Networks and Heterogeneous Media

https://www.aimspress.com/journal/nhm

NHM, 21(4): 1197–1226.
DOI: 10.3934/nhm.2026048
Received: 27 November 2025
Revised: 15 June 2026
Accepted: 17 June 2026
Published: 24 June 2026

Research article

Novel insights into the truncated M-fractional Shynaray-IIA equation: New wave solutions and its dynamic behavior

Kun Zhang[1,2,*], **Zihao Ge**[1,2] **and Jiangping Cao**[3]

[1] College of Computer Science, Chengdu University, Chengdu 610106, China
[2] Key Laboratory of Pattern Recognition and Intelligent Information Processing of Sichuan, Chengdu University, Chengdu 610106, China
[3] National Institute of Measurement and Testing Technology, Chengdu 610021, China

* **Correspondence:** Email: zhangkun@cdu.edu.cn.

Abstract: This article investigates the truncated M-fractional Shynaray-IIA equation, which has important applications in many fields, such as fiber optic communication, magnetodynamics of ferromagnetic materials, and fluid dynamics. First, the truncated M-fractional Shynaray-IIA equation is transformed into a nonlinear ordinary differential equation through a fractional-order derivative and the traveling wave transformation. Then, the generalized (G'/G)-expansion method is adopted to get multiple groups of coefficient solutions based on the auxiliary equation. The conditions satisfied by the coefficients of the auxiliary equation are classified, and all novel wave solutions of the TMF-SIIAE are obtained, including bright solitons, dark solitons, and breathers. Finally, in order to better understand the behavior of the solutions, a 3D graph, cartesian and polar graph, contour graph and density graph of the solutions are drawn using Python software. Specifically, we also conducted a comparative analysis between the newly obtained solution and the existing solution, and quantitatively discussed the parameters of fractional-order derivatives and analyzed their physical applications.

Keywords: truncated M-fractional Shynaray-IIA equation; truncated M-fractional derivative; wave solution; generalized (G'/G)-expansion method; dynamics

1. Introduction

With the extensive discovery of nonlinear phenomena and the widespread application of nonlinear tools, nonlinear theory [1, 2] has been widely applied in multiple disciplines, and theoretical and applied research through nonlinear partial differential equations (NLPDEs) has become quite common. The theoretical system of nonlinear integer-order partial differential equations (a unified abbreviation for NLPDEs) has been fully developed, and local approximation [3] or an iterative method [4] can effectively handle nonlinear terms. However, for nonlinear fractional-order partial differential equations (NFPDEs), the coupling of nonlinear terms and fractional derivatives significantly increases complexity, which requires in-depth research by combining various fractional calculus rules [5] and nonlinear analysis techniques [6]. For solving NLPDEs, there are many classical methods, such as the complete discriminant system method [7], dynamical systems method [8, 9], Adomian decomposition method [10], improved nonlinear Riccati equation method [11], Hirota method [12, 13], Kudryashov approach [14], and neural networks method [15, 16]. Although there are existing techniques for solving NFPDEs, such as Lie symmetry analysis [17], special transformations [18] and the homotopy analysis method [19], these methods have limitations. Meanwhile, some researchers are also focusing on the soliton resolution conjecture and the asymptotic stability of solutions of the modified Camassa-Holm equation [20]. Currently, research on NFPDEs continues to deepen.

We have noticed that a class of NLPDEs with coupling properties, the Shynaray IIA equation (S-IIAE), has been widely studied by researchers. In the field of fiber optic communication, the ϕ^6-model expansion method has been utilized with the S-IIAE and some new solutions of the perturbed form are obtained, including various types of soliton solutions which are stable forms of corresponding optical pulses under dispersion and nonlinear equilibrium [21]. In the field of magnetodynamics of ferromagnetic materials, analytical methods such as the improved modified Sardar sub-equation method have been used to derive solutions for bright solitons, dark solitons, and anti-kink solitons. Through these solutions, they can predict the soliton like motion modes of domain walls driven by magnetic fields, providing a reference for optimizing the writing speed and stability of magnetic storage devices [22]. In fluid mechanics, the S-IIAE is extended to the dynamic reconstruction of catastrophic waves such as tsunamis. The bright and dark solitons represented by hyperbolic functions have been obtained via the improved tan(Φ/2)-expansion method. Solitary wave solutions such as trigonometric function solutions, hyperbolic function solutions, and Jacobi function solutions were derived through utilizing the improved Sardar sub-equation method and the new extended direct algebraic method [23].

Here, we focus on the truncated M-fractional Shynaray-IIA equation (TMF-SIIAE) given as [24]:

$$\begin{aligned}
&iD^{\alpha,\beta}_{M,t}g + D^{2\alpha,\beta}_{M,xt}g - iD^{\alpha,\beta}_{M,x}(g \cdot h) = 0,\\
&iD^{\alpha,\beta}_{M,t}f - D^{2\alpha,\beta}_{M,xt}f - iD^{\alpha,\beta}_{M,x}(f \cdot h) = 0,\\
&D^{\alpha,\beta}_{M,x}h - 2\mu D^{\alpha,\beta}_{M,t}(f \cdot g) = 0.
\end{aligned} \tag{1.1}$$

If $f = \lambda\bar{g}(\lambda = \pm 1)$, then Eq (1.1) is simplified to

$$\begin{aligned}
&iD^{\alpha,\beta}_{M,t}g + D^{2\alpha,\beta}_{M,xt}g - iD^{\alpha,\beta}_{M,x}(g \cdot h) = 0,\\
&D^{\alpha,\beta}_{M,x}h - 2\mu\lambda D^{\alpha,\beta}_{M,t}(|g|^2) = 0,
\end{aligned} \tag{1.2}$$

where the function $g = g(x, t)$ is a complex-valued function, while $h = h(x, t)$ is a real-valued function.

The TMF-SIIAE is a partial differential equation model combining the truncated M-fractional derivative and strong nonlinearity. Due to its ability to accurately describe memory, nonlocal properties, and complex coupled behaviors, it exhibits more significant physical characteristics in optical fiber communication, oceanic catastrophic simulation, and magnetic domain wall dynamics.

Especially, Eq (1.1) possesses integrable characteristics through the inverse scattering transform and has important applications in nonlinear optics.

In this paper, we focus on solving the TMF-SIIAE. The exp_α function, modified simplest equation and Kudryashov technique have already been utilized to solve problems. The generalized (G$'$/G)-expansion method we use is based on the classical (G$'$/G)-expansion method [25, 26], which has the advantage of being able to obtain a variety of wave solutions, including trigonometric function solutions, hyperbolic function solutions, rational function solutions, etc.

The structure of this paper is as follows: In Section 2, the TMF-SIIAE is transformed into a nonlinear ordinary differential equation via fractional-order derivative and the traveling wave transformation. In Section 3, all new wave solutions of TMF-SIIAE are obtained through the generalized (G$'$/G)-expansion method. In Section 4, a 3D graph, cartesian and polar graph, contour graph and density graph of the solutions are drawn using Python software, and the dynamics of the solutions are analyzed. In Section 5, a comparison and novelty analysis of the solutions is given. In Section 6, a comprehensive conclusion is presented.

2. Mathematical techniques

Definition 2.1. [27] The truncated M-fractional derivative for $f : [0, +\infty) \to \mathbf{R}$ is defined as:

$$D^{\alpha,\beta}_{M,x} f(x) = \lim_{\epsilon\to 0} \frac{f(x \cdot {}_jE_\beta(\epsilon x^{-\alpha})) - f(x)}{\epsilon}, \tag{2.1}$$

where $0 < \alpha \leq 1, \beta > 0$, and ${}_jE_\beta(x)$ is a truncated Mittag-Leffler (TML) function which is defined as ${}_jE_\beta(x) = \sum\limits_{k=0}^{j} \frac{x^k}{\Gamma(\beta k+1)}$.

Some operational properties of the TML function are applied during the calculation process.

Theorem 2.1. Let $0 < \alpha \leq 1, \beta > 0, a, b$ are all real constants, and f, g are α-differentiable functions with independent variable $x > 0$.

(1) $D^{\alpha,\beta}_{M,x}(af(x) + bg(x)) = aD^{\alpha,\beta}_{M,x} f(x) + bD^{\alpha,\beta}_{M,x} g(x)$.

(2) $D^{\alpha,\beta}_{M,x}(f(x) \cdot g(x)) = f(x) \cdot D^{\alpha,\beta}_{M,x} g(x) + g(x) \cdot D^{\alpha,\beta}_{M,x} f(x)$.

(3) $D^{\alpha,\beta}_{M,x} \frac{f(x)}{g(x)} = \frac{g(x)\cdot D^{\alpha,\beta}_{M,x} f(x) - f(x)\cdot D^{\alpha,\beta}_{M,x} g(x)}{g^2(x)}$.

(4) $D^{\alpha,\beta}_{M,x}(C) = 0$, C is a constant.

(5) $D^{\alpha,\beta}_{M,x}(f \circ g(x)) = f'(g(x)) \cdot D^{\alpha,\beta}_{M,x} g(x)$, f is differentiable at $g(x)$.

(6) $D^{\alpha,\beta}_{M,x} f(x) = \frac{x^{1-\alpha}}{\Gamma(\beta+1)} \cdot \frac{df(x)}{dx}$.

The sixth rule of the Theorem 2.1 states the relationship between the truncated M-fractional derivative and the classical derivative.

Remark 1. On the basis of the definition of the truncated M-fractional derivative, the truncated M-fractional second derivative can be obtained as $D^{2\alpha,\beta}_{M,xt} f(x) = D^{\alpha,\beta}_{M,t}(D^{\alpha,\beta}_{M,x} f(x))$.

There are many definitions and forms of fractional derivatives. Different fractional derivatives have different limitations. For example, the integral forms of Caputo and Riemann-Liouville fractional derivatives are complex, and it is very difficult to construct analytical traveling wave solutions after substituting a partial differential. The differential operation of Atangana-Baleanu fractional derivative has no elementary explicit simplification, and the transformed equation is highly complex, so it is difficult to find the exact solution by using the algebraic expansion method, such as the (G'/G)-expansion method. The conformable fractional derivative only contains a single parameter α, which cannot independently control the two kinds of physical quantities of medium damping or dispersion, and the degree of freedom of the physical description is insufficient. The He fractional derivative [28] and the local fractional derivative [29] are mostly used in fractal continuous media. The operator depends on the fractal dimension and is suitable for narrow scenes. Compared with these fractional derivatives, the truncated M-fractional derivative has the following advantages: first, the two free parameters α, β are more flexible in the physical description, in which parameter α is used to describe the strength of the medium memory effect, and parameter β is used to characterize the equivalent dispersion and damping. Second, the truncated Mittag-Leffler (TML) function in the truncated M-fractional derivative is an infinite series. The truncated M-fractional derivative is defined by finite term TML truncation to avoid infinite series operation. Through traveling wave transformation, fractional partial differential equations can be transformed into integer ordinary differential equations. Third, the truncated M-fractional derivative can quickly eliminate the coupling term and reduce the order of the equation based on the differential definition, so as to obtain the univariate ordinary differential equation. Fourth, when $\alpha = \beta = 1$, the classical integer-order derivative is strictly regressed, which is convenient for the horizontal comparison and degradation verification of the solution with the results of the existing integer-order literature.

The transformation is applied as:

$$g(x,t) = P(\xi) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \quad h(x,t) = Q(\xi), \quad \xi = \frac{\Gamma(\beta+1)}{\alpha}(x^{\alpha} - \delta t^{\alpha}), \tag{2.2}$$

where $P(\xi)$ is the amplitude of the pulse $g(x,t)$, η is the frequency and ω denotes the wave number. ν indicates the initial phase component, and δ is the velocity of the soliton.

By substituting the above transformation into Eq (1.2), the equation can be transformed into

$$\begin{aligned} &\delta P'' + \omega(1-\eta)P + \eta PQ + i(\delta - \eta\delta - \omega)P' + iQP' + iQ'P = 0, \\ &Q' + 4\mu\lambda\delta PP' = 0. \end{aligned} \tag{2.3}$$

By integrating the second equation of Eq (2.3), we can get $Q = -2\mu\lambda\delta P^2$. Then we substitute the expressions of Q and Q' into the first equation of Eq (2.3), and obtain

$$\delta P'' + \omega(1-\eta)P - 2\mu\lambda\delta\eta P^3 + i(\delta - \eta\delta - \omega)P' - 6i\mu\lambda\delta P^2 P' = 0. \tag{2.4}$$

The real part of Eq (2.4) is

$$\delta P'' + \omega(1-\eta)P - 2\mu\lambda\delta\eta P^3 = 0, \tag{2.5}$$

and from the imaginary part of Eq (2.4), we can obtain

$$\omega = \delta(1-\eta). \tag{2.6}$$

Next, in order to obtain the exact traveling wave solutions of TMF-SIIAE, different methods are considered.

3. New wave solutions

3.1. (G'/G)-expansion method applied to TMF-SIIAE

Assume that the solutions of Eq (2.5) can be indicated by a polynomial in $(\frac{G'}{G})$ as $P(\xi) = a_m(\frac{G'}{G})^m + \cdots + a_0, a_m \neq 0$, where $G = G(\xi)$ satisfies the second-order LODE in the form $G'' + d_1G' + d_2G = 0$, and then

$$
\begin{aligned}
P^3(\xi) &= a_m^3(\frac{G'}{G})^{3m} + \cdots + a_0^3, \\
P' &= -ma_m(\frac{G'}{G})^{m+1} - \cdots - a_1d_2, \\
P'' &= m(m+1)a_m(\frac{G'}{G})^{m+2} + \cdots + a_1d_1d_2.
\end{aligned} \tag{3.1}
$$

Considering the homogeneous balance and balancing the highest-order derivative term P'' and the highest-order nonlinear term P^3, that is, $m + 2 = 3m$, we get $m = 1$. So we can get

$$
\begin{aligned}
P(\xi) &= a_1(\frac{G'}{G}) + a_0, a_1 \neq 0, \\
P^3(\xi) &= a_1^3(\frac{G'}{G})^3 + 3a_0a_1^2(\frac{G'}{G})^2 + 3a_0^2a_1(\frac{G'}{G}) + a_0^3, \\
P' &= -a_1(\frac{G'}{G})^2 - a_1d_1(\frac{G'}{G}) - a_1d_2, \\
P'' &= 2a_1(\frac{G'}{G})^3 + 3a_1d_1(\frac{G'}{G})^2 + (2a_1d_2 + a_1d_1^2)(\frac{G'}{G}) + a_1d_1d_2.
\end{aligned} \tag{3.2}
$$

We substitute Eq (3.2) into Eq (2.5), and combine similar terms of $(\frac{G'}{G})$. Next by making the coefficients of the obtained polynomial zero, the algebraic equations can be obtained:

$(\frac{G'}{G})^3$: $2\delta a_1 - 2\mu\lambda\delta\eta a_1^3 = 0$,
$(\frac{G'}{G})^2$: $3a_1d_1\delta - 6a_0a_1^2\mu\lambda\delta\eta = 0$,
$(\frac{G'}{G})^1$: $2a_1d_2\delta + a_1d_1^2\delta + a_1\omega(1-\eta) - 6a_0^2a_1\mu\lambda\delta\eta = 0$,
$(\frac{G'}{G})^0$: $a_1d_1d_2\delta + \omega(1-\eta)a_0 - 2a_0^3\mu\lambda\delta\eta = 0$.

By solving the above algebraic equation system, we can obtain $\delta = \frac{\omega(\eta-1)}{2d_2 - \frac{1}{2}d_1^2}$, $a_1 = \frac{1}{\sqrt{\mu\lambda\eta}}$, $a_0 = \frac{d_1}{2\sqrt{\mu\lambda\eta}}$.

Suppose that $\Delta = d_1^2 - 4d_2$, and then the travelling wave solutions of the truncated M-fractional S-IIA equation can be expressed in the following three cases.

Case 1 $\Delta > 0$.

The travelling wave solution of Eq (1.2) is

$$
\begin{aligned}
g(x,t) &= \frac{\sqrt{\Delta}}{2\sqrt{\mu\lambda\eta}} \cdot \frac{C_1 \sinh\frac{\sqrt{\Delta}}{2}\xi + C_2\cosh\frac{\sqrt{\Delta}}{2}\xi}{C_1\cosh\frac{\sqrt{\Delta}}{2}\xi + C_2\sinh\frac{\sqrt{\Delta}}{2}\xi} \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h(x,t) &= -\frac{\delta\Delta}{2\eta} \cdot (\frac{C_1 \sinh\frac{\sqrt{\Delta}}{2}\xi + C_2\cosh\frac{\sqrt{\Delta}}{2}\xi}{C_1\cosh\frac{\sqrt{\Delta}}{2}\xi + C_2\sinh\frac{\sqrt{\Delta}}{2}\xi})^2,
\end{aligned} \tag{3.3}
$$

where $\xi = \frac{\Gamma(\beta+1)}{\alpha}(x^\alpha - \delta t^\alpha)$.

Case 2 $\Delta < 0$.

The travelling wave solution of Eq (1.2) is

$$\begin{aligned} g(x,t) &= \frac{\sqrt{-\Delta}}{2\sqrt{\mu\lambda\eta}} \cdot \frac{-C_1 \sin\frac{\sqrt{-\Delta}}{2}\xi + C_2 \cos\frac{\sqrt{-\Delta}}{2}\xi}{C_1 \cos\frac{\sqrt{-\Delta}}{2}\xi + C_2 \sin\frac{\sqrt{-\Delta}}{2}\xi} \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha}+\omega t^{\alpha})+\nu)}, \\ h(x,t) &= \frac{\delta\Delta}{2\eta} \cdot (\frac{-C_1 \sin\frac{\sqrt{-\Delta}}{2}\xi + C_2 \cos\frac{\sqrt{-\Delta}}{2}\xi}{C_1 \cos\frac{\sqrt{-\Delta}}{2}\xi + C_2 \sin\frac{\sqrt{-\Delta}}{2}\xi})^2, \end{aligned} \tag{3.4}$$

where $\xi = \frac{\Gamma(\beta+1)}{\alpha}(x^{\alpha} - \delta t^{\alpha})$.

Case 3 $\Delta = 0$.

The travelling wave solution of Eq (1.2) is

$$\begin{aligned} g(x,t) &= \frac{C_2}{\sqrt{\mu\lambda\eta}(C_1 + C_2\xi)} \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha}+\omega t^{\alpha})+\nu)}, \\ h(x,t) &= -\frac{2\delta}{\eta} \cdot (\frac{C_2}{C_1 + C_2\xi})^2, \end{aligned} \tag{3.5}$$

where $\xi = \frac{\Gamma(\beta+1)}{\alpha}(x^{\alpha} - \delta t^{\alpha})$.

3.2. Generalized ($G^{'}/G$)-expansion method applied to the TMF-SIIAE

Due to the different auxiliary equations, Eq (1.2) may obtain more types of solutions. Below, we consider another form of solution as follows:

$$P(\xi) = \sum_{i=0}^{m} a_i(l + \frac{G'}{G})^i + \sum_{i=1}^{m} b_i(l + \frac{G'}{G})^{-i}, \tag{3.6}$$

where a_i, b_i $(i = 1 \ldots m)$ are constant coefficients, a_m, b_m are not zero simultaneously, and m is a positive integer. In Eq (3.6), $G(\xi)$ satisfies the auxiliary equation

$$K_1GG^{''} - K_2GG^{'} - K_3(G^{'})^2 - K_4G^2 = 0, \tag{3.7}$$

where K_1, K_2, K_3, K_4 are real constants.

Substitute the solution Eq (3.6) into Eq (2.5). Then consider the homogeneous balance and balancing the highest-order derivative term $P^{''}$ and the highest-order nonlinear term P^3, that is, $m + 2 = 3m$, and we get $m = 1$.

Thus we can get

$$\begin{aligned} P(\xi) &= a_0 + a_1(l + \frac{G^{'}}{G}) + b_1(l + \frac{G^{'}}{G})^{-1}, \\ P^{'}(\xi) &= a_1(\frac{G^{'}}{G})^{'} - b_1(l + \frac{G^{'}}{G})^{-2}(\frac{G^{'}}{G})^{'}, \\ P^{''}(\xi) &= a_1(\frac{G^{'}}{G})^{''} + 2b_1(l + \frac{G^{'}}{G})^{-3}((\frac{G^{'}}{G})^{'})^2. \end{aligned} \tag{3.8}$$

Combining Eq (3.7) and the fact that $(\frac{G'}{G})' = \frac{G''G-(G')^2}{G^2}$, we can obtain

$$(\frac{G'}{G})' = (\frac{K_3}{K_1} - 1)(\frac{G'}{G})^2 + \frac{K_2}{K_1}\frac{G'}{G} + \frac{K_4}{K_1}. \tag{3.9}$$

Next, Combining Eqs (2.5), (3.8) and (3.9), the coefficients of each power of $(\frac{G'}{G})$ are obtained and set to zero, resulting in a system of algebraic equations as follows:

$(\frac{G'}{G}+l)^3$: $\frac{2a_1\delta(K_3-K_1)^2}{K_1^2} - 2\mu\lambda\delta\eta a_1^3 = 0,$

$(\frac{G'}{G}+l)^2$: $\frac{3a_1\delta(K_3-K_1)(K_2-2lK_3+2lK_1)}{K_1^2} - 6\mu\lambda\delta\eta a_0 a_1^2 = 0,$

$(\frac{G'}{G}+l)^1$: $\delta\frac{2a_1(K_3-K_1)(K_4-l^2K_1-lK_2+l^2K_3)+a_1(K_2-2lK_3+2lK_1)^2}{K_1^2} + \omega(1-\eta)a_1 - 6\mu\lambda\delta\eta(a_0^2a_1 + a_1^2b_1) = 0,$

$(\frac{G'}{G}+l)^0$: $\delta\frac{a_1(K_2-2lK_3+2lK_1)(K_4-l^2K_1-lK_2+l^2K_3)+b_1(K_2-2lK_3+2lK_1)(K_3-K_1)}{K_1^2} + \omega(1-\eta)a_0 - 2\mu\lambda\delta\eta(a_0^3 + 6a_0a_1b_1) = 0,$

$(\frac{G'}{G}+l)^{-1}$: $\delta\frac{b_1(K_2-2lK_3+2lK_1)^2+2b_1(K_4-l^2K_1-lK_2+l^2K_3)(K_3-K_1)}{K_1^2} + \omega(1-\eta)b_1 - 6\mu\lambda\delta\eta(a_0^2b_1 + a_1b_1^2) = 0,$

$(\frac{G'}{G}+l)^{-2}$: $\frac{3b_1\delta(K_4-l^2K_1-lK_2+l^2K_3)(K_2-2lK_3+2lK_1)}{K_1^2} - 6\mu\lambda\delta\eta a_0 b_1^2 = 0,$

$(\frac{G'}{G}+l)^{-3}$: $\frac{2b_1\delta(K_4-l^2K_1-lK_2+l^2K_3)^2}{K_1^2} - 2\mu\lambda\delta\eta b_1^3 = 0.$

There are several possible solutions to algebraic systems of above equations.

Group 1: $a_0 = \pm\frac{K_2-2lK_3+2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$, $a_1 = \pm\frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}$, $b_1 = 0$, $\delta = \frac{2\omega(\eta-1)K_1^2}{4(K_3-K_1)(K_4-l^2K_1-lK_2+l^2K_3)-(K_2-2lK_3+2lK_1)^2}$, $l = l$, $K_3 \neq K_1$.

Group 2: $a_0 = \pm\frac{K_2-2lK_3+2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$, $a_1 = 0$, $b_1 = \pm\frac{K_4-l^2K_1-lK_2+l^2K_3}{K_1\sqrt{\mu\lambda\eta}} \neq 0$, $\delta = \frac{2\omega(\eta-1)K_1^2}{4(K_4-l^2K_1-lK_2+l^2K_3)-(K_2-2lK_3+2lK_1)^2}$, $l = l$.

Group 3: $a_0 = 0$, $a_1 = \pm\frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}$, $b_1 = 0$, $\delta = \frac{\omega(\eta-1)K_1^2}{2(K_3-K_1)(K_4-l^2K_1-lK_2+l^2K_3)}$, $l = \frac{K_2}{2(K_3-K_1)}$, $K_3 \neq K_1$.

Group 4: $a_0 = a_0$, $a_1 = 0$, $b_1 = \pm\frac{K_4-l^2K_1-lK_2+l^2K_3}{K_1\sqrt{\mu\lambda\eta}} \neq 0$,
$\delta = \frac{\omega(\eta-1)K_1^2}{(K_2-2lK_3+2lK_1)^2+2(K_3-K_1)(K_4-l^2K_1-lK_2+l^2K_3)-6\mu\lambda\eta a_0^2K_1^2}$, $l = \frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta}-K_2}{2(K_1-K_3)}$, $K_3 \neq K_1$.

Group 5: $a_0 = a_0$, $a_1 = \pm\frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}$, $b_1 = 0$, $\delta = \frac{\omega(\eta-1)K_1^2}{(K_2-2lK_3+2lK_1)^2+2(K_3-K_1)(K_4-l^2K_1-lK_2+l^2K_3)-6\mu\lambda\eta a_0^2K_1^2}$,
$l = \frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta}-K_2}{2(K_1-K_3)}$, $K_3 \neq K_1$.

Group 6: $a_0 = \pm\frac{K_2-2lK_3+2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$, $a_1 = \pm\frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}$, $b_1 = \pm\frac{K_4-l^2K_1-lK_2+l^2K_3}{K_1\sqrt{\mu\lambda\eta}}$,
while $K_3 = K_1$, $\delta = -\frac{2\omega(\eta-1)K_1^2}{K_2^2}$, $l = l$,
while $K_3 \neq K_1$, $\delta = -\frac{2\omega(\eta-1)K_1^2}{(K_2-2lK_3+2lK_1)^2}$, $l = \frac{K_2\pm\sqrt{K_2^2-4K_4(K_3-K_1)}}{2(K_3-K_1)}$.

Moreover, the solutions $\frac{G'}{G}$ of Eq (3.9) can be divided into the following cases:

Case 1 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1-K_3) > 0$.

$$\frac{G'}{G} = \frac{K_2}{2(K_1-K_3)} + \frac{\sqrt{\Delta}}{2(K_1-K_3)} \cdot \frac{C_1\sinh\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi + C_2\cosh\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi}{C_1\cosh\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi + C_2\sinh\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi}, \tag{3.10}$$

where $\Delta = K_2^2 + 4K_4(K_1-K_3)$.

Case 2 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1-K_3) < 0$.

$$\frac{G'}{G} = \frac{K_2}{2(K_1-K_3)} + \frac{\sqrt{-\Delta}}{2(K_1-K_3)} \cdot \frac{-C_1\sin\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi + C_2\cos\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi}{C_1\cos\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi + C_2\sin\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi}. \tag{3.11}$$

Case 3 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) = 0$.

$$\frac{G'}{G} = \frac{K_2}{2(K_1 - K_3)} + \frac{C_2}{C_1 + C_2\xi}. \tag{3.12}$$

Case 4 $K_2 = 0$ and $K_4(K_1 - K_3) > 0$.

$$\frac{G'}{G} = \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \cdot \frac{C_1 \sinh \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi + C_2 \cosh \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi}{C_1 \cosh \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi + C_2 \sinh \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi}. \tag{3.13}$$

Case 5 $K_2 = 0$ and $K_4(K_1 - K_3) < 0$.

$$\frac{G'}{G} = \frac{\sqrt{-K_4(K_1 - K_3)}}{K_1 - K_3} \cdot \frac{-C_1 \sin \frac{\sqrt{-K_4(K_1-K_3)}}{K_1-K_3}\xi + C_2 \cos \frac{\sqrt{-K_4(K_1-K_3)}}{K_1-K_3}\xi}{C_1 \cos \frac{\sqrt{-K_4(K_1-K_3)}}{K_1-K_3}\xi + C_2 \sin \frac{\sqrt{-K_4(K_1-K_3)}}{K_1-K_3}\xi}. \tag{3.14}$$

By combining the classification of parameter variables and the classification of solutions to auxiliary Eq (3.7), we can obtain the new travelling wave solutions of the original Eq (1.2) as:

For Group 1

Case 1 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_1(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\coth\Big(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi\Big)\Big)\Big) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\ h_1(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_3 - K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\ &+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\coth\Big(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi\Big)\Big)\Big)^2, \end{aligned} \tag{3.15}$$

where l is arbitrary constant, and $\Delta = K_2^2 + 4K_4(K_1 - K_3)$.

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned} g_2(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh\Big(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi\Big)\Big)\Big) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\ h_2(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_3 - K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\ &+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh\Big(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi\Big)\Big)\Big)^2. \end{aligned} \tag{3.16}$$

Case 2 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_3(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1-K_3)} + \frac{\sqrt{-\Delta}}{2(K_1-K_3)}\cot\Big(\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi\Big)\Big)\Big) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha)+\nu)}, \\ h_3(x,t) &= -\frac{4\omega(\eta-1)\mu\lambda K_1^2}{4(K_3-K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\ &+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1-K_3)} + \frac{\sqrt{-\Delta}}{2(K_1-K_3)}\cot\Big(\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi\Big)\Big)\Big)^2. \end{aligned} \tag{3.17}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned} g_4(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1-K_3)} - \frac{\sqrt{-\Delta}}{2(K_1-K_3)}\tan\Big(\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi\Big)\Big)\Big) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha)+\nu)}, \\ h_4(x,t) &= -\frac{4\omega(\eta-1)\mu\lambda K_1^2}{4(K_3-K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\ &+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1-K_3)} - \frac{\sqrt{-\Delta}}{2(K_1-K_3)}\tan\Big(\frac{\sqrt{-\Delta}}{2(K_1-K_3)}\xi\Big)\Big)\Big)^2. \end{aligned} \tag{3.18}$$

Case 3 $K_2 = 0$ and $K_4(K_1 - K_3) > 0$.
When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_5(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\coth\Big(\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi\Big)\Big)\Big) \\ &e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha)+\nu)}, \\ h_5(x,t) &= -\frac{4\omega(\eta-1)\mu\lambda K_1^2}{4(K_3-K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\ &+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\coth\Big(\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi\Big)\Big)\Big)^2. \end{aligned} \tag{3.19}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned} g_6(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\tanh\Big(\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi\Big)\Big)\Big) \\ &e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha)+\nu)}, \\ h_6(x,t) &= -\frac{4\omega(\eta-1)\mu\lambda K_1^2}{4(K_3-K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\ &+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\tanh\Big(\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi\Big)\Big)\Big)^2. \end{aligned} \tag{3.20}$$

Case 4 $K_2 = 0$ and $K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$g_7(x,t) = \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)))$$
$$e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$
$$h_7(x,t) = -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_3 - K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$$
$$+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)))^2. \tag{3.21}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$g_8(x,t) = \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)))$$
$$e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$
$$h_8(x,t) = -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_3 - K_1)(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$$
$$+ \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)))^2. \tag{3.22}$$

For Group 2

Case 1 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$g_9(x,t) = \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}$$
$$\coth(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$
$$h_9(x,t) = -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$$
$$+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1})^2, \tag{3.23}$$

where l is arbitrary constant, and $\Delta = K_2^2 + 4K_4(K_1 - K_3)$.

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$g_{10}(x,t) = \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}$$
$$\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$
$$h_{10}(x,t) = -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$$
$$+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1})^2. \tag{3.24}$$

Case 2 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned}
g_{11}(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\\
&\cot\big(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi\big)\Big)^{-1}\Big) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},\\
h_{11}(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}}\\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\cot\big(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi\big)\Big)^{-1}\Big)^2,
\end{aligned} \tag{3.25}$$

where l is arbitrary constant, and $\Delta = K_2^2 + 4K_4(K_1 - K_3)$.

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned}
g_{12}(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\\
&\tan\big(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi\big)\Big)^{-1}\Big) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},\\
h_{12}(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}}\\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\tan\big(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi\big)\Big)^{-1}\Big)^2.
\end{aligned} \tag{3.26}$$

Case 3 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) = 0$.

When $C_1 = C_2 = 1$, the solution is

$$\begin{aligned}
g_{13}(x,t) &= \pm\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{1}{1 + \xi}\Big)^{-1}\Big)\\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},\\
h_{13}(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}\Big(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}}\\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}\Big(l + \frac{K_2}{2(K_1 - K_3)} + \frac{1}{1 + \xi}\Big)\Big)^{-1}\Big)^2.
\end{aligned} \tag{3.27}$$

Case 4 $K_2 = 0$ and $K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned}
g_{14}(x,t) &= \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h_{14}(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned} \tag{3.28}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned}
g_{15}(x,t) &= \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h_{15}(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned} \tag{3.29}$$

Case 5 $K_2 = 0$ and $K_4(K_1 - K_3) < 0$.
When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned}
g_{16}(x,t) &= \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h_{16}(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned} \tag{3.30}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned}
g_{17}(x,t) &= \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h_{17}(x,t) &= -\frac{4\omega(\eta - 1)\mu\lambda K_1^2}{4(K_4 - l^2K_1 - lK_2 + l^2K_3) - (K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} \\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned} \tag{3.31}$$

For Group 3

Case 1 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_{18}(x,t) &= \pm\frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi)) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\ h_{18}(x,t) &= -\frac{\omega(\eta - 1)(K_3 - K_1)}{2\eta(K_4 - l^2K_1 - lK_2 + l^2K_3)}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^2, \end{aligned} \tag{3.32}$$

where $l = \frac{K_2}{2(K_3 - K_1)}$, and $\Delta = K_2^2 + 4K_4(K_1 - K_3)$.

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned} g_{19}(x,t) &= \pm\frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi)) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\ h_{19}(x,t) &= -\frac{\omega(\eta - 1)(K_3 - K_1)}{2\eta(K_4 - l^2K_1 - lK_2 + l^2K_3)}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^2. \end{aligned} \tag{3.33}$$

Case 2 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_{20}(x,t) &= \pm\frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\cot(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi)) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\ h_{20}(x,t) &= -\frac{\omega(\eta - 1)(K_3 - K_1)}{2\eta(K_4 - l^2K_1 - lK_2 + l^2K_3)}(l + \frac{K_2}{2(K_1 - K_3)} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\cot(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))^2. \end{aligned} \tag{3.34}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned} g_{21}(x,t) &= \pm\frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{K_2}{2(K_1 - K_3)} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\tan(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi)) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\ h_{21}(x,t) &= -\frac{\omega(\eta - 1)(K_3 - K_1)}{2\eta(K_4 - l^2K_1 - lK_2 + l^2K_3)}(l + \frac{K_2}{2(K_1 - K_3)} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\tan(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))^2. \end{aligned} \tag{3.35}$$

Case 3 $K_2 = 0$ and $K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_{22}(x,t) &= \pm\frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(l + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi)) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\ h_{22}(x,t) &= -\frac{\omega(\eta - 1)(K_3 - K_1)}{2\eta(K_4 - l^2K_1 - lK_2 + l^2K_3)}(l + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^2. \end{aligned} \tag{3.36}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned} g_{23}(x,t) &= \pm\frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(l+\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\tanh(\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi)) \\ &\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha+\omega t^\alpha)+\nu)}, \\ h_{23}(x,t) &= -\frac{\omega(\eta-1)(K_3-K_1)}{2\eta(K_4-l^2K_1-lK_2+l^2K_3)}(l+\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\tanh(\frac{\sqrt{K_4(K_1-K_3)}}{K_1-K_3}\xi))^2. \end{aligned} \tag{3.37}$$

Case 4 $K_2 = 0$ and $K_4(K_1-K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_{24}(x,t) &= \pm\frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(l+\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\cot(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi))e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha+\omega t^\alpha)+\nu)}, \\ h_{24}(x,t) &= -\frac{\omega(\eta-1)(K_3-K_1)}{2\eta(K_4-l^2K_1-lK_2+l^2K_3)}(l+\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\cot(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi))^2. \end{aligned} \tag{3.38}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned} g_{25}(x,t) &= \pm\frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(l+\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\tan(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi))e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha+\omega t^\alpha)+\nu)}, \\ h_{25}(x,t) &= -\frac{\omega(\eta-1)(K_3-K_1)}{2\eta(K_4-l^2K_1-lK_2+l^2K_3)}(l+\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\tan(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi))^2. \end{aligned} \tag{3.39}$$

For Group 4

Case 1 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1-K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned} g_{26}(x,t) &= (a_0 \pm \frac{K_4-l^2K_1-lK_2+l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1-K_3}+\frac{\sqrt{\Delta}}{2(K_1-K_3)} \\ &\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi))^{-1})\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha+\omega t^\alpha)+\nu)}, \\ h_{26}(x,t) &= -\frac{\omega\mu\lambda(\eta-1)K_1^2}{(K_2-2lK_3+2lK_1)^2+2(K_4-l^2K_1-lK_2+l^2K_3)(K_3-K_1)-6\mu\lambda\eta a_0^2K_1^2} \\ &(a_0 \pm \frac{K_4-l^2K_1-lK_2+l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1-K_3}+\frac{\sqrt{\Delta}}{2(K_1-K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi))^{-1})^2, \end{aligned} \tag{3.40}$$

where $l = \frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta}-K_2}{2(K_1-K_3)}$, a_0 is an arbitrary constant, and $\Delta = K_2^2 + 4K_4(K_1-K_3)$.

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned}
&g_{27}(x,t) = (a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)} \\
&\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
&h_{27}(x,t) = -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1})^2.
\end{aligned} \tag{3.41}$$

Case 2 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned}
&g_{28}(x,t) = (a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\cot(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))^{-1}) \\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
&h_{28}(x,t) = -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\cot(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))^{-1})^2.
\end{aligned} \tag{3.42}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned}
&g_{29}(x,t) = (a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\tan(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))^{-1}) \\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
&h_{29}(x,t) = -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\tan(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))^{-1})^2.
\end{aligned} \tag{3.43}$$

Case 3 $K_2 = 0$ and $K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$
\begin{aligned}
g_{30}(x,t) &= (a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h_{30}(x,t) &= -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned} \tag{3.44}
$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$
\begin{aligned}
g_{31}(x,t) &= (a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h_{31}(x,t) &= -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned} \tag{3.45}
$$

Case 4 $K_2 = 0$ and $K_4(K_1 - K_3) < 0$.
When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$
\begin{aligned}
g_{32}(x,t) &= (a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)}, \\
h_{32}(x,t) &= -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned} \tag{3.46}
$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$
\begin{aligned}
g_{33}(x,t) &= (a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1-K_3)} + \frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3} \\
&\tan(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha)+\nu)}, \\
h_{33}(x,t) &= -\frac{\omega\mu\lambda(\eta-1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3-K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1-K_3)} + \frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3} \\
&\tan(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi))^{-1})^2.
\end{aligned}
\tag{3.47}
$$

For Group 5

Case 1 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$
\begin{aligned}
g_{34}(x,t) &= (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1-K_3} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi))) \\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha)+\nu)}, \\
h_{34}(x,t) &= -\frac{\omega\mu\lambda(\eta-1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3-K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1-K_3} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi)))^2,
\end{aligned}
\tag{3.48}
$$

where $l = \frac{\pm a_0K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1-K_3)}$, a_0 is an arbitrary constant, and $\Delta = K_2^2 + 4K_4(K_1 - K_3)$.

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$
\begin{aligned}
g_{35}(x,t) &= (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1-K_3} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi))) \\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha)+\nu)}, \\
h_{35}(x,t) &= -\frac{\omega\mu\lambda(\eta-1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3-K_1) - 6\mu\lambda\eta a_0^2K_1^2} \\
&(a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0K_1\sqrt{\mu\lambda\eta}}{K_1-K_3} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi)))^2.
\end{aligned}
\tag{3.49}
$$

Case 2 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$g_{36}(x,t) = (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0 K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\cot(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$

$$h_{36}(x,t) = -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2 K_1^2} (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0 K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} + \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\cot(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi)))^2. \tag{3.50}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$g_{37}(x,t) = (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0 K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\tan(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi))) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$

$$h_{37}(x,t) = -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2 K_1^2} (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm a_0 K_1\sqrt{\mu\lambda\eta}}{K_1 - K_3} - \frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\tan(\frac{\sqrt{-\Delta}}{2(K_1 - K_3)}\xi)))^2. \tag{3.51}$$

Case 3 $K_2 = 0$ and $K_4(K_1 - K_3) > 0$.
When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$g_{38}(x,t) = (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0 K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$

$$h_{38}(x,t) = -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2 K_1^2} (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0 K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi)))^2. \tag{3.52}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$g_{39}(x,t) = (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0 K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^\alpha + \omega t^\alpha) + \nu)},$$

$$h_{39}(x,t) = -\frac{\omega\mu\lambda(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2 + 2(K_4 - l^2K_1 - lK_2 + l^2K_3)(K_3 - K_1) - 6\mu\lambda\eta a_0^2 K_1^2} (a_0 \pm \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0 K_1\sqrt{\mu\lambda\eta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi)))^2. \tag{3.53}$$

Case 4 $K_2 = 0$ and $K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned}
g_{40}(x,t) &= (a_0 \pm \frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta}-K_2}{2(K_1-K_3)} + \frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\cot(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi)))\\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha}+\omega t^{\alpha})+\nu)},\\
h_{40}(x,t) &= -\frac{\omega\mu\lambda(\eta-1)K_1^2}{(K_2-2lK_3+2lK_1)^2+2(K_4-l^2K_1-lK_2+l^2K_3)(K_3-K_1)-6\mu\lambda\eta a_0^2K_1^2}\\
&(a_0 \pm \frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta}-K_2}{2(K_1-K_3)} + \frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\cot(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi)))^2.
\end{aligned} \tag{3.54}$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$\begin{aligned}
g_{41}(x,t) &= (a_0 \pm \frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta}-K_2}{2(K_1-K_3)} + \frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\tan(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi)))\\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha}+\omega t^{\alpha})+\nu)},\\
h_{41}(x,t) &= -\frac{\omega\mu\lambda(\eta-1)K_1^2}{(K_2-2lK_3+2lK_1)^2+2(K_4-l^2K_1-lK_2+l^2K_3)(K_3-K_1)-6\mu\lambda\eta a_0^2K_1^2}\\
&(a_0 \pm \frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm 2a_0K_1\sqrt{\mu\lambda\eta}-K_2}{2(K_1-K_3)} + \frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\tan(\frac{\sqrt{K_4(K_3-K_1)}}{K_1-K_3}\xi)))^2.
\end{aligned} \tag{3.55}$$

For Group 6

Case 1 $K_2 \neq 0$ and $K_2^2 + 4K_4(K_1-K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$\begin{aligned}
g_{42}(x,t) &= \pm(\frac{K_2-2lK_3+2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1-K_3)} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi))\\
&+ \frac{K_4-l^2K_1-lK_2+l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1-K_3)} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi))^{-1})\\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha}+\omega t^{\alpha})+\nu)},\\
h_{42}(x,t) &= \frac{4\mu\lambda\omega(\eta-1)K_1^2}{(K_2-2lK_3+2lK_1)^2}(\frac{K_2-2lK_3+2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3-K_1}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1-K_3)} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\\
&\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi) + \frac{K_4-l^2K_1-lK_2+l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1-K_3)} + \frac{\sqrt{\Delta}}{2(K_1-K_3)}\\
&\coth(\frac{\sqrt{\Delta}}{2(K_1-K_3)}\xi))^{-1})^2,
\end{aligned} \tag{3.56}$$

where $l = \frac{\pm\sqrt{\Delta}-K_2}{2(K_1-K_3)}$, $a_0 = \pm\frac{K_2-2lK_3+2lK_1}{2K_1\sqrt{\mu\lambda\eta}}$, and $\Delta = K_2^2 + 4K_4(K_1-K_3)$.

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$
\begin{aligned}
g_{43}(x,t) &= \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi)) \\
&+ \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)}\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1}) \\
&\cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\
h_{43}(x,t) &= \frac{4\mu\lambda\omega(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)} \\
&\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\pm\frac{\sqrt{\Delta}}{2(K_1 - K_3)} + \frac{\sqrt{\Delta}}{2(K_1 - K_3)} \\
&\tanh(\frac{\sqrt{\Delta}}{2(K_1 - K_3)}\xi))^{-1})^2.
\end{aligned}
\tag{3.57}
$$

Case 2 $K_2 = 0$ and $K_4(K_1 - K_3) > 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$
\begin{aligned}
g_{44}(x,t) &= \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\
h_{44}(x,t) &= \frac{4\mu\lambda\omega(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
&\coth(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned}
\tag{3.58}
$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$
\begin{aligned}
g_{45}(x,t) = & \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
& \tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
& \tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\
h_{45}(x,t) = & \frac{4\mu\lambda\omega(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
& \tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3} \\
& \tanh(\frac{\sqrt{K_4(K_1 - K_3)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned}
\tag{3.59}
$$

Case 3 $K_2 = 0$ and $K_4(K_1 - K_3) < 0$.

When $C_1 = 0$ and $C_2 \neq 0$, the solution is

$$
\begin{aligned}
g_{46}(x,t) = & \pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
& \cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
& \cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\
h_{46}(x,t) = & \frac{4\mu\lambda\omega(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
& \cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
& \cot(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned}
\tag{3.60}
$$

When $C_1 \neq 0$ and $C_2 = 0$, the solution is

$$
\begin{aligned}
g_{47}(x,t) = &\pm(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1}) \cdot e^{i(\frac{\Gamma(\beta+1)}{\alpha}(-\eta x^{\alpha} + \omega t^{\alpha}) + \nu)}, \\
h_{47}(x,t) = &\frac{4\mu\lambda\omega(\eta - 1)K_1^2}{(K_2 - 2lK_3 + 2lK_1)^2}(\frac{K_2 - 2lK_3 + 2lK_1}{2K_1\sqrt{\mu\lambda\eta}} + \frac{K_3 - K_1}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi)) + \frac{K_4 - l^2K_1 - lK_2 + l^2K_3}{K_1\sqrt{\mu\lambda\eta}}(\frac{\pm\sqrt{\Delta} - K_2}{2(K_1 - K_3)} + \frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3} \\
&\tan(\frac{\sqrt{K_4(K_3 - K_1)}}{K_1 - K_3}\xi))^{-1})^2.
\end{aligned}
\tag{3.61}
$$

4. Numerical calculation and visualization

4.1. Graphical representation of solutions based on specific parameter values

In order to better understand the dynamic behavior and characteristics of the solution of Eq (1.2), the parameters are assigned and dynamically changed through Python software programming to observe the shape of the solution. In particular, due to the physical significance of the truncated M-fractional derivative, the ranges of both the parameters x and t should satisfy $x > 0, t > 0$.

When the parameter values are set to the following: $K_1 = 2, K_2 = 1, K_3 = 0, K_4 = 1, \lambda = 1, \mu = 1, \eta = 1 + \frac{3}{4}\sqrt{2}, \omega = 1, \delta = -\frac{2}{3}\sqrt{2}, l = 1, \alpha = 0.5, \beta = 0.5$, the representation of the solution $g_1(x,t)$ can be obtained as shown in Figure 1. From 3D and Cartesian/polar graphs, it can be seen that the solution $g_1(x,t)$ has the localization, stability, and particle-like behavior of a soliton, and $g_1(x,t)$ is a bright soliton with an isolated peak appearing in a field with a zero background value.

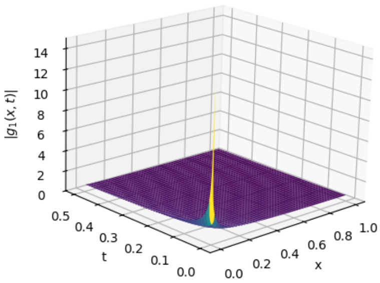
When the parameter values are set to the following: $K_1 = 2, K_2 = -1, K_3 = 1, K_4 = -1, \lambda = 1, \mu = 1, \eta = 1 - \frac{\sqrt{10}}{4}, \omega = \sqrt{10}, \delta = 4, l = 1, \alpha = 0.5, \beta = 0.5$, the representation of the solution $h_{11}(x,t)$ can be obtained as shown in Figure 2. From 3D and Cartesian/polar graphs, it can be seen that the solution $h_{11}(x,t)$ has periodicity. It is a soliton that oscillates and fluctuates over time, and is a breather.

When the parameter values are set to the following: $K_1 = 1, K_2 = 2, K_3 = 0, K_4 = -1, \lambda = 1, \mu = 1, \eta = 5, \omega = -4, \delta = 1, l = 1, \alpha = 0.5, \beta = 0.5$, the representation of the solution can be obtained as $g_{13}(x,t)$ shown in Figure 3. Solution $g_{13}(x,t)$ is a rational function wave. From 3D and Cartesian/polar graphs, it can be seen that it is a bright soliton, and its energy density exhibits symmetry.

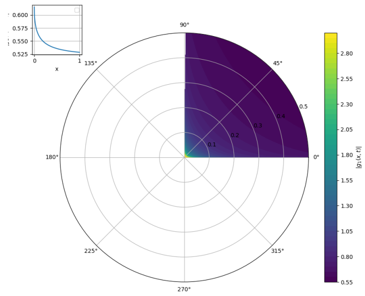
When the parameter values are set to the following: $K_1 = 1, K_2 = 0, K_3 = 3, K_4 = 1, \lambda = 1, \mu = 1, \eta = 2, \omega = -1, \delta = 1, l = \frac{\sqrt{3}}{2}, \alpha = 0.5, \beta = 0.5$, the representation of the solution $h_{17}(x,t)$ can be obtained as shown in Figure 4. Solution $h_{17}(x,t)$ is a breather. It can be seen from the polar graph that it has periodicity.

When the parameter values are set to the following: $K_1 = 2, K_2 = 1, K_3 = 0, K_4 = 1, \lambda = 1, \mu = 1, \eta = 1 + \frac{3}{4}\sqrt{2}, \omega = \sqrt{2}, \delta = -\frac{4}{3}, l = -\frac{1}{4}, \alpha = 0.5, \beta = 0.5$, the representation of the solution $g_{19}(x,t)$ can be obtained as shown in Figure 5. From 3D and Cartesian/polar graphs, it can be concluded that

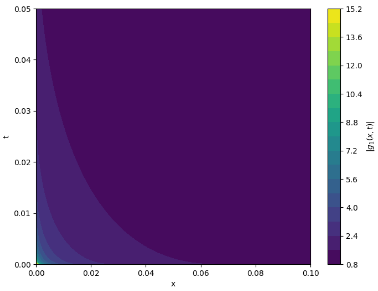
$g_{19}(x,t)$ exhibits a local depression on a uniform field with non-zero background values. Its intensity is minimal at the center and gradually increases to a constant value on both sides. It is a dark soliton.



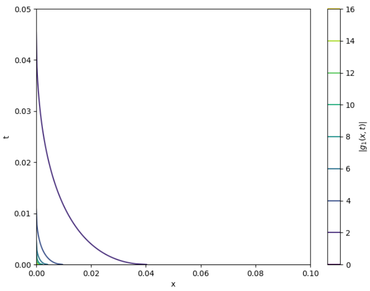
(a) 3D graph



(b) 2D Cartesian and polar graph

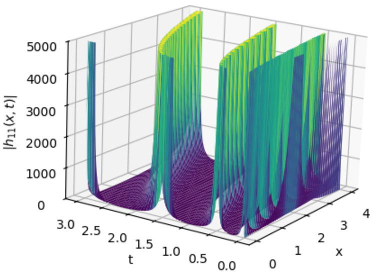


(c) Contour graph

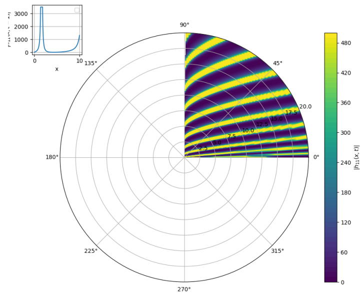


(d) Density graph

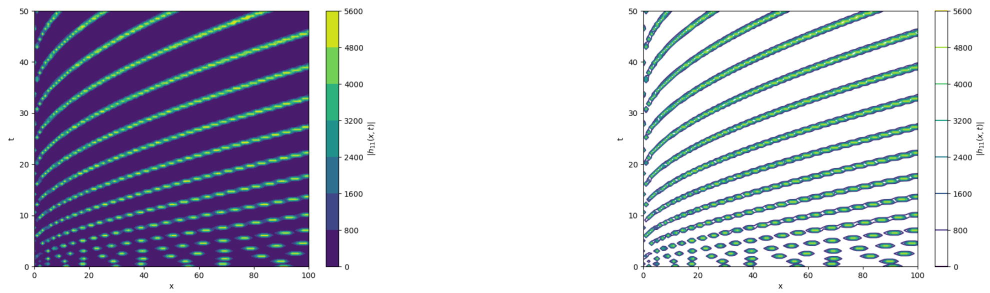
Figure 1. The solution $g_1(x,t)$ of Eq (1.1) when $K_1 = 2, K_2 = 1, K_3 = 0, K_4 = 1, \lambda = 1, \mu = 1, \eta = 1 + \frac{3}{4}\sqrt{2}, \omega = 1, \delta = -\frac{2}{3}\sqrt{2}, l = 1, \alpha = 0.5, \beta = 0.5$.



(a) 3D graph

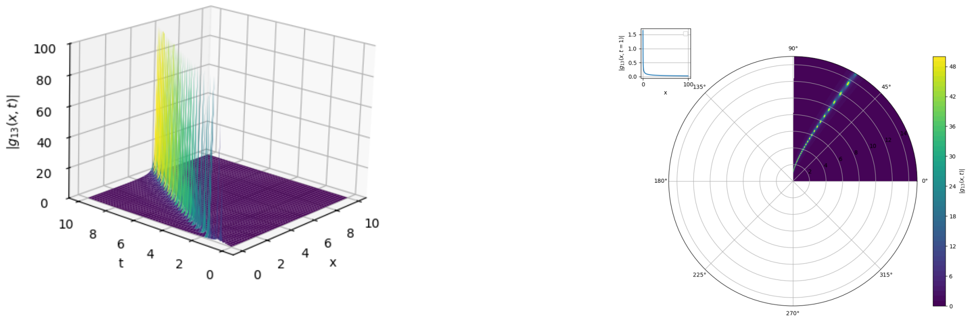


(b) 2D Cartesian and polar graph

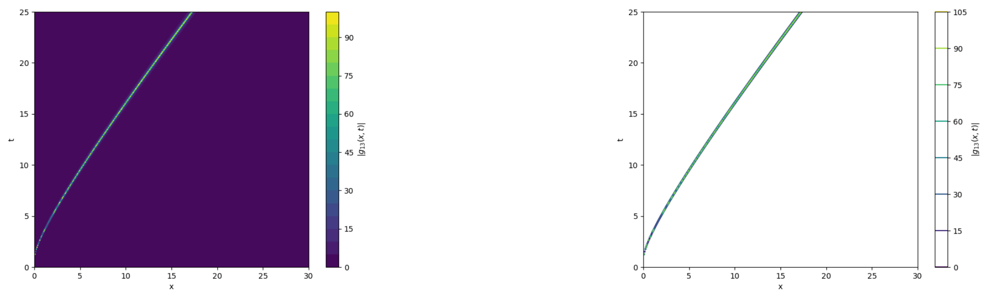


(c) Contour graph (d) Density graph

Figure 2. The solution $h_{11}(x,t)$ of Eq (1.1) when $K_1 = 2, K_2 = -1, K_3 = 1, K_4 = -1, \lambda = 1, \mu = 1, \eta = 1 - \frac{\sqrt{10}}{4}, \omega = \sqrt{10}, \delta = 4, l = 1, \alpha = 0.5, \beta = 0.5$.

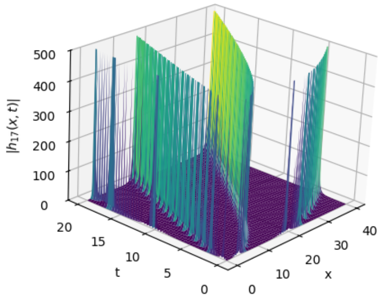


(a) 3D graph (b) 2D Cartesian and polar graph

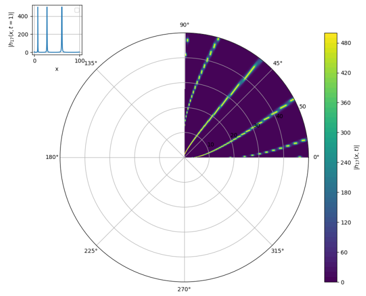


(c) Contour graph (d) Density graph

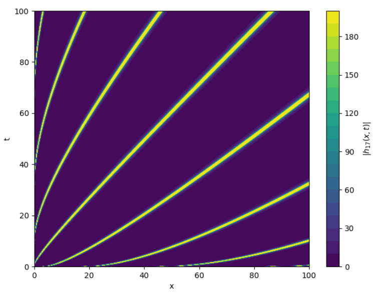
Figure 3. The solution $g_{13}(x,t)$ of Eq (1.1) when $K_1 = 1, K_2 = 2, K_3 = 0, K_4 = -1, \lambda = 1, \mu = 1, \eta = 5, \omega = -4, \delta = 1, l = 1, \alpha = 0.5, \beta = 0.5$.



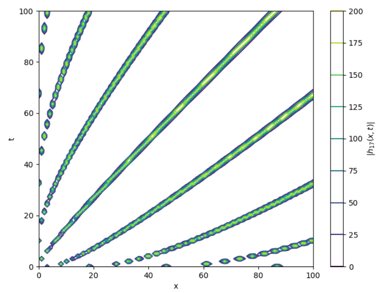
(a) 3D graph



(b) 2D Cartesian and polar graph

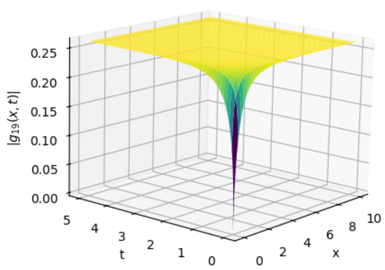


(c) Contour graph

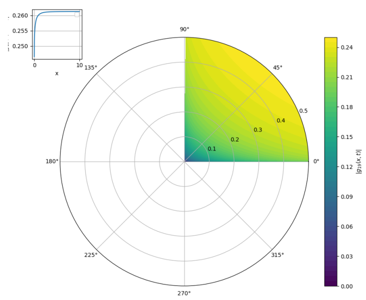


(d) Density graph

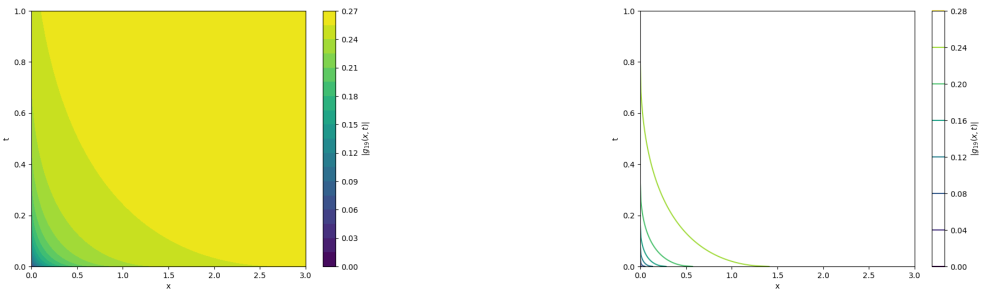
Figure 4. The solution $h_{17}(x, t)$ of Eq (1.1) when $K_1 = 1, K_2 = 0, K_3 = 3, K_4 = 1, \lambda = 1, \mu = 1, \eta = 2, \omega = -1, \delta = 1, l = \frac{\sqrt{3}}{2}, \alpha = 0.5, \beta = 0.5$.



(a) 3D graph



(b) 2D Cartesian and polar graph



(c) Contour graph (d) Density graph

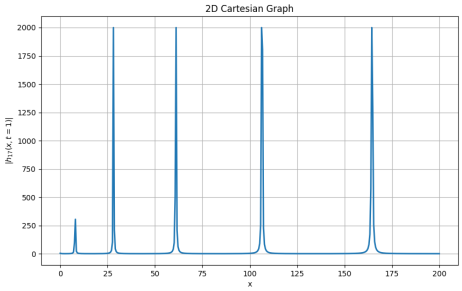
Figure 5. The solution $g_{19}(x,t)$ of Eq (1.1) when $K_1 = 2, K_2 = 1, K_3 = 0, K_4 = 1, \lambda = 1, \mu = 1, \eta = 1 + \frac{3}{4}\sqrt{2}, \omega = \sqrt{2}, \delta = -\frac{4}{3}, l = -\frac{1}{4}, \alpha = 0.5, \beta = 0.5$.

4.2. The influence pattern of fractional-order numbers α and β on the solutions

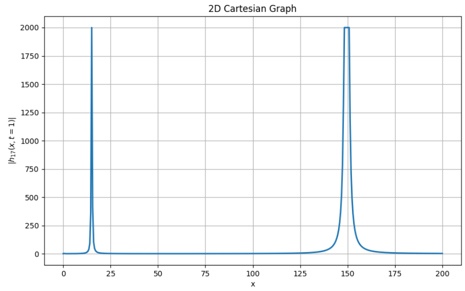
The parameters α and β in the truncated M-fractional derivative $D_{M,x}^{\alpha,\beta} f(x)$ are the core control parameters for waveform evolution, parameter α regulates spatial scale changes through the power function term $x^{1-\alpha}$, while β alters the equivalent derivative coefficient through the gamma function $\Gamma(\beta + 1)$. Together, they modify the construction form of the transformation variable $\xi = \frac{\Gamma(\beta+1)}{\alpha}(x^\alpha - \delta t^\alpha)$, ultimately altering the independent variable structure of all analytical solutions. Keeping all other coefficients of the fixed model $K_1, K_2, K_3, K_4, \lambda, \mu, \eta, \omega, \delta, l$ unchanged, we employ the control variable method to conduct grouped simulation experiments to analyze the impact of parameters α and β on the system.

Comparing Figures 6(a) and (b) in the diagram, when $\beta = 0.5$ and α changes from 0.5 to 0.2, the oscillation period of the breather becomes longer. This change carries different implications in different physical contexts. For instance, in fluid mechanics, early disturbances continue to accumulate, solitary waves continuously expand, and dissipation slows down. In optical fiber communication, the memory effect is pronounced, historical optical fields continue to couple and overlap, solitons are broadened, peak values decrease, and pulse dispersion intensifies. In the dynamics of ferromagnetic materials, excessive memory leads to the dispersion of domain wall solitons and the broadening of waveforms.

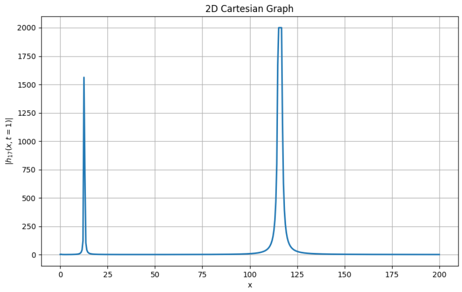
Comparing Figure 6(a) and (d) in the diagram, when $\alpha = 0.5$ and β changes from 0.5 to 0.9, the amplitude of the breather increases. In fluid mechanics, an increase in *beta* leads to an increase in fluid viscous damping, which in turn accelerates the periodic loss and replenishment of water energy, resulting in violent oscillations of fluid breathing waves. In optical fiber communication, the dispersion effect of the optical fiber is enhanced, leading to an increased imbalance between dispersion and nonlinearity. This results in stronger periodic stretching and contraction of the optical field, as well as a larger amplitude and shorter period of breathing mode oscillation. In the dynamics of ferromagnetic materials, the increase of β enhances the domain wall damping, leading to the periodic accumulation of magnetic energy, which in turn results in a more intense release and a stronger oscillation of the magnetic breathing sub-oscillation.



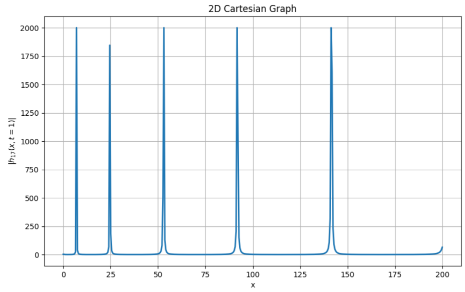
(a) $\alpha = 0.5, \beta = 0.5$



(b) $\alpha = 0.2, \beta = 0.5$



(c) $\alpha = 0.2, \beta = 0.9$



(d) $\alpha = 0.5, \beta = 0.9$

Figure 6. 2D Cartesian graph of the solution $h_{17}(x, t)$ when $K_1 = 1, K_2 = 0, K_3 = 3, K_4 = 1, \lambda = 1, \mu = 1, \eta = 2, \omega = -1, \delta = 1, l = \frac{\sqrt{3}}{2}$.

5. Comparison and novelty analysis of solutions

This study is the first to employ the generalized (G$'$/G)-expansion method within the truncated M-fractional derivative to solve the coupled TMF-SIIAE, introducing a novel solution structure due to the coupling terms. Compared to previous research on integer-order or other fractional-order Shynaray-IIA equations in literature [21–24], which mainly utilized the ϕ^6-model expansion method, the improved modified Sardar sub-equation and the improved tan(Φ/2)-expansion method, yielding only hyperbolic function and trigonometric function solitons, this paper adopts the generalized (G$'$/G)-expansion method to obtain over forty seven sets of six major categories of novel analytical solutions, encompassing rational-type solitons, composite oscillatory breathing solitons, and mixed-form bright-dark solitons.

6. Conclusions

In this research work, we focus on a class of NFPEDs (named the TMF-SIIAE) that has important applications in fields such as fiber optic communication, magnetodynamics of ferromagnetic materials, and fluid dynamics. By using the mathematical tool (the truncated M-fractional derivative and the special traveling wave transformation), the TMF-SIIAE is transformed from a nonlinear fractional-order equation to a nonlinear integer-order equation, and then the form and properties of the solutions to the equation are discussed. By transitioning from classical (G$'$/G)-expansion method to the generalized

(G'/G)-expansion method, we have obtained the method based on auxiliary equations, which has a wide range of application scenarios. By establishing algebraic equation systems for the coefficients in the auxiliary equations and the coefficients in the original equations, and solving them, we obtained six groups of coefficient solution systems. Based on the classification of the coefficient conditions in the auxiliary equations, we obtain the novel wave solutions of the original equations of different classifications under the six groups of coefficient solution systems, including bright solitons, dark solitons, and breathers. Finally, the behavior of five types of solutions are depicted through numerical calculations and Python software visualization. A 3D graph, Cartesian/polar graph, contour graph, and density graph better reflect the characteristics of the novel wave solutions.

Authors contributions

Kun Zhang: Writing—original draft; Zihao Ge: Software; Jiangping Cao: Writing—review and editing.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This project is supported by Key Laboratory of Pattern Recognition and Intelligent Information Processing of Sichuan Province (MSSB-2025-08).

Conflict of interest

The authors declare there is no conflict of interest.

References

1. M. M. Al-Gharabli, Stability results for a system of nonlinear viscoelastic plate equations with nonlinear frictional damping and logarithmic source terms, *J. Dyn. Control Syst.*, **30** (2024), 3. https://doi.org/10.1007/s10883-023-09676-8
2. H. Su, Y. Yu, J. Guo, Z. Yan, Prolongation structures of the super mixed derivative nonlinear Schrödinger equation, *Phys. Scr.*, **100** (2025), 035225. https://doi.org/10.1088/1402-4896/adb224
3. L. Song, J. Fan, D. R. Chen, D. X. Zhou, Approximation of nonlinear functionals using deep ReLU networks, *J. Fourier Anal. Appl.*, **29** (2023), 50. https://doi.org/10.1007/s00041-023-10027-1
4. L. Yu, X. Wang, Semi-local convergence of a sixth-order iterative method for solving nonlinear systems, *Electron. Res. Arch.*, **33** (2025), 3794–3810. https://doi.org/10.3934/era.2025168
5. A. Mateen, W. Haider, A. Shehzadi, H. Budak, B. Bin-Mohsin, Numerical approximations and fractional calculus: Extending Boole's rule with Riemann–Liouville fractional integral inequalities, *Fractal Fract.*, **9** (2025), 52. https://doi.org/10.3390/fractalfract9010052

6. L. Mohan, A. Prakash, Analysis of nonlinear convection-diffusion model with fractional operator by two efficient numerical techniques, *Mod. Phys. Lett. A*, **40** (2025), 26. https://doi.org/10.1142/S0217732325500981

7. Z. Li, E. Hussain, Qualitative analysis and traveling wave solutions of a (3+1)-Dimensional generalized nonlinear Konopelchenko-Dubrovsky-Kaup-Kupershmidt system, *Fractal Fract.*, **9** (2025), 285. https://doi.org/10.3390/fractalfract9050285

8. K. Zhang, J. Cao, J. Lyu, Dynamic behavior and modulation instability for a generalized nonlinear Schrödinger equation with nonlocal nonlinearity, *Phys. Scr.*, **100** (2025), 015262. https://doi.org/10.1088/1402-4896/ad9cfa

9. K. Zhang, J. Cao, Dynamic behavior and modulation instability of the generalized coupled fractional nonlinear Helmholtz equation with cubic-quintic term, *Open Phys.*, **23** (2025), 20250144. https://doi.org/10.1515/phys-2025-0144

10. H. F. Ismael, İ. Okumuş, T. Aktürk, H. Bulut, M. S. Osman, Analyzing study for the 3D potential Yu-Toda-Sasa-Fukuyama equation in the two-layer liquid medium, *J. Ocean Eng. Sci.*, **9** (2024), 454–460. https://doi.org/10.1016/j.joes.2022.03.017

11. L. Akinyemi, F. Erebholo, V. Palamara, K. Oluwasegun, A study of nonlinear Riccati equation and its applications to multi-dimensional nonlinear evolution equations, *Qual. Theory Dyn. Syst.*, **23** (2024), 296. https://doi.org/10.1007/s12346-024-01137-2

12. K. J. Wang, X. L. Liu, W. D. Wang, S. Li, H. W. Zhu, Novel singular and non-singular complexiton, interaction wave and the complex multi-soliton solutions to the generalized nonlinear evolution equation, *Mod. Phys. Lett. B*, **39** (2025), 2550135. https://doi.org/10.1142/S0217984925501350

13. K. J. Wang, K. H. Yan, S. Li, Multi-rogue wave, generalized breathers wave, bell shape and singular wave solutions to the (3+1)-dimensional Yu-Toda-Sasa-Fukuyama equation, *Math. Methods Appl. Sci.*, (2026), 1–14. https://doi.org/10.1002/mma.70663

14. K. J. Wang, K. H. Yan, J. Cheng, Y. B. Zheng, F. Shi, H. W. Zhu, et al., Bilinear form, Bäcklund transformation to the Kairat-II-X-extended equation: N-soliton, anti-kink soliton, novel soliton molecule, multi-lump and travelling wave solutions, *Mod. Phys. Lett. B*, **48** (2026), 2650057. https://doi.org/10.1142/S0217984926500570

15. Z. Li, M. Wu, E. Hussain, Y. Yildirim, Exactly explicit solutions of (2+1)-dimensional conformable fractional diffusive Predator-Prey model via neural networks method, *Front. Phys.*, **14** (2026), 1824530. https://doi.org/10.3389/fphy.2026.1824530

16. K. J. Wang, Exploring exact wave solutions of the Cahn-Allen equation via a novel Bernoulli sub-equation neural networks method, *Mod. Phys. Lett. B*, **40** (2026), 2650062. https://doi.org/10.1142/S0217984926500624

17. P. Wang, X. Feng, S. He, Lie symmetry reduction for (2+1)-dimensional fractional Schrödinger equation, *J. Appl. Anal. Comput.*, **15** (2025), 502–516. https://doi.org/10.11948/20240133

18. P. R. Gordoa, A. Pickering, The extended second Painlevé hierarchy: Auto-Bäcklund transformations and special integrals, *J. Differ. Equ.*, **417** (2025), 132–163. https://doi.org/10.1016/j.jde.2024.11.017

19. I. Addai, H. K. Asiedu, B. Barnes, The exact solution of the composite fractional differential equation, *Chaos Solitons Fractals*, **195** (2025), 116288. https://doi.org/10.1016/j.chaos.2025.116288
20. J. J. Yang, S. F. Tian, Z. Q. Li, The modified Camassa-Holm equation with nonzero background: Soliton resolution conjecture and asymptotic stability of N-soliton solutions, *Adv. Math.*, **484** (2026), 110552. https://doi.org/10.1016/j.aim.2025.110552
21. G. H. Tipu, W. A. Faridi, D. Rizk, Z. Myrzakulova, R. Myrzakulov, L. Akinyemi, The optical exact soliton solutions of Shynary-IIA equation with ϕ^6-model expansion approach, *Opt. Quantum Electron.*, **56** (2024), 226. https://doi.org/10.1007/s11082-023-05814-5
22. M. I. Khan, D. N. K. Marwat, J. Sabi'u, M. Inc, Exact solutions of Shynaray-IIA equation (S-IIAE) using the improved modified Sardar sub-equation method, *Opt. Quantum Electron.*, **56** (2024), 459. https://doi.org/10.1007/s11082-023-06051-6
23. Y. Sağlam Özkan, The exact solutions of the Shynaray-IIA equation along with analysis of bifurcation and chaotic behaviors, *Qual. Theory Dyn. Syst.*, **23** (2024), 288. https://doi.org/10.1007/s12346-024-01153-2
24. A. Altalbe, A. Taishiyeva, R. Myrzakulov, A. Bekir, A. A. Zaagan, Effect of truncated M-fractional derivative on the new exact solitons to the Shynaray-IIA equation and stability analysis, *Results Phys.*, **57** (2024), 107422. https://doi.org/10.1016/j.rinp.2024.107422
25. M. Wang, X. Li, J. Zhang, The $(\frac{G'}{G})$-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, *Phys. Lett. A*, **372** (2008), 417–423. https://doi.org/10.1016/j.physleta.2007.07.051
26. Z. Li, Optical solutions of the nonlinear Kodama equation with the M-Truncated derivative via the extended $(\frac{G'}{G})$-expansion method, *Fractal Fract.*, **9** (2025), 300. https://doi.org/10.3390/fractalfract9050300
27. M. A. Alomair, M. D. Junjua, Analysis of truncated M-fractional mathematical and physical (2+1)-dimensional nonlinear Kadomtsev-Petviashvili-Modified Equal-Width model, *Fractal Fract.*, **8** (2024), 442. https://doi.org/10.3390/fractalfract8080442
28. K. Wang, Exploring the fractal exothermic reactions model with constant heat source and porous media via a novel fractal physics-informed neural networks method, *Fractals*, (2026), 2650087. https://doi.org/10.1142/S0218348X26500878
29. K. Wang, On a high-pass filter described by local fractional derivative, *Fractals*, **28** (2020), 2050031. https://doi.org/10.1142/S0218348X20500310