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*Research article*

# Intrinsic risks of managing multiple enterprises: a simple deterministic model highlighting unstable patterns

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**Abstract:** In this paper, we present a simplified mathematical model using a system of ordinary differential equations to describe the relationships between money flow, staff size, and the number of enterprises managed by a corporation. Despite ignoring external shocks, our stability analysis reveals inherent instabilities in the system, highlighting significant risks for both venture success and staff employment. This model aims to support risk management by emphasizing the intrinsic sensitivity and potential instability in managing multiple enterprises. The methodology is based on the analysis of a system of differential equations, and the major contributions are a novel mathematical model and an explicit stability analysis with relevance to real-world situations.

**Keywords:** corporate growth; multiple enterprises; risk management; intrinsic instability; ordinary differential equations

**JEL Codes:** L25, M54, D21, C02, C62, C30

## 1. Introduction: corporate growth through ventures

Corporations undertake a variety of new ventures that can generate significant cash flow, contributing to growth in terms of both operations and staff size. Typical examples include professional soccer teams, which participate in numerous tournaments that yield substantial revenue and require the recruitment of large player rosters, or universities, which often expand their academic offerings or establish new national and international campuses, using part of the income to hire additional administrative and academic staff.

Although potentially lucrative, these ventures are subject to significant risks, including, but not limited to, economic fluctuations (Acs and Audretsch (2005)), financial market instability (Sahlman (1990)), competition with other organizations (Porter (1998)), operational challenges (Chandler and

Hanks (1994)), and regulatory constraints (Djankov et al. (2002)). Additionally, (un)predictable events such as wars (Hymer (1976)) or pandemics (Kuckertz et al. (2020)) can negatively impact outcomes.

Moreover, the launch of new enterprises is often financed through borrowing, which can increase the asset-liability ratio and expose the corporation to interest rate fluctuations (Berger and Udell (1998)), thereby introducing further financial risk.

The risks associated with managing an excessive number of enterprises are also arguably diverse. They may include rising operational, maintenance, and staffing costs; market saturation; inconsistencies in standards across enterprises; overextension and reputational damage; as well as political and exchange rate risks, particularly if the enterprises operate in unstable regions or in foreign currencies (Bartram et al. (2010); Kogut and Singh (1988).)

Additionally, aggressive or inconsistent marketing linked to new ventures can erode long-term brand value for short-term gains (see e.g. (Kotler and Keller, 2006, Chapter 9) and (Keller, 2013, Part III)).

However, possibly given the complexity and variety of stakeholders involved, it is rare to reflect on the intrinsic risks that arise solely from the internal relationship between new enterprises, revenue generation, performance, and staffing. The objective of this paper is to present a simple example in which no major external disruptions occur, yet the system exhibits a high sensitivity to specific parameters, causing the system to transition from stability to instability.

Interestingly, a relevant parameter describing this kind of instability can be linked to declining employee performance, which may result from fatigue, stress, overwork, or a diminished work-life balance, often caused by the launch of new ventures, especially when these are not coordinated with the available personnel.

## 2. Methodology: the mathematical model

In our simplified analysis, we consider the interaction among three structural variables: the money flow $m$, the number of staff $s$ employed by the corporation, and the number of enterprises $e$ it operates.

More specifically, we posit that the money flow is governed by an equation of the type

$$\dot{m} = \alpha e - \beta s + p + \kappa m, \tag{1}$$

where $p$ stands for the staff productivity (intended as an overall proxy for motivation, fatigue, skill improvements, etc.).

In (1), the term $\alpha e$ displays* the fact that each enterprise produces some revenue $\alpha$ (since we are considering here only the "best possible scenario", we suppose $\alpha > 0$, not comprising the possibility that enterprises are, in fact, at a loss). Additionally, the negative term $\beta s$ is related to salary costs (e.g., each employee receiving a gross salary $\beta > 0$). Finally, the term $\kappa m$, with $\kappa > 0$, accounts for the income produced directly by the capital (e.g., it includes interest rates for money deposited in a bank). Indirectly, this term could also account for the mediated benefits for a corporation in having a solid financial state (e.g., in terms of attractiveness for investors, public reputation, and also in the possibility of affording auxiliary managing costs which produce secondary revenue, such as, for a soccer team, training centers, medical facilities, and analytics rooms, or, for a university, laboratories, equipment, and libraries).

*We observe that our model implicitly assumes a separation of time scales, in which suitable parameters are treated as constants. In practice, these parameters (such as the revenue $\alpha$) do fluctuate, but we consider them as "slow variables" in the time scale under consideration, which mostly focuses on the opportunity of opening a new venture.

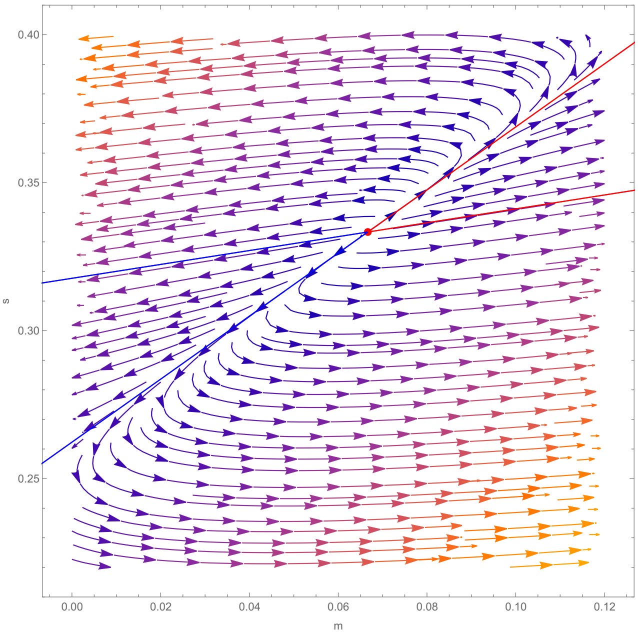


**Figure 1.** Example of streamlines of (12) when $\alpha := \delta := \eta := \zeta := \mu := 1$, $\epsilon := \frac{1}{5}$, and $\beta := \kappa := 5$.

The dynamic equation for cash flow (1) is a drastic simplification of more complex financial models, since, for example, it does not include the return of capital which could come from real estate investments related to the venture. As (1) is stated, each enterprise undertaken requires only staff, and cash flow is required to pay wages.

On the one hand, this simplification cannot capture the financial complexity related to future capital gains coming from the revaluation of assets.

On the other hand, return of capital is not often considered a priority for a new venture, which is mostly focused on achieving rapid growth and market share, thus requiring investment that may not generate a positive operating profit for several years. In this sense, we also recall that classical financial models sometimes intentionally omit distributions, dividends, or return of capital in their model assumptions, see e.g. (Black and Scholes, 1973, Assumption (c)). Along the same lines, taxes and transaction costs are sometimes omitted, see Modigliani and Miller (1958). These additional features can also be incorporated at a later stage to either correct or consolidate the model, see e.g. Assumption (3) on page 126 and the subsequent discussion in Merton (1976).

We will also describe productivity in terms of staff and enterprises, and specifically through the employee-to-venture ratio $\frac{s}{e}$. The simple ansatz is that a higher employee-to-venture ratio indicates larger teams per venture, enabling division of labor, specialization, and focused skill application. In contrast, a low ratio, corresponding to many ventures with few employees, may reflect the absence of a critical mass of employees necessary for effective specialization. It may also suggest that each worker must handle an overly broad set of responsibilities, potentially leading to excessive work hours and, consequently, reduced efficiency that drags down average productivity.

With these considerations in mind, we assume here that

$$p = \frac{\mu s}{e}, \tag{2}$$

with $\mu > 0$.

This simple linear relation must be understood as a convenient mathematical simplification, since real-world productivity is rarely exactly proportional to the number of employees per venture. In practice, productivity may saturate or even decline with too many employees due to coordination losses, and it may be constrained with too few employees due to understaffing. Moreover, the precise measurement of productivity often depends on the type of venture and the output metric used.

In this respect, empirical studies on productivity have frequently employed linear regression methods to estimate how inputs, including labor, relate to productivity measures such as revenue per worker, see Shevelova et al. (2023); Wallskog et al. (2024). However, in practical applications, it is also important to enable for more complex functional relationships than those implied by a simple proportional model.

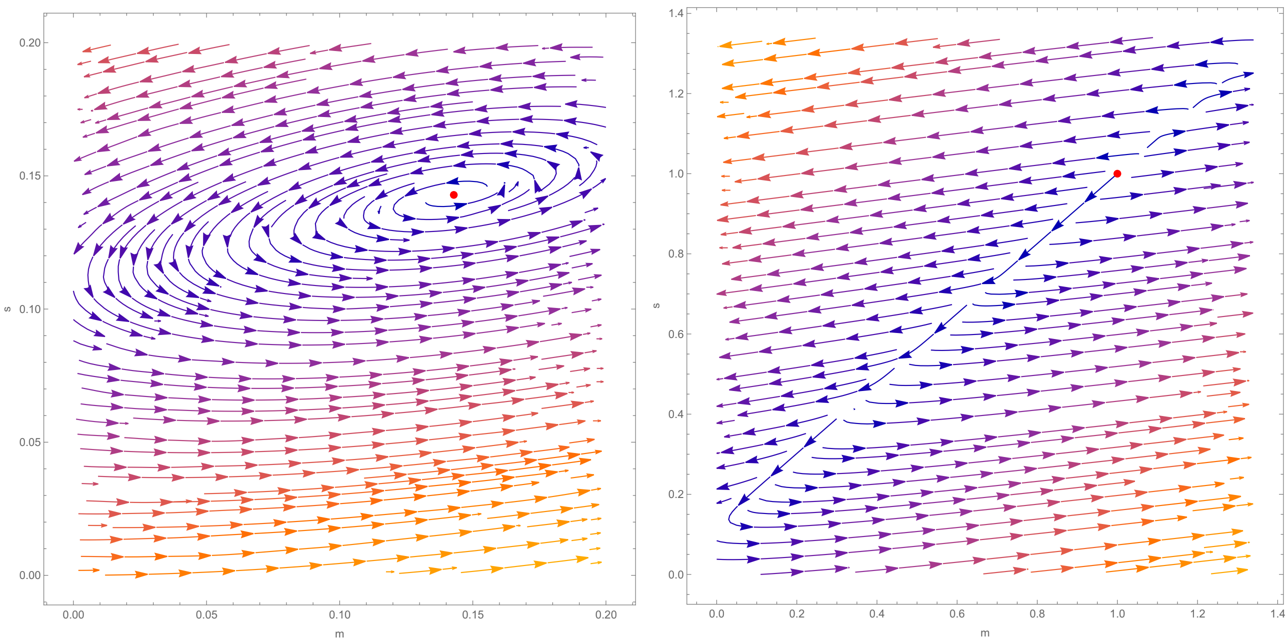


**Figure 2.** Example of streamlines of (12) when $\alpha := \delta := \epsilon := \eta := \zeta := \mu := 1$, $\kappa := 2$ (left) and $\kappa := 8$ (right), and $\beta := 10$.

We also assume that the establishment of new ventures is favored by their profitability, and that a certain proportion of enterprises close, or are "converted", or "restricted" (e.g., due to lack of product-market fit, poor financial planning, high customer acquisition costs, poor management, economic downturns, shifts in market's trends, etc.). This leads to the equation

$$\dot{e} = \alpha\zeta - \eta e, \tag{3}$$

where the parameter $\zeta > 0$ models the fact that the higher the revenue $\alpha$ generated by the ventures in (1), the more likely it is that new enterprises will be founded. Moreover, the parameter $\eta > 0$ accounts for the closure of the existing ventures.

To keep the analysis as simple as possible, the dynamic equation for enterprises (3) does not depend explicitly on production or opportunity costs. In a sense, however, additional costs can be implicitly included in the revenue parameter $\alpha$ (since higher costs can reduce profit). In any case, the analysis of additional costs can be separately included at a later stage of the study: This practice is, in fact, not uncommon in the developments of economic theories, see e.g. page 93, lines 10–11 from the bottom, in Wernerfelt (1990), as well as lines 2–3 of Section 2 in Beccuti and Möller (2018).

Some further remarks about our model. The cash flow $m$ is not considered as an incentive in itself to open new ventures (since executives can decide to use this money elsewhere, e.g., to reward the staff, to refurbish facilities, etc.).

We also stress that the assumption that the money flow does not impact the establishing of new ventures is a simplification of reality. Generally, one might think that cash availability is a favoring factor, while a money deficit prevents new ventures. However, there are documented situations in which financially distressed firms continue to open new ventures regardless of their deficit, even if their individual business units are not all generating a surplus. The impact of cash flow on the founding of new ventures is therefore a very complex phenomenon, which is not addressed in this paper and deserves additional study. It may reflect various market trends, including the tendency to diversify across business units to reduce the risk of bankruptcy or the intention of managers to protect their own careers by expanding the firm's scope. See e.g. Dimitrov and Tice (2006); Singhal and Zhu (2013) for further information on these aspects.

The decay modeled by the parameter $\eta$ accounts also indirectly for the costs needed to maintain a venture (which in turn leads to the closure of enterprises in the absence, or in the insufficiency, of the revenue $\alpha$).

Finally, we hypothesize that the increase in staff number occurs as a consequence of the money availability and of the number of existing enterprises. Namely, we suppose that the increase of staff number is proportional to the product of $e$ and $m$, through a constant of proportionality $\delta > 0$ (the ansatz here is that if no money is available, or if no enterprise is in place, no new staff can be hired). We also suppose that there is a natural decay in staff number (e.g., due to retirement) with a rate of proportionality given by a parameter $\epsilon > 0$. These assumptions lead to the equation

$$\dot{s} = \delta em - \epsilon s. \tag{4}$$

The dynamic equations for staff (4) do not depend on wages, labor productivity, aggregate unemployment, and vacancy rates. This is useful to keep the analysis manageable at a mathematical level. Besides, this kind of simplifying assumptions are not uncommon in the literature, see e.g. Section 2.7 in Prettner and Strulik (2017) or footnote 29 in Mechanick and Weber (2024) regarding the possibility of incorporating unemployment only at a later stage of the analysis. See also footnote 12 in Podrecca and Rossini (2012) regarding the omission of wages and unemployment benefits.

Gathering (1), (2), (3), and (4), we obtain the system of ordinary differential equations which is the main object of interest in this paper, namely

$$\begin{cases} \dot{e} = \alpha\zeta - \eta e, \\ \dot{m} = \alpha e - \beta s + \dfrac{\mu s}{e} + \kappa m, \\ \dot{s} = \delta em - \epsilon s. \end{cases} \tag{5}$$

In terms of the economic foundations of our model, we adopt a different approach from the common assumption that a firm aims solely to maximize profits. Indeed, the system in (5) is not introduced through an optimization problem, but rather via the identification of suitable quantities that represent incentives and disincentives for opening new enterprises. We believe this approach is sufficiently realistic, especially in light of several examples of economic models in which firms do not strictly pursue profit maximization.

For instance, firms may seek satisfactory profits rather than optimal ones to remain within a safer range of operation (see Giarlotta and Petralia (2024); Curwen (1976)). Managers may also prioritize job security, corporate growth, or personal utility over strict profit maximization (see Koutsoyiannis (1975); Zhou et al. (2025)). On this matter, while our model does not focus explicitly on the principal-agent problem, in which a conflict of interest arises and a manager acts on behalf of a principal but may have different goals, our equations do not forcibly impose the realization of the highest profit. Instead, they deal with impulses and potential downsides in the opening of new enterprises.

Likewise, the model does not explicitly incorporate additional issues arising from possible information asymmetry, where one party has more or better information about a company's condition and prospects than others. This factor must also be considered a source of further hazards in corporate finance and complicates any ansatz attempting to reduce the analysis to that of maximizing profit.

Furthermore, government agencies, universities, and other public organizations (many of which also engage in multiple entrepreneurial activities) are subject to distinct objective functions (see Winston (1999)).

In this spirit, the system in (5) is not intended to be an ultimate model encompassing all aspects of all types of firms. Rather, it provides a simple yet sufficiently general framework that seeks to capture some basic and common characteristics. One of the major reasons for adopting such a simplified system as in (5) is that it considers only the most essential features, deliberately ignoring the complex impacts of numerous stakeholders.

This approach creates a "caricature" model, which is much simpler than reality and is assumed to have the highest possible degree of stability. However, we will showcase that even such a purely deterministic toy model, devoid of complex external interactions, can develop intrinsic instability. This finding is, in our view, thought-provoking and should serve as a reminder to managers and staff of the significant risks inherently associated with certain enterprises.

## 3. Main results and discussion of their managerial implications

The result that we present is the following.

**Theorem 1.** *The only equilibrium* $(e_*, m_*, s_*)$ *of* (5) *is*

$$\begin{cases} e_* = \dfrac{\alpha\zeta}{\eta}, \\ m_* = \dfrac{\alpha^2\epsilon\zeta}{c_0}, \\ s_* = \dfrac{\alpha^3\delta\zeta^2}{c_0\eta}, \end{cases} \tag{6}$$

*where*

$$c_0 := \alpha\beta\delta\zeta - \epsilon\eta\kappa - \delta\eta\mu. \tag{7}$$

*This equilibrium is admissible if and only if*

$$c_0 > 0. \tag{8}$$

*The eigenvalues of the corresponding linearized system are*

$$\begin{aligned} \lambda_1 &:= -\eta, \\ \lambda_2 &:= \frac{\kappa - \epsilon - \sqrt{(\kappa-\epsilon)^2 - \frac{4c_0}{\eta}}}{2}, \\ \textit{and} \quad \lambda_3 &:= \frac{\kappa - \epsilon + \sqrt{(\kappa-\epsilon)^2 - \frac{4c_0}{\eta}}}{2}. \end{aligned} \tag{9}$$

*In particular, when*

$$\kappa > \epsilon \tag{10}$$

*the above equilibrium is unstable.*

The structural condition for instability (10), which is related to the real part of $\lambda_2$ and $\lambda_3$ being positive, is also implied by the stronger condition

$$\kappa > \epsilon + \sqrt{\frac{4c_0}{\eta}}, \tag{11}$$

which ensures that $\lambda_2$ and $\lambda_3$ are real and positive.

These conditions are interesting, because they can occur, compatibly with (8), in several possible scenarios. For instance, when $\kappa := \frac{1}{\epsilon}, \beta := \delta := \eta := \zeta := \mu := 1$, $\alpha := 4$, and $\epsilon$ is small enough, then both (8) and (11) are satisfied, and this corresponds to a situation in which there are compounding effects related to a high return on capital investment (large $\kappa$) and low retirement rates or delayed retirement (small $\epsilon$).

Another scenario of interest corresponds to $\beta := \delta := \zeta := \eta := \mu := 1$, $\epsilon := \frac{1}{2}$, and $\alpha := \kappa$ large, which also fulfills both (8) and (11), showcasing a situation in which the establishment of new ventures is very convenient (large $\alpha$) but also the bank investments return a high income (large $\kappa$, which could also exemplify the situation in which secondary revenues for capital are relevant; see the comment after (1)).

Another situation arises from $\alpha := 3\mu$, $\beta := \eta := \zeta := \kappa := 1$, $\delta := \frac{1}{\mu^2}$, $\epsilon := \frac{1}{\mu}$, and $\mu$ large, which also satisfies both (8) and (11). This corresponds to a scenario in which staff fluctuations are reduced (small $\delta$ and $\epsilon$) but the performance is highly impacted by the employee-to-venture ratio $\frac{s}{e}$ (large $\mu$).

It is also helpful to observe that the first equation in (5) can be solved independently, leading to

$$e(t) = \frac{\alpha\zeta}{\eta} + \left(e(0) - \frac{\alpha\zeta}{\eta}\right)\exp(-\eta t)$$

and, in particular,

$$\lim_{t\to+\infty} e(t) = \frac{\alpha\zeta}{\eta},$$

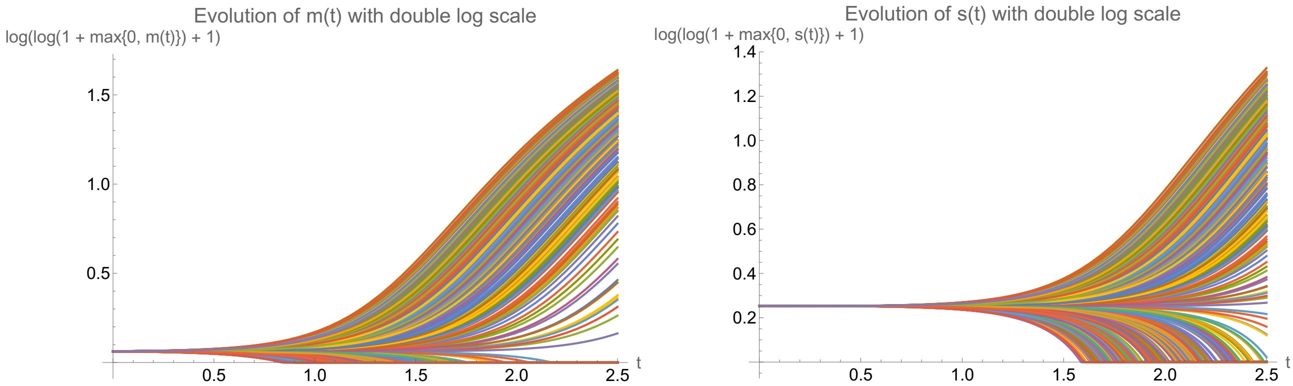


**Figure 3.** Evolution of $m(t)$ and $s(t)$ for (12) when $\alpha := \delta := \eta := \zeta := \mu := 1$, $\epsilon := \frac{1}{5}$, and $\beta := \kappa := 5$, with double logarithmic scale for 500 random perturbations of the equilibrium (the maximal size of the perturbation is $\pm 0.001$, and the trajectory hits the horizontal axis when it reaches zero, corresponding, respectively, to bankruptcy and staff dissolution).

showing that the market has a strong tendency to drive the number of enterprises toward equilibrium.

It is thereby somewhat compelling to study a "two-dimensional" reduction of (5) in which the orbit of $e$ is replaced by its limit state. This leads to the system of ordinary differential equations

$$\begin{cases} \dot{m} = \dfrac{\alpha^2\zeta}{\eta} - \theta s + \kappa m, \\ \dot{s} = \dfrac{\alpha\delta\zeta}{\eta} m - \epsilon s, \end{cases} \tag{12}$$

with

$$\theta := \frac{\alpha\beta\zeta - \eta\mu}{\alpha\zeta} = \frac{c_0 + \epsilon\eta\kappa}{\alpha\delta\zeta}.$$

The system in (12), however, is different from that in (5), and the counterpart of Theorem 1 would read as follows:

**Theorem 2.** *The only equilibrium* $(m_o, s_o)$ *of* (12) *is*

$$\begin{cases} m_o = \dfrac{\alpha^2\epsilon\zeta}{c_0}, \\ s_o = \dfrac{\alpha^3\delta\zeta^2}{c_0\eta}. \end{cases}$$

*This equilibrium is admissible if and only if* (8) *holds true.*

*The two eigenvalues of the corresponding linearized system are*

$$\lambda_\pm := \frac{\kappa - \epsilon \pm \sqrt{(\kappa-\epsilon)^2 - \dfrac{4c_0}{\eta}}}{2}.$$

*In particular, when* (10) *holds true, the above equilibrium is unstable.*

Figures 1 and 3 show sketches of this instability when $\alpha := \delta := \eta := \zeta := \mu := 1$, $\epsilon := \frac{1}{5}$, and $\beta := \kappa := 5$ (in which case (8), and in fact (10), is fulfilled; of course, a technical advantage of the reduced system in (12) is to enable easier visualizations than the one in (5), at the price of one additional simplification).

Examples of this kind should be of some concern when planning the establishment of new ventures, since, for this choice of parameters, the velocity field at points of the form $(\overline{m}, 0)$ has a first component of the form $1 + 5\overline{m} > 1$, suggesting that *there are solutions of* (12) *which can reach high monetary values passing through small staff numbers*.

Furthermore, the velocity field at points of the form $(0, \overline{s})$ with $\overline{s} > \frac{1}{4}$ has strictly negative first component and, accordingly, *there are solutions of* (12) *which reach the bankruptcy threshold* $m = 0$ *in finite time*.

Figures 2 and 4 show sketches of the instability given by Theorem 2 when $\alpha := \delta := \epsilon := \eta := \zeta := \mu := 1$, $\kappa \in \{2, 8\}$, and $\beta := 10$. We point out that, when $\kappa = 2$, the equilibrium is a spiral source, with oscillatory instability arising from an eigenvalue with a positive real part and a nonzero imaginary part, while, in the case $\kappa = 8$, the equilibrium is an unstable node, with exponential divergence without oscillation along the direction of the eigenvector corresponding to a real, positive eigenvalue.

The analogues of Figures 3 and 4 for the system in (5) is showcased in Figures 5 and 6, respectively. Consistently with the model, this set of images shows that the number of enterprises $e(t)$ approaches the asymptotic equilibrium value $e_*$, which is prescribed by market convenience. However, the money flow $m(t)$ and the staff size $s(t)$ exhibit significant instabilities, reflecting underlying dynamical fluctuations despite the stabilization of $e(t)$.

Concerning the roots displayed in Theorems 1 and 2, we observe that multiple roots are allowed. These correspond to double real roots occurring when $\kappa = \epsilon + 2\sqrt{\frac{c_0}{\eta}}$. Interestingly, these "resonant" states do not significantly alter the dynamics, in the sense that they always take place within the unstable regime prescribed by (10).

The condition for instability in (10) has an intuitive mathematical meaning. Indeed, looking at (5), as observed on page 230, we have that the evolution of the enterprise number $e$ is independent of the other flowing variables and stabilizes exponentially fast. Heuristically, this reduces the system to

$$\begin{cases} \dot{m} = \kappa m + C_m, \\ \dot{s} = -\epsilon s + C_s, \end{cases}$$

where $C_m$ and $C_s$ denote some kind of coupling between the flowing variables, with $C_m$ independent of $m$, and $C_s$ independent of $s$.

The corresponding Jacobian thus takes the form

$$\mathcal{J} := \begin{pmatrix} \kappa & * \\ * & -\epsilon \end{pmatrix},$$

with "$*$" representing some off-diagonal terms produced by the coupling (by the above-mentioned structure, the coupling does not produce diagonal terms).

This shows that the money variable $m$ has intrinsic growth rate $\kappa$, and the staff numerosity variable $s$ has intrinsic decay rate $\epsilon$ (with this regard, $\kappa$ plays the role of an amplifier and $\epsilon$ that of a damper).

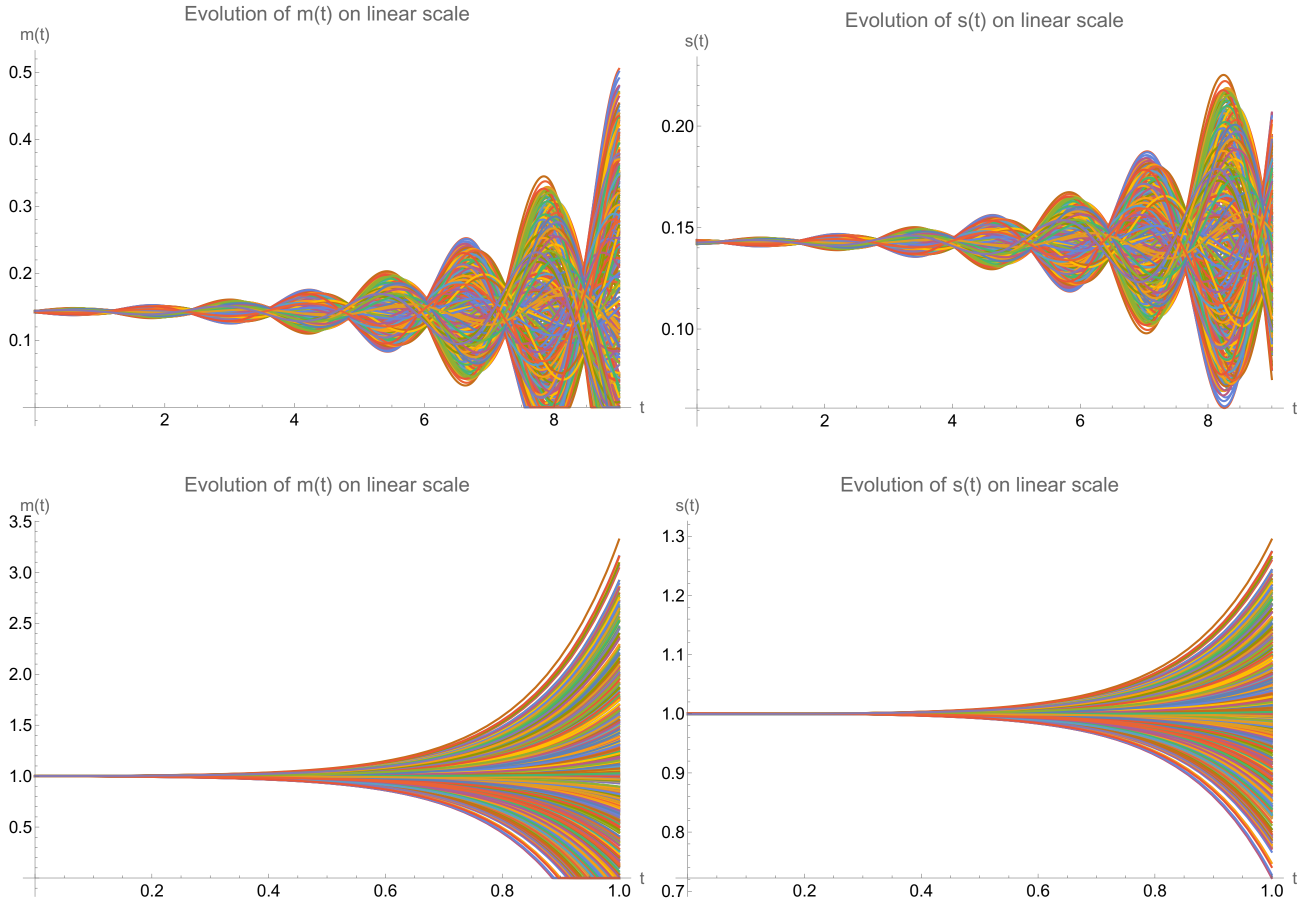


**Figure 4.** Evolution of $m(t)$ and $s(t)$ for (12) when $\alpha := \delta := \epsilon := \eta := \zeta := \mu := 1$, $\kappa := 2$ (top), and $\kappa := 8$ (bottom), and $\beta := 10$, with a linear scale for 500 random perturbations of the equilibrium (the maximal size of the perturbation is $\pm 0.001$, and the trajectory hits the horizontal axis when it reaches zero, corresponding, respectively, to bankruptcy and staff dissolution).

More explicitly, since linear systems expand or contract volume at a rate equal to the trace of the Jacobian, we have that the infinitesimal rate of volume change equals

$$\mathrm{tr}(\mathcal{J}) = \kappa - \epsilon,$$

showing that the off-diagonal terms do not play any role. The instability is thus encoded in the volume expansion corresponding to the case

$$\mathrm{tr}(\mathcal{J}) > 0,$$

which is (10).

The condition for instability in (10) has also managerial implications. In a nutshell, this condition detects a potential source of instability when the income produced directly from the capital is higher than the natural decay in staff number. This is somewhat a new ingredient in corporation decision-making, since a high income from capital and a low rate of staff retirement is not usually considered as a source of risk, and in fact it may be commonly assessed as a risk-reducing situation, since the company can rely

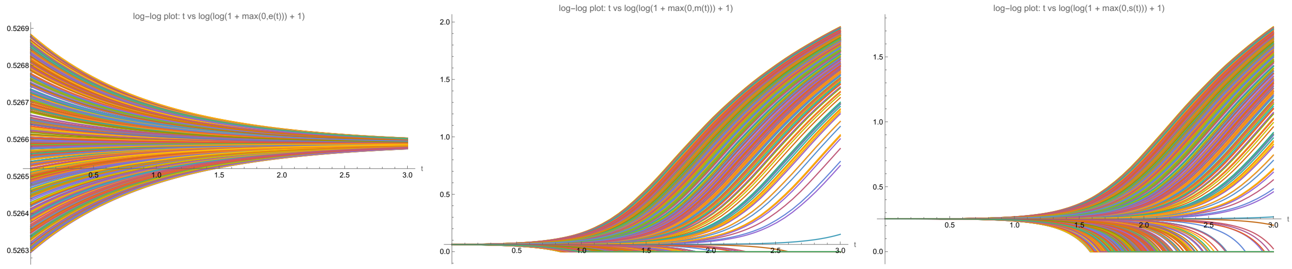


**Figure 5.** Evolution of $e(t)$, $m(t)$, and $s(t)$ for (5) when $\alpha := \delta := \eta := \zeta := \mu := 1$, $\epsilon := \frac{1}{5}$, and $\beta := \kappa := 5$, with a double logarithmic scale for 500 random perturbations of the equilibrium (the maximal size of the perturbation is $\pm 0.001$, and the trajectory hits the horizontal axis when it reaches zero, corresponding, respectively, to bankruptcy and staff dissolution).

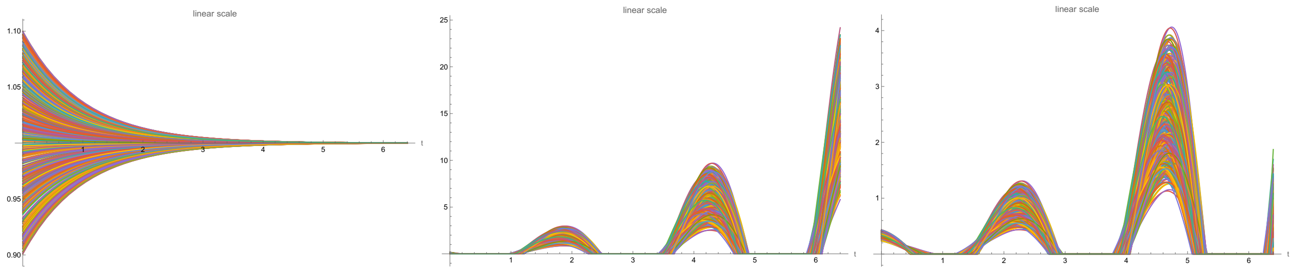


**Figure 6.** Evolution of $e(t)$, $m(t)$, and $s(t)$ for (5) when $\alpha := \delta := \epsilon := \eta := \zeta := \mu := 1$, $\kappa := 2$, and $\beta := 10$, with a linear scale for 500 random perturbations of the equilibrium (the maximal size of the perturbation is $\pm 0.1$, and the trajectory hits the horizontal axis when it reaches zero, corresponding, respectively, to bankruptcy and staff dissolution).

on financial resources deposited in a bank at a high interest and retains its workforce and institutional knowledge.

On the other hand, our model can detect this subtle source of instability, related to the situation in which high labor costs due to overstaffing (linked to the parameter $\beta$) and lower productivity due to an excessive number of enterprises (linked to the parameter $\mu$) can eat into profits.

We emphasize that our model does not incorporate option prices or volatility, and is therefore conceptually distinct from standard financial models based on diffusions and stochastic differential equations. Instead, our risk assessment is grounded in the stability analysis of ordinary differential equations, as our aim is to capture a more "fundamental" type of risk, namely one intrinsic to the initiation of ventures, even under the idealized assumption that market stability renders risks from fluctuations and uncertainties only marginal.

It would be of interest to extend the model in future work to incorporate external shocks and stochastic components. Additionally, calibrating the model using available data, for example from public companies, would be a valuable direction for further research.

Let us, however, mention that, though it would certainly be interesting to compare theoretical results with real-world data, an accurate collection and analysis is not always practical. Indeed, on the one hand, there is a significant amount of data[†] publicly available on the internet that displays money flow,

[†]For example, money flow can be monitored through annual reports available on company investor relations websites or public

staff size, and the number of enterprises managed by corporations. On the other hand, differently from publicly traded companies, private companies may have few disclosure requirements, and obtaining detailed information can be impossible without direct contact. On top of this, specifying the notion of "number of enterprises managed" can be challenging to unambiguously quantify as a single, consistent metric, as it often involves a dynamic list of subsidiaries rather than a simple count that changes incrementally.

For all these reasons, models of this type should be regarded more as auxiliary tools to stimulate comprehensive risk assessments, rather than as panaceas capable of fully addressing a highly complex problem. At any rate, notwithstanding that an exhaustive analysis of data goes beyond the goals of this paper, we outline, as a concrete example, that explicit information can be obtained regarding the Starbucks Corporation (SBUX), as displayed[‡] in Table 1 and Figure 7. In this case, since Starbucks operates globally through a vast network of company-operated or licensed stores, which are effectively their "enterprises", one can use the number of total stores as a sufficiently consistent[§] and plottable metric.

**Table 1.** Starbucks Corporation data for fiscal years 2020–2024.

| Fiscal Year End | Stores | Revenue | Employees |
|---|---|---|---|
| Sep 27, 2020 | 32660 | 23518 | 349000 |
| Oct 3, 2021 | 33833 | 29061 | 383000 |
| Oct 2, 2022 | 35711 | 32250 | 402000 |
| Oct 1, 2023 | 38096 | 35979 | 381000 |
| Sep 29, 2024 | 40199 | 36176 | 361000 |

Though this is a very small data set not enabling statistically significant results unaffected by temporary deviations from broader trends, it is interesting to note that the total revenue and the number of managed enterprises are consistently increasing, but the staff number experiences significant oscillations, with the employee count declining by approximately 10.2% from its peak in 2022 to the end of fiscal year 2024.

It is intriguing to compare the data in Table 1 and Figure 7 with the trajectories of the system in (5).

access databases such as https://www.investor.gov/ and https://www.annualreports.com/. Financial data and analyst reports are also available on platforms such as https://www.google.com/finance/, https://finance.yahoo.com/, https://www.factset.com/, and https://www.bloomberg.com/. An estimated employee count can be sometimes obtained from the company websites, in the "About Us" or "Careers" pages, or platforms of general use such as https://www.linkedin.com. Government agencies may also publish information on enterprises by employee size, though these are usually aggregate data for broader industry or regional analyses rather than specific corporations.

[‡]The data in Table 1 fit in the following categories:

Number of enterprises/subsidiaries (source: Starbucks Annual Reports, Investor Releases https://investor.starbucks.com/financials/annual-reports/default.aspx).

Total revenue in millions of USD (source: Starbucks' Annual Reports https://investor.starbucks.com/financials/annual-reports/default.aspx and https://s203.q4cdn.com/326826266/files/doc_financials/2024/q4/Q4-and-Full-FY24-Earnings-Release-Final-10-30-24.pdf).

Total Employees (source: Macrotrends https://www.macrotrends.net/stocks/charts/SBUX/starbucks/number-of-employees and StockAnalysis https://stockanalysis.com/stocks/sbux/employees/).

Starbucks filings are available in the EDGAR database https://www.sec.gov/edgar/search/ (one can search for "SBUX" and then filter by "Form Type" and "Filing Date", e.g. https://www.sec.gov/Archives/edgar/data/829224/000082922424000057/sbux-20240929.htm).

[§]For simplicity, the number of stores computed in Table 1 is an aggregate of company-operated and licensed.

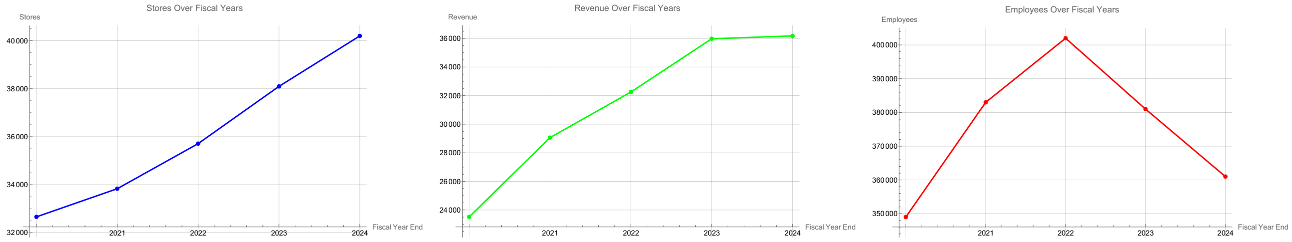


**Figure 7.** Visualization of the data in Table 1.

First, the data in Table 1 and Figure 7 point out that the increase in enterprises and money flow does not necessarily imply an increase in employees, and that the employee graph can exhibit turning points. This is consistent with the system in (5), see e.g. Figure 8 for a situation in which the trajectories of the system are increasing for $e(t)$ and $m(t)$, but exhibit a turning point and a decreasing range in $s(t)$.

It is also interesting to note a structural difference between the data in Table 1 and Figure 7 with the trajectories of the system in (5). Indeed, unlike Figure 7, in an interval of increase of enterprises and cash flow, the staff evolution described by (5) can transition only from decreasing to increasing, and never from increasing to decreasing:

**Theorem 3.** *Let $t_0 < t_1$. Suppose that $\dot{m} \geq 0$ and $\dot{e} \geq 0$ in the interval $(t_0, t_1)$. Then, $s$ cannot have a local maximum in the interval $(t_0, t_1)$ (unless the solution is constantly equal to the equilibrium $(e_*, m_*, s_*)$ in (6)).*

The structural difference outlined in Theorem 3 is important because it highlights that real-world enterprises face more instability than the simplified model suggests. It also helps us understand the possible cause and extent of these additional unpredictable factors. In fact, the datasets in Table 1 and Figure 7 are arguably significantly impacted by COVID-19, since the 2020 revenue was negatively affected by store closures and reduced customer traffic globally, while a successive strong rebound was fueled by the reopening of stores. The strong rebound of staffing in 2021 was also due to the resumed operations, which was later compensated by restructuring, leading to a decrease in staff.

## 4. Literature review and organization of the paper

Even though our modeling approach is distinctive, and the results presented here are new in the literature, our paper sits at the intersection of organizational growth, internal firm dynamics, instability, and non-

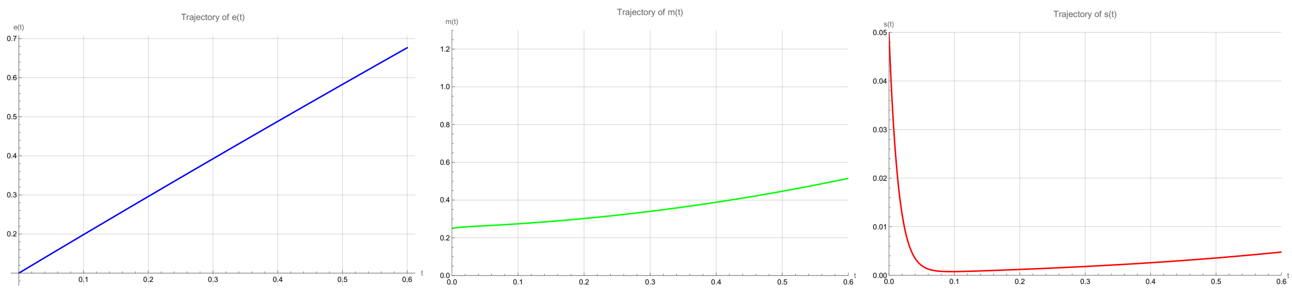


**Figure 8.** Evolution of $e(t)$, $m(t)$, and $s(t)$ for (5) when $\alpha := \delta := \zeta := \mu := 1, \beta := \eta := \kappa := 0.1$, and $\epsilon := 70$, with initial conditions $e(0) := 0.1$, $m(0) := 0.25$, and $s(0) := 0.05$.

profit-maximizing behavior. As such, this article can be compared with the specialized literature dealing with growth, diversification, and internal instability of firms. In particular, the classical book Penrose (1995) discusses the theory of the growth of firms, describing an interacting process influenced by internal managerial and organizational capacity, not just market conditions.

On a related note, the model in Marris (1964) shifted the focus of the firm from traditional profit maximization to growth maximization, arguing also that in modern corporations where ownership is separated from control, managers have different priorities than shareholders (tying managers' objectives to salary, power, and status, which are more closely correlated with the size and growth rate of the firm than with absolute profit, dividends, and share prices). According to Marris (1964), the firm reaches equilibrium when the rate at which managers create new demand for products matches the rate at which they can supply capital to fund that expansion. In this scenario, managers must maintain a minimum level of profit to prevent a "hostile takeover", that stops them from pursuing growth at any cost.

From this perspective, the works Penrose (1995); Marris (1964) support the idea that expansion can generate internal fragility, even in the absence of major external shocks, in line with the findings of this paper.

Critical dynamics in the "enterprise-staff interaction" are also evidenced in Rajan et al. (2000), showing that funds are often transferred from efficient divisions to inefficient ones, and that misallocation is most severe when there is high diversity in the investment opportunities and resources across divisions. These internal inefficiencies faced by multi-division firms can be related to the enterprise-staff interaction mechanism highlighted in this paper.

Additionally, see e.g. Lang and Stulz (1994), there is empirical evidence of the "diversification discount", i.e., of the fact that diversification is often associated with lower firm value, which aligns with the idea that spreading resources across multiple enterprises can lead to instability rather than "synergy": Interestingly, the drop in value was most significant when a firm moved from being a single-segment firm to a multi-segment firm.

Regarding the traditional view that economic fluctuations must be caused by external shocks, it can be shown, see Day (1982) that even a simple deterministic model can generate chaotic behaviors. A common feature between Day (1982) and our approach is the idea of detecting stability in the neighborhood of stationary states. However, the setting of Day (1982) is structurally different from ours, since it focuses on a single difference equation in the capital-labor ratio (within the theory of iterated maps of the interval), rather than a system of ordinary differential equations involving multiple evolving structural quantities.

A related topic in corporate growth literature is open innovation, see Costa et al. (2023), where companies leverage a variety of internal and external knowledge to accelerate development. Despite some similarities in short- and long-term cost-benefit structures, this represents a distinct domain that does not directly address the dynamic mechanisms modeled in this paper; thus, it requires a separate analysis.

Hence, in comparison with the existing literature, our approach deviates from the standard paradigm, which emphasizes external shocks, optimization-based behavior, or stochastic environments as necessary sources of instability. By contrast, our analysis highlights how intrinsic instability can arise from internal structural interactions alone, even in a deterministic setting and without profit-maximizing assumptions.

At the same time, the existing literature does justify our deviation from the main paradigm, since related approaches have been considered in the past. However, the model proposed here is new: It accounts for several evolving quantities and leads to a novel stability analysis.

In summary:

- The *purpose* of this paper is to develop a simplified mathematical framework that captures the internal dynamics between money flow, staff size, and the number of enterprises managed by a corporation, independent of external shocks and stochastic events.
- The *objective* is to analyze the stability of these internal relationships using a system of ordinary differential equations and to identify whether inherent risks arise purely from the corporation's internal structure and management dynamics.
- The assumed *hypotheses* are based on simple proportionality relations linking the main quantities under consideration, in the absence of random components.
- The main *novelty* of this paper is the introduction of a new mathematical model showing that, even in the absence of external shocks, the internal interactions between financial flow, staffing levels, and enterprise count can generate instability. Such intrinsic instability poses significant risks to both venture success and staff employment.

This paper is organized as follows: In Section 5.1, we present the proof of Theorem 1. The proof of Theorem 2 is technically simpler than that of Theorem 1 and is omitted. Section 5.2 contains the proof of Theorem 3.

## 5. Mathematical proofs

Here below, we present the detailed proofs of the main results of this paper.

### *5.1. Stability analysis and proof of Theorem 1*

The possible equilibria of (5) correspond to solutions of

$$\begin{cases} \alpha\zeta - \eta e_* = 0, \\ \alpha e_* - \beta s_* + \dfrac{\mu s_*}{e_*} + \kappa m_* = 0, \\ \delta e_* m_* - \epsilon s_* = 0. \end{cases}$$

This yields that $e_* = \frac{\alpha\zeta}{\eta}$ and, therefore,

$$m_* = \frac{\epsilon s_*}{\delta e_*} = \frac{\epsilon\eta s_*}{\alpha\delta\zeta}. \tag{13}$$

Accordingly,

$$\begin{aligned} 0 &= \alpha e_* - \beta s_* + \frac{\mu s_*}{e_*} + \kappa m_* \\ &= \frac{\alpha^2\zeta}{\eta} - \beta s_* + \frac{\eta\mu s_*}{\alpha\zeta} + \frac{\epsilon\eta\kappa s_*}{\alpha\delta\zeta} \\ &= \frac{\alpha^2\zeta}{\eta} - \frac{(\alpha\beta\delta\zeta - \delta\eta\mu - \epsilon\eta\kappa)s_*}{\alpha\delta\zeta} \\ &= \frac{\alpha^2\zeta}{\eta} - \frac{c_0 s_*}{\alpha\delta\zeta} \end{aligned}$$

where the notation in (7) has been used.

From this, we arrive at $s_* = \frac{\alpha^3\delta\zeta^2}{c_0\eta}$ and, as a result, by (13), $m_* = \frac{\alpha^2\epsilon\zeta}{c_0}$, completing the proof of (6).

Moreover, the Jacobian matrix associated with the system of ordinary differential equations (5) is

$$J = \begin{bmatrix} -\eta & 0 & 0 \\ \alpha - \dfrac{\mu s}{e^2} & \kappa & -\beta + \dfrac{\mu}{e} \\ \delta m & \delta e & -\epsilon \end{bmatrix}.$$

Evaluated at the equilibrium in (6), this gives

$$J_* = \begin{bmatrix} -\eta & 0 & 0 \\ \dfrac{\alpha\,(c_0 - \delta\eta\mu)}{c_0} & \kappa & \dfrac{\eta\mu - \alpha\beta\zeta}{\alpha\zeta} \\ \dfrac{\alpha^2\delta\epsilon\zeta}{c_0} & \dfrac{\alpha\delta\zeta}{\eta} & -\epsilon \end{bmatrix}.$$

The characteristic polynomial of $J_*$ is

$$P(t) := \det(t\mathbf{1} - J_*) = t^3 + c_2 t^2 + c_1 t + c_0,$$

where $\mathbf{1}$ denotes the $(3 \times 3)$-identity matrix,

$$c_1 := \epsilon\eta - \eta\kappa - \epsilon\kappa - \delta\mu + \frac{\alpha\beta\delta\zeta}{\eta} = \eta(\epsilon - \kappa) + \frac{c_0}{\eta},$$

$$\text{and} \qquad c_2 := \epsilon - \kappa + \eta.$$

Hence, in the setting of (9),

$$P(t) = (t - \lambda_1)(t - \lambda_2)(t - \lambda_3).$$

Notice also that, when (10) holds true, we have that $\lambda_3 > 0$, and the proof of Theorem 1 is thereby complete.

### *5.2. Local maxima of staff and proof of Theorem 3*

For the sake of contradiction, we suppose that $s$ possesses a local maximum at $t_\sharp \in (t_0, t_1)$ (without the solution being constantly equal to the equilibrium $(e_*, m_*, s_*)$).

We use the short notation

$$X := (e, m, s) \qquad \text{and} \qquad X_\sharp := X(t_\sharp) = (e(t_\sharp), m(t_\sharp), s(t_\sharp)),$$

and rewrite (5) in the concise form $\dot{X}(t) = f(X(t))$, for a suitable function $f$ which is real analytic in $(0, +\infty)^3$.

We claim that

$$X_\sharp \in (0, +\infty)^3. \tag{14}$$

To achieve this aim, we first check that

$$e(t_\sharp) > 0. \tag{15}$$

Indeed, if $e(t_0) > 0$, then the monotonicity of $e$ gives that $e(t_\sharp) \geq e(t_0) > 0$, and we are done. If instead $e(t_0) = 0$, we infer from (5) that $\dot{e}(t_0) = \alpha\zeta > 0$. In this case, we can find $t_0' \in (0, t_\sharp)$ such that $e(t_0') > 0$ and then, again by the monotonicity of $e$, we find that $e(t_\sharp) \geq e(t_0') > 0$. The proof of (15) is thereby complete.

Now we check that

$$m(t_\sharp) > 0. \tag{16}$$

Indeed, suppose not. Then, by the monotonicity of $m$, we get that $m = 0$ in $(t_0, t_\sharp]$. But then, by (5), we have that $\dot{s} = -\epsilon s \leq 0$ in $(t_0, t_\sharp]$. Since $s$ has a local maximum at $t_\sharp$, necessarily $s$ is constant in $(t_0, t_\sharp]$.

Thus, using again (5), in $(t_0, t_\sharp)$,

$$0 = \dot{m} = \alpha e - \beta s + \frac{\mu s}{e} + \kappa m = \alpha e - \beta s + \frac{\mu s}{e},$$

yielding that $e$ is also constant in $(t_0, t_\sharp)$.

This says that the trajectory under consideration is constant in $(t_0, t_\sharp)$ and then, by the uniqueness of the equilibrium discussed in Theorem 1, we get that

$$m(t_\sharp) = m_* = \frac{\alpha^2 \epsilon\zeta}{c_0} > 0,$$

which goes against our assumption, and this proves (16).

Now we prove that

$$s(t_\sharp) > 0. \tag{17}$$

Indeed, if not, since $s$ has a local maximum at $t_\sharp$, necessarily $s(t) = 0$ in $(t_0, t_\sharp]$. This reduces (5) in this interval to

$$\begin{cases} \dot{e} = \alpha\zeta - \eta e, \\ \dot{m} = \alpha e + \kappa m, \\ 0 = em. \end{cases}$$

Accordingly,

$$0 = \dot{e}m + e\dot{m} = (\alpha\zeta - \eta e)m + e(\alpha e + \kappa m) = \alpha\zeta m + \alpha e^2 \geq \alpha e^2.$$

This implies that $e$ vanishes identically in $(t_0, t_\sharp)$ and then, in this interval,

$$0 = \dot{e} = \alpha\zeta - \eta e = \alpha\zeta,$$

which is a contradiction. This completes the proof of (17).

Thus, the claim in (14) follows by putting together (15), (16), and (17).

Now, we observe that, since $t_\sharp \in (t_0, t_1)$ is a local maximum for $s$, by (5),

$$0 = \dot{s}(t_\sharp) = \delta e(t_\sharp) m(t_\sharp) - \epsilon s(t_\sharp) \tag{18}$$

and

$$\begin{aligned} 0 &\geq \ddot{s}(t_\sharp) \\ &= \delta\dot{e}(t_\sharp)m(t_\sharp) + \delta e(t_\sharp)\dot{m}(t_\sharp) - \epsilon\dot{s}(t_\sharp) \\ &= \delta\dot{e}(t_\sharp)m(t_\sharp) + \delta e(t_\sharp)\dot{m}(t_\sharp). \end{aligned}$$

From the monotonicity assumption on $e$ and $m$, we evince that the latter term is nonnegative, therefore, recalling (14),

$$\dot{e}(t_\sharp) = \dot{m}(t_\sharp) = 0. \tag{19}$$

By virtue of (18) and (19),

$$f(X_\sharp) = f(X(t_\sharp)) = \dot{X}(t_\sharp) = (\dot{e}(t_\sharp), \dot{m}(t_\sharp), \dot{s}(t_\sharp)) = 0$$

and consequently

the function constantly equal to $X_\sharp$ is a solution of (5).

Then, by the uniqueness of the solution of the Cauchy problem, it follows that the solution under consideration $X$ is also constantly equal to $X_\sharp$ (which is necessarily the equilibrium in (6), since we know by Theorem 1 that there is only one equilibrium), which goes against our original assumption.

## 6. Conclusions

Opening new enterprises enables corporations to grow, enter new markets, and innovate. However, they face major risks such as economic changes, financial instability, competition, operational difficulties, regulatory hurdles, and unpredictable events like wars or pandemics. Additionally, borrowing costs and currency fluctuations can increase financial risks.

In these situations, it is natural to think that the risks involved are merely a byproduct of the abundance, variety, and complexity of the factors at play.

Instead, in this paper, we present a simple mathematical model that reduces this complexity to its core, accounting only for deterministic relationships among money flow, staff size, and the number of enterprises managed by a corporation, without considering major external threats.

Even in this highly simplified scenario, we show that, while the market drives the number of enterprises toward equilibrium, severe instability arises in money flow and staff size, posing concrete risks to the practical success of the venture and the employability of staff.

The results of this study have relevant implications for economic policy, corporate governance, and public oversight, particularly in contexts where organizations manage multiple ventures simultaneously. These implications can be summarized as follows:

- *Growth-oriented policies can generate intrinsic instability*, even in the absence of external shocks, purely due to internal structural dynamics: Policies that strongly incentivize expansion may therefore unintentionally increase volatility in employment and financial performance if they are not accompanied by appropriate safeguards.
- *Capital-biased incentives may undermine employment stability*, since the analysis shows that high returns on capital are a key driver of instability, particularly when staff adjustment is slow: This phenomenon may increase the likelihood of employment volatility or workforce contraction even during periods of rising revenues and expanding operations.
- Because the model *does not assume profit maximization*, the results are particularly relevant for universities, public agencies, hospitals, and state-owned enterprises, which often pursue growth, diversification, or mission expansion rather than profits alone. This highlights the importance of governance rules that link expansion decisions to staffing and operational capacity, rather than to revenue growth alone.

- From a regulatory standpoint, the findings suggest that *traditional risk assessments, focused mostly on external shocks, market volatility, or financial leverage, may overlook important and intrinsic sources of instability*.
- The results *caution against performance frameworks that reward organizations primarily for increasing the number of ventures, subsidiaries, or projects*: Such metrics may mask rising internal instability and lead to *short-term success followed by sharp employment or financial disruptions*.

We hope this simplified model aids the challenging task of risk management by reminding corporate executives of the intrinsic instability and high sensitivity inherent in managing multiple enterprises.

## Data availability statement

All the data used in this paper are publicly available.

## Author contributions

In collaborative and inclusive fields such as mathematics, authors' contributions are understood to be equal.

## Use of AI tools declaration

Not relevant for this paper.

## Conflict of interest

The authors declare no conflicts of interest.

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