STEM Education

STEM Education, 5(5): 836–854
DOI: 10.3934/steme.2025037
Received: 11 March 2025
Revised: 09 May 2025
Accepted: 20 May 2025
Published: 22 July 2025

https://www.aimspress.com/journal/steme

Research article

Actualization, activation, and potentialization of spatial imagination in geometry among students

Gulnisa Borboeva[1,*], Gulbadan Matieva[1], Venera Isakova[2], Cholpon Mustapakulova[1] and Gulshana Omurzakova[1]

[1] Institute of Mathematics, Physics, Technics and Information Technologies, Osh State University, 723500, 331 Lenin Str., Osh, Kyrgyz Republic; gulnisaborboeva36@gmail.com, gu-matieva@outlook.com, c_mustapakulova@outlook.com, gulomurzakova@hotmail.com

[2] Faculty of Mathematics and Computer Technologies, Osh State Pedagogical University, 723500, 73 Isanov Str., Osh, Kyrgyz Republic; v.isakova422@hotmail.com

* **Correspondence:** Email: gulnisaborboeva36@gmail.com.

Academic Editor: William Guo

Abstract: This study investigates the realization of spatial imagination in the independent creative development of students. It identifies learning activities as a key factor in shaping a student's thinking through knowledge acquisition. Spatial representation is defined as the ability to create, modify, and manipulate spatial images in the imagination, which is essential to solve geometric problems. Success in building models, analyzing structures, and performing practical tasks largely depends on this ability. The study focuses on the organization of educational activities for future mathematics teachers to foster spatial imagination. Three stages of developing spatial images in solving geometric problems are distinguished: actualization, activation, and potentialization. Actualization restores the existing knowledge, activation encourages its application in new contexts, and potentialization aims at advancing imagination to the level of independent creative thinking. The study emphasizes the role of creative assignments, independent activities, and problem-based approaches in enhancing learning. The integration of these stages facilitates a more effective understanding of geometric concepts and the development of stable spatial representations. Furthermore, the research introduces a theoretical model for the development of spatial imagination, based on three interrelated components: cognitive, analytical, and practical, which together ensure the transition towards independent creative thinking.

Keywords: future teacher, visualisation, development zones, self-development, mathematical discipline

1. Introduction

Global changes are occurring at an increasingly rapid pace, driven by the introduction of artificial intelligence, which is gradually replacing humans in many areas of activity. However, artificial intelligence cannot completely replace the human mind due to its software limitations. Humans possess such unique properties as intuition, the ability to understand and compare situations, make creative decisions, and apply spatial thinking, which help solve problems in various life and professional contexts. Spatial thinking is one of the key components of intelligence and the basis for creating new ideas, products, and spiritual values. The development of this type of thinking begins in childhood but requires special conditions in the education system to improve it. As one of the main disciplines, geometry plays an important role in forming a student's spatial thinking. The problem of developing this type of thinking is relevant not only for students but also for mathematics teachers, who often do not work enough in this direction. Pedagogical observations, results of diagnostic surveys, and training seminars show an insufficient level of spatial thinking development among students and teachers, which complicates their geometric activities. Traditional education, which focuses on the accumulation of knowledge, no longer meets modern requirements. In a world where information is rapidly updated, students need to develop the competencies of self-learning, adaptation, and practical use of knowledge. This is determined by the need to develop new skills that allow for a better integration into today's dynamic society.

Despite the considerable interest in the psychology of spatial thinking, the issue of its development in future mathematics teachers remains relevant. Furthermore, the issues of professional training of teachers for the development of spatial thinking in students within the modern two-level education system should be addressed. Many scientists have studied the issues of actualization, activation, and potentialization of a student's spatial imagination in geometry. Gagnier et al. [1] and Lee [2] highlighted the role of spatial thinking in science, technology, engineering, and mathematics (STEM), thereby exploring primary school teachers' beliefs, perceptions, and self-esteem in teaching spatial thinking. The study demonstrated that spatial thinking is a key component of STEM education that can be developed from an early age. The authors emphasized the need to integrate spatial skills into pedagogical strategies and assess teachers' competencies, which contribute to the development of spatial skills in students. Goyibnazarova [3] analyzed methods of developing a student's spatial representations by teaching geometric problems. The author analyzed how problem-solving helps to form spatial thinking and imagination, in particular, through the use of interactive and visual approaches in geometry. This study emphasized the importance of visualization and creative thinking in the development of spatial representation in the learning process. Hickman [4] investigated the role of geographic information systems (GIS) in the development of a student's spatial thinking. The author assessed how GIS technologies contribute to the formation and assessment of spatial thinking competencies. Kahharov [5] analyzed intensive methods of developing a student's spatial imagination in the process of teaching graphic disciplines. The author proposed the integration of innovative approaches, such as the use of digital technologies and special training exercises, to improve the level of graphic training and emphasized the importance of developing spatial thinking for the successful mastery of graphic sciences and professional activities in technical fields. McLaughlin and Bailey [6] investigated the need to improve spatial thinking practices in geoscience. They found that students need more practical tasks and interactive approaches to help develop spatial analysis skills. Additionally, the authors emphasized that shortcomings in educational methods significantly limit a student's ability to effectively use spatial thinking in real-world geoscience problems.

Prihandika et al. [7] and Sonneveld et al. [8] investigated how different task orientations in geometry can promote spatial thinking in learning. The study highlighted how the use of constructive and narrative approaches can adapt teaching methods for different types of learners, which can improve their spatial skills. In addition, Prihadi et al. [9] investigated the impact of online geometry learning during the COVID-19 pandemic on a student's spatial reasoning ability. The study examined how changes in learning modalities and access to online resources affected the development of a student's spatial skills, particularly in the context of geometry. Thayaseelan et al. [10] conducted a study to improve methods for measuring spatial thinking among students. The authors focused on validating and improving methods for assessing cognitive skills, particularly in the context of disciplines such as geometry and other STEM sciences, thus providing more accurate and reliable tools to measure spatial ability.

The aforementioned studies did not sufficiently address the influence of cultural factors on the development of spatial representation in geometry in different educational systems. Moreover, the integration of technologies, specifically virtual reality, to improve a student's spatial thinking in geometry was not highlighted.

The study aims to identify a theoretical model for the development of a student's spatial representation in geometry.

The objectives of the study are as follows:

1. To describe the interpretation of the concept of "spatial representation" and its impact on solving geometric problems.
2. To test the peculiarities of methods for the development of spatial imagination in students, including checking the performance of practical tasks.
3. To form a theoretical model of the development of a student's spatial representation in geometry.

2. Materials and methods

Such techniques as dynamic geometric environments for the visualization and manipulation of geometric objects were used in the study, which contributed to the development of a student's spatial thinking [11]. The method of real-life tasks was used to adapt the educational material to practical needs, thus increasing a student's motivation and engagement [12]. Moreover, a manipulative model in geometry was used, which enabled interactive learning through physical and virtual tools, thus helping to deepen understanding of geometric concepts and their applications [13].

The study on the formation of a student's spatial thinking using geometric tasks was conducted in three stages. It was attended by 120 third-year students of the future mathematics teachers at Osh State University. The age of the participants ranged from 19 to 22 years, including 98 girls and 22 boys. The sample inclusion criteria are as follows: students had to be third-year students of the pedagogical specialty; participants had to have a basic knowledge of geometry obtained during the first two years of study; and voluntary consent to participate in the study. At the beginning of the study, the students were expected to possess basic knowledge of plane and solid geometry, including an understanding of geometric figures, an ability to perform simple constructions, and a familiarity with coordinate transformations, in accordance with the educational program for Mathematics teacher training.

The tests were created to incrementally evaluate and enhance spatial reasoning abilities. The initial phase involved exercises centered on recognizing and forming basic geometric shapes, including triangles and squares, in accordance with precise instructions. The second stage encompassed more intricate activities, including the construction of three-dimensional objects from two-dimensional pictures and the execution of mental rotations. The third level encompassed challenges that necessitated modeling three-dimensional objects from various planes and the resolution of practical difficulties utilizing geometry.

Pilot research was done with a smaller cohort of students (n = 20) prior to the main study to ascertain the reliability and validity of the tasks. Three experts in mathematics education evaluated the activities to verify their relevance and suitability for testing spatial thinking skills. The internal consistency reliability was evaluated using Cronbach's alpha, which resulted in a value of 0.85, signifying high reliability. Validity was confirmed via expert evaluations and by juxtaposing the task results with recognized metrics of spatial skills.

In the first stage, the students completed tasks aimed at identifying their initial level of spatial thinking. The exercises included the following:

1. Identification of the shapes of geometric figures;
2. Construction of simple objects such as triangles and squares following instructions; and
3. Identification of relationships between two-dimensional objects (for instance, area and perimeter).

The assessment was conducted using tests in which the students were to correctly name, construct, and compare geometric shapes.

The second stage consisted of more complex tasks that activated the skills of mental manipulation of geometric objects. The students performed the following tasks:

1. Solution of problems by constructing three-dimensional objects from two-dimensional images (e.g., cube scans);
2. A task for imaginary rotation: rotation of an object in space to determine its projections; and
3. Three-dimensional model reconstruction using the data provided.

The assessment included checking the accuracy of the constructed objects, the speed of the exercises, and the correctness of spatial transformations.

The third stage aimed to develop the potential of spatial thinking by working with the most complex tasks. The exercises included the following:

1. Modeling three-dimensional objects from multiple planes;
2. Performing operations with three-dimensional objects, such as folding, rotating, and zooming in/out; and
3. Solving practical problems using geometry in real life, such as calculating volumes or creating packaging scans.

The results of the exercises were assessed according to the following scale: high level – performance of 80% or more, sufficient level – performance of 60% to 79%, and average level – performance of 40% to 59%. The categorization into high, sufficient, and average levels was chosen based on established standards of educational assessment that differentiate between confident mastery of material, basic functional competence, and minimal acceptable understanding. Such a division allows a more precise differentiation of a student's spatial thinking abilities and aligns with commonly accepted benchmarks in pedagogical diagnostics, thus ensuring the comparability of the results with other educational research. This approach was used to comprehensively assess a student's ability to apply spatial thinking and geometric skills in practice. Next, a theoretical model of the development of a student's spatial representation in geometry was formed, and consisted of three main components: cognitive, analytical, and practical.

3. Results

As the sphere of human activity gradually expands, a person is required to have developed spatial thinking, which is considered to be the main component of intelligence. In addition, the success of the professional activities of many specialists depends not only on their professional training but also on their level of spatial thinking.

Spatial thinking is a type of cognitive activity that creates spatial images and performs operations on them in the process of solving various practical and theoretical problems. The use of methods for the development of spatial representation in geometry is a key element of the educational process, as they contribute to the formation of a student's ability to imagine, analyze, and construct geometric objects. Methods such as dynamic geometric environments, manipulative geometry, and real-life problems not only help students to understand the theoretical aspects of geometry, but also to develop the spatial thinking skills needed to solve practical problems. The interactive approach and the use of modern technologies enable students to visualize abstract concepts, which contribute to better learning and an increased interest in geometry, in particular, the method of using dynamic geometric environments (DGE). This technique is based on the use of interactive software such as GeoGebra, Cabri Geometry, or SketchUp. It allows students to explore the properties of geometric objects in real-time, change their parameters, and explore the relationships between two-dimensional and three-dimensional shapes. For example, with GeoGebra, students can build shapes, manipulate their elements, and visualize transformations. This helps to develop not only spatial reasoning but also analytical skills, as tasks in such environments often involve more complex mathematical concepts such as rotation, symmetry, or the scaling of shapes. DGEs increase the interactivity of learning by allowing students to explore geometric phenomena on their own.

Manipulative geometry focuses on the use of physical objects, such as paper models, geometric constructors, or 3D puzzles, to develop an understanding of three-dimensional objects [14,15]. For instance, making a paper model of a pyramid or prism helps students better understand the concepts of surface area and volume, while working with 3D construction sets develops skills in understanding the structure of three-dimensional objects. The methodology stimulates the visualization of spatial relationships through physical interactions, which is especially effective for students with an initial level of spatial thinking. The use of such tools in the learning process makes it more practical and engaging. The methodology of real-life tasks reflects everyday situations, such as interior design, furniture layout planning, or calculating the area for construction projects, aimed at developing applied spatial thinking [16]. Such tasks not only help to reinforce basic geometry skills but also demonstrate the practical value of geometric knowledge. For instance, students can model the three-dimensional space of a room using specialized software or by creating building plans, applying geometric laws in practice. This approach promotes critical thinking and creative problem-solving and significantly increases motivation to learn geometry.

The level of a student's spatial thinking is determined by solving specially designed tasks, and this intelligence is formed by solving such tasks. The results of completing the tasks according to the first stage in the quantitative ratio of participants in three aspects of assessment—correct definition of geometric shapes, construction of objects, and the relationship between shapes—are shown in Figure 1.

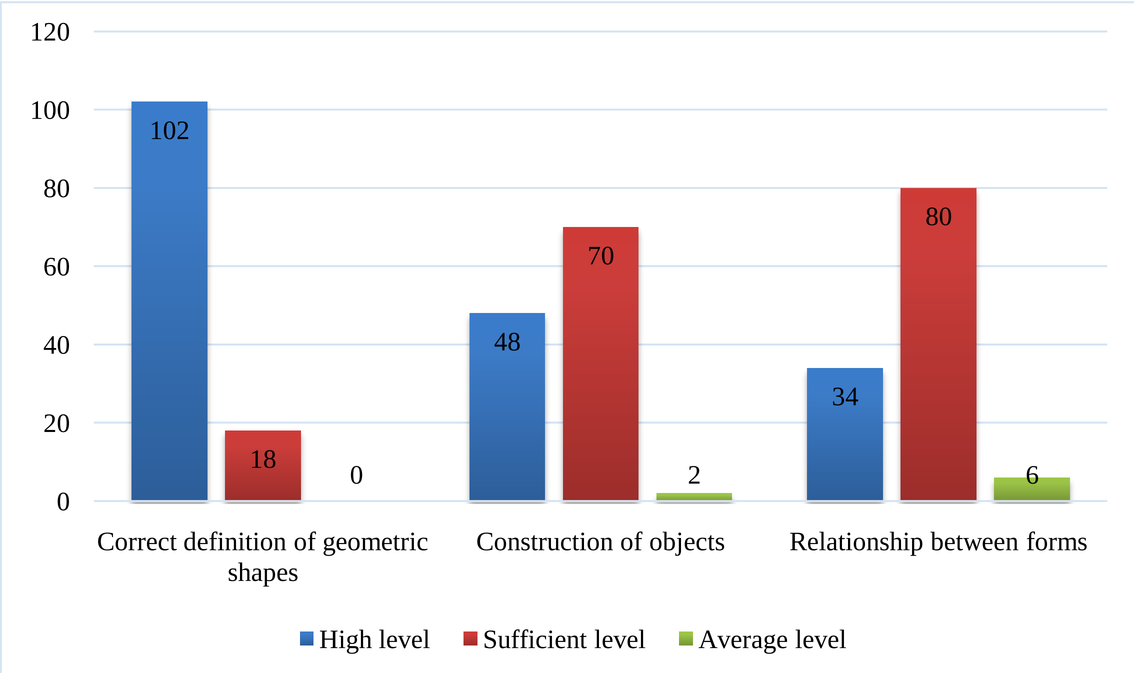

Figure 1. Results of the assessment of the basic level of spatial thinking according to the first stage.

In the aspect of correctly identifying geometric shapes, the majority of participants achieved a high level, thus demonstrating a high accuracy in identifying shapes based on two-dimensional images. Another 15% of participants performed at a sufficient level, which indicates a good but not perfect understanding of geometric shapes. Notably, none of the participants demonstrated an intermediate level. However, even with these results, some of the participants may benefit from additional exercises to improve their ability to quickly and accurately recognize complex geometric structures. The results of the next aspect were less optimistic. Only 40% of students achieved a high level, while the majority, 60%, performed at a sufficient level. Only two students fell into the intermediate level category; however, the proportion of students with a high level was much lower than in the first aspect. This indicates some difficulties in completing tasks related to the construction of objects. The difficulty is probably due to the need to precisely follow instructions and to the spatial representation when moving from a two-dimensional to a three-dimensional representation. Further training could focus on developing practical skills in creating objects in real space or through visualization. The lowest results were observed in the relationship between shapes. Only 30% of students demonstrated a high level of proficiency, while 70% completed the task at a sufficient level. The results indicate that students had significant difficulties in establishing relationships between two-dimensional and three-dimensional shapes, particularly in tasks involving imaginary rotation or reconstruction. This aspect is one of the most critical since the skills of analyzing relationships between shapes are necessary to solve many practical problems in real life.

Nevertheless, the students had difficulties with the imaginary rotation of objects in space, which manifested in errors in determining the changes in the appearance of three-dimensional models after rotation. The study participants often spent more time completing tasks, thus demonstrating slowness in solving problems with spatial transformations. When constructing three-dimensional objects from two-dimensional images, numerous errors were observed due to the complexity of the final result. The difficulty of analyzing the relationships between two-dimensional and three-dimensional shapes was a common problem, especially for students with insufficient practice. In tasks involving the analysis of complex shapes, students paid excessive attention to details, which slowed down the overall process of completion. Intermediate students were unable to effectively use the instructions for building models, which affected their performance.

Thus, the students did well in tasks that required a basic understanding of geometric shapes. However, their skills in constructing objects and analyzing the relationships between them need to be improved. This is especially true for tasks that require deeper spatial thinking, such as the mental manipulation of objects or their reconstruction from two-dimensional images.

Then, the assessment was carried out again in three aspects, but in a different way. Each of these aspects reflected certain aspects of a student's spatial thinking and was key to identifying the strengths and weaknesses of their skills (Figure 2).

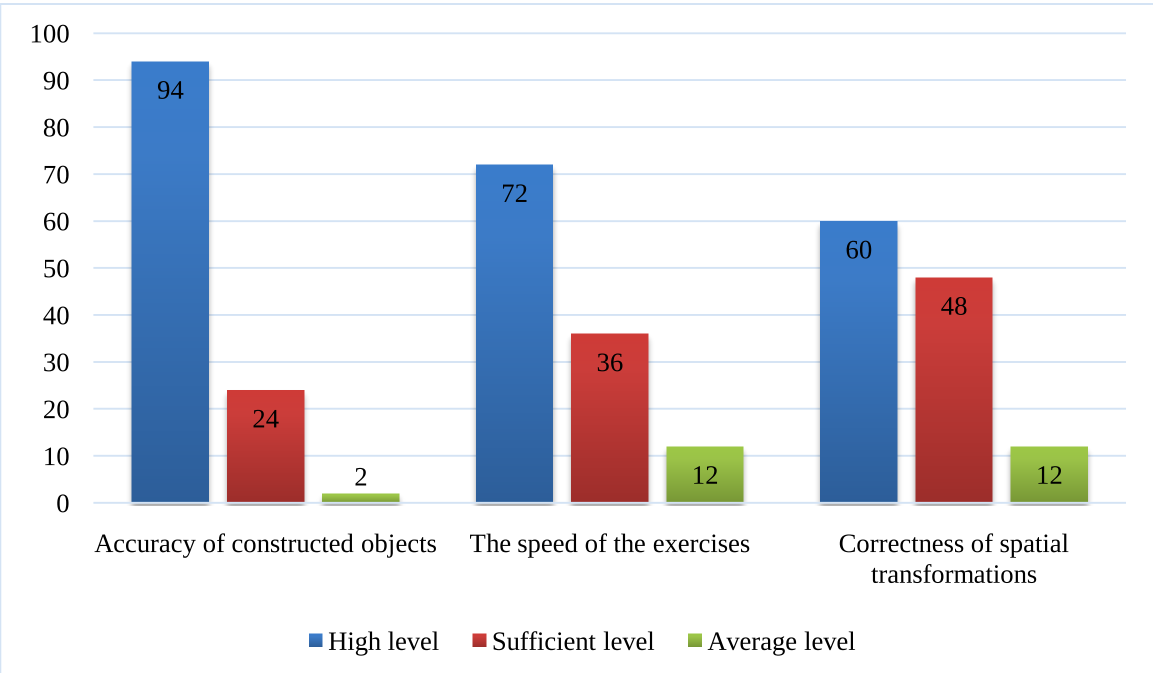

Figure 2. Results of the assessment of the second stage of students' spatial thinking by grade levels.

The results showed that most students completed tasks related to the construction of three-dimensional objects. In particular, 80% of the students demonstrated a high level of accuracy, thereby confidently completing tasks with complex instructions and using data from two-dimensional scans. Another 20% of students completed the task at a sufficient level, thereby making minor errors, which were mostly related to inaccurate instructions or mistakes in the construction of parts. The average level in this aspect was not found, which indicates that students are generally ready to work with the basic components of spatial thinking. This aspect was the most successful for students, thus indicating their ability to carefully follow instructions and use basic geometry knowledge. This result can be attributed to the fact that the students probably already had some experience with simple geometric objects at school or in previous stages of their education.

The speed of completing tasks was less stable than the accuracy. More than half of the students (60%) performed at a high level (i.e., completing tasks quickly and without errors). Another 30% of students performed at a sufficient level (i.e., with some delays due to the need for additional analyses or calculations). At the same time, 10% of higher education students were at the intermediate level, thus demonstrating difficulties in making quick decisions and the need for additional time to complete the exercises. These results point to the importance of developing automated skills and spatial thinking strategies that could speed up task completion. The speed of the exercises showed the need for further development. Although most students completed the tasks on time, some participants needed more time, which indicated that their skills were not sufficiently automated. Moreover, it is important to note that the speed of performance may depend on the level of stress or unfamiliarity with the complexity of the tasks [17].

The aspect of correct spatial transformations proved to be the most difficult for students. Exactly half of the study participants reached a high level, thereby confidently performing tasks related to the imaginary rotation of objects and determining their projections. Another 40% of students were at a sufficient level, thus making some inaccuracies in calculations or interpretations, but generally completing the task successfully. However, 10% of students showed an average level, which indicated significant difficulties with spatial transformations. This was the most challenging aspect, as these tasks required not only a basic level of spatial thinking but also the ability to mentally manipulate objects, which is a higher level of cognitive activity. The complexity of these tasks suggests that the development of such skills requires a special approach, including the use of virtual reality technologies or software for visualizing three-dimensional objects.

The students were well-versed in basic geometric shapes but needed more practice in building models and making connections between different aspects. To improve the results, tasks that develop analytical thinking and mental manipulation of objects should be prioritized. To develop the accuracy of constructed objects, it is necessary to introduce tasks with more complex shapes and objects that require the integration of various knowledge and skills [18]. As for the speed of the exercises, it is recommended to conduct training sessions aimed at developing quick decision-making, for example, through the use of games or simulations that require a quick reaction. The development of spatial transformation skills requires special approaches, such as the introduction of courses using CAD software (e.g., AutoCAD, Blender) that allow students to work with three-dimensional models.

The results of the third stage of the study showed that students generally demonstrate a sufficient level of spatial thinking, but some aspects need to be improved (Figure 3).

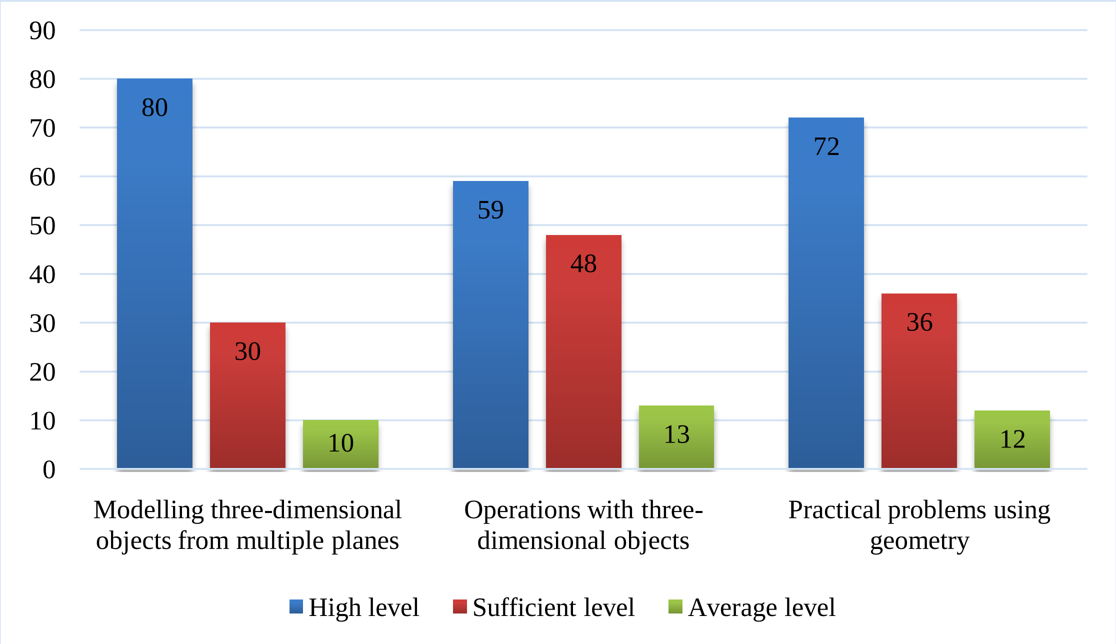

Figure 3. Assessment results of the third stage of students' spatial thinking by grade levels.

In the task of modeling three-dimensional objects from several planes, 70% of the participants achieved a high level, thus demonstrating accuracy and the ability to combine different elements. The remaining 30% showed a sufficient level, while the average level of performance was not recorded, thus indicating an overall positive result. The operations with three-dimensional objects proved to be more difficult: only 50% of participants achieved a high level, while 40% performed at a sufficient level. Another 10% of students showed an average level, which indicates some difficulties in manipulating complex shapes. The practical tasks involving geometry also demonstrated a distribution of results: 60% of the participants performed the task at a high level, 30% at a sufficient level, and 10% at an intermediate level. The students encountered difficulties such as the complexity of integrating several geometric shapes into a single model, difficulties with the spatial rotation of objects, and limited experience with abstract problems. Some participants required more time to solve problems and lacked confidence in applying theoretical knowledge. To improve the results, it is recommended to increase the number of practical exercises aimed at developing spatial thinking skills, in particular, through the use of three-dimensional modeling software. Additionally, it is worth introducing a gradual increase in the complexity of tasks, which will allow students to improve their analytical skills and spatial imagination.

To verify the significance of changes in a student's spatial thinking development between the stages, a non-parametric Wilcoxon signed-rank test was performed. The evaluation was based on a three-point scale, where a high level of task completion was assigned 3 points, sufficient level – 2 points, and average level – 1 point. The results demonstrated a statistically significant improvement in the students' performance from Stage 1 to Stage 2 ($p < 0.01$) and from Stage 2 to Stage 3 ($p < 0.01$). These findings confirm that the observed improvements in spatial thinking were not random and can be attributed to the educational interventions introduced.

The general analysis of the results of practical tasks was used to identify both strengths and difficulties in the formation of a student's spatial thinking, which became the basis for the development of a theoretical model of its development. This model aims to develop the cognitive, analytical, and practical skills necessary for successful work with geometric shapes and three-dimensional objects. Its main goal is to provide a harmonious combination of theoretical knowledge, analytical thinking, and practical actions that allow students to solve complex problems in three-dimensional space (Figure 4).

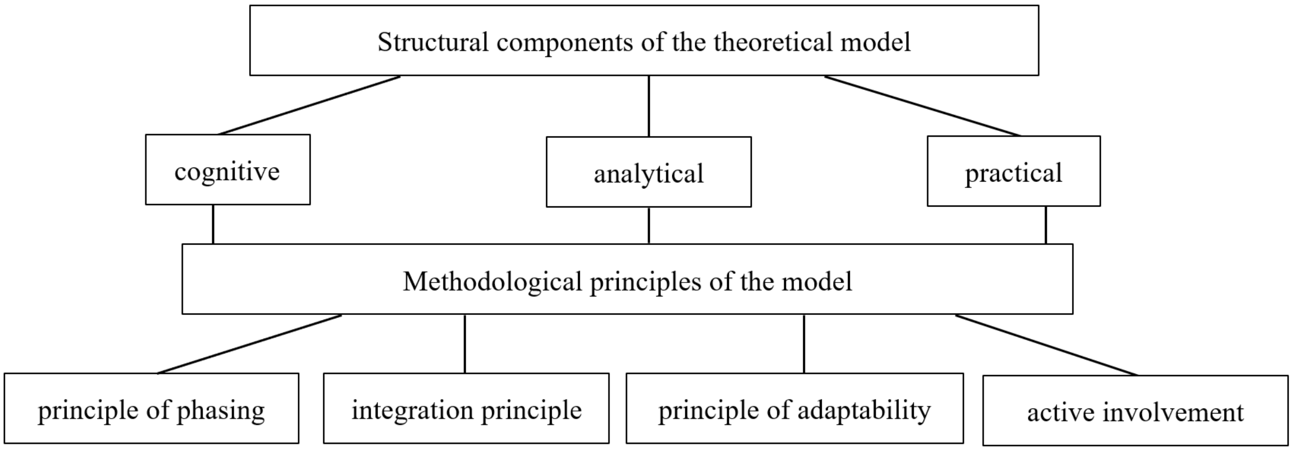

Figure 4. Theoretical model of development of spatial thinking formation in higher education students.

The model consists of three main components: cognitive, analytical, and practical. The cognitive component addresses the basic theoretical foundations of geometry, mathematical principles, and laws of three-dimensional space. The analytical component includes the development of the ability to analyze shapes, structures, and relationships between objects using logical thinking, abstraction, and spatial modeling. The practical component ensures the integration of the acquired knowledge into the process of building three-dimensional models and performing practical tasks using digital tools such as modeling software (for example, Autodesk or Blender).

The model is based on several methodological principles. The principle of phasing involves the gradual complication of tasks, which helps students learn new material without overload. The principle of integration focuses on an interdisciplinary approach that allows one to combine knowledge from different fields, such as geometry, architecture, or physics. The principle of active engagement aims to provide students with the opportunity to explore, experiment, and solve problems on their own. Finally, the principle of adaptability addresses the level of preparation of each student, thus offering tasks that match their capabilities.

The stages of model implementation include three main phases: theoretical, analytical, and practical. In the first stage, students study the theoretical foundations of three-dimensional space and get acquainted with modeling tools. In the second stage, they complete tasks aimed at analyzing two-dimensional shapes and their relationship to three-dimensional objects. The practical stage involves completing tasks to build three-dimensional models and working with digital tools for modeling and design work.

The expected results of the model implementation are an increased modeling accuracy, the development of analytical thinking to establish connections between objects, and the effective use of modern tools to solve practical problems. To achieve these outcomes, it is recommended to use modern technologies such as 3D printers and graphics tablets, conduct spatial modeling training, and introduce gamification and group projects to increase student motivation.

The theoretical model provides a systematic approach to the formation of spatial representation, thus helping to prepare students to solve complex professional problems in the real world. However, this requires specific stages of spatial thinking formation, for example, in the areas of development of the topic "Cartesian coordinate system in space" in the geometry lesson of a future mathematics teacher (Table 1).

Table 1. Stages of formation of spatial thinking in development zones.

Development zones	Spatial thinking stages	Exercises performed using imagination to develop spatial thinking
Zone of actual development	Actualization of imagination.	Task-1. The point A (2, 3) is given in the rectangular coordinate system. 1. Determine the point that is symmetrical to: a) (0x) axis; b) (0y) axis; 2. Determine the point that appears when moving parallel to the direction of: a) (1,0); b) (1,1) vectors; 3. What point does point A move to when turning by: a) $90°$; b) $-90°$ in the coordinate system about the origin (0,0)?
A zone of near-term development	Activate imagination	Task-2. In a rectangular coordinate system, there is a square with vertices A (0,0), B (1,0), C (1,1), D (0,1). 1. Determine the coordinates of the vertices of a square symmetrical about axis: a) (Ox); b) (Oy); 2. The square is rotated by $90°$ around the center at the origin (0,0). Name the coordinates of the vertices of the resulting square.
Zone of creative self-development	Potentialisation of imagination	Assignment. Create spatial analogues of the above problems and propose solutions.

Note: The concept of "developmental zones" in this study is based on the staged development of cognitive and creative abilities in the educational process. In the zone of actual development, students consolidate existing skills; in the zone of near-term development, they apply skills in more complex contexts, activating imagination; in the zone of creative self-development, they independently generate and solve new spatial problems. These stages are interrelated and support the systematic growth of spatial imagination competencies.

To achieve the goal of the theoretical model, it is necessary to identify three key developmental zones: the zone of actual development, the zone of immediate development, and the zone of creative self-development. In the actual development zone, the students update their basic knowledge and understanding of spatial objects by performing elementary tasks in the Cartesian coordinate system. For instance, they identify symmetrical points and find the coordinates of points after rotation or parallel transfer, thus taking the first step in developing systematic spatial thinking. The zone of immediate development is focused on activating the imagination and developing the ability to perform complex spatial transformations. Tasks in this zone include working with geometric shapes, such as squares, triangles, or other polygons, in the coordinate plane. The students determine the new coordinates of the vertices of the shapes after symmetry, the rotation of axes, or other manipulations. This stage promotes a deeper understanding of the connections between mathematical objects and their spatial properties. In the area of creative self-development, the students move on to generating ideas and independently creating tasks. The students not only complete the proposed tasks, but also create similar or more complex problems, develop algorithms to solve them, and propose solutions. This approach develops critical thinking, creativity, and the ability to independently analyze spatial problems.

The integration of digital technologies is an integral part of this process. Modern tools such as GeoGebra, WolframAlpha, or other modeling and analysis software allow students to not only visualize spatial objects but also to carry out their transformations in an interactive environment. The use of virtual laboratories makes it possible to perform tasks without the risk of errors that can occur in real life and contributes to a deeper understanding of theoretical aspects. Augmented and virtual realities provide a unique opportunity to study objects in a three-dimensional space, which significantly increases the effectiveness of learning [19]. The practical orientation of the tasks includes both individual and group work. The discussion of tasks in small groups contributes to a better understanding of the material and the development of communication and collaboration skills. In addition, students can participate in the creation of problems that require the integration of knowledge from different mathematical sections, as well as in the process of solving them, which stimulates their activity and creativity.

To assess progress in the development of spatial thinking, it is necessary to develop clear criteria, which may include test tasks, the observation of practical work, an analysis of self-created tasks, and the evaluation of the effectiveness of proposed solutions. This can be used to track the dynamics of a student's development and adjust the learning process in a timely manner. Following the above information, the proposed model of spatial thinking development ensures the high-quality training for future mathematics teachers to solve problems of increased complexity. The use of interactive technologies, multi-level tasks, and a systematic approach makes this process effective and adapted to the modern requirements of the educational environment.

4. Discussions

The study determined that the development of spatial imagination is a multifaceted process that requires integration into various educational disciplines. The use of technologies, creative tasks, and specialized methods create conditions to improve these skills in students of different specialties. The results of this study have shown that the integration of interactive tools significantly increases the level of a student's spatial imagination. According to the findings of Tursunov [20], the use of GeoGebra not only activates a student's interest but also contributes to solving complex geometric problems due to the ability to visualize three-dimensional objects. Based on this, this study recommends introducing such tools into the educational process to develop a student's spatial thinking. Moreover, Berdibekova et al. [21] emphasized the importance of developing teaching guidelines that promote the development of spatial imagination. Their task-based approaches demonstrate the possibility of integrating these skills into science disciplines. Another important aspect is the development of spatial structural skills through STEM education. This allows one to not only improve their understanding of physical concepts but also to develop a student's ability to think logically and critically. The integration of such skills into the curriculum ensures the all-round development of the students, thus increasing their ability to solve complex problems. For instance, Sofiboyeva [22] noted that STEM education develops the ability of higher education students to abstractly think and spatially visualize, which is critical in teaching geometry. The findings of this paper confirm this. Similar results were found by Qodirovich et al. [23], who determined that computer graphics improved the understanding of three-dimensional objects through their realistic representation. The results of this study pointed to a similar trend: the use of interactive 3D models made it much easier for students to understand spatial relationships and structures.

In addition to activating spatial imagination, it is important to consider the issue of potentialization. Pavlovičová et al. [24] noted that the development of geometric thinking depends not only on technology but also on pedagogical strategies. This study determined that interactive technologies are more effective if teachers use differentiated teaching methods. This approach allows students to keep their interest and adapt learning to their individual needs. Additionally, it promotes a deeper understanding of the material and the development of critical thinking. As a result, the students can better grasp geometric concepts, which helps to develop their spatial understanding. At the same time, it is necessary to constantly improve the pedagogical methods to ensure a maximum effect from the use of modern technologies. Additionally, Sagitova [25] emphasized the importance of a phased formation of spatial imagination, starting with basic concepts. The present study confirmed these findings, as higher education students who gradually mastered visualization skills made more progress than those who started their studies with complex tasks. This suggests the need to structure the learning process in geometry.

Another important aspect identified in this study was the overcoming of epistemological obstacles that hinder the understanding of three-dimensional geometry. Sudirman et al. [26] described the students' difficulties in representing and structuring spatial objects. These results confirmed this, especially among students who had an average level of spatial imagination. The use of interactive technologies, such as 3D modeling, helps to reduce these difficulties. According to a study by Rohmah et al. [27], a student's spatial intelligence develops much faster if teachers include practical tasks that simulate real-life situations in their teaching. In the present study, a similar approach proved to be effective, as the use of real-life examples (e.g., architectural objects) stimulated the development of imagination in higher education students. These findings are consistent with the study by Urazbaev [28], who focused on the problems of determining the level of a student's spatial imagination in the study of geometry and engineering graphics. The author highlighted the need for special assessment methods that would help teachers adapt teaching materials to the level of students.

The results of this study confirmed that the use of creative tasks and technologies helps to improve spatial thinking, especially in the context of solving geometry problems and working with graphic materials. This correlates with the findings of other researchers who emphasize the importance of integrating these skills across disciplines; in particular, Choriyevich [29] studied the role of composition in the formation of spatial imagination and confirmed that creative tasks can significantly increase the level of spatial thinking. The importance of creative approaches was also emphasized by Honzíková et al. [30], who suggested using SketchUp to assess a student's creativity and spatial imagination. The use of digital tools has become an effective means of developing these skills.

This study confirmed that the introduction of innovative methods, such as virtual reality and special graphic programs, can significantly develop a student's spatial imagination. This is consistent with the study by Ishikawa and Newcombe [31], who emphasized the importance of spatial skills in learning, and the findings of Li [32], who developed strategies to use virtual reality to develop spatial imagination in artistic disciplines. The study emphasized the importance of practical tasks and specialized methods in teaching graphic arts. Additionally, Mirzaliyeva [33] and Odiljanovna [34] emphasized that creative tasks and visual materials effectively contribute to the formation of spatial imagination during drawing. Khalimov [35] confirms this position, arguing that elements of spatial imagination should be integrated into graphic arts to achieve better results. Moreover, the results of the study showed that solving geometry problems is a primary component of improving a student's learning process. This is confirmed by the works of Izzati [36], who investigated the relationship between spatial skills and mathematical abilities. The study showed that the development of spatial thinking significantly affects the ability to solve complex mathematical problems. Izbosarov and Jamilova [37] and Matieva and Borboeva [38] noted that learning the elements of geometry contributes to a deeper understanding of mathematical concepts. They emphasized that geometric problems help students visualize abstract mathematical concepts. In addition, Yurmalia and Hasanah [39] investigated gender differences in a student's spatial thinking. Their study revealed statistically significant differences in approaches to solving geometric problems between men and women. These results indicate that gender differences can affect the effectiveness of geometry learning and the development of spatial skills. In their theory of geometric representations for spatial navigation, Zeng et al. [40] examined how different types of space representation can affect spatial orientation ability. Their research, as well as the present study, confirms the importance of visual and cognitive skills in the development of spatial imagination, which is the basis for successful learning in engineering and graphic arts disciplines [41].

A comparison of the results of this study with the findings of other authors demonstrates a significant level of agreement on the key aspects of a student's spatial imagination development. All the authors emphasized the importance of interactive technologies, a structured approach to teaching, and a consideration of a student's cognitive characteristics. The uniqueness of this study lies in the comprehensive approach to actualization, activation, and potentialization of spatial imagination, which was used to address these aspects as an integrated system.

5. Conclusions

The formation of a student's spatial thinking with the help of geometric tasks is a complex and multi-stage process that includes the actualization, activation, and potentialization of spatial imagination. The study confirmed that the development of these abilities occurs through the implementation of complex geometric tasks that do not solely train spatial imagination. The proposed model of spatial thinking formation aimed to create conditions for a student's development in three areas: current development, near-term development, and creative self-development.

The first stage of the study aimed to assess the initial level of students' spatial thinking. The results showed that most participants have basic skills in working with geometric shapes. However, gaps were identified in analyzing relationships between objects and building three-dimensional models. This stage demonstrated the need to activate spatial thinking through practical tasks aimed at developing analytical thinking. This includes exercises on visualizing space, recognizing shapes, and creating simple models. The second stage of the study focused on activating spatial representation. By completing more complex tasks that required analyzing the relationships between objects, the students showed improvements in working with three-dimensional models. Approximately 60% of students completed the modeling tasks, thus demonstrating a sufficient level. At the same time, 40% of the participants needed additional instructions and support. This stage confirmed the effectiveness of integrating theoretical knowledge with practical exercises, which helps to form connections between abstract concepts and real objects. The third stage of the study was key to assessing the potential of spatial thinking. The tasks included modeling three-dimensional objects from several planes, performing operations with geometric shapes, and solving practical problems. In modeling, most students achieved a high level of accuracy, thus demonstrating the ability to synthesize information and effectively use theoretical knowledge.

The developed theoretical model of spatial thinking formation can be used as a basis for building the educational process. The model includes cognitive, analytical, and practical components that ensure the gradual development of spatial skills. The application of the principles of phasing, integration, active involvement, and adaptability contributed to the effective formation of students' spatial imagination. The tasks proposed within the model addressed the individual capabilities of the participants and ensured a combination of theoretical knowledge and practical activities. The developed model for the formation of spatial thinking provides a high level of training for future mathematics teachers to solve problems of increased complexity. The integration of interactive technologies, multi-level tasks, and a systematic approach makes this process effective and in line with modern educational requirements. Nevertheless, it is recommended to improve teaching materials by focusing on the integration of modern technologies, such as 3D modeling software. This will not only improve the quality of the educational process but also provide students with the necessary skills to solve complex professional problems in the future.

The study results indicated that the proportion of students that achieved a high level of spatial thinking consistently increased across the three stages: from 30–40% at the first stage to 60–80% at the final stage. The average level of spatial thinking, which was calculated based on a three-point evaluation scale, significantly improved over time. A statistical analysis using the Wilcoxon signed-rank test confirmed the significance of these improvements ($p < 0.01$), thus indicating the effectiveness of the implemented teaching methods and the proposed theoretical model for enhancing spatial imagination.

The study was conducted among students of one specialty, which may not account for the peculiarities of the development of spatial thinking of students of other specialties. Prospects for further research are the development of digital learning platforms and programs for self-training in spatial thinking, which will be beneficial for students and teachers.

Author contributions

Gulnisa Borboeva: Conceptualization, Methodology, Writing – Review & Editing; Gulbadan Matieva: Writing – Original Draft, Formal analysis, Investigation; Venera Isakova: Resources, Data Curation, Writing – Original Draft; Cholpon Mustapakulova: Writing – Review & Editing, Visualization, Supervision; Gulshana Omurzakova: Project administration, Validation, Software.

Use of Generative-AI tools declaration

The authors declare that generative AI or AI-assisted technologies were not used in any way to prepare, write, or complete this manuscript.

Conflict of interest

The authors declare that there is no conflict of interest in this manuscript.

Ethics declaration

All procedures performed in the study were in accordance with the ethical standards of the institutional research committee and with the 1964 Helsinki Declaration and its later amendments.

References

1. Gagnier, K.M., Holochwost, S.J. and Fisher, K.R., Spatial thinking in science, technology, engineering, and mathematics: Elementary teachers' beliefs, perceptions, and self-efficacy. *Journal of Research in Science Teaching,* 2021, 59(1): 95–126. https://doi.org/10.1002/tea.21722
2. Lee, C., Documenting children's spatial reasoning through art: A case study on play-based STEAM education. *Sustainability,* 2023, 15(19): 14051. https://doi.org/10.3390/su151914051
3. Goyibnazarova, G., Methods of development spatial representation of students by teaching solving geometric problems. *Tuijin Jishu/Journal of Propulsion Technology,* 2024, 45(4): 623–629. https://www.propulsiontechjournal.com/index.php/journal/article/view/8118
4. Hickman, J., Spatial thinking and GIS: Developing and assessing student competencies. *International Research in Geographical and Environmental Education,* 2023, 32(2): 140–158. https://doi.org/10.1080/10382046.2022.2138172
5. Kahharov, A.A., Intensive methods of developing students' spatial imagination in the teaching of graphic sciences. *Annals of R.S.C.B.,* 2021, 25(4): 11885–11892. http://annalsofrscb.ro/index.php/journal/article/view/4042/3273
6. McLaughlin, J.A. and Bailey, J.M., Students need more practice with spatial thinking in geoscience education: A systematic review of the literature. *Studies in Science Education,* 2022, 59(2): 147–204. https://doi.org/10.1080/03057267.2022.2029305
7. Prihandika, Y.A.P., Triyanto and Saputro, D.R.S., Analysis of students' representation skills on geometry material viewed from the spatial intelligence level. *AIP Conference Proceedings,* 2022, 2566(1): 020019. https://doi.org/10.1063/5.0114186
8. Sonneveld, L., Klapwijk, R.M. and Stappers, P.J., Constructing and storytelling: accommodating different play orientations in learning spatial thinking. *Frontiers in Education,* 2024, 9: 1307951. https://doi.org/10.3389/feduc.2024.1307951
9. Prihadi, S., Sajidan, S., Siswandari, S. and Sugiyanto, S., Students' Spatial Thinking Ability on Online Geography Learning During the COVID-19 Pandemic. In *Proceedings of the 5th International Conference on Learning Innovation and Quality Education*, 2022, 1–5. https://doi.org/10.1145/3516875.3516903
10. Thayaseelan, K., Zhai, Y., Li, S. and Liu, X., Revalidating a measurement instrument of spatial thinking ability for junior and high school students. *Disciplinary and Interdisciplinary Science Education Research*, 2024, 6: 3. https://doi.org/10.1186/s43031-024-00033-3
11. Sträßer, R., Research on dynamic geometry software (DGS) – An introduction. *ZDM: International Journal on Mathematics Education,* 2002, 34(3): 65. https://doi.org/10.1007/BF02655707
12. Stoll, T., Stockinger, A. and Wartzack, S., Geometric manipulation method for evaluation of aesthetic quality in early design phases. In: *S.J. Culley, B.J. Hicks, T.C. McAloone, T.J. Howard, A. Dong (eds), Proceedings of the 18th International Conference on Engineering Design*, 2011, 153–164. https://www.designsociety.org/publication/30793/

13. Math manipulatives and tools for teaching geometry. 2024. Available from: https://saddleupfor2ndgrade.com/math-manipulatives-and-tools-for-teaching-geometry/#google_vignette
14. Kyrychok, T., Korotenko, O., Baglai, V. and Kyrychok, A., Investigation of quality recognition of banknotes marks for visually impaired people. *Proceedings of SPIE - The International Society for Optical Engineering*, 2024, 12938: 1293817. https://doi.org/10.1117/12.3012707
15. Kyrychok, T., Korotenko, O., Talimonov, Y. and Kyrychok, A., Improving a method for determining the level of wear of the mark for people with visual impairments on Ukrainian hryvnia banknotes. *Eastern-European Journal of Enterprise Technologies*, 2023, 5(1(125)): 92–103. https://doi.org/10.15587/1729-4061.2023.287746
16. Cherniha, R. and Pliukhin, O., New conditional symmetries and exact solutions of reaction-diffusion-convection equations with exponential nonlinearities. *Journal of Mathematical Analysis and Applications*, 2013, 403(1): 23–37. https://doi.org/10.1016/j.jmaa.2013.02.010
17. Oliinyk, O., Zholdoshbaev, D., Koshonova, S., Kravtsov, Y. and Bocheliuk, V., Psychology of stress and adaptation during complex crises: Practical aspects of assisting people in difficult circumstances. *European Journal of Trauma and Dissociation*, 2025, 9(2): 100541. https://doi.org/10.1016/j.ejtd.2025.100541
18. Cherniha, R., Serov, M. and Rassokha, I., Lie symmetries and form-preserving transformations of reaction-diffusion-convection equations. *Journal of Mathematical Analysis and Applications*, 2008, 342(2): 1363–1379. https://doi.org/10.1016/j.jmaa.2008.01.011
19. Dahan, E., Aviv, I. and Diskin, T., Aerial Imagery Redefined: Next-Generation Approach to Object Classification. *Information*, 2025, 16(2): 134. https://doi.org/10.3390/info16020134
20. Tursunov, I.E., Development of spatial imagination of students using GeoGebra program in geometry lessons. *Eurasian Journal of Physics, Chemistry and Mathematics*, 2024, 28: 54–56. https://geniusjournals.org/index.php/ejpcm/article/view/5872/4909
21. Berdibekova, S., Aldashov, M., Asanbekova, D., Ismailova, G. and Attokurov, A., Development of methodological recommendations for teaching physics based on the development of spatial imagination. *Scientific Herald of Uzhhorod University*, 2024, 55: 2878–2885. https://doi.org/10.54919/physics/55.2024.287hd7
22. Sofiboyeva, G.M., The content of developing students' spatial imaginations through STEM education. *International Journal of Scientific Trends*, 2024, 3(2): 33–35. https://scientifictrends.org/index.php/ijst/article/view/180
23. Qodirovich, M.D., Jalolovich, Y.N., Samadovich, A.S. and Abdurazzakovna, R.N., Methods of developing students' spatial imagination using computer graphics in the teaching of drawing. *Journal of Contemporary Issues in Business and Government*, 2021, 27(1): 1522–1528. https://cibgp.com/au/index.php/1323-6903/article/view/650
24. Pavlovičová, G., Bocková, V. and Lassová, K., Spatial ability and geometric thinking of the students of teacher training for primary education. *TEM Journal*, 2022, 11(1): 388–395. https://doi.org/10.18421/TEM111-49
25. Sagitova, S., Formation of spatial imaginations in students. *Alatoo Academic Studies*, 2022, 22(2): 110–115. https://doi.org/10.17015/aas.2022.222.14
26. Sudirman, S., Kusumah, Y.S., Martadiputra, B.A.P. and Runisah, R., Epistemological obstacle in 3D geometry thinking: Representation, spatial structuring, and measurement. *Pegem Journal of Education and Instruction*, 2023, 13(4): 292–301. https://doi.org/10.47750/pegegog.13.04.34

27. Rohmah, M., Budiyono and Indriati, D., Hass's theory: How is the Students' Spatial Intelligence in Solving Problems? In *Proceedings of the International Conference of Mathematics and Mathematics Education*, 2021, 169–175. https://doi.org/10.2991/assehr.k.211122.024
28. Urazbaev, B.T., Problems of determining the level of spatial imagination of students in the process of teaching the science of drawing geometry and engineering graphics. *Innovative Technologica: Methodical Research Journal,* 2023, 4(11): 23–26. https://it.academiascience.org/index.php/it/article/view/603
29. Choriyevich, J.S., The role of composition in increasing the spatial imagination of students and students. *Central Asian Journal of Arts and Design,* 2021, 2(11): 15–17. https://cajad.centralasianstudies.org/index.php/CAJAD/article/view/69
30. Honzíková, J., Fadrhonc, J., and Krotký, J., Testing the level of creativity and spatial imagination in the SketchUp program using a modified urban test of creative thinking. *Digital,* 2024, 4(3): 804–820. https://doi.org/10.3390/digital4030040
31. Ishikawa, T. and Newcombe, N.S., Why spatial is special in education, learning, and everyday activities. *Cognitive Research: Principles and Implications,* 2021, 6(1): 20. https://doi.org/10.1186/s41235-021-00274-5
32. Li, J., Strategies for cultivating students' spatial imagination in art courses of colleges and universities assisted by virtual reality technology. *Applied Mathematics and Nonlinear Sciences*, 2024, 9(1): 1–15. https://doi.org/10.2478/amns-2024-3380
33. Mirzaliyeva, S.Z.Q., The importance of creative tasks in improving the spatial imagination of students in drawing. *Archive of Conferences,* 2022, 65–70. Available from: https://conferencepublication.com/index.php/aoc/article/view/2183
34. Odiljanovna, Y.N., Teaching students to think creatively (in drawing classes). *Emergent: Journal of Educational Discoveries and Lifelong Learning,* 2024, 3(1): 7. https://doi.org/10.47134/emergent.v3i1.40
35. Khalimov, M.K., Elements of student space imagination in the teaching of graphic sciences and methods of using it. *Current Research Journal of Pedagogics*, 2022, 3(2): 103–116. https://doi.org/10.37547/pedagogics-crjp-03-02-19
36. Izzati, N., Influence of spatial ability on students' mathematical representation ability in the spatial geometry course. *Journal of General Education and Humanities,* 2024, 3(4): 433–442. https://doi.org/10.58421/gehu.v3i4.332
37. Izbosarov, I.U. and Jamilova, D.T., Developing spatial imagination by teaching students the elements of geometry. *International Journal on Integrated Education,* 2021, 4(12): 216–218.
38. Matieva, G. and Borboeva, G., The Significance of Interpretation in the Development of Spatial Thinking. In *Materials of the VII World Congress of Mathematicians of the Turkic World (TWMS Congress-2023) (Part II),* Turkestan: Kh.A. Yassawi International Kazakh-Turkish University, 2023, 342–347.
39. Yurmalia, D. and Hasanah, A., Student spatial visual in geometry: The case of gender differences. *Journal of Physics: Conference Series,* 2021, 1806: 012083. https://doi.org/10.1088/1742-6596/1806/1/012083
40. Zeng, T., Si, B. and Feng, J., A theory of geometry representations for spatial navigation. *Progress in Neurobiology,* 2022, 211: 102228. https://doi.org/10.1016/j.pneurobio.2022.102228
41. Habovda, O., Latest technologies in production, automated design systems, computer modelling, innovative teaching methods, graphic disciplines. *Scientific Bulletin of Mukachevo State University. Series "Pedagogy and Psychology"*, 2022, 8(4): 66–72. https://doi.org/10.52534/msu-pp.8(4).2022.66-72

Author's biography

Gulnisa Borboeva is a PhD, Associate Professor at the Institute of Mathematics, Physics, Technics and Information Technologies, Osh State University. She is specialized in mathematical modeling and applied physics. Her research interests include computational fluid dynamics, numerical methods for partial differential equations, and their applications in engineering.

Gulbadan Matieva is a Full Doctor, Professor at the Institute of Mathematics, Physics, Technics and Information Technologies, Osh State University. She is specialized in theoretical physics and quantum mechanics. Her research interests focus on quantum information theory, condensed matter physics, and the interaction of light with matter.

Venera Isakova is a PhD, Associate Professor at the Faculty of Mathematics and Computer Technologies, Osh State Pedagogical University. She is specialized in applied mathematics and computer science. Her research interests include algorithm design, machine learning, and the development of software systems for data processing.

Cholpon Mustapakulova is a Master, Lecturer at the Institute of Mathematics, Physics, Technics and Information Technologies, Osh State University. She is specialized in mathematical analysis and optimization techniques. Her research interests are focused on optimization problems in various fields, including economics and industrial applications.

Gulshana Omurzakova is a Master, Lecturer at the Institute of Mathematics, Physics, Technics and Information Technologies, Osh State University. She is specialized in computational mathematics and simulation. Her research interests include high-performance computing, simulation of complex systems, and computational methods in physics and engineering.