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Research article

Pricing hybrid-triggered catastrophe bonds based on copula-EVT model

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Abstract: This paper presents a hybrid-triggered catastrophe bond (CAT bond) pricing model. We take earthquake CAT bonds as an example for model construction and numerical analysis. According to the characteristics of earthquake disasters, we choose direct economic loss and magnitude as trigger indicators. The marginal distributions of the two trigger indicators are depicted using extreme value theory, and the joint distribution is established by using a copula function. Furthermore, we derive a multi-year hybrid-triggered CAT bond pricing formula under stochastic interest rates. The numerical experiments show that the bond price is negatively correlated with maturity, market interest rate and dependence of trigger indicators, and positively correlated with trigger level and coupon rate. This study can be used as a reference for formulating reasonable CAT bond pricing strategies.

Keywords: catastrophe bonds; hybrid trigger mechanism; Archimedean copula; extreme value theory; stochastic interest rates

JEL Codes: G15, G22

1. Introduction

The year 2020 will be remembered for the global financial crisis triggered by COVID-19. Against the backdrop of disruption and upheaval caused by the pandemic, millions of people also experienced severe natural and man-made catastrophes, such as bush fires in Australia, derecho storms, Hurricane Laura and a Beirut explosion. In inflation-adjusted terms, global economic losses from natural and man-made catastrophes were USD 202 billion in 2020. The insurance industry has played an essential role, which covered USD 89 billion of the absolute economic costs incurred in 2020 on account of disaster events. Notwithstanding, the global catastrophe protection gap was around USD 113 billion (Swiss Re, 2020). Insurance-linked securities (ILSs), which connect both the capital market and the insurance market, provide an effective channel for the insurance industry to raise funds from the capital market. Catastrophe bonds (CAT bonds) are the most active ILS to date; they not only enhance the catastrophe underwriting capacity of the insurance industry, but also enrich the investment portfolios for investors.

The successful issuance of CAT bonds is inseparable from reasonable pricing. The default of the CAT bonds' principal or remaining interest payment is triggered if a catastrophic event exceeds a certain pre-suppositional condition (Smack, 2016). Clearly, defining the default trigger event plays an important role in pricing CAT bonds. In general, there are three trigger types: indemnity triggers, index triggers and hybrid triggers (Ma and Ma, 2013). The indicator of indemnity triggers is the level of actual economic loss suffered by the sponsor. There are three broad types of index triggers: industry loss indices, modeled loss indices and parametric indices. Industry loss index-triggered CAT bonds take the estimated industry-wide losses from a catastrophic event as the trigger indicator. Published data for industry loss index triggers are obtained from an index maintained by an independent third party, such as the Sigma index and Property Claim Service (PCS) index. The modeled loss index is calculated using the model provided by a catastrophe modeling agency, such as RMS and EQECAT (Ma and Ma, 2013). A parameter trigger mechanism refers to taking the physical parameters of catastrophic events as trigger indicators, such as the earthquake magnitude, typhoon wind speed and flood flow level. The moral hazard faced by bond investors and the basis risk borne by bond sponsors are two important factors to be considered in determining the trigger type for CAT bonds. When an indemnity trigger is used to insure the sponsor's actual loss, it results in the highest degree of moral hazard. The application of the three types of index triggers can eliminate the moral hazard and enhance the investment attraction for investors, but the basic risk is inevitable. It can be seen that there are obvious deficiencies in CAT bonds with a single trigger type, no matter whether it is an indemnity trigger or index trigger. A hybrid trigger mechanism refers to the simultaneous application of two or more types of trigger indicators in the contract, which can be a hybrid of different index triggers, or a hybrid of index triggers and indemnity triggers (Bouriaux and MacMinn, 2009). In practice, it is more advantageous to combine indemnity triggers and index triggers, because this combination can achieve the purpose of balancing moral hazard and basic risk.

In the last two decades, several research studies have been conducted on CAT bonds. Mainly, these studies have focused on the pricing of indemnity-triggered CAT bonds (e.g., Cox and Pedersen, 2000; Lee and Yu, 2002; Nowak and Romaniuk, 2013; Chen et al., 2013; Romaniuk, 2017; Zhang and Tsai, 2018; Kurniawan et al., 2021) and index-triggered CAT bonds (e.g., Zimbidis et al., 2007; Ma and Ma, 2013; Shao et al., 2015; Ma et al., 2017; Mousavi et al., 2019). Nevertheless, studies on the pricing of hybrid-triggered CAT bonds are not very numerous. Taking terrorism risk CAT bonds as an example, Woo (2004) proposed the concept of bonds triggered by multiple events. Reshetar (2008) developed a framework for the pricing of multiple-event CAT bonds that provides a theoretical basis for the construction of the hybrid-triggered CAT bonds pricing model. In this study, the payoffs on the bond are linked to catastrophic property losses and daily deaths, in which the attachment point for property losses is given as a threshold total annual loss, and the attachment point for death is defined as a threshold number of deaths per day (Reshetar, 2008). It should be noted that most insurance companies expect to be supplemented with capital in a timely manner after a single disaster with extreme losses to avoid bankruptcy due to substantial claims. Therefore, this study takes the property loss and magnitude of a single earthquake disaster as the hybrid trigger indicators.

CAT bonds can be considered as zero-beta assets, and the structure of a bond's cash flow depends on the occurrence of the predefined catastrophe and the attachment point of trigger indicators (e.g., Litzenberger et al., 1996; Cummins and Weiss, 2009; Tao, 2011). Therefore, the accurate assessment of catastrophe risk is a crucial step. Catastrophe risks have obvious heavy tail characteristics, and the pricing of CAT bonds is mainly related to the extremes of the tail. Extreme value theory (EVT) is an obbligato tool in the study of catastrophe; it can effectively describe the tail characteristics of the trigger indicators' distribution. It is applied widely in various fields, such as earth science, meteorology and hydrology (Ma et al., 2021). By using the PCS loss index data from 2001 to 2010, Ma et al. (2017) adopted the peaks-over-threshold (POT) model in EVT to characterize the tail characteristics of catastrophic loss distribution when evaluating zero-coupon CAT bonds. Chao and Zou (2018) evaluated the distributions of flood catastrophe losses and deaths according to EVT by using the global flood data since 1985. Based on the data on global drought catastrophe losses from 1900 to 2018, Deng et al. (2020) studied the pricing of drought CAT bonds by using EVT and high quantile estimation.

When multiple variables related to a catastrophic event are used as trigger indicators, the correlation between them should be considered. The copula modeling is a more general method that can be used to describe the dependence of a set of random variables (Chebbi and Hedhli, 2020). Frees and Valdez (1998) were among the first to use it in insurance applications. Then, the literature on the applications of copulas in catastrophe insurance has been developing rapidly in recent years. Bokusheva (2014) presented a copula-based approach for rating weather index insurance designed to provide coverage for yield losses due to extreme weather events; their results indicated that the application of the copula approach might improve the performance of weather index insurance. Xu et al. (2013) built a drought disaster evaluation model based on the copula-EVT model; it reflected the extreme process about drought duration and drought intensity. Liu et al. (2019) modeled the direct economical losses and tolls of earthquakes by using a copula-mixed distribution model. Chao (2021) employed the copula function to describe the dependence of damage areas and deaths due to flood catastrophe; they developed a framework for valuing multirisk catastrophe reinsurance contracts.

Earthquake disaster is the most severe of all disasters, as it has caused considerable losses to human beings. In this paper, we take the earthquake disaster as the insurance subject and construct the pricing model of multi-year hybrid-triggered CAT bonds under stochastic interest rates. According to the characteristics of a earthquake disaster, we choose the economic loss and magnitude of a single earthquake disaster as trigger indicators, and their marginal distributions are depicted according to EVT. By introducing an Archimedean copula function and its survival function, the valuation formula for CAT bonds' price is derived. Furthermore, the impacts of the maturity, trigger level, dependence of trigger indicators, market interest rate and coupon rate on CAT bond pricing are analyzed.

Compared with previous research, the main contributions of this paper to the field of CAT bond pricing are twofold. First, a framework is developed for valuing CAT bonds based on the hybrid trigger mechanism; it combines the indemnity trigger and index trigger. The application of the hybrid trigger mechanism can effectively enhance the investment attractiveness and market competitiveness of CAT bonds. However, compared with indemnity-triggered or index-triggered CAT bonds, the payment structure of hybrid-triggered CAT bonds is more complicated, and there are relatively few related studies. Hence, this study aims to develop a model to value hybrid-triggered CAT bonds. Second, a multi-year hybrid-triggered CAT bond pricing formula is derived using the copula-EVT model. The general statistical distributions are not suitable for fitting the heavy-tailed data since the catastrophe risk data contains some extreme values.

And, there may be a certain degree of correlation between the related variables of the same catastrophic event. Therefore, the goal was to combine the EVT and copula function to analyze the distribution characteristics and the dependence of the catastrophe variables.

The rest of the paper is organized as follows. The hybrid-triggered CAT bond pricing formula is introduced and proved in Section 2. Section 3 shows the parameters estimation of the pricing model. Section 4 presents the numerical analysis. Finally, we end our work with a conclusion in Section 5.

2. Valuation framework

We consider a coupon-paying CAT bond with maturity T years, and choose the economy loss and magnitude as trigger indicators. The principal is F and coupon payments C_t are paid annually at the times $t = 1, 2, ..., T$. The coupon rate is denoted by R. Let $(\Omega, \mathcal{F}, \mathbb{P})$ define a probability space, where Ω is the sample space, $\mathcal{F}$ is a σ-algebra of subsets of Ω and $\mathbb{P}$ is the probability measure. Let $\{N_t, t = 1, 2, ..., T\}$ denote the number of earthquake disasters that occurred in the interval $[t-1, t]$, which is a sequence of independent and identically distributed (*i.i.d.*) random variables that obeys a Poisson distribution with an intensity $\lambda > 0$. The occurrence time of the i-th earthquake disaster is denoted as d_i, $i = 1, 2, ..., m$ ($m = \sum_{t=1}^{T} N_t$). Let $\{X_d, d = d_1, d_2, ..., d_m\}$ denote the economy loss caused by the earthquake disaster that occurred at the time d, and let $\{Y_d, d = d_1, d_2, ..., d_m\}$ denote the magnitude of the earthquake disaster that occurred at the time d. $\{X_d, d = d_1, d_2, ..., d_m\}$ and $\{Y_d, d = d_1, d_2, ..., d_m\}$ are two sequences of *i.i.d.* positive random variables with cumulative distribution functions F_X and F_Y, respectively. Both of them are independent of $\{N_t, t = 1, 2, ..., T\}$.

2.1. Trigger conditions and payoff structures

In the event of a catastrophe, the coupons and principal are at risk. When one of the indicators is triggered, the current and future coupons will no longer be paid. When both indicators are triggered simultaneously, the principal at maturity will no longer be returned to investors.

We define the stopping time τ_C as the first time when the economy loss exceeds the trigger level Tr_X or the magnitude exceeds the trigger level Tr_Y:

$$\tau_C = min\{d = d_1, d_2, ..., d_m | X_d > Tr_X \ or \ Y_d > Tr_Y\}, \tag{1}$$

Then, the payoff structure of coupons can be described as

$$C_t = FR\mathbb{I}_{\{\tau_C > t\}}, \quad t = 1, 2, ..., T, \tag{2}$$

where $\mathbb{I}_{\{\cdot\}}$ is an indicator function.

Next, we define the stopping time τ_F as the first time when the economy loss and magnitude exceed their trigger levels simultaneously:

$$\tau_F = min\{d = d_1, d_2, ..., d_m | X_d > Tr_X \ and \ Y_d > Tr_Y\}. \tag{3}$$

Then, the payoff structure of the principal is given by the following:

$$F_T = F\mathbb{I}_{\{\tau_F > T\}}. \tag{4}$$

Combining Equations (2) and (4), the cash flows to the bondholder can be described as

$$CF(t) = \begin{cases} FR\mathbb{I}_{\{\tau_C > t\}}, & t = 1, 2, ..., T-1, \\ FR\mathbb{I}_{\{\tau_C > t\}} + F\mathbb{I}_{\{\tau_F > t\}}, & t = T. \end{cases} \tag{5}$$

2.2. Distribution model for the trigger indicators

2.2.1. Extreme value theory and modeling

There are two principal kinds of models for extreme values, namely, the block maxima (BM) model and POT model. The focus of the former is on modeling the BM of the variables with the generalized extreme value distribution (Acero et al., 2018). However, it is only interested in the behavior of the sample maximum, which could result in cause vast amounts of missing valid data (Chao and Zou, 2018). The POT model is based on the asymptotic convergence of the exceedances of a high threshold u to a generalized Pareto distribution (GPD) when u tends to infinity. It is generally considered the most useful for practical applications due to its more efficient use of data with extreme values. Hence, the GPD is chosen to analyze the marginal distributions of trigger indicators.

The GPD is a two-parameter distribution with distribution function:

$$G_{\xi,\beta}(y) = 1 - (1 + \xi y/\beta)^{-1/\xi}, \tag{6}$$

where $\beta > 0$, and the support is $y \geq 0$ when $\xi \geq 0$ and $0 \leq y \leq -\beta/\xi$ when $\xi < 0$.

This distribution is generalized in the sense that it subsumes certain other distributions under a common parametric form (McNeil and Frey, 2000). If $\xi > 0$, then $G_{\xi,\beta}$ is a reparametrized version of the ordinary Pareto distribution; $\xi < 0$ is known as a Pareto type II distribution; $\xi = 0$ corresponds to the exponential distribution, that is, $G_{0,\beta} = 1 - exp(-y/\beta)$.

Consider $X_1, X_1, ..., X_n$ to be a sequence of *i.i.d.* random variables with a cumulative continuous distribution function $F(x)$. We choose a sufficiently large value threshold u, and the number of observations exceeding u are set as N_u. $Y_i = X_i - u$ denotes the excesses of X_i over the threshold u. Then, the distribution of Y_i, known as the "conditional excess distribution function", is given by

$$F_u(y) = P(X - u \leq y | X > u) = \frac{F(y+u) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)}. \tag{7}$$

When the threshold u is large enough, $F_u(y)$ can be approximated by the GPD (see Balkema and Haan, 1974; Pickands, 1975). Substituting Equation (6) for $F_u(y)$ into Equation (7), we obtain

$$F(x) = (1 - F(u))(1 - (1 + \xi(x-u)/\beta)^{-1/\xi}) + F(u), \quad x > u, \tag{8}$$

There is an empirical estimator of $F(u)$, that is, $(n - N_u)/n$, which can be substituted into Equation (8) to obtain the following expression (see McNeil and Frey, 2000):

$$\hat{F}(x) = 1 - \frac{N_u}{n}(1 + \xi(x-u)/\beta)^{-1/\xi}, \quad x > u. \tag{9}$$

2.2.2. Copula function

A copula is a function that factors a joint function into a marginal distribution and a dependence function C. The concept of copula goes back to Sklar (see Sklar, 1959). For two random variables X and Y, the joint distribution function is defined as

$$F_{XY}(x, y) = P(X \leq x, Y \leq y). \tag{10}$$

The corresponding survival function is defined as

$$\overline{F}_{XY}(x, y) = P(X > x, Y > y). \tag{11}$$

The dependence function C is defined as

$$F_{XY}(x, y) = C(w, v), \tag{12}$$

where $w = F_X(x)$ and $v = F_Y(y)$, and $F_X(x)$ and $F_Y(y)$ are the marginal distribution functions of X and Y, respectively. If $F_X(x)$ and $F_Y(y)$ are continuous, then the copula function C is unique. Conversely, when w, v are marginal distribution functions and C is a copula, the function F defined by Equation (12) is a multivariate distribution with marginal distributions $F_X(x)$ and $F_Y(y)$ (Yao et al., 2017).

Another notation that is used in this paper is a survival copula of C, denoted by $\overline{C}$:

$$\overline{F}_{XY}(x, y) = \overline{C}(\overline{F}_X(x), \overline{F}_Y(y)). \tag{13}$$

where $\overline{F}_X(x)$ and $\overline{F}_Y(y)$ are the marginal survival functions of X and Y, respectively. The survival copula $\overline{C}$ is related to the copula C by

$$\overline{C}(w, v) = C(1 - w, 1 - v) + w + v - 1. \tag{14}$$

Combining Equation (13) with Equation (14), we obtain

$$\overline{F}_{XY}(x, y) = C(F_X(x), F_Y(y)) - F_X(x) - F_Y(y) + 1. \tag{15}$$

2.3. CAT bond pricing formula

Our aim in this section is to prove the CAT bond pricing formula. We follow Merton (1976) to assume that the overall economy is only marginally influenced by localized catastrophic events and the catastrophe losses pertain to idiosyncratic shocks to the capital markets. That is, the catastrophic shocks will represent as "nonsystematic" and have a zero-risk premium. Under the risk-neutral probability measure $\mathbb{Q}$, those events that depend only on financial variables are independent of those events that depend on catastrophic risk variables. As a result, the stochastic processes and distributions under $\mathbb{Q}$ retain the same characteristics as those under $\mathbb{P}$ (for a detailed discussion of this point, please refer to Cox and Pedersen, 2000; Lee and Yu, 2002; Braun, 2011; Ma and Ma, 2013). Assuming that the cash flows $CF(t)$ on CAT bonds depend only on the catastrophic risk variables, then the price of CAT bonds can be calculated by the following formula:

$$\begin{aligned} P_0(CF) &= E_{\mathbb{Q}}\left[\sum_{t=1}^{T} e^{-\int_0^t r_s ds} CF(t)\right] \\ &= p(0, t) E_{\mathbb{Q}}\left[\sum_{t=1}^{T} CF(t)\right], \end{aligned} \tag{16}$$

where r_s denotes the instantaneous interest rate and $p(0, t) = E_{\mathbb{Q}}\left[e^{-\int_0^t r_s ds}\right]$ stands for the price of a non-defaultable zero-coupon bond with a face value of 1 maturing at a time t.

In this paper, the instantaneous interest rate is set to be governed by a Cox-Ingersoll-Ross (CIR) model (see Cox et al., 1985). Then, the dynamics of the instantaneous interest rate under the risk-neutral probability measure $\mathbb{Q}$ can be described as

$$dr_t = \kappa(\theta - r_t)dt + \varepsilon\sqrt{r_t}dW_t. \tag{17}$$

where $\kappa > 0$ is the mean-reverting force measurement, $\varepsilon > 0$ is the volatility parameter, $\theta > 0$ is the long-run mean of the interest rate, and W_t denotes a standard Wiener process under $\mathbb{Q}$. The initial interest rate r_0 is labeled as r; then, $p(0,t)$ can be expressed as

$$p(0,t) = A(0,t)e^{-B(0,t)r}, \tag{18}$$

where

$$\begin{aligned} A(0,t) &= \left[\frac{2\eta e^{(\kappa+\eta)t/2}}{(\kappa+\eta)(e^{\eta t}-1)+2\eta}\right]^{2\kappa\theta/\varepsilon^2}, \\ B(0,t) &= \frac{2(e^{\eta t}-1)}{(\kappa+\eta)(e^{\eta t}-1)+2\eta}, \\ \eta &= \sqrt{\kappa^2+2\varepsilon^2}. \end{aligned}$$

Combining Equation (5) and Equation (16), the price of the bond at time $t = 0$ is given by

$$\begin{aligned} P_0(CF) &= \sum_{t=1}^{T} E_{\mathbb{Q}}\left[C_t\right] p(0,t) + E_{\mathbb{Q}}\left[F_T\right] p(0,T) \\ &= \sum_{t=1}^{T} FRE_{\mathbb{Q}}\left[\mathbb{I}_{\{\tau_C > t\}}\right] p(0,t) + FE_{\mathbb{Q}}\left[\mathbb{I}_{\{\tau_F > T\}}\right] p(0,T) \\ &= F(R\sum_{t=1}^{T} P(\tau_C > t)p(0,t) + P(\tau_F > T)p(0,T)). \end{aligned} \tag{19}$$

The probabilities that a single earthquake disaster satisfies the trigger conditions for a coupon and principal are marked as γ_C and γ_F, respectively. According to Equation (12) and Equation (15), we obtain

$$\gamma_C = 1 - C(F_X(Tr_x), F_Y(Tr_y)), \tag{20}$$

and

$$\gamma_F = C(F_X(Tr_x), F_Y(Tr_y)) - F_X(Tr_x) - F_Y(Tr_y) + 1. \tag{21}$$

The probability that the trigger condition for the coupon is satisfied in the interval $[t-1, t]$ is marked as υ_C; then, it can be expressed as follows:

$$\begin{aligned}
\upsilon_C &= 1 - E_{\mathbb{Q}}\left[\prod_{i=1}^{N_t}(1-\gamma_C)\right] \\
&= 1 - \sum_{n=0}^{\infty} E_{\mathbb{Q}}\left[\prod_{i=1}^{N_t}(1-\gamma_C)|N_t = n\right] P(N_t = n) \\
&= 1 - \sum_{n=0}^{\infty} E_{\mathbb{Q}}\left[\prod_{i=1}^{n}(1-\gamma_C)\right] P(N_t = n) \\
&= 1 - \sum_{n=0}^{\infty}(1-\gamma_C)^n \frac{\lambda^n e^{-\lambda}}{n!} \\
&= 1 - e^{-\lambda}\sum_{n=0}^{\infty}\frac{[\lambda(1-\gamma_C)]^n}{n!} \\
&= 1 - e^{-\lambda}e^{\lambda(1-\gamma_C)} \\
&= 1 - e^{-\lambda\gamma_C}.
\end{aligned} \tag{22}$$

Similarly, the probability that the trigger condition for the principal is satisfied in the interval $[0, T]$ is marked as υ_F. Since $\{N_t, t = 1, 2, ..., T\}$ is a sequence of *i.i.d.* random variables that obeys a Poisson distribution with an intensity λ, then $\sum_{t=1}^{T} N_t$ obeys a Poisson distribution with the intensity λT. Then, we can get

$$\begin{aligned}
\upsilon_F &= 1 - E_{\mathbb{Q}}\left[\prod_{i=1}^{\sum_{t=1}^{T} N_t}(1-\gamma_F)\right] \\
&= 1 - \sum_{n=0}^{\infty} E_{\mathbb{Q}}\left[\prod_{i=1}^{\sum_{t=1}^{T} N_t}(1-\gamma_F)|\sum_{t=1}^{T} N_t = n\right] P(\sum_{t=1}^{T} N_t = n) \\
&= 1 - \sum_{n=0}^{\infty} E_{\mathbb{Q}}\left[\prod_{i=1}^{n}(1-\gamma_F)\right] P(\sum_{t=1}^{T} N_t = n) \\
&= 1 - \sum_{n=0}^{\infty}(1-\gamma_F)^n \frac{(\lambda T)^n e^{-\lambda T}}{n!} \\
&= 1 - e^{-\lambda T}\sum_{n=0}^{\infty}\frac{[\lambda T(1-\gamma_F)]^n}{n!} \\
&= 1 - e^{-\lambda T}e^{\lambda T(1-\gamma_F)} \\
&= 1 - e^{-\lambda T\gamma_F}.
\end{aligned} \tag{23}$$

Combining Equations (19)–(23), we can derive the CAT bond pricing formula as follows:

$$
\begin{aligned}
P_0(CF) &= F(R\sum_{t=1}^{T}(1-\upsilon_C)^t p(0,t) + (1-\upsilon_F)p(0,T)) \\
&= F(R\sum_{t=1}^{T} e^{-\lambda t\gamma_C} p(0,t) + e^{-\lambda T\gamma_F} p(0,T)) \\
&= F(R\sum_{t=1}^{T} e^{-\lambda t(1-C(F_X(Tr_x),F_Y(Tr_y)))} p(0,t) + e^{-\lambda T(C(F_X(Tr_x),F_Y(Tr_y))-F_X(Tr_x)-F_Y(Tr_y)+1)} p(0,T)).
\end{aligned} \tag{24}
$$

3. Parameter estimation for the pricing model

3.1. Data description

The data cover economy losses and magnitudes resulting from earthquake events on the Chinese mainland that occurred between 1990 and 2020. The data comes from the "Review of earthquake damage losses in the mainland of China (1990–2020)" compiled by the China Earthquake Networks Center; a total of 344 pairs of observations for economy losses and magnitudes were picked out. To eliminate the impact of time, the GDP for 2020 in China was chosen as the fixed base index to deal with the economy loss data.

3.2. Parameter estimation for marginal distributions

In this section, we construct the marginal distributions of economy losses and magnitudes based on the POT model. In the field of EVT, threshold selection is critical, especially when the threshold-based methods are considered (Chukwudum et al., 2020). Hence, threshold selecting has to be done first before we can fit the GPD for the given data. We combine the mean excess function (MEF) plot and Hill plot to select the optimal threshold.

Figure 1 shows the MEF plot of economic losses, which is approximately linearly inclined upward, indicating that this group of data is a heavy tailed-distribution ($\xi > 0$). Figure 2 is the Hill plot of economic losses, and the image tends to be stable roughly from the 50th point. To select the optimal threshold, the sequence statistics corresponding to each point are set as the threshold starting from the 45th point, and the maximum likelihood estimation is adopted for fitting. According to the results of Kolmogorov-Smirnov (K-S) testing, it can be known that the model fitting is optimal at the 48th point, that is, the threshold is $u_X = 336975.39$. Figure 3 shows the MEF plot of magnitudes, which is approximately linearly inclined downward, indicating that the data is light-tailed. Figure 4 is the Hill plot of magnitudes. Using a similar method to that used to determine the threshold of economic losses, the threshold of magnitudes can be determined as $u_Y = 6.6$. Other parameter estimation results are shown in Table 1.

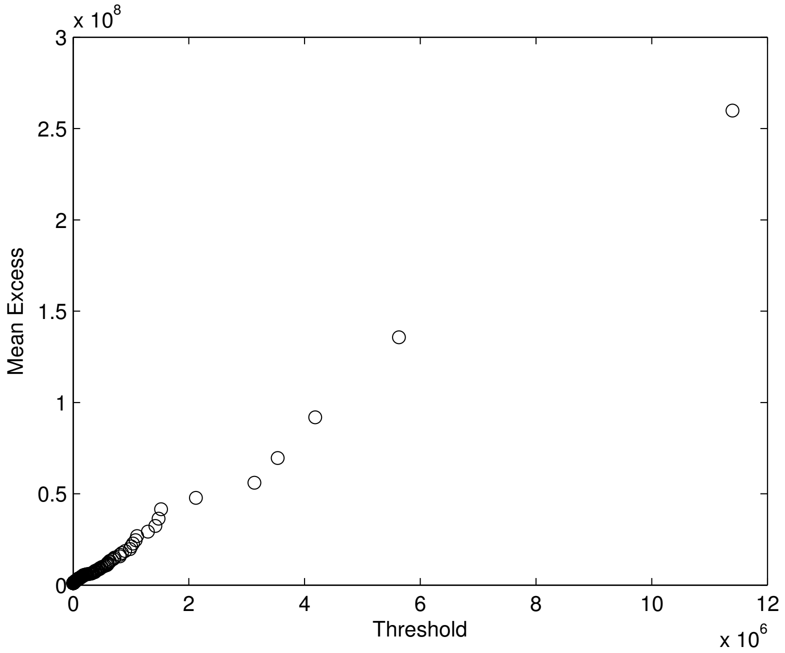

Figure 1. MEF plot of economic losses.

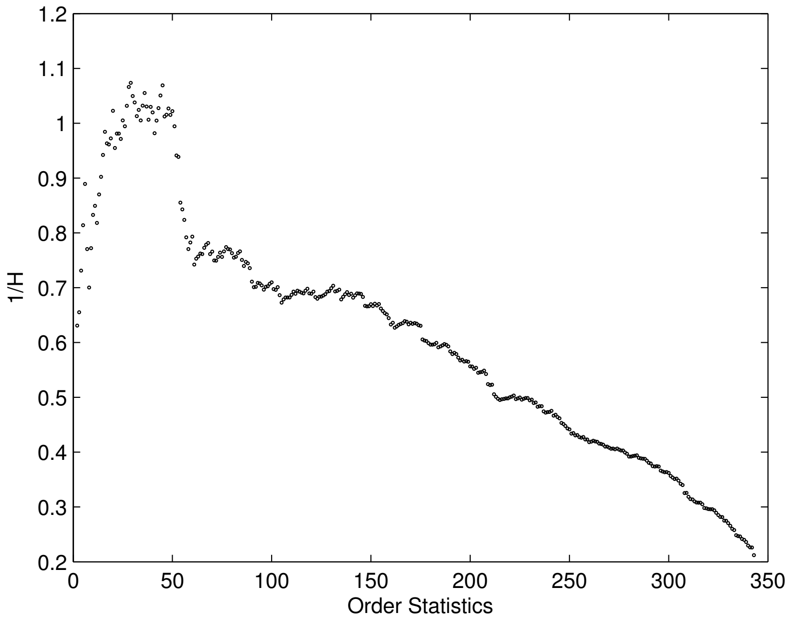

Figure 2. Hill plot of economic losses.

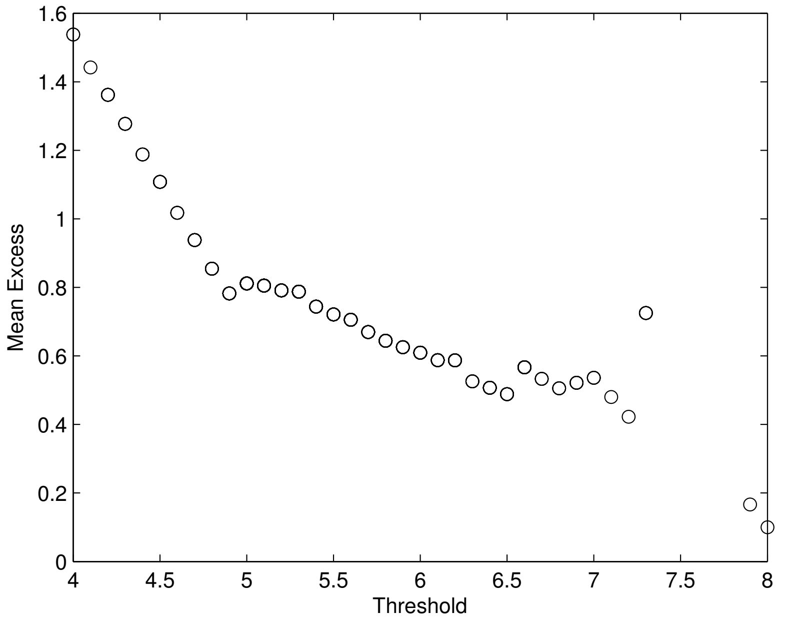

Figure 3. MEF plot of magnitudes.

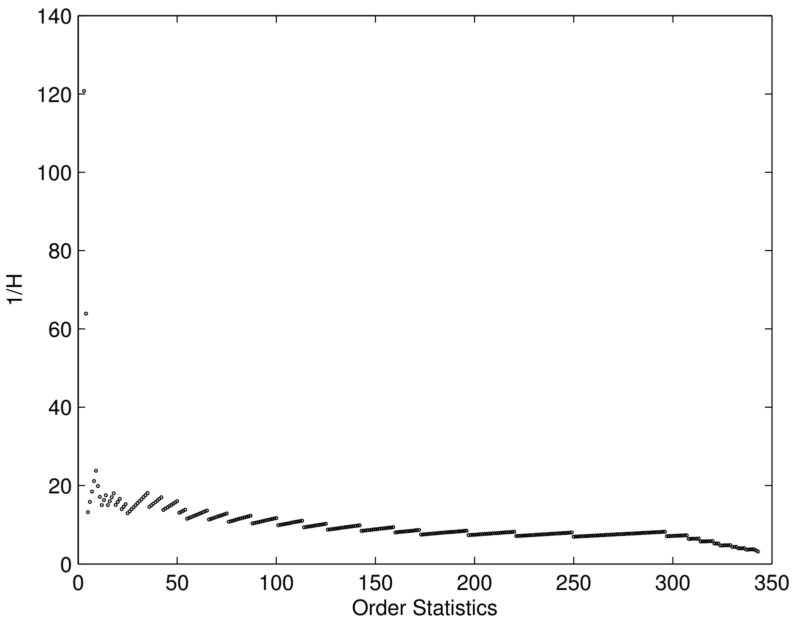

Figure 4. Hill plot of magnitudes.

Substituting the estimated values of parameters into equation (9), the marginal distribution functions of economic losses and magnitudes can be obtained as follows:

$$F_X(x) = 1 - \frac{47}{344}(1 + 1.1266(x - 336975.39)/280471.11)^{-1/1.1266}, \tag{25}$$

and

$$F_Y(y) = 1 - \frac{24}{344}(1 - 0.4789(y - 6.6)/0.8650)^{1/0.4789}. \tag{26}$$

Table 1. Parameters estimates of POT and K-S test results.

	ξ	β	u	N_u	K-S statistics	p value
Economic losses	1.1266	280471.11	336975.39	47	0.0659	0.9765
Magnitudes	−0.4789	0.8650	6.6	24	0.1926	0.2961

3.3. Parameter estimation for the joint distribution

Archimedean copula functions have the advantage of better describing the asymmetric correlation between variables and tail features. In this section, the maximum likelihood method is used to estimate the parameters of three commonly used binary Archimedean copula functions. The results are listed in Table 2. The empirical copula function is introduced to calculate the squared Euclidean distance between the estimation results for the three copula functions and the empirical copula function of the sample. The smaller the squared Euclidean distance, the better the fitness of the copula function (Shen et al., 2018; Gu et al., 2019; Cai et al., 2019). The empirical copula function is defined as

$$\hat{C}(w,v) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{I}_{\{F_X(x_i)\leqslant w\}}\mathbb{I}_{\{F_Y(y_i)\leqslant v\}}. \tag{27}$$

Then, the squared Euclidean distance can be given by

$$d^2 = \sum_{i=1}^{n} |\hat{C}(w,v) - C(w,v)|^2. \tag{28}$$

As shown in Table 3, the square Euclidean distance d^2 of the Gumbel copula function is the smallest. Thus, the Gumbel copula function is the optimal joint distribution function; its function form is as follows:

$$C(w,v) = exp\left\{-\left[(-\ln w)^{1.6176} + (-\ln v)^{1.6176}\right]^{1/1.6176}\right\}. \tag{29}$$

The correlation coefficient between economic losses and magnitudes can be measured by the Kendall rank correlation coefficient, which can be calculated as $\rho = 1 - 1/\alpha = 0.3818$.

Table 2. Parameter estimation results for the three Archimedean copula functions.

Copula family	Function expression	Parameter α
Frank copula	$C(w,v) = -\frac{1}{\alpha}\ln\left\{1 + \frac{[exp(-\alpha w)-1][exp(-\alpha v)-1]}{exp(-\alpha)-1}\right\}, \alpha \neq 0$	4.2634
Clayton copula	$C(w,v) = (w^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}, \alpha \in [-1,\infty)\backslash\{0\}$	1.0696
Gumbel copula	$C(w,v) = exp\left\{-[(-\ln w)^{\alpha} + (-\ln v)^{\alpha}]^{1/\alpha}\right\}, \alpha \in [1,\infty)$	1.6176

Table 3. Squared Euclidean distance of three Archimedean copula functions.

Copula family	Squared Euclidean distance d^2
Frank copula	0.0773
Clayton copula	0.2035
Gumbel copula	0.0266

4. Numerical analysis

For a reference point of numerical analysis, we establish the set of parameters summarized in Table 4. The attachment points of the two trigger indicators are measured by the risk value VaR_p, which refers to the maximum loss that a risk product may incur in a specified future period of time under the confidence level p. The risk value VaR_P can be described as

$$VaR_p(X) = F^{-1}(p) = inf\{x \in R : F(X) \geq p\}, \quad 0 < p < 1. \tag{30}$$

Combining Equation (9) and Equation (30), the estimate of the risk value VaR_p based on the GPD can be obtained as

$$VaR_p(X) = u + \frac{\beta}{\xi}\left\{[\frac{n}{N_u}(1-p)]^{-\xi} - 1\right\}. \tag{31}$$

In our case, the confidence level p is set equal to 99%. Then, the attachment points of the two trigger indicators can be calculated as shown in Table 4. The expectation and variance of the Poisson distribution are equal. Based on the data for the frequency of the earthquake disaster, we calculated the mean to be approximately equal to 11 for the estimation. Then, K-S testing was done; the result shows that the distribution of the frequency of earthquake disasters in China obeys the Poisson distribution. The interest rate parameters for the CIR model were set to $r = 0.04$, $\kappa = 0.2$, $\theta = 0.05$ and $\varepsilon = 0.1$. These interest rate parameter values are all within the range typically used in previous studies (e.g., Lee and Yu, 2007; Lo, et al., 2013; Chao, 2021).

Table 4. Parameter definitions and base values.

Parameter	Definitions	Base values
T	Maturity (year)	1–5
F	Principal (RMB)	100
R	Coupon rate	0.06
r	Initial interest rate value	0.04
κ	Magnitude of mean-reverting force	0.2
θ	Long-run mean of the interest rate	0.05
ε	Volatility of the interest rate	0.1
λ	Poisson intensity	11
ρ	Kendall rank correlation coefficient	0.3818
Tr_X	Attachment point of economic losses (million RMB)	4824080.67
Tr_Y	Attachment point of magnitudes	7.7

4.1. CAT bond prices at different maturities

The prices of CAT bonds with maturities of 1–5 years are shown in Table 5. The CAT bond prices tend to decrease with the maturity period. The main reason is that, as the maturity period increases, there will be more catastrophe events that meet the trigger conditions, and the probability of losing coupons and the principal will be higher. The decline of the CAT bond price is slower than that of the principal; it is due to the coupon payment structure, that is, with the extension of the maturity period, the accumulated coupon amount also increases, making up for part of the principal loss.

Table 5. CAT bond prices at different maturities.

Price	T =1	T =2	T =3	T =4	T =5
Cumulative discounted value of coupon	4.87	8.06	10.00	11.03	11.41
Discounted value of principal	91.17	83.00	75.50	68.63	62.37
CAT bond price	96.04	91.06	85.50	79.66	73.78

4.2. Effects of the attachment point and dependence of trigger indicators

The correlation between the two trigger indicators and the trigger levels are both important factors affecting the pricing of CAT bonds, which are shown in Figure 5. When the trigger levels of the two trigger indicators are increased simultaneously, that is, when the confidence level p of VaR is increased, the price is significantly increased. As can be seen in Figure 6, CAT bond prices are positively correlated with Tr_X and Tr_Y. Raising the trigger levels means that the probability that the trigger conditions are met becomes smaller, and the possibility of the loss of coupons and principal is also reduced. Furthermore, we pay attention to the respective sensitivities of CAT bond prices to Tr_X and Tr_Y.

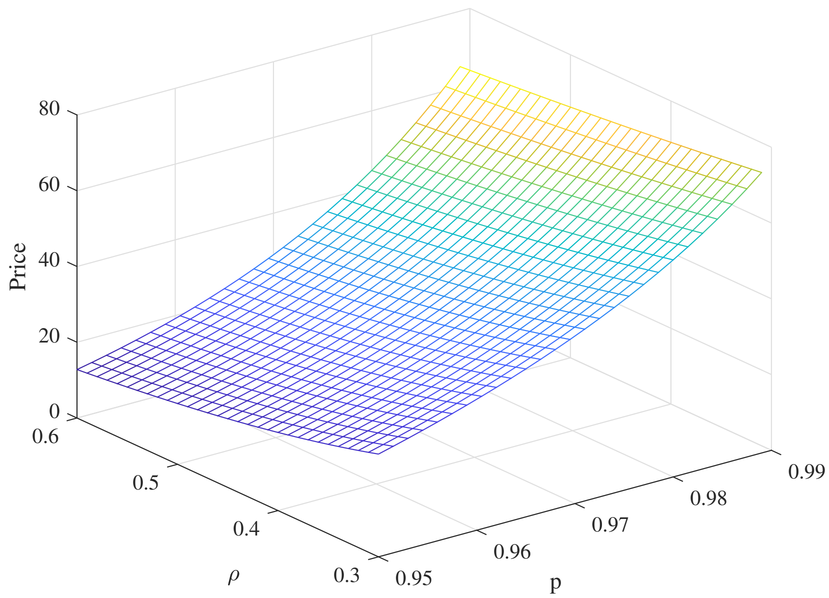

Figure 5. CAT bond prices for alternative sets of ρ and p (T = 5).

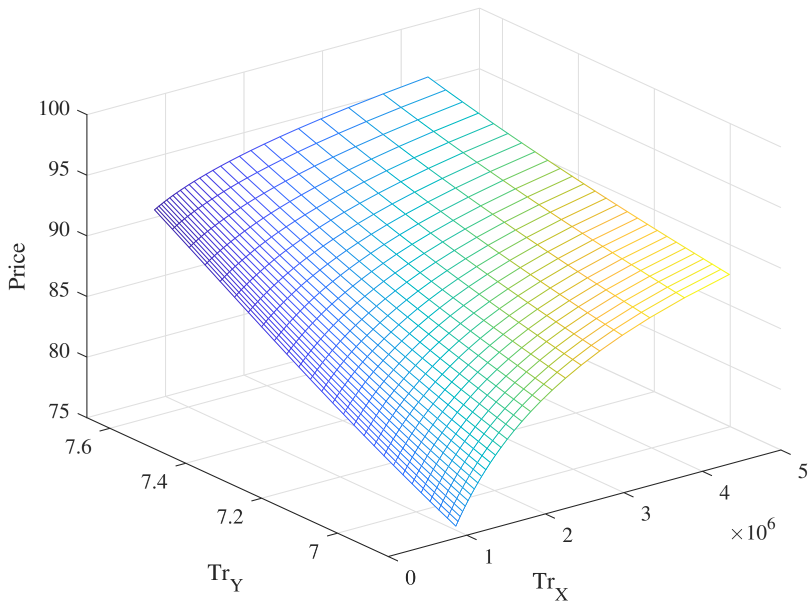

Figure 6. CAT bond prices for alternative sets of Tr_X and Tr_Y ($T = 1$).

Tables 6 and 7 show the changes in 1-year CAT bond prices after the trigger levels of the two trigger indicators were increased or decreased by 2.5% and 5%, respectively. It can be seen that, under the adjustment ratio, Tr_Y has a greater impact on the CAT bond price.

Table 6. Sensitivity of Tr_X to CAT bond price.

Adjustment ratio for Tr_X	$T = 1$	$T = 2$	$T = 3$	$T = 4$	$T = 5$
+5%	+0.13%	+0.27%	+0.43%	+0.60%	+0.77%
+2.5%	+0.06%	+0.14%	+0.22%	+0.30%	+0.39%
−2.5%	−0.06%	−0.14%	−0.23%	−0.31%	−0.40%
−5%	−0.13%	−0.29%	−0.46%	−0.64%	−0.82%

Table 7. Sensitivity of Tr_Y to CAT bond price.

Adjustment ratio for Tr_Y	$T = 1$	$T = 2$	$T = 3$	$T = 4$	$T = 5$
+5%	+3.63%	+7.55%	+11.72%	+16.13%	+20.76%
+2.5%	+1.77%	+3.73%	+5.84%	+8.06%	+10.35%
−2.5%	−1.49%	−3.26%	−5.18%	−7.15%	−9.12%
−5%	−2.72%	−6.06%	−9.61%	−13.15%	−16.53%

It can also be seen in Figure 5 that the CAT bond price is negatively correlated with ρ. Table 8 shows the price of the 5-year CAT bonds at different values of the Kendall rank correlation coefficient ρ for the two trigger indicators. As ρ increases, the coupon discount changes less, while the principal discount decreases significantly. The stronger correlation between losses and magnitudes implies a greater probability of triggering two trigger indicators at the same time during a catastrophe event, and a greater possibility of principal damage.

Table 8. Effect of dependence ρ ($T = 5$).

Price	ρ =0.2818	ρ =0.3818	ρ =0.4818	ρ =0.5818	ρ =0.6818
Cumulative discounted value of coupon	10.7437	11.4073	12.0641	12.7116	13.3477
Discounted value of principal	66.2205	62.3683	58.9730	55.9689	53.3018
CAT bond price	76.9643	73.7757	71.0370	68.6805	66.6495

4.3. Effects of interest rate and coupon rate

Both the market interest rate and the coupon rate are important factors that directly affect the pricing of CAT bonds. According to the structure of the CIR model, the market interest rate is further divided into the initial interest rate and the long-run mean of the interest rate. As shown in Tables 9 and 10, CAT bond prices are negatively correlated with both r and θ, which is consistent with bond pricing theory. The adjustment of r has a greater impact on the pricing of CAT bonds, while the change of θ has little impact in the short term. It can be ascertained from Table 11 that there is a positive correlation between CAT bond price and R. Raising the coupon rate can increase the cash flow, and the CAT bond price will increase accordingly. Similarly, the adjustment of R has a greater impact on the price of multi-year CAT bonds, which is related to the pricing structure of CAT bonds. According to Table 5, the longer the maturity period, the larger the proportion of the cumulative discounted value of the coupon in the bond price, so the change of R will have a greater impact on the bond price.

Table 9. Sensitivity of r to CAT bond price.

Adjustment ratio for r	$T = 1$	$T = 2$	$T = 3$	$T = 4$	$T = 5$
+20%	−0.72%	−1.28%	−1.71%	−2.05%	−2.31%
+10%	−0.36%	−0.64%	−0.86%	−1.03%	−1.16%
−10%	+0.36%	+0.64%	+0.87%	+1.04%	+1.18%
−20%	+0.73%	+1.29%	+1.74%	+2.09%	+2.37%

Table 10. Sensitivity of θ to CAT bond price.

Adjustment ratio for θ	$T = 1$	$T = 2$	$T = 3$	$T = 4$	$T = 5$
+20%	−0.09%	−0.34%	−0.70%	−1.14%	−1.65%
+10%	−0.05%	−0.17%	−0.35%	−0.57%	−0.83%
−10%	+0.06%	+0.17%	+0.35%	+0.57%	+0.83%
−20%	+0.09%	+0.34%	+0.70%	+1.15%	+1.67%

Table 11. Sensitivity of R to CAT bond price.

Adjustment ratio for R	$T = 1$	$T = 2$	$T = 3$	$T = 4$	$T = 5$
+20%	+1.01%	+1.77%	+2.34%	+2.77%	+3.09%
+10%	+0.51%	+0.88%	+1.17%	+1.38%	+1.55%
−10%	−0.51%	−0.88%	−1.17%	−1.38%	−1.55%
−20%	−1.01%	−1.77%	−2.34%	−2.77%	−3.09%

5. Conclusions

We developed a hybrid-triggered CAT bond pricing model and derived the pricing formula based on the copula-EVT model. We took earthquake CAT bonds as an example for model construction and numerical analysis. Taking economic loss and magnitude as dual trigger indicators, the advantages of the indemnity trigger and index trigger have been combined to increase the interest of investors with low-risk preference. The marginal distribution of the two trigger indicators were fitted to a POT model, and the joint distribution was established by using a copula function. Furthermore, we illustrated an application to the earthquake data for China from 1990 to 2020 and analyzed the sensitivity of CAT bond prices to model parameters.

The numerical experiments showed that the maturity, trigger level, dependence of trigger indicators, market interest rate and coupon rate have important implications for the CAT bond pricing model. First, the extension of the maturity period increases the probability of catastrophic events that meet the trigger conditions, and the CAT bond price decreases. In the development of multi-year CAT bond products, we can increase the yield of investors by raising the coupon rate or strengthening the trigger conditions so that CAT bonds will be more attractive in the investment market. Second, the stronger the correlation between the two trigger indicators, the more likely they will be triggered at the same time during a catastrophe event, which will weaken the advantages of the hybrid trigger mechanism. The Kendall rank correlation coefficient between economic loss and magnitude was 0.3818. It is appropriate to choose them as trigger indicators. Third, adjusting the trigger values of the two trigger indicators will change the price of CAT bonds in the opposite direction. Under the same adjustment ratio, a change in the magnitude trigger level has a greater impact on the price of CAT bonds. It is essential to select a reasonable attachment point of magnitude for the pricing of CAT bonds. Furthermore, the CAT bond price is positively correlated with the coupon rate and negatively correlated with the market interest rate. In the pricing of multi-year CAT bonds, the impact of a forward rate of interest on pricing should be fully considered.

There are several possible extensions and potential improvements in this direction. Firstly, the hybrid trigger indicator applied in our model is composed of an indemnity trigger indicator and a parametric trigger indicator. In the follow-up study, one can consider the application of other index triggers, such as the industry loss index and modeled loss index, in the construction of a hybrid-triggered CAT bond pricing model. Secondly, the periodicity and trend of the frequency of catastrophic disasters can be taken into consideration in our research. It is more realistic to apply the stochastic process with uncertain intensity to describe the frequency of catastrophic events. Thirdly, in addition to earthquakes, other disasters such as floods, typhoons and pandemics can also cause substantial economic losses. The characteristics of various disasters are different, and the CAT bonds with different catastrophe risks as the guarantee object have differences in the selection of trigger indicators and trigger thresholds, which will be the focus of future research.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

References

Acero FJ, Parey S, Garcia JA, et al. (2018) Return level estimation of extreme rainfall over the Iberian Peninsula: Comparison of methods. *Water* 10: 179. https://doi.org/10.3390/w10020179

Balkema AA, Haan L (1974) Residual lifetime at great age. *Ann Probab* 2: 792–804. https://www.jstor.org/stable/2959306

Bokusheva R (2014) Improving the effectiveness of weather-based insurance: An application of copula approach. MPRA Paper 62339, University Library of Munich, Germany. https://mpra.ub.uni-muenchen.de/62339/

Bouriaux S, MacMinn R (2009) Securitization of catastrophe risk: New developments in insurance-linked securities and derivatives. *J Insur Iss* 32: 1–34. http://www.jstor.org/stable/41946289

Braun A (2011) Pricing catastrophe swaps: A contingent claims approach. *Insur Math Econ* 49: 520–536. https://doi.org/10.1016/j.insmatheco.2011.08.003

Cai Y, Cai J, Xu L, et al. (2019) Integrated risk analysis of water-energy nexus systems based on systems dynamics, orthogonal design and copula analysis. *Renew Sust Energ Rev* 99: 125–137. https://doi.org/10.1016/j.rser.2018.10.001

Chao W (2021) Valuing multirisk catastrophe reinsurance based on the Cox-Ingersoll-Ross (CIR) model. *Discrete Dyn Nat Soc* 2021: 8818486. https://doi.org/10.1155/2021/8818486

Chao W, Zou HW (2018) Multiple-event catastrophe bond pricing based on CIR-Copula-POT model. *Discrete Dyn Nat Soc* 2018: 5068480. https://doi.org/10.1155/2018/5068480

Chebbi A, Hedhli A (2020) Revisiting the accuracy of standard VaR methods for risk assessment: Using the copula-EVT multidimensional approach for stock markets in the MENA region. *Q Rev Econ Financ*. https://doi.org/10.1016/j.qref.2020.09.005

Chen JF, Liu GY, Yang L, et al. (2013) Pricing and simulation for extreme flood catastrophe bonds. *Water Resour Manag* 27: 3713–3725. https://doi.org/10.1007/s11269-013-0376-2

Chukwudum QC, Mwita P, Mung'atu JK (2020) Optimal threshold determination based on the mean excess plot. *Commun Stat-Theor M* 49: 5948–5963. https://doi.org/10.1080/03610926.2019.1624772

Cox SH, Pedersen HW (2000) Catastrophe risk bonds. *N Am Actuar J* 4: 56–82. https://dx.doi.org/10.1080/10920277.2000.10595938

Cox JC, Ingersoll JE, Ross SA (1985) A theory of the term structure of interest rates. *Econometrica* 53: 385–407. https://doi.org/10.2307/1911242

Cummins JD, Weiss MA (2009) Convergence of insurance and financial markets: Hybrid and securitized risk-transfer solutions. *J Risk Insur* 76: 493–545. https://doi.org/10.1111/j.1539-6975.2009.01311.x

Deng GQ, Liu SQ, Deng CS (2020) Research on the pricing of global drought catastrophe bonds. *Math Probl Eng* 2020: 3898191. https://doi.org/10.1155/2020/3898191

Frees EW, Valdez EA (1998) Understanding relationships using copulas. *N Am Actuar J* 2: 1–25. https://doi.org/10.1080/10920277.1998.10595667

Gu YK, Fan CJ, Liang LQ, et al. (2019) Reliability calculation method based on the copula function for mechanical systems with dependent failure. *Ann Oper Res* 311: 99–116. https://doi.org/10.1007/s10479-019-03202-5

Kurniawan H, Putri ER, Imron C, et al. (2021) Monte Carlo method to valuate CAT bonds of flood in Surabaya under jump diffusion process. *J Phys Conf Ser* 1821: 012026. http://dx.doi.org/10.1088/1742-6596/1821/1/012026

Lee JP, Yu MT (2002) Pricing default-risky CAT bonds with moral hazard and basis risk. *J Risk Insur* 69: 25–44. https://doi.org/10.1111/1539-6975.00003

Lee JP, Yu MT (2007) Valuation of catastrophe reinsurance with catastrophe bonds. *Insur Math Econ* 41: 264–278. https://doi.org/10.1016/j.insmatheco.2006.11.003

Litzenberger RH, Beaglehole DR, Reynolds CE (1996) Assessing catastrophe reinsurance-linked securities as a new asset class. *J Portfolio Manage* 23: 76–86. https://doi.org/10.3905/jpm.1996.076

Liu XH, Meng SW, Li ZX (2019) Copula-mixed distribution model and its application in modeling earthquake loss in China. *Syst Eng Theor Pract* 39: 1855–1866. https://doi.org/10.12011/1000-6788-2017-2116-12

Lo CL, Lee JP, Yu MT (2013) Valuation of insurers' contingent capital with counterparty risk and price endogeneity. *J Bank Financ* 37: 5025–5035. https://doi.org/10.1016/j.jbankfin.2013.09.007.

Ma N, Bai YB, Meng SW (2021) Return period evaluation of the largest possible earthquake magnitudes in mainland China based on extreme value theory. *Sensors* 21: 3519. https://doi.org/10.3390/s21103519

Ma ZG, Ma CQ (2013) Pricing catastrophe risk bonds: A mixed approximation method. *Insur Math Econ* 52: 243–254. https://doi.org/10.1016/j.insmatheco.2012.12.007

Ma ZG, Ma CQ, Xiao SS (2017) Pricing zero-coupon catastrophe bonds using EVT with doubly stochastic poisson arrivals. *Discrete Dyn Nat Soc* 2017: 3279647. https://doi.org/10.1155/2017/3279647

McNeil AJ, Frey R (2000) Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. *J Empir Financ* 7: 271–300. https://doi.org/10.1016/S0927-5398(00)00012-8

Merton RC (1976) Option prices when underlying stock returns are discontinuous. *J Financ Econ* 3: 125–144. https://doi.org/10.1016/0304-405X(76)90022-2

Mousavi M, Akkar S, Erdik M (2019) A candidate proxy to be used in intensity-based triggering mechanism for parametric CAT-bond insurance: Istanbul case study. *Earthq Spectra* 35: 565–588. https://doi.org/10.1193/081018EQS201M

Nowak P, Romaniuk M (2013) Pricing and simulations of catastrophe bonds. *Insur Math Econ* 52: 18–28. https://doi.org/10.1016/j.insmatheco.2012.10.006

Pickands J (1975) Statistical inference using extreme order statistics. *Ann Stat* 3: 119–131. https://www.jstor.org/stable/2958083

Reshetar G (2008) Pricing of multiple-event coupon paying CAT bond. *SSRN Electron J.* http://dx.doi.org/10.2139/ssrn.1059021

Romaniuk M (2017) Analysis of the insurance portfolio with an embedded catastrophe bond in a case of uncertain parameter of the insurer's share. *Adv Intel Syst Comput* 524: 33–43. http://dx.doi.org/10.1007/978-3-319-46592-0_3

Shao J, Pantelous A, Papaioannou AD (2015) Catastrophe risk bonds with applications to earthquakes. *Eur Actuar J* 5: 113–138. https://doi.org/10.1007/s13385-015-0104-9

Shen L, Zhang Y, Zhuang X, et al. (2018) Reliability modeling for gear door lock system with dependent failures based on copula. *ASME J Risk Uncertainty Part B* 4: 041003. https://doi.org/10.1115/1.4039941

Sklar M (1959) Fonctions de repartition an dimensions et leurs marges. *Publ Inst Statist Univ Paris* 8: 229–231.

Smack L (2016) Catastrophe bonds-Regulating a growing asset class. *Risk Manage Insur Rev* 19: 105–125. https://doi.org/10.1111/rmir.12057

Swiss Re (2020) Natural catastrophes in 2020: Secondary perils in the spotlight, but don't forget primary-peril risks. Sigma 1/2021, Zurich, Switzerland. Available from: `https://www.swissre.com/institute/research/sigma-research/sigma-2021-01.html`

Tao Z (2011) Zero-beta characteristic of CAT bonds. 2011 Fourth International Conference on Business Intelligence and Financial Engineering, 641–644. https://doi.ieeecomputersociety.org/10.1109/BIFE.2011.159

Woo G (2004) A catastrophe bond niche: Multiple event risk. Working Paper, NBER Insurance Group Work-Shop, Cambridge, UK. Available from: `https://conference.nber.org/confer/2004/insw04/woo.pdf`

Xu LY, Wang HM, Chen JF (2013) Research of drought disaster risk assessment based on copula-EVT model. *Appl Stat Manage* 32: 284–294.

Yao CZ, Sun BY, Lin JN (2017) A study of correlation between investor sentiment and stock market based on copula model. *Kybernetes* 46: 550-571. https://doi.org/10.1108/K-10-2016-0297

Zhang XL, Tsai CCL (2018) The optimal write-down coefficients in a percentage for a catastrophe bond. *N Am Actuar J* 22: 1–21. https://doi.org/10.1080/10920277.2017.1283236

Zimbidis AA, Frangos NE, Pantelous AA (2007) Modeling earthquake risk via extreme value theory and pricing the respective catastrophe bonds. *Astin Bull* 37: 163–183. https://doi.org/10.1017/S0515036100014793

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