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EVALUATING THE EFFECTS OF PARKING PRICE AND LOCATION IN MULTI-MODAL TRANSPORTATION NETWORKS

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Abstract. In urban transportation, combined mode trips are increasing in importance due to current urban transportation polices which encourage the use of transit through the creation of apposite parking lots and improvements in the public transportation system. It is widely recognized that parking policy plays an important role in urban management: parking policy measures not only affect the parking system, but also generate impacts to the transport and socioeconomic system of a city. The present paper attempts to expand on previous research concerning the development of models to capture drivers' parking behavior. It introduces in the modeling structure additional variables to the ones usually employed, with which the drivers' behavior to changes in prices and distances (mainly walking) are better captured. We develop a network model that represents trips as a combination of private and transit modes. A graph representing four different modes (car, bus, metro and pedestrian) is defined and a set of free park and ride facilities is introduced to discourage the use of private cars. An algorithm that evaluates the location and the effects of the parking price variation using multi-modal shortest paths is proposed together with an application to the City of Rome. Computational results are shown.

1. Introduction.

Relevant distinctions between private and public transportation paths are the result of the continued expansion of cities and the chaotic growth of traffic: on the one hand, there is the difficulty of reaching suburban areas using public transportation, and, on the other hand, there is traffic congestion that has led to a strong limitation of private transportation in downtown areas and the creation of preferential lanes for public transportation.

The effort to limit private traffic as much as possible in downtown areas has made the private mode and transit mode complementary, defining any trip between two points of the city as a combination of different transportation systems. In fact, many trips within urban areas are taken by using more than one mode, where, in particular, the first part of the trip is made by private car and then completed by

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one or more transit modes. In general, we shall refer to a trip that occurs having more than one mode as a *combined mode trip* [34].

In order to encourage people to leave their private cars and use the public transportation network, some parking lots located close to stops along public transportation lines have been identified which allow travellers to change from the private to the transit mode. In particular, in the following, we shall refer to a *park and ride facility* (PRF), as an organized free parking lot where the car driver can change from the car mode to the pedestrian or public transit mode without paying any parking fee. While we shall refer to an *on-street* parking as a parking lot not free so that the driver has to pay a fee when leaving the car. Variations of parking price significantly modify the car user behavior.

Park and ride (e.g., see [80]) consists of parking facilities at transit stations, bus stops and highway onramps to encourage public transit use and ridesharing. Parking is free or significantly less expensive than on-street parking. With respect to traffic impacts, park and ride facilities can increase transit and rideshare travel. Morrall and Bolger [73, 74] find that the supply of park and ride facilities can have a major influence on the portion of downtown commute trips made by transit.

It is widely recognized that parking policy plays an important role in urban management [101]. Pricing parking is one of the most effective ways to change travel behavior (e.g., see [58, 65, 84]) and can have various transportation and land use impacts. Charging for parking may cause spillover impacts onto other streets, or onto other off-street parking facilities where parking is free. If parking is only priced in certain areas, some motorists may shift destinations, particularly for discretionary trips. This may encourage shifts of business activity and development away from cities to urban fringe locations where most parking is free.

When the economic analysis on downtown parking is concerned, there has been very little formal economic analysis of even the most obvious issues. If traffic congestion is efficiently priced, how should parking fees be set? Alternatively, what are the second-best parking fees when traffic congestion is underpriced? Depending on the pricing of auto congestion and public parking, should private, off-street parking fees be taxed or regulated? For various pricing regimes, how much land should be allocated to parking, both on- and off-street? What is the value of information concerning parking availability?

Among the mentioned aspects of parking those that have been considered in the literature mainly refer to the descriptions of parking patterns, the effects of on-street parking on traffic circulation, and the technology of off-street parking appear e.g., [49, 54], as well as discussions of parking policy e.g., [1, 71, 86, 87, 96]. Some empirical work has been done identifying the determinants of modal choice and parking location e.g., [39, 40, 52, 105]. Numerous city-specific parking studies have been undertaken, and there are high-quality, non-technical economic discussions of parking policy, notably Vickrey [100] and Roth [85]. But with the exception of a note by Douglas [31] and papers by Arnott et al. [2], Glazer and Niskanen [41], and Verhoef et al. [99] no economic model has been developed that considers the potential efficiency gains from parking fees or that incorporates the effects of parking on travel congestion. The effects may be substantial, for in major urban areas the time to find a parking spot and walk from there to work can be an appreciable fraction of total travel time, and parking fees may be comparable to vehicle operating costs [39]. In [5] the authors present a simple model of parking congestion focusing on drivers' search for a vacant parking space in a spatially homogeneous metropolis. The mean density of vacant parking spaces is endogenous. A parking externality arises because individuals neglect the effect of their parking on this mean density. They examine stochastic stationary-state equilibria and optima in the model. Due to the model's nonlinearity, multiple equilibria may exist and the effects of parking fees are complex.

Arnott et al. [2] explored the effects of parking fees in a deterministic model of the morning auto commute to the central business district, with bottleneck congestion. They showed that parking fees which vary over location can significantly reduce total travel costs. Glazer and Niskanen [41] examined simple partial equilibrium models to demonstrate that raising parking fees may increase both local traffic by encouraging shorter visits and through traffic. And Verhoef et al. [99] compared parking fees and parking regulations.

In [45] the authors investigate the role of parking pricing and supply by time of day in whether to drive and park in the central business district (CBD). A stated preference survey of car drivers and public transport users was undertaken at a number of parking locations, public transit interchanges, and shopping centres in Sydney CBD during 1998. In the context of a current trip to the CBD, respondents were asked to consider six alternatives, including three parking locations in the CBD, park outside of the CBD with public transport connection to the CBD, switch to public transport, or forego that trip to the CBD. The three parking locations were defined by hours of operation, a tariff schedule, and access time to the final destination from the parking station. Data from the survey were then used to estimate a nested logit model of mode and parking choices, which was then used to simulate the impacts of supply pricing scenarios on CBD parking share.

In [95] the author estimates drivers' behavior for changing an already chosen parking location and the thresholds of current parking fare increases that would make them shift to another mode from the currently used private car. The models are calibrated with data from the Central Business Area of Athens employing revealed and stated preference methods. They can be employed to estimate the impacts of a specific transport policy related to parking fares, and as such they are useful policy tools providing the means to estimate changes in car usage and parking locations utilization.

Parking policy measures not only affect the parking system, but also generate impacts to the transport and socioeconomic system of a city. As for parking, several methodologies have developed dealing with the process the drivers follow to find a parking space. They assume that the search for a parking space is not a random procedure. In reality, it is an activity undertaken by the driver for a specific purpose having a pre-set objective. Experimental measurements, as presented in other sectors of the economy, i.e. in geography, support the notion that seeking for a parking space is an activity based on certain decision rules [48]. The most important factors of these decision rules are: the cost of parking; the walking distance to the final destination; and the time needed to search for a parking space.

Until now, the most comprehensive research effort on simulating parking behavior is the model CLAMP [83]. It provides a detailed and dynamic representation of parking type and mode choice decision making, and it also quantifies the effects of these decisions on traffic speeds and flows. The model had been successfully applied in suburban town centres, e.g. in the Royal Borough of Kingston upon Thames (Bradley et al. [18]), where stated preference methods for data collection [7] were used.

Concerning the parking location, several models were developed based on the random utility framework [32, 40]. The most comprehensive model used is the single-level logit formulation, which includes more variables than the ones most commonly employed, such as money cost and proximity to final destination [97].

Another study, using revealed preference data [53] developed a nested logit model for parking location choice on work trips in a large central business district. Included in the choice set are the selection of off-street and on-street (kerb parking) parking options, as well as parking supplied by employers.

With the development of advanced real-time traffic information, new attitudes of drivers' behavior are emerging. This is also acknowledged for parking related information. Results from an experimental information system in Nottingham and other relevant research [55] suggest that the dynamic parking information probably increased drivers' knowledge of parking locations. For this, a dedicated software PARKIT [17] has been developed, that captures the drivers' response to dynamic parking guidance information system (PGI).

Another study of Kurauchi et al. [59] provides valuable input for the drivers' behavior to the provision of PGI on parking availability and expected waiting time. However, a recent simulation experiment of Collura et al. [25] has found that drivers follow the simplest decision rule for the choice of a parking space: the reduction of total expected travel time. They do not pay much attention to any advanced information systems, even to the information indicating that the parking is full. In any case, it is recognized that more research is needed in this area, especially on the effect of parking policies. Perhaps, this is one reason that the most widely used parking information is the static one, which is provided through advertisements and leaflets.

As for the effects of parking on car use, studies indicate that the single most important factor determining the reduction of car usage is the level of parking fares and therefore, it is suggested to use it as an effective policy tool [50]. However, limited research has been carried out regarding the effect of parking measures on modal choice [33]. Moreover, the disaggregate models developed for parking location choice follow the principle that parking prices and supply restrictions have considerable impacts on selecting the parking location, although quite a few mode-choice studies deal adequately with parking factors. Nevertheless, there is enough evidence to support the view that parking policy measures have a relatively important influence on modal choice. In addition, recently concluded research [22] proves that the second-best urban travel pricing measure is that of pricing the parking spaces. It produces higher welfare gains than the use of road pricing measures. Despite the fact that few studies are available, there is a general consensus that parking prices and supply have considerable impacts on parking location choice, since market prices allocate parking spaces fairly and efficiently [88].

The present paper attempts to expand on previous research concerning the development of models to capture drivers' parking behavior. It introduces in the modelling structure additional variables to the ones usually employed (see e.g. [60, 88]), with which the drivers' behavior to changes in prices and distances (mainly walking) are better captured - an area that needs more in depth analysis [60].

We present a unified approach that consists in designing a network that allows people to use combined mode trips. In our representation we set a number of free park and ride facilities, so commuters who use other modes receive comparable benefits, and charge on-street parking in order to evaluate the use of a park and ride facility with respect to on-street parking fee variations.

Referring to our task of designing a unified network representation for intermodal trips, Spiess and Florian [93] proposed an algorithm that expands the original transit network by adding arcs to represent waiting, walking and line switching. Probabilistic waiting and deterministic travel times are associated with every arc of the transformed network and the expected least time path is calculated on the expanded network with a stochastic shortest path algorithm. However, in [93] neither private transportation nor parking lots nor the number and kinds of transfers are considered in the expanded representation.

The attempt of modelling transfers can be found in [63] where the authors developed a formulation that takes into account the number and kinds of transfers to catch the actual behavior of the user in a multi-modal network. A previous work in [66] attempted to deal with this issue. Here the authors proposed a method to find shortest viable path in a multi-modal network studying the relation between the number of transfers in that path and the path impedance. Another paper in which multi-modal trips are considered can be found in [51]. The latter study proposed a model for planning passenger journeys in a intra-urban context where walking, fixed-route public transport, and demand-responsive modes are considered.

Referring to the problem of the location of PRFs, in [102] the authors investigate the problem in a linear city, i.e., in a city where residences are uniformly distributed from the center to the exogenous city boundary, and all trips are from home to center. Beyond the linearity of the city, the proposed model includes several restrictive features such as the existence of only one PRF, and the fact that the represented network does not entirely capture the multi-modal nature of the trips.

In this work we present a more general scenario than those presented in the aforementioned articles, since we deal with a multi-modal network with both private and transit modes, with more than one PRF, and take into account the number of transfers and the path composition.

The paper is organized as follows. In Section 2 we formulate a general description of the public and the private transportation. The unified representation is discussed in Section 3; in Section 4 we propose the path composition on the unified representation network. In Section 5 we propose a mathematical model and discuss our algorithm, and finally in Section 6 a real application to the City of Rome is presented. Final comments can be found in the last section.

2. Transit and private transportation networks.

An urban private transportation network can be modelled by means of a directed graph $G = (N, E)$, being N a set of nodes and E a set of directed arcs whose generic element $(i, j) \in E$ is the link between node i and node j. With each arc $e \in E$ are associated some characteristics such as the flow v_e of vehicles traversing e and the cost, e.g., the travelling time, to traverse e. Costs can be either constant functions or continuous non decreasing functions of the corresponding link flows. Each trip is identified in G by a path from an origin node to a destination node, and it is assumed that all trips have origins and destinations belonging exclusively to a given subset of nodes in N called *centroids*.

Differently to a private transportation network, the representation of a transit transportation network needs the introduction of other ingredients such the bus lines and their frequencies. In particular, a transit network consists of a set of distinct lines and stops where passengers board and alight carries. Each line l of the public transportation system is associated with a frequency f_l and a travel time t_l. Frequency f_l in general is a function $f_l : [0, \bar{v}_l) \to (0, \infty)$ with $f_l'(\cdot) < 0$ and $f_l(v_l) \to 0$ as $v_l \to \bar{v}_l$. The value $\bar{v}_l$ is called the saturation flow of line l.

At a given stop, a passenger may have a choice of several lines and itineraries to reach his/her destination. The decision faced by the passenger in this case is whether to board the incoming carrier or to wait for another one and, clearly, different strategies are used by different passengers. However, in the paper we consider the simplified situation where at a given stop passengers with a common destination are homogeneous with respect to the various criteria. The behavior of the passenger is characterized, at every reachable node of the network, by a given set of attractive lines. The user always boards the first incoming carrier with positive residual capacity among this set. This simplifying assumption allows for mathematically tractable assignment models, which are discussed in the following.

Earlier attempts to provide realistic assignment models for transit networks failed to overcome the modelling limitations of the classical single path framework. Dial [30] proposed to bundle the common portion of overlapping lines into a single line bearing a frequency equal to the sum of the frequencies of each individual line, while LeClercq [61] called upon a line-node description of the transit network, in contrast with the traditional node-arc representation. In general, they neglected congestion and assumed that passengers traveled along shortest paths on each origin-destination (OD) pair. The length of a path in this context corresponds to the total transit time including waiting as well as in-vehicle travel time. Later on Chriqui and Robillard [24] introduced the notion of common-lines suggesting that passengers could bundle together a subset of the available lines in order to reduce the waiting and hence the overall transit time. They seem to be the first to formulate and solve the selection problem, i.e., the problem of selecting, at a given boarding or transfer node, a subset of overlapping lines that minimizes the local waiting time. However this local model does not consider subsequent transfers and thus may fail to provide globally optimal choice sets at every stop node of the network.

The extension of the common-line idea to general networks led Spiess [91] and Spiess and Florian [93] to introduce the notion of strategy, which was later expressed by Nguyen and Pallottino [77] under the denomination of hyperpath. Spiess and Florian were the first to propose a model of assignment in uncongested transit networks, based on the concept of optimal strategy. In this model - which can handle simultaneously several OD pairs, overlapping bus lines, and transfers at intermediate nodes on each trip - passengers are assumed to travel along shortest hyperpaths. Despite this generality, the model did not consider explicitly the increase in waiting times induced by congestion, and the assignment of passengers to bus lines was done proportionally to nominal frequencies. However, in [77] the authors consider flow-dependent travel-times, modelling the on-board crowding of buses which may affect the passenger's choice. The first attempt to incorporate the congestion effects on the passenger distribution and waiting times at bus stops seems to be that in [38].

According to [24], for the purpose of travelling from an origin to a destination, passengers select a subset of common lines boarding the first incoming bus from this set. The chosen strategy s should minimize the expected transit time T_s, including the waiting time $\frac{1}{\sum_{i \in s} f_i}$ and the expected in-vehicle travel time $\sum_{i \in s} t_i \pi_i^s$, where

$\pi_i^s = \frac{f_i}{\sum_{j\in s} f_j}$ is the probability of boarding line l_i, i.e., $T_s = \frac{1+\sum_{i\in s} t_i f_i}{\sum_{i\in s} f_i}$. Similarly, a waiting time of the form $\frac{\alpha}{\sum_{i\in s} f_i}$, with $\alpha \in (0,1)$ can be used, by changing in the previous setting the frequencies to $\frac{f_i}{\alpha}$. Note that the combination of a strategy s and its arc probabilities define a hyperpath (e.g., see [77]).

3. **A unified representation of the transportation networks.** Once the private and the transit networks have been defined we look for a unified representation that allows the definition of multi-modal trips, where, for instance, the optimum path between two nodes may consist of driving to a parking lot close to the destination node and then walking to the ultimate destination, or it may consist of driving to a park and ride facility, riding a bus or a train to a transit stop close to the destination, and from there walking to the ultimate destination.

The multi-modal network so defined allows us to propose an algorithm to evaluate the use of free parking lots as opposed to on-street parking, and thus to identify which ones of the former are more convenient for different values of on-street parking fees.

The multi-modal network is built by using three different levels of representation, each corresponding to different modes, i.e., private car, transit and pedestrian modes. For ease of presentation we will consider separately the three levels in the following order: first, third and second.

The *first level* describes the transportation network available to private cars. In the definition of the graph we have to:

- Identify the nodes of the street-network;
- Define the links between the street-network nodes which can be used by the private flow;
- Define the connection arcs among centroid nodes and the street-network nodes.

The network so obtained is denoted as *car network*. *Car network* users can park their cars in the neighborhood of each node (if it is allowed) paying a fee, except for PRFs.

The *third level* consists of the transit network and thus it is formed by the sequences of line nodes associated with bus lines, and the corresponding boarding and alighting arcs. The line nodes belonging to different paths are represented separately (see Figure 1).

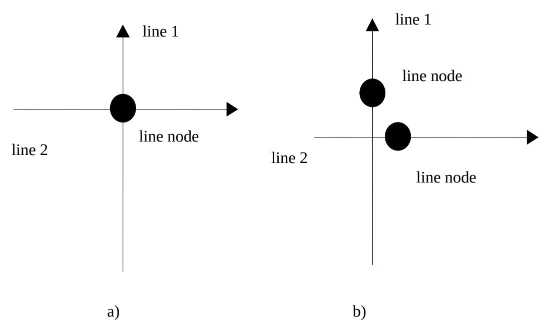

FIGURE 1. Representation of nodes belonging to different paths.

Metro lines are also considered in the public transportation network, since bus and metro paths are distinct. An in-vehicle travel time is associated with each arc and, as for the buses, a frequency is associated with each line. The network obtained is denoted as *metro-bus* network.

The *second level*, denoted as *pedestrian network*, is composed by the arcs available to the pedestrian mode and by a set of connection arcs that allow users to switch among modes. For the sake of completeness, in this model one can assume that the pedestrian mode is representative of all non-motorized modes, e.g., cycling, which, in some cities, represents a significant transportation mode.

The access to the public transportation network is assured by the introduction of special nodes called *stop nodes* (see Figure 2) and by *boarding* and *alighting* arcs.

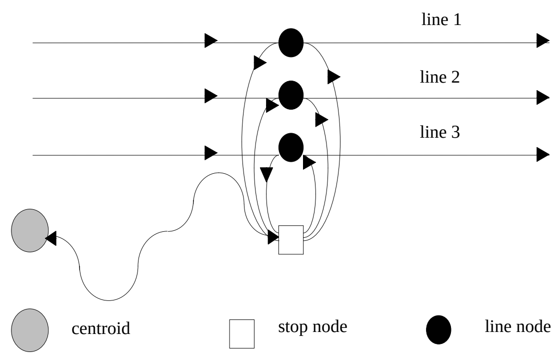

FIGURE 2. Representation of *boarding* and *alighting* arcs.

Each boarding arc is assigned a cost equal to the average waiting time at that stop. Each alighting arc is assigned a null cost. Moreover, for each boarding arc corresponding to a metro node, the time needed to move from the surface transportation system to the metro is summed to the average waiting time.

In order to complete the description of the *second level*, we introduce connection arcs that allow users to switch from the car mode to the pedestrian mode. We call a *c-p connection arc* a link between the car mode and the pedestrian mode, and a *p-c connection arc* a link between the pedestrian mode and the car mode. The cost associated with a *c-p* connection arc takes into account the time needed to get from the parking lot to the bus stop (or the destination centroid) and the parking price if it is an on-street parking. On the other hand, the cost associated with a *p-c* connection arc depends solely on the time needed (by walking) to get from the bus stop to the parking lot.

Since some users can dislike transfers (either between transit routes and modes), one can limit the number of possible mode changes. This undesirability can be incorporated into the model by:

1. Charging waiting times at a higher rate (e.g., twice or three times);
2. Assigning an additional in-vehicle time to each additional transfer;
3. Imposing a constraint which avoids trips formed by more than a given number of mode changes.

In our model we choose to charge the average waiting time at a higher rate by means of a function w_{ij}, where (i, j) is a boarding arc, defined as $w_{ij} = c_{ij} \cdot n_m$ where c_{ij} is the average waiting time at a stop node, and n_m is the number of modes used in a given trip (except for the pedestrian mode). Note that if $n_m = 1$ then the waiting times are not overcharged, i.e., they remain unvaried with respect to the waiting times computed for boarding arcs, while tend to grow by the second transfer on.

Once all the above details have been defined, each arc of the multi-modal network is representative of a single mode drawn from the following set:

c: car
p: pedestrian
b: bus
m: metro

The multi-modal network described is depicted in Figure 3. Note that centroids have been added also in the pedestrian network in order to allow a path to start and/or finish with the p mode as described in the next section.

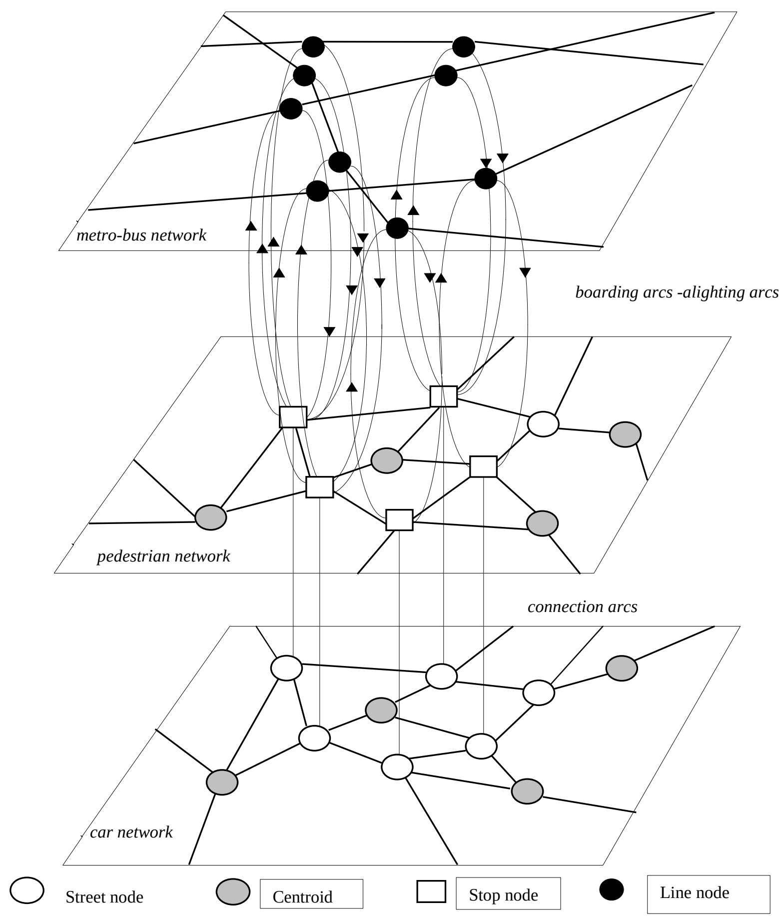

FIGURE 3. The multi-modal network.

4. **Path composition.** In a multi-modal network, a path is obtained through a concatenation of a certain number of subpaths, each one formed by a single mode trip [9] and we will denote such a path *multi-modal path*. In order to describe urban

trips as realistically as possible, we give some properties on the path composition in our multi-modal network.

Assumption 1. Paths can start or finish only with the pedestrian or car mode.

Assumption 2. Car mode in a multi-modal path appears at most once, either at the beginning or at the end of the path.

These assumptions are realistic, as in "usual" multi-modal trips, the private car is used either at the beginning or at the end of the path. In particular, referring to Assumption 2, we introduce two classes of paths: *car-first paths* and *car-last paths*.

Definition 1. A car-first is a path in which c is used only at the beginning of the path and p appears at the end. A car-last is a path in which p is used at the beginning of the path and c is used at the end.

Remark 1. Note that it is allowed that one drives from the origin to a parking facility and then walks to the ultimate destination (centroid) node in the second level, just traversing a unique *c-p* connection arc. On the contrary, one can walk from the origin, traverse a *p-c* connection arc, take the car from the parking lot, and never change mode until the destination (centroid) node.

Assumption 3. All combinations between modes p, b and m are allowed.

Assumption 4. p is a separator between c, b and m.

By the assumptions we made, we can state the following theorem.

Theorem 1. *A path is feasible if and only if it uses at most either one c-p connection arc or one p-c connection arc.*

Proof. (⇒) If the path is feasible the following cases are possible:

i) c is not used;
ii) c is used only at the beginning of the path;
iii) c is used only at the end of the path.

In case (i) neither a *c-p* nor a *p-c* connection arc will be used and the path will be formed by b, p and m only.

If we are in (ii), the path starts in the car network; if the path is made up of other *pure modes*, then a *c-p* connection arc must be used to get to the pedestrian network. Therefore, c will no longer be used and no *p-c* connection arc will be used.

We stated that c can appear at most once. In (iii) it is used at the end of the path. Thus, a switch from the pedestrian network to the car network via a *p-c* connection arc has occurred, and the path will end with that mode.
(⇐) If the path does not use any *p-c* or *c-p* connection arcs, it is feasible as it is made up of c or b, p and m only. If a *c-p* connection arc is used then there will be a path beginning with c, i.e., a *car-first* path; otherwise, if a *p-c* connection arc is used there will be a path ending with c, i.e., a *car-last* path. □

Definition 2. A path on a multi-modal network is feasible if the sequence of its arcs is compatible with one of the following expressions*:

** defines the repetition of the expression which it refers to n times with $n \geq 0$ (0 implies the absence of the expression)

$$c((pb)^*(pm)^*)^*p$$
$$p((bp)^*(mp)^*)^*$$
$$p((bp)^*(mp)^*)^*c$$

Definition 3. The node state is a binary variable assuming value 0 if in the path containing that node neither a c-p nor a p-c connection arc has been used, and assuming value 1 otherwise.

The following corollary follows directly from Theorem 1 and Definition 3:

Corollary 1. *If in a path a node belonging to the pedestrian network has state* 1 *then it is no longer possible to use c; if a node belonging to the car network has state* 1 *then it is no longer possible to use p, b and m.*

5. **The assignment algorithm and the choice of PRFs.** Many papers in the literature have dealt with multi-modal transportation algorithms (see, e.g., [28, 51, 66, 67, 108]) and these algorithms are mainly based on shortest paths or shortest hyperpaths. In this section we propose an algorithm capable of managing the multi-modal network defined in Section 3 in order to assign traffic flows with the objective of minimizing the total travelling time of the network users and respecting the path composition defined in the previous section. A by product of this algorithm will be the possibility to derive the location of PRFs.

The algorithm works by alternately executing two functions. The first function is an all-or-nothing assignment of the OD demand on feasible shortest multi-modal paths computed with fixed arc costs (travelling times). The second function computes new travelling times for each arc in the multi-modal network taking into account the congestion produced by the flow assignment output by the former function. Thus, these two functionalities, i.e., the assignment algorithm and the travelling time function, stop when the difference between two successive iterations in terms of total travelling time does not vary significantly, i.e., it is smaller than or equal to a prefixed arbitrarily small threshold. Once the algorithm stops we are able to determine PRFs; moreover, by changing the price of on-street parking (see the next section for implementation details) one is able to perform a sensitivity analysis on their location.

Before starting with the algorithms description, we recall that the behavioral assumption of the equilibrium traffic assignment problem is that each user chooses the route that he/she perceives the best; if there is a shorter route than the one that he/she is using, he/she will choose it. This results in flows that satisfy Wardrop's "user optimal" principle, that no user can improve his/her travel time by changing routes [104]. The consequence is that the equilibrium traffic assignment corresponds to a set of flows such that all paths used between an origin-destination pair are of equal time.

Let

- C be the set of nodes in the private network;
- P be the set of nodes in the pedestrian network;
- T be the set of nodes in the transit network;
- N be the set of nodes in the multi-modal network, i.e., $N = C \cup P \cup T$;
- $E(C)$ be the set of arcs in the private network;

- $E(P)$ be the set of arcs in the pedestrian network;
- $E(T_b)$ be the set of arcs associated with bus lines in the transit network;
- $E(T_m)$ be the set of arcs associated with metro lines in the transit network;
- CP be the set of *c*-*p* connection arcs;
- PC be the set of *p*-*c* connections arcs;
- Bo be the set of boarding arcs;
- Al be the set of alighting arcs;
- E be the set of arcs in the multi-modal network.

In the multi-modal network model, since pedestrian network arcs, *c*-*p* and *p*-*c* connections and arcs in $E(T_m)$ do not suffer from congestion, we can associate with each of them a transferring time t_e defined as follows:

(a) t_e, with $e \in PC$, is equal to the time needed (by walking) to get from the stop node to the parking lot;
(b) t_e, with $e \in Al$, is equal to zero;
(c) t_e, with $e \in CP$, is equal to the sum of the parking cost if the tail of e is not a PRF and the time needed to get from the parking lot to the stop node;
(d) t_e, with $e \in Bo$, is equal to the average waiting time at that stop;
(e) t_e, with $e \in E(T_m)$, is equal to the in-vehicle travelling time for traversing arc e with the metro mode: this travelling time is not affected by congestion and can be obtained from the company managing the transportation service;
(f) t_e, with $e \in P$, is equal to $\frac{arc\_length_e}{4}$, where arc_length_e is the length of the pedestrian arc e, and 4 stands for a walking speed of 4 km/h.

For those arcs belonging to the car network and the transit network, we must consider the congestion effect deriving from the flow of vehicles. To this aim, we consider the following non-linear cost function for arcs $e \in E(C) \cup E(T_b)$:

$$t_e(v_e) = t_e^0 \left[1 + \left(\frac{v_e}{C_e} \right)^{\alpha} \right],$$

where v_e and C_e are, respectively, the flow and the capacity of arc e (the latter is measured as the number of lanes times the number of vehicles per hour per lane) and α is a parameter that characterizes the physical structure of the street [20], e.g., the numbers of lanes and the presence of on-street parking. Trivially, t_e^0 is the travelling time associated with $v_e = 0$ and is computed by simply dividing the length of e by the maximum speed allowed on that street.

The mathematical model associated with the equilibrium assignment problem is given below, where

- D is the set of destination nodes;
- O is the set of origin nodes;
- E_i^+ is the set of arcs with tail i;
- E_i^- is the set of arcs with head i;
- v_e is the flow of arc e, and $v_e^{(ij)}$ is the amount of users among v_e travelling from origin i to destination j;
- g_{ij} is the demand from origin i to destination j;
- y_e is a binary variable that equals 1 if $v_e > 0$, and is 0 if $v_e = 0$;
- M is a big number, e.g., M is greater than $\sum_{i \in O, j \in D} g_{ij}$;

$$\min \sum_{e\in E} t_e(v_e)\cdot y_e \tag{1}$$

$$\text{s.t.} \quad v_e = \sum_{i\in O, j\in D} v_e^{(ij)},\ \forall e\in E \tag{2}$$

$$\sum_{e\in E_j^-} v_e^{(ij)} - \sum_{e\in E_j^+} v_e^{(ij)} = g_{ij},\ \forall i\in O,\ j\in D \tag{3}$$

$$\sum_{e\in E_j^-} v_e - \sum_{e\in E_j^+} v_e = 0,\ \forall j\in N\setminus D \tag{4}$$

$$v_e^{(ij)} \le y_e\cdot M,\ e\in E,\ \forall i\in O,\ j\in D \tag{5}$$

$$\sum_{e\in CP} y_e \le 1 \tag{6}$$

$$\sum_{e\in PC} y_e \le 1 \tag{7}$$

$$\sum_{e\in PC} y_e + \sum_{e\in CP} y_e \le 1 \tag{8}$$

$$v_e,\ v_e^{(ij)} \ge 0,\ e\in E,\ \forall i\in O,\ j\in D \tag{9}$$

The objective function (1) is the minimization of the total travelling time of the users in the network; note that the presence of the binary variables y_e and of constraints (5) allow us to consider only travelling times associated with arcs whose flow is greater than zero. Constraints (3) and (4) are typical conservation flow constraints.

Constraints (6)-(8) are path composition constraints. In fact, (6) says that at most a *c-p* connection arc can appear in a feasible path, (7) says that at most a *p-c* connection arc can be used, and constraint (8) allows at most either one *c-p* or one *p-c* connection arc. These three constraints define the feasibility condition of Theorem 1.

The above formulation has a non-linear objective function and linear constraints with both discrete and continuous variables, and is difficult to solve for networks of large sizes. Even if we consider a weaker formulation where we use the unfair assumption of $\alpha = 1$ in the definition of $t_e(v_e)$, and we remove path composition constraints the above model remains non-linear. Thus, to cope with problem (1)-(9) we propose the heuristic algorithm reported in Table 1.

In Step 1, index i counts the number of iterations and $t_e^{(i)}$ is the travelling time associated with arc e at iteration i. In the initialization stage $t_e^{(0)}$ is set equal to t_e^0 for congested arcs, i.e., for $e \in E(C)\cup E(T_b)$; all the other arcs are assigned travelling times as reported above in (a)-(f), and these travelling times do not vary during the algorithm progresses since these arcs are not affected by congestion. Moreover, in the algorithm we denote with $T^{(i)}$ the value of the objective function (1). In particular, in the initialization step, $T^{(0)}$ is set to $+\infty$.

Let i be a generic iteration. In Step 2 we solve the all-or-nothing assignment of the OD demand on shortest paths computed with arc costs $t_e^{(i)}$. This assignment algorithm will be discussed next. The result is a vector of flows which has as many components as the number of arcs. Let $v_e^{(i)}$ be the flow of arc e after this assignment.

(1. Initialization) $i = 0$; let $t_e^{(i)} = t_e^0$ with $e \in E(C) \cup E(T_b)$; let $T^{(0)} = +\infty$;
(2. Flow Assignment) $i = i + 1$; solve the all-or-nothing assignment of the O/D demand on shortest paths computed with arc costs $t_e^{(i-1)}$, and let $v^{(i)}$ be the vector of flows so obtained whose generic component is $v_e^{(i)}$ with $e \in E$; let $T^{(i)} = \sum_{e \in E} t_e(v_e^{(i)}) \cdot y_e$;
(3. Stopping Criterion) If $(T^{(i)} - T^{(i-1)}) \leq \epsilon$ (or a maximum number of iterations has been met) then $v^* = v^{(i)}$, $t_e^* = t_e^{(i-1)}$ and stop; otherwise go to Step 4;
(4. Update Link Costs) Compute travelling times $t_e^{(i)} = t_e(v_e^{(i)})$ for the arcs in $E(C) \cup E(T_b)$ and goto Step 2.

TABLE 1. The proposed algorithm.

Step 3 is the stopping criterion, i.e., if the difference between the total travelling times in two successive iterations is sufficiently small than the algorithm stops (the threshold ϵ is arbitrarily chosen); otherwise, Step 4 is invoked, and new travelling times for the arcs in $E(C) \cup E(T_b)$ are computed taking into account flows $v_e^{(i)}$ computed in Step 2. Then, the process keeps on iterating between Step 2 and Step 4 until the stopping rule is met. Note that, due to the heuristic nature of the algorithm, we added in Step 3 a further possible stopping criterion that occurs when a given maximum number of iterations is met.

As shown in Step 2, to solve this problem one needs to calculate an all-or-nothing assignment of the demand on shortest paths. Although all-or-nothing assignment algorithms are well known for transit or private transportation, our goal is to propose a new all-or-nothing algorithm that can be used in the multi-modal network defined in Section 3 to take into account the presence of parking lots, different mode switchings, and the path composition constraints.

In Table 2 we reported the algorithm for a given origin-destination pair (o, d). The following parameters need to be defined:

- $dist$ is a vector that stores the distance of each node to the destination d, i.e., $dist_j$ is the distance from node j to destination d at a certain iteration;
- F is a vector that stores the number of transfers in a path;
- pr is a vector that stores the predecessor node of a given node, i.e., pr_d is the predecessor of node d;
- B is the set of stop nodes;
- L is a list used to store nodes;
- c_{ij} is the average waiting time associated with boarding arc (i, j);
- $state_j$, with $j \in N$, is the state of node j; it is equal to 1 if the car mode is in the path to which j belongs; it is equal to 0 otherwise;
- A_i^+ is the set of immediate successors of node i;
- A_i^- is the set of immediate predecessors of node i;
- t_{ij} is the same as t_e where $e = (i, j)$.

The algorithm proposed works as follows. First (see Step 1) the successor of each node is set equal to destination node d, the distance of each node to d is set equal to $+\infty$, and, trivially, the distance from d to d is set equal to zero. Then, we put d in the list L of the nodes to be processed. Furthermore, the number of transfers is set equal to 1, i.e., $F_j = 1$, $\forall j \in N$.

(1. Initialization) $pr_j = d,\ \forall j \in N$; $dist_j = +\infty,\ \forall j \in N \setminus \{d\}$; $dist_d = 0$; $L = \{d\}$; $F_j = 1,\ \forall j \in N$; $state_j = 0,\ \forall j \in N$;

(2. Iterative) Let j be a node in list L with the minimum value $dist_j$. For each node $k \in A_j^-$ do:

- **(2.1)** if (($j \notin B$) or ($j \in B$ and $k \notin T$)) then
  - **(2.1.1)** if ($dist_k > dist_j + t_{kj}$) then
    - **(2.1.1.1)** if (not (($k \in C$) and ($j \notin C$) and ($state_j = 1$))) then
      - **(2.1.1.1.1)** $pr_k = j$
      - **(2.1.1.1.2)** $dist_k = dist_j + t_{kj}$;
      - **(2.1.1.1.3)** if ($k \in C$) then $state_k = 1$; else $state_k = state_j$;
      - **(2.1.1.1.4)** $F_k = F_j$;
- **(2.2)** if (($j \in B$) and ($k \in T$))
  - **(2.2.1)** if ($dist_k > dist_j + F_j \cdot c_{kj}$) then
    - **(2.2.1.1)** $dist_k = dist_j + F_j \cdot c_{kj}$;
    - **(2.2.1.2)** $F_k = F_k + 1$;
    - **(2.2.1.3)** $pr_k = j$
    - **(2.2.1.4)** $state_k = state_j$;

(3. Update the list L**)** Put in L all the nodes $k \in A_j^-$ such that $dist_k > dist_j + t_{kj}$ if either $j \notin B$ or ($j \in B$ and $k \notin T$), and all the nodes $k \in A_j^-$ such that $dist_k > dist_j + F_j \cdot c_{kj}$ if $j \in B$ and $k \in T$; if L is empty then go to Step 4, otherwise go to Step 2;

(4. Network loading) Load the flow g_{od} on each arc of the shortest path from o to d found. If all the OD pairs have been considered stop. Otherwise, restart the algorithm with another origin-destination pair summing up each time the flows on the paths found.

TABLE 2. The assignment algorithm on feasible shortest multi-modal paths.

In the iterative step (see Step 2) a node, say j, is selected from list L and all the predecessors $k \in A_j^-$ are considered. Now, if $j \notin B$ or $j \in B$ and $k \notin T$ then (k, j) is not a boarding arc, and thus the update of the label *dist* of node k is done according to the Bellman's optimality condition. Before examining what if node $j \in B$ and $k \in T$, we notice that the variable *state* is used in Step 2.1.1.1 and then updated in Step 2.1.1.1.3. The latter is a binary variable assuming value 1 if at least once the path to which k belongs has encountered the car mode, and assumes value 0 otherwise. The correct use of this variable ensures that if in the path from a certain node k to d mode c has already been used, then it can no longer be used. This condition is expressed in Step 2.1.1.1, where we say that if $k \in C$ and its successor $j \notin C$ and has $state_j = 1$ then we cannot choose node k as predecessor of j otherwise the car more will be considered again in a new (disjoint) subpath.

Step 2.2 updates, if necessary, vector *dist* when (k, j) is a boarding arcs. In this case the update is executed not only on *dist*, but also on F in order to manage the penalty on the number of transfers, as reported in Section 3. Note that F_j is initialized to one, for each $j \in N$, and increases each time a boarding arc is traversed. With this variable one can penalize c_{kj} in Step 2.2.1.1 by a factor F_j.

Step 3 puts in list L those nodes which do not verify the Bellman's optimality condition. If L is not empty then Step 3 invokes Step 2 which restarts the updating process; otherwise, i.e., L is empty, we go to Step 4 that loads the flow on the shortest multi-modal (o, d) path found. Now, if all the origin-destination pairs have been considered the whole process stops, otherwise a new origin-destination pair is considered and the algorithm is restarted.

Once the algorithm stops, the nodes of the shortest multi-modal path used for switching from c to p, or from p to c, locate the facilities where it is more convenient for car users to leave their cars. Moreover, by varying the cost of on-street parking one can observe how the use and the location of PRFs change.

Remark 2. Note that the updating of F and *state* done in Step 2.1.1.1.3, Step 2.1.1.1.4, Step 2.2.1.2 and Step 2.2.1.4 is such that the number of transfers and the state of a node refer each time to the current optimal subpath to which that node belongs. In fact, when the label of a node k changes and therefore its predecessor changes as well, values F_k and $state_k$ are updated according to the values of such variables associated with the new predecessor thus transferring to k the history of the new subpath to which k now belongs.

6. An application to the City of Rome.

6.1. The network.

We have applied the proposed method to the City of Rome. In our application, we use a graph representing the viability inside the convex polygon defined by the nodes Piazza San Giovanni - Stazione Termini - Piazza del Colosseo - Piazza Venezia (see Figure 4 for a map of the area).

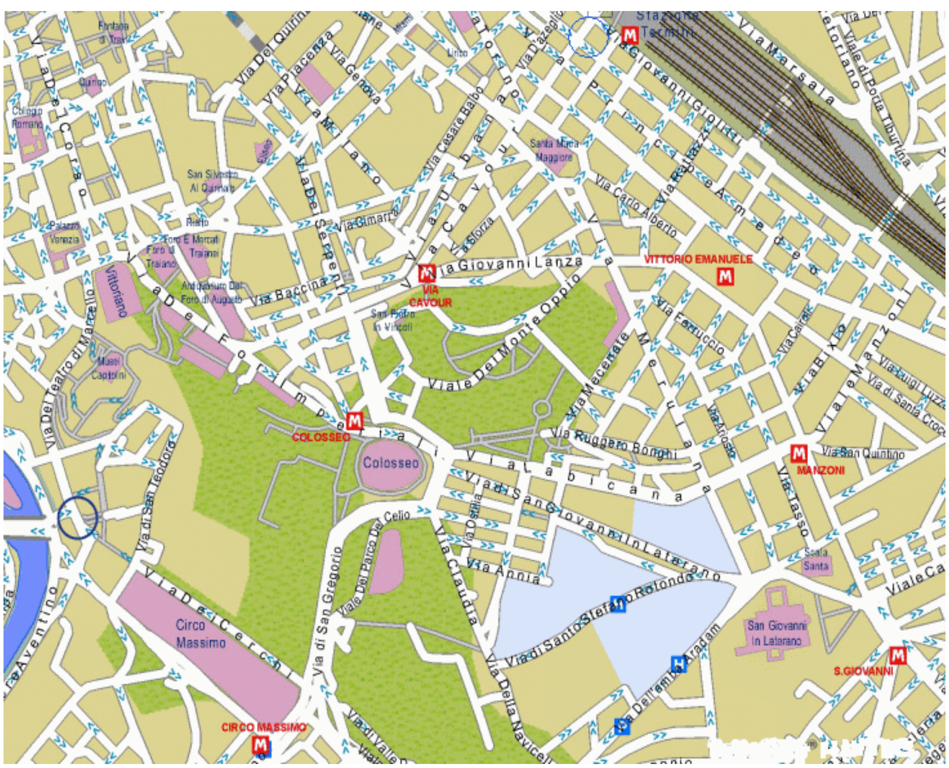

FIGURE 4. Area considered in the experimentation.

This graph is formed by 890 arcs and 226 nodes, 20 of which are centroid nodes and 8 nodes of which refer to two metro lines inside the considered area. We have set an average speed of 4 km/h for the pedestrian mode and real data for the metro mode. The number of trains for the two metro lines are, respectively, 19 and 11 per hour. There are 6 bus lines serving the considered zone and their frequencies vary from 7 to 11 per hour.

Based on real scenarios suggested by planners, we have identified two PRFs located in the nodes close to Stazione Termini (ST) and Via Petroselli (VP), respectively (see the two blue circles in Figure 4). We have performed tests for each couple of centroid nodes and reported the most significant experimental values in the following tables.

6.2. **Determination of the *c-p connection arc* weights.** One of the most difficult setting in our approach is surely the determination of the *c-p connection arc* weights. In Section 3 we have said that the components giving rise to the mentioned weights are both the time to reach the bus stop (or the destination centroid) from the parking lot where the car is left, and the parking price. The first term is easily achievable by measuring the distance in the street network and dividing it by the average walking speed, e.g., 4 km/h. Denote this first component as c_1. Our concern is the second term. Indeed, the importance of this term is also in the fact that it could affect the time that a car user waste to find a place where he/she can leave the car in an on-street parking lot. In general, a higher price makes room for a larger number of places where one can leave the car. In our experiments, we considered a daily parking price equal to 0 Euro, 3 Euros, 6 Euros and 9 Euros, respectively.

Destination centroid	Total path cost	Use of free park	Car cost	Pedestrian cost	Transit cost
1	191	no	114	77	0
2	388	yes	132	256	0
3	203	no	108	95	0
4	315	no	126	189	0
5	305	no	192	113	0
6	435	no	210	225	0
7	161	no	84	77	0
8	137	no	60	77	0
9	185	no	108	77	0
10	285	no	150	135	0
11	315	no	180	135	0
12	137	no	60	77	0
13	165	no	30	135	0
14	245	no	168	77	0
15	570	no	210	360	0
16	636	no	186	450	0
17	213	no	60	153	0
18	144	no	54	90	0
19	234	no	144	90	0

TABLE 3. The parking price outside PRFs is 0 Euro.

Although the choice of these values is based on a calibration of the model, it is realistic since, currently, in Rome, the on-street parking lots admitting a daily parking fee (most of them are managed with a per hour fee) ask for about 3 Euros. Denote the component associated with the parking fee as c_2. For the sake of homogeneity, we have introduced a factor $\alpha = 100\frac{seconds}{Euro}$ which allows one to compare and unify the component c_2 with c_1 (as for the parking fee, the choice of the value of α is based on a model calibration). This means that with each one of the above prices, we have associated a time in seconds c_2 equal to 0, 300, 600 and 900, being, respectively, α times 0 Euro, 3 Euros, 6 Euros and 9 Euros. It is worth mentioning that the factor α may vary changing for example from Euro to Dollar, or to another unit of money.

In the experiments, the contribution of the first component c_1, will appear in the so called *Pedestrian cost*, while the component c_2, strictly related to the parking fee, will be considered in the so called *Total path cost* which includes also the *Pedestrian cost* as well as the term *Car cost* and *Transit cost* (see the next paragraph for details).

6.3. **Results and analysis.** Tables 1, 2, 3, and 4 report the cost, in seconds, of shortest multi-modal paths from an origin located in Piazza Labicana (PL) to each centroid node.

Destination centroid	Total Path cost	Use of free park	Car cost	Pedestrian cost	Transit cost
1	344	yes	132	212	0
2	388	yes	132	256	0
3	503	no	108	95	0
4	441	yes	90	189	162
5	605	no	192	113	0
6	735	no	210	225	0
7	461	no	84	77	0
8	441	no	60	77	0
9	485	no	108	77	0
10	447	yes	132	315	0
11	615	no	180	135	0
12	437	no	60	77	0
13	465	no	30	135	0
14	545	no	168	77	0
15	870	no	210	360	0
16	936	no	186	450	0
17	513	no	60	153	0
18	440	no	54	90	0
19	450	yes	90	90	270

TABLE 4. The parking price outside PRFs is 3 Euros.

The tables contain six columns: the first, *Destination centroid*, contains the destination node; the second, *Total path cost*, contains the cost (in seconds) of the shortest path from the origin to the destination; the third *Use of free park* says whether a free parking is used or not; the forth, *Car cost*, represents the cost (in

Destination Centroid	Total path cost	Use of free park	Car cost	Pedestrian cost	Transit cost
1	344	yes	132	212	0
2	388	yes	132	256	0
3	803	no	108	95	0
4	441	yes	90	189	162
5	905	no	192	113	0
6	855	yes	90	423	342
7	761	no	84	77	0
8	741	no	60	77	0
9	504	yes	90	198	216
10	447	yes	132	315	0
11	915	no	180	135	0
12	737	no	60	77	0
13	587	yes	132	275	180
14	611	yes	90	257	264
15	990	yes	90	558	342
16	1079	yes	90	617	372
17	813	no	60	153	0
18	744	no	54	90	0
19	450	yes	90	90	270

TABLE 5. The parking price outside PRFs is 6 Euros.

seconds) related to the car mode as the sum of the costs of the car mode arcs in the path; the fifth column, *Pedestrian cost*, represents the cost (in seconds) related to the pedestrian mode as the sum of the costs of the pedestrian mode arcs in the path; the sixth column, *Transit cost*, represents the cost (in seconds) related to the transit mode as the sum of the costs of the metro and/or bus mode arcs in the path. For example, the first raw in Table 1 says that the path to centroid 1 has a total cost of 191, 114 of which are paid by car and 77 by feet (the transit cost is 0). Moreover, the path does not include a free parking.

Values in Table 5 refer to the use of the PRF at ST and VP. Columns *% of use of ST parking*, *% of use of VP parking* and *% of total* show the percentage of use. The column *Cost without using PRF*, contains the sum of the costs of the paths from the origin to all the destinations where no PRF is considered. The column *Cost using PRF*, contains the sum of the costs of the paths from the origin to all the destinations where PRFs are located at ST and VP. The last column, *Gain*, considers the advantage, in terms of cost, related to the use of PRFs compared to not using them. Tables 6 and 7 contain the same columns as Table 5, but the values refer, respectively, to the only PRF at ST (Table 6) and VP (Table 7).

Analyzing the computational results we see that when the cost outside a PRF is zero, i.e., when all the parking areas are free, only the 5.2% of the users will park their cars in a PRF. As the cost is increased to 300 this percentage grows to about 26.3% and when we set to 600 such a cost, 57.8% of the users are encouraged to leave their cars at PRFs. The saturation cost is 900, i.e, when the cost equals such a value all the users are discouraged to take a trip made up of only the c mode.

Destination centroid	Total path cost	Use of free park	Car cost	Pedestrian cost	Transit cost
1	344	yes	132	212	0
2	388	yes	132	256	0
3	785	yes	132	293	360
4	441	yes	90	189	162
5	947	yes	132	653	162
6	855	yes	90	423	342
7	887	yes	90	527	270
8	912	yes	90	450	372
9	504	yes	90	198	216
10	447	yes	132	315	0
11	1023	yes	132	711	180
12	764	yes	90	410	264
13	587	yes	132	275	180
14	611	yes	90	257	264
15	990	yes	90	558	342
16	1079	yes	90	617	372
17	836	yes	90	374	372
18	669	yes	90	315	264
19	450	yes	90	90	270

TABLE 6. The parking price outside PRFs is 9 Euros.

Parking price outside PRFs	% of use of ST parking	% of use of VP parking	% of total	Cost without using PRFs	Cost using PRFs	Gain
0 Euro	0	5	5	5591	5591	0
3 Euros	10	15	25	10664	10007	657
6 Euros	35	20	55	15737	12912	2825
9 Euros	70	30	100	20810	14140	6670

TABLE 7. Percentage values of the use of PRFs ST and VP.

Parking price outside PRF	% of use of ST parking	Cost without using PRF	Cost using PRF	Gain
0 Euro	0	5591	5591	0
3 Euros	15	10664	10301	363
6 Euros	45	15737	13553	2184
9 Euros	100	20810	15962	4848

TABLE 8. Percentage values of the use of PRF ST.

7. **Conclusions.** In this paper we proposed a model which helps evaluate the effects of the parking price on travel choice. Besides a network representation able to capture the multi-modal nature of trips in an urban scenario, we proposed an

Parking price outside PRF	% of use of VP parking	Cost without using PRF	Cost using PRF	Gain
0 Euro	5	5591	5591	0
3 Euros	15	10664	10322	342
6 Euros	20	15737	14291	1446
9 Euros	50	20810	17672	3138
15 Euros	100	30956	19788	11168

TABLE 9. Percentage values of the use of PRF VP.

all-or-nothing algorithm for the assignment of the OD demand on feasible shortest multi-modal paths used as a subroutine of a heuristic algorithm for the minimization of the total travelling times of the users. The proposed algorithm was tested on a real scenario. Computational results showed that the method is sensitive to the variation of the *c*-*p* connection arc costs, modifying the choice of the area where modes are switched.

REFERENCES

[1] Adiv, A., Wang, W., On-street parking meter behavior, *Transportation Quarterly*, 41 (1987), 281–307.

[2] Arnott, R., de Palma, A., Lindsey, R., A temporal and spatial equilibrium analysis of commuter parking, *Journal of Public Economics*, 45 (1991), 301–337.

[3] Arnott, R., Grieson, R., Optimal tolls with high-peak travel demand. In: The Urban Economy and Housing, Heath, Lexington, MA, 1982.

[4] Arnott, R., Rowse, R., On comparative-static analysis in numerical nonlinear economic models, *Computational Economics*, 8 (1995), 95–105.

[5] Arnott, R., Rowse, J., Modeling Parking, *Journal of Urban Economics*, 45 (1999), 97–124.

[6] Asakura, Y., Kashiwadani, M., Parking availability information as a measure for reducing congestion: A simulation approach, mimeo, 7th World Conference on Transport Research, Sydney, 1995.

[7] Axhausen, K.W., Polak, J.W., et al., Choice of parking: stated preference approach. TSU Report 560. Oxford University, Oxford, U.K, 1990.

[8] Bates, J., Econometric issues in stated preference analysis, *Journal of Transport Economics and Policy*, (1) 22 (1988), 59–69.

[9] Battista, M.G., Lucertini, M., Simeone, B., Path Composition and Multiple Choice in a Bimodal Transportation Network, Proc. 7th WCTR, Sydney, 1995.

[10] Beaton, P., Meghdir, H., Carragher, J., Assessing the effectiveness of transportation control measures: use of stated preference models to project mode split for work trips. Transportation Research Record, 1346, TRB, National Research Council, Washington, DC, pp. 44-52, 1992.

[11] Ben Akiva, M., Lerman, S., Discrete Choice Analysis, MIT Press, Cambridge, 1985.

[12] Ben Akiva, M., Morikowa, T., et al., Estimation of travel demand models from multiple data sources. In: Koshi, M. (Ed.). Transportation and Traffic Theory, Proceedings of the 11th ISTTT. Elsevier, pp. 461- 476, 1991.

[13] Ben-Akiva, M., Morikawa, T., et al., Estimation of switching models from revealed preferences and stated intentions, *Transportation Research A*, (6) 24A (1990), 485–495.

[14] Bianco, M.J., Dueker, K., Strathman, J., Parking Strategies to Attract Auto Users to Transit. Discussion Paper 95-5, The Center for Urban Studies, Portland State University, Portland, Oregon, January 1997.

[15] Bifulco, G.N., Stochastic models for the simulation of parking choices: a non-network approach, Proc. 24th PTRC -Seminar D&E, Part 2, 1996.

[16] Bhat, C., Recent methodological advances relevant to activity and travel behavior analysis. In: IATBRO97, The Eighth Meeting of the International Association of Travel Behavior Research, Austin, Texas, September (Conference Pre-prints), 1997.

[17] Bonsall, P., Palmer, I., Balmforth, P., PARKIT - a simulated world for parking choice research. Presentation at the 8th WCTR, Antwerp, 1998.

[18] Bradley, M.A., Goodwin, P.D., Layzell, A.D., Bates, J.J., CLAMP: a model to simulate parking behavior in town centres. Proceedings of the 1986 UTSG Conference, Cardiff, 1986.

[19] Brownstone, D., Bunch, D.S., Train, K., Joint mixed logit models of stated and revealed preferences for alternative-fuel vehicles, *Transportation Research Part B*, 34 (2000), 315–338.

[20] Bureau of Public Roads, Traffic Assignment Manual U.S. Dept. of Commerce, Urban Planning Division, Washington D.C., 1964.

[21] Button, K.J, Hensher, D.A, (Eds), Handbook of Transport Systems and Traffic Control, Pergamon, 2001.

[22] Calthrop, E., Proost, S., van Dender, K., Parking policies and road pricing, *Urban Studies*, (1) 37 (2000), 63–76.

[23] Cascetta, E. Nuzzolo, A, Bifulco, G.N., Some Application of a Stochastic Assigment Model for the Evaluztion of Parking Policy, 19th Summer Annual Meeting of PTRC, Proceedings of Seminar G: Tranportation Planning Methods, 1991.

[24] Chriqui, C., Robillard, P., Common bus lines, *Transporation Science*, 9 (1975), 115–121.

[25] Collura, J., Fisher, D.L., Holton, A., 1998. Advanced parking management systems: strategic management of intermodal junctions. Publication of Transportation Center of University of Massachusetts and New England University Transportation Center of Massachusetts Institute of Technology, Massachusetts, USA.

[26] Cominetti, R., Correa, J., Common-lines and passenger assignment in congested transit networks, *Transportation Science*, (3) 35 (2001), 250–267.

[27] Cooper, R.B., Introduction to Queuing Theory, 2nd ed., North-Holland, Amsterdam 1981.

[28] Crainic, T., Rousseau, J.M., Multicommodity, multimode freight transportation: A general modeling and algorithmic framework for the service network design problem, *Transportation Research B*, 20 (1986), 225–242.

[29] Daly, A.J., ALOGIT 3.2. User's Guide, Hague Consulting Group, The Hague, 1992.

[30] Dial, R.B., A Probabilistic multipath traffic assignment model which obviates path enumeration, *Transportation Research*, 5 (1971), 83–111.

[31] Douglas, R.W., A parking model - the effect of supply on demand, *American Economist*, (1) 19 (1975), 85–86.

[32] Ergun, S., Development of a downtown parking model, Highway Research Record. 369, TRB, National Research Council, Washington, DC, pp. 118-134, 1971.

[33] Feeney, B.A., Review of the impact of parking policy measures on travel demand, *Transportation Planning and Technology*, (4) 13 (1989), 229–244.

[34] Fernandez, E., De Cea, J., Florian, M., Cabrera, E., Network Equilibrium Models with Combined Modes, *Transportation Science*, 28 (1994), 182–192.

[35] Frank, M., Wolfe, P., An algorithm for quadratic programming, *Naval Research Logistics Quartely*, 3 (1956), 95–110.

[36] Frantzeskakis, J.M., Tsamboulas, D.A., et al., Supply, demand and characteristics of parking in the Athens area. Final Research Report, National Technical University of Athens, Department of Transportation Planning and Engineering, Athens, 1990.

[37] Gallo, G., Longo, G., Nguyen, S., Pallottino, S., Directed Hypergraphs and Applications, *Discrete Applied Mathematics*, 42 (1993), 177–201.

[38] Gendreau, M., Etude approfondie d'un modele d'equilibre pour l'affectation de passagers dans les reseaux de transports en commun, Publication 384, Centre de Recherche sur les Transport, U. de Montreal, 1984.

[39] Gillen, D.W., Estimation and specification of the effects of parking costs on urban transport mode choice, *Journal of Urban Economics*, 4 (1977), 186–199.

[40] Gillen, D.W., Parking policy, Parking location decisions and the distribution of congestion, *Transportation*, 7 (1978), 69–85.

[41] Glazer, A., Niskanen, E., Parking fees and congestion, *Regional Science and Urban Economics*, 22 (1992), 123–132.

[42] Hensher, D.A., Stated preference analysis of travel choices: the state of the practice, *Transportation*, 21 (1994), 107–133.

[43] Hensher, D.A., Understanding travel behavior: some appealing research direction, Working Paper 868, Transport Studies Unit, University of Oxford, Oxford 1998.

[44] Hensher, D., Bernard, P., Truong, T., The role of stated preference methods in studies of travel choice, *Journal of Transport Economics and Policy*, (1) 22 (1988), 45–58.

[45] Hensher, D.A., King, J., Parking demand and responsiveness to supply, pricing and location in the Sydney central business district, *Transportation Research Part A*, 35 (2001), 177–196.

[46] Hensher, D.A., Louviere, J.J., A comparison of elasticities derived from multinomial logit, nested logit and heteroscedastic extreme value SP-RP discrete choice models, Proceedings of the Eighth World Conference on Transport Research, Antwerp, July 1999.

[47] Hensher, D., Louviere, J., Swait, J., Combining sources of preference data, *Journal of Econometrics*, 89 (1999), 197–221.

[48] Hey, A., 1980. Are optimal search rules reasonable? (and does it matter anyway?) *Journal of Economic Behavior and Organization*, 2 (1980), 47–70.

[49] Highway Research Board, Parking Principles, Special Report 125, Highway Research Board, Washington, DC, 1971.

[50] Higgins, D., Parking taxes: effectiveness, legality and implementation, some general considerations, *Transportation*, (3) 19 (1992), 221–230.

[51] Horn, M.E.T., An extended model and procedural framework for planning multi-modal passenger journeys, *Transportation Research B*, 37 (2003), 641–660.

[52] Hunt, J.D., Parking location choice: Insights and representations based on observed behavior and the hierarchical logic modeling formulation, presented at the Institute of Transportation Engineers International Conference in Vancouver, BC, Canada, Sept. 26-29 1988.

[53] Hunt, E., Teply, S., et al., A nested logit model of parking location choice, *Transportation Research*, 27B (1993), 253–265.

[54] Institute of Transportation Engineers, Transportation and Traffic Engineering Hand- book, Institute of Transportation Engineers, 1982.

[55] Khattak, F., Polak, J., et al., Effect of parking information on travellers' knowledge and behavior, *Transportation*, (4) 20 (1993), 373–393.

[56] Keane, M.P., Current issues in discrete choice modelling, *Marketing Letters*, (3) 8 (1997), 307–322.

[57] Kroes, E., Sheldon, R., et al., Stated preference methods: an introduction, *Journal of Transport Economics and Policy*, (1) 22 (1988), 11–26.

[58] K.T. Analytics, Inc., Parking Management Strategies: A Handbook For Implementation, Regional Transportation Authority, Chicago, 1995.

[59] Kurauchi, F., Iida, Y., Yoshiya, Y., Experimental analysis on drivers parking behavior under information provided, 8th WCTR, Antwerp, 1998.

[60] Lambe, T.A., Driver choice of parking in the city, *Socio-economic Planning Science*, (3) 30 (1996), 207–219.

[61] Le Clercq, F., A public transport assignment model, *Traffic engineering and control*, (1972), 91–96.

[62] Lennie, O.S., Smith, R.A., Practicalities of CBD fringe area car parking policy, Proceedings of the 11th Australian Transport Research Forum, vol. 1, Darwin, pp. 141-165, 1986.

[63] Lo, H.K., Yip, C.W., Wan, K.H., Modelling transfer and non-linear fare structure in multimodal network, *Transportation Research B*, 37 (2003), 147–170.

[64] Louviere, J.J., Hensher, D.A., Swait, J., (in press). Stated Choice Methods: Analysis and Applications in Marketing. Transportation and Environmental Valuation, Cambridge University Press, Cambridge.

[65] Litman, T., Pavement Busters Guide, VTPI (http://www.islandnet.com/ litman)), 1999.

[66] Lozano, A., Storchi, G., Shortest viable path algorithm in multimodal networks, *Transportation Research A*, (3) 35 (2001), 225–241.

[67] Lozano, A., Storchi, G., Shortest viable hyperpath in multimodal networks, *Transportation Research B*, (10) 36 (2002), 853–874.

[68] Marcotte, P., Nguyen, S., Hyperpath Formulations of Traffic Assignment Problems, in Equilibrium and Advanced Transportation Modelling, P. Marcotte and S. Nguyen (Editors), Kluwer Academic Publishers, pp. 175–199, 1998.

[69] McFadden, D., Conditional logit analysis of qualitative choice behavior. In: Zarembka, P. (Ed.). Frontiers in Econometrics. Academic Press, 1974.

[70] McFadden, D., Econometric models of probabilistic choice. In: Manski, C.F., McFadden, D. (Eds.), Structural Analysis of Discrete Data with Econometric Applications. MIT Press, Cambridge, MA, pp. 198-272, 1981.

[71] Miller, G.K., Everett, C.T., Raising commuter parking prices: an empirical study, *Transportation*, 11 (1982), 105–129.

[72] Modesti, P., Sciomachen, A., A utility measure for finding multiobjective shortest paths in urban multimodal transportation networks, *European Journal of Operational Research*, 111 (1998), 495–508.

[73] Morrall, J., Bolger, D., The relationship between downtown parking supply and transit use, *ITE Journal*, (1996), 32–36.

[74] Morrall, J., Bolger, D., Park-and-Ride: Canada's Most Cost Effective TDM Strategy, Transportation Association of Canada Annual Conference, Charlottetown, 1996.

[75] Munizaga, B.G., Heydecker, M.A., Ortuzar, J. de D., 2000. Representation of heteroskedasticity in discrete choice models, *Transportation Research Part B*, 34 (2000), 219–240.

[76] Nathan, V.P.R., Welsh, K., A Review of Parking Control Policies and Guidelines as Applied to the City of Sydney, January 1997.

[77] Nguyen, S., Pallottino, S., Equilibrium traffic assignment in large scale transit networks, *European Journal of Operational Research*, (2) 37 (1988), 176–186.

[78] Nguyen, S., Pallottino, S., Hyperpaths and shortest hyperpaths, in: B. Simeone (Ed.), Combinatorial optimization, Lecture Notes in Mathematics 1403, Springer-Verlag, Berlin, pp.258–271, 1989.

[79] Nour Eldin, M.S., El.Reedy, T.Y., Ismail, H.K., A Combined Parking and Traffic Assigment Model, *Traffic Enginnering and Control*, 22 (1981), 524–530.

[80] Online TDM Encyclopedia, http://www.vtpi.org/tdm.

[81] Ortuzar, J. de D., Willumsen, L.G., Modelling Transport. 2nd ed. Wiley, 1994.

[82] Parkhurst, G., Influence of Bus-Based Park and Ride Facilities on Users' Car Traffic, *Transport Policy*, (2) 7 (2000), 159–172.

[83] Polak, J., Axhausen, K.W., et al., CLAMP: a macroscopic simulation model for parking policy analysis. Proceedings of the 68th Annual Meeting of Transportation Research Board, National Research Council, Washington, DC, 1989.

[84] Pratt, R.H., Traveler Response to Transportation System Changes, Interim Handbook, TCRP Web Document 12, www4.nationalacademies.org/trb/, 1999

[85] Roth, G.J., Paying for Parking, The Institute of Economic Affairs, London, 1965.

[86] Segelhorst, E.W., Kirkus, L.D., Parking bias in transit choice, *Journal of Transport Economics and Policy*, (1973), 58–70.

[87] Shoup, D., Cashing out free parking, *Transportation Quarterly*, 35 (1982), 351–364.

[88] Shoup, D.C., The trouble with minimum parking requirements, *Transportation Research, Part A*, 33 (1999), 549–574.

[89] Slavin, H., An Integrated Dynamic Approach to Travel Demand Forecasting, *Transportation*, 23 (1996), 313–250.

[90] Small, K., Noland, K., Koskenoje, P., Socio-economic attributes and impacts of travel reliability: A stated preference approach, unpublished report, University of California, Irvine, 1995.

[91] Spiess, H., Contribution á la thèorie et aux outils de planification des rèseaux de transport urbains, Publication 382, Centre de Recherche sur les Transports, U. de Montreal, 1984.

[92] Spiess, H., Conical Volume-Delay Functions, *Transportation Science*, (2) 24 (1990), 153–158.

[93] Spiess, H., Florian, M., Optimal Strategies: a new assignment model for transit networks, *Transportation Research*, (2) 23B (1989), 83–102.

[94] Syski, R., Congestion Theory, North-Holland, Amsterdam, 1986.

[95] Tsamboulas, D.A., Parking fare thresholds, *Transport Policy*, 8 (2001), 115–124.

[96] U.S. Department of Transportation, *Innovations in Parking Management*, DOT-1-83-51, 1982.

[97] Van der Goot, D., A model to describe the choice of parking places, *Transportation Research*, 16A (1982), 109–115.

[98] Van der Waerden, P., Oppewal, H., Timmermans, H., Adaptive choice behavior of motorists in congested shopping centre parking lots, *Transportation*, 20 (1993), 395–408.

[99] Verhoef, R., Nijkamp, P., Rietveld, P., The economics of regulatory parking policies: The im possibilities of parking policies in traffic regulation, *Transportation Research A*, 29 (1995), 141–156.

[100] Vickrey, W.S., Statement on the pricing of urban street use, Hearings, U.S. Congress Joint Committee on Metropolitan Washington Problems, November 11, 1959. Reprinted in Journal of Urban Economics 36, pp. 42-65, 1994.

[101] Visser, J., Van der Mede P., The effects of parking measures on trafc congestion. Proceedings of the 1986 PTRC Summer Annual Meeting, Brighton England, 1986.

[102] Wang, J.Y.T., Yang, H., Lindsey, R., Locating and pricing park-and-ride facilities in a linear monocentric city with deterministic mode choice, *Transportation Research B*, 38 (2004), 709–731.

[103] Wardman, M., A comparison of revealed preference and stated preference models of travel behavior, *Journal of Transport Economics and Policy*, (1) 22 (1988), 71–91.

[104] Wardrop, J.G., Some theoretical aspects of read traffic research, Proc. Inst. Civil Engineers, Part II 1, pp. 325-378, 1952.

[105] Westin, R.B., Gillen, D.W., Parking location and transit demand: A case study of endogenous attributes in disaggregate mode choice models, *Journal of Econometrics*, 8 (1978), 75–101.

[106] Young, R., PARKSIM/1: a network model for parking location designing, *Traffic Engineering and Control*, 27 (1986), 606–613.

[107] Young, W., Thompson, R.G., Taylor, M.A.P., A review of urban parking models, *Transport Reviews*, (1) 11 (1991), 63–84.

[108] Ziliaskopoulos, A., Wardell, W., An intermodal optimum path algorithm for multimodal networks with dynamic arc travel times and switching delays, *European Journal of Operational Research*, 125 (2000), 486–502.

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