Networks and Heterogeneous Media

https://www.aimspress.com/journal/nhm

NHM, 21(1): 198–212.
DOI: 10.3934/nhm.2026009
Received: 20 December 2025
Revised: 20 January 2026
Accepted: 27 January 2026
Published: 03 February 2026

Research article

Reachable set bounding for delayed memristive neural networks via adaptive control

Jiemei Zhao, Ning Wu and Xiaowu Zhou*

School of Mathematics and Computer Science, Wuhan Polytechnic University, Wuhan 430023, China

* **Correspondence:** Email: xiaowuzhou102@163.com; Tel: +8613349851121.

Abstract: This article is concerned with the reachable set estimation (RSE) for delayed memristive neural networks (MNNs). By exploiting the differential inclusion theory and inequality techniques, the RSE problem of MNNs was investigated. A memoryless adaptive controller was designed to realize that states of MNNs converge to a bounded region. Based on this result, an updated memoryless adaptive controller was designed, which further removed the restriction that the delay derivative must be less than 1, leading to a more general result. The new results were presented in the form of algebraic criteria, which were straightforward to verify. Ultimately, the effectiveness of the proposed criteria was demonstrated through two numerical simulations.

Keywords: memristive neural networks; reachable set; adaptive control; time delay; Filippov solution

1. Introduction

Neural networks, as computational models designed to mimic the information-processing mechanisms of biological nervous systems, have become one of the core architectures in artificial intelligence and high-efficiency computing research [1, 2]. Memristive neural networks (MNNs) have emerged as a transformative architecture in neuromorphic computing, leveraging the unique properties of memristors (nonlinear resistive elements with memory) to emulate synaptic behavior in artificial neural systems [3]. The discovery of memristors at the nano-scale has enabled the development of neural networks that closely mimic biological brain functions, offering advantages such as non-volatile memory, analog tunability, and high density-integration. These characteristics make MNNs particularly suitable for applications in artificial intelligence, image recognition, and personalized medicine, where energy efficiency and compact design are critical [4–6]. Moreover, MNNs address the limitations of von Neumann architectures by integrating memory and processing, thus overcoming latency and energy bottlenecks. As can be seen from the above, MNNs not only provide an ideal physical carrier for realizing spiking neural networks that more closely approximate the neurodynamic characteristics of biological systems, but also lay an important theoretical and hardware foundation for exploring cutting-edge directions such as neuromorphic computing, memory-computing integrated architectures, and edge intelligence [7]. They represent an indispensable research pathway for artificial intelligence to move toward an efficient, adaptive, and low-power future.

The presence of time delays further complicates MNNs, as delays amplify uncertainties and may lead to oscillatory or chaotic behaviors [8]. In general, the analysis of delayed MNNs employs methods such as those based on linear matrix inequalities and algebraic approaches. Owing to the simpler formulation, the algebraic methods are generally easier to verify and hence find wider application. Algebraic methods have been effectively applied in MNNs, such as stability and stabilization analysis [9, 10], finite-time synchronization [11, 12], as well as quasi-projective synchronization [13].

Various disturbances and uncertainties exist in practical systems, making robustness analysis crucial [14, 15]. Reachable set estimation (RSE) is often used to quantitatively assess the robustness of systems. RSE is a critical tool in control theory for analyzing the behavior of dynamic systems under uncertainties and external disturbances. It involves computing a bounded set that contains all possible system state trajectories starting from a given set of initial conditions, subject to constrained inputs or perturbations [16]. For instance, in aircraft landing systems, reachable set analysis ensures that flight trajectories remain within safe operational envelopes under fault conditions. The RSE problem finds applications in many systems, such as singular systems [16, 17], Markov jump systems [18], switched systems [19, 20], genetic regulatory networks [21, 22], etc. In the context of neural networks, RSE helps evaluate safety margins and resilience against destabilizing factors [23, 24]. The problem of RSE of complex-valued neural networks using an event-triggered approach and cyber-attacks was addressed in [23]. The problem of RSE for nonlinear Markovian networked systems subject to denial-of-service (DoS) attacks was investigated in [24].

Similarly, for MNNs, it provides guarantees about the network's stability and convergence, which are vital for applications like neural inference or real-time signal processing. The study of reachable set bounding for delayed MNNs sit at the intersection of neuromorphic engineering and robust control theory. By developing accurate bounding methods, researchers can enhance the reliability of MNNs in applications ranging from autonomous systems to neuroprosthetics. Therefore, it has attracted significant research interest in recent years. A nonreduced-order method was employed to investigate the reachable set bounding problem of inertial MNNs with bounded input disturbances in [25]. The RSE of complex-valued inertial MNNs with bounded disturbances was studied in [26]. Based on the Gronwall-Bellman inequality, the result on the states of complex-valued MNNs converging within a sphere was derived in [27]. By employing the nonreduced-order method and reduced-order method, novel algebraic conditions were derived to estimate the states of the considered inertial MNNs in [28]. Reference [29] focused on reachable set bounding for MNNs with bounded input disturbances. However, the literature [25–29] imposes relatively strict requirements on the time delay, specifically requiring that the derivative of the time delay be less than one.

Based upon the arguments, deriving tight and computable bounds for the reachable set is crucial to ensuring system reliability while minimizing computational conservatism. Therefore, we attempt to investigate the RSE problem for MNNs. The main contributions of this paper are listed as follows:

i) By employing inequality techniques and differential inclusion theory, algebraic conditions are performed to ascertain the RSE stabilization criteria of the underlying MNNs in Filippovs sense.

ii) A memoryless adaptive controller and a memory adaptive controller are proposed to ensure that the states of MNNs converge to a bounded region.

iii) Two new criteria for RSE are presented. One result is the condition that the derivative of the delay is less than 1. The other removes the constraint on the delay derivative. In addition, a feature of the proposed approach is that the resulting criteria are expressed in algebraic forms, facilitating straightforward verification.

Notations: $\mho = \{1, 2, \cdots, n\}$. A_{mj} denotes the maximum value of $|\bar{a}_{mj}|$ and $|\underline{a}_{mj}|$. B_{mj} is the maximum of $|\bar{b}_{mj}|$ and $|\underline{b}_{mj}|$. C_{mj} denotes the maximum of $|\bar{c}_{mj}|$ and $|\underline{c}_{mj}|$. $\bar{F} = max\{F_m^2 | m \in \mho\}, \bar{G} = max\{G_m^2 | m \in \mho\}$, $r(\nu) = (r_1(\nu), r_2(\nu), \cdots, r_n(\nu))^T$, and $\psi(s) = (\psi_1(s), \psi_2(s), \cdots, \psi_n(s))^T$.

2. Preliminaries

The delayed MNN model is described as follows:

$$\begin{aligned} \dot{r}_m(\nu) &= -d_m r_m(\nu) + \sum_{j=1}^{n} a_{mj}(r_m(\nu)) f_j\left(r_j(\nu)\right) \\ &\quad + \sum_{j=1}^{n} b_{mj}(r_m(\nu)) g_j\left(r_j\left(\nu - \rho_j(\nu)\right)\right) + \sum_{j=1}^{n} c_{mj}(r_m(\nu))\vartheta_j(\nu) + U_m(\nu), \nu \geq 0, m \in \mho, \end{aligned} \tag{2.1}$$

where $r_m(\nu)$ is the voltage of the capacitor $\mathbf{C}_m$ at time ν, $f_j(r_j(\nu))$ and $g_j(r_j(\nu - \rho_j(\nu)))$ represent the activation functions, and $\rho_j(\nu)$ is the delay and satisfies $0 \leq \rho_j(\nu) \leq \rho$. $d_m > 0$ denotes the mth neuron self-inhibitions at time t. $a_{mj}(r_m(\nu))$, $b_{mj}(r_m(\nu))$, and $c_{mj}(r_m(\nu))$ are the state-based memristors synaptic connection weights, and $a_{mj}(r_m(\nu)) = \frac{\mathbf{M}_{f,mj}}{\mathbf{C}_m} \times \text{sign}_{mj}$, $b_{mj}(r_m(\nu)) = \frac{\mathbf{M}_{g,mj}}{\mathbf{C}_m} \times \text{sign}_{mj}$ with $\text{sign}_{f,mj} = \begin{cases} 1, m \neq j \\ -1, m = j \end{cases}$. $\mathbf{M}_{f,mj}$ and $\mathbf{M}_{g,mj}$ are the memductances of memristors. $\vartheta_j(\nu)$ represents the bounded peak disturbance and

$$|\vartheta_j(\nu)| \leq \bar{\vartheta}, \tag{2.2}$$

with a constant scalar $\bar{\vartheta} > 0$.

The parameters governing the memristor's behavior, as derived from its intrinsic properties and current-voltage relationship, are as follows:

$$a_{mj}(r_m(\nu)) = \begin{cases} a_{mj}^*, |r_m(\nu)| \leq \Gamma_m, \\ a_{mj}^{**}, |r_m(\nu)| > \Gamma_m, \end{cases} \quad b_{mj}(r_m(\nu)) = \begin{cases} b_{mj}^*, |r_m(\nu)| \leq \Gamma_m, \\ b_{mj}^{**}, |r_m(\nu)| > \Gamma_m, \end{cases} \quad c_{mj}(r_m(\nu)) = \begin{cases} c_{mj}^*, |r_m(\nu)| \leq \Gamma_m, \\ c_{mj}^{**}, |r_m(\nu)| > \Gamma_m, \end{cases}$$

where $\Gamma_m > 0$ are switching jumps. $a_{mj}^*, a_{mj}^{**}, b_{mj}^*, b_{mj}^{**}, c_m^*, c_m^{**}, m, j \in \mho$ are constants.

In the analysis of the delayed MNN (2.1), we make the following assumptions to facilitate the proof of the main theorems.

Assumption 1. Time-varying delay $\rho_j(\nu)$ satisfies $\dot{\rho}_j(\nu) \leq \sigma < 1$, where σ is a positive constant.

Assumption 2. It is assumed that the activation functions f_j and g_j ($j \in \mho$) are bounded and further, that for all $\varsigma_1, \varsigma_2 \in \mathcal{R}$, the following condition holds:

$$\left|\frac{f_j(\varsigma_1) - f_j(\varsigma_2)}{s_1 - s_2}\right| \leq F_j, \left|\frac{g_j(\varsigma_1) - g_j(\varsigma_2)}{\varsigma_1 - \varsigma_2}\right| \leq G_j, \tag{2.3}$$

with positive constants $\varsigma_1 \neq \varsigma_2$, F_j, G_j for all $j \in \mho$, and with the initial condition $f_j(0) = g_j(0) = 0$ assumed for each $j \in \mho$.

Definition 1. [30] Let $E \subset \mathcal{R}^n$ and $r \mapsto F(r)$ is called a set-valued map from $E \hookrightarrow \mathcal{R}^n$, if for each point r of a set $E \subset \mathcal{R}^n$, there corresponds a nonempty set $F(r) \subset \mathcal{R}^n$. A set-valued map F with nonempty values is said to be upper semicontinuous at $r_0 \in E \subset \mathcal{R}^n$ if, for any open set N containing $F(r_0)$, there exists a neighborhood N_0 of r_0 such that $F(N_0) \subset N$. $F(r)$ is said to have a closed (convex, compact) image if for each $r \in E$, $F(r)$ is closed (convex, compact).
Definition 2. [30] For system $(dr/d\nu) = F(r)$, $r \in \mathcal{R}^n$, with discontinuous right-hand sides, a set-valued map is defined as $F(r) = \bigcap_{\delta>0} \bigcap_{\mu(N)=0} \overline{co}\,[F(B(r,\delta) \setminus N)]$, where $\overline{co}$ is the closure of the convex hull, $B(r,\delta) = \{y : \|y - r\| \le \delta\}$, and $\mu(N)$ is the Lebesgue measure of set N.
Definition 3. [30] A function $r(\nu)$ (in Filippov's sense) is a solution of MNN (2.1) with initial conditions $\psi(s)$, if $r(\nu)$ is an absolutely continuous function and satisfies the differential inclusion

$$\begin{aligned} \dot{r}_m(\nu) \in\; & -d_m r_m(\nu) + \sum_{j=1}^{n} \mathrm{co}[\underline{a}_{mj}, \bar{a}_{mj}] f_j\left(r_j(\nu)\right) \\ & + \sum_{j=1}^{n} \mathrm{co}[\underline{b}_{mj}, \bar{b}_{mj}] g_j\left(r_j\left(\nu - \rho_j(\nu)\right)\right) + \sum_{j=1}^{n} \mathrm{co}[\underline{c}_{mj}, \bar{c}_{mj}] \vartheta_j(\nu) + U_m(\nu), t \ge 0, \end{aligned} \tag{2.4}$$

where

$$\mathrm{co}[\underline{a}_{mj}, \bar{a}_{mj}] = \begin{cases} a^{*}_{mj}, & \text{if } |r_m(\nu)| < \Gamma_m, \\ [\underline{a}_{mj}, \bar{a}_{mj}], & \text{if } |r_m(\nu)| = \Gamma_m, \\ a^{**}_{mj}, & \text{if } |r_m(\nu)| > \Gamma_m, \end{cases} \tag{2.5}$$

$$\mathrm{co}[\underline{b}_{mj}, \bar{b}_{mj}] = \begin{cases} b^{*}_{mj}, & \text{if } |r_m(\nu)| < \Gamma_m, \\ [\underline{b}_{mj}, \bar{b}_{mj}], & \text{if } |r_m(\nu)| = \Gamma_m, \\ b^{**}_{mj}, & \text{if } |r_m(\nu)| > \Gamma_m, \end{cases} \tag{2.6}$$

$$\mathrm{co}[\underline{c}_{mj}, \bar{c}_{mj}] = \begin{cases} c^{*}_{mj}, & \text{if } |r_m(\nu)| < \Gamma_m, \\ [\underline{c}_{mj}, \bar{c}_{mj}], & \text{if } |r_m(\nu)| = \Gamma_m, \\ c^{**}_{mj}, & \text{if } |r_m(\nu)| > \Gamma_m, \end{cases} \tag{2.7}$$

with

$$\bar{a}_{mj} = \max\{a^{*}_{mj}, a^{**}_{mj}\}, \underline{a}_{mj} = \min\{a^{*}_{mj}, a^{**}_{mj}\},$$
$$\bar{b}_{mj} = \max\{b^{*}_{mj}, b^{**}_{mj}\}, \underline{b}_{mj} = \min\{b^{*}_{mj}, b^{**}_{mj}\},$$
$$\bar{c}_{mj} = \max\{c^{*}_{mj}, c^{**}_{mj}\}, \underline{c}_{mj} = \min\{c^{*}_{mj}, c^{**}_{mj}\},$$

for $m, j \in \mho$. Or equivalently, for $m, j \in \mho$, there exist $\tilde{a}_{mj} \in \mathrm{co}[\underline{a}_{mj}, \bar{a}_{mj}], \tilde{b}_{mj} \in \mathrm{co}[\underline{b}_{mj}, \bar{b}_{mj}]$, and $\tilde{c}_{mj} \in \mathrm{co}[\underline{c}_{mj}, \bar{c}_{mj}]$ such that

$$\begin{aligned} \dot{r}_m(\nu) =\; & -d_m r_m(\nu) + \sum_{j=1}^{n} \tilde{a}_{mj} f_j\left(r_j(\nu)\right) \\ & + \sum_{j=1}^{n} \tilde{b}_{mj} g_j\left(r_j\left(\nu - \rho_j(\nu)\right)\right) + \sum_{j=1}^{n} \tilde{c}_{mj} \vartheta_j(\nu) + U_m(\nu), \nu \ge 0, m \in \mho. \end{aligned} \tag{2.8}$$

Lemma 1. [30] Under Assumption 1, the delayed MNN model (2.1) with initial condition $\psi(s) \in C(-\rho, 0], \mathcal{R}^n)$ admits at least one local solution $r(\nu)$. Furthermore, this local solution can be extended to the entire interval $[0, +\infty)$ in the sense of Filippov.

The RSE problem aims to find an optimally small region such that it bounds the reachable set of the delayed MNN (2.1) under the constraint specified in condition (2.2). Generally, the RSE of delayed MNN (2.1) contains the following problems.

Problem 1. The objective is to bound the reachable set (or design a controller) such that all state trajectories of the delayed MNN (2.1) are confined within an ellipsoid.

The reachable set can be formally defined as the collection of all possible system states reachable from a given initial set under specified inputs and constraints, denoted typically by

$$\mathcal{Z}_r := \{r(\nu) \in \mathcal{R}^n | r(\nu), \vartheta(\nu) \text{ satisfy Eqs (2.1) and (2.2)}, \nu \geq 0\}. \tag{2.9}$$

Given a positive definite matrix $P > 0$, an ellipsoid $o(P, 1)$ with the purpose of enclosing the reachable set Eq (2.9) is given by

$$o(P, 1) := \{r(\nu) \in \mathcal{R}^n | r^T(\nu) P r(\nu) \leq 1\}. \tag{2.10}$$

Problem 2. The ellipsoid of RSE should be as small as possible. To minimize the size of the ellipsoid $\mathcal{Z}_r$ defined in Eq (2.9), we apply the optimization method from [31]. This involves maximizing ϱ subject to $\varrho I \leq P$. Equivalently, the problem can be transformed to minimize $\bar{\varrho}$ (where $\bar{\varrho} = 1/\varrho$) subject to

$$\begin{pmatrix} \bar{\varrho} I & I \\ I & P \end{pmatrix} \geq 0. \tag{2.11}$$

To establish the RSE of MNN (2.1), we next introduce a key mathematical tool.

Lemma 2. [32] Let $\mathbb{V}(r(\nu))$ be a non-negative function, $\mathbb{V}(r(\nu_0)) \leq \frac{\alpha\bar{\vartheta}^2}{\varepsilon}$, $\nu_0 \geq 0, \varepsilon > 0$, and $\alpha > 0$. If

$$\dot{\mathbb{V}}(\nu) + \varepsilon \mathbb{V}(\nu) - \alpha \vartheta^2(\nu) \leq 0, \tag{2.12}$$

then $\mathbb{V}(r(\nu)) \leq \frac{\alpha\bar{\vartheta}^2}{\varepsilon}, \forall \nu \geq 0$.

3. Main results

In the following, two kinds of adaptive controllers will be proposed. Theorem 1 analyzes the RSE problem of delayed MNN (2.1) under Assumptions 1 and 2. Theorem 2 removes the constraint on the delay derivative in Assumption 1.

3.1. RSE for MNN (2.1) with memoryless adaptive control

Now, a memoryless adaptive controller of MNN (2.1) is considered. It is designed as follows:

$$U_m(\nu) = -\beta_m(\nu) r_m(\nu), \tag{3.1}$$

and the update law $\dot{\beta}_m(\nu) = \lambda_m \gamma_m e^{\varepsilon t} r_m^2(\nu)$, where λ_m, γ_m, and ε are positive constants.

Theorem 1. Given positive integers $\sigma, \lambda_m, \gamma_m, F_m, G_m, m \in \mho$ and ε, consider the delayed MNN (2.1) under Assumptions 1 and 2, and bounded peak disturbance (2.2). If

$$\sum_{j=1}^{n} \frac{\lambda_m C_{mj}}{2} - \frac{\varepsilon}{\bar{\vartheta}^2} < 0 \tag{3.2}$$

holds, then the ellipsoid reachable set $o(P,1)$ of MNN (2.1) with the adaptive scheme (3.1) can be obtained.

Proof. Define a non-negative function

$$\mathbb{V}(\nu) = \sum_{m=1}^{n} \mathbb{V}_m(\nu), \tag{3.3}$$

where

$$\mathbb{V}_m(\nu) = \frac{1}{2}\lambda_m r_m^2(\nu) + \frac{e^{-\varepsilon\nu}}{2\gamma_m}(\beta_m^* - \beta_m(\nu))^2 + \frac{1}{2(1-\sigma)}\sum_{j=1}^{n}\int_{\nu-\rho_j(\nu)}^{\nu} e^{\varepsilon(s-\nu)} g_j^2(r_j(s))ds$$

with β_m^* as a constant to be determined.

Calculating the derivative of $\mathbb{V}_m(\nu)$, we have

$$\begin{aligned}
\dot{\mathbb{V}}_m(\nu) = {} & \lambda_m r_m(\nu)\dot{r}_m(\nu) - \frac{\varepsilon e^{-\varepsilon\nu}}{2\gamma_m}(\beta_m^* - \beta_m(\nu))^2 - \frac{e^{-\varepsilon\nu}}{\gamma_m}(\beta_m^* - \beta_m(\nu))\dot{\beta}_m(\nu) \\
& - \frac{\varepsilon}{2(1-\sigma)}\sum_{j=1}^{n}\int_{\nu-\rho_j(\nu)}^{\nu} e^{\varepsilon(s-\nu)} g_j^2(r_j(s))ds + \frac{1}{2(1-\sigma)}\sum_{j=1}^{n} g_j^2(r_j(\nu)) \\
& - \frac{1-\dot{\tau}_j(\nu)}{2(1-\sigma)}\sum_{j=1}^{n} g_j^2(r_j(\nu-\rho_j(\nu)))e^{-\varepsilon\rho_j(\nu)} \\
= {} & \lambda_m r_m(\nu)[-(d_m + \beta_m(\nu))r_m(\nu) + \sum_{j=1}^{n}\tilde{a}_{mj}(r_m(\nu))f_j(r_j(\nu)) + \sum_{j=1}^{n}\tilde{b}_{mj}(r_m(\nu))g_j(r_j(\nu-\rho_j(\nu))) \\
& + \sum_{j=1}^{n}\tilde{c}_{mj}(r_m(\nu))\vartheta_j(\nu)] - \frac{\varepsilon e^{-\varepsilon\nu}}{2\gamma_m}(\beta_m^* - \beta_m(\nu))^2 - \frac{e^{-\varepsilon\nu}}{\gamma_m}(\beta_m^* - \beta_m(\nu))\dot{\beta}_m(\nu) \\
& - \frac{\varepsilon}{2(1-\sigma)}\sum_{j=1}^{n}\int_{\nu-\rho_j(\nu)}^{\nu} e^{\varepsilon(s-\nu)} g_j^2(r_j(s))ds + \frac{1}{2(1-\sigma)}\sum_{j=1}^{n} g_j^2(r_j(\nu)) \\
& - \frac{1-\dot{\rho_j}(\nu)}{2(1-\sigma)}\sum_{j=1}^{n} g_j^2(r_j(\nu-\rho_j(\nu)))e^{-\varepsilon\rho_j(\nu)} \\
\leq {} & -\lambda_m(d_m + \beta_m^*)r_m^2(\nu) + \sum_{j=1}^{n}\lambda_m A_{mj}|r_m(\nu)||f_j(r_j(\nu))| + \sum_{j=1}^{n}\lambda_m B_{mj}|r_m(\nu)||g_j(r_j(\nu-\rho_j(\nu)))| \\
& + \sum_{j=1}^{n}\lambda_m C_{mj}|r_m(\nu)||\vartheta_j(\nu)| - \frac{\varepsilon e^{-\varepsilon\nu}}{2\gamma_m}(\beta_m^* - \beta_m(\nu))^2 - \frac{\varepsilon}{2(1-\sigma)}\sum_{j=1}^{n}\int_{\nu-\rho_j(\nu)}^{\nu} e^{\varepsilon(s-\nu)} g_j^2(r_j(s))ds \\
& + \frac{1}{2(1-\sigma)}\sum_{j=1}^{n} g_j^2(r_j(\nu)) - \frac{1}{2}\sum_{j=1}^{n} g_j^2(r_j(\nu-\rho_j(\nu)))e^{-\varepsilon\sigma}.
\end{aligned} \tag{3.4}$$

According to the conditions of Assumptions 1 and 2, there are positive real constants F_j and G_j such that

$$\lambda_m A_{mj}|r_m(\nu)||f_j(r_j(\nu))| \leq \frac{\lambda_m^2 A_{mj}^2}{2}r_m^2(\nu) + \frac{\bar{F}}{2}r_j^2(\nu), \tag{3.5}$$

$$\lambda_m B_{mj}|r_m(\nu)||g_j(r_j(\nu-\rho_j(\nu)))| \leq \frac{\lambda_m^2 B_{mj}^2 e^{\varepsilon\sigma}}{2} r_m^2(\nu) + \frac{e^{-\varepsilon\sigma}}{2} g_j^2(r_j(\nu-\rho_j(\nu))), \tag{3.6}$$

$$\lambda_m C_{mj}|r_m(\nu)||\vartheta_j(\nu)| \leq \frac{C_{mj}\lambda_m}{2}[r_m^2(\nu)+\vartheta_j^2(\nu)], \tag{3.7}$$

$$g_j^2(r_j(\nu)) \leq \bar{G} r_j^2(\nu). \tag{3.8}$$

According to Eqs (2.3) and (3.5)–(3.8), we obtain

$$\begin{aligned}\dot{\mathbb{V}}(\nu)+\varepsilon\mathbb{V}(\nu)-\frac{\varepsilon}{\bar{\vartheta}^2}\vartheta^T(\nu)\vartheta(\nu) \leq & \sum_{m=1}^{n}\left[-\lambda_m d_m-\lambda_m\beta_m^*+\frac{1}{2}\lambda_m+\sum_{j=1}^{n}\frac{\lambda_m^2A_{mj}^2+\lambda_m^2B_{mj}^2e^{\varepsilon\sigma}+\lambda_m C_{mj}}{2}\right]r_m^2(\nu)\\ &+\sum_{m=1}^{n}\sum_{j=1}^{n}\left[\frac{\bar{F}}{2}+\frac{\bar{G}}{2(1-\sigma)}\right]r_j^2(\nu)+\sum_{m=1}^{n}\sum_{j=1}^{n}\left[\frac{\lambda_m C_{mj}}{2}-\frac{\varepsilon}{\bar{\vartheta}^2}\right]\vartheta_j^2(\nu). \end{aligned}\tag{3.9}$$

Let constant $\beta_m^* = -d_m + \max_{1\leq m\leq n}\left[\sum_{j=1}^{n}\frac{\lambda_m A_{mj}^2+\lambda_m B_{mj}^2 e^{\varepsilon\sigma}+C_{mj}}{2}\right]+\frac{n\bar{F}}{2\lambda_m}+\frac{n\bar{G}}{2\lambda_m(1-\sigma)}+1$. Combining inequality (3.2) with the definition of $\mathbb{V}(\nu)$, we obtain $\dot{\mathbb{V}}(\nu)+\varepsilon\mathbb{V}(\nu)-\frac{\varepsilon}{\bar{\vartheta}^2}\vartheta^T(\nu)\vartheta(\nu)<0$. Applying Lemma 2 to this result yields $\mathbb{V}(r(\nu))\leq 1$. Consequently, from Eq (3.3), it follows that

$$\sum_{m=1}^{n}\frac{1}{2}\lambda_m r_m^2(\nu)\leq 1,$$

i.e., $r^T(\nu)Pr(\nu)\leq 1$, where $P=\frac{1}{2}\mathrm{diag}(\lambda_1,\lambda_2,\cdots,\lambda_n)$. This implies that all state trajectories of the MNN (2.1) originating from the origin remain confined to the ellipsoid $o(P,1)$. In another words, the reachable set bounding is obtained.

Remark 1. In contrast to the state feedback control in [26, 27], the adaptive control in Theorem 1 can automatically adjust its parameters to cope with uncertainties such as an unknown system model or parameter variations, thereby maintaining superior control performance.

3.2. RSE for MNN (2.1) with memory adaptive control

A memory adaptive controller of MNN (2.1) is designed as follows:

$$U_m(\nu) = -\bar{\beta}_m(\nu)r_m(\nu)-k_m(\nu)\sum_{j=1}^{n}r_j(\nu-\rho_j(\nu)), \tag{3.10}$$

with the update law $\dot{\bar{\beta}}_m(\nu)=\bar{\lambda}_m\bar{\gamma}_m e^{\bar{\varepsilon}\nu}r_m^2(\nu)$, $\dot{k}_m(\nu)=\bar{\lambda}_m\bar{\gamma}_m e^{\bar{\varepsilon}\nu}\sum_{j=1}^{n}r_m(\nu)r_j(\nu-\rho_j(\nu))$, where $\bar{\lambda}_m$, $\bar{\gamma}_m$, and $\bar{\varepsilon}$ are positive constants.

Theorem 2. Under Assumption 2, the reachable set of the delayed MNN (2.1) with adaptive controller (3.10) under disturbance (2.2) is bounded by the ellipsoid $o(P,1)$, if there exist positive scalars $\bar{\varepsilon}$, $\bar{F}_m$, $\bar{G}_m$, $\bar{\lambda}_m$, and $\bar{\gamma}_m$, $m\in\mho$ satisfying

$$\sum_{j=1}^{n}\frac{\bar{\lambda}_m C_{mj}^2}{2}-\frac{\bar{\varepsilon}}{\bar{\vartheta}^2}<0. \tag{3.11}$$

Proof. Define a non-negative function

$$\mathbb{W}(\nu) = \sum_{m=1}^{n} \mathbb{W}_m(\nu), \tag{3.12}$$

where

$$\mathbb{W}_m(\nu) = \frac{1}{2}\bar{\lambda}_m r_m^2(\nu) + \frac{e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(\bar{\beta}_m^* - \bar{\beta}_m(\nu))^2 + \frac{e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(k_m^* - k_m(\nu))^2. \tag{3.13}$$

Calculating the derivative of $\mathbb{W}_m(\nu)$, and according to the condition of Assumption 2, there are positive real constants F_j and G_j such that

$$\begin{aligned}
\dot{\mathbb{W}}_m(\nu) =\ & \bar{\lambda}_m r_m(\nu)\dot{r}_m(\nu) - \frac{\bar{\varepsilon}e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(\bar{\beta}_m^* - \bar{\beta}_m(\nu))^2 - \frac{e^{-\bar{\varepsilon}\nu}}{\bar{\gamma}_m}(\bar{\beta}_m^* - \bar{\beta}_m(\nu))\dot{\bar{\beta}}_m(\nu) \\
& - \frac{\bar{\varepsilon}e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(k_m^* - k_m(\nu))^2 - \frac{e^{-\bar{\varepsilon}\nu}}{\bar{\gamma}_m}(k_m^* - k_m(\nu))\dot{k}_m(\nu) \\
=\ & \bar{\lambda}_m r_m(\nu)[-(d_m + \bar{\beta}_m(\nu))r_m(\nu) + \sum_{j=1}^{n} \tilde{a}_{mj}(r_m(\nu))f_j(r_j(\nu)) + \sum_{j=1}^{n} \tilde{b}_{mj}(r_m(\nu))g_j(r_j(\nu - \rho_j(\nu))) \\
& + \sum_{j=1}^{n} \tilde{c}_{mj}(r_m(\nu))\vartheta_j(\nu) - k_m(\nu)\sum_{j=1}^{n} r_j(\nu - \rho_j(\nu))] - \frac{\bar{\varepsilon}e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(\bar{\beta}_m^* - \bar{\beta}_m(\nu))^2 \\
& - \frac{e^{-\bar{\varepsilon}\nu}}{\bar{\gamma}_m}(\bar{\beta}_m^* - \bar{\beta}_m(\nu))\dot{\bar{\beta}}_m(\nu) - \frac{\bar{\varepsilon}e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(k_m^* - k_m(\nu))^2 - \frac{e^{-\bar{\varepsilon}\nu}}{\bar{\gamma}_m}(k_m^* - k_m(\nu))\dot{k}_m(\nu) \\
\leq & - \bar{\lambda}_m(d_m + \bar{\beta}_m^*)r_m^2(\nu) + \sum_{j=1}^{n} \bar{\lambda}_m A_{mj}|r_m(\nu)||f_j(r_j(\nu))| + \sum_{j=1}^{n} \bar{\lambda}_m B_{mj}|r_m(\nu)||g_j(r_j(\nu - \rho_j(\nu)))| \\
& + \sum_{j=1}^{n} \bar{\lambda}_m C_{mj}|r_m(\nu)||\vartheta_j(\nu)| - \frac{\bar{\varepsilon}e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(\bar{\beta}_m^* - \bar{\beta}_m(\nu))^2 \\
& - \frac{\bar{\varepsilon}e^{-\bar{\varepsilon}\nu}}{2\bar{\gamma}_m}(k_m^* - k_m(\nu))^2 - \bar{\lambda}_m \sum_{j=1}^{n} k_m^* r_m(\nu) r_j(\nu - \rho_j(\nu)).
\end{aligned} \tag{3.14}$$

By Young's inequality and Assumption 1, we have

$$\bar{\lambda}_m A_{mj}|r_m(\nu)||f_j(r_j(\nu))| \leq \frac{\bar{\lambda}_m^2 A_{mj}^2}{2} r_m^2(\nu) + \frac{\bar{F}}{2} r_j^2(\nu),$$

$$\bar{\lambda}_m B_{mj}|r_m(\nu)||g_j(r_j(\nu - \tau(\nu)))| \leq \bar{\lambda}_m B_{mj}\bar{G}|r_m(\nu)||r_j(\nu - \tau(\nu))|,$$

$$\bar{\lambda}_m C_{mj}|r_m(\nu)||\vartheta_j(\nu)| \leq \frac{\bar{\lambda}_m}{2}(C_{mj}^2 r_m^2(\nu) + \vartheta_j^2(\nu)).$$

According to Eq (2.3), we obtain

$$\dot{\mathbb{W}}(\nu) + \bar{\varepsilon}\mathbb{W}(\nu) - \frac{\bar{\varepsilon}}{\bar{\vartheta}^2}\vartheta^T(\nu)\vartheta(\nu) \tag{3.15}$$

$$\le \sum_{m=1}^{n}\left[-\bar{\lambda}_m d_m - \bar{\lambda}_m \bar{\beta}_m^* + \frac{1}{2}\bar{\lambda}_m + \frac{n\bar{F}}{2} + \sum_{j=1}^{n}\frac{\bar{\lambda}_m^2 A_{mj}^2 + \bar{\lambda}_m C_{mj}^2}{2}\right] r_m^2(\nu)$$
$$+ \sum_{m=1}^{n}\sum_{j=1}^{n}\left[\bar{\lambda}_m B_{mj}\bar{G} + \bar{\lambda}_m k_m^*\right]|r_m(\nu)||r_j(\nu-\rho_j(\nu))| + \sum_{m=1}^{n}\sum_{j=1}^{n}\left[\frac{\bar{\lambda}_m C_{mj}^2}{2} - \frac{\bar{\varepsilon}}{\bar{\vartheta}^2}\right]\vartheta_j^2. \tag{3.16}$$

Let $\bar{\beta}_m^* = -d_m + \max\limits_{1\le m\le n}\left[\sum\limits_{j=1}^{n}\frac{\bar{\lambda}_m A_{mj}^2 + C_{mj}^2}{2}\right] + \frac{n\bar{F}}{2} + 1$ and $k_m^* = -\sum\limits_{j=1}^{n} B_{mj}\bar{G}$. Combining inequality (3.11) with the definition of $\mathbb{W}(\nu)$, we obtain $\dot{\mathbb{W}}(\nu) + \bar{\varepsilon}\mathbb{W}(\nu) - \frac{\bar{\varepsilon}}{\bar{\vartheta}^2}\vartheta^T(\nu)\vartheta(\nu) < 0$. Applying Lemma 2 to this result yields $\mathbb{W}(r(\nu)) \le 1$. Consequently, from Eq (3.13), it follows that

$$\sum_{m=1}^{n}\frac{1}{2}\bar{\lambda}_m r_m^2(\nu) \le V_m(\nu) \le 1,$$

i.e., $r^T(\nu)\bar{P}r(\nu) \le 1$, where $\bar{P} = \frac{1}{2}\mathrm{diag}(\bar{\lambda}_1, \bar{\lambda}_2, \cdots, \bar{\lambda}_n)$. This implies that all state trajectories of the MNN (2.1) originating from the origin remain confined to the ellipsoid $o(P, 1)$.

Remark 2. By adopting the memory adaptive controller, Theorem 2 relaxes the restriction on the time delay. The condition that the derivative of the time delay is less than 1 has been removed. This indicates that the result in Theorem 2 has an advantage over the literature [25–29].

4. Numerical simulations

Here, we provide a simulation example to verify the efficacy of the sufficient criteria derived in Theorems 1 and 2.

Example 1: Consider the delayed MNN as follows:

$$\dot{r}_m(\nu) = -d_m r_m(\nu) + \sum_{j=1}^{n} a_{mj}(r_m(\nu)) f_j(r_j(\nu))$$
$$+ \sum_{j=1}^{n} b_{mj}(r_m(\nu)) g_j(r_j(\nu-\rho_j(\nu))) + \sum_{j=1}^{n} c_{mj}(r_m(\nu))\vartheta_j(\nu), \nu \ge 0, m = 1, 2, \tag{4.1}$$

where

$$d_1 = 2.3,\ d_2 = 4.5,$$
$$a_{11}(r_1(\nu)) = \begin{cases} 1.07, & |r_1(\nu)| \le 1, \\ 1, & |r_1(\nu)| > 1, \end{cases} \quad a_{12}(r_1(\nu)) = \begin{cases} -0.2, & |r_1(\nu)| \le 1, \\ -0.3, & |r_1(\nu)| > 1, \end{cases}$$
$$a_{21}(r_2(\nu)) = \begin{cases} 2.6, & |r_2(\nu)| \le 1, \\ 2.4, & |r_2(\nu)| > 1, \end{cases} \quad a_{22}(r_2(\nu)) = \begin{cases} 1.5, & |r_2(\nu)| \le 1, \\ 0.3, & |r_2(\nu)| > 1, \end{cases}$$
$$b_{11}(r_1(\nu)) = \begin{cases} 1, & |r_1(\nu)| \le 1, \\ -1, & |r_1(\nu)| > 1, \end{cases} \quad b_{12}(r_1(\nu)) = \begin{cases} -0.08, & |r_1(\nu)| \le 1, \\ -1, & |r_1(\nu)| > 1, \end{cases}$$
$$b_{21}(r_2(\nu)) = \begin{cases} -0.15, & |r_2(\nu)| \le 1, \\ -0.2, & |r_2(\nu)| > 1, \end{cases} \quad b_{22}(r_2(\nu)) = \begin{cases} -2, & |r_2(\nu)| \le 1, \\ -2.05, & |r_2(\nu)| > 1, \end{cases}$$

$$c_{11}\left(r_1(\nu)\right)=\begin{cases}-0.185, & |r_1(\nu)|\le 1,\\ -0.13, & |r_1(\nu)|>1,\end{cases}\qquad c_{12}\left(r_1(\nu)\right)=\begin{cases}-0.182, & |r_1(\nu)|\le 1,\\ -0.15, & |r_1(\nu)|>1,\end{cases}$$

$$c_{21}\left(r_2(\nu)\right)=\begin{cases}-0.186, & |r_2(\nu)|\le 1,\\ -0.12, & |r_2(\nu)|>1,\end{cases}\qquad c_{22}\left(r_2(\nu)\right)=\begin{cases}-0.174, & |r_2(\nu)|\le 1,\\ -0.11, & |r_2(\nu)|>1,\end{cases}$$

and the activation function is $f_j(r_j) = g_j(r_j) = \tanh(r_j), j = 1, 2$. Then $F_1 = F_2 = G_1 = G_2 = 1$. By simple computation, we have $A_{11} = 1.07, A_{12} = 0.3, A_{21} = 2.6, A_{22} = 1.5, B_{11} = 1, B_{12} = 1, B_{21} = 0.2, B_{22} = 2, C_{11} = 0.185, C_{12} = 0.182, C_{21} = 0.186, C_{22} = 0.174$.

Case (1): The derivative of the time delay is less than 1. Let $\rho_1(\nu) = \rho_2(\nu) = \frac{e^\nu}{1+e^\nu}$. It can be obtained that $\dot{\rho}_1(\nu) = \dot{\rho}_2(\nu) \le 0.25 < 1$. We choose $\varepsilon = 1.5, \gamma_1 = \gamma_2 = 1, \vartheta_j(\nu) = 0.3\sin(\nu)$. Then $\bar{\vartheta} = 0.3$. From Theorem 1, using MATLAB to solve Eqs (3.11) and (3.14), we obtain $\lambda_1 = 67.0178, \lambda_2 = 66.9786$. The state behaviors of MNN (4.1) are depicted in Figure 1. As shown in Figure 1, the states $r_1(\nu)$ and $r_2(\nu)$ are bounded as time goes on. Figure 2 presents the phase diagram and reachable set of the delayed MNN (4.1). As shown in Figure 2, the MNNs' states are bounded within an elliptical set.

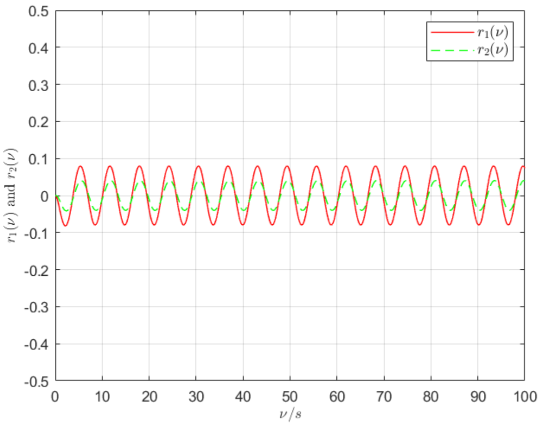

Figure 1. $r_1(\nu)$ and $r_2(\nu)$ of MNN (4.1) in Case (1).

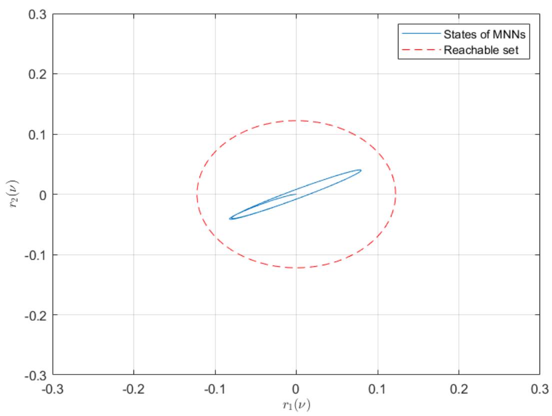

Figure 2. The states phase plot of MNN (4.1) and the elliptical reachable set in Case (1).

Case (2): The derivative of the time delay is more than 1. Let $\rho_1(\nu) = \frac{6e^\nu}{1+e^\nu}$ and $\rho_2(\nu) = \frac{5e^\nu}{1+e^\nu}$. It can be obtained that $\dot{\rho}_1(\nu) \le 1.5, \dot{\rho}_2(\nu) \le 1.25$. So the methods in [25–29] are invalid. We choose $\bar{\varepsilon} = 1.5$,

$\gamma_1 = \gamma_2 = 2, \vartheta_j(\nu) = 0.2\sin(\nu)$. Then $\bar{\vartheta} = 0.2$. From Theorem 2, using MATLAB to solve Eqs (3.10) and (3.11), we obtain $\bar{\lambda}_1 = 38.724, \bar{\lambda}_2 = 38.782$. The state behaviors of MNN (4.1) are depicted in Figure 3. As shown in Figure 3, the states $r_1(\nu)$ and $r_2(\nu)$ are bounded as time goes on. Figure 4 presents the phase diagram and reachable set of the delayed MNN (4.1). As shown in Figure 4, the MNNs' states are bounded within an elliptical set.

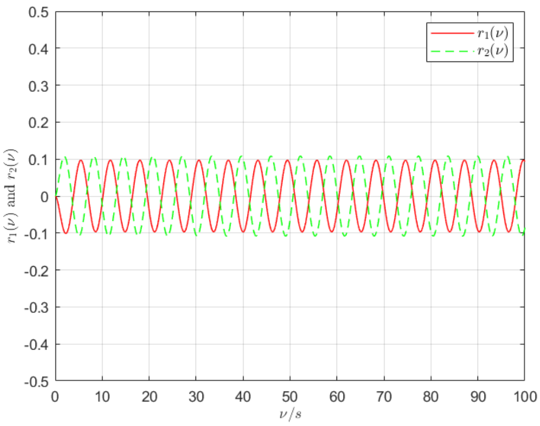

Figure 3. $r_1(\nu)$ and $r_2(\nu)$ of MNN (4.1) in Case (2).

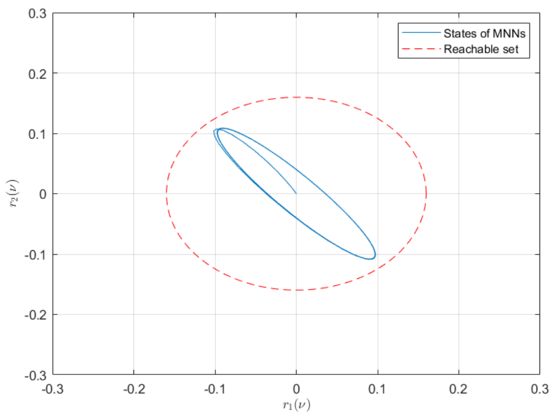

Figure 4. The states phase plot of MNN (4.1) and the reachable set in Case (2).

5. Conclusions

In this paper, the problem of RSE of a class of MNNs is investigated. By adopting the inequality techniques in Filippov's sense, algebraic criteria are given to guarantee the states of the addressed MNNs are contained in an ellipsoid set by introducing a memoryless and a memory adaptive controller. The considered models are general since they relax the conditions of time delay, so better results are obtained. In addition, the criteria are presented in the form of algebraic conditions, and they are verified easily. In future work, it would be meaningful to investigate event-triggered reachable set estimation, which could reduce control update frequency and optimize communication resource utilization.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

The work was supported by the Natural Science Foundation of Wuhan (Chenguang Project) under Grant 2024040801020332.

Conflict of interest

Jiemei Zhao is a Guest Editor for *Networks and Heterogeneous Media* and was not involved in the editorial review or the decision to publish this article. All authors declare that there are no competing interests.

Author contributions

Jiemei Zhao: Conceptualization, Methodology, Investigation, Review and Editing, Visualization, Funding acquisition, Supervision. Ning Wu: Writing–original draft, Investigation, Formal analysis, Programming. Xiaowu Zhou: Investigation, Methodology, Writing, Review, Revision, Validation. All authors read and approved the manuscript.

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