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Research article

Exploring creativity in pattern generalization: A study of future teachers in Brazil and Portugal

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Abstract: Addressing the needs of talented students presents a significant challenge in today's society and consequently in schools, where creativity plays an important role. Creativity is a dynamic characteristic that students can develop, particularly in mathematics education, where solving problems involving the generalization of mathematical relations through the identification of figurative patterns is key. These patterns are the foundation of algebra, which is important across many disciplines and in some professional challenges. This study investigated how future teachers from Brazil and Portugal, during their initial teacher training, solve problems involving the generalization of mathematical patterns, with an emphasis on multiple solutions and creativity. The research adopted a qualitative and exploratory approach, characterizing the participants' performance in solving tasks involving figurative patterns, identifying dimensions of creativity that emerge from their solutions, and analyzing distinctions between the two contexts. The findings highlight the importance of integrating visualization and the exploration of multiple representations in the training of future teachers. This approach contributes to the identification of some dimensions of creativity (fluency, flexibility, and originality) that emerged in different ways in the participants' productions. It is evident that the focus on visualization in Portugal promoted a more robust understanding of regularities and algebraic generalizations, while in Brazil, the use of symbolic notation favors the early development of formal skills.

Keywords: pre-service mathematics teachers, creativity, figurative patterns, representations, generalization, algebraic thinking

1. Introduction

The primary objective of educators is to help students enhance their mathematical skills, enabling them to tackle a variety of challenges both in and out of school. In addition to societal demands, schools also emphasize the importance of innovation and creativity, which are essential traits that students must develop. Creativity, often associated with the ability to generate new and original ideas, has been studied and analyzed through the lens of mathematical task solutions. There is consensus that creativity begins with curiosity, engaging students in exploration and experimentation tasks where they can express imagination and originality [1,2].

Mathematics, when developed through challenging tasks that involve the exploration of patterns, enables the construction and expansion of mathematical concepts. This approach not only gives meaning to mathematical procedures and ideas—often learned in isolation or in a decontextualized manner—but also promotes problem-solving in various ways. Moreover, it enhances transversal skills, such as communication, representations, connections, and reasoning. Patterns, as a valuable resource in the mathematics classroom, offer various approaches and permeate all areas of mathematics. Their study makes it possible to access powerful mathematical ideas, as generalization and algebraic thinking, where visualization can play an important role [3]. Numerous authors highlight the creative potential of tasks involving patterns, as they can be open-ended and facilitate a wide range of connections across all mathematical topics. These tasks not only prepare students for advanced mathematical learning but also foster problem-solving and problem-posing skills, as well as communication abilities.

Both the Brazilian (National Common Curricular Base) [4] and the Portuguese curriculum (Essential Learnings of Mathematics for Basic Education) [5] advocate for the inclusion of tasks promoting the development of algebraic thinking from the early years (in Brazil) and in the first cycle of basic education (in Portugal). Providing students with opportunities to develop creativity through situations that interconnect mathematical concepts [6] can be enriching, especially regarding multiple solutions and the generalization of patterns. To achieve this, such approaches must be part of teacher training programs, especially initial training. By incorporating tasks that promote multiple solutions, facilitating the generalization of patterns, and stimulating creativity, teachers can help students develop a deeper understanding of mathematics and foster algebraic thinking [7–12].

Grounded on the previous ideas about algebraic thinking, visualization, creativity, and multiple solutions, we developed a qualitative study aimed at investigating how future teachers—both Brazilian and Portuguese—solve problems involving the generalization of patterns in figurative contexts during their initial training. With a focus on multiple solutions, we also seek to understand how these tasks can contribute to the development of creativity.

The following sections present the theoretical framework, methodology, results, and data analysis, as well as the main conclusions drawn from this study.

2. Theoretical framework

2.1. Algebraic thinking and visualization

Algebraic thinking is a form of mathematical reasoning that extends beyond the mere manipulation of symbols and equations. It involves the ability to identify patterns, generalize mathematical relationships, and solve problems in an abstract and generalized way [13]. For students to be competent in algebra, they must learn to reason about mathematical relationships across a variety of contexts, from the earliest years of basic education. The traditional approach to algebra, often focused on symbolic manipulations with no significant connection to conceptual thinking, has been criticized for failing to foster a deep understanding of algebraic reasoning [13].

One of the most effective strategies to introduce and develop algebraic thinking is the exploration of patterns—a fundamental process for understanding relationships between variable quantities. In fact, the development of robust algebraic thinking requires students to explore patterns and build generalizations that allow them to apply mathematical concepts in a broader way, which is one of the pillars of contemporary mathematics education [14]. According to Kieran [13], algebraic thinking should not be seen as an isolated practice. On the contrary, it must be introduced into the curriculum from the earliest years of schooling, allowing students to gradually develop their ability to abstract and generalize. Working with patterns and generalization helps establish a foundation for formal and functional algebraic thinking, contributing to a more meaningful transition to the study of traditional algebra [15].

Visualization plays an essential role in the development of algebraic thinking, especially in tasks involving figurative patterns, which are recognized for facilitating more natural transitions from arithmetic to algebra [16]. Usiskin points out that one of the conceptions of algebra is that it is a generalized arithmetic. In this conception, "it is natural to think of variables as generalizer models" [16, p. 13].

Rivera [3] argues that visualization allows students to build a clear mental structure of patterns, relating them to generalized mathematical expressions. This makes it easier to identify regularities and structure throughout the construction of a sequence. When working with figurative patterns, students can simultaneously explore visual and numerical regularities, associating them with each other, formulating generalizations, and justifying their solutions through functional relationships [12]. In addition, the ability to transition between different forms of representation—visual, numerical, and algebraic—promotes cognitive flexibility and helps formulate more robust and meaningful generalizations.

Visualization thus facilitates the understanding of the relationships between the elements of a pattern and their representation in terms of equations or functions, which is fundamental for solving algebraic problems [17]. This connection between visualization and algebra is reinforced when students are encouraged to use visual strategies to understand patterns and generalize. When solving a task, visual strategies are those that include the use of different visual representations (pictures, drawings, diagrams, graphs) as an essential part of the solving process [18,19]. Therefore, visual representations can help students progress in their understanding of mathematical concepts and procedures, giving meaning to the content involved in the problems, as well as contributing to both problem-solving and collective discussions [19–21].

The teaching sequence proposed by Vale et al. [10] exemplifies the connection that can be established between visualization, generalization, and algebraic thinking. Structured into three fundamental phases, this approach allows students to explore patterns and formulate conjectures and generalizations in visual contexts, promoting a richer understanding of the underlying mathematical relationships (Figure 1).

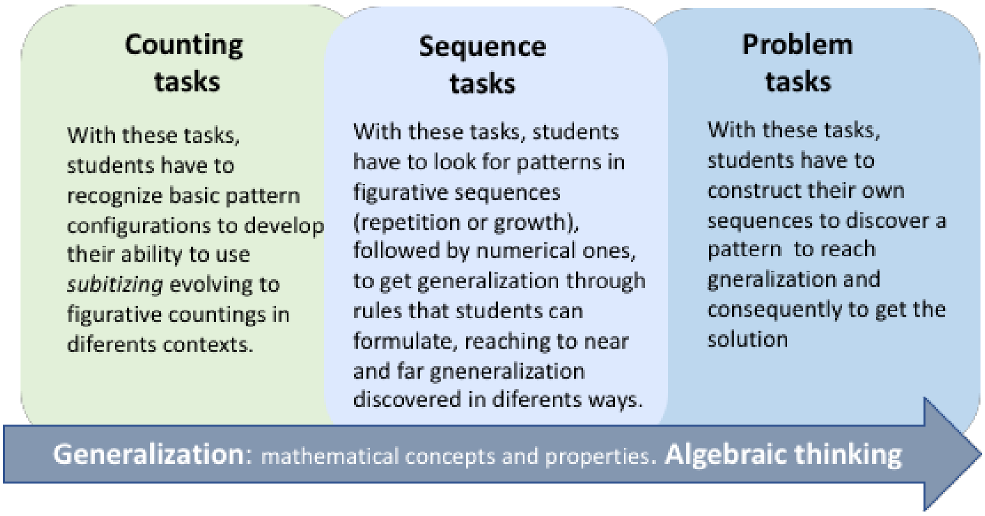

Figure 1. Didactical sequence for algebraic thinking.

The first phase of the sequence focuses on visual/figurative counting, starting with the recognition of basic configurations, enabling solvers to develop fundamental counting skills in different contexts, such as subitizing (instantly seeing). The second phase is based on the exploration of figurative sequences, including repetition and growth, followed by numerical sequences, which aims to recognize, discover, and generalize patterns, giving priority to figurative patterns. This phase facilitates the formulation of generalization through rules that students can formulate, enabling them to achieve near and far generalization through different approaches. Finally, the third phase involves solving problems where the underlying sequence is not explicitly presented. In this way, students have to discover, explore, and construct their own sequences in order to establish generalizations to achieve a solution. This didactic sequence shows how the integration of visualization and algebraic thinking can enrich mathematical learning, expanding students' ability to connect concepts and develop more creative and innovative solving strategies.

Reflecting on these ideas, the development of algebraic thinking, framed by the presented perspectives, is closely aligned with the development of the so-called 4Cs: creativity, critical thinking, communication, and collaboration [22]. In particular, creativity is evident when students explore different problem-solving strategies and develop their own generalizations; they become more fluent when generating different solutions to the same problem, more flexible when using multiple approaches, and more original when creating their own representations [2,23]. Critical thinking is encouraged when students are challenged to justify their solutions and refine their algebraic representations based on visual and numerical evidence. In addition, communication is essential in expressing mathematical ideas clearly and coherently, using various types of representations. Collaboration is important to foster an environment where students can exchange ideas, compare strategies, and work together to solve complex problems [14].

Algebraic thinking is central to the development of mathematical reasoning. Its introduction from the earliest years, through the exploration of patterns—particularly figurative ones—leverages the potential of visualization to facilitate a more natural transition to the study of formal algebra. On the other hand, the use of multiple representations and the integration of the 4Cs in the teaching and learning process of algebra are fundamental to promote more effective teaching.

2.2. Algebraic thinking and algebra: curricular guidelines in Portugal and Brazil

Algebra holds a central position in mathematics curricula, as it is one of the main pillars for the development of abstract and relational thinking. In both Portugal and Brazil, curricular reforms have reinforced the importance of algebraic thinking from the early years of basic education (Portugal) and primary education (Brazil). The main aim of this study is to provide a comparative analysis between the current curricular guidelines in the two countries, focusing on the *Essential Mathematical Learning for Basic Education* [5] in the Portuguese context and the *National Common Curricular Base* [4] in the Brazilian context.

In Portugal, the *Essential Mathematical Learning for Basic Education* emphasizes the development of algebraic thinking from the earliest years. The curriculum moves beyond a sole focus on symbolic manipulation to value generalization, pattern exploration, and problem-solving in both figurative and numerical contexts. In the first cycle of basic education (6–10 years), algebra is presented as an independent yet integrated mathematical topic, highlighting its transversal nature and its connections with other mathematical topics, especially numbers, promoting an approach oriented toward generalized arithmetic. The curriculum emphasizes the gradual development of algebraic thinking in students, encouraging students to understand variation in different contexts, formulate conjectures, and express relationships and generalizations, both numerically and algebraically. To do this, age-appropriate representations, such as diagrams and tables, play a key role in helping students make sense of their mathematical reasoning. Additionally, the curriculum values the ability to construct and apply mathematical models to real-life situations, enabling students to describe these situations, make predictions, and understand the relevance of mathematics.

In the second cycle of basic education (10–12 years old), algebraic thinking continues to be developed, with an introduction to symbolic representations, namely algebraic expressions, within contexts that encourage meaningful use of letters. The curriculum also introduces the concept of direct proportionality, a context that promotes the idea of variation and functional thinking. In this sense, algebra is approached in an integrated way with arithmetic and is introduced through tasks involving the visualization of patterns and the formulation of generalizations. A striking feature of the Portuguese curriculum guidelines is the emphasis on visualization, which is considered a fundamental tool for the development of algebraic thinking, allowing students to establish connections between different representations and understand the underlying structure of algebraic problems [12]. This approach is based on the use of figurative patterns, especially in growth sequences, which facilitates the transition from arithmetic to algebraic thinking, promoting inductive reasoning and the formulation of generalized expressions [12].

In the third cycle of basic education (12–15 years old), the curriculum focuses on the development of algebraic thinking, promoting a progressive transition from concrete to abstract reasoning. In this cycle, students explore more systematically algebraic expressions, equations, and inequalities, as well as the application of concepts of direct and inverse proportionality, preparing them to understand functions and functional relationships. Throughout the third cycle of basic education, algebra teaching seeks to strengthen the understanding of the algebraic and functional structure of mathematical expressions, providing students with opportunities to work with formulas, symbolic manipulation, and problem-solving in meaningful contexts. The emphasis on sequences and patterns continues, enabling the development of abilities to identify and generalize regularities, as well as to formulate and solve equations as a way of modeling real-world problems.

In Brazil, the *National Common Curricular Base (BNCC)* [4] also promotes the development of algebraic thinking from the beginning of primary school, with a focus on pattern recognition and generalization as core strategies. The BNCC outlines that the primary goal of algebra in the first years of schooling is to enable students to identify regularities in numerical and non-numerical sequences and use these regularities as a foundation for algebraic generalizations [24]. One of the key aspects of the Brazilian curriculum is the emphasis on the generalization of mathematical patterns as one of the main ways to develop algebraic thinking. According to the BNCC, students are encouraged to reason beyond particular cases and construct general rules that can be expressed through algebraic or arithmetic language [25].

This approach recognizes that the use of visualization and pattern exploration is not restricted to formal algebra but permeates various areas of mathematics, allowing the construction of a strong foundation for advanced mathematical thinking [26]. However, in the Brazilian context, formal symbolic notation is introduced earlier compared to Portugal, starting from the 7th year of elementary school. The BNCC promotes the use of symbolic representations as a way of structuring algebraic thinking; however, visual representations and natural language are also encouraged in the early years of elementary school [24]. Thus, the Brazilian curriculum seeks to balance the use of multiple representations with the gradual introduction of algebraic symbols, preparing students for the formal manipulation of these representations as they progress through education.

2.3. The influence of visualization and mathematical representations on the development of creativity

The importance of visualization in mathematics, particularly its relationship with solving tasks with multiple solutions and mathematical creativity, is a field of research that has been expanded in recent times, with significant implications for professional practice. The ability to visualize mathematical concepts allows students to make deeper connections between different representations and approaches, which simultaneously promotes creativity in mathematical thinking across its dimensions. Zimmermann and Cunningham [27] defined visualization as the process of forming images (mentally, with pencil and paper, with manipulatives, with teaching materials, or with the aid of technology) and using these images to discover and understand mathematics.

In mathematics, visualization is often associated with the ability to present multiple solutions to a problem, encouraging divergent thinking and, consequently, the development of creativity. The use of visualization facilitates the emergence of approaches alternative to analytical ones, based on the manipulation of figures and the identification of spatial relationships, which highlight different aspects of the same problem. In this context, Vale and Barbosa [2,11] argued that challenging tasks with multiple solutions are essential for stimulating creativity. These tasks allow students to approach problems from different perspectives and integrate different representations, thereby strengthening their conceptual understanding and facilitating the solution of more complex problems.

The concept of creativity in mathematics is complex and multidimensional, often categorized according to three dimensions: fluency, flexibility, and originality. Together, these dimensions support problem-solving from a perspective that enhances divergent thinking [2,28,29]. Fluency refers to the ability to generate a variety of ideas or solutions to a single problem. In mathematics, it is demonstrated by the ability to explore different solving strategies, especially when it comes to tasks that allow multiple approaches [30]. In addition, fluency is one of the first signs of creativity and is particularly important in problem-solving tasks where there is more than one possible solution. To develop fluency, the use of varied representations—such as diagrams, graphs, algebraic formulas, and verbal narratives—is essential, as it allows students to construct different paths to reach the solution. Flexibility is the ability to switch between different perspectives or strategies to solve a problem. In mathematics, this can mean the ability to articulate different representations or approaches. According to Leikin [7], flexibility is a distinctive characteristic of creative thinkers, as it involves adapting quickly to new information and the ability to reconsider and restructure problems from new angles. The use of visual representations in combination with other forms of representation allows students to uncover previously unnoticed connections, fostering a richer and more creative understanding of mathematical concepts. Originality is the ability to produce new or unconventional ideas. In the context of mathematics, originality manifests itself when students find innovative solutions to known problems or approach new problems in non-traditional ways [31]. This dimension of creativity can be cultivated when students are encouraged to explore unusual methods, rather than following a standardized path. By providing tasks that involve complex and challenging representations, teachers create opportunities for students to demonstrate originality.

Mathematical representations can be categorized in different ways depending on the nature of the problem and the approach adopted. Bruner [32] proposed a foundational categorization of representations based on concrete (action-based), iconic (visual), and symbolic (abstract) representations, which continues to be relevant in current discussions in the field of mathematics education. However, other authors such as Matteson [33] and Tripathi [8] and organizations such as NCTM [21] have expanded this classification to include other categories of representations.

The terms numerical, graphic, verbal, symbolic, and dual representation were used in a study developed by Matteson [33]. According to the author, numerical representations refer to the use of numbers or numerical lists, graphic representations include a variety of distinct visual representations, such as pictorial representations, models, diagrams or graphs, verbal representations require the use of speech, symbolic representations focus on the use of symbolic notation, and, finally, dual representations do not constitute a distinct category but are rather seen as a link between the four previous categories, applied when using representations from two different representational categories, neither of which presents sufficient details to be considered independent.

Tripathi [8] and the NCTM [21] advocated a model consisting of five forms of representation associated with mathematics learning and problem-solving: contextual (real-life situations), concrete/physical (manipulative materials/objects), semi-concrete/visual (pictorial), verbal (language), and symbolic (mathematical notation). Using these categorizations as a basis, Vale and Barbosa [11] proposed a representational system that integrates elements from the prior classifications, summarized in the diagram shown in Figure 2. In this diagram, five main categories are considered (active, verbal, visual, numerical, and symbolic), as well as the dual representations resulting from the complementary use of two main ones.

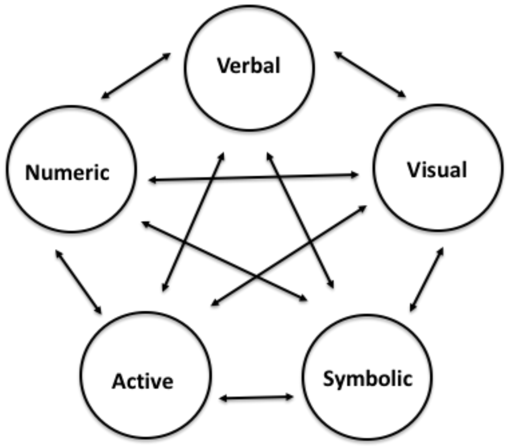

Figure 2. Multiple representations [11].

The integration of different representations is recognized as a fundamental practice for the construction of mathematical knowledge. This integration enables students to explore concepts from different perspectives, fostering a richer, more interrelated understanding of mathematical ideas. This practice is particularly valuable when solving tasks with multiple solutions, where the transition between different representations can lead to new solutions or unexpected insights [2,34]. This reinforces the importance for teachers to create learning environments that favor the use of visualization and multiple representations. Moreover, teacher training should intentionally include experiences that highlight the importance of visualization as an essential tool in problem-solving, thereby contributing to more dynamic and innovative teaching [11].

3. Methodology

This study was guided by an interpretative paradigm, assuming a qualitative nature with an exploratory approach [35,36]. The methodological choices were informed by the nature of the research problem, which aims to analyze the creativity of future Portuguese and Brazilian teachers, higher education students, in solving tasks centered on algebraic thinking in figurative contexts. This implies the development of an in-depth understanding of the participants' experiences and the construction of new knowledge, with the potential to inform and guide future research [36]. Based on the defined problem, we structured the following research questions: 1) How is the performance of students from both countries characterized in solving these tasks? 2) What dimensions of creativity emerge in solving these tasks, and how do they differ between the two contexts?

The participants included 18 Portuguese and 22 Brazilian students enrolled in initial teacher training courses. The Portuguese participants were third-year students in a six-semester Bachelor's degree in Basic Education, preparing them to teach children aged 3 to 12 years. This degree program included subjects in the areas of Didactics, General Education, and Teaching and Practices in formal and non-formal educational contexts. During the study, the participants were enrolled in a Didactics of Mathematics course focused on the exploration of mathematical topics such as Numbers and Operations, Geometry, Algebra, and Data and Probability. The Portuguese researchers, who also taught these students at the Mathematics Education Laboratory (LEM), emphasized the use of visual strategies, particularly in algebraic contexts, as visualization outside geometry is rarely prioritized in the students' prior academic experiences. Before carrying out the tasks, the Portuguese students had already covered the topic of algebra, with a strong focus on the exploration of figurative patterns and generalization.

The Brazilian students were in the fifth period of the Mathematics Bachelor's degree program, which lasts eight semesters and trains future teachers for the final years of elementary school and high school. This degree program consists of subjects that cover the areas of Pure and Applied Mathematics, Didactics, Pedagogy, and Teaching Practices. During the study, the participants were enrolled in the Instrumentation for Teaching subject, which addresses pedagogical strategies and teaching tools aimed at teaching mathematics. The curricular unit explores topics such as the use of educational technologies, manipulation and preparation of teaching materials, and lesson planning with a focus on promoting active and meaningful learning. It is worth noting that this subject is also taught at a Mathematics Education Laboratory (LEM). The subjects of both courses were the responsibility of the researchers and served as a context for data collection. During this study, the Brazilian students were attending an algebra course, in which they studied fundamental topics such as algebraic structures (groups, rings, and fields), polynomial theory and its applications, as well as approaches to teaching these contents in a school context. It is also clear that algebraic thinking is explored in an integrated manner with topics from other disciplines. Although visualization is primarily addressed in the Fundamentals of Plane Geometry and Fundamentals of Spatial Geometry courses, a recent focus has emerged on integrating visualization tasks into algebraic thinking. Instrumentation for teaching discipline is a more comprehensive curricular component, covering topics that explore arithmetic, geometry, quantities and measurements, algebra, and data processing, in order to promote explorations in different areas; the process of developing algebraic thinking can be encompassed as a transversal theme that permeates all the topics discussed in the discipline.

The participants came from two different teacher education programs. The Portuguese students were in the final year of a three-year bachelor’s degree in Basic Education and had already completed most of their mathematics and didactics modules. The Brazilian participants, by contrast, were in the fifth semester of an eight-semester bachelor’s program in mathematics education. Although their academic progression levels differ, our intention with this work was not to conduct a comparative study or make generalizations. Instead, through this exploratory approach, we aimed to understand how students from both countries engage with algebraic development during their initial exposure to the topic, within their respective learning environments and stages of training. We expect that these findings can provide important insights for teacher education on this subject and can guide future orientations for both countries, contributing to more effective teaching practices.

Data were collected and analyzed in a holistic, descriptive, and interpretative way, and included classroom observations and written productions of the proposed tasks. Portuguese students worked in pairs, while Brazilian students formed six pairs, three trios, and one individual participant. All written records produced by the participants were analyzed and triangulated with the field notes regarding the observed classes. The analysis of data was guided by two main categories: performance and dimensions of creativity. Performance included the task solution (correct solution, incorrect solution, does not present a solution), the representations, and the strategies used (analytical, visual, mixed). In creativity, we considered the dimensions fluency, flexibility, and originality.

4. Findings

Below, we present the tasks used in the study and highlight some of the solutions provided by the

Portuguese and Brazilian students. These productions allowed us to identify the strategies and representations used, as well as evaluate the tasks' potential in fostering creativity across its dimensions. These visual tasks correspond to the last two phases of the didactic sequence previously described. They were designed to encourage the use of multiple representations by involving different solution processes. This perspective promotes inclusivity, enabling a wider group of students to perform better by selecting the strategy that is most favorable to them.

It should be noted that the results presented for each task are not intended to compare the responses of students from both countries. Rather, our goal is to understand and identify the solution processes they employed. Nevertheless, we aimed to highlight particular aspects of the responses from Portuguese and Brazilian students that we considered noteworthy and deserving of attention.

4.1. Task 1

The statement of Task 1 is provided in Figure 3.

One of the topics covered in this discipline has been algebraic thinking through the exploration of patterns. Consider the following task:

a) Imagine the figure provided is the first term of a sequence.
Draw the next terms of a sequence of your choice.

b) How many cubes will there be in the 16th term? Explain your reasoning.

c) Discover a numerical expression that represents a way to calculate the nth term of the sequence you created. Explain your reasoning.

d) Imagine that the sequence you created in a) starts in the second term. Draw the first term of the adjusted sequence.

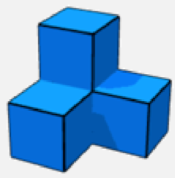

Figure 3. Statement of Task 1.

This is an open-ended task with a problem formulation component, covering several objectives. Initially, the task aims to assess students' ability to construct a growth sequence, providing them with the chance to be imaginative, while also evaluating their understanding of the concept of growth sequence. This task is versatile and can be used at different grade levels, starting in the early years, with possible adjustments. Overall, 89% of Portuguese (P) and 45% of Brazilian (B) students responded correctly to the task requirements. This difference in the results can be related to the type of task. It clearly emphasizes visual aspects, particularly 3D representations, which may pose a challenge for students who tend to work in a more analytical manner—a factor that may have influenced their performance.

There are some aspects that we would like to highlight. Figure 4 illustrates the different sequences constructed in question 1a), as well as the representation of the first term (in green) for question 1d). Figure 4 also identifies the number of occurrences of each sequence by the students from Portugal (P) and from Brazil (B). It is important to note that students who did not identify the first term (item d) were also unable to construct a valid sequence. Considering all the students and observing the solutions presented in Figure 4, we can identify that the Brazilian students proposed two original solutions (the last two).

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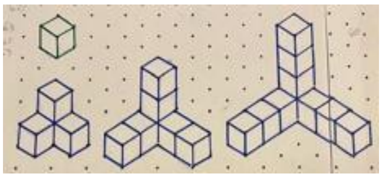
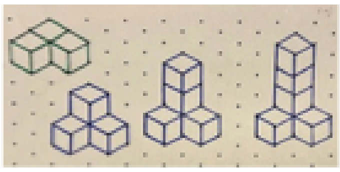
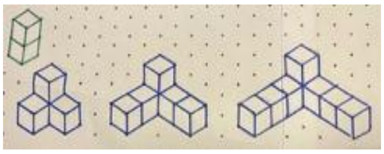
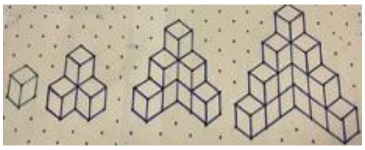
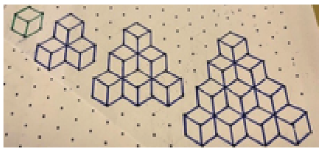
<tr><td colspan="3">
4 Portuguese students provided this solution
1 Brazilian student provided this solution</td><td colspan="3">
3 Portuguese students provided this solution
1 Brazilian student provided this solution</td></tr>
<tr><td colspan="2">
1 Portuguese student provided this solution
1 Brazilian student provided this solution</td><td colspan="2">
1 Brazilian student provided this solution</td><td colspan="2">
1 Brazilian student provided this solution</td></tr>
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Figure 4. Different types of sequences proposed by the students for questions 1.a) and 1.d).

Considering that, in question 1.d), the first element of the proposed sequence is a 3D object, the task also allowed us to analyze students' visualization abilities. They were given the possibility to resort to manipulative material (MM) if needed, specifically Polycubes. The availability of MM aimed to facilitate the construction of the sequence, offering a faster alternative to drawing. On the other hand, manipulation and construction with MM would facilitate the design of the constructed sequence.

Several students had difficulty drawing the sequences. Only two groups from (P) and four groups from (B) used MM, recognizing it as a tool for visualizing and constructing the sequence. This observation reinforced the notion that MM, when integrated into classroom practices, can significantly contribute to the teaching and learning of mathematics [37]. Figure 5 illustrates examples of sequences constructed with the help of MM.

Figure 5. Three sequences proposed by the students built with Polycubes.

This situation showed a recurring difficulty with 3D geometric drawing—a skill that appears to be increasingly underdeveloped among students, future teachers, and even school-aged learners. This observation reinforces the need to introduce tasks from an early age that challenge learners to address this gap and develop their spatial reasoning and geometric visualization skills. One common issue observed was the drawing of sequences constructed with isolated cubes (first image constructed by one Brazilian student) or the inability to distinguish hidden and visible edges (the second and third images constructed by Portuguese students). Figure 6 presents three such situations.

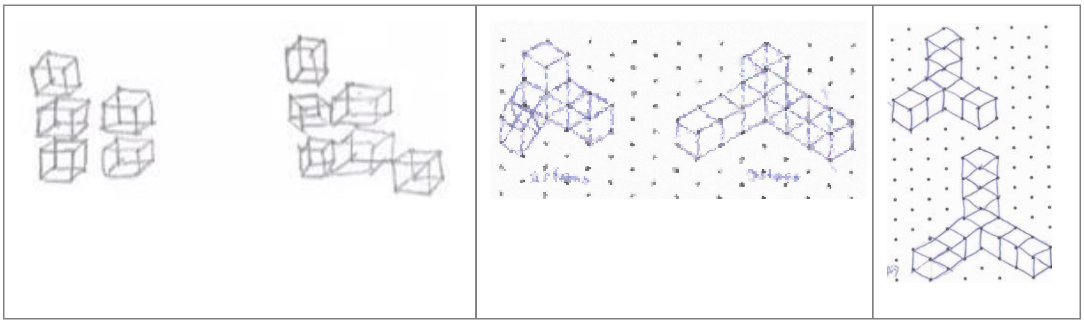


Figure 6. Examples presented by students that illustrate difficulties in 3D drawing.

Regarding the challenges related to visualization, the final discussion of the results revealed that some of the Portuguese students (P) struggled to perceive that two different proposals represented the same sequence, despite being drawn from different perspectives (Figure 7). This shows that their geometric eye still needs further development. In the process of identifying an expression that translates the generalization of the sequence, the first and most important step is the ability to see and interpret the underlying pattern.

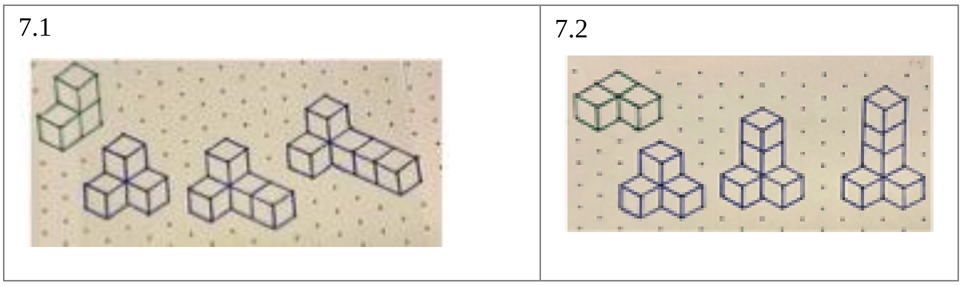


Figure 7. The same sequence drawn in two different perspectives.

In relation to the dimensions of creativity, Task 1, situated in the scope of problem posing, involved the creation of a sequence starting from a static situation—the given first term of the sequence. This approach led to the emergence of five different sequences. One of these solutions, when drawn in a different way, could influence the discovery of the expression of generality, as the change in visual perception alters how the pattern is interpreted (see Figures 7 and 12.2).

Task 1 demonstrates strong potential to stimulate students' fluency in problem-solving. It cannot be considered a low cognitive level task, as it requires significant skills, such as visualizing and drawing in 3D from a 2D model—an inherently challenging task, particularly for students who lack strong visualization skills. Furthermore, the task fosters flexibility of thought in constructing the sequence. For instance, students demonstrated different strategies: fixing a cube in one of the arms related to the given term; fixing two cubes; or changing the different arms. Although in these two questions (1a; 1d), the privileged representations were visual, some students showed flexibility as they simultaneously used active and visual representations to create a sequence.

On the other hand, the task allowed the identification of two constructions that could be considered original (Figure 8). These constructions stood out not only because of their low frequency ($n = 1$) but also for their level of complexity, which involved a more sophisticated way of thinking. Despite being similar in appearance, identifying the general term in the second construction is more challenging due to its mathematical complexity.

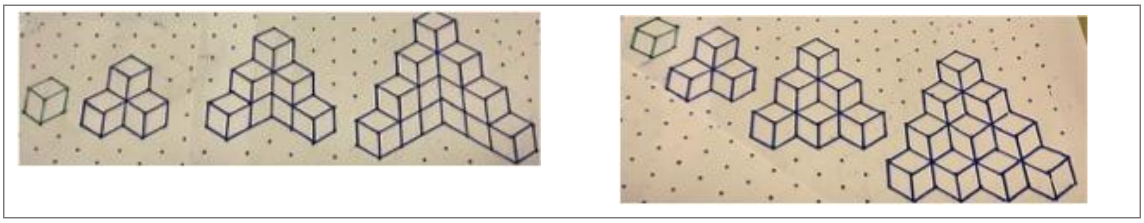


Figure 8. The two sequences considered to be original.

An interesting case that deserves to be analyzed is that of a Brazilian student (B) who, in the answer regarding the construction of a sequence (1a), presented the sequence illustrated in Figure 9.

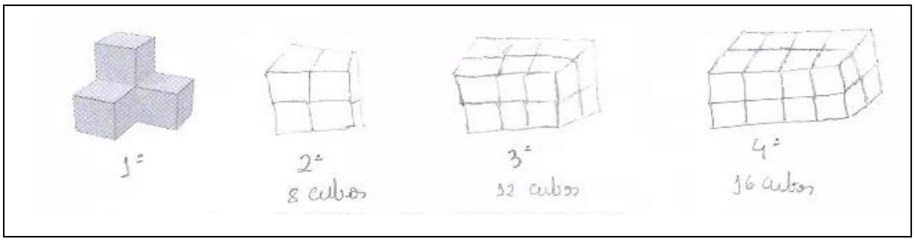


Figure 9. Sequence presented by one of the Brazilian students.

Initially, this solution to Task 1 was considered unsuccessful. Upon observing that the given element had four cubes, the students reimagined a different arrangement for this term. Based on this new arrangement of the first term, they proceeded to design the second term and continued building the sequence. In Figure 10, we show what could be a correct solution using this strategy. First, we start by transforming the given first term into the one generated in the sequence constructed in Figure 9 (Figure 10, left image). The image on the right in Figure 10 shows the complete correct sequence that should be constructed, including an adequate first term, but not the one that was given.

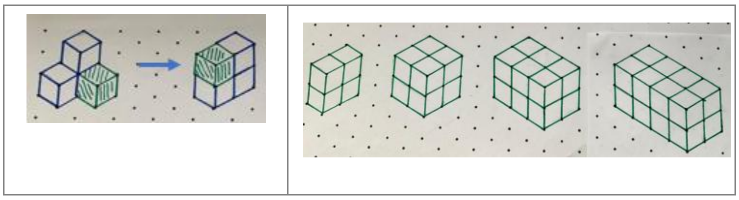


Figure 10. Correct sequence for the sequence presented in Figure 9.

The sequence in Figure 9 cannot be considered correct, as it does not demonstrate a completely defined figurative pattern. However, the student successfully answered the subsequent questions. This suggests that, while the student struggled to visualize and construct the sequence based on the given model, they displayed flexibility of thought by presenting an alternative. Even so, the initial configuration provided in the task is crucial to fulfill the request, as it specifies starting from a previously defined structure. The presented sequence would have been correct if the task had been stated as follows: *Construct a figurative sequence with cubes, where the first term is formed by four cubes*.

In Task 1, students were also expected to discover the number of cubes in the 16th figure of the sequence (question 1.b). All Portuguese students successfully completed the task, whereas only half of the Brazilian students did (B 50%; P 100%). Those who were unable to answer questions 1.b) and 1.d) often failed because they either did not construct a sequence or constructed an incorrect sequence. Let's analyze some of the answers. In this question, the students were divided between those who resorted to a close generalization and those who solved by exhaustion, confirming the solution by also determining the general expression, that is, giving an answer to item 1.c). This approach led to a predominance of analytical solutions, supported by representations such as words, tables, and arithmetic and/or algebraic expressions, and recursive and functional reasoning. A distinctive aspect between the two groups is that no Brazilian student used a table, while no Portuguese student used knowledge of arithmetic progressions to calculate any term.

One of the questions [1.c)] aimed to lead students to discover the algebraic expression generating any term in the drawn sequence. This required them to engage in algebraic generalization [17], which leverages functional reasoning, a richer approach than arithmetic generalization.

The arithmetic generalization based on recursive reasoning is limited to obtaining consecutive terms and does not allow directly discovering any term of the sequence. Thus, when searching for the algebraic generalization—or distant generalization [3,10,38,39]—geometric patterns are very useful. Students must identify a common characteristic that is repeated in the formation of the terms of the sequence and realize that this characteristic applies to any term of that sequence. Figurative pattern tasks offer students the opportunity to observe and verbalize their own generalizations and translate them into more formal language, according to their age [9,10,17].

Determining the general term for any sequence can be complex for students lacking strong mathematical foundations. However, analyzing the construction of the figure in the sequence can make this process simpler. Figure 11 illustrates this concept, showing one of the sequences in which the pattern formation is visualized, leading to the expression of generality. It is important to highlight that, in order to achieve successful generalization, it is essential to establish a well-defined pattern that remains constant throughout the sequence. As argued by Stylianides and Silver [40], the structure provided by the figurative representation of the sequence—when its construction is uniquely determined—ensures that the discovered rule can be both justified and reliably applied [39].

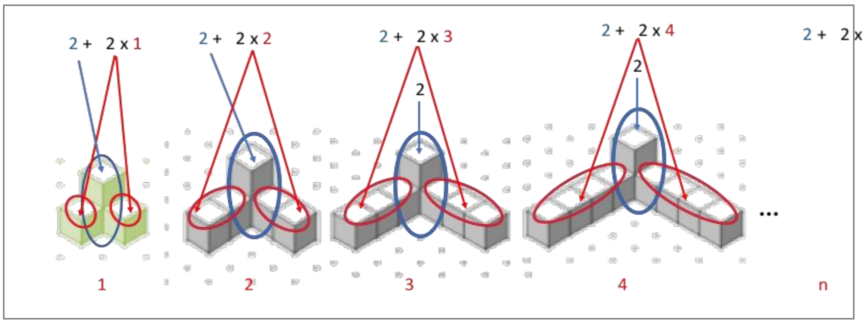


Figure 11. Visual discovery of the general term of the sequence.

Figure 12 presents the different sequences constructed by the students, as well as the corresponding expressions and general term (Tn), followed by the identification of the number of occurrences of each sequence by the students from Portugal (P) and from Brazil (B). The Brazilian students (B) who proposed the last sequence (12.5), despite having made an effort to formulate an answer to find Tn, were unable to go beyond attempting to construct the first terms of the sequence. The complexity of the expression likely hindered their progress, as it does not involve a linear pattern.

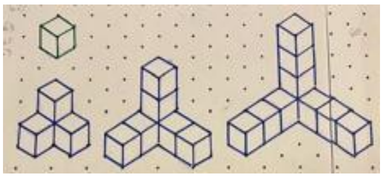
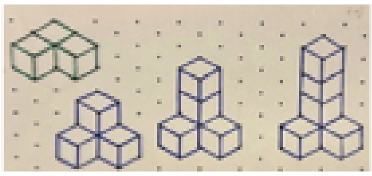
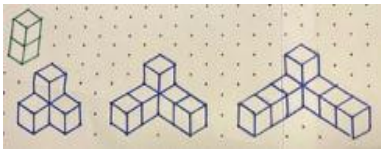
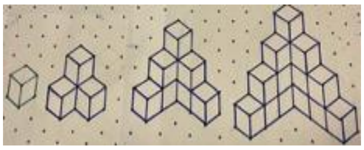
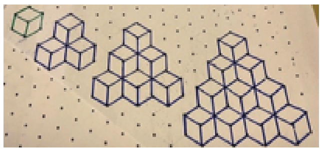
<table>
<tr><td colspan="2">12.1

Tn= 1+3xn
4 Portuguese students provided this solution
1 Brazilian student provided this solution
Tn= 3x(n-1)+4
1 Brazilian student provided this solution</td><td>12.2

Tn= 3+ n.
2 Portuguese students provided this solution
Tn= 4+ (n-1)
1 Portuguese student provided this solution</td></tr>
<tr><td>12.3

Tn= 2+2xn
1 Portuguese student provided this solution</td><td>12.4

Tn= $(n+1)^2$
1 Brazilian student provided this solution</td><td>12.5

----</td></tr>
</table>

Figure 12. Sequences constructed and respective general terms of question 1.c).

It is evident that some sequences (12.1; 12.2) are associated with more than one general expression. This variation arises from the way each student interprets the formation of the sequence, even when observing the same pattern. It is important to emphasize this aspect to students, especially younger ones, highlighting that although the expressions are different, they are equivalent. Both expressions ultimately represent the same number of cubes for any given term of the sequence (Figure 13).

(from 12.1) **3x(n-1) + 4** = 3xn-3 + 4 = 3xn + 1= **1 + 3xn**	(from 12.2) **4 + (n-1)** = 4 + n-1 =**3 + n**

Figure 13. Equivalence of the algebraic expressions.

Fluency is evident in the task, as a variety of solutions and general expressions emerged. Flexibility was more apparent in some solutions than others; for instance, some students determined the terms using one process and then confirmed their results through a different approach. On the other hand, students resorted to different representations in their solutions, especially words, numerical expressions, and algebraic expressions. However, none of the solutions could be considered truly original, as the approaches were quite similar across both countries.

One group of Brazilian students (B), which constructed sequence 12.4, answered both questions without any visual support. This approach is an example of a purely analytical solution. The group transformed the sequence of cubes into the equivalent numerical sequence: 4, 9, 16, 25, ... and realized that they had a sequence of perfect squares, where each term was the square of the term number in the sequence plus 1. Thus, they immediately concluded that the 16th term would have 289 cubes, calculated as $(n+1)^2$. By using an analytical solution, the students resorted to words and numerical expressions and were able to generalize and obtain the general expression.

One group of Portuguese students (P), which constructed sequence 12.1, solved the two questions with visual support. To approach the questions, they created an organized list, recording each term along with the corresponding number of cubes. They identified the number of cubes added at each step in the sequence. For example, they found that the third term had 10 cubes, which resulted from 4+3+3+3. We considered this to be a visual solution, in which the expressions reflect the way of understanding the formation of the sequence, having been the main strategy to reach the solution. The generalization immediately followed from the analysis of the list.

Overall, the results show that Task 1 effectively stimulated creative thinking and highlighted key differences in representation preferences and generalization strategies. Portuguese students benefited from curricular practices that value visualization, while Brazilian students demonstrated stronger symbolic manipulation skills. Specifically, Portuguese students favored visual strategies, using drawings and, in a few cases, manipulative materials to support their constructions. These strategies often facilitated accurate sequence development and more intuitive identification of general terms. In contrast, Brazilian students predominantly relied on analytical methods, focusing on numerical reasoning and symbolic notation. While this approach aligned with their curricular exposure, it sometimes led to difficulties in representing or constructing sequences visually, especially with 3D elements. Nonetheless, some Brazilian students displayed creative adaptability.

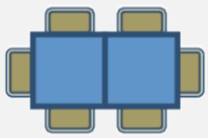
Let's now look at the performance of both groups of students (B and P) on Task 2.

4.2. Task 2

The statement for Task 2 is provided in Figure 14.

Solve the following problem:

Manuel and his father are arranging the tables for his birthday party. They started by placing the tables together side by side. The tables are square and only one person can sit on each side. For example, when Manuel placed two tables together, he found that he could seat six of his classmates, as shown in the picture.



a) How many people can be seated if 55 tables are arranged in this way? Explain your reasoning.

b) With this arrangement of tables, is it possible to seat 71 people without leaving any empty seats? Explain your reasoning.

c) How would you calculate the number of people who can be seated for any given number of tables? Explain your reasoning.

Figure 14. Statement of Task 2.

Task 2 focuses on problem-solving, in which pattern discovery is the primary strategy for its solution. This task has the same objective as Task 1: to reach generalization. However, unlike Task 1, students must independently construct the sequence that leads to the general rule. This requirement makes Task 2 a high cognitive demand task for most of the students. They may also arrive at the generalization by using more analytical processes, such as applying knowledge of arithmetic progressions. In this case, the process becomes simpler after identifying the first term and the common difference of the progression. The ultimate goal is for these future teachers to solve the problem using tools accessible to elementary students with basic mathematical knowledge. Therefore, they should follow similar strategies to those used in the previous task. Like Task 1, this task can be adapted for use across various school levels, with possible adaptations.

In general, across the three questions, both countries showed a generally higher performance in Task 2 compared to Task 1: 80% (B) successfully solved it, with only two groups failing: one for not completing the task and another for solving it incorrectly; and 77% (P) were successful. In this task, it was interesting to see the difference in approaches among students from the two countries.

The strategies employed by the students were diverse, ranging from visual to analytical approaches, as well as mixed solutions, utilizing both words, arithmetic, and algebraic expressions. In the case of the Brazilian students, only one group (B) (Figure 15) presented a visual solution for question 2.a). However, they also developed an analytical solution, suggesting the use of an expression to calculate a term of an arithmetic progression. During this process, they made a mistake in using the formula, resulting in an incorrect answer of 55 instead of 54. The order of the two solutions suggests that the students approached the task using the process they were most familiar or comfortable with, and only afterward did they turn to a visual approach.

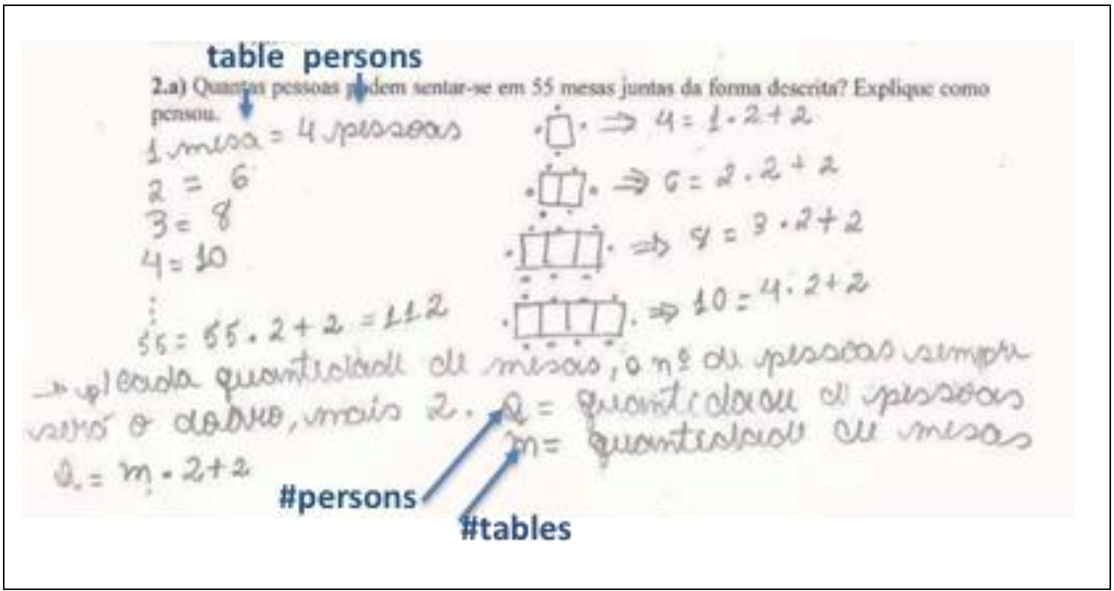


Figure 15. Visual solution of question 2.c) of one group of Brazilian students.

As in Task 1, many groups responded to question 2.a) by looking, more or less explicitly, for the general expression requested in 2.c). For question 2.b), most students (both B and P) discovered that the sequence of chairs (or people) corresponds to even numbers. Based on this observation, they immediately concluded that it would not be possible to fulfill the condition with 71 chairs, as 71 is an odd number.

Some groups provided an analytic solution, explaining their reasoning using words. Figure 16 illustrates two such responses (transcribed from the original, as it was not legible), one from a Brazilian (B) student and another from a Portuguese (P) student.

B
It's not possible. Either there would be 35 tables for 72 people, but there would be 1 empty seat, or there would be 34 tables for 70 people, but there would be 1 person standing.

P
It's not possible. It is not possible to seat 71 people without leaving any empty seats at this table arrangement, because the number of people is even: 1 table has 4 people, 2 tables have 6 people, 3 tables have 8 people and so on. Therefore, there would always be one empty seat, since the number 71 is odd. 35 tables would be needed to give 72 people seating, that is, there would be 1 person more.

Figure 16. Analytic solutions of question 2.c) of two groups, one Brazilian and one Portuguese.

However, some students used different approaches to explain their reasoning, including deriving the general expression through algebraic methods (B), making attempts (guess-and-check) (P), or using visual approaches (P) (Figure 17).

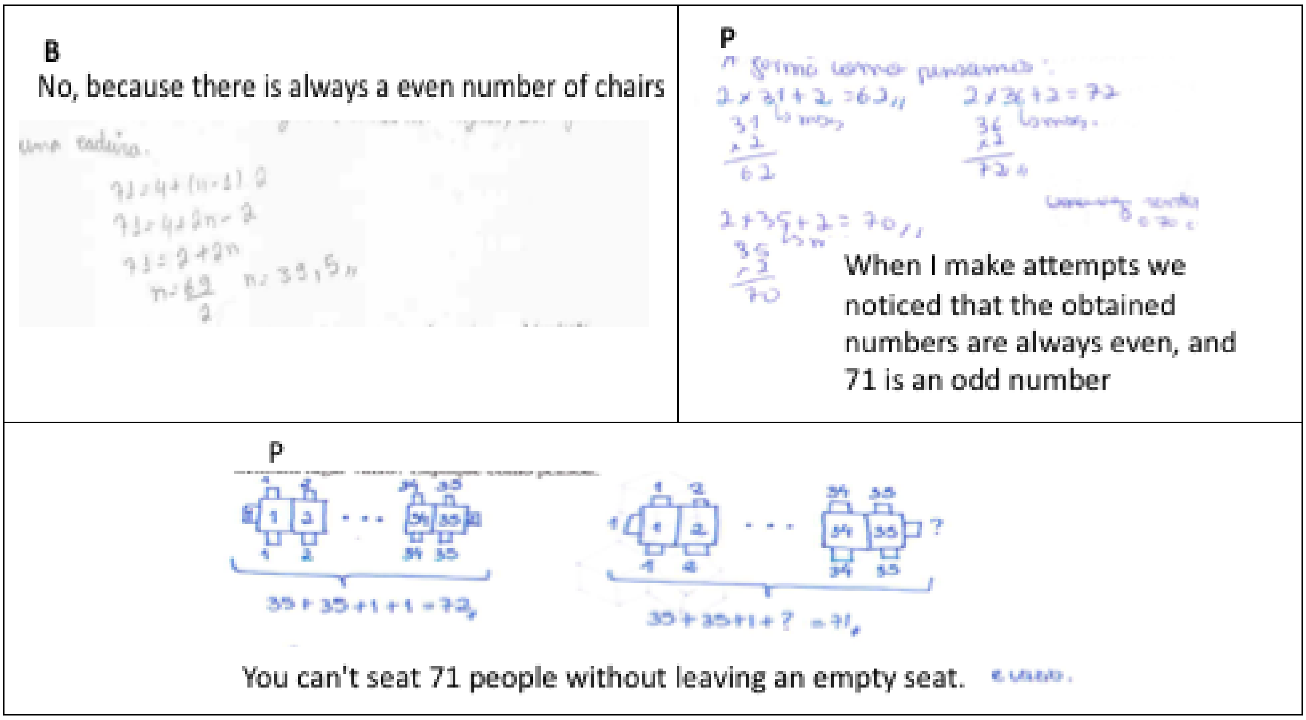
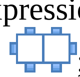


Figure 17. Two analytical solutions (algebraic and arithmetic expressions) and a visual solution (drawings).

As mentioned, most students solved question 2.c) with verbal justifications based on their analysis of the figure, arriving at the most commonly presented general expression, $T_n = 2 + 2n$. Other students used arithmetic progressions to obtain the expression $u_n = 4+(n-1) \times 2$. Some students interpreted the given image as the first term of the sequence ; while others considered the given image to be the

2nd term of the sequence, so they drew the 1st term . However, one group (P) presented another general expression, $T_n = 6 + (n-2) \times 2$, using a visual solution (Figure 18, transcribed from the original as it was not legible). This group first identified the pattern visually and made an arithmetic generalization to solve question 2.a), followed by an algebraic generalization to answer question 2.c).

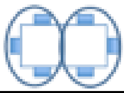
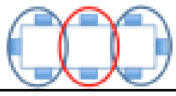
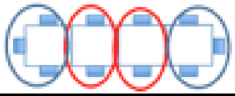
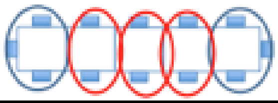
#Tables		#Persons
2		6 =3+3
3		8 =3+3+2 = 6+2
4		10=3+3+4 = 6+2+2
5		12=3+3+6 = 6+2+2+2
...		...
55		6+ 53x2= 112
...		...
n		6+ (n-2) x2

Figure 18. Visual solution.

Regarding the dimensions of creativity, the proposed task, situated in the context of problem-solving and aligned with the final phase of the didactical sequence (Figure 1), encouraged the emergence of varied expressions of generality and justifications, including verbal, arithmetic, algebraic, and visual approaches. These justifications often complemented each other, and, in some cases, two different justifications were provided for the same question. This task demonstrated its potential to stimulate fluency, as evidenced by the diversity of strategies employed by the students. It also fostered flexibility, as participants used different ways of thinking to solve the same task. Notably, one solution (Figure 18) stood out as original, presenting a unique general expression emerging through the use of a table and visual analysis of the sequences. This originality highlights the potential of such tasks to encourage creative thinking.

5. Final considerations

When students enter school, they bring with them significant potential in areas such as visualization, imagination, generalization, and ways of expression. To help them move beyond a purely procedural understanding of mathematics, it is crucial to provide opportunities for students to explore intuitive ways of reasoning and solving mathematical problems. This approach also allows them to appreciate and enjoy mathematics. In this context, teachers should explore these skills in different ways, adjusting to the natural abilities of each individual. Therefore, teachers must be attentive and use a variety of approaches that make it possible to explore this potential in each student and in each task.

This study analyzed the creativity of future teachers in solving tasks focused on algebraic thinking in figurative contexts, looking to understand the students' performance and identify the dimensions of creativity that emerged. The results, grounded in the detailed analysis of the participants' productions, highlight the theoretical framework presented. Two tasks were implemented, one focusing on the construction of a growth sequence and its generalization, and the other centered on problem-solving, in which pattern discovery was the primary strategy to solve the problem.

When solving the first task, the Portuguese and Brazilian future teachers performed differently. The Portuguese students showed a stronger integration of visual strategies, using the structure of figurative patterns to explore regularities and formulate generalizations. They stood out for their use of drawings and diagrams to explore this type of pattern. This approach reflects the emphasis of the Portuguese curriculum [5] on visualization as a means for more meaningful transitions between arithmetic and algebra, as indicated by Vale and Barbosa [12], who highlighted the central role of visualization in the development of algebraic thinking. On the other hand, the Brazilian students showed greater difficulty in constructing visual sequences and presented predominantly analytical solutions, often related to recursive reasoning. They used formal algebraic notation, looking for direct connections with algebraic concepts, as evidenced in the BNCC [4]. Although this promoted efficiency in symbolic manipulation, there was less diversity in the presented strategies, limiting the flexibility of reasoning.

In the second task, the gap between both groups narrowed, although differences in strategies remained: the Portuguese students continued to favor visual representations, often combined with verbal representations, while the Brazilian students predominantly used arithmetic progressions to generalize the patterns observed, a powerful tool for achieving the expression of generality, demonstrating mastery of formal concepts but less versatility in employing multiple representations. The results from both tasks confirm the importance of teaching strategies that integrate visualization, symbolic manipulation, and functional reasoning in order to strengthen both the performance and the conceptual understanding of future teachers [3,10].

The study revealed how the dimensions of creativity—fluency, flexibility, and originality—emerged in different ways in the participants' productions. Fluency was demonstrated through the diversity of solutions, with multiple approaches to generalize patterns (visual, verbal, or algebraic). Both groups presented varied solutions, but the Portuguese students presented more solutions per task, driven by the use of multiple representations (visual, numerical, and symbolic) [8,11,17]. The flexibility of the Portuguese students was highlighted by the alternation between different forms of representation, such as the combined use of organized lists and diagrams to justify solutions [19]. The Brazilian students' flexibility was identified in the adaptation of analytical strategies to less structured contexts, such as the application of arithmetic progressions to generalize figurative sequences. Although less frequent, originality was observed, and unique solutions were more evident in the Brazilian future teachers when they proposed innovative solutions that reinterpreted the figurative patterns in a creative way, even with the emphasis on formal symbolic notation.

These results corroborate the literature that associates mathematical creativity with the exploration of multiple representations and divergent thinking, as advocated, for example, by Leikin [28] and Vale and Barbosa [2].

This study highlights the importance of integrating visualization and the exploration of multiple representations in the training of future teachers. The findings suggest that the focus on visualization in Portugal promotes a more robust understanding of algebraic regularities and generalizations. The use of symbolic notation in Brazil favors the development of formal skills from very early on. The curricular choices in each country reflect underlying pedagogical philosophies: constructivist and discovery-oriented in Portugal, and more structured and content-focused in Brazil. Such orientations shape the training of future teachers, influencing the types of strategies they adopt and the forms of creativity they are likely to express. A hybrid approach can enhance the creativity and performance of future teachers, balancing fluency, flexibility, and originality in algebraic thinking [7,19,28,29,30].

The study provides useful data that can inform both teacher education and the development of algebraic thinking and creativity. We observed that the differences observed in the participants' performance from the two countries reflect the specific characteristics of their respective training programs. This highlights the importance of aligning teacher education with the mathematical content and approaches that future teachers will later use with their own students, particularly by considering the specific curricular guidelines of each national context.

The findings of this study underscore the importance of integrating visualization and multiple representations in the development of algebraic thinking and creativity among future teachers. However, further research is needed to build on these insights. For instance, a longitudinal study can be useful to analyze how initial teacher training programs that emphasize visual strategies influence classroom practices and student outcomes once these future teachers enter the profession. Another possibility is to investigate the transferability of strategies across mathematical domains and across cultures, particularly in contexts beyond Brazil and Portugal. Moreover, considering the distinct approaches observed between Brazilian and Portuguese students, comparative studies using mixed methods could explore the effects of curriculum design and instructional emphasis on creativity and algebraic generalization. These lines of inquiry could significantly contribute to the design of more effective teacher education programs that foster deeper conceptual understanding and creative mathematical thinking.

Author contributions

Isabel Vale: Conceptualization, Methodology creation, Investigation, Resources, Validation, Formal analysis, Data curation, Writing - original draft, Writing – review & editing, Supervision; Ana Barbosa: Conceptualization, Methodology creation, Investigation, Resources, Validation, Formal analysis, Data curation, Writing – review & editing; Jorge Gualandi: Conceptualization, Methodology creation, Investigation, Writing – review & editing. All authors have read and approved the final version of the manuscript for publication.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest

Ethics declaration

The ethics principles for conducting research in education have been followed.

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