



Research article

Forecasting volatility indices in stock and gold markets: Synergistic effects of the GARCH-MIDAS model and economic policy uncertainty

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Abstract: Volatility indices reflect the risk-neutral expectation of future volatility implied in option prices, differing from the volatility predicted by historical volatility forecasting frameworks. Prior studies have overlooked the influence of low-frequency macroeconomic factors on the volatility of the derivatives market. This study applied the GARCH-MIDAS model to forecast the VIX (equities) and GVZ (gold) volatility indices, highlighting the synergistic effects of mixed-frequency modeling and economic policy uncertainty (EPU) indices. After risk neutralization, we accounted for the forward-looking nature of volatility indices by incorporating cross-month adjustments to long-term variances under a risk-neutral framework. Empirical results show that incorporating mixed-frequency components improves forecasting accuracy. The results of the model confidence set (MCS) test verify statistical robustness, whereas the evaluation of economic significance highlights practical relevance. Overall, the integration of EPU into risk-neutral GARCH-MIDAS frameworks provides superior predictive performance compared to other approaches and reveals the critical role of macroeconomic uncertainty in volatility forecasting.

Keywords: volatility indices; economic policy uncertainty; GARCH-MIDAS model; risk-neutral pricing

JEL Codes: G10, G13, G17

1. Introduction

The implied volatility indices calculated by the CBOE (Chicago Board Options Exchange) represent the market's expectations for future volatility over 30 days. These indices are derived from current option prices and reflect expected price fluctuations. As of today, the CBOE has released multiple volatility indices, including stock markets, precious metals, currencies, interest rates, and crude oil. Volatility indices are considered forward-looking barometers of market sentiment and risk appetite. When the indices increase, market participants are more pessimistic and concerned about future market trends, whereas a decline in the indices suggests that investors are more optimistic about future market movements (Whaley, 2000). Volatility indices contain richer information than the implied volatility calculated through option pricing models, the former of which can more effectively predict volatility and improve the pricing accuracy of derivatives (Kanniainen et al., 2014; Bollerslev et al., 2015; Pan et al., 2019). Effective modeling and forecasting of volatility indices are also crucial for hedging volatility risks (Degiannakis et al., 2018; Vrontos et al., 2021); some scholars have recognized the role of the important information contained in these indices in asset pricing and investment portfolios (Ruan, 2020; Das et al., 2022; Dutta and Das, 2022).

The literature on volatility index forecasting has evolved through the successive improvement of discrete-time GARCH-type models. Since the foundational studies by Hao and Zhang (2013) and Liu et al. (2015), researchers have extended GARCH modeling for volatility index forecasting in multiple dimensions. Qiao et al. (2020, 2022) incorporated realized variance decomposition into GARCH structures to study VIX and VIX term structure forecasting. Wu et al. (2023a) considered the impact of time-varying risk aversion on VIX forecasting on the basis of the REGARCH-MIDAS model, whereas Wu et al. (2023b) extended the realized EGARCH model to consider long- and short-term volatility in VIX prediction. Hansen et al. (2024) noted that the realized GARCH model can flexibly characterize the dual impact of returns and volatility, demonstrating superior predictive performance over standard GARCH specifications. Qiao et al. (2024) examined the predictive effect of asymmetric positive and negative jump volatility in the equity market on the crude oil volatility index. Most recently, Qiao et al. (2025) proposed an integrated framework by combining HAR-DJI-GARCH and GARCH-MIDAS models, where the decomposed volatilities of VIX are incorporated into a new model and achieve improved forecasting accuracy.

Previous studies have shown that combining the GARCH model with the MIDAS method yields more accurate volatility predictions (Asgharian et al., 2013; Nieto et al., 2015; Conrad and Loch, 2015; Fang et al., 2018). This finding has been further supported by subsequent studies using extended models and different datasets (Pan et al., 2020; Li et al., 2021; Guo et al., 2022). Additionally, such models provide a flexible framework for integrating low-frequency macroeconomic variables, including economic policy uncertainty (EPU) and geopolitical risk indices (GRI), into volatility analysis. This ability effectively connects macroeconomic fundamentals with financial market behavior, as demonstrated in prior research (Fang et al., 2018; Salisu et al., 2020; Asgharian et al., 2023). More recent studies further emphasize the role of uncertainty-related factors in shaping volatility dynamics (Yao and Liu, 2023; Liu et al., 2023; Li et al., 2023). In addition to macroeconomic factors, market-based measures have been incorporated. For instance, Amendola et al. (2021) extended the double asymmetric GARCH-MIDAS model by incorporating VIX, markedly improving forecasts of stock market volatility.

While GARCH-MIDAS models have proven successful in volatility forecasting, their application in forecasting volatility indices has received relatively little attention, trailing their well-established use in modeling historical volatility. Unlike traditional GARCH-MIDAS models, which focus on capturing historical volatility, volatility indices such as the VIX index instead reflect the risk-neutral expectations of future volatility embedded in option prices (Demeterfi et al., 1999; Bollerslev et al., 2009; Drechsler and Yaron, 2011). This situation creates a critical disconnect, as the model's focus under the physical measure does not account for the forward-looking risk premiums required for derivatives pricing. Adaptation to the risk-neutral framework necessitates the derivation of the market price of risk adjustments, which are not inherently incorporated into the standard GARCH-MIDAS structure. Moreover, the literature has largely overlooked how low-frequency macroeconomic factors influence volatility index dynamics and their pricing implications. These factors may influence derivative markets through risk premium transmission (e.g., monetary policy affecting discount rates) and pricing model foundations (e.g., dividend yields as critical inputs). Although much of the literature has focused on empirical analysis, theoretical studies also provide important insights into the relationship between policy uncertainty and market volatility. Pastor and Veronesi (2012, 2013) developed influential models showing that when future policy direction is uncertain, investors demand higher risk premia, which in turn amplifies stock price fluctuations. Their framework highlights the policy uncertainty channel through which monetary policy uncertainty (MPU) can directly drive market volatility by affecting both expected cash flows and discount rates. This theoretical foundation offers a solid basis for subsequent empirical studies. Despite these insights, integrating these macroeconomic factors into risk-neutral frameworks is challenging due to data frequency mismatches between derivative prices and macroeconomic variables. Although emerging studies have attempted to apply GARCH-MIDAS-type models to VIX forecasting, they have ignored the cross-month¹ effects when introducing long-term variances (Wu et al., 2023a) or are confined only to time-series forecasting frameworks rather than to addressing derivative pricing perspectives (Qiao et al., 2025).

Against this background, a growing body of research has examined the link between economic policy uncertainty (EPU) and market volatility. The EPU index, as an important early warning signal in the financial market, has proven to be correlated with volatility. Studies have shown that high EPU signals increase economic uncertainty, which dampens business investment and consumer spending, leading to greater economic volatility. This uncertainty often extends to financial markets, heightening investor risk aversion, amplifying stock market fluctuations, and increasing volatility (Baker et al., 2016; Brogaard and Detzel, 2015; Kelly et al., 2016). Jones and Sackley (2014) demonstrated that incorporating the EPU index into the modeling of gold pricing models reveals a positive relationship between policy uncertainty and gold price fluctuations. Overall, these findings suggest that changes in EPU serve as a reliable predictor of future market volatility (Antonakakis et al., 2013; Liu and Zhang, 2015; Ma et al., 2019). Building on this, several studies have integrated EPU into the GARCH-MIDAS framework, demonstrating its validity as a predictor. For instance, Yu and Huang (2021) integrated EPU into the GARCH-MIDAS model and reported that EPU substantially improves stock volatility forecasts. Li et al. (2023) formulated a smoothed-transformation GARCH-MIDAS model to assess the impact of EPU on equity volatility, emphasizing its pivotal role in driving market fluctuations. Apergis et al. (2023) reported that the COVID-19 pandemic considerably increased the VIX index under conditions of great uncertainty,

¹ The cross-month effect refers to the influencing factors of volatility indices whose calculation window spans two months (e.g., from day i of month t to day j of month $t+1$).

while the effect is negligible under conditions of low uncertainty. Le et al. (2024) utilized a GARCH-MIDAS model to assess how five key variables influence WTI crude oil volatility and reported that macroeconomic uncertainty (EPU) has the greatest effect on price dynamics. Zeng et al. (2024) employed a mixed-frequency dynamic threshold MS-MIDAS model to reveal time-varying threshold effects between EPU and stock market volatility, notably enhancing forecasting performance. Yang (2025) reported that EPU increases foreign exchange implied volatility in developed economies and some emerging markets at longer horizons and during extreme market conditions. Botta and Sakariyahu (2025) demonstrated that U.S. elections substantially amplify abnormal volatility across asset classes, with the effects being stronger during recessions. Collectively, these studies emphasize the central role of EPU in shaping volatility dynamics across markets and conditions.

This study focuses on forecasting two volatility indices—the VIX index (linked to S&P 500 index options) and the GVZ index (tied to gold futures options). Unlike in previous research, we incorporate three EPU indices as low-frequency macroeconomic drivers within the GARCH-MIDAS framework, seeking to capture how they influence the dynamics of risk-neutral volatility. Our analysis addresses the gap in the literature concerning the limited integration of macroeconomic variables into risk-neutral volatility modeling and forecasting. The proposed framework explicitly links low-frequency uncertainty to volatility index dynamics, providing a more comprehensive understanding of how macroeconomic fundamentals shape the volatility expectations of derivative markets.

This paper makes several contributions. First, we expand the GARCH-MIDAS framework to forecast volatility indices in a risk-neutral setting, deriving analytical expressions for volatility indices that account for their forward-looking characteristics. Departing from Engle et al.'s (2013) original physical-measure specification, our formulation incorporates risk-neutral valuation principles, a critical requirement for the application of volatility derivatives. Given the inherently forward-looking nature of volatility indices, we address the challenge of cross-month effects arising from the introduction of long-term variances. Specifically, the calculation of volatility indices is decomposed into two components—a short-term GARCH-driven component and a long-term EPU-sensitive component. To ensure the feasibility of this decomposition, we develop a novel method for estimating the conditional expectation for long-term variance for the subsequent month, which is a critical parameter in the risk-neutral pricing framework. This decomposition is a key methodological innovation and thereby resolves the challenge of how to incorporate low-frequency data into volatility index forecasts.

Second, we integrate EPU indices as the macroeconomic factors within the GARCH-MIDAS framework. This approach confirms the effectiveness of these EPU indices in predicting the VIX and GVZ indices, highlighting asset-specific responses to uncertainty. This inclusion demonstrates that macroeconomic uncertainty, as measured by EPU indices, is a significant factor in volatility index forecasting. Moreover, low-frequency macroeconomic variables are rarely considered in volatility index forecasting and derivative pricing models. By embedding EPU into a derivative pricing-compatible GARCH-MIDAS framework, our model enables the quantification of volatility risk premiums attributable to macroeconomic uncertainty. The model separates total volatility into a short-term component, driven by short-term GARCH dynamics, and a long-term component, estimated via the MIDAS approach, to capture the impact of macroeconomic uncertainty, such as changes in the EPU index. This framework allows us to precisely quantify how much macroeconomic uncertainty contributes to the volatility index. This study addresses the underexplored integration of low-frequency variables into risk-neutral pricing frameworks, offering new insights into how policy uncertainty propagates through derivative markets.

Third, through an extensive empirical analysis, the performance of the GARCH-MIDAS-type model in forecasting the VIX and GVZ volatility indices is evaluated, and the robustness and reliability of the model's predictability under the MCS test and economic significance evaluation are assessed. Our analysis reveals that the EPU index improves forecasts for both the VIX and the GVZ indices, although different EPU indices have varying impacts. Robustness tests confirm these results, highlighting the model's ability to disentangle macroeconomic influences across asset classes, thus contributing to understanding how macroeconomic uncertainty shapes implied volatility differently under risk-neutral pricing, with implications for derivative pricing and risk management.

The rest of the article is structured as follows: Section 2 outlines the methodology, including detailed explanations of risk neutralization, the calculation of volatility indices, comparative models, parameter estimation, and error evaluation. Section 3 offers a thorough analysis of the empirical results. Sections 4 and 5 present assessments of economic significance and robustness tests, respectively. Finally, the conclusions synthesize the findings from the previous sections.

2. Methodology

2.1. Risk neutralization

The GARCH-MIDAS framework, developed by Engle et al. (2013), brings together GARCH-type short-term volatility modeling and MIDAS-based long-term component analysis in an innovative way. By combining these two elements, the model effectively captures both transient fluctuations and persistent movements in financial market volatility, thereby improving forecast performance (Pan et al., 2017; Mo et al., 2018; Ma et al., 2019). Below, we outline the specific formulation of the GARCH-MIDAS-type model used for forecasting volatility indices.

Following the option pricing framework of Duan (1995), under physical measure P , it is assumed that the daily return of the underlying asset² follows:

$$R_{i,t} = r - \frac{1}{2} h_{i,t} + \lambda \sqrt{h_{i,t}} + z_{i,t} \quad (1)$$

where $R_{i,t}$ represents the daily logarithmic return on day i in month t , r represents the risk-free rate, and the equity risk premium λ , linked to conditional variance $h_{i,t}$, is $\lambda > 0$. The innovation term $z_{i,t}$ follows $z_{i,t} = \sqrt{h_{i,t}} \varepsilon_{i,t}$, with $\varepsilon_{i,t}$ drawn from the standard normal distribution as follows: $\varepsilon_{i,t} \sim N(0,1)$

Total conditional variance $h_{i,t}$ is expressed as follows:

$$h_{i,t} = \tau_t g_{i,t} \quad (2)$$

where τ_t is the long-term component, and $g_{i,t}$ is the short-term component.

² The assumption of daily returns is a little different from that of Engle et al. (2013), primarily because our framework builds on the option pricing model developed by Duan (1995).

The short-term components are specified according to the option pricing framework of Duan (1995), which has been applied to VIX forecasting, providing a unified representation of short-term volatility dynamics and risk-neutral valuation. This approach has been widely adopted in subsequent studies on option valuation and VIX modeling (e.g., Hao and Zhang, 2013; Christoffersen et al., 2014; Kannianen et al., 2014; Qiao et al., 2020).

The dynamics of the short-term components are as follows:

$$g_{i,t} = \omega + \beta g_{i-1,t} + \alpha g_{i-1,t} \varepsilon_{i-1,t}^2 \quad (3)$$

where $\omega = 1 - \alpha - \beta$ and $\omega > 0, \alpha \geq 0, \beta \geq 0$.

In this study, we specify the long-term component in a linear form rather than an exponential specification. This choice follows prior GARCH-MIDAS applications (e.g., Engle et al., 2013) and ensures both tractability for deriving option-based volatility forecasts and preservation of the sign information of low-frequency predictors. The long-term component is expressed as follows:

$$\tau_t = m + \eta_{RV} \sum_{k=1}^{K_{RV}} \phi_k(\omega_{RV,1}, \omega_{RV,2}) RV_{t-k}^m \quad (4)$$

where $RV_t^m = \sum_{i=1}^N R_{i,t}^2$ represents the monthly realized variance, N is the number of trading days in month t , and K_{RV} is the length of the MIDAS lag terms for RV_t^m . $\phi_k(\cdot)$ is the weighting function, and we use a beta-weighting scheme, which is very flexible and can accommodate different lag orders, calculated as follows:

$$\phi_k(\omega_1, \omega_2) = \frac{(k/K)^{\omega_1-1} (1-k/K)^{\omega_2-1}}{\sum_{j=1}^K (j/K)^{\omega_1-1} (1-j/K)^{\omega_2-1}} \quad (5)$$

Under risk-neutrality measure Q , we can prove that asset returns become

$$R_{i,t} = r - \frac{1}{2} h_{i,t} + z_{i,t}^* \quad (6)$$

where $z_{i,t}^* = z_{i,t} + \lambda \sqrt{h_{i,t}}$ and $z_{i,t}^* = \sqrt{h_{i,t}} \varepsilon_{i,t}^*, \varepsilon_{i,t}^* \sim N(0,1)$. See Appendix A for details.

Since there is no random term in long-term variance, it maintains the same form under the risk-neutral measure. The short-term conditional variance becomes the following:

$$g_{i,t} = \omega + \beta g_{i-1,t} + \alpha g_{i-1,t} (\varepsilon_{i-1,t}^* - \lambda)^2 \quad (7)$$

where $\sqrt{g_{i-1,t}} \varepsilon_{i-1,t}^* = \frac{R_{i-1,t} - r + \frac{1}{2} h_{i-1,t}}{\tau_t}$. In this study, we also examine the information on EPU indices³

for volatility index forecasting. In this case, the long-term variance becomes

$$\tau_t = m + \eta_{RV} \sum_{k=1}^{K_{RV}} \phi_k(\omega_{RV,1}, \omega_{RV,2}) RV_{t-k}^m + \eta_X \sum_{k=1}^{K_X} \phi_k(\omega_{X,1}, \omega_{X,2}) X_{t-k} \quad (8)$$

where X_t represents an exogenous variable, taken as one of the EPU indices in our study, and K_X is the length of the MIDAS lag terms for X_t .

2.2. Calculation of the volatility index

In terms of the CBOE, VIX squared represents the risk-neutral variance of the S&P 500 Index for the next 30 days. However, in the GARCH-MIDAS framework, the methodology for computing the VIX index differs from that used in conventional, same-frequency GARCH models (Liu et al., 2015; Wang et al., 2017; Yang and Wang, 2018). Subsequent studies have further explored and extended this computational framework across various scenarios (Yang et al., 2019; Guo and Liu, 2020; Xie et al., 2020; Qiao et al., 2020). As shown in Figure 1, considering the forward-looking characteristic of volatility indices, the calculation of VIX squared for day i of month t requires the conditional variance of the next N (taken as 22 here) days, and the long-term variance component τ_t is usually modeled as “backward looking” on the basis of historical data [such as realized volatility (RV) and economic policy uncertainty (EPU)], which may span two months, namely, the current (t) and following ($t+1$) months. This means that long-term variances τ_t and τ_{t+1} involving two months are needed for VIX_{it} . To address this challenge, we split the calculation of VIX squared into the sum of two components: one that captures intramonth effects and another that captures intermonth effects⁴. Mathematically, this can be represented as follows:

³ Assessed primarily through the following aspects: (1) frequency of relevant terms: the frequency of words associated with “economy”, “policy”, and “uncertainty” in newspaper articles; (2) tax policy uncertainty: changes in tax rates and the introduction of new taxes; (3) federal government expenditure uncertainty: scale, structure, and timing of federal government expenditures; and (4) inflation expectation uncertainty: uncertainty regarding inflation expectations. A higher EPU index signifies greater economic policy uncertainty, which may stem from factors such as disputes in the policy formulation process, anticipated policy changes, and insufficient coordination among government departments.

⁴ Intramonth effects refer to the influencing factors of the volatility index within the same month, where the calculation window is entirely contained within a single month (e.g., from day i to day j of a month, with $j \leq$ the last day of that month). Intermonth effects refer to volatility index-influencing factors, where the calculation window spans two consecutive months (e.g., from day i of month t to day j of month $t+1$).

$$\begin{aligned} \frac{1}{252} \left(\frac{VIX_{i,t}^{Model}}{100} \right)^2 &\cong \frac{1}{22} \left[\sum_{k=1}^{M_t-i} E_{i,t}^Q[h_{i+k,t}] + \sum_{m=1}^{22-(M_t-i)} E_{i,t}^Q[h_{m,t+1}] \right] \\ &= \frac{1}{22} \left[\sum_{k=0}^{M_t-1-i} \tau_t E_{i,t}^Q[g_{i+k+1,t}] + \sum_{m=0}^{21-(M_t-i)} E_{i,t}^Q[\tau_{t+1}] E_{i,t}^Q[g_{m+1,t+1}] \right] \end{aligned} \tag{9}$$

where M_t represents the cardinality of trading days within month t 's observation window, and the two components represent the contributions of conditional variance within the given month and due to transitions between months.

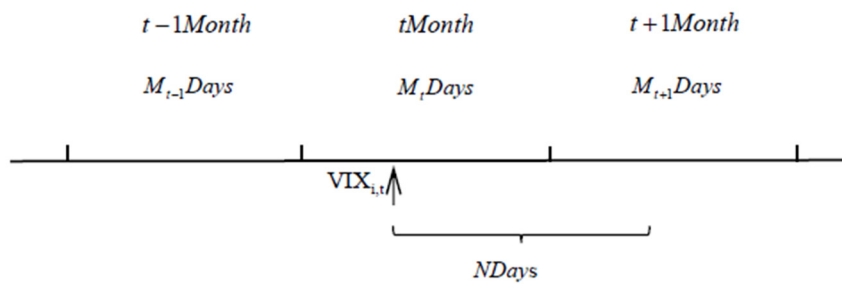


Figure 1. Timeline for the VIX calculation.

In the second term of Equation (9),

$$\begin{aligned} E_{i,t}^Q[\tau_{t+1}] &= m + \eta'_{RV} \sum_{k=2}^{K_{RV}} \phi_k(\omega_{RV,1}, \omega_{RV,2}) RV_{t+1-k}^m + \eta'_{RV} \phi_k(\omega_{RV,1}, \omega_{RV,2}) E_{i,t}^Q[RV_t^m] \\ \text{or} \\ E_{i,t}^Q[\tau_{t+1}] &= m + \eta'_{RV} \sum_{k=2}^{K_{RV}} \phi_k(\omega_{RV,1}, \omega_{RV,2}) RV_{t+1-k}^m + \eta'_X \sum_{k=2}^{K_X} \phi_k(\omega_{X,1}, \omega_{X,2}) X_{t+1-k} \\ &\quad + \eta'_{RV} \phi_k(\omega_{RV,1}, \omega_{RV,2}) E_{i,t}^Q[RV_t^m] + \eta'_X \phi_k(\omega_{X,1}, \omega_{X,2}) E_{i,t}^Q[X_t] \end{aligned} \tag{10}$$

However, the calculation of $E_{i,t}^Q[RV_t^m]$ requires the total returns of month t , which cannot be resolved on the basis of the information up to the i -th day of the t -th month. Here, we apply the AR(1) regression model for RV_t^m and obtain the estimated value of

$$E_{i,t}^Q[RV_t^m] = \nu + \zeta \cdot RV_{t-1}^m \tag{11}$$

where ν and ζ are the regression coefficients of the AR(1) model and are estimated via the least squares method. Then, the estimated $\hat{\tau}_{t+1} \triangleq E_{i,t}^Q[\tau_{t+1}]$ is obtained.

Finally, the analytical expression of VIX squared can be calculated as follows:

$$\frac{1}{252} \left(\frac{VIX_{i,t}^{Model}}{100} \right)^2 = \tau_t [A + Bg_{i+1,t}] + \tau_{t+1} [C + Dg_{i+1,t}] \tag{12}$$

where A , B , C , and D are parameter-dependent functions that satisfy $A = \frac{\Omega^*}{1-\Gamma^*} \left(\frac{M_t-i}{22} - B \right)$,

$$B = \frac{1-\Gamma^{*M_t-i}}{22(1-\Gamma^*)}, \quad C = \frac{\Omega^*}{1-\Gamma^*} \left(\frac{22-M_t+i}{22} - D \right), \quad \text{and} \quad D = \frac{\Gamma^{*M_t-i}(1-\Gamma^{*22-M_t+i})}{22(1-\Gamma^*)},$$

respectively, where

$\Omega^* > 0$, and $\Omega^* = \omega$. The persistence of the conditional variance satisfies $\Gamma^* = \beta + \alpha(1 + \lambda^2) < 1$.

See Appendix B for the detailed deduction.

If we consider the exogenous variable EPU in the long-term variance [Equation (8)], then we use the same processing method to obtain the predicted $E_{i,t}[X_t]$, as well as the estimated long-term variance τ_{t+1} .

The representative indicators of the gold market’s implied volatility indices, GVZ, are calculated in a similar way to VIX.

In addition, our study differs from Wu et al.’s (2023a) initial utilization of the mixed-frequency model for VIX forecasting, in which the latter verifies the effectiveness of the GARCH-MIDAS model and time-varying risk aversion. We further improve the calculation of volatility indices under the GARCH-MIDAS model by considering the cross-month effects of long-term variances due to the special forward-looking characteristic of volatility indices and propose a reasonable estimation of $E_{i,t}[\tau_{t+1}]$, which is needed in the calculation. In addition, we consider two kinds of volatility index forecasting and investigate the predictability of three kinds of EPU indices.

2.3. Competing models

Duan (1995) proposed and proved that under specific preference and distribution assumptions, the local risk-neutral valuation relationship (LRNVR) can be established. The introduction of the LRNVR enables risk-neutral valuation under the GARCH framework, which is an extension of the traditional risk-neutral valuation method. Under physical measure P , the GARCH(1,1) model specifies the asset’s daily return and conditional variance as follows:

$$R_t = r - \frac{1}{2}h_t + \lambda\sqrt{h_t} + z_t \tag{13}$$

$$h_t = \omega + \beta h_{t-1} + \alpha z_{t-1}^2 \tag{14}$$

where r denotes the risk-free interest rate of return, and λ represents the equity risk premium, where $\lambda > 0$. z_t is innovation that satisfies $z_t = \sqrt{h_t}\varepsilon_t$, with $\varepsilon_t \sim N(0,1)$. The parameter

constraints are $\omega > 0$, α , β , and $\gamma > 0$, and persistence $\Gamma = \alpha + \beta < 1$. Unconditional variance is given by $\Omega / (1 - \Gamma)$, where $\Omega = \omega$.

After the LRNVR under the GARCH framework is introduced, the daily return and conditional variance are given as follows:

$$R_t = r - \frac{1}{2}h_t + z_t^* \quad (15)$$

$$h_t = \omega + \beta h_{t-1} + \alpha(z_{t-1}^* - \lambda\sqrt{h_{t-1}})^2 \quad (16)$$

where $z_t^* = z_t + \lambda\sqrt{h_t}$, $z_t^* = \sqrt{h_t}\varepsilon_t^*$, $\varepsilon_t^* \sim N(0,1)$, the steady-state conditional constraints satisfy $\Gamma^* = \beta + \alpha(1 + \lambda^2) < 1$, and unconditional variance $\Omega^* / (1 - \Gamma^*)$, $\Omega^* = \omega$.

Heston and Nandi (2000) developed a closed-form solution option pricing formula suitable for spot assets whose variance follows the GARCH process, which is referred to as the HN-GARCH model. Under physical measure P , the daily return and conditional variance are as follows:

$$R_t = r - \left(\lambda - \frac{1}{2}\right)h_t + z_t \quad (17)$$

$$h_t = \omega + \beta h_{t-1} + \alpha(\varepsilon_{t-1} - \gamma\sqrt{h_{t-1}})^2 \quad (18)$$

where the parameters satisfy $\omega > 0$, α , $\beta \geq 0$, $\gamma > -0.5$, $z_t = \sqrt{h_t}\varepsilon_t$, $\Omega = \omega + \alpha > 0$, and $\Gamma = \beta + \alpha\gamma^2 < 1$.

In the risk-neutral framework (Q), the return and variance dynamics satisfy the following:

$$R_t = r - \frac{1}{2}h_t + z_t^* \quad (19)$$

$$h_t = \omega + \beta h_{t-1} + \alpha(\varepsilon_{t-1}^* - \gamma^*\sqrt{h_{t-1}})^2 \quad (20)$$

Transformation is carried out by $\varepsilon_t^* = \varepsilon_t + \lambda\sqrt{h_t}$, $\varepsilon_t^* \sim N(0,1)$, where $\Omega^* = \omega + \alpha > 0$, $\gamma^* = \gamma + \lambda$, and $\Gamma^* = \beta + \alpha\gamma^{*2} < 1$.

Glosten et al. (1993) proposed the GJR-GARCH model, an enhanced GARCH specification incorporating dummy variables, to examine the relationship between conditional expected returns and conditional variance. To capture seasonal effects, nominal interest rates are incorporated into the forecast conditional variance, while the impacts of positive and negative innovations on volatility are differentiated. Under physical measure P , the return and conditional variance are modeled as follows:

$$R_t = r - \frac{1}{2}h_t + \lambda\sqrt{h_t} + z_t \quad (21)$$

$$h_t = \omega + \beta h_{t-1} + z_{t-1}^2(\alpha + \gamma I_{\{z_{t-1} < 0\}}) \quad (22)$$

where $z_t = \sqrt{h_t}\varepsilon_t$, $\varepsilon_t \sim N(0,1)$, and $I_{\{z_{t-1} < 0\}}$ is the indication function, used to distinguish between the effects of positive and negative innovation. Specifically, the value is 1 when $z_{t-1} < 0$; otherwise, it is 0. The parameters satisfy $\omega > 0$, α , β , $\gamma > 0$, $\Omega = \omega > 0$, and $\Gamma = \alpha + \beta + \frac{1}{2}\gamma < 1$.

Under the risk-neutral measure Q , the return and its associated conditional variance are expressed as follows:

$$R_t = r - \frac{1}{2}h_t + z_t^* \quad (23)$$

$$h_t = \omega + \beta h_{t-1} + (z_{t-1}^* - \lambda\sqrt{h_{t-1}})^2(\alpha + \gamma I_{\{z_{t-1}^* - \lambda\sqrt{h_{t-1}} < 0\}}) \quad (24)$$

where the parameters satisfy $\omega > 0$, α , β , $\gamma \geq 0$, and $\Omega^* = \omega$. $z_t^* = z_t + \lambda\sqrt{h_t}$, where $\varepsilon_t^* \sim N(0,1)$. Persistence satisfies $\Gamma = \beta + \alpha(1 + \lambda^2) + \gamma[\lambda n(\lambda) + (1 + \lambda^2)N(\lambda)] < 1$, where $N(\cdot)$ is the standard normal cumulative distribution, and $n(\cdot)$ represents its density distribution.

VIX squared is a linear function of conditional variance under simple GARCH-type models (Hao and Zhang, 2013; Liu et al., 2015; Hansen et al., 2024). Specifically, VIX squared is calculated as follows:

$$VIX_t^2 / (252 \times 100^2) = \frac{1}{n} \sum_{k=1}^n E_t^Q [h_{t+k}] = A + B h_{t+1} \quad (25)$$

where $A = \Omega^*(1 - B) / (1 - \Gamma^*)$ and $B = (1 - \Gamma^{*n}) / (n(1 - \Gamma^*))$.

2.4. Parameters estimation

Maximum likelihood estimation (MLE) is a widely used statistical technique in economics and finance and is known for its validity and consistency in large sample sizes. MLE aims to provide the most accurate parameter estimates by optimizing the likelihood function on the basis of the observed data. This property is particularly crucial for the GARCH-MIDAS model, which integrates both GARCH and MIDAS frameworks. Given the model's complexity, involving multiple volatility components, MLE offers a robust approach to parameter estimation. Through the optimization of the likelihood function, MLE effectively accounts for the influence of each volatility component, ensuring that the estimation process comprehensively captures the underlying model dynamics.

As an indicator of implied volatility, the VIX index not only contains information about the underlying stock index but also includes additional information related to the variance risk premium.

Christoffersen et al. (2015) advocated estimating model parameters using both index returns and option pricing. Since the VIX index is derived on the basis of option prices, parameter estimation via the VIX index should yield comparable outcomes while being more computationally efficient. This approach has been widely utilized in the literature (Hao and Zhang, 2013; Kannianen et al., 2014; Liu et al., 2015; Qiao et al., 2020) to streamline the modeling process. Therefore, the same method is used here. It is assumed that forecast errors for VIX follow a Gaussian distribution, i.e., that $VIX^{Mar} - VIX^{Mod} = \mu$, $\mu \sim N(0, s_v^2)$ (Hao and Zhang, 2013; Kannianen et al., 2014), with VIX^{Mar} denoting observed market VIX, VIX^{Mod} denoting model-implied VIX, and s_v^2 denoting constant error variance. The corresponding log-likelihood function is as follows:

$$\ln L^{RV, VIX} = -\frac{T}{2} \ln(2\pi s_v^2) - \frac{1}{2s_v^2} \sum_{t=1}^T (VIX^{Mar} - VIX^{Mod})^2 \quad (26)$$

The maximum likelihood function estimation method for GVZ is the same as that for VIX.

2.5. Measures of forecast accuracy

To assess the effectiveness of the model, we utilize the following commonly employed metrics: The mean absolute error is as follows:

$$MAE = \frac{1}{l} \sum_{i=1}^l |VIX_i^{Mar} - VIX_i^{Mod}| \quad (27)$$

The root mean squared error is as follows:

$$RMSE = \sqrt{\frac{1}{l} \sum_{i=1}^l (VIX_i^{Mar} - VIX_i^{Mod})^2} \quad (28)$$

The heteroscedasticity-adjusted mean squared error is as follows:

$$HMSE = \frac{1}{l} \sum_{i=1}^l (1 - VIX_i^{Mod} / VIX_i^{Mar})^2 \quad (29)$$

The mean absolute percentage error is as follows:

$$MAPE = \frac{1}{l} \sum_{i=1}^l |1 - VIX_i^{Mod} / VIX_i^{Mar}| \quad (30)$$

where l is the length of the sample. The forecasting errors of GVZ are calculated similarly.

3. Empirical analysis

3.1. Data description

The VIX and GVZ indices used in this study are retrieved from the CBOE website. Specifically, the VIX data cover the period from January 4, 2000, to February 23, 2024, while the GVZ series data span from September 18, 2009, to March 22, 2024. S&P 500 index prices are obtained from Yahoo Finance, and daily returns are calculated on the basis of closing prices. Finally, the 3-month Treasury yield, used as the risk-free rate, is downloaded from the Federal Reserve Economic Data (FRED) database.

Baker et al. (2016) developed the EPU index, which has been shown to be a strong predictor of stock market volatility (Liu and Zhang, 2015; Arouri et al., 2016; Li et al., 2023). This index measures economic policy uncertainty by tracking how often news articles include keywords related to policy and uncertainty. Specifically, this index identifies key terms such as “economic policy”, “uncertainty”, and “government” in news articles and then counts the number of reports containing these terms in major news databases. To reduce the impact of fluctuations in overall news volume, the counts are standardized. Additionally, to better capture long-term trends in policy uncertainty, the index uses smoothing techniques such as moving averages. The theoretical basis of the EPU index lies in Knightian uncertainty, which suggests that policy changes increase decision-making uncertainty and in turn amplify market volatility. In this context, the EPU index quantifies how policy uncertainty affects economic and financial outcomes.

Data for the EPU indices are sourced from the FRED website. Within the sample data, the Equity Market Volatility: Infectious Disease Tracker (EMV) quantifies infectious disease influences on equity market volatility and the influence of infectious disease occurrences on equity market uncertainty and volatility by analyzing news reports and stock market data related to infectious diseases (Baker et al., 2020). The Equity Market-Related Economic Uncertainty Index (EMr) is an index that quantifies economic uncertainty related to the stock market, measuring investors’ concerns about the economic outlook by analyzing discussions on “uncertain”-related words on social media and combining equity market volatility data (Altig et al., 2020). The U.S. Economic Policy Uncertainty Index (EPUI) quantifies policy uncertainty by analyzing the news frequency of relevant keywords and policy variables (Baker et al., 2016). In the empirical analysis, the monthly variables are calculated as the average of their daily observations over the corresponding month. The Augmented Dickey–Fuller (ADF) unit root test for the monthly variables indicates that all three variables are stationary over our sample period.

Table 1 gives the descriptive statistics of the S&P 500 index’s return, VIX, EPUI, EMV, EMr, and RV^m data. The results show that all variables have positive skewness except returns, which has negative skewness, indicating a right-skewed distribution. High-level kurtosis indicates that there may be outliers in the trend, whereas low values indicate that there are no outliers. The results show that the kurtosis of VIX is greater than 3, which indicates leptokurtic distributions that are longer than the normal distribution and have thicker tails, whereas the kurtosis of GVZ has a flat peak that is shorter and thicker than that of the normal distribution. The higher LB statistic shows that the data exhibit significant autocorrelations. We also test the interrelationships among EPUI, EMV, and EMr to check for redundancy and multicollinearity. Spearman rank correlations reveal weak to moderate positive associations among EPUI, EMV, and EMr, indicating that the indices are related but not redundant. In

line with this, all VIF values are well below the conventional threshold of 10 (specifically, less than 2), confirming the absence of multicollinearity concerns.

Table 1. Descriptive statistics.

	Returns	VIX	GVZ	EPUI	EMV	EMr	RV^m
N	5971	5971	3619	272	272	272	272
Min	-0.13	9.14	8.88	37.05	0.08	13.10	1.24 E-04
Max	0.10	82.69	48.98	534.21	44.93	483.66	0.067
Mean	1.76E-04	20.02	17.31	111.19	2.54	71.10	3.34E-03
Std	0.01	8.55	4.81	62.44	5.43	63.18	6.40-03
Skewness	-0.50	2.18	1.14	2.70	3.47	2.61	6.67
Kurtosis	9.59	7.89	2.68	15.12	19.28	13.09	57.63
Q(10)	105.01	5.06E+04	2.93E+04	2.00E+03	3.55E+03	1.46E+03	3.58E+04

Note: The table shows descriptive statistics for the following variables: the S&P500 returns, the VIX and GVZ, the Economic Policy Uncertainty Index for the United States (EPUI), the Equity Market Volatility: Infectious Disease Tracker (EMV), equity descriptive statistics of the Market-Related Economic Uncertainty Index (EMr), and realized measures of the S&P500. N represents the total number of samples, and the maximum, minimum, mean, standard deviation, kurtosis, skewness, and LB statistic are reported separately. Q(10) represents the Ljung–Box test statistic for continuous correlation with a maximum of 10 lags.

3.2. Parameter estimation

Table 2 reports the full parameter estimates for the VIX models, with several key findings worth highlighting. First, when different models are compared, lower AIC (Akaike information criterion) and BIC (Bayesian information criterion) values indicate better performance. For VIX prediction, the GARCH-MIDAS-EPUI model is clearly advantageous. Second, the estimated volatility persistence parameter is very close to one, providing strong evidence of persistent volatility effects. Additionally, parameters η_{RV} and η_X assess the explanatory power of realized measures and the EPU indices on the current long-term volatility component. The long-term volatility components η_{RV} and η_X in all GARCH-MIDAS models are positive, indicating that realized measures and policy uncertainty factors positively impact long-term volatility. Finally, in the GARCH-MIDAS model, the parameter α is noticeably larger than in the basic GARCH version, which suggests that short-term shocks play a more pronounced role in shaping current volatility than do long-term shocks. In contrast, in the GARCH-MIDAS framework, the parameter β is estimated at a lower level than in the standard GARCH model, indicating that the model more effectively captures long-term volatility dynamics through its incorporation of low-frequency macroeconomic indicators. These findings underscore that market volatility is shaped not only by the inertia of past fluctuations but also by macroeconomic conditions.

Table 2. Parameter estimation of VIX.

	GARCH	GJR-GARCH	HN-GARCH	GARCH-MIDAS	GARCH-MIDAS-EPUI	GARCH-MIDAS-EMV	GARCH-MIDAS-EMr
ω	1.7909E-06 (0.7097E-04)	1.8280E-06 (0.0056)	2.8235E-06 (0.0118)	0.0672	0.0667	0.0658	0.0690
α	0.0382 (10.5302)	0.0388 (21.5865)	2.82E-06 (0.0177)	0.1117 (0.2566)	0.1117 (0.3819)	0.1155 (0.4710)	0.1161 (1.6734)
β	0.9502 (4.8999)	0.9494 (95.3952)	0.9846 (157.6718)	0.8211 (1.0282)	0.8215 (0.1995)	0.8187 (3.0735)	0.8149 (194.7121)
γ	2.8805E-09 (218.6125)	4.7734E-11 (3.6287)	2.0282 (6.3564E+04)				
λ		6.6415E-10 (59.8506)	-0.3194 (4.5289E+04)	3.5957E-10 (2.6666)	1.9603e-04 (2.1496)	1.56444e-04 (47.2213)	1.37895e-04 (62.6570)
m				3.4900E-05 (0.0006)	-3.5303E-07 (2.9169e-05)	1.5775E-05 (0.0003)	7.8695E-06 (0.0230)
η_{RV}				0.0423 (0.3044)	0.0389 (0.0101)	0.0433 (0.3144)	0.0387 (107.1463)
η_X					4.7086E-07 (5.0079e-07)	2.5767E-05 (0.0022)	5.7304E-07 (0.0033)
$\omega_{RV,1}$				1.0000 (59.9170)	1.0026 (26.9154)	1.0011 (3.8429)	1.0033 (3110.7950)
$\omega_{RV,2}$				9.4523 (715.2521)	10.7781 (98.9318)	9.8215 (174.0433)	11.5079 (2.4852E+04)

Continued on next page

	GARCH	GJR-GARCH	HN-GARCH	GARCH-MIDAS	GARCH-MIDAS-EPUI	GARCH-MIDAS-EMV	GARCH-MIDAS-EMr
$\omega_{X,1}$					4.9384 (92.6669)	31.0750 (4.1277E+03)	1.1278 (1.7284E+04)
$\omega_{X,2}$					4.4049 (41.1752)	21.9429 (2.4868E+03)	1.4312 (1.5860E+04)
Persistence	0.9884	0.9882	0.9846	0.9328	0.9333	0.9342	0.9310
Log-likelihoods	1.8369E+04	1.8369E+04	1.7365E+04	1.8771E+04	1.9584E+04	1.9792E+04	1.9589E+04
AIC	-3.6730E+04	-3.6728E+04	-3.4720E+04	-3.9153E+04	-3.9563E+04	-3.9157E+04	-3.9432E+04
BIC	-3.6705E+04	-3.6696E+04	-3.4688E+04	-3.9102E+04	-3.9493E+04	-3.9087E+04	-3.9362E+04

Note: This table shows the parameter estimates of VIX that are obtained using the sample period from January 4, 2000, to December 30, 2016.

3.3. Forecasting error analysis

Table 3 summarizes the in-sample forecasting accuracy for the VIX and GVZ indices across seven models. In Panel A (VIX), the GARCH-MIDAS-EPUI variant records the lowest error under all four loss criteria. Notably, both the baseline GARCH-MIDAS and its extensions using EPU indices outperform the three traditional GARCH models (standard GARCH, GJR-GARCH, and HN-GARCH). Panel B (GVZ) shows the same among simple GARCH-type models. However, after incorporating EPU into the GARCH-MIDAS model, the model has the smallest forecasting error under the MAE and RMSE loss functions, whereas the GARCH-MIDAS-EPUI model performs best under the HMSE and MAPE loss functions. These results underscore the effectiveness of incorporating EPU and the MIDAS framework into volatility indices to enhance forecast accuracy.

Table 3. In-sample forecasting errors of VIX and GVZ.

	MAE	HMSE	RMSE	MAPE
Panel A: VIX (January 4, 2000, to December 30, 2016)				
GARCH	3.7074	0.0784	4.8312	0.2002
GJR-GARCH	3.7085	0.0785	4.8311	0.2003
HN-GARCH	4.2969	0.1026	6.1371	0.2303
GARCH-MIDAS	2.6323	0.0311	3.5337	0.1347
GARCH-MIDAS-EPUI	2.4761	0.0274	3.3620	0.1267
GARCH-MIDAS-EMr	2.5464	0.0279	3.4149	0.1298
GARCH-MIDAS-EMV	2.6322	0.0304	3.5294	0.1342
Panel B: GVZ (September 18, 2009, to December 30, 2022)				
GARCH	3.1033	0.0465	4.2294	0.1742
GJR-GARCH	3.1033	0.0465	4.2294	0.1742
HN-GARCH	3.2274	0.0730	4.1963	0.1972
GARCH-MIDAS	2.6940	0.0368	3.5389	0.1526
GARCH-MIDAS-EPUI	2.5246	0.0328	3.3860	0.1424
GARCH-MIDAS-EMr	2.5856	0.0356	3.4427	0.1472
GARCH-MIDAS-EMV	2.5135	0.0353	3.3290	0.1454

Note: This table presents the in-sample forecasting errors of the seven individual models for VIX and GVZ forecasting, with sample periods ranging from January 4, 2000, to December 30, 2016, and from September 18, 2009, to December 30, 2022, respectively. The error measures are the mean absolute error (MAE), the heteroscedasticity-adjusted mean squared error (HMSE), the root mean squared error (RMSE), and the mean absolute percentage error (MAPE). The numbers in bold denote the lowest errors among the seven individual models for VIX and GVZ.

Table 4 displays the out-of-sample forecast errors derived from the parameters calculated in-sample. In Panel A (VIX), the GARCH-MIDAS-EPUI model achieves the lowest RMSE, whereas the GARCH-MIDAS-EMr variant performs best on the MAE and MAPE criteria. In Panel B (GVZ), both the GARCH-MIDAS-EPUI and GARCH-MIDAS-EMV models deliver the strongest predictive

accuracy. Overall, these results demonstrate that embedding realized volatility measures and the EPU indices in the long-term component markedly enhances forecasting performance. Moreover, models that include EPU consistently outperform the baseline GARCH-MIDAS model, emphasizing the value added of incorporating low-frequency policy uncertainty in volatility index modeling. This situation highlights the critical importance of macroeconomic variables—especially policy-related uncertainty—for improving the precision of market volatility index predictions.

Table 4. Out-of-sample forecasting errors of VIX and GVZ.

	MAE	HMSE	RMSE	MAPE
Panel A: VIX (January 3, 2017, to February 23, 2024)				
GARCH	3.9736	0.0745	5.1345	0.2180
GJR-GARCH	3.9700	0.0744	5.1234	0.2181
HN-GARCH	4.4762	0.1055	5.9259	0.2619
GARCH-MIDAS	3.1329	0.0412	4.3752	0.1628
GARCH-MIDAS-EPUI	3.0404	0.0433	4.2360	0.1636
GARCH-MIDAS-EMr	3.0379	0.0431	4.3851	0.1569
GARCH-MIDAS-EMV	6.4879	0.2487	8.7366	0.3553
Panel B: GVZ (January 3, 2023, to March 22, 2024)				
GARCH	2.6968	0.0461	3.0672	0.1886
GJR-GARCH	2.6969	0.0461	3.0672	0.1886
HN-GARCH	5.2889	0.2125	5.8466	0.3992
GARCH-MIDAS	2.7431	0.0683	3.2471	0.2114
GARCH-MIDAS-EPUI	1.7247	0.0258	2.1278	0.1264
GARCH-MIDAS-EMr	3.5948	0.0735	4.1452	0.2418
GARCH-MIDAS-EMV	1.7886	0.0214	2.2241	0.1206

Note: This table presents the out-of-sample forecasting errors of the seven individual models for VIX and GVZ forecasting, which are calculated ranging from January 3, 2017, to February 23, 2024, and from January 3, 2023, to March 22, 2024, respectively, based on estimation parameters in-sample. The error measures are the mean absolute error (MAE), the heteroscedasticity-adjusted mean squared error (HMSE), the root mean squared error (RMSE), and the mean absolute percentage error (MAPE). The numbers in bold denote the lowest errors among the seven individual models for VIX and GVZ.

3.4. MCS test

The model confidence set (MCS) test is a key statistical method in VIX prediction research and is used to systematically assess and rank the predictive power of competing models. Developed by Hansen et al. (2011), this test addresses the challenge of model selection by building a confidence set that includes models with statistically similar performance. The MCS test works through a sequential elimination process; it starts by using the predictive accuracy test (PAT) to compare forecast errors between pairs of models. By closely examining differences in loss functions, such as the mean squared

errors or mean absolute deviations, it identifies models with weaker performance. These underperforming models are removed step by step until only a subset remains, all of which show comparable predictive ability at a given confidence level.

This process builds on earlier theoretical work (Giacomini and White, 2006; White, 2000; Sullivan et al., 1999), which introduced key ideas for evaluating forecast accuracy and significance. The MCS framework has two main benefits—it reduces overfitting risk by weeding out poorly performing models in out-of-sample tests and provides a clear way in which to determine true predictive skill apart from random variation. This framework aligns with the work of Hansen and Lunde (2005), who stressed the need for robust model evaluation in financial forecasting. In VIX prediction, where accurate volatility estimates are vital for risk management and derivatives pricing, the MCS test is an essential tool for researchers and practitioners aiming to identify the most reliable predictive models. The range test statistic is defined as follows:

$$T_{R,M} = \frac{\max_{i,j \in M} \overline{d_{ij}}}{\sqrt{\widehat{\text{var}}(\overline{d_{ij}})}} \quad (31)$$

where the average loss error is $\overline{d_{ij}} = \frac{1}{m} \sum_{t=1}^n d_{ij,t}$, and $d_{ij,t}$ represents the loss difference for each pair of models i and j at a certain point in time.

Table 5 displays the MCS evaluation outcomes for the VIX and GVZ forecasts across four error metrics (MAE, MAPE, HMSE, and RMSE). Statistically superior models are identified through boldfaced p-values in each comparison set. In Panel A, the GARCH-MIDAS-EPUI model is consistently selected under all four loss functions, with p-values close to or equal to 1. The GARCH-MIDAS-EMr model is the next best model. These findings are aligned with the previous forecast error results, indicating that the incorporation of the macroeconomic factors EPUI and EMr enhances the ability to capture the implied volatility of the stock market. Notably, the GARCH-MIDAS model in Panel A is also selected under three loss functions, suggesting that the mixed-frequency models demonstrate optimal performance in predicting VIX. Consistent with our theoretical expectations, the results in Panel B confirm that the GARCH-MIDAS extensions incorporating EPUI and EMV factors exhibit notable forecasting advantages over competing specifications.

Table 5. MCS testing of seven out-of-sample forecasting models under two volatility indices.

	MAE	HMSE	RMSE	MAPE
Panel A: VIX (January 3, 2017, to February 23, 2024)				
GARCH				
GJR-GARCH				
HN-GARCH				
GARCH-MIDAS	<u>1.0000</u>	0.0918	<u>1.0000</u>	
GARCH-MIDAS-EPUI	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>
GARCH-MIDAS-EMr	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>	0.8305
GARCH-MIDAS-EMV				
Panel B: GVZ (January 3, 2023, to March 22, 2024)				
GARCH				
GJR-GARCH				
HN-GARCH				
GARCH-MIDAS				
GARCH-MIDAS-EPUI	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>
GARCH-MIDAS-EMr				
GARCH-MIDAS-EMV	0.5553		0.5625	0.1003

Note: Panels A and B present the p-values from the MCS test for out-of-sample forecasting by VIX and GVZ across seven models, with only results showing p-values > 0.01 displayed. Four error metrics are used: MAE, MAPE, HMSE, and RMSE.

3.5. High- and low-uncertainty regimes

To analyze the regime-dependent performance of the models, we classify the periods into low- and high-uncertainty regimes and systematically evaluate the predictive differences among the seven models, with a focus on the enhancement effects of three exogenous variables. Tables 6 and 7 present the forecasting errors of each model for the VIX and GVZ indices under two distinct regimes, respectively, revealing significant regime-dependent characteristics in model performance.

As presented in Table 6, during periods of low uncertainty, the extended GARCH-MIDAS models clearly exhibit advantages. Specifically, the EPUI-augmented model achieves the fewest forecasting errors across all three loss metrics during the 2000–2006 period, whereas the EMr-augmented model performs best during the 2017–2019 period. In contrast, the high-uncertainty regime reveals more complex dynamics. During the global financial crisis (2007–2012), the EPUI-augmented model demonstrates particularly strong performance. Notably, in the pandemic and postpandemic inflationary phase (2020–2023), the EMV-augmented model fails entirely, whereas the EPUI-based model remains relatively robust.

Table 6. VIX prediction of GARCH-MIDAS models with EPU indices across uncertainty periods.

VIX	MAE	RMSE	HMSE	MAPE
2000–2006 (low-uncertainty regime)				
GARCH	4.4963	5.3967	0.1200	0.2631
GJR-GARCH	4.4969	5.3978	0.1202	0.2632
HN-GARCH	4.4654	5.6743	0.1477	0.2748
GARCH-MIDAS	2.9401	3.7178	0.0428	0.1619
GARCH-MIDAS-EPUI	2.6602	3.4922	0.0351	0.1438
GARCH-MIDAS-EMr	2.7224	3.4130	0.0367	0.1517
GARCH-MIDAS-EMV	2.8581	3.6530	0.0401	0.1565
2007–2012 (high-uncertainty regime)				
GARCH	4.0674	5.2874	0.0688	0.1859
GJR-GARCH	4.0676	5.2871	0.0688	0.1858
HN-GARCH	4.9391	7.7951	0.0799	0.1929
GARCH-MIDAS	2.9440	3.9540	0.0277	0.1271
GARCH-MIDAS-EPUI	2.7447	3.7287	0.0242	0.1182
GARCH-MIDAS-EMr	2.9171	3.9251	0.0252	0.1243
GARCH-MIDAS-EMV	2.9559	3.9567	0.0271	0.1271
2017–2019 (low-uncertainty regime)				
GARCH	2.3400	2.7854	0.0419	0.1746
GJR-GARCH	2.3485	2.7884	0.0422	0.1755
HN-GARCH	4.0338	4.4331	0.1224	0.3117
GARCH_MIDAS	1.8832	2.3652	0.0257	0.1325
GARCH-MIDAS-EPUI	1.9099	2.3503	0.0271	0.1371
GARCH-MIDAS-EMr	1.6506	2.1877	0.0191	0.1117
GARCH-MIDAS-EMV	1.7322	2.2671	0.0214	0.1187
2020–2023 (high-uncertainty regime)				
GARCH	5.0473	6.2331	0.0880	0.2364
GJR-GARCH	5.0344	6.2158	0.0877	0.2358
HN-GARCH	4.6498	6.7216	0.0809	0.2100
GARCH-MIDAS	3.9936	5.3815	0.0498	0.1785
GARCH-MIDAS-EPUI	3.7727	5.1491	0.0503	0.1738
GARCH-MIDAS-EMr	3.9429	5.3998	0.0549	0.1794
GARCH-MIDAS-EMV	9.7295	11.1850	0.3907	0.5064

Note: Regimes are classified based on historical market uncertainty levels. The low-uncertainty regime (2000–2006, 2017–2019) and high-uncertainty regime (2007–2012, 2020–2023) are identified by macroeconomic events (e.g., financial crises, pandemic) and volatility clustering.

Table 7. GVZ prediction of GARCH-MIDAS models with EPU indices across uncertainty periods.

GVZ	MAE	RMSE	HMSE	MAPE
2013–2019 (low-uncertainty regime)				
GARCH	2.7942	3.5294	0.0518	0.1844
GJR-GARCH	2.7942	3.5294	0.0518	0.1844
HN-GARCH	3.5142	4.4588	0.1121	0.2499
GARCH-MIDAS	2.2767	2.9276	0.0348	0.1479
GARCH-MIDAS-EPUI	2.2201	2.9184	0.0322	0.1422
GARCH-MIDAS-EMr	2.2953	3.0151	0.0368	0.1494
GARCH-MIDAS-EMV	2.3311	3.0085	0.0399	0.1547
2020–2022 (high-uncertainty regime)				
GARCH	3.2404	4.4641	0.0421	0.1708
GJR-GARCH	3.2404	4.4641	0.0421	0.1708
HN-GARCH	2.5284	3.2683	0.0296	0.1350
GARCH-MIDAS	3.0748	3.8280	0.0433	0.1670
GARCH-MIDAS-EPUI	2.6039	3.4473	0.0347	0.1406
GARCH-MIDAS-EMr	2.5469	3.3761	0.0327	0.1361
GARCH-MIDAS-EMV	2.1262	2.9098	0.0223	0.1123

Note: Regimes are classified based on historical market uncertainty levels. The low-uncertainty (2013–2019) and high-uncertainty regimes (2020–2022) are identified by macroeconomic events (e.g., financial crises, pandemic) and volatility clustering.

In Table 7, during the “high-uncertainty” period from 2020 to 2022, which includes the COVID-19 shock, the GARCH-MIDAS-EMV model demonstrates superior forecasting performance, with all four losses outperforming those of alternative models. This finding stands in sharp contrast to the findings in the equity market and reflects the unique role of gold as a safe-haven asset—its volatility is more closely linked to spot market fluctuations (captured by EMV) than to policy uncertainty (measured by EPUI). Notably, the HN-GARCH model also delivers unexpectedly strong results during this period, with an RMSE of 3.2683, suggesting the presence of a distinct leverage effect structure in gold volatility dynamics. In contrast, during the low-uncertainty period of 2013–2019, models augmented with EPUI regain their dominance. This cyclical variation highlights the dual nature of the gold market—as a safe-haven asset, compared with equity volatility indices, GVZ is less sensitive to policy shifts but responds more acutely to real-market disturbances.

4. Economic significance evaluation

To evaluate the economic significance of the predictions generated by each estimation model, we implement two volatility-based trading strategies (Vrontos et al., 2021; Qiao et al., 2022). These strategies are chosen because they are simple, easy to implement, and logically clear.

Strategy 1: Long positions are initiated when the $t+1$ predicted volatility index exceeds the realized t -period volatility index; otherwise, we stay out of the market.

Strategy 2: A long position is taken in the VIX when the predicted VIX for time $t + 1$ exceeds the

realized VIX at time t ; otherwise, a short position is taken.

These trading strategies evaluate model efficacy by translating volatility forecasts into tradable returns, thereby quantifying the economic value of directional predictability. Assuming zero trading costs, we calculate the annualized average daily return and Sharpe ratio (with a risk-free rate of zero) of each model to capture their risk-adjusted profitability (Vrontos et al., 2021).

According to the data in Table 8, the results of Strategy 1 reveal the following distinct performance hierarchies: for VIX (Panel A), the GARCH-MIDAS-EMr specification is dominant in terms of both daily returns and Sharpe ratios, with GARCH-MIDAS ranking second. Panel B reveals that the GARCH-MIDAS-EMr specification generates statistically superior risk-adjusted performance in GVZ volatility trading under Strategy 1, with the EMV variant ranking second but maintaining competitive economic significance. The results of Strategy 2 demonstrate the EPUI-augmented model's consistent dominance across both indices, achieving peak performance metrics (VIX: 3.9217 daily return/0.1993 Sharpe ratio; GVZ: 2.4570 daily return/0.1858 Sharpe ratio) that outperform competing specifications. Overall, GARCH-MIDAS models can provide valuable supplements under different economic meanings and investment strategies, achieving greater daily returns and Sharpe ratios in key cases such as the GARCH-MIDAS-EPUI model under Strategy 2 and the GARCH-MIDAS-EMr model under Strategy 1.

Table 8. Evaluation of the economic significance of the two indices under the seven models.

	Strategy 1		Strategy 2	
	Daily return	Sharpe ratio	Daily return	Sharpe ratio
Panel A: VIX (January 3, 2017, to February 23, 2024)				
GARCH	2.0191	0.1070	2.4078	0.1207
GJR-GARCH	2.0039	0.1063	2.3941	0.1201
HN-GARCH	1.5963	0.0870	2.0804	0.1042
GARCH-MIDAS	2.7378	0.1404	3.4309	0.1735
GARCH-MIDAS-EPUI	2.7095	0.1403	3.9217	0.1993
GARCH-MIDAS-EMr	3.0462	0.1566	3.7193	0.1886
GARCH-MIDAS-EMV	1.6964	0.0866	2.6889	0.1352
Panel B: GVZ (January 3, 2023, to March 22, 2024)				
GARCH	0.4248	0.0347	0.6537	0.0486
GJR-GARCH	0.4248	0.0347	0.6537	0.04865
HN-GARCH	0.1862	0.0140	0.4706	0.0350
GARCH-MIDAS	1.2465	0.0978	2.2858	0.1724
GARCH-MIDAS-EPUI	1.9005	0.1439	2.4570	0.1858
GARCH-MIDAS-EMr	5.0323	0.2691	1.1063	0.0825
GARCH-MIDAS-EMV	2.6479	0.2035	1.9126	0.1436

Note: This table reports Sharpe ratios and annualized average daily return for VIX and GVZ based on out-of-sample forecasts for seven models. Strategy 1 takes a long position only when the predicted volatility index at time $t + 1$ is higher than the actual volatility index at time t and does nothing else. Strategy 2 selects long or short positions based on whether the volatility index at time $t + 1$ is higher than the actual volatility index at time t . Bold numbers indicate the performance of these models with the highest Sharpe ratio and daily return. Since the predicted value at each $t + 1$ time point is less than time t , the failure to output daily return and Sharpe ratio is represented by "/".

Additionally, we calculate the certainty equivalent return (CER) gain for mean-variance investors. Drawing on previous research on volatility forecasting (Guidolin and Timmermann, 2006; Zhang et al., 2019; Mei et al., 2020), mean-variance investors are assumed to optimally allocate their resources between risky assets and risk-free assets. This framework is based on classical Markowitz portfolio theory (Markowitz, 1952; Markowitz, 1971), where investors seek to maximize expected returns for a given level of risk or, conversely, minimize risk for a given expected return, thus maximizing utility. In this context, the accuracy of volatility forecasts is crucial for determining optimal portfolio weights and enhancing the effectiveness of asset allocation.

The expected utility of investing in this portfolio is given as follows:

$$U_t(\hat{r}_t) = E_t(w_t^* \hat{r}_t + r_{t,f}) - \frac{1}{2} \gamma \text{Var}_t(w_t^* \hat{r}_t + r_{t,f}), \quad (32)$$

where w_t^* is the optimal weight in this portfolio, \hat{r}_t is the excess return, $r_{t,f}$ is the risk-free rate, and γ is a risk aversion coefficient.

Portfolio returns are $w_t^* \hat{r}_t + r_{t,f}$, and the ex ante optimal weight of the risk asset on day $t+1$ is calculated as follows:

$$w_t^* = \frac{1}{\gamma} \left(\frac{\hat{r}_{t+1}}{\hat{\sigma}_{t+1}^2} \right), \quad (33)$$

where \hat{r}_{t+1} represents the excess return of the risk asset on day $t+1$, which can be evaluated by the prevailing AR(1) model. $\hat{\sigma}_{t+1}^2$ represents the forecasted volatility on day $t+1$ based on one of the volatility index models. The mean-variance investor who allocates assets using Equation (38) realizes a CER of

$$\text{CER} = \bar{R}_p - 0.5\gamma\sigma_p^2, \quad (34)$$

where \bar{R}_p and σ_p^2 are the sample mean and variance of the portfolio return over the out-of-sample evaluation period, respectively. The CER quantifies the economic superiority of the target forecasting model relative to a specified benchmark. When annualized, this differential represents the maximum hypothetical management fee that a rational utility-maximizing investor would sacrifice to access the target model's forecasts while maintaining indifference between the two strategies.

Table 9. CER gains of VIX and GVZ.

	CER		
	Gamma = 1	Gamma = 3	Gamma = 5
Panel A: VIX (January 3, 2017, to February 23, 2024)			
GARCH	1.3479	1.2893	1.2776
HN-GARCH	1.3478	1.2893	1.2776
GJR-GARCH	1.3409	1.2870	1.2762
GARCH-MIDAS	1.3493	1.2898	1.2779
GARCH-MIDAS-EPUI	1.3453	1.2884	1.2771
GARCH-MIDAS-EMr	1.3496	1.2899	1.2779
GARCH-MIDAS-EMV	1.3373	1.2858	1.2755
Panel B: GVZ (January 3, 2023, to March 22, 2024)			
GARCH	1.3667	1.2956	1.2813
HN-GARCH	1.3667	1.2956	1.2813
GJR-GARCH	1.3394	1.2865	1.2759
GARCH-MIDAS	1.3541	1.2914	1.2788
GARCH-MIDAS-EPUI	1.3663	1.2954	1.2813
GARCH-MIDAS-EMr	1.4093	1.3098	1.2899
GARCH-MIDAS-EMV	1.3776	1.2992	1.2835

Note: This table reports certainty equivalent returns (CER) with risk-aversion coefficients (gamma) of 1, 3, and 5. Bold values mean higher economic significance for the corresponding model.

Table 9 focuses on the VIX and GVZ indices and compares the CER gains of different GARCH-type models for mean-variance investors. From the perspective of the risk aversion coefficient gamma (1, 3, 5), in the VIX scenario, MIDAS-extended models such as GARCH-MIDAS-EMr model perform better. The effects of classic models (such as GARCH and HN-GARCH models) are similar, and as gamma increases (investors become more risk averse), the CER of each model generally decreases. In the GVZ scenario, the GARCH-MIDAS-EMr model has a significant advantage, reflecting its adaptability to the volatility characteristics of the gold volatility index. Overall, the results show that MIDAS-extended models have more potential in characterizing the returns of volatility indices than do other models.

5. Robustness

To rigorously evaluate the predictive power of the GARCH-MIDAS framework, we conduct segmented analysis by repartitioning the sample period. Tables 10–11 systematically present both in-sample and out-of-sample forecast errors across the VIX and GVZ indices, enabling a comprehensive model assessment. As shown in Table 10, in Panel A, the EPUI- and EMr-augmented GARCH-MIDAS variants perform best within the sample with the smallest error. In Panel B, the EPUI-enhanced model consistently outperforms the others across all four loss functions. According to Panel A in Table 11, the GARCH-MIDAS-EMV model has the smallest error in terms of MAE, HMSE and MAPE. While the GARCH-MIDAS-EPUI model is dominant in Panel B, Table 11 reveals that compared with alternative models, the GARCH-MIDAS model produces a notably higher RMSE when forecasting the VIX index. This discrepancy may be due to the influence of extreme data, and the results after extreme values are removed are presented in the following tables.

Table 10. In-sample forecasting errors of VIX and GVZ (robustness).

Models	MAE	HMSE	RMSE	MAPE
Panel A: VIX (January 4, 2000, to December 31, 2014)				
GARCH	3.9241	0.0856	5.0396	0.2104
GJR-GARCH	3.9235	0.0856	5.0396	0.2104
HN-GARCH	4.4576	0.1122	6.3917	0.2361
GARCH-MIDAS	2.7263	0.0322	3.6364	0.1373
GARCH-MIDAS-EPUI	2.5809	0.0287	3.4597	0.1307
GARCH-MIDAS-EMr	2.6141	0.0257	3.5541	0.1261
GARCH-MIDAS-EMV	2.7173	0.0317	3.6341	0.1363
Panel B: GVZ (September 18, 2009, to December 30, 2016)				
GARCH	2.6889	0.0297	3.3862	0.1412
GJR-GARCH	2.6758	0.0280	3.4113	0.1378
HN-GARCH	3.0934	0.0379	3.8902	0.1616
GARCH-MIDAS	2.6736	0.0292	3.3373	0.1408
GARCH-MIDAS-EPUI	2.3155	0.0232	2.9817	0.1218
GARCH-MIDAS-EMr	2.5391	0.0285	3.1788	0.1357
GARCH-MIDAS-EMV	2.5556	0.0272	3.2329	0.1347

Note: This table presents the in-sample forecasting errors of seven individual models for VIX (January 4, 2000, to December 31, 2014) and GVZ (September 18, 2009, to December 30, 2016). Bold values indicate the lowest errors among the seven models for VIX forecasts.

Table 11. Out-of-sample forecasting errors of VIX and GVZ (robustness).

	MAE	HMSE	RMSE	MAPE
Panel A: VIX (January 5, 2015, to February 23, 2024)				
GARCH	3.4975	0.0609	4.7117	0.1919
GJR-GARCH	3.4989	0.0609	4.7125	0.192
HN-GARCH	4.1949	0.0949	5.5239	0.25
GARCH-MIDAS	3.1147	0.0439	6.5026	0.1515
GARCH-MIDAS-EPUI	3.0776	0.0455	6.2359	0.1552
GARCH-MIDAS-EMr	3.2973	0.0488	6.2570	0.1673
GARCH-MIDAS-EMV	3.0560	0.0419	6.4079	0.1473
Panel B: GVZ (January 3, 2017, to March 22, 2024)				
GARCH	6.5449	0.3186	7.7259	0.4710
GJR-GARCH	6.5991	0.3340	7.7482	0.4786
HN-GARCH	7.4026	0.4749	8.5005	0.5653
GARCH-MIDAS	6.4775	0.3038	7.8259	0.4624
GARCH-MIDAS-EPUI	5.6595	0.2414	7.3618	0.3816
GARCH-MIDAS-EMr	5.7583	0.2450	7.5054	0.3817
GARCH-MIDAS-EMV	6.4936	0.3060	7.4478	0.4674

Note: This table presents the out-of-sample forecasting errors of seven individual models for VIX (January 5, 2015, to February 23, 2024) and GVZ (January 3, 2017, to March 22, 2024), based on in-sample parameter estimation. Bold values indicate the lowest errors among the seven models for VIX forecasts.

Tables 12, 13, and 14 present the MCS test results and economic significance evaluations for the VIX and GVZ indices under an alternative sample partition. The analysis reveals several key findings. In Table 12 (Panel A), the GARCH-MIDAS-EMV specification demonstrates superior predictive accuracy, outperforming all competing models. Panel B shows that the GARCH-MIDAS class of models achieves particularly strong results, with the EPUI variant emerging as the top performer, followed closely by the EMr specification. The economic evaluation in Table 13 yields additional insights. Panel A indicates that the EMV model generates the greatest returns under Strategy 1, as measured by both daily returns and Sharpe ratios. For Strategy 2, the EPUI variant delivers optimal economic performance. These findings in Panel B align consistently with the out-of-sample forecasting results presented earlier. Table 14 reports the CER gains for the VIX and GVZ indices across GARCH-type models with varying levels of risk aversion ($\gamma = 1, 3, 5$). For VIX, the GARCH-MIDAS-EMV model outperforms in most cases; for GVZ, the GARCH-MIDAS-EMr model leads. MIDAS-extended models show greater potential in capturing volatility index returns, with performance differing by index, highlighting the value of multifactor integration for volatility modeling.

Table 12. MCS testing of seven out-of-sample forecasting models under VIX and GVZ (robustness).

	MAE	HMSE	RMSE	MAPE
Panel A: VIX (January 5, 2015, to February 22, 2024)				
GARCH	0.7657	0.1829	0.7506	
GJR-GARCH	0.0549		0.0626	
HN-GARCH	0.0118		0.0116	
GARCH-MIDAS	0.1705		0.1823	0.0555
GARCH-MIDAS-EPUI	0.9997	0.2237	0.9995	0.3473
GARCH-MIDAS-EMr				
GARCH-MIDAS-EMV	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>
Panel B: GVZ (January 3, 2017, to March 22, 2024)				
GARCH	0.3869	0.2687	0.3706	0.0402
GJR-GARCH	0.3591	0.0490	0.3342	0.0233
HN-GARCH				
GARCH-MIDAS	0.6197	0.6741	0.6086	0.1753
GARCH-MIDAS-EPUI	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>
GARCH-MIDAS-EMr	0.9999	<u>1.0000</u>	0.9999	<u>1.0000</u>
GARCH-MIDAS-EMV	0.6995	0.7936	0.6952	0.2289

Note: Panels A and B present the p-values from the MCS test for out-of-sample forecasting by VIX and GVZ across seven models, with only results showing p-values > 0.01 being displayed. Four error metrics are used: MAE, MAPE, HMSE, and RMSE.

Table 13. Evaluation of the economic significance of VIX and GVZ under the seven models (robustness).

	Strategy 1		Strategy 2	
	Daily return	Sharpe ratio	Daily return	Sharpe ratio
Panel A: VIX (January 5, 2015, to February 22, 2024)				
GARCH	2.0385	0.1123	2.5237	0.1264
GJR-GARCH	2.0815	0.1143	2.5800	0.1293
HN-GARCH	1.4119	0.0776	2.0161	0.1007
GARCH-MIDAS	2.7905	0.1484	3.3424	0.1684
GARCH-MIDAS-EPUI	2.8130	0.1481	3.7492	0.1896
GARCH-MIDAS-EMr	2.4715	0.1310	3.2694	0.1646
GARCH-MIDAS-EMV	3.1832	0.1646	3.6456	0.1842
Panel B: GVZ (January 3, 2017, to March 22, 2024)				
GARCH	0.1468	0.0114	0.2919	0.0226
GJR-GARCH	0.1674	0.0198	0.3256	0.0252
HN-GARCH	0.2880	0.0230	0.5256	0.0407
GARCH-MIDAS	0.1778	0.0138	0.3476	0.0269
GARCH-MIDAS-EPUI	0.4375	0.0344	0.7841	0.0608
GARCH-MIDAS-EMr	0.2789	0.0219	0.4326	0.3350
GARCH-MIDAS-EMV	0.1099	0.0086	0.2285	0.0177

Note: This table reports Sharpe ratios and annualized average daily return for VIX and GVZ based on out-of-sample forecasts for seven models. Bold numbers indicate the performance of these models with the highest Sharpe ratio and daily return.

Table 14. CER gains of VIX and GVZ (robustness).

	CER		
	Gamma = 1	Gamma = 3	Gamma = 5
Panel A: VIX (January 5, 2015, to February 22, 2024)			
GARCH	1.3510	1.2903	1.2782
HN-GARCH	1.3510	1.2903	1.2782
GJR-GARCH	1.3424	1.2875	1.2765
GARCH-MIDAS	1.3532	1.2911	1.2786
GARCH-MIDAS-EPUI	1.3503	1.2901	1.2781
GARCH-MIDAS-EMr	1.3505	1.2902	1.2781
GARCH-MIDAS-EMV	1.3543	1.2914	1.2789
Panel B: GVZ (January 3, 2017, to March 22, 2024)			
GARCH	1.3365	1.2855	1.2753
HN-GARCH	1.3364	1.2855	1.2753
GJR-GARCH	1.3329	1.2843	1.2746
GARCH-MIDAS	1.3371	1.2857	1.2754
GARCH-MIDAS-EPUI	1.3456	1.2885	1.2771
GARCH-MIDAS-EMr	1.3523	1.2908	1.2785
GARCH-MIDAS-EMV	1.3369	1.2856	1.2754

Note: This table reports certainty equivalent returns (CER) with risk-aversion coefficients (gamma) of 1, 3, and 5. Bold values mean higher economic significance for the corresponding model.

As previously noted, compared with competing models, the GARCH-MIDAS model has markedly higher RMSE. To reduce extreme-value bias, we exclude VIX observations above 65 and reassess the forecasting accuracy of the model. After extreme values are removed, VIX forecasting performance errors under the second sample division are summarized in Table 15. The in-sample and out-of-sample forecasting errors reveal that both GARCH-MIDAS-EPUI and GARCH-MIDAS-EMr specifications demonstrate superior in-sample performance, whereas the GARCH-MIDAS-EMr specification achieves optimal out-of-sample accuracy (Table 15). Notably, the exclusion of extreme values yields a measurable reduction in RMSE for the in-sample period, while almost all the out-of-sample forecasting errors decrease.

Table 15. In-sample and out-of-sample forecasting errors of VIX after removing extreme values.

	MAE	HMSE	RMSE	MAPE
In-sample				
GARCH	3.9192	0.0864	5.0228	0.2110
GJR-GARCH	3.9203	0.0865	5.0227	0.2111
HN-GARCH	4.3042	0.1060	5.9848	0.2307
GARCH-MIDAS	2.6505	0.0295	3.5439	0.1328
GARCH-MIDAS-EPUI	2.5943	0.0255	3.5070	0.1269
GARCH-MIDAS-EMr	2.5497	0.0266	3.4226	0.1275
GARCH-MIDAS-EMV	2.6437	0.0288	3.5329	0.1321
Out-of-sample				
GARCH	3.4346	0.0610	4.5651	0.1910
GJR-GARCH	3.4340	0.0610	4.5544	0.1914
HN-GARCH	4.0538	0.0893	5.1408	0.2435
GARCH-MIDAS	2.9074	0.0385	4.4867	0.1513
GARCH-MIDAS-EPUI	2.9010	0.0392	4.4658	0.1528
GARCH-MIDAS-EMr	2.7911	0.0367	4.3765	0.1440
GARCH-MIDAS-EMV	3.4720	0.0655	5.2255	0.1848

Note: This table presents the in-sample and out-of-sample forecasting errors of the seven individual models for VIX forecasting, the in-sample periods ranged from January 4, 2000, to December 31, 2014, where the VIX extremes were removed. The out-of-sample is from January 5, 2015, to February 23, 2024. Numbers in bold denote the lowest errors among the seven individual models for VIX.

Further validation is provided by the results of the MCS test in Table 16, which confirms the dominant predictive performance of the GARCH-MIDAS-EMr model. The economic evaluation in Table 17 shows that this specification generates the greatest returns across both investment strategies. These results collectively demonstrate that the GARCH-MIDAS framework's decomposition of volatility into distinct temporal components, coupled with macroeconomic factor integration, enhances out-of-sample forecast precision. This evidence strongly supports the effectiveness of the model in

terms of implied volatility prediction. Table 18 shows the CER gains for the VIX index under different GARCH-type models, with risk-aversion coefficients $\gamma = 1, 3, 5$, and compares models such as GARCH, HN-GARCH, GJR-GARCH, and MIDAS-extended models (e.g., EMr-augmented GARCH-MIDAS variants), among which EMr-augmented models exhibit better performance.

Table 16. MCS test for VIX after removing the extreme values.

	MAE	HMSE	RMSE	MAPE
GARCH				
GJR-GARCH				
HN-GARCH				
GARCH-MIDAS				
GARCH-MIDAS-EPUI				
GARCH-MIDAS-EMr	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>	<u>1.0000</u>
GARCH-MIDAS-EMV				

Note: This table presents the p-values from the MCS test for out-of-sample forecasting by VIX after removing extremes across seven models, with only results showing p-values > 0.01 being displayed. Four error metrics are used: MAE, MAPE, HMSE, and RMSE.

Table 17. Evaluation of the economic significance of VIX forecasting after removing extreme values.

	Strategy 1		Strategy 2	
	Daily return	Sharpe ratio	Daily return	Sharpe ratio
GARCH	1.9881	0.1109	2.4325	0.1231
GJR-GARCH	1.9709	0.1099	2.4187	0.1224
HN-GARCH	1.5294	0.0842	2.1509	0.1087
GARCH-MIDAS	2.8992	0.1567	3.4327	0.1750
GARCH-MIDAS-EPUI	2.6046	0.1414	2.8349	0.1438
GARCH-MIDAS-EMr	3.4605	0.1844	3.8236	0.1957
GARCH-MIDAS-EMV	2.8231	0.1505	3.2878	0.1674

Note: This table reports the Sharpe ratio and annualized average daily return of the VIX based on out-of-sample projections for seven models after the extremes are removed. Numbers in bold indicate the performance of these models, with Sharpe ratios and daily returns being the highest.

Table 18. CER gains of VIX.

	CER		
	Gamma = 1	Gamma = 3	Gamma = 5
GARCH	1.351549	1.290516	1.278310
HN-GARCH	1.351396	1.290465	1.278279
GJR-GARCH	1.343313	1.287771	1.276663
GARCH-MIDAS	1.353838	1.291279	1.278768
GARCH-MIDAS-EPUI	1.354526	1.291509	1.278905
GARCH-MIDAS-EMr	1.355892	1.291964	1.279178
GARCH-MIDAS-EMV	1.352647	1.290882	1.278529

Note: This table reports certainty equivalent returns (CER) with risk-aversion coefficients (gamma) of 1, 3, and 5. Bold values mean higher economic significance for the corresponding model.

6. Conclusions

This paper aims to explore the forecasting of volatility indices within the GARCH-MIDAS framework, with a particular focus on assessing whether the inclusion of EPU factors enhances the predictive accuracy of volatility indices, including the VIX and GVZ indices. A central challenge is how to address the cross-month effects that arise from the “forward-looking” nature of volatility indices. To mitigate this issue, we propose a novel approach that decomposes the volatility index calculation into two distinct components, allowing for a more precise estimation of the conditional expectation of long-term variance for the upcoming month. Additionally, we integrate the EPU within the GARCH-MIDAS framework, demonstrating how the inclusion of macroeconomic uncertainty can improve forecast accuracy and offer a more nuanced perspective on volatility index predictions.

Our findings show that the GARCH-MIDAS model, which takes both long- and short-term volatility into account, produces more accurate forecasts than do traditional models. Adding the EPU index notably improves forecast accuracy, especially for VIX and GVZ, underscoring how important macroeconomic uncertainty is for volatility forecasting. Moreover, our results indicate that the impact of EPU differs across various volatility indices. These conclusions highlight the value of including low-frequency macroeconomic variables in volatility forecasting, offering insights that are important for traders, risk managers, and policymakers alike. Our study provides actionable insights for both policymakers and investors. In equity markets, GARCH-MIDAS-EPUI forecasts highlight the role of policy uncertainty, guiding central banks to strengthen communication when predicted and realized VIX diverge. In gold markets, the GARCH-MIDAS-EMV model is dominant under high uncertainty, underscoring the need for market-functioning tools beyond communication. For investors, simple forecast-based trading rules increase the Sharpe ratios and CER gains—the GARCH-MIDAS-EPUI model dominates for VIX, whereas the GARCH-MIDAS-EMV model improves GVZ timing during 2020–2022. Overall, the results advocate uncertainty-targeted communication and regime-aware hedging strategies, balancing policy- and market-driven volatility protection.

This study examines primarily how EPU indices affect the forecasting of volatility indices. Future research could include more macroeconomic variables, such as monetary policy, fiscal policy, and

international trade policy, to more fully capture how macroeconomic factors dynamically influence volatility indices. Methodologically, improvements may also come from hybrid models that combine the GARCH-MIDAS framework with advanced computational techniques, especially machine learning algorithms. Moreover, these hybrid models could be paired with behavioral financial indicators such as investor sentiment and market attention to analyze how their interaction with macroeconomic uncertainty impacts volatility indices. This approach would help reveal the role of irrational investor behavior in volatility index forecasting.

Author contributions

Gaoxiu Qiao: conceptualization, methodology, supervision, formal analysis, writing - reviewing and editing, funding acquisition.

Yunli Bi: conceptualization, investigation, methodology, data curation, methodology, software, visualization, writing—review & editing.

Wanmei Cui: software, visualization, Writing—review & editing.

Yaxuan Wang: software, visualization, Writing—review & editing.

Use of AI tools declaration

The authors used AI tools for language editing and grammar review of this manuscript. The authors are fully responsible for the content of this publication.

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Conflict of interest

The authors declare no conflicts of interest.

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