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Research article

Volatility estimation and crash risk forecasting using unified GARCH models: Empirical evidence from the Russian invasion of Ukraine

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Abstract: Given the significant impact on economies and financial markets, this study aims to effectively estimate stock return volatilities related to the Russian invasion of Ukraine on February 24, 2022. To achieve this, we developed fully unified generalized autoregressive conditional heteroscedasticity (GARCH) models based on behavioral finance theories that incorporate asymmetry, skewed distribution errors, and structural breaks. Using daily samples from July 2020 to March 2025, which include the impact of the Russian invasion, we first revealed that our fully unified GARCH models are effective in estimating Russian stock market index return volatilities, demonstrating the efficacy of our model unification approach. Additionally, using two other sets of daily 1,000 samples up until one week before or four weeks before the Russian invasion, we found that both one week and four weeks before the invasion, our fully unified GARCH models more effectively predicted the soaring Russian stock market volatilities affected by the Russian invasion. This shows the ability of our fully unified GARCH models to predict crash risk in the Russian stock market, which is one of the most novel pieces of evidence from our work. Furthermore, for the Russian stock index, the volatility estimates from our fully unified GARCH models have shown predictive power for the volatilities of non-fully unified models, while those from non-fully unified models do not predict the volatilities of the fully unified models. This finding also demonstrates the novelty of our work. Moreover, we provided numerous valuable interpretations, implications, and innovative perspectives for future research and practice of quantitative finance and economics. Furthermore, we emphasize that, in addition to extending GARCH models, this paper offers: (*i*) insight into the link between models, theory, and real-world data; (*ii*) perspectives on the relationship between information, human psychology, and structural breaks; and (*iii*) valuable broader implications of the impacts of Russia's invasion of Ukraine. These novel insights are crucial and will inspire future research across multiple disciplines.

Keywords: adaptive markets hypothesis; artificial intelligence; crash risk; efficient market hypothesis; GARCH; skew-GED error; skew-*t* error; Russian invasion of Ukraine; structural break; volatility

JEL Codes: G10, G15, G32

1. Introduction

Russia launched an invasion of Ukraine on Thursday, February 24, 2022, by sending troops into the former Soviet republic and firing missiles at several locations near Kyiv, the capital of Ukraine (Cable News Network, 2022). In response, the price of the Russian representative stock index, the Russian Trading System (RTS) index, plummeted by about 38.3% to 742.91 points from 1,204.11 points on the previous day. As a result, the Russian invasion of Ukraine had a significant impact on the Russian stock market on the same day, greatly increasing volatility risk—referred to as *crash risk* in this paper. The invasion also had serious effects on world economies later on (Financial Times, 2022; Pestova et al., 2022; World Bank, 2022). This event is also important in terms of the environment, resources, and building a future sustainable economy (e.g., Aziz and Sarwar, 2023; Aziz et al., 2023).

Recently, the academic literature has begun to examine this significant new issue. In these studies, investigations of the impact of the Russian invasion of Ukraine on stock markets have been conducted using various methods, including (*i*) event studies (e.g., Boubaker et al., 2022); (*ii*) vector autoregression (VAR) (e.g., Beraich et al., 2022); (*iii*) heterogeneous autoregressive (HAR) modeling (Izzeldin et al., 2023); (*iv*) the time-varying parameter VAR connectedness approach (e.g., Alam et al., 2022); and (*v*) panel data regression and standard regression (e.g., Boungou and Yatié, 2022). It is important to note that despite its effectiveness, generalized autoregressive conditional heteroscedasticity (GARCH) analysis has been limited in existing studies.

Therefore, considering this research background, this study is motivated by the importance of estimating the volatilities affected by the Russian invasion of Ukraine using extended GARCH models in a more timely manner. The motivation for this research stems from the importance and necessity of utilizing extended GARCH models to effectively estimate surging volatilities caused by the significant event of the Russian invasion of Ukraine. We note that similar to other existing volatility studies, the focus of this study is on total volatility. We are not interested in isolating the pure impact of the Russian invasion. Most practitioners pay attention to total volatility in markets. To address this, we develop and utilize unified GARCH models. It is worth noting that while the effectiveness of GARCH models has been supported by many existing studies (e.g., Bollerslev, 1986; Nelson, 1991; Awartani and Corradi, 2005; Tsuji, 2016; Ardia et al., 2019; Luan et al., 2022), we further develop unified GARCH models in this paper. We believe that this different advanced approach from previous studies strongly fills the research gap left by existing studies.

Specifically, to estimate the volatilities affected by the Russian invasion of Ukraine in a more timely manner, our study suggests that unifying consideration of the following factors is crucial: (*i*) the asymmetric feature of return–volatility relations, (*ii*) skewed distribution errors, and (*iii*) structural breaks. Therefore, we propose fully unified GARCH models that incorporate all three factors into one model. Our research questions are as follows: (*i*) "Do our fully unified GARCH models, which incorporate asymmetry, skewed errors, and structural breaks, estimate the surging volatilities affected by the Russian invasion of Ukraine more effectively?" and (*ii*) "Do the volatility estimates from our fully unified GARCH models possess predictive power for the extremely increased volatilities—crash risk—caused by the Russian invasion of Ukraine?" The goal of this study is to clarify these issues.

To achieve this, we model Russian stock market return volatilities by applying fully unified GARCH models, i.e., (*i*) the autoregressive (AR) mean–skew-*t* (ST) error and structural break (SB) incorporated Glosten–Jagannathan–Runkle (GJR)–GARCH model and (*ii*) the AR–skew-generalized error distribution (SGED) error and SB incorporated GJR–GARCH models (hereinafter, AR–ST–SB–GJR–GARCH and AR–SGED–SB–GJR–GARCH models, respectively). We note that these fully unified models incorporate not only GJR asymmetry (Glosten et al., 1993) but also skewed distribution errors and structural breaks.

Through our meticulous examinations, we uncover new findings that highlight the significant contributions of this study. First, we clarify that even after including the effective GJR asymmetry, incorporating skew-*t* or skew-GED errors is effective in estimating the volatilities of Russian stock returns. Second, we discover that even after incorporating GJR asymmetry and skew-*t* or skew-GED errors, integrating structural breaks remains valid for estimating the volatilities of Russian stock returns. Third, we also find that the volatility estimates from our two advanced fully unified models above have predictive power for fluctuations in Russian stock returns. In contrast, the volatility estimates from the non-fully unified models lack predictive power for fluctuations in Russian stock returns and fall behind those from our fully unified models.

Fourth, we also discover that the estimates from our two fully unified models respond to the extreme volatility increases resulting from the Russian invasion significantly earlier than the estimates from the non-fully unified models. This is evident not only one week prior to the Russian invasion but also four weeks prior. This evidence suggests that both one week and four weeks before the invasion, our two fully unified GARCH models more strongly predicted the extreme volatility increases—crash risk—stemming from the Russian invasion. Fifth, we have also developed and presented numerous valuable interpretations, implications, and innovative perspectives. This signifies another contribution of our study. These include theoretical advancements in asset pricing models, implications for managerial finance, and the effective application of artificial intelligence (AI) for risk management. These insights will be beneficial for both academic research and industry practices of quantitative finance and economics.

The rest of the paper is organized as follows: Section 2 provides theoretical background, Section 3 conducts a literature review, Section 4 explains the data, and Section 5 describes our model unification procedures and their effectiveness. Following that, Section 6 scrutinizes the forecasting ability of the volatility estimates from both fully and non-fully unified models, and Section 7 provides discussions. Lastly, Section 8 concludes the paper.

2. Theoretical background

This section presents the theoretical background of our extended models. The purpose of this section is to establish connections between each element of our model extension and existing theories. Initially, Ross (1989) provided a theoretical explanation of the meaning of volatility, stating that asset price volatility is directly related to the rate of information flow to the market. In response to this, Campbell and Hentschel (1992) expressed that "no news is good news," implying that volatility will remain stable in the absence of news. In our current context, Russia's invasion of Ukraine is a significant piece of negative news. Consequently, we can theoretically anticipate a strong reaction from the stock market to this event, resulting in a significant spike in stock return volatility.

In our context, the psychological aspect is crucial, as Whaley (2000, 2009) describes volatility as a fear gauge. The theory of loss aversion suggests that people react more strongly to losses than to gains, displaying asymmetrical behavior (Tversky and Kahneman, 1991; Kahneman et al., 1986, 1991; Benartzi and Thaler, 1995; Berkelaar et al., 2004; Blavatskyy, 2011). This theoretically explains why the return distribution is negatively skewed, making it sensible to include negative return skewness in our models. In other words, because humans react more strongly to negative return shocks than to positive return shocks, it is theoretically considered that stock returns are negatively skewed and volatility fluctuates asymmetrically. This suggests that the market reacts more strongly to negative information, providing theoretical support for incorporating volatility asymmetry (Engle and Ng, 1993; Zakoian, 1994; Bekaert and Wu, 2000; Wu, 2001) into the models. There are also other theoretical explanations for volatility asymmetry, such as the leverage effect (Black, 1976; Christie, 1982) and the volatility feedback effect (French et al., 1987; Campbell and Hentschel, 1992). However, we believe the above psychological explanations based on loss aversion are more reasonable.

After Fama (1970) proposed the efficient market hypothesis (EMH), Lo (2004, 2012) introduced the adaptive markets hypothesis (AMH). Fama's EMH focuses on the relationship between information and returns, rather than risk, and therefore cannot adequately explain stock market volatilities during negative events such as Russia's invasion of Ukraine. According to the AMH, markets adapt and evolve in response to various factors, ultimately becoming more efficient. In recent years, technological advancements and financial innovations like exchange-traded funds (ETFs) have led to faster trading frequencies. As the AMH suggests, this increased efficiency means that markets will react more sensitively to relevant news, leading to more frequent structural breaks, especially in recent years. Therefore, it is theoretically sound to incorporate these structural breaks into the models. We note that we are discussing AMH in the context of behavioral finance, in the sense that it allows human emotions and fluctuations during market turmoil to be more strongly expressed in the markets, demonstrating the importance of considering structural breaks especially in recent years.

The theoretical background provided above serves as the foundation for the development of our unified GARCH models. These theories also serve as the motivation behind the creation of our extended models. Specifically, in light of Russia's invasion of Ukraine, which is the focal point of our study, it is believed that stock return distributions will be negatively distorted. Additionally, volatility is expected to react asymmetrically to the sudden influx of negative information stemming from this event. Structural breaks are also anticipated to occur. Therefore, we emphasize that our extended unified GARCH models, which will be elaborated on later, are robust models with a strong theoretical foundation.

In summary, the main issue is that standard finance theory does not adequately explain the risk—the volatility of stock markets, especially when unexpected negative events occur. Human psychology becomes especially apparent during times of market stress. This is why behavioral finance is more suitable than standard finance theory in explaining increases in risk—volatility during market turmoil, such as Russia's invasion of Ukraine. During negative events, economic agents become even more irrational. This is why we rely on behavioral finance theory. As previously mentioned, it is important to emphasize that our extended models are based on key theories and therefore have strong connections to existing, practically useful significant theories. It is important to note that this advantageous characteristic of our current study sets it apart from mere statistical analysis, showing the high value of this paper.

3. Literature review

3.1. Review of existing studies

Recently, the academic literature has begun to explore the effects of the Russian invasion of Ukraine on February 24, 2022, on stock markets using various methods. This section reviews these studies, focusing on methodology. The first category of existing studies investigated the impact of the Russian invasion of Ukraine through event studies. The following studies utilized event study analysis. In a study by Boubaker et al. (2022), it was found that the invasion led to negative cumulative abnormal returns for global stock market indices, although the effects varied. Next, Nerlinger and Utz (2022) found that cumulative average abnormal returns of energy firms were positive around the event.

Furthermore, Yousaf et al. (2022) found that the Russian invasion had a strong negative impact on the stock markets of developed countries. Ahmed et al. (2023) found that European stocks experienced a significant negative abnormal return just a few days before the invasion, while Kumari et al. (2023) found different directional cumulative abnormal stock returns in European countries. Assaf et al. (2023) found that stock indices exhibited negative abnormal returns following the announcement of the invasion. The magnitude of these returns varied across different regions, including the US, EMEA (Europe, the Middle East, and Africa), and the Asia-Pacific regions. More recently, Kakhkharov et al. (2024) found that, overall, renewable energy and oil and gas producers performed better than the general stock market during the Russia–Ukraine conflict. Höhler et al. (2026) discovered that agribusiness firms in Europe and Japan were more negatively impacted by the Russia–Ukraine war than those in the United States.

The second category investigated the effects using VAR analysis. A study by Beraich et al. (2022) found that the connectivity between American, European, and Chinese stock markets became stronger before and after the Russian invasion using simple VAR analysis. Through quantile VAR analysis, Manelli et al. (2024) found a specific link between European stock prices, wheat prices, and gas futures prices during the invasion. The third category utilized a HAR modeling approach. Through HAR analysis, a study by Izzeldin et al. (2023) found that, overall, international stock markets responded rapidly to the Russian invasion. Moreover, the fourth category utilized a time-varying parameter VAR connectedness approach. Alam et al. (2022) discovered that the US, Canadian, Chinese, and Brazilian stock markets were influenced by other markets during the Russian invasion. Umar et al. (2022) also noted that European equities and Russian bonds impacted other markets during the invasion. Chowdhury et al. (2025) investigated the transmission of geopolitical risk to the stock, commodity, and energy markets, finding an increase in spillover during the Russia–Ukraine war. The three studies mentioned above employed time-varying parameter VAR connectedness analysis.

The fifth category utilized either panel data regression or standard regression. Boungou and Yatié (2022) discovered a negative relationship between the Russian invasion and world stock market returns during that period. Silva et al. (2023) found that trade exposures to countries involved in the conflict were crucial in determining the impact of the Russian invasion on international stock markets. Both studies mentioned above employed panel data analysis. Using regressions, Das et al. (2023) found evidence of the negative impact of the Ukraine–Russia conflict on stock returns in European stock markets, particularly affecting the mining, construction, and manufacturing sectors. Hasan et al. (2024) found that, internationally, traditional stock markets were more volatile and had lower returns than Fintech stocks during the Russia–Ukraine war compared to before the war. Federle et al. (2026) discovered that in countries geographically close to the war, markets suffered from sharply negative returns during the first couple of weeks of the conflict.

3.2. Research gap in literature

This section will discuss the research gap in current literature. As shown in the literature review above, there is a lack of research on the impacts of the Russian invasion of Ukraine in February 2022 using advanced GARCH models. Furthermore, quantitative models that incorporate not only GJR asymmetry but also skewed distribution errors and structural breaks, like the models we introduce in our current study, are scarce. In summary, previous studies have not attempted to forecast volatilities associated with the Russian invasion using extended advanced GARCH models like ours.

Furthermore, we emphasize that our study not only extends GARCH models but also offers insights into the connection between models, theory, and real-world data. We provide perspectives on the relationship between information, human psychology, and structural breaks, as well as valuable broader implications of the impacts of Russia's invasion of Ukraine. These novel insights are crucial and will inspire future research across multiple disciplines. Therefore, our new investigation into the volatilities and crash risk linked to the Russian invasion of Ukraine using the highly effective models developed in our current study significantly fills the research gap in existing literature. As a result, our research will provide fresh knowledge and insights not only to the field of quantitative economics and finance but also to other related fields more broadly.

4. Data

This study utilizes daily stock price data for the RTS index. Using the index price series at time t, denoted as p_t^{Index}, we calculate the daily log-difference percentage return series as $dlr_t^{Index} = \ln(p_t^{Index}/p_{t-1}^{Index}) \times 100$, following previous research (e.g., Tsuji, 2018, 2020, 2024; Mensi et al., 2019; Abakah et al., 2020). To evaluate the effectiveness of our unified models described later, we analyze the price (return) data from July 2, 2020 (July 3, 2020), to March 28, 2025. The RTS index price data from July 2, 2020, to June 20, 2024, is sourced from Bloomberg. However, from that date until March 28, 2025, it is obtained from Investing.com due to availability. This sample period includes the Russian invasion of Ukraine on February 24, 2022, and excludes the COVID-19 outbreak in early 2020. We emphasize that this is to focus on the impact of the Russian invasion and exclude the effect of the COVID-19 pandemic. Therefore, this sample period is ideal for analyzing the impact of the Russian invasion on Russian stock market volatility.

Table 1. Summary statistics for RTS index returns, July 2020 to March 2025.

Statistics	Values	Statistics	Values
Mean	−0.008	SD	2.451
Minimum	−48.292	Maximum	23.204
Skewness	−6.077	Kurtosis	139.293
ADF	−17.674	p-value (ADF)	0.000
JB	911,990.700	p-value (JB)	0.000

Notes: SD, ADF, and JB represent the standard deviation, the augmented Dickey–Fuller test statistic, and Jarque–Bera statistic, respectively. The sample period ranges from July 3, 2020, to March 28, 2025.

Figures 1 and 2 plot the price and return series, respectively, of the RTS index for our analysis period. Figure 1 shows that the RTS index plunged in early 2022, coinciding with the Russian invasion of Ukraine. This clearly indicates a rapid increase in volatility risk in the Russian stock market. Furthermore, Figure 2 illustrates that the RTS index returns exhibit huge fluctuations in early 2022, at the time of the Russian invasion. This clearly reflects a surge in crash risk in the Russian stock market. From these observations, we can assume that the Russian invasion of Ukraine had a great impact on the Russian stock market. Table 1 provides summary statistics for the RTS index return series. As shown, the returns have negative skewness and high kurtosis values, indicating the importance of using non-normal and skewed distribution errors when analyzing the series. Furthermore, the augmented Dickey–Fuller (ADF) test statistics indicate that the returns do not have a unit root. This suggests that the RTS index returns are stationary, supporting the use of GARCH analysis in our study.

Next, as the RTS index returns in Figure 2 indicate the presence of structural breaks, we identify these break points using the iterated cumulative sums of squares (ICSS) algorithm (Inclán and Tiao, 1994). It is important to note that the ICSS algorithm is a highly effective method for detecting structural breaks in financial time series (e.g., Wang and Moore, 2009; Ross, 2013; Malik, 2022). Specifically, the Bayesian information criterion (BIC) suggests that for our analysis period, the appropriate autoregressive lag order for the RTS index return is three. Therefore, we conduct the regression by specifying the intercept and first- to third-return lags for the RTS index returns: $dlr_t^{Index} = \eta + \sum_{p=1}^{3} \psi_p dlr_{t-p}^{Index} + \kappa_t$, and identify the structural break points in the return residuals, κ_t. Note that all model notations are fully described in the following section. Table 2 presents the identified break points for our analysis period. Specifically, there are 10 break points in the RTS index return residuals. These break points are also visually represented using ± 3 standard deviation bands of the return residuals in Figure 3. This figure clearly illustrates the importance of considering structural breaks when analyzing the effect of the Russian invasion of Ukraine on the Russian stock market.

Figure 1. Daily price evolution of the RTS index, July 2020 to March 2025.

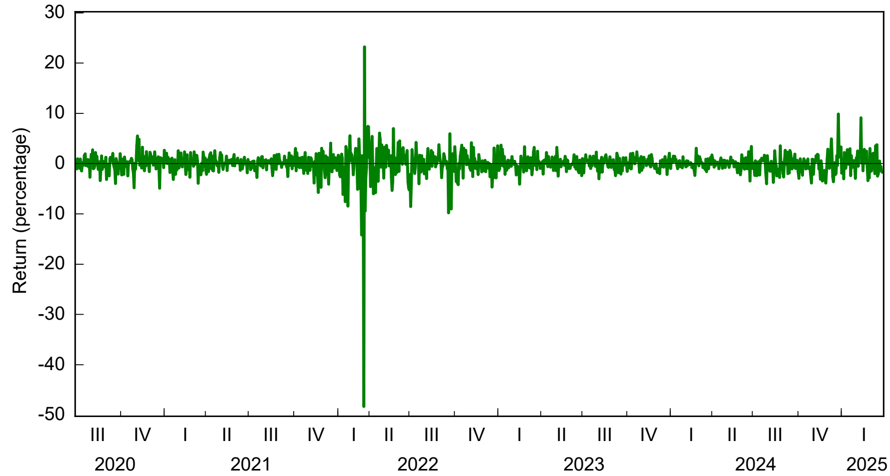

Figure 2. Daily return evolution of the RTS index, July 2020 to March 2025.

Table 2. Structural break points for the RTS index return residuals, July 2020 to March 2025.

No.	Date	No.	Date
1	April 22, 2021	6	July 7, 2022
2	November 11, 2021	7	January 11, 2023
3	January 12, 2022	8	September 22, 2023
4	February 23, 2022	9	June 18, 2024
5	February 25, 2022	10	November 22, 2024

Notes: Break points are identified by the ICSS algorithm. The sample period ranges from July 3, 2020, to March 28, 2025.

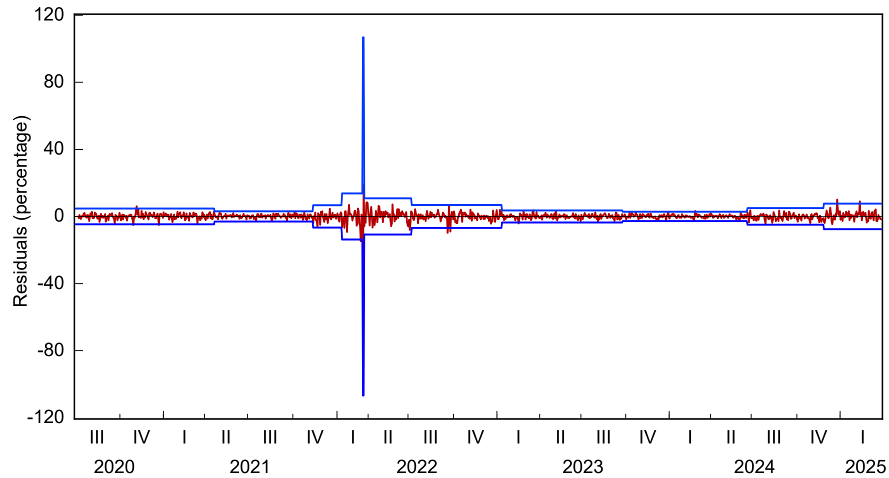

Figure 3. Structural breaks in the RTS index return residuals, July 2020 to March 2025.
Note: ±3 standard deviation bands and change points were identified by the ICSS algorithm.

5. Unified GARCH models

5.1. Model construction

In this section, we will develop unified models to estimate volatility. In this study, we define "unification" as the integration of two or three of the following elements: GJR asymmetry, skewed distribution errors, and structural breaks in a single model. Due to the non-normality of the RTS index returns as shown in Table 1, we will begin by using Student's t (T) distribution or GED errors in our model development. Our methodology for unifying models is outlined in Table A2 of the Appendix.

The first two base models for building our unified models are the following AR–T–GARCH and AR–GED–GARCH models:

$$\begin{aligned} dlr_t^{Index} &= \eta + \sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index} + \kappa_{t,t}, \\ h_{t,t} &= \omega + \xi \kappa_{t,t-1}^2 + \phi h_{t,t-1}, \end{aligned} \tag{1}$$

and

$$\begin{aligned} dlr_t^{Index} &= \eta + \sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index} + \kappa_{g,t}, \\ h_{g,t} &= \omega + \xi \kappa_{g,t-1}^2 + \phi h_{g,t-1}, \end{aligned} \tag{2}$$

where dlr_t^{Index} denotes the returns of the RTS index at time t; $\kappa_{t,t}$ ($\kappa_{g,t}$) is Student's t distribution (GED) errors at time t with distribution shape parameter ν (τ); and $\sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index}$ are autoregressive variables with coefficients ψ_p, where $k = 3$ for the RTS index lagged returns. Additionally, $h_{t,t}$ ($h_{g,t}$) is the variance to be estimated by model (1) (model (2)) at time t. Furthermore, η denotes the mean equation intercept, ω is the variance equation intercept, ξ is the ARCH-term coefficient, and ϕ is the GARCH-term coefficient.

The first step in our model building process is to include the asymmetric term, known as the GJR term, into the variance equations in models (1) and (2). Therefore, in this initial stage, our unified models are the following AR–T–GJR–GARCH and AR–GED–GJR–GARCH models:

$$\begin{aligned} dlr_t^{Index} &= \eta + \sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index} + \kappa_{t,t}, \\ h_{t,t} &= \omega + \xi \kappa_{t,t-1}^2 + \phi h_{t,t-1} + \delta S_{t,t-1}^{-} \kappa_{t,t-1}^2, \end{aligned} \tag{3}$$

and

$$
\begin{aligned}
dlr_t^{Index} &= \eta + \sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index} + \kappa_{g,t}, \\
h_{g,t} &= \omega + \xi \kappa_{g,t-1}^2 + \phi h_{g,t-1} + \delta S_{g,t-1}^{-} \kappa_{g,t-1}^2,
\end{aligned} \tag{4}
$$

where $\delta S_{t,t-1}^{-} \kappa_{t,t-1}^2$ and $\delta S_{g,t-1}^{-} \kappa_{g,t-1}^2$ represent the asymmetric terms, and $S_{t,t-1}^{-} = 1$ ($S_{g,t-1}^{-} = 1$) if the one-day lagged return residual $\kappa_{t,t-1} < 0$ ($\kappa_{g,t-1} < 0$) and is zero otherwise. δ is the coefficient such that a positive δ indicates the asymmetric effect of return residuals on the next day's variances. Note that the remaining notations for models (3) and (4) are the same as those for models (1) and (2), respectively.

The second step in our model building process is to incorporate skewness into the error distributions in models (3) and (4). Therefore our further unified models in this second step are the following AR–ST–GJR–GARCH and AR–SGED–GJR–GARCH models:

$$
\begin{aligned}
dlr_t^{Index} &= \eta + \sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index} + \kappa_{st,t}, \\
h_{st,t} &= \omega + \xi \kappa_{st,t-1}^2 + \phi h_{st,t-1} + \delta S_{st,t-1}^{-} \kappa_{st,t-1}^2,
\end{aligned} \tag{5}
$$

and

$$
\begin{aligned}
dlr_t^{Index} &= \eta + \sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index} + \kappa_{sg,t}, \\
h_{sg,t} &= \omega + \xi \kappa_{sg,t-1}^2 + \phi h_{sg,t-1} + \delta S_{sg,t-1}^{-} \kappa_{sg,t-1}^2,
\end{aligned} \tag{6}
$$

where $\kappa_{st,t}$ in model (5) ($\kappa_{sg,t}$ in model (6)) denotes the skew-t distribution (Hansen, 1994) errors (skew-GED (Theodossiou, 2015) errors) at time t with the shape parameter ν (τ) and skewness parameter χ (γ). In model (5), $\chi > 0$ ($\chi < 0$) means right-skewed (left-skewed) errors; and in model (6), $\gamma > 0$ ($\gamma < 0$) indicates right-skewed (left-skewed) errors.

The exact density function of the skew-t distribution is as follows:

$$
f(z_t|\nu,\chi) = \begin{cases} bc\left(1 + \dfrac{1}{\nu-2}\left(\dfrac{bz_t + a}{1-\chi}\right)^2\right)^{-(\nu+1)/2} & z_t < -a/b, \\ bc\left(1 + \dfrac{1}{\nu-2}\left(\dfrac{bz_t + a}{1+\chi}\right)^2\right)^{-(\nu+1)/2} & z_t \geq -a/b, \end{cases}
$$

where $z_t = (\kappa_{st,t} - \mu_{st})/\sigma_{st}$. μ_{st} is the mean of $\kappa_{st,t}$ and σ_{st} is the standard deviation of $\kappa_{st,t}$. The constants a, b, and c are given by

$$
a = 4\chi c\left(\frac{\nu-2}{\nu-1}\right),
$$

$$
b^2 = 1 + 3\chi^2 - a^2,
$$

and

$$c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)}.$$

In addition, the exact density function of the skew-GED is as follows:

$$f(u_t|\tau,\gamma,\varphi) = \frac{\tau^{1-\frac{1}{\tau}}}{2\varphi}\Gamma\left(\frac{1}{\tau}\right)^{-1}\exp\left(-\frac{1}{\tau}\frac{|u_t|^\tau}{(1+sgn(u_t)\gamma)^\tau\varphi^\tau}\right),$$

where $u_t = \kappa_{sg,t} - n$ and n is the mode of $\kappa_{sg,t}$. $sgn(u_t)$ is the sign function taking the value of -1 for $u_t < 0$ and 1 for $u_t \geq 0$, while $\Gamma(\cdot)$ is the gamma function. Further, φ is a scaling constant related to the standard deviation of $\kappa_{sg,t}$, given by

$$\varphi = q\sigma_{sg}.$$

σ_{sg} is the standard deviation of $\kappa_{sg,t}$ and

$$q = 1\Big/\sqrt{A_2 - A_1^2},$$

where

$$A_1 = 2\gamma\tau^{\frac{1}{\tau}}\Gamma\left(\frac{2}{\tau}\right)\Gamma\left(\frac{1}{\tau}\right)^{-1},$$

$$A_2 = (1+3\gamma^2)\tau^{\frac{2}{\tau}}\Gamma\left(\frac{3}{\tau}\right)\Gamma\left(\frac{1}{\tau}\right)^{-1}.$$

Furthermore, $h_{st,t}$ ($h_{sg,t}$) represents the variance to be estimated by model (5) (model (6)) at time t. Additionally, as in models (3) and (4), $\delta S^-_{st,t-1}\kappa^2_{st,t-1}$ and $\delta S^-_{sg,t-1}\kappa^2_{sg,t-1}$ are the GJR asymmetric terms. Note that the other notations for model (5) (model (6)) are the same as those for model (3) (model (4)).

The final step in our model unification process is to incorporate structural break effects into models (5) and (6). Specifically, the fully unified models resulting from this final step are the following AR–ST–SB–GJR–GARCH and AR–SGED–SB–GJR–GARCH models:

$$\begin{aligned} dlr_t^{Index} &= \eta + \sum_{p=1}^{k}\psi_p dlr_{t-p}^{Index} + \kappa_{stsb,t}, \\ h_{stsb,t} &= \omega + \xi\kappa^2_{stsb,t-1} + \phi h_{stsb,t-1} \\ &\quad + \delta S^-_{stsb,t-1}\kappa^2_{stsb,t-1} + \sum_{j=1}^{m}\lambda_j D_{j,t}^{SB}, \end{aligned} \tag{7}$$

and

$$\begin{aligned} dlr_t^{Index} &= \eta + \sum_{p=1}^{k} \psi_p dlr_{t-p}^{Index} + \kappa_{sgsb,t}, \\ h_{sgsb,t} &= \omega + \xi \kappa_{sgsb,t-1}^2 + \phi h_{sgsb,t-1} \\ &\quad + \delta S_{sgsb,t-1}^- \kappa_{sgsb,t-1}^2 + \sum_{j=1}^{m} \lambda_j D_{j,t}^{SB}, \end{aligned} \tag{8}$$

where $\kappa_{stsb,t}$ ($\kappa_{sgsb,t}$) in model (7) (model (8)) represents the skew-*t* distribution (skew-GED) errors at time t, and $h_{stsb,t}$ ($h_{sgsb,t}$) is the variance to be estimated by model (7) (model (8)) at time t.

In addition, $D_{j,t}^{SB}$ represents the j-th structural break dummy variable, m represents the number of structural breaks, and λ_j is the coefficient of the j-th structural break dummy. Our j-th structural break dummy variable remains at zero until the break point is identified for the return residuals using the ICSS algorithm, and one thereafter. Additionally, as in models (3)–(6), $\delta S_{stsb,t-1}^- \kappa_{stsb,t-1}^2$ and $\delta S_{sgsb,t-1}^- \kappa_{sgsb,t-1}^2$ are the GJR asymmetric terms. Note that including the parameters of skewed distributions, ν, τ, χ, and γ, the remaining notations in models (7) and (8) are the same as those in models (5) and (6), respectively. The skewed density for model (7) (model (8)) is the same as in model (5) (model (6)). Accordingly, we refer to models (7) and (8) as “fully unified” models whereas models (1)–(6) as “non-fully unified” models.

5.2. *Empirical evidence*

In this section, we present empirical evidence supporting the effectiveness of our model unifications. Table 3 displays the estimation results of models (1)–(8) for the RTS index returns. Panels A–D in Table 3 demonstrate that all these models are well estimated. Panels C and D in Table 3 reveal that all the parameters of the skew-*t* and skew-GED errors—ν, τ, χ, and γ—are consistently statistically significant. It is worth noting that the skewness parameters—χ and γ—all have negative signs. This indicates that our incorporation of skew-*t* and skew-GED errors effectively captures the negative skewness of the RTS index returns.

Additionally, Panel D of Table 3 shows that all coefficients of our structural break dummy variables, λ, are statistically significant for the RTS stock index. These findings highlight the effectiveness of integrating structural breaks into our GJR–GARCH analysis, as it accurately captures the structural breaks in the RTS index returns. Furthermore, the coefficients of GJR asymmetry, δ, consistently display statistically significant positive values in Panels B–D of Table 3, indicating that negative asymmetries are consistently observed in the RTS index return volatilities. Overall, our findings demonstrate the robustness of our unification of GJR asymmetry, skewness, and structural breaks in estimating RTS index volatilities affected by the significant event, the Russian invasion of Ukraine.

To further evaluate model effectiveness, we also conduct likelihood ratio (LR) tests to compare model performance and present the results in Table 4. As shown in Panel A–1, the null hypothesis that the AR–T–GARCH model (1) is superior to the AR–T–GJR–GARCH model (3) is strongly rejected, indicating the effectiveness of employing GJR asymmetry in our Student’s *t* error model. Panel A–2 shows that the null hypothesis that the AR–T–GJR–GARCH model (3) is superior to the AR–ST–GJR–GARCH model (5) is also strongly rejected, signifying the effectiveness of incorporating skew-*t* errors instead of simple Student’s *t* errors into our models.

Table 3. Estimation results for model unifications for the RTS index: July 2020 to March 2025.

Panel A. Base models					
AR–T–GARCH			AR–GED–GARCH		
Mean equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
η	0.064	0.101	η	0.085**	0.018
ψ_1	0.053*	0.071	ψ_1	0.027	0.282
ψ_2	−0.038	0.207	ψ_2	−0.032	0.240
ψ_3	0.024	0.405	ψ_3	0.013	0.632
Variance equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
ω	0.079***	0.010	ω	0.071**	0.033
ξ	0.099***	0.000	ξ	0.119***	0.000
ϕ	0.882***	0.000	ϕ	0.869***	0.000
ν	4.831***	0.000	τ	1.699***	0.000
LL	−2,196.254		LL	−2,210.696	
Panel B. Asymmetry-incorporated models					
AR–T–GJR–GARCH			AR–GED–GJR–GARCH		
Mean equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
η	0.043	0.276	η	0.052	0.190
ψ_1	0.062**	0.037	ψ_1	0.041	0.111
ψ_2	−0.031	0.308	ψ_2	−0.024	0.530
ψ_3	0.031	0.290	ψ_3	0.022	0.446
Variance equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
ω	0.074***	0.009	ω	0.070**	0.033
ξ	0.042**	0.032	ξ	0.046**	0.036
ϕ	0.884***	0.000	ϕ	0.876***	0.000
δ	0.102***	0.003	δ	0.117***	0.002
ν	5.136***	0.000	τ	1.644***	0.000
LL	−2,190.683		LL	−2,204.694	
Panel C. Asymmetry- and skewed error-incorporated models					
AR–ST–GJR–GARCH			AR–SGED–GJR–GARCH		
Mean equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
η	−0.005	0.906	η	0.210***	0.002
ψ_1	0.051*	0.083	ψ_1	0.038	0.181
ψ_2	−0.034	0.263	ψ_2	−0.022	0.465
ψ_3	0.030	0.301	ψ_3	0.023	0.407
Variance equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value

ω	0.070***	0.007	ω	0.052*	0.059
ξ	0.045**	0.014	ξ	0.052**	0.033
ϕ	0.886***	0.000	ϕ	0.885***	0.000
δ	0.098***	0.003	δ	0.086**	0.011
ν	5.305***	0.000	τ	1.231***	0.000
χ	−0.136***	0.000	γ	−0.104***	0.003
LL	−2,185.107		LL	−2,199.344	
Panel D. Asymmetry-, skewed error-, and structural break-incorporated models					
AR–ST–SB–GJR–GARCH			AR–SGED–SB–GJR–GARCH		
Mean equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
η	0.030	0.455	η	0.177***	0.007
ψ_1	0.071**	0.017	ψ_1	0.061**	0.034
ψ_2	−0.037	0.204	ψ_2	−0.034	0.239
ψ_3	0.038	0.179	ψ_3	0.029	0.274
Variance equations					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
ω	0.864***	0.000	ω	0.855***	0.000
ξ	−0.031	0.121	ξ	−0.029	0.180
ϕ	0.615***	0.000	ϕ	0.617***	0.000
δ	0.141***	0.002	δ	0.129***	0.002
λ_1	−0.470***	0.000	λ_1	−0.493***	0.001
λ_2	1.224**	0.011	λ_2	1.215**	0.024
λ_3	4.195***	0.000	λ_3	4.479***	0.000
λ_4	287.879***	0.000	λ_4	258.735***	0.000
λ_5	−290.426***	0.000	λ_5	−261.679***	0.000
λ_6	−2.010***	0.005	λ_6	−1.797**	0.037
λ_7	−0.742***	0.003	λ_7	−0.827***	0.006
λ_8	−0.203***	0.006	λ_8	−0.203**	0.018
λ_9	0.575***	0.002	λ_9	0.575***	0.002
λ_{10}	1.330***	0.002	λ_{10}	1.447***	0.004
ν	9.998***	0.000	τ	1.566***	0.000
χ	−0.080*	0.054	γ	−0.072**	0.025
LL	−2,146.366		LL	−2,148.211	

Notes: LL represents the log-likelihood value. ***, **, and * denote the 1%, 5%, and 10% significance levels, respectively. All parameters in the eight models in this table are clearly defined in Section 5.1.

Table 4. LR tests and AIC values for model selection for the RTS index: July 2020 to March 2025.

Panel A. LR tests	
A–1. AR–T–GARCH vs. AR–T–GJR–GARCH	
χ^2 statistic	11.141***
p-value	0.001
A–2. AR–T–GJR–GARCH vs. AR–ST–GJR–GARCH	
χ^2 statistic	11.154***
p-value	0.001
A–3. AR–ST–GJR–GARCH vs. AR–ST–SB–GJR–GARCH	
χ^2 statistic	77.480***
p-value	0.000
A–4. AR–GED–GARCH vs. AR–GED–GJR–GARCH	
χ^2 statistic	12.004***
p-value	0.001
A–5. AR–GED–GJR–GARCH vs. AR–SGED–GJR–GARCH	
χ^2 statistic	10.700***
p-value	0.001
A–6. AR–SGED–GJR–GARCH vs. AR–SGED–SB–GJR–GARCH	
χ^2 statistic	102.267***
p-value	0.000
Panel B. AIC values	
B–1. AR–T–GARCH	
AIC	4,408.507
B–2. AR–T–GJR–GARCH	
AIC	4,399.367
B–3. AR–ST–GJR–GARCH	
AIC	4,390.213
B–4. AR–ST–SB–GJR–GARCH	
AIC	4,332.733
B–5. AR–GED–GARCH	
AIC	4,437.393
B–6. AR–GED–GJR–GARCH	
AIC	4,427.389
B–7. AR–SGED–GJR–GARCH	
AIC	4,418.688
B–8. AR–SGED–SB–GJR–GARCH	
AIC	4,336.422

Notes: *** denotes the 1% significance level. The test statistic follows a χ^2 distribution. The null hypotheses are as follows. The AR–T–GARCH model is superior to the AR–T–GJR–GARCH model (A–1); the AR–T–GJR–GARCH model is superior to the AR–ST–GJR–GARCH model (A–2); the AR–ST–GJR–GARCH model is superior to the AR–ST–SB–GJR–GARCH model (A–3). The AR–GED–GARCH model is superior to the AR–GED–GJR–GARCH model (A–4); the AR–GED–GJR–GARCH model is superior to the AR–SGED–GJR–GARCH model (A–5); the AR–SGED–GJR–GARCH model is superior to the AR–SGED–SB–GJR–GARCH model (A–6). AIC stands for Akaike information criteria.

Moreover, Panel A–3 of Table 4 displays the null hypothesis that the AR–ST–GJR–GARCH model (5) is superior to the AR–ST–SB–GJR–GARCH model (7) is also strongly rejected, indicating the meaningful incorporation of structural breaks in the analysis. These results for the LR tests clearly demonstrate the superiority of our first fully unified model, i.e., the AR–ST–SB–GJR–GARCH model (7). Additionally, Panel A–4 in Table 4 indicates that our LR test strongly rejects the null hypothesis that the AR–GED–GARCH model (2) is superior to the AR–GED–GJR–GARCH model (4), showing the effectiveness of employing GJR asymmetry for our GED-error model. Furthermore, Panel A–5 shows that the null hypothesis that the AR–GED–GJR–GARCH model (4) is superior to the AR–SGED–GJR–GARCH model (6) is strongly rejected, suggesting the effectiveness of incorporating skew-GED errors rather than simple GED errors in our models.

Finally, Panel A–6 shows that the null hypothesis that the AR–SGED–GJR–GARCH model (6) is superior to the AR–SGED–SB–GJR–GARCH model (8) is strongly rejected, indicating the meaningful incorporation of structural breaks in the SGED-error model. These results for the LR tests strongly demonstrate the superiority of our second fully unified model, i.e., the AR–SGED–SB–GJR-GARCH model (8). Therefore, the results of our LR tests once again demonstrate the robustness of our model unification approach in estimating volatilities affected by the significant event, the Russian invasion. We also displayed the Akaike information criteria (AIC) values for all estimated models in Panel B of Table 4. As the complexity of models increases, the AIC values consistently decrease, suggesting that over-parameterization is not a concern in our model unifications.

In summary, the skewed errors and asymmetry included in the models effectively captured the negative skewness and shocks of stock returns. The structural breaks incorporated into the models also effectively predicted the increase in the volatility of stock returns associated with the negative event. These factors are the reasons why the fully unified models we developed produced superior estimation results.

5.3. *Analyzing volatilities affected by the Russian invasion of Ukraine*

To analyze the volatilities affected by the Russian invasion of Ukraine, we provided the annualized volatilities of the RTS index returns derived from the AR–T–GJR–GARCH, AR–ST–GJR–GARCH, and AR–ST–SB–GJR–GARCH models using samples from July 3, 2020, to March 28, 2025, in Figure 4. Additionally, we presented the annualized volatilities from the AR–GED–GJR–GARCH, AR–SGED–GJR–GARCH, and AR–SGED–SB–GJR–GARCH models using the same samples in Figure 5.

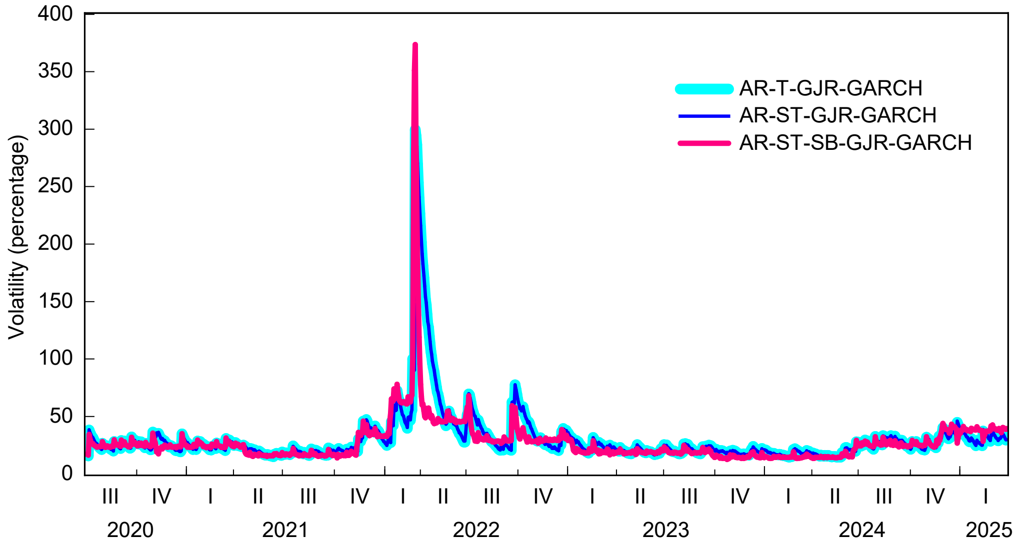

Figure 4. Annualized volatilities of RTS index returns from the AR–T–GJR–GARCH, AR–ST–GJR–GARCH, and AR–ST–SB–GJR–GARCH models: July 2020 to March 2025.

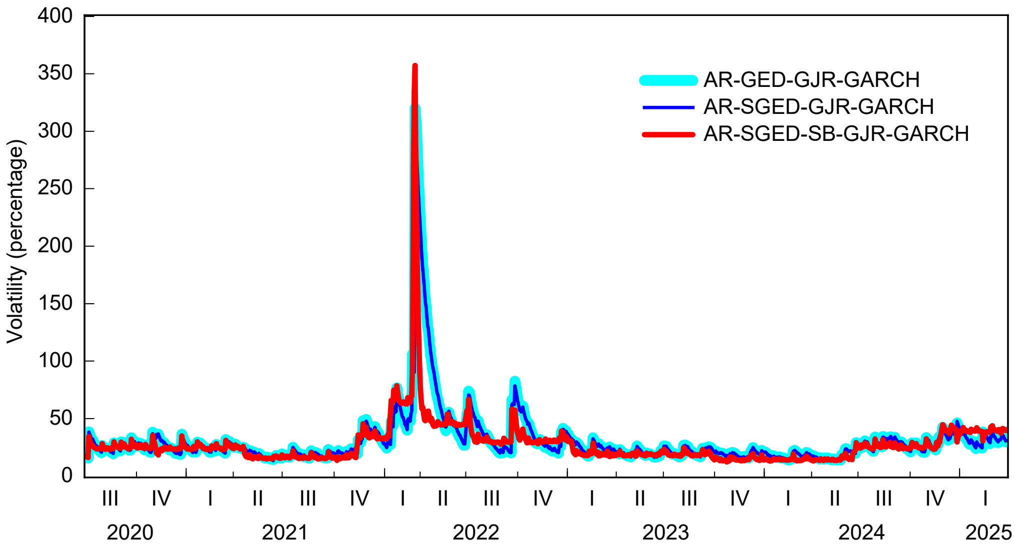

Figure 5. Annualized volatilities of RTS index returns from the AR-GED–GJR–GARCH, AR–SGED–GJR–GARCH, and AR–SGED–SB–GJR–GARCH models: July 2020 to March 2025.

Most importantly, Figures 4 and 5 indicate that for the RTS index, the volatilities calculated from our two fully unified models precede the volatilities derived from the other non-fully unified models, especially when Russia invaded Ukraine. We emphasize that these forward-looking characteristics of the volatilities from our two fully unified models suggest the possibility of forecasting the crash risk associated with the Russian invasion of Ukraine.

6. Was the crash risk related to the Russian invasion predictable?

Up to this point, we have demonstrated the effectiveness of our two fully unified GARCH models. This indicates that using the volatility estimates from our extended models for practical risk management is worthwhile. With that in mind, this section empirically illustrates the benefits and advantages of our fully unified GARCH models in practical risk management. This empirical analysis and discussion will provide more comprehensive and practical implications, showing how our extended models can be applied in real-world volatility predictions. This further emphasizes the significance of this study.

Table 5. LR tests and AIC values for model selection of the RTS index: One week before the Russian invasion.

Panel A. LR tests	
A–1. T–GARCH vs. T–GJR–GARCH	
χ^2 statistic	13.202***
p-value	0.000
A–2. T–GJR–GARCH vs. ST–GJR–GARCH	
χ^2 statistic	24.041***
p-value	0.000
A–3. ST–GJR–GARCH vs. ST–SB–GJR–GARCH	
χ^2 statistic	38.807***
p-value	0.000
A–4. GED–GARCH vs. GED–GJR–GARCH	
χ^2 statistic	15.965***
p-value	0.000
A–5. GED–GJR–GARCH vs. SGED–GJR–GARCH	
χ^2 statistic	26.381***
p-value	0.000
A–6. SGED–GJR–GARCH vs. SGED–SB–GJR–GARCH	
χ^2 statistic	61.880***
p-value	0.000
Panel B. AIC values	
B–1. T–GARCH	
AIC	3,559.068
B–2. T–GJR–GARCH	
AIC	3,547.866
B–3. ST–GJR–GARCH	
AIC	3,525.826
B–4. ST–SB–GJR–GARCH	
AIC	3,503.019
B–5. GED–GARCH	
AIC	3,587.231

B–6. GED–GJR–GARCH	
AIC	3,573.266
B–7. SGED–GJR–GARCH	
AIC	3,548.885
B–8. SGED–SB–GJR–GARCH	
AIC	3,503.005

Notes: Results from 1,000 samples up to one week before the Russian invasion of Ukraine. *** denotes the 1% significance level. The test statistic follows a χ^2 distribution. The null hypotheses are as follows. The T–GARCH model is superior to the T–GJR–GARCH model (A–1); the T–GJR–GARCH model is superior to the ST–GJR–GARCH model (A–2); the ST–GJR–GARCH model is superior to the ST–SB–GJR–GARCH model (A–3). The GED–GARCH model is superior to the GED–GJR–GARCH model (A–4); the GED–GJR–GARCH model is superior to the SGED–GJR–GARCH model (A–5); the SGED–GJR–GARCH model is superior to the SGED–SB–GJR–GARCH model (A–6). AIC stands for Akaike information criteria.

6.1. Prediction as of one week before the Russian invasion

This section will empirically demonstrate the predictive power of the volatility estimates from our fully unified models. As documented, the Russian invasion on February 24, 2022, has led to increased crash risk in the Russian stock market. Taking this into account and considering the ease of application, we estimated the ST–SB–GJR–GARCH model (7) and SGED–SB–GJR–GARCH model (8) using the historical 1,000 daily samples up to one week before the Russian invasion (February 17, 2022). For the 1,000 samples, it was determined that the appropriate autoregressive lag order for the RTS index return is zero. Subsequently, we conducted LR tests as before and presented the results in Table 5.

Panels A–1 to A–3 of Table 5 show that the T–GJR–GARCH model (3) is superior to the T–GARCH model (1), the ST–GJR–GARCH model (5) is superior to the T–GJR–GARCH model (3), and the ST–SB–GJR–GARCH model (7) is superior to the ST–GJR–GARCH model (5). Additionally, Panels A–4 to A–6 of Table 5 show that the GED–GJR–GARCH model (4) is superior to the GED–GARCH model (2), the SGED–GJR–GARCH model (6) is superior to the GED–GJR–GARCH model (4), and the SGED–SB–GJR–GARCH model (8) is superior to the SGED–GJR–GARCH model (6). We also displayed the AIC values for all estimated models in Panel B of Table 5. As the complexity of models increases, the AIC values consistently decrease, indicating that there is no over-parameterization. Therefore, these results demonstrate the robustness of our model-building approach.

Next, we evaluate the predictability of the volatility estimates from our fully unified models (7)–(8) using these 1,000 historical samples. This is achieved by applying the following regression model:

$$y_t \quad = \quad \chi_0^{1week} + \chi_1^{1week} x_{t-k} + \tau_t^{1week}, \tag{9}$$

where y_t represents the estimated volatility from the non-fully unified model (1), (2), (3), (4), (5), or (6) at time t, while x_{t-k} represents the estimated volatility from our fully unified model (7) or (8) at time $t-k$. The value of k can be 1, 2, or 3, allowing us to test the forecasting power from one to three days ahead. χ_0^{1week} denotes the constant term, χ_1^{1week} is the coefficient of x_{t-k}, and τ_t^{1week} is the residual of the regression. Therefore, a statistically significant positive value of χ_1^{1week} indicates predictability in the volatility estimates from our fully unified models (7)–(8).

Table 6. Predictive power of the volatility estimates from fully unified GARCH models of the RTS index: One week before the Russian invasion.

Panel A. Predictive power of the estimates from the ST–SB–GJR–GARCH model					
Prediction target volatilities					
T–GJR–GARCH			GED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
χ_0^{1week}	0.013	0.871	χ_0^{1week}	0.013	0.879
χ_1^{1week}	0.139**	0.023	χ_1^{1week}	0.148**	0.024
$Adj.R^2$	0.014		$Adj.R^2$	0.014	
Two-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
χ_0^{1week}	0.007	0.918	χ_0^{1week}	0.007	0.927
χ_1^{1week}	0.337***	0.003	χ_1^{1week}	0.359***	0.003
$Adj.R^2$	0.089		$Adj.R^2$	0.088	
Three-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
χ_0^{1week}	0.018	0.834	χ_0^{1week}	0.019	0.840
χ_1^{1week}	0.012	0.800	χ_1^{1week}	0.010	0.844
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Prediction target volatilities					
ST–GJR–GARCH			SGED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
χ_0^{1week}	0.013	0.867	χ_0^{1week}	0.013	0.873
χ_1^{1week}	0.137**	0.021	χ_1^{1week}	0.137**	0.022
$Adj.R^2$	0.015		$Adj.R^2$	0.014	
Two-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
χ_0^{1week}	0.007	0.913	χ_0^{1week}	0.007	0.917
χ_1^{1week}	0.330***	0.003	χ_1^{1week}	0.325***	0.003
$Adj.R^2$	0.090		$Adj.R^2$	0.083	
Three-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
χ_0^{1week}	0.018	0.831	χ_0^{1week}	0.018	0.833
χ_1^{1week}	0.012	0.786	χ_1^{1week}	0.009	0.844
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Panel B. Predictive power of the estimates from the SGED–SB–GJR–GARCH model					

Prediction target volatilities					
T–GJR–GARCH			GED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{1week}	0.013	0.871	χ_0^{1week}	0.013	0.878
χ_1^{1week}	0.131**	0.026	χ_1^{1week}	0.140**	0.027
$Adj.R^2$	0.015		$Adj.R^2$	0.014	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{1week}	0.006	0.926	χ_0^{1week}	0.006	0.935
χ_1^{1week}	0.351***	0.006	χ_1^{1week}	0.375***	0.005
$Adj.R^2$	0.111		$Adj.R^2$	0.109	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{1week}	0.019	0.833	χ_0^{1week}	0.019	0.839
χ_1^{1week}	0.004	0.928	χ_1^{1week}	0.002	0.970
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Prediction target volatilities					
ST–GJR–GARCH			SGED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{1week}	0.013	0.866	χ_0^{1week}	0.013	0.873
χ_1^{1week}	0.129**	0.025	χ_1^{1week}	0.129**	0.025
$Adj.R^2$	0.015		$Adj.R^2$	0.014	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{1week}	0.007	0.922	χ_0^{1week}	0.006	0.925
χ_1^{1week}	0.345***	0.006	χ_1^{1week}	0.339***	0.005
$Adj.R^2$	0.113		$Adj.R^2$	0.104	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{1week}	0.019	0.829	χ_0^{1week}	0.019	0.832
χ_1^{1week}	0.005	0.913	χ_1^{1week}	0.002	0.971
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	

Notes: Results from 1,000 samples up to one week before the Russian invasion of Ukraine. $Adj.R^2$: adjusted R-squared value. *** and ** denote the 1% and 5% significance levels, respectively. Every regression is conducted using the Newey–West heteroscedasticity- and autocorrelation-consistent covariance matrix. All parameters in this table are clearly defined in Section 6.1.

Table 6 displays the regression results. We used the Newey and West (1987) heteroscedasticity- and autocorrelation-consistent (HAC) covariance matrix for robust estimation in every regression specified in equations (9) through (12). In Panels A and B of Table 6, the estimates of χ_1^{1week} for x_{t-1} and x_{t-2} all show statistically significant positive values. This indicates that for the historical 1,000 daily samples up to one week before the Russian invasion, the volatility estimates from our fully unified models (7)–(8) have predictive power for the volatilities from the non-fully unified models (1)–(6) one to two days ahead.

In contrast, we also assess the predictability of the volatility estimates from the non-fully unified models (1)–(6) by using the following regression model and the same 1,000 samples:

$$y_t \quad = \quad \gamma_0^{1week} + \gamma_1^{1week} x_{t-k} + \nu_t^{1week}, \tag{10}$$

where y_t is the estimated volatility from our fully unified model (7) or (8) at time t, while x_{t-k} is the estimated volatility from the non-fully unified model (1), (2), (3), (4), (5), or (6) at time $t-k$. The value of k can be 1, 2, or 3, allowing us to test the forecasting power from one to three days ahead. γ_0^{1week} denotes the constant term, γ_1^{1week} is the coefficient of x_{t-k}, and ν_t^{1week} is the residual of the regression. Therefore, a statistically significant positive value of γ_1^{1week} indicates predictability in the volatility estimates from the non-fully unified models (1)–(6).

Table 7 displays the regression results. In Panels A–D, no statistically significant positive value for the estimates of γ_1^{1week} is obtained. This indicates that the volatility estimates from the non-fully unified models (1)–(6) have no forecasting power for the volatilities of our fully unified models (7)–(8). Therefore, our results empirically prove that the volatilities from our fully unified models have a strong predictive power of Russian stock return fluctuations, whereas the non-fully unified models do not possess such strong power.

Furthermore, we plotted the volatility estimates in Figure 6, which were estimated using the 1,000 daily samples up to one week before the Russian invasion. These plots cover the 30 business days leading up to February 17, 2022, which is one week before the Russian invasion. The volatility pairs shown are estimates from the ST–SB–GJR–GARCH model and T–GJR–GARCH model (Panel A), the ST–SB–GJR–GARCH model and ST–GJR–GARCH model (Panel B), the SGED–SB–GJR–GARCH model and GED–GJR–GARCH model (Panel C), and the SGED–SB–GJR–GARCH model and SGED–GJR–GARCH model (Panel D).

Notably, Panels A and B in Figure 6 demonstrate that the ST–SB–GJR–GARCH model estimates had a significantly earlier response compared to the other model estimates, which exhibited slower and smaller responses. The volatility estimates from the non-fully unified models decrease as we approach the date one week before the Russian invasion. Similarly, Panels C and D in Figure 6 show that the SGED–SB–GJR–GARCH model estimates also responded significantly earlier compared to the other model estimates, which showed slower and smaller responses. It is worth noting again that the volatility estimates from the models that are not fully unified decrease as we approach the date one week before the Russian invasion. This evidence suggests that as of one week prior, the ST–SB–GJR–GARCH and SGED–SB–GJR–GARCH models more strongly predicted the crash risk due to the Russian invasion of Ukraine.

Table 7. Predictive power of the volatility estimates from non-fully unified GARCH models of the RTS index: One week before the Russian invasion.

Panel A. Predictive power of the estimates from the T–GJR–GARCH model					
Prediction target volatilities					
ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{1week}	0.034	0.745	γ_0^{1week}	0.036	0.749
γ_1^{1week}	−0.134***	0.000	γ_1^{1week}	−0.157***	0.000
$Adj.R^2$	0.022		$Adj.R^2$	0.026	
Two-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{1week}	0.034	0.733	γ_0^{1week}	0.035	0.740
γ_1^{1week}	−0.028	0.443	γ_1^{1week}	−0.034	0.347
$Adj.R^2$	−4.5E−05		$Adj.R^2$	2.7E−04	
Three-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{1week}	0.035	0.723	γ_0^{1week}	0.036	0.732
γ_1^{1week}	−0.047	0.203	γ_1^{1week}	−0.043	0.270
$Adj.R^2$	0.002		$Adj.R^2$	0.001	
Panel B. Predictive power of the estimates from the GED–GJR–GARCH model					
Prediction target volatilities					
ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{1week}	0.034	0.745	γ_0^{1week}	0.035	0.750
γ_1^{1week}	−0.124***	0.000	γ_1^{1week}	−0.146***	0.000
$Adj.R^2$	0.021		$Adj.R^2$	0.026	
Two-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{1week}	0.034	0.733	γ_0^{1week}	0.035	0.740
γ_1^{1week}	−0.026	0.436	γ_1^{1week}	−0.032	0.340
$Adj.R^2$	−2.7E−05		$Adj.R^2$	3.0E−04	
Three-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{1week}	0.035	0.724	γ_0^{1week}	0.036	0.732
γ_1^{1week}	−0.043	0.211	γ_1^{1week}	−0.039	0.281
$Adj.R^2$	0.002		$Adj.R^2$	0.001	
Panel C. Predictive power of the estimates from the ST–GJR–GARCH model					
Prediction target volatilities					
ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					

Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{1week}	0.034	0.744	γ_0^{1week}	0.036	0.749
γ_1^{1week}	−0.137***	0.000	γ_1^{1week}	−0.162***	0.000
$Adj.R^2$	0.021		$Adj.R^2$	0.026	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{1week}	0.034	0.733	γ_0^{1week}	0.035	0.740
γ_1^{1week}	−0.027	0.468	γ_1^{1week}	−0.033	0.370
$Adj.R^2$	−1.4E−04		$Adj.R^2$	1.6E−04	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{1week}	0.035	0.723	γ_0^{1week}	0.036	0.732
γ_1^{1week}	−0.047	0.215	γ_1^{1week}	−0.043	0.283
$Adj.R^2$	0.002		$Adj.R^2$	0.001	
Panel D. Predictive power of the estimates from the SGED–GJR–GARCH model					
Prediction target volatilities					
ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{1week}	0.034	0.745	γ_0^{1week}	0.036	0.749
γ_1^{1week}	−0.133***	0.000	γ_1^{1week}	−0.159***	0.000
$Adj.R^2$	0.021		$Adj.R^2$	0.027	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{1week}	0.034	0.733	γ_0^{1week}	0.035	0.740
γ_1^{1week}	−0.032	0.372	γ_1^{1week}	−0.039	0.280
$Adj.R^2$	2.6E−04		$Adj.R^2$	0.001	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{1week}	0.035	0.723	γ_0^{1week}	0.036	0.732
γ_1^{1week}	−0.049	0.180	γ_1^{1week}	−0.045	0.239
$Adj.R^2$	0.002		$Adj.R^2$	0.001	

Notes: Results from 1,000 samples up to one week before the Russian invasion of Ukraine. $Adj.R^2$: adjusted R-squared value. *** denotes the 1% significance level. Every regression is conducted using the Newey–West heteroscedasticity- and autocorrelation-consistent covariance matrix. All parameters in this table are clearly defined in Section 6.1.

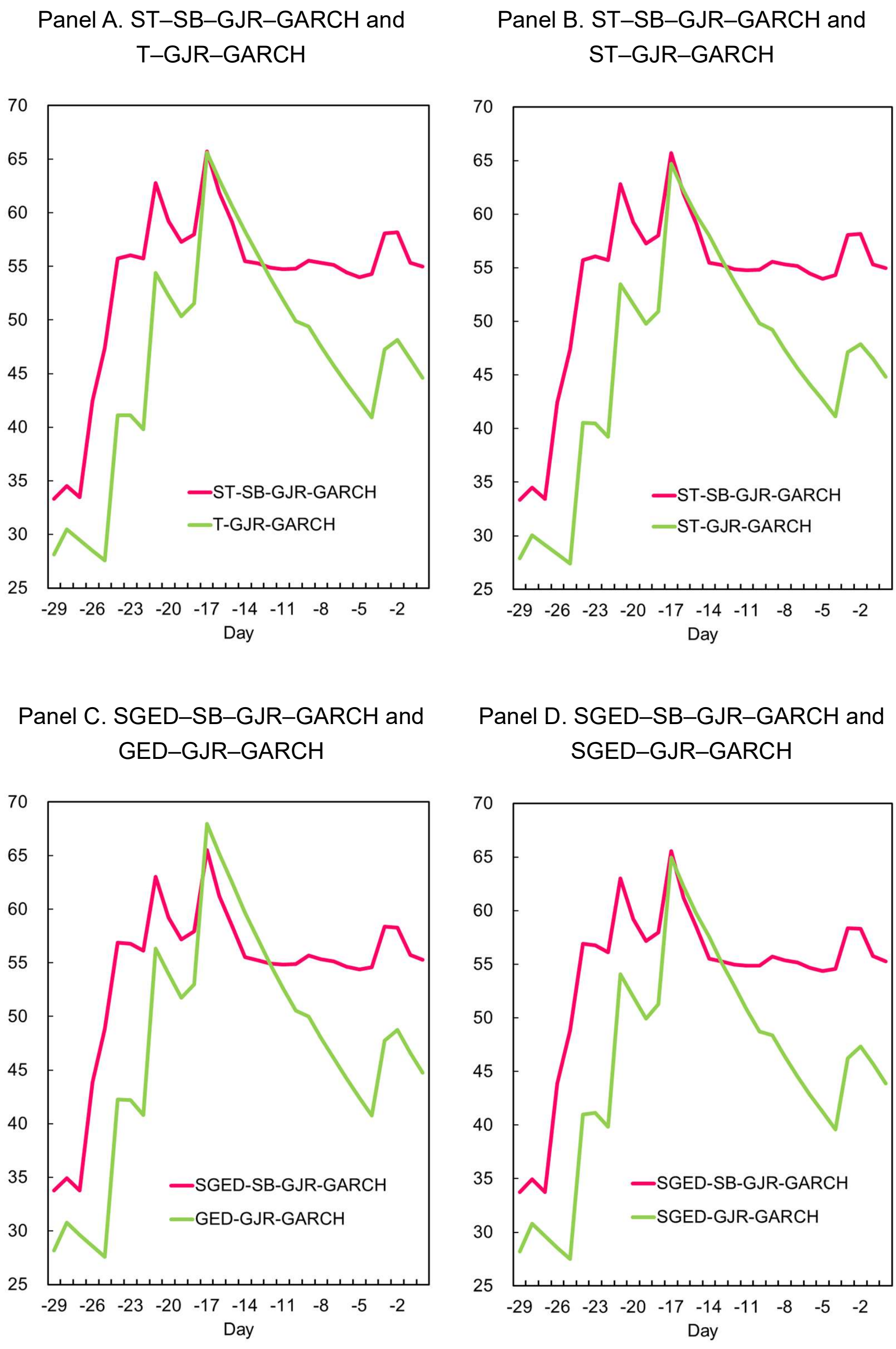

Figure 6. Predictability comparisons of daily volatilities of the RTS index: One week before the Russian invasion.
Note: Estimated volatilities are plotted for the 30 business days leading up to one week before the Russian invasion of Ukraine.

6.2. Prediction as of four weeks before the Russian invasion

For robustness checks and again considering ease of application, we estimated the ST–SB–GJR–GARCH model (7) and SGED–SB–GJR–GARCH model (8) using the historical 1,000 daily samples up to four weeks before the Russian invasion (January 27, 2022). After analyzing the 1,000 samples, we determined that the appropriate autoregressive lag order for the RTS index return is zero. We then conducted LR tests as before and presented the results in Table 8.

Table 8. LR tests and AIC values for model selection of the RTS index: Four weeks before the Russian invasion.

Panel A. LR tests	
A–1. T–GARCH vs. T–GJR–GARCH	
χ^2 statistic	13.945***
p-value	0.000
A–2. T–GJR–GARCH vs. ST–GJR–GARCH	
χ^2 statistic	23.317***
p-value	0.000
A–3. ST–GJR–GARCH vs. ST–SB–GJR–GARCH	
χ^2 statistic	39.502***
p-value	0.000
A–4. GED–GARCH vs. GED–GJR–GARCH	
χ^2 statistic	16.730***
p-value	0.000
A–5. GED–GJR–GARCH vs. SGED–GJR–GARCH	
χ^2 statistic	27.213***
p-value	0.000
A–6. SGED–GJR–GARCH vs. SGED–SB–GJR–GARCH	
χ^2 statistic	62.355***
p-value	0.000
Panel B. AIC values	
B–1. T–GARCH	
AIC	3,539.652
B–2. T–GJR–GARCH	
AIC	3,527.707
B–3. ST–GJR–GARCH	
AIC	3,506.389
B–4. ST–SB–GJR–GARCH	
AIC	3,482.888
B–5. GED–GARCH	
AIC	3,568.447
B–6. GED–GJR–GARCH	
AIC	3,553.717

B–7. SGED–GJR–GARCH	
AIC	3,528.504
B–8. SGED–SB–GJR–GARCH	
AIC	3,482.150

Notes: Results from 1,000 samples up to four weeks before the Russian invasion of Ukraine. *** denotes the 1% significance level. The test statistic follows a χ^2 distribution. The null hypotheses are as follows. The T–GARCH model is superior to the T–GJR–GARCH model (A–1); the T–GJR–GARCH model is superior to the ST–GJR–GARCH model (A–2); the ST–GJR–GARCH model is superior to the ST–SB–GJR–GARCH model (A–3). The GED–GARCH model is superior to the GED–GJR–GARCH model (A–4); the GED–GJR–GARCH model is superior to the SGED–GJR–GARCH model (A–5); the SGED–GJR–GARCH model is superior to the SGED–SB–GJR–GARCH model (A–6). AIC stands for Akaike information criteria.

Panels A–1 to A–3 of Table 8 show that the T–GJR–GARCH model (3) is superior to the T–GARCH model (1), the ST–GJR–GARCH model (5) is superior to the T–GJR–GARCH model (3), and the ST–SB–GJR–GARCH model (7) is superior to the ST–GJR–GARCH model (5). Furthermore, Panels A–4 to A–6 of Table 8 indicate that the GED–GJR–GARCH model (4) is superior to the GED–GARCH model (2), the SGED–GJR–GARCH model (6) is superior to the GED–GJR–GARCH model (4), and the SGED–SB–GJR–GARCH model (8) is superior to the SGED–GJR–GARCH model (6). We also displayed the AIC values for all estimated models in Panel B of Table 8. As the complexity of models increases, the AIC values consistently decrease, indicating no over-parameterization. Therefore, these results once again demonstrate the robustness of our model unification approach.

Furthermore, we next evaluate the predictability of the volatility estimates from our fully unified models (7)–(8) by using these 1,000 historical samples and the following regression model:

$$y_t = \chi_0^{4weeks} + \chi_1^{4weeks} x_{t-k} + \tau_t^{4weeks}, \tag{11}$$

where y_t is the estimated volatility from the non-fully unified model (1), (2), (3), (4), (5), or (6) at time t while x_{t-k} is the estimated volatility from our fully unified model (7) or (8) at time $t-k$. k takes a value of 1, 2, or 3; hence, we test the forecasting power from one to three days ahead. χ_0^{4weeks} denotes the constant term, χ_1^{4weeks} is the coefficient of x_{t-k}, and τ_t^{4weeks} is the residual. Therefore, a statistically significant positive value of χ_1^{4weeks} indicates predictability in the volatility estimates from our fully unified models (7)–(8).

Table 9 displays the regression results. In Panels A and B, the estimates of χ_1^{4weeks} for x_{t-1} and x_{t-2} all show statistically significant positive values. This indicates that for the historical 1,000 daily samples up to four weeks before the Russian invasion, the volatility estimates from our fully unified models (7)–(8) have forecasting power for the volatilities of the non-fully unified models (1)–(6) one to two days ahead.

In contrast, we also assess the predictability of the volatility estimates from the non-fully unified models (1)–(6) using the same 1,000 samples and the following regression model:

$$y_t = \gamma_0^{4weeks} + \gamma_1^{4weeks} x_{t-k} + \nu_t^{4weeks}, \tag{12}$$

where y_t is the estimated volatility from our fully unified model (7) or (8) at time t while x_{t-k} is the estimated volatility from the non-fully unified model (1), (2), (3), (4), (5), or (6) at time $t-k$. k takes a value of 1, 2, or 3. Hence, we test the forecasting power from one to three days ahead. γ_0^{4weeks} denotes the constant term, γ_1^{4weeks} is the coefficient of x_{t-k}, and v_t^{4weeks} is the residual. Therefore, a statistically significant positive value of γ_1^{4weeks} indicates predictability in the volatility estimates from the non-fully unified models (1)–(6).

Table 9. Predictive power of the volatility estimates from fully unified GARCH models of the RTS index: Four weeks before the Russian invasion.

Panel A. Predictive power of the estimates from the ST–SB–GJR–GARCH model					
Prediction target volatilities					
T–GJR–GARCH			GED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.026	0.743	χ_0^{4weeks}	0.027	0.743
χ_1^{4weeks}	0.149**	0.019	χ_1^{4weeks}	0.158**	0.020
$Adj.R^2$	0.015		$Adj.R^2$	0.015	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.015	0.817	χ_0^{4weeks}	0.016	0.819
χ_1^{4weeks}	0.349***	0.001	χ_1^{4weeks}	0.373***	0.001
$Adj.R^2$	0.089		$Adj.R^2$	0.088	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.034	0.695	χ_0^{4weeks}	0.037	0.694
χ_1^{4weeks}	0.021	0.687	χ_1^{4weeks}	0.019	0.728
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Prediction target volatilities					
ST–GJR–GARCH			SGED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.025	0.744	χ_0^{4weeks}	0.025	0.745
χ_1^{4weeks}	0.148**	0.017	χ_1^{4weeks}	0.145**	0.019
$Adj.R^2$	0.016		$Adj.R^2$	0.015	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.015	0.818	χ_0^{4weeks}	0.015	0.815
χ_1^{4weeks}	0.343***	0.001	χ_1^{4weeks}	0.336***	0.001
$Adj.R^2$	0.091		$Adj.R^2$	0.083	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.034	0.695	χ_0^{4weeks}	0.034	0.693

χ_1^{4weeks}	0.021	0.682	χ_1^{4weeks}	0.017	0.734
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Panel B. Predictive power of the estimates from the SGED–SB–GJR–GARCH model					
Prediction target volatilities					
T–GJR–GARCH			GED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.031	0.717	χ_0^{4weeks}	0.033	0.717
χ_1^{4weeks}	0.055**	0.020	χ_1^{4weeks}	0.058**	0.021
$Adj.R^2$	0.004		$Adj.R^2$	0.004	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.021	0.778	χ_0^{4weeks}	0.022	0.779
χ_1^{4weeks}	0.269**	0.016	χ_1^{4weeks}	0.289**	0.015
$Adj.R^2$	0.129		$Adj.R^2$	0.128	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.036	0.689	χ_0^{4weeks}	0.038	0.689
χ_1^{4weeks}	−0.007	0.726	χ_1^{4weeks}	−0.008	0.702
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Prediction target volatilities					
ST–GJR–GARCH			SGED–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.030	0.718	χ_0^{4weeks}	0.030	0.719
χ_1^{4weeks}	0.055**	0.017	χ_1^{4weeks}	0.055**	0.016
$Adj.R^2$	0.005		$Adj.R^2$	0.005	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.020	0.778	χ_0^{4weeks}	0.021	0.777
χ_1^{4weeks}	0.264**	0.017	χ_1^{4weeks}	0.261**	0.015
$Adj.R^2$	0.130		$Adj.R^2$	0.122	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
χ_0^{4weeks}	0.035	0.690	χ_0^{4weeks}	0.035	0.688
χ_1^{4weeks}	−0.007	0.719	χ_1^{4weeks}	−0.008	0.672
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	

Notes: Results from 1,000 samples up to four weeks before the Russian invasion of Ukraine. $Adj.R^2$: adjusted R-squared value. *** and ** denote the 1% and 5% significance levels, respectively. Every regression is conducted using the Newey–West heteroscedasticity- and autocorrelation-consistent covariance matrix. All parameters in this table are clearly defined in Section 6.2.

Table 10. Predictive power of the volatility estimates from non-fully unified GARCH models of the RTS index: Four weeks before the Russian invasion.

Panel A. Predictive power of the estimates from the T–GJR–GARCH model					
Prediction target volatilities					
ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{4weeks}	0.056	0.597	γ_0^{4weeks}	0.056	0.645
γ_1^{4weeks}	−0.105***	0.002	γ_1^{4weeks}	−0.235***	0.000
$Adj.R^2$	0.014		$Adj.R^2$	0.030	
Two-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{4weeks}	0.054	0.594	γ_0^{4weeks}	0.048	0.679
γ_1^{4weeks}	−0.010	0.777	γ_1^{4weeks}	0.024	0.454
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Three-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{4weeks}	0.056	0.585	γ_0^{4weeks}	0.049	0.670
γ_1^{4weeks}	−0.035	0.377	γ_1^{4weeks}	0.001	0.984
$Adj.R^2$	0.001		$Adj.R^2$	−0.001	
Panel B. Predictive power of the estimates from the GED–GJR–GARCH model					
Prediction target volatilities					
ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{4weeks}	0.056	0.597	γ_0^{4weeks}	0.056	0.645
γ_1^{4weeks}	−0.097***	0.002	γ_1^{4weeks}	−0.024***	0.000
$Adj.R^2$	0.014		$Adj.R^2$	0.031	
Two-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{4weeks}	0.054	0.594	γ_0^{4weeks}	0.048	0.679
γ_1^{4weeks}	−0.009	0.775	γ_1^{4weeks}	0.022	0.460
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Three-day-ahead predictive power					
Coeff.	Estimates	*p*-value	Coeff.	Estimates	*p*-value
γ_0^{4weeks}	0.056	0.585	γ_0^{4weeks}	0.049	0.670
γ_1^{4weeks}	−0.031	0.390	γ_1^{4weeks}	0.001	0.970
$Adj.R^2$	5.0E−04		$Adj.R^2$	−0.001	
Panel C. Predictive power of the estimates from the ST–GJR–GARCH model					
Prediction target volatilities					

ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{4weeks}	0.056	0.597	γ_0^{4weeks}	0.056	0.646
γ_1^{4weeks}	−0.108***	0.002	γ_1^{4weeks}	−0.229***	0.000
$Adj.R^2$	0.014		$Adj.R^2$	0.027	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{4weeks}	0.054	0.594	γ_0^{4weeks}	0.047	0.680
γ_1^{4weeks}	−0.009	0.800	γ_1^{4weeks}	0.027	0.423
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{4weeks}	0.056	0.585	γ_0^{4weeks}	0.049	0.670
γ_1^{4weeks}	−0.035	0.390	γ_1^{4weeks}	0.002	0.965
$Adj.R^2$	0.001		$Adj.R^2$	−0.001	
Panel D. Predictive power of the estimates from the SGED–GJR–GARCH model					
Prediction target volatilities					
ST–SB–GJR–GARCH			SGED–SB–GJR–GARCH		
One-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{4weeks}	0.056	0.597	γ_0^{4weeks}	0.056	0.645
γ_1^{4weeks}	−0.105***	0.002	γ_1^{4weeks}	−0.238***	0.000
$Adj.R^2$	0.014		$Adj.R^2$	0.031	
Two-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{4weeks}	0.055	0.593	γ_0^{4weeks}	0.048	0.678
γ_1^{4weeks}	−0.014	0.685	γ_1^{4weeks}	0.021	0.516
$Adj.R^2$	−0.001		$Adj.R^2$	−0.001	
Three-day-ahead predictive power					
Coeff.	Estimates	p-value	Coeff.	Estimates	p-value
γ_0^{4weeks}	0.056	0.585	γ_0^{4weeks}	0.049	0.670
γ_1^{4weeks}	−0.036	0.348	γ_1^{4weeks}	1.3E−04	0.998
$Adj.R^2$	0.001		$Adj.R^2$	−0.001	

Notes: Results from 1,000 samples up to four weeks before the Russian invasion of Ukraine. $Adj.R^2$: adjusted R-squared value. *** denotes the 1% significance level. Every regression is conducted using the Newey–West heteroscedasticity- and autocorrelation-consistent covariance matrix. All parameters in this table are clearly defined in Section 6.2.

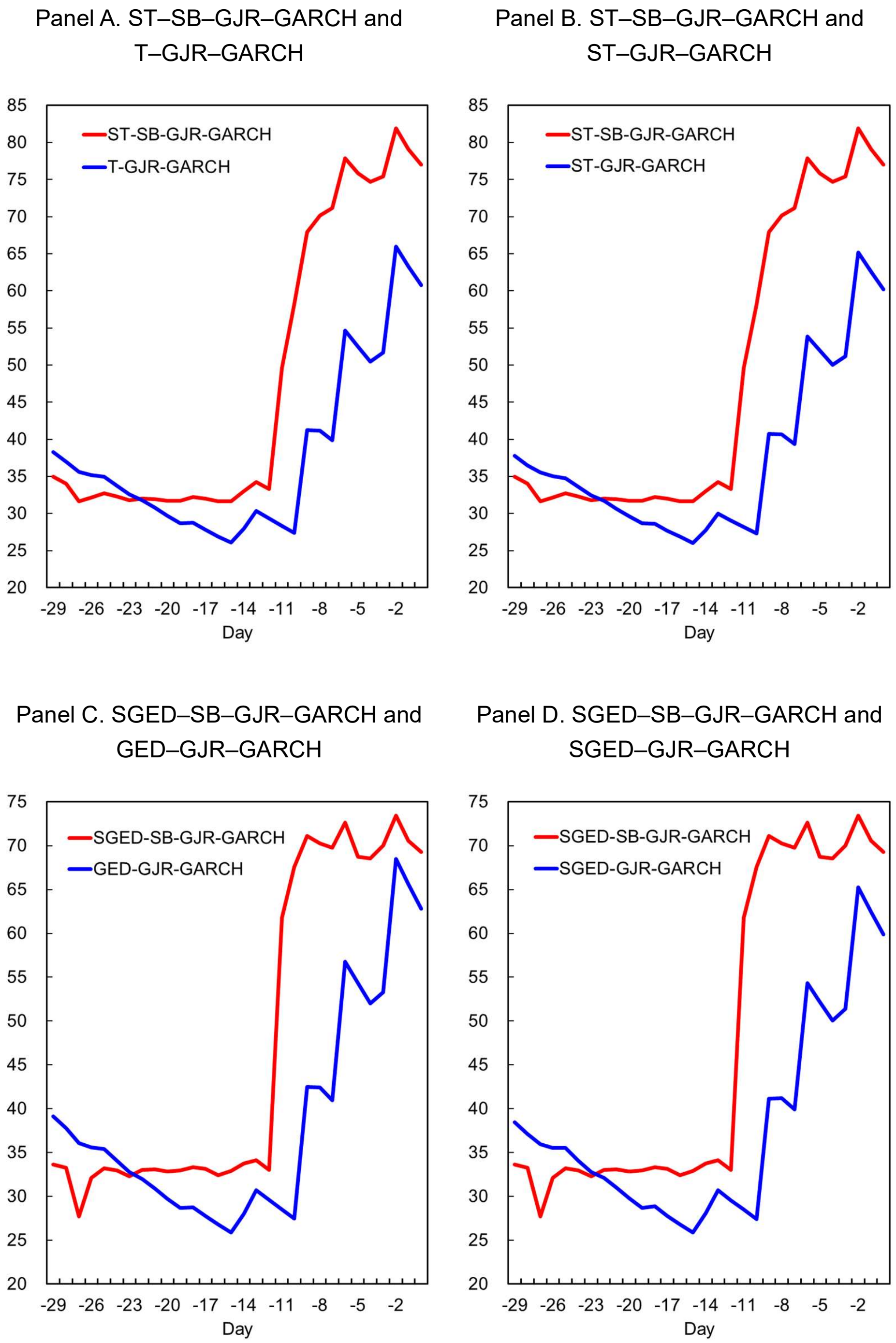

Figure 7. Predictability comparisons of daily volatilities of the RTS index: Four weeks before the Russian invasion.

Note: Estimated volatilities are plotted for the 30 business days leading up to four weeks before the Russian invasion of Ukraine.

Table 10 presents the regression results. In Panels A–D, no statistically significant positive value for the estimates of γ_1^{4weeks} can be found. This indicates that the volatility estimates from the non-fully unified models (1)–(6) have no forecasting power for the volatilities of our fully unified models (7)–(8). Therefore, the results from another 1,000 historical samples empirically prove once again that the volatilities from our fully unified models have a strong forecasting power of Russian stock return fluctuations, while the non-fully unified models do not possess such strong forecasting power.

Furthermore, we plotted the volatility estimates in Figure 7, which were estimated using the 1,000 daily samples up to four weeks before the Russian invasion. These plots cover the 30 business days leading up to January 27, 2022, which is four weeks before the Russian invasion. The volatility pairs shown in Panels A–D in Figure 7 are the same as those in Figure 6. It is noteworthy once again that Panels A and B in Figure 7 demonstrate that the ST–SB–GJR–GARCH model estimates had a significantly earlier response compared to the other model estimates, which exhibited slower and smaller responses. Similarly, Panels C and D in Figure 7 show that the SGED–SB–GJR–GARCH model estimates also responded significantly earlier compared to the other model estimates, which showed slower and smaller responses. This evidence suggests that as of four weeks prior, the ST–SB–GJR–GARCH and SGED–SB–GJR–GARCH models more strongly predicted the crash risk associated with the Russian invasion of Ukraine.

In conclusion, the skewed errors and asymmetry in our fully unified GARCH models effectively captured the negative skewness and shocks in Russian stock returns. Additionally, the structural breaks incorporated into our fully unified GARCH models timely predicted the increase in Russian stock volatilities associated with the Russian invasion of Ukraine. These factors are why our fully unified GARCH models produced superior results. We emphasize that the analyses above clearly demonstrate that our fully unified models and their volatility estimates are highly effective for out-of-sample predictions of volatility and crash risks related to Russia's invasion.

We note that volatility is an unobserved variable, making it difficult to measure forecasting accuracy using metrics such as mean absolute errors in practice. Therefore, the out-of-sample testing method conducted and displayed in Tables 6, 7, 9, and 10, as well as Figures 6 and 7, is an effective approach for testing the accuracy of out-of-sample volatility forecasting. We believe that these results offer comprehensive and strong practical implications, providing valuable insights into the application of our extended models in real-world scenarios. This highlights the significance of the present study. We also note that structural breaks often occur before abnormal events. Therefore, using our models beforehand may assist in predicting volatility and crash risks in stock markets. As a result, these empirical analyses have significant implications. Ultimately, our fully unified GARCH models can be used for real-world risk and volatility management in stock markets worldwide.

Furthermore, as discussed in Section 3, numerous previous studies have examined the effects of Russia's invasion of Ukraine on financial markets and the economy. These sudies have typically analyzed trends in financial market returns post-invasion or explored the relationships between different markets following the invasion. In contrast, our current study focuses on predicting the shocks of Russia's invasion of Ukraine before the clear impacts on the Russian stock market emerged. In other words, the aim of our study is to forecast the surging volatility of the Russian stock market caused by the invasion beforehand. Therefore, this unique focus sets our study apart from existing research, allowing us to offer fresh insights and results, demonstrating the novelty of our research focus and findings.

7. Discussion—Implications and perspectives

This section delves into the significant implications discussed in the previous section. The following further discussion offers valuable perspectives for research not only in quantitative finance and economics but also in financial innovation in the post-COVID-19 era. As highlighted in Fethi and Pasiouras (2010), Dirican (2015), Ahmed et al. (2022), and Chaklader et al. (2023), AI should play a more prominent role in the future of business and financial industries. The implications of our study and our innovative views are as follows.

First, as we have demonstrated, capturing the effects of significant events on stock markets in a timely manner requires estimating stock return volatilities by incorporating asymmetry, skewed errors, and structural breaks. We believe that capturing structural breaks promptly is particularly crucial for volatility modeling and future risk management. This is because, as shown in Figures 4 to 7, the volatilities derived from models that are not fully unified lag behind the volatilities from our two fully unified models, which incorporate structural breaks. This lag is especially evident during significant events such as the Russian invasion of Ukraine. Therefore, it is necessary for us to accurately forecast volatilities that do not lag behind significant events by applying effective econometric models like ours or innovative AI techniques such as machine learning. This will lead to more effective risk management in the post-COVID-19 era.

Second, with the increasing importance of structural breaks in financial markets, as we have shown, it is now more critical than ever for us to accurately predict these breaks. We should strive to achieve this by utilizing innovative AI technologies, which will enhance risk management in the unpredictable post-COVID-19 world. We also believe that accurately predicting structural breaks enables us to forecast volatilities without lagging behind sudden changes in financial asset prices triggered by serious events. This study has adequately demonstrated the appropriateness of our unifying process for developing new GARCH models and the effectiveness of our unified GARCH models. However, future research, such as comparisons with other types of GARCH models or machine learning methods like long short-term memory and gated recurrent unit, would be interesting.

Third, the effectiveness of our fully unified models, which also incorporate GJR asymmetry, indicates that accurate recognition of volatility asymmetries is also crucial for risk management. This suggests that being more cautious about negative return shocks is highly important for future risk management. Therefore, it is also vital for us to carefully forecast the direction and magnitude of asset price movement by utilizing effective econometric models like ours or innovative AI techniques. We can then use these forecast results to improve risk management practices in the post-COVID-19 era.

Fourth, in terms of managerial implications, we believe that corporate managers should ensure they have enough shareholder equity to withstand sudden negative events that may cause structural breaks and subsequent recessions on a global scale. It is crucial for all firms to have sufficient equity, especially in preparation for such events. Therefore, we stress the importance, especially in the uncertain post-COVID-19 era, of maintaining a strong capital buffer as a form of risk management for all industries. This involves carefully considering aspects of their operational management, such as payout policies, capital structure management, and cash management. More specifically, when structural breaks in financial markets are perceived as a sign of economic stagnation, companies will need to focus on policies such as maintaining shareholder payouts without increasing them, carefully assessing their fundraising and capital investment decisions, and ensuring adequate liquidity.

Furthermore, as a theoretical implication, it is important to further examine and develop theoretical models that consider the third and fourth moments of stock returns. This is crucial because, as we have seen, considering fat-tailed and skewed distribution errors has been proven to be effective, especially during times of market stress. Instead of relying on traditional theories that assume a normal distribution in asset returns, it is essential to explore alternative asset pricing models. Additionally, as we have demonstrated, considering negative asymmetry and structural breaks has also been shown to be effective. Therefore, it is important to study and develop theoretical models that incorporate return jumps, asymmetric reactions, and loss-averse preferences of economic agents. These models can offer valuable insights into market behavior during periods of stress.

The comprehensive inspections conducted in this paper provide numerous insightful interpretations and implications, as well as several innovative viewpoints. Collectively, these demonstrate the importance of our current work for future related research and practice. Therefore, we believe that our empirical findings, along with the significant interpretations, implications, and innovative views discussed above, all demonstrate our significant contribution to current and future research on quantitative economics and finance.

8. Contributions and conclusions

This study aimed to more effectively estimate the volatilities in the Russian stock market, with a specific focus on the Russian invasion of Ukraine on February 24, 2022. To achieve this goal, we developed and utilized fully unified volatility models: the AR–ST–SB–GJR–GARCH and the AR–SGED–SB–GJR–GARCH models. Both models incorporate not only asymmetry but also skewed distribution errors and structural breaks. We emphasize that our extended models are expected to be applicable to other stock markets all over the world. Our comprehensive analysis has revealed the following new findings.

First, our analyses revealed that incorporating skewed-distribution errors was effective in modeling fluctuations in Russian stock returns. Specifically, we found that even after including the effective GJR-asymmetric effect, incorporating skew-*t* or skew-GED errors was effective in estimating stock return volatilities in a timely manner during the Russian invasion of Ukraine. Second, our examinations revealed that incorporating structural breaks was effective in modeling Russian stock return volatilities. Specifically, we found that even after incorporating GJR asymmetry and skew-*t* or skew-GED errors, including structural breaks was effective in estimating stock return volatilities in a timely manner during the Russian invasion.

Third, we also discovered that the volatility estimates from our superior fully unified models, the AR–ST–SB–GJR–GARCH and AR–SGED–SB–GJR–GARCH models, have predictive power for fluctuations in Russian stock returns. Conversely, the volatility estimates from the non-fully unified models lack predictive power for fluctuations in Russian stock returns and trail behind those from our fully unified models. Fourth, we also found that our two fully unified model estimates responded to the crash risk associated with the Russian invasion significantly earlier than the estimates from the non-fully unified models. This was evident not only one week prior to the Russian invasion but also four weeks prior. This evidence suggests that both one week and four weeks before the invasion, our two fully unified GARCH models more effectively predicted the increased crash risk associated with the Russian invasion of Ukraine.

Fifth, in addition to the points documented above, we have also developed and shared numerous valuable interpretations, implications, and innovative perspectives for future research and practice in quantitative finance and economics. This represents another significant contribution of our work. These interpretations, implications, and innovative perspectives including the effective use of AI for future risk management will be valuable not only for academic researchers but also for industry practitioners.

Furthermore, we finally emphasize the following points. In addition to extending GARCH models, this paper makes three conceptually and empirically significant contributions. First, it offers insights into the connection between quantitative models, financial theory, and real-world data. Second, it provides perspectives on the relationship between information, human psychology, and structural breaks in financial markets. Lastly, it provides valuable insights into the profound impacts of major events like Russia's invasion of Ukraine. These insights are crucial in various areas, including financial markets, human behavior, and the economy, as well as information-related domains. We believe that these contributions are substantial and will likely inspire future research across multiple disciplines.

This study focused on the impact of the Russian invasion of Ukraine. Economies worldwide have felt the effects of this invasion, capturing the attention of scholars and professionals. Due to its significance and gravity, we are confident that the numerous valuable pieces of evidence and insights uncovered through our novel approach will greatly contribute not only to the body of academic literature but also to practical applications in various industries around the world.

Appendix

Table A1. Abbreviations.

ADF: Augmented Dickey–Fuller
AI: Artificial intelligence
AIC: Akaike information criterion
AMH: Adaptive market hypothesis
AR: Autoregressive
BIC: Bayesian information criterion
EMH: Efficient market hypothesis
ETF: Exchange-traded fund
GARCH: Generalized autoregressive conditional heteroscedasticity
GED: Generalized error distribution
GJR: Glosten–Jagannathan–Runkle
HAC: Heteroscedasticity and autocorrelation consistent
HAR: Heterogeneous autoregressive
ICSS: Iterated cumulative sums of squares
LR: Likelihood ratio
RTS: Russian Trading System
SB: Structural break
SGED: Skew-generalized error distribution
ST: Skew-*t*
VAR: Vector autoregression

Table A2. Methodology for model unification.

Panel A. Extension of Student's *t* distribution error models
Step 1: Incorporating GJR asymmetry into the AR–T–GARCH model
Step 2: Incorporating skew-*t* distribution errors into the AR–T–GJR–GARCH model
Step 3: Building the AR–ST–SB–GJR–GARCH model by incorporating structural breaks into the AR–ST–GJR–GARCH model
Panel B. Extension of GED error models
Step 1: Incorporating GJR asymmetry into the AR–GED–GARCH model
Step 2: Incorporating skew-GED errors into the AR–GED–GJR–GARCH model
Step 3: Building the AR–SGED–SB–GJR–GARCH model by incorporating structural breaks into the AR–SGED–GJR–GARCH model

Author contributions

The author designed and conducted all of the research.

Use of AI tools declaration

The author declares that he has not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The author declares no conflicts of interest in this paper.

Data availability

The data used in this study were obtained from commercial third-party providers and are not publicly available due to licensing restrictions. The data may be obtained from the providers, subject to their terms and conditions.

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