Approaching proficiency* level in generalized arithmetic point to stronger abilities in identifying patterns and generalizing numerical relationships. In contrast, the lower performance of engineering and computer science students reflects that, while they possessed procedural or computational ability rooted in arithmetic, they struggled in identifying underlying, universal structures, relationships, or properties to form a general rule [25].

While teacher education students showed a modest advantage over engineering and computer science students in functional thinking, the overall performance across disciplines demonstrated that they need further support in formulating relationships involving covariation and rule-based representations. Analysis of responses showed that, while students can identify implicit rules behind patterns and generate succeeding terms in a sequence, they manifest conceptual limitations in generalizing such patterns. These limitations may have constrained their transition to functional thinking [25]. This finding is consistent with established literature indicating that functional thinking does not emerge solely from procedural algebraic instruction that is focused on symbol manipulation, rule application, and routine problem-solving [19,23]. Thus, looking into the impact of targeted instructional design is necessary, as the ability to formulate functional relationships is foundational both to computational and algebraic thinking [21,65]. This may include learning tasks on relational and pattern-based explorations that support conceptual shifts to enhance relational and overall algebraic thinking [8,9,24].

Modeling scores showed the lowest performance of students across programs, reflecting students' difficulty in meaningfully translating contextual information into algebraic models, which requires representation, interpretation, and application of these models [17,22,75]. Analysis of responses from students who successfully demonstrated algebraic thinking across the three strands indicates that modeling is an iterative process that applies fundamental algebraic abilities such as generalized arithmetic. It draws on prior knowledge and integrates it with contextual information to generate appropriate interpretations, which in turn guide the construction, representation, and application of the resulting model. Accordingly, studies highlight that modeling fluency benefits from explicit emphasis in connecting context with content structure, abstraction scaffolding, strategic visual representations, and model construction [17,26].

In light of discipline-specific learning trajectories defined by learning goals, content, and task design of STEM programs, future research may examine how these instructional strategies support the development of more complex strands of algebraic thinking, such as functional thinking and mathematical modeling.

5.3. Mapping algebraic and geometric thinking within van Hiele levels

The statistically significant associations between college students' geometric thinking and their performance across key strands of algebraic thinking provide empirical support for a substantive cognitive interdependence between these domains. Such results reinforce the position that algebra and geometry function as complementary and mutually reinforcing structures rather than as compartmentalized areas of learning. In particular, it has been argued theoretically and demonstrated empirically that integrated engagement with algebraic and geometric thinking processes can foster deeper conceptual understanding and more coherent knowledge construction in mathematics education [5,44,47].

Relative to the van Hiele levels, data show that the distribution of algebraic thinking strands was not uniform. Generalized arithmetic appears comparatively strong at lower geometric levels, consistent with prior research characterizing it as the earliest-emerging strand associated with pattern generalization [18,20,23]. On the other hand, the relative proportions of higher proficiency levels in functional thinking and modeling tend to increase as distribution progresses to the analysis and deduction levels of the van Hiele hierarchy. This pattern suggests that students who attain more advanced geometric thinking are also more likely to demonstrate higher proficiency in later-developing strands of algebraic thinking. Simple regression analysis confirmed this association, indicating that geometric thinking, operationalized through van Hiele levels, accounted for a substantial proportion of the variance in college students' algebraic thinking.

These findings are consistent with prior studies demonstrating that achievement in geometry provides an essential cognitive foundation for the development of algebraic thinking [5-8,18,48,49,76]. Empirical evidence shows that, in cultivating mathematical thinking through cognitively rich geometric contexts, the multiple dimensions of algebraic abilities, including pattern generalization and relational and functional thinking, can subsequently develop [5,45,47]. The study findings further point out the importance of cognitively aligning algebraic and geometric contexts and thinking skills within instruction. For instance, to address the learning needs of those operating at lower van Hiele levels, future research may examine how the development of algebraic thinking can be enhanced through learning experiences that allow learners to progress from visualization, pattern recognition, and basic analysis toward informal deduction and formulation of generalized functional relationships. STEM curricula should, therefore, intentionally provide students' coordinated development of algebraic and geometric thinking to promote conceptual integration across algebraic thinking strands and the van Hiele levels of geometric thinking.

6. Conclusions

The study contributes to contemporary STEM education discourse by explicitly situating the algebraic thinking strands within van Hiele levels of geometric thinking of college students and establishing disciplinary and cognitive patterns between the two domains. Findings revealed that much remained to be desired in both geometric and algebraic thinking abilities of college students, and that variations in the achievement levels in both domains were discipline-specific. Prospective mathematics teachers were better positioned to demonstrate proficiency in higher van Hiele levels, such as deductive reasoning and formal proof construction, which are essential for understanding axiomatic systems, as well as in complex algebraic thinking strands such as functional thinking and modeling, than engineering and computer science students. Furthermore, the significant predictive relationship between geometric and algebraic thinking supports the study's assumption that progression in geometric thinking, represented by van Hiele levels, corresponds with an increasingly complex algebraic thinking. Learners who demonstrate higher levels of geometric thinking also tend to exhibit more advanced algebraic thinking abilities. These underscore the interconnected nature of geometric and algebraic domains and suggest that the development of geometric thinking may play an important role in fostering late-developing algebraic thinking strands. While current mathematics education literature offers a wide range of conceptually focused strategies that are grounded in geometric contexts and intended to scaffold students' engagement in both algebraic and geometric thinking through mathematical deductions, quantitative change-relationship analyses, and symbolic or mathematical representations, future research should also focus on how these strategies come into play within disciplinary learning trajectories of students in higher-education STEM programs.

7. Limitations and future research directions

The study acknowledges several limitations on sampling coverage, locale, instrumentation, and participant representation that may restrict the generalizability of the findings beyond the immediate context of the study. A larger sample size and varied representation by other mathematics-oriented programs in various university settings would likely provide sufficient data for comprehensive analysis, as students' achievement in algebraic and geometric thinking from other institutions and degree programs may have been shaped by their respective academic environments and learning trajectories. As the researcher-developed algebraic thinking test underwent expert validation and only limited pilot testing, using a standardized instrument would provide more comprehensive information and support a wider range of data analysis approaches. Future research should address these limitations by incorporating more diverse samples and considering mixed-methods, multivariate large-scale surveys, or experimental designs in investigating the dynamics between algebraic thinking strands and geometric thinking levels in an integrated instructional design.

Use of Generative-AI tools declaration

The author declares the use of Artificial Intelligence (AI) tools in the creation of this article. Grammarly and ChatGPT were used for language clarity and coherence in the presentation and discussion of results. The development of the study including its conceptualization and points of inquiry, review of literature, analysis and interpretation of results remains the intellectual contribution of the author.

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Conflict of interest

The author declares no conflict of interest.

Ethics declaration

The research panel of the university reviewed and approved the conduct of the study. The participants were made fully aware on their voluntary participation and their right to withdraw should their participation may cause harm to their well-being. They also provided written informed consent prior to their participation and their institutional heads issued approval for the administration of research instruments.

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Author's biography

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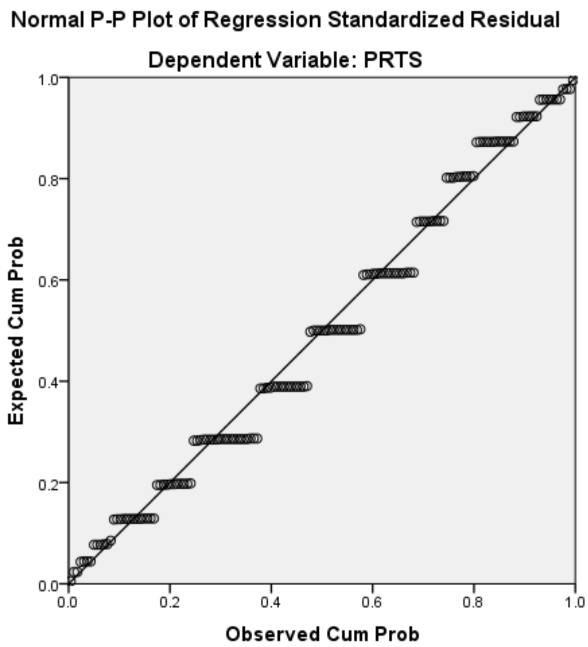
Appendix 1. Preliminary analyses on the properties of residuals.

Standardized residual		Statistic	Std. Error
Mean		0.0000000	0.08084169
95% confidence interval for mean	Lower bound	-0.1597269	
	Upper bound	0.1597269	
5% trimmed mean		-0.0034986	
Median		0.0002879	
Variance		0.993	
Std. deviation		0.99668324	
Minimum		-2.56673	
Maximum		2.56415	
Range		5.13089	
Interquartile range		1.42273	
Skewness		0.148	0.197
Kurtosis		-0.497	0.391

Appendix 2. Normal P-P plot of regression standardized residual



Appendix 3. Tests of normality results

Tests of normality			
	Shapiro–Wilk		
	Statistic	df	Sig.
Standardized residual	0.984	152	0.077
Unstandardized residual	0.984	152	0.077