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Research article

Modeling exchange rate volatility: application of GARCH models with a Normal Tempered Stable distribution

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Abstract: The aim of this paper is to examine exchange rate volatility using GARCH models with a new innovation distribution, the Normal Tempered Stable. We estimated daily exchange rate volatility using different distributions (Normal, Student, NIG) in order to specify the performed model. In addition, a forecasting analysis is performed to check which distribution reveals the best out-of-sample results. We found that the estimated parameters of GARCH-NTS model outperform the GARCH-N and GARCH-t ones for all currencies. Besides, we asserted that GARCH-NTS and EGARCH-NTS are the preferred models in terms of out-of sample forecasting accuracy. Our results indicating the performance of GARCH models with NTS distribution contribute to increase the accuracy of risk measures which is very important for international traders and investors.

Keywords: GARCH; Normal Tempered Stable distribution; exchange rate volatility

JEL Codes: C52, C580, E44, E47

1. Introduction

Exchange rate volatility stands for a key factor that influences the portfolio diversification, risk management and option pricing. Therefore, forecasting financial volatility is an important issue in the empirical analysis.

Several volatility models have been developed to predict the exchange rate volatility. Over the past few decades, the Generalized Auto-Regressive Conditional Heteroscedastic (GARCH) model has become the widely used one to asses this volatility.

Introduced by Bollerslev (1986), the GARCH model was designed to capture many properties of financial time series such as the volatility persistence and the heteroscedasticity. However, the main drawback of this model is the normality assumption of its residual terms. Empirical studies revealed that financial time series are characterized by a fat-tailed distribution. As a matter of fact, they rejected the hypothesis of Normal innovations (Baillie et al. (2002), Danielsson et al. (2000), Ho et al. (2013) and Podobnik et al (2009)).

To avoid the underestimation or the overestimation of conditional volatility, it is important to clearly identify the appropriate distribution of innovations. Therefore, the empirical literature suggested alternative distributions as solutions such as the Student's t distribution and the General Error Distribution (GED) (Abounoori et al. (2016) and Zhu and Galbraith (2011)). However, multiple studies assume that the Student's t and the General Error Distribution (GED) distributions display some drawbacks due to their lack of stability under aggregation. These drawbacks may influence the financial forecasting's accuracy which is crucial for risk analysis (Kim (2010), Calzolari et al. (2014)).

Recently, some studies have focused on the application of the GARCH model with a Tempered Stable distribution. The results disclosed that this distribution can capture the asymmetric distribution of financial time series and provide a defined moment unlike the Student's t-distribution. Kim (2010) found that using GARCH model with Tempered Stable distribution (GARCH-TS) model in estimating volatility of stocks, market index and option prices provides better results compared to the Normal-GARCH model (GARCH-N). In addition, Feng et al. (2017) applied the FIGARCH model with a Tempered Stable distribution to model the S&P500 return series. They provided empirical evidence that the (FIGARCH-TS) model outperforms with the Gaussian and the commonly used fat-tailed distributions: Student's t and GED. Moreover, Shi (2012) investigated the MRS-GARCH model with a Tempered Stable distribution to estimate daily S&P500 return series. Theirs empirical results highlighted that the Tempered Stable distribution outperforms the Student's t and the GED distribution. Therefore, they assumed that this distribution could be a widely applied tool in modeling the financial volatility in general contexts with a MRS-GARCH-type of specification. Thus, it is obvious that the Tempered Stable distribution contributes to ameliorate the GARCH models performance when modeling financial volatility series. The aim of this paper is to estimate and forecast exchange rates volatility using GARCH and EGARCH models with a Normal Tempered Stable distribution.

The innovation of this paper is the application of GARCH models with the Normal Tempered Stable, new innovation distribution, to estimate and forecast daily exchange rate volatility. The attractive statistics properties of this new distribution provide more estimation accuracy which allow to capture the fat-tailed properties.

To the best of our knowledge, it is the first study which applied this kind of distribution to overcome the problem of fat-tailed residuals of exchange rates. Our motivation of introducing NTS distribution with GARCH models lies in the fact that it retains the most attractive properties of the TS distribution and also it has a defined moment. In addition, the importance of applying the NTS distribution refers to the fact that its variance is random which allows more flexibility to the statistical study of data compared to the Normal distribution which has a fixed variance. All these features allow providing more forecasting accuracy.

To empirically check the performance of GARCH-NTS model, daily exchange rate data are used over the period 1999–2019. For this reason, we first report the density plot of different residuals distributions and we compare it to the real data. Secondly, we estimate GARCH and EGARCH models with different distributions (Normal, Student's t-distribution and NTS) to specify the best model. Finally, we display a forecasting analysis in order to check which distribution reveals the best out-of-sample results.

The paper is organized as follows. Models and Data analysis methods are explained in Section 2, followed by Results and Discussions in Section 3. The paper is closed with Conclusion in Section 4.

2. Materials and methods

2.1. The GARCH model

In 1986, Bollerslev (1986) proposed the GARCH model as a generalized form of the ARCH model developed by Engle (1982). The GARCH model has proved its ability to accurately capture some stylized features of exchange rate volatility, such as the flatter tails, the time varying heteroscedasticity and the volatility clustering. The GARCH(1,1) model is defined by the following equation:

$$r_t = \mu + \epsilon_t = \mu + \sigma_t \eta_t, \text{ with } \eta_t \sim N(0,1) \tag{1}$$

$$\sigma_t{}^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

where r_t is the series of the exchange rate returns, ϵ_t is the residual series, σ_t^2 is the conditional variance of returns and innovations and η_t is the identically independent innovation sequence with zero mean and unit variance. Therefore, the size of parameters α and β should satisfy the stationarity and the positive variance constraints: $\alpha + \beta < 1$ and ω, α and $\beta \geq 0$ which indicate the persistence of shocks to the exchange rate volatility.

2.2. The EGARCH model

The main weakness of the traditional GARCH model is its inability to detect the presence of a leverage effect in the financial time series which is regarded as the most important stylized facts of volatility. The pioneer work of Black (1976) provided a description of this leverage effect phenomenon in terms of asymmetric relationship between returns and volatility. As a matter of fact, it was revealed that future volatilities are mainly influenced by negative rather than positive returns. Thus, in order to overcome this problem, many empirical studies have proposed alternative GARCH models like EGARCH, TGARCH, and GJR-GARCH to capture the leverage effect (Cont (2001) and Rodriguez et al. (2012)).

The exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model was proposed by Nelson (1991) to account for an asymmetric response to shocks. This model is expressed as follows:

$$r_t = \mu + \epsilon_t \tag{3}$$

$$\ln(\sigma_t{}^2) = \omega + \alpha \left\{ \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta \ln(\sigma_{t-1}^2) \tag{4}$$

where ω is a constant term, α defines the size parameter which measures the magnitude of shocks, β represents the conditional volatility persistence and γ stands for the asymmetry coefficient which indicates the presence of the leverage effect. In fact, when γ is negative, this indicates that negative return shocks generate more volatility than those generated by positive ones Besides, parameters ω, α, γ and β may take negative values without affecting the positive variance assumption because σ_t^2 takes the logarithmic form.

2.3. *Distribution assumptions of residuals*

The original innovation distribution of the GARCH models is assumed to be a Normal one. However, it is well known that empirical distribution of financial returns is leptokurtic, fat-tailed and rarely Normal. Therefore, to better accommodate the fat-tailed feature of distribution behavior, many researchers have considered alternative distributions like Student's t and GED. From this perspective, our empirical contribution consists in introducing a new alternative residual's distribution which is the Normal Tempered Stable one in order to better fit the tails.

2.3.1. Student's t distribution

The corresponding density of Student's t distribution applied with the GARCH models, proposed by Bollerslev (1986), is indicated as follows:

$$f(\eta_t;\nu) = \frac{\Gamma((\nu+1)/2))}{\Gamma(\nu/2)\sqrt{\pi(\nu-2)}}\left(1+\frac{\eta_t^2}{\nu-2}\right)^{-(\nu+1)/2)} \tag{5}$$

where $\Gamma(\nu) = \int \exp^{-x} x^{\nu-1} dx$ is the gamma function, $\eta_t = \dfrac{\epsilon_t}{\sigma_t}$.

2.3.2. General Error Distribution (GED)

Hamilton (1994) introduced GARCH model with the GED distribution the density of which is provided by:

$$f(\eta_t;\nu) = \frac{\nu \exp^{\frac{-1}{2})|\eta_t|^{\nu}}}{\lambda 2^{(\nu+1/\nu)}\Gamma(1/\nu)} \tag{6}$$

where $\lambda = \left(\frac{2^{-2/\nu}\Gamma(1/\nu)}{\Gamma(3/\nu)}\right)^{1/2}$.

2.3.3. Normal Tempered Stable distribution

The normal tempered stable distribution represents a normal variance-mean mixture model with tempered stable random variance. It covers many distributions that widely used in stochastic volatility modeling such as the normal inverse Gaussian and the variance gamma ones. In this framework, we may cite the research works of Barndorff-Nielsen (1997), Barndorff-Nielsen and Levendorskii (2001), Barndorff-Nielsen and Schmiegel (2008), Fiorani et al. (2010), Hirsa and Madan (2003), Madan et al. (1998) and Madan and Seneta (1990). This kind of distributions has curves with the same shape as the normal one but with semi-heavy tails (Barndorff and Shephard (2001)). In terms of processes, it represents a Brownian motion subordinated by the tempered stable subordinator and fit better for non-Gaussian data. More exactly, Marinelli et al. (2001) have examined this kind of problems and explained the use of subordinated Brownian motion by the tempered stable subordinator in financial modeling by the following way: the subordinated models the market activity, which changes over time, and the Brownian motion represents the exchange rate process in market time. In fact, the Normal Tempered Stable distribution was defined by Barndorff and Shephard (2001) as a mixture between a Normal distribution and a class of Tempered Stable Mixing. It is well known that the tempered stable distribution has a complicated probability density function which represents infinite series (Nolan (2007)). Then, the normal tempered stable distribution has a density with complicated structure that constitutes an in integral of Gaussian kernel with a tempered stable mixing (Barndorff and Shephard (2001)). This complexity of the probability density function has an impact on the likelihood function which used in the estimation of AIC and BIC criteria. It is important to notice that the maximum likelihood estimation of the normal tempered stable distribution is lacking in the literature. For this reason, we have just used the mean squared error as a comparison criterion in this paper. The tempered stable family represents an exponentially titled version of the stable one with parameter $\nu \in (0, 1)$ (Letac et al (2005)). Therefore, it is characterized by its Laplace transform which is given by the following expression:

$$L(\theta) = \exp(1 - (1 - \delta\theta)^{\nu}), \text{ for all } \delta > 0 \text{ and } \theta < 1/\delta \tag{7}$$

This implies that a NTS random variable X has the following Laplace transform:

$$L_X(\theta) = \exp\left(1 - \left(1 - \delta\frac{\theta^2}{2}\right)^{\nu}\right), \text{ for all } -\sqrt{2/\delta} < \theta < \sqrt{2/\delta} \tag{8}$$

Departing from this transformation, we infer that $\mathbf{E}(X) = \mathbf{E}(X^3) = 0$. Hence, we deduce that the Skewness is equal to 0. Moreover, we obtain:

$$\mathbf{E}(X^2) = \nu\delta \quad \text{and} \quad \mathbf{E}(X^4) = 3\nu\delta^2 \tag{9}$$

As a result, the kurtosis of the NTS distribution, with $\nu \in (0, 1)$, is equal to

$$\frac{\mathbf{E}(X^4)}{\mathbf{E}(X^2)^2} = \frac{3}{\nu} > 3 \tag{10}$$

It follows that the NTS distribution is Leptokurtic. Let's denote by $\overline{x}$ the empirical mean, S_2 the variance and S_4 the centered moment with an order 4:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \; S_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \text{ and } S_4 = \frac{1}{n}\sum_{i=1}^{n} x_i^4 \tag{11}$$

According to Louati et al. (2020) and using the method of moments, we deduce that the estimators of ν and δ, namely $\widehat{\nu}$ and $\widehat{\delta}$, are given by:

$$\widehat{\nu} = \frac{3S_2^2}{S_4} \quad \text{and} \quad \widehat{\delta} = \frac{S_4}{3S_2} \tag{12}$$

It is important to notice that, due to the complexity of the likelihood function, we use the method of moments for the estimation of the parameters.

2.4. *The GARCH and EGARCH model with a Normal Tempered Stable*

After identifying the NTS, we represent the NTS-GARCH as follows:

$$r_t = \mu + \epsilon_t = \mu + \sigma_t\eta_t, \text{ with } \eta_t \sim NTS(\nu, \delta) \tag{13}$$

$$\sigma_t{}^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{14}$$

Hence, the NTS-EGARCH model is expressed as:

$$r_t = \mu + \epsilon_t = \mu + \sigma_t \eta_t, \text{ with } \eta_t \sim NTS(\nu, \delta) \tag{15}$$

$$\ln({\sigma_t}^2) = \omega + \alpha_1 \left\{ \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \sigma_{t-1}^2 \tag{16}$$

2.5. Data

In this empirical study, we use daily exchange rate returns of the major currencies against the US dollar: the Euro (EUR/USD), the British pound sterling (GBP/USD), the Canadian dollar (CAD/USD), and the Tunisian dinar (TND/USD). The sample spans over the period 1999–2019. This period is divided into two sub-samples: The first sub-sample, the in-sample period, spans from 1 January 2016 to 31 December 2016. The second sub-sample, the out-of-sample period, spans from 1 January 2017 to 31 December 2019. We split the sample data in two sub-samples in order to evaluate the performance of GARCH models across different currencies. Thus, the daily exchange rate returns are calculated in terms of:

$$r_t = 100 \times (log(P_t/P_{t-1})) \tag{17}$$

where r_t denotes the daily exchange rate return at time t, P_t represents the nominal exchange rate at time t and P_{t-1} corresponds to the nominal exchange rate at time $t-1$.

Table 1. Descriptive statistics of exchange rate returns.

	Mean	Standard Deviation	Skwness	Kurtosis	JB	ARCH(20)
EUR/USD	−0.0005	0.2775	−0.1688	5.1296	845.81*	18.2*
GBP/USD	−0.0013	0.2507	−0.1056	7.2821	3343.88*	50.11*
CAD/USD	0.0014	0.25003	0.0967	8.8364	6203.6*	51.42*
TND/USD	−0.0067	0.2138	0.1319	6.0975	1758*	10.67*

This table reports descriptive statistics of exchange rate returns. The in-sample covers the periods from January 1999 to December 2016. JB is the statistic of the Jarque-Bera normality test. ARCH (20) refers to the ARCH test with twenty lags.

*significance of test statistics at 1% level

Table 1 provides the mean, the standard deviation, the skewness value, the kurtosis value, the Jarque Bera test (a normality test statistics) and the ARCH-LM test (a residual heteroscedasticity test) for each set of exchange rates.

As it can be detected in Table 1, the EUR/USD, CAD/USD, TND/USD are positively skewed, except for the GBP/USD. In addition, all the return series shows excessive kurtosis. Moreover, statistics of the Jarque Bera test are all significant at 1% level which indicates that the distribution of all the exchange rate returns is not Normal.

In addition, we applied the ARCH-LM test to verify the presence of heteroscedasticity of residuals. Statistics of this test are significant and the null hypothesis, which indicates the absence of the ARCH effects, is rejected. Generally, we can conclude according to the findings of Table 1 that all exchange rates returns exhibit fatter tails than normal distributions.

3. Results and discussion

3.1. Analysis of the residual's distribution

In this section, we examine the distribution of the exchange rate returns for different currencies. For this purpose, we first compare the density of real data with the Normal density and then with the NTS density. The results of this comparison are presented in Figure 1.

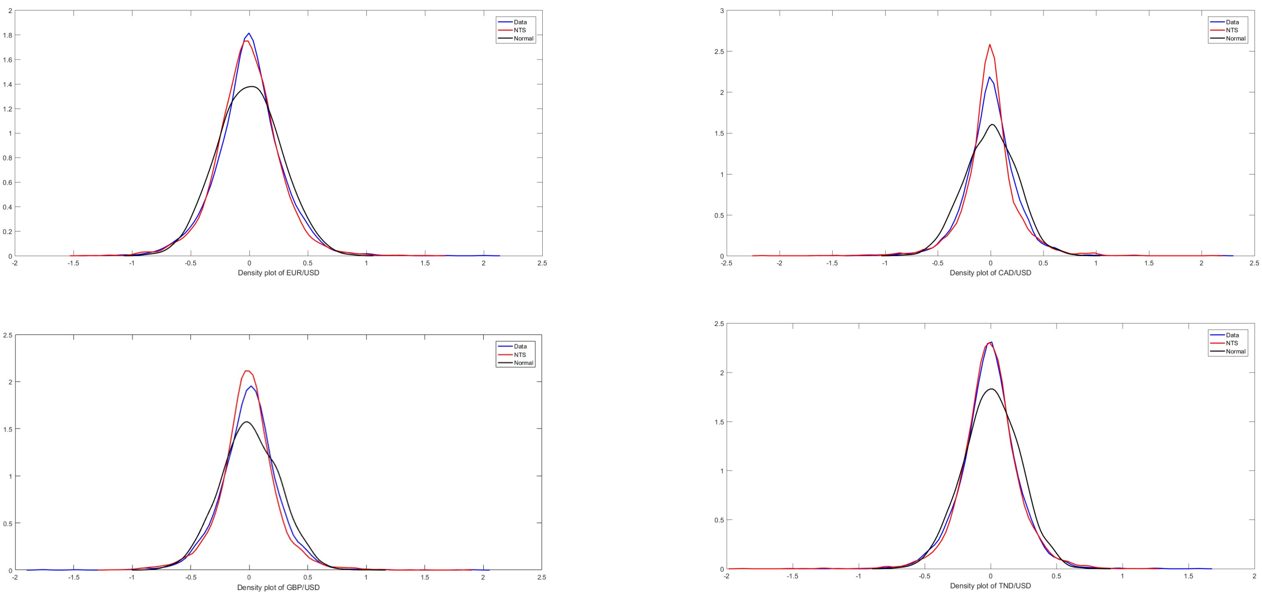

Figure 1. Density plots of exchange rates residuals

As it can be noticed in Figure 1, the NTS density plot is very close to that of the real data for all currencies, particularly for the EUR/USD and the TND/USD. Hence, to better verify the performance of the Normal Tempered Stable distribution with respect to the Normal distribution in detecting the characteristics of real data, we attempt to apply the MSE criterion presented below:

$$MSE = n^{-1} \sum_{i=1}^{n} (r_{t+1}^2 - \epsilon_{t+1})^2 \tag{18}$$

Table 2. MSE of density's distribution.

MSE	EUR/USD	GBP/USD	CAD/USD	TND/USD
Normal	0.1541	0.1274	0.1281	0.0925
Normal tempred stable	0.1529	0.1245	0.1249	0.0894

MSE refers to the Mean Squared Errors criterion applied for the Norman and Normal Tempered Stable distributions of exchange rates returns. The *in − sample* covers the period from January 1999 to December 2016.

According to Table 2, the MSE of the NTS distribution is smaller than the MSE of the Normal distribution for all the studied currencies, which confirms that the NTS distribution is more adequate than the Normal one for the exchange rate returns.

Moreover, to evaluate the relevance of an adjustment of real data to the NTS distribution, we graphically depicts the distributions through quartiles using the boxplot method. The results are illustrated in Figure.2.

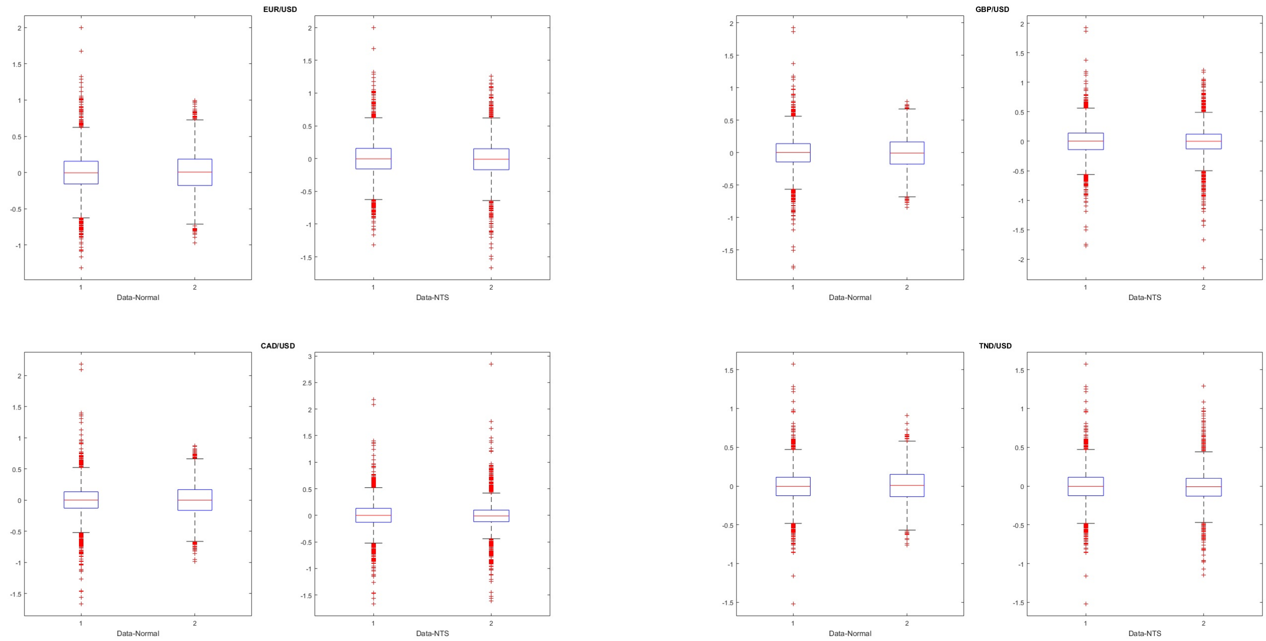

Figure 2. Boxplot of exchange rate distribution.

Figure 2 portrays that the boxplot of real data is comparatively the same with the boxplot of NTS distribution. This suggests that tails of real data are well captured by the NTS distribution. Therefore, it is clear that for all currencies, the NTS distribution density outperforms the Normal distribution density in detecting the real data features.

These findings also confirm conclusions of Abdullah et al. (2017) who underlined that the normal distributions doesn't fit well the exchange rate innovations using ARCH, APARCH, EGARCH, TGARCH and IGARCH models. The authors found that, in contrast to the normal distribution, the application of Student's t-distribution for errors ameliorates the models' estimation and show improved forecasting accuracy. In addition, Atabani Adi (2019) found that exchange rate returns were not normally distributed using GARCH family models (APARCH, GJR-GARCH and I-GARCH).

3.2. Estimating exchange rate volatility

In this section, we estimate the exchange rate volatility using GARCH and EGARCH models with different assumptions: Normal, Student's t-distribution and NTS. The application of the EGARCH model enables us to capture the asymmetric effect of the exchange rate volatility.

The parameters estimation of GARCH and EGARCH models with Normal, Student's t-distribution and NTS for EUR/USD, GBP/USD, CAD/USD and TND/USD currencies are summarized in Tables 3–6, respectively.

It is noteworthy that literature provides a wide set of criteria to select the model with the highest quality of competing volatility (Bollerslev (1986)). Therefore, we used the MSE criterion to select the most convenient model.

Table 3. Estimation of the EUR/USD exchange rate volatility.

	GARCH			EGARCH		
	GARCH-N	GARCH-t	GARCH-NTS	EGARCH-N	EGARCH-t	EGARCH-NTS
μ	0.0023	-0.0013	0.002	0.0018	0.001	0.0012
	(0.0036)	(0.0038)	(0.0037)	(0.0036)	(0.0035)	(0.0036)
ω	0.0002*	0.0023*	0.0029*	−0.1534*	−0.1534*	−0.1574*
	(4.75E-0.5)	(0.0111)		(0.006)	(0.0101)	
α	0.0301*	0.0151	0.0139*	0.0241*	0.0241*	0.023*
	(0.003)	(0.0051)		(0.0063)	(0.0102)	
β	0.9670*	0.9383*	0.9383*	0.9542*	0.9542*	0.9542*
	(0.0031)	(0.1245)		(0.0035)	(0.0035)	
γ				0.0038*	0.0038**	0.0037*
				(0.0009)	(0.0017)	
MSE	0.0826	0.0826	0.0817	0.0802	0.08028	0.0797

Parameter estimation of GARCH and EGARCH modes. A Normal, Student-t, and Normal Tempered Stable distributionsare chosen respectively. The in-sample covers the periods from January 1999 to December 2016. MSE refers to the Mean Squared Errors criterion. Standard deviations are reported in parentheses

*,** and***significance of test statistics at 1%, 5% and 10%level, respectively

According to Table 3, the coefficients of GARCH and EGARCH models for the EUR/USD are all significant and quite close. This implies the presence of volatility clustering in GARCH (1,1) model with different distributions. For GARCH model, the sum of α and β is close to one, which indicates that the volatility shocks are exceptionally persistent. According to MSE criterion value, it can be observed that GARCH-NTS outperforms GARCH-N and GARCH-t.

Regarding the EGARCH model, the estimated asymmetry parameter γ is significant and positive. This implies that good news have a greater effect on the EUR/USD exchange rate volatility than bad news. In addition, it can be observed that EGARCH-NTS outperforms EGARCH-N as well as EGARCH-t.

Comparing GARCH to EGARCH model with different distributions using MSE criterion, we can conclude that EGARCH-NTS performs the best among them.

According to Table 4, the coefficients of GARCH and EGARCH models for GBP/USD are all significant and quite close. In addition, GARCH-NTS model has the lowest MSE criterion value (0.06709) which implies that the GARCH-NTS model outperforms GARCH-N and GARCH-t in estimating the GBP/USD exchange rate volatility.

Table 4. Estimation of the GBP/USD exchange rate volatility.

	GARCH			EGARCH		
	GARCH-N	GARCH-t	GARCH-NTS	EGARCH-N	EGARCH-t	EGARCH-NTS
μ	0.0011	0.0017	0.0016	-0.0014	0.00008	0.00008
	(0.0032)	(0.0032)	(0.0032)	(0.0032)	(0.0032)	(0.0032)
ω	0.0003*	0.0018**	0.0017*	−0.0954*	−0.0954*	−0.0971*
	(9.76E-05)	(0.00013)		(0.0086)	(0.0113)	
α	0.0381*	0.0242*	0.0222*	0.0186*	0.0186*	0.0179*
	(0.0039)	(0.0049)		(0.0076)	(0.0101)	
β	0.9555*	0.942*	0.9420*	0.9744*	0.9744*	0.9744*
	(0.0047)	(0.0057)		(0.0048)	(0.0059)	
γ				−0.0029*	−0.0029**	−0.0028*
				(0.0015)	(0.0019)	
MSE	0.06784	0.0678	0.06709	0.0652	0.0652	0.0649

Parameter estimation of GARCH and EGARCH modes. A Normal, Student-t, and Normal Tempered Stable distributionsare chosen respectively. The in-sample covers the periods from January 1999 to December 2016. MSE refers to the Mean Squared Errors criterion. Standard deviations are reported in parentheses

*,** and*** significance of test statistics at 1%, 5% and 10%level, respectively

Moreover, the estimated leverage effect coefficient γ of EGARCH model is significant and negative. This implies the presence of leverage effect: negative shocks have a greater effect on future volatility than positive shocks.When comparing GARCH to EGARCH model using MSE criterion, it can be inferred that EGARCH-NTS is the best among them. This result is in line with the work of Shi (2012) who applied MRS-GARCH model with tempered stable to estimate SP 500 daily return volatility distribution. Their empirical results showed that MRS-GARCH model with tempered stable consistently outperform Student's t-distribution and GED. Thus, they provided evidence that the tempered stable distribution could be a widely useful tool for modeling the financial volatility in general contexts with GARCH models. Furthermore, Feng et al. (2017) proved that the tempered stable distribution could be a widely useful tool for modeling the high-frequency financial volatility in general contexts with a GARCH models. Moreover, based on information criteria, Tserakh et al. (2019) found that tempered stable GARCH(1,1) models model with $\alpha - stable$ and tempered stable residuals have the good statistical results regarding the GARCH-N model in estimating financial data volatility.

According to Table 5, the parameters estimation of GARCH and EGARCH models for CAD/USD currency are all significant and quite close. In addition, the GARCH-NTS model displays the lowest MSE criterion value (0.065) which implies that the GARCH-NTS model outperforms GARCH-N and GARCH-t in estimating the CAD/USD currency volatility.

Furthermore, the estimated asymmetry parameter γ of EGARCH model is significant and negative. This implies the presence of leverage effect: negative shocks generate more volatility than positive ones. When comparing GARCH to EGARCH model using MSE criterion, we can deduce that EGARCH-NTS provides the best estimated volatility among them.

Table 5. Estimation of the CAD/USD exchange rate volatility.

	GARCH			EGARCH		
	GARCH-N	GARCH-t	GARCH-NTS	EGARCH-N	EGARCH-t	EGARCH-NTS
μ	0.0012	0.001	0.001	−0.0008	$-2E$	$-0.3E9$
	(0.0029)	(0.0028)	(0.0028)	(0.0028)	(0.0028)	(0.0028)
ω	0.0002^{*}	0.00187^{***}	0.0017^{*}	-0.1098^{*}	-0.1098^{*}	-0.1121^{*}
	(6.77E-0.5)	(8.47E-05)		(0.0116)	(0.011)	
α	0.0439^{*}	0.0199^{*}	0.0182^{*}	0.0201^{*}	0.02^{*}	0.0193^{*}
	(0.0041)	(0.00527)		(0.011)	(0.011)	
β	0.9521^{*}	0.946^{*}	0.946^{*}	0.913^{*}	70.9713^{**}	0.9713^{*}
	(0.0046)	(0.0057)		(0.0063)	(0.006)	
γ				-0.0007^{*}	-0.0007^{**}	-0.0006^{*}
				(0.0018)	(0.001)	
MSE	0.0665	0.0665	0.065	0.0641	0.064	0.063

Parameter estimation of GARCH and EGARCH modes. A Normal, Student-t, and Normal Tempered Stable distributionsare chosen respectively. The in-sample covers the periods from January 1999 to December 2016. MSE refers to the Mean Squared Errors criterion. Standard deviations are reported in parentheses

*,** and***significance of test statistics at 1%, 5% and 10%level, respectively

Table 6. GARCH estimation of TND/USD volatilityty.

	GARCH			EGARCH		
	GARCH-N	GARCH-t	GARCH-NTS	EGARCH-N	EGARCH-t	EGARCH-NTS
μ	-0.0044^{***}	-0.0047^{***}	-0.0044^{***}	−0.0044	-0.0046^{***}	−0.0044
	(0.0026)	(0.0025)	(0.0026	(0.0027)	(0.0025)	(0.0026)
ω	0.0001^{*}	0.0035^{**}	0.003^{*}	-0.1320^{*}	-0.132^{*}	-0.1344^{*}
	(4.71E-0.5)	(7.69E-0.5)		(0.006)	(0.0119)	
α	0.02886^{*}	0.003^{*}	0.0027^{*}	0.0152^{*}	0.0152^{*}	0.0147^{*}
	(0.0025)	(0.0043)		(0.0061)	(0.0104)	
β	0.9675^{*}	0.9090^{*}	0.9090^{*}	0.9648^{*}	0.9648^{*}	0.9648^{*}
	(0.0026)	(0.0048)		(0.0031)	(0.0052)	
γ				-0.0009	−0.0009	−0.00008
				(0.0011)	(0.0021)	
MSE	0.0478	0.04787	0.0471	0.0473	0.0473	0.045721

Parameter estimation of GARCH and EGARCH modes. A Normal, Student-t, and Normal Tempered Stable distributionsare chosen respectively. The in-sample covers the periods from January 1999 to December 2016. MSE refers to the Mean Squared Errors criterion. Standard deviations are reported in parentheses

*,** and***significance of test statistics at 1%, 5% and 10%level, respectively

According to Table 6, the coefficients of GARCH and EGARCH models for TND/USD currency are all significant and quite close. The MSE criterion values indicate that the GARCH-NTS model outperforms GARCH-N and GARCH-t in estimating the TND/USD exchange rate volatility.

For the EGARCH model, the estimated asymmetry parameter γ is negative but insignificant, which implies the possible absence of an asymmetry effect on volatility. Using the MSE criterion, it is evident that GARCH-NTS provides the best performed estimations compared with the EGARCH model with different distributions.

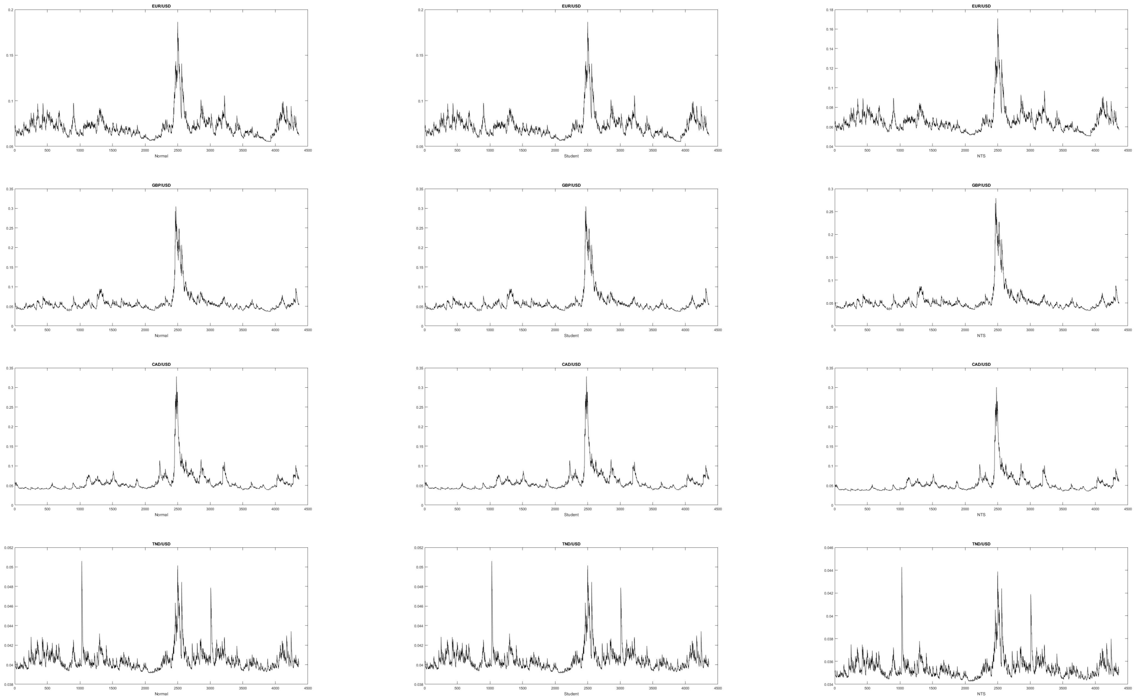

Figure 3. Estimation of the GARCH model with different distributions.

Finally, we report the pattern of the estimated conditional volatility of GARCH and EGARCH models with different distributions for different studied currencies in Figures 3–4 respectively. At the first sight, we detect that the estimated conditional volatility by the GARCH model has the same shape with different distributions for all currencies. This is the same conclusion for theEGARCH model (see Figure 4). Besides, the exchange rate volatility trend increased during 2008-2010, which coincided with the turbulent period of 2008 financial crisis.

3.3. Exchange rate volatility forecasting

Results of volatility estimation using GARCH and EGARCH models with NTS distribution match the exchange rate volatility properly. However, investors and practitioners are more interested in out-of-sample forecasting than in-sample estimation. Therefore, the purpose of this section is to evaluate the out-of-sample forecasts of competing models. According to Lopez (2001),various criteria can be used to select the most accurate model. To that purpose, we applied the Mean Squared Error (MSE) and the Mean absolute error (MAE).

The loss function MSE is defined by:

$$MSE = n^{-1} \sum_{t=1}^{n} (\hat{\sigma}_{t+1}^2 - \hat{h}_{t+1\backslash t})^2 \tag{19}$$

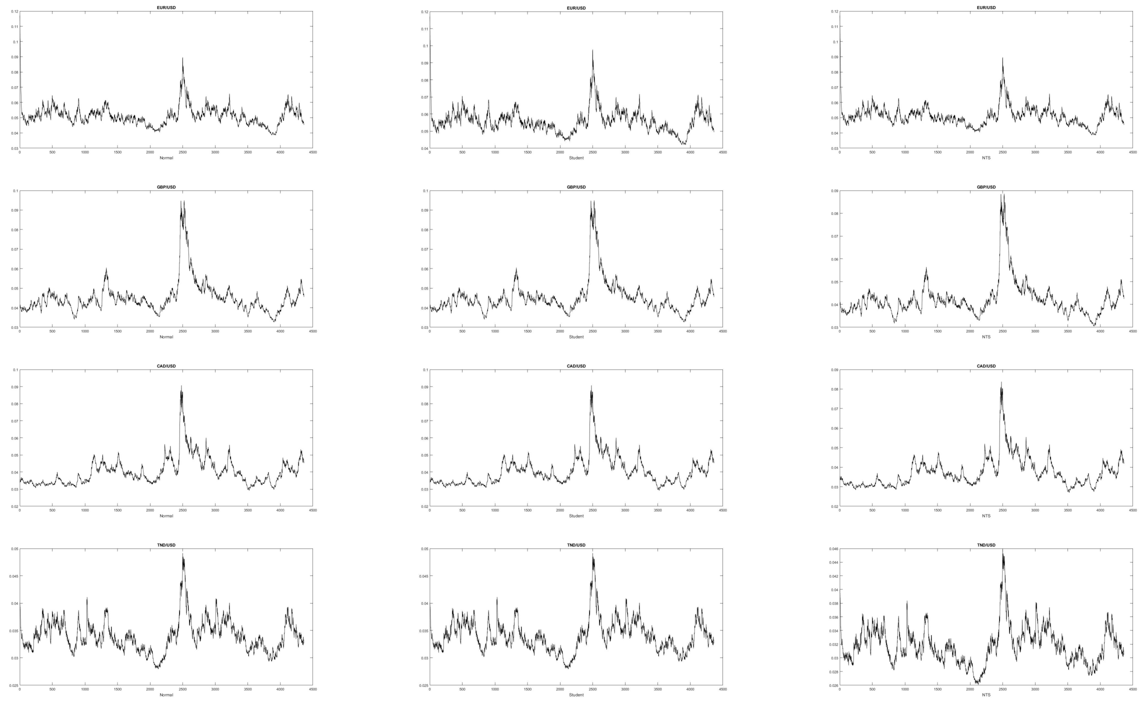

Figure 4. Estimation of the EGARCH model with different distributions.

Then, the Mean absolute error (MAE) is defined by:

$$MAE = n^{-1} \sum_{t=1}^{n} |\hat{\sigma}^2_{t+1} - \hat{h}_{t+1\backslash t}| \tag{20}$$

where $\hat{\sigma}^2_{t+1}$ represents the computed volatility over the in-sample period (from 1 January 1999 to 31 December2016), $\hat{h}_{t+1\backslash t}$ stands for the forecasted volatility over the out-of-sample period (from 1 January 2017 to 31 December 2019). The 1-day, 5-day and 10-day step ahead are applied for forecasting horizons.

Table 7. Out-of-sample forecasting of exchange rate volatility.

		1-Day horzion		5-Day horzion		10-Day horzion	
		MSE	MAE	MSE	MAE	MSE	MAE
GBP/USD	GARCH-N	0.0013	0.0205	0.0029	0.0432	0.0027	0.0381
	GARCH-t	0.0013	0.0205	0.012	0.0432	0.0027	0.0384
	GARCH-NTS	0.0012	0.0155	0.0025	0.0382	0.084	0.043
	EGARCH-N	0.1640	0.4047	0.0341	0.0993	0.0206	0.0761
	EGARCH-t	0.1640	0.4047	0.0341	0.0993	0.0206	0.0761
	EGARCH-NTS	0.1666	0.4079	0.0346	0.0999	0.02	0.0761
CAD/USD	GARCH-N	0.1441	0.3793	0.0312	0.1083	0.0182	0.0831
	GARCH-t	0.1441	0.3793	0.0313	0.1083	0.0183	0.0831
	GARCH-NTS	0.1488	0.3854	0.0318	0.1046	0.0184	0.0806
	EGARCH-N	0.1633	0.4041	0.0332	0.0994	0.0197	0.0761
	EGARCH-t	0.1643	0.4041	0.0330	0.0994	0.0197	0.0761
	EGARCH-NTS	0.1673	0.4088	0.0347	0.1001	0.0200	0.0761
EUR/USD	GARCH-N	0.0045	0.0601	0.0041	0.065	0.0034	0.055
	GARCH-t	0.0045	0.0061	0.0041	0.065	0.0034	0.055
	GARCH-NTS	0.0039	0.0547	0.0037	0.0527	0.0031	0.051
	EGARCH-N	0.1603	0.4014	0.0337	0.0986	0.0194	0.0761
	EGARCH-t	0.1603	0.4014	0.0337	0.0986	0.0194	0.0761
	EGARCH-NTS	0.1638	0.4057	0.0343	0.0995	0.0197	0.0761
TND/USD	GARCH-N	0.0023	0.0378	0.00019	0.284	0.0055	0.0494
	GARCH-t	0.0023	0.0378	0.00019	0.284	0.0055	0.0494
	GARCH-NTS	0.0020	0.0328	0.0016	0.0254	0.0056	0.0474
	EGARCH-N	0.1770	0.4205	0.0366	0.1025	0.0211	0.0761
	EGARCH-t	0.1770	0.4205	0.0366	0.1025	0.0211	0.0761
	EGARCH-NTS	0.1788	0.4227	0.0370	0.1029	0.0213	0.0763

This table reports the predictive power of GARCH and EGACRCH models Normal with $Student-t$, and Normal Tempered Stable distributions for 1 day, 5 days and 10 days horizon. The out of ample period spans from 1 January 2017 to 31 December 2019 and is used for forecasting analysis validation.

Results of the out-of-sample exchange rate forecasts are reported in Table 7. At first sight, we infer that GARCH-NTS performs better than with GARCH-N for all currencies at every forecast horizon.

For one-step-ahead volatility forecasts, the GARCH-NTS model holds the best forecasting accuracy criteria (has the lowest MSE and MAE citerion's value) followed closely by the GARCH-N model. In general, results are similar for the five-step-ahead and the ten-step-ahead volatility. However, it is obvious that gains accomplished by GARCH and EGARCH models with NTS distribution are little in forecasting accuracy compared to the Normal and Student-t distributions.

4. Conclusions

The GARCH models are commonly used to examine the exchange rate volatility. However, the Normal assumption of its innovations represents a crucial drawback. Even the alternative proposed distributions such as Student's t and GED failed to capture the fat-tailed properties.

To solve this issue, we introduced in this research paper the NTS, as a new distribution for GARCH and EGARCH models'innovations. To the best of our knowledge, this is the first application of GARCH and EGARCH models with NTS distribution to estimate exchange rate volatility.

To empirically examine the performance of GARCH-NTS and EGARCH-NTS regarding the other traditional distributions, we have used daily exchange rates over the period 1999–2019.

The analysis of exchange rate distributio ns using both the density plot and the Boxplot method show that the NTS distribution fits better the real data than the Normal one. Particularly, the NTS density plot of EUR/USD and TND/USD exchange rates returns is very close to that of the real data.

In addition, our findings demonstrate that the estimated parameters of GARCH-NTS model outperform the GARCH-N and GARCH-t ones for all currencies. Besides, EGARCH-NTS proves to have consistently better estimation results in comparison with the other distributions. Furthermore, it is noteworthily that the presence of leverage effect for EUR/USD, GBP/USD and CAD/USD exchange rates is significant. For GBP/USD and CAD/USD exchange rates the leverage effect is negative which implies that negative shocks generate more volatility than does positive shocks. However, for the EUR/USD the effect is positive. This means that good news have a greater effect on the EUR/USD volatility than does bad news.

At this stage of analysis, we would assert that GARCH-NTS and EGARCH-NTS are the preferred models in terms of out-of-sample forecasting accuracy.

To sum up, our results demonstrate that none of GARCH and EGARCH models with traditional distributions (The Normal and the Student-t distributions) can outperform in and out-of-sample the GARCH-NTS and EGARCH-NTS in estimating exchange rate volatility.

For financial practitioners, our findings indicating the performance of GARCH models with NTS distribution may largely increase the accuracy of risk measures. Eventually, the achieved results are promising and beneficial especially for portfolio management and investment analysis.

Conflict of interest

The authors declare no conflict of interest.

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