

## EMPIRICAL STUDIES ON INTERNATIONALLY DIVERSIFIED INVESTMENT USING A STOCK-BOND INTEGRATED MODEL

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**ABSTRACT.** This paper presents the result of extensive computational experiments on an internationally diversified investment model using a large number of individual stocks and bonds in a single stock-bond integrated mean risk framework. This model was proposed by one of the authors in 1997 and was shown to perform better than standard asset allocation strategy when the universe is the set of stocks and bonds of Japan and U.S. In this paper, we extend the universe to over 3500 assets consisting of stocks of 46 countries and bonds of 20 countries and compare the integrated approach with other well used methods. Computational experiments show that the integrated approach is superior to the traditional methods.

**1. Introduction.** Mean-Variance(MV) model and other mean-risk models can, in principle, be applied to any kind of assets as long as the return-risk relation among each assets are available[6]. In fact, these methods are being used in the first stage of asset allocation strategy where the universe is the set of indexes of various asset classes.

However, the use of mean-risk model is largely restricted to a single asset class, usually stocks. Instead, people use two stage asset allocation strategy when the universe covers different classes of assets. Reasons are two-folds.

First, it was not easy to formulate and solve a large scale mean-risk model consisting of over a few thousands assets, at least until early 1990's. Second, people appear to believe that different asset classes had better be handled by fund managers with sufficient knowledge and skills to handle each asset class.

Asset allocation is a sort of divide-and-conquer strategy where one first classifies assets into a number of asset classes and solve the resulting mean-variance model consisting of indexes of each class[8,10,12]. The number of asset classes is typically less than one hundred, so that the resulting problem can be solved fast. Once the proportion of the fund to be allocated to each asset class is determined, one invests into index itself or construct a portfolio simulating an index.

This is a very useful and practical method when it was not easy to solve large scale mean-risk models and when it was expensive to collect data of individual assets.

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However, it is more desirable, if possible to handle all individual assets in a single mean-risk model, according to the fundamental principle of diversification of investment[6]. In fact, we can achieve much better risk-return structure by using individual assets instead of indexes of asset classes.

In this paper, we will apply a stock-bond integrated model[1] to an internationally diversified investment where the universe is 3500 assets consisting of stocks of 46 countries and bonds of 20 countries. We compare the performance of stock-bond integrated models with asset allocation models and other benchmark portfolios.

Typical benchmark portfolios are subject to so-called "home country bias". For example, as can be seen in Figure 1, Pension Fund Association of Japan invests 70 percent of the fund into Japanese stocks and bonds, which is hardly authorized from the theoretical point of view. One of the reasons behind this bias is the fact that there is almost no literature on the simulation about large scale internationally diversified investment.

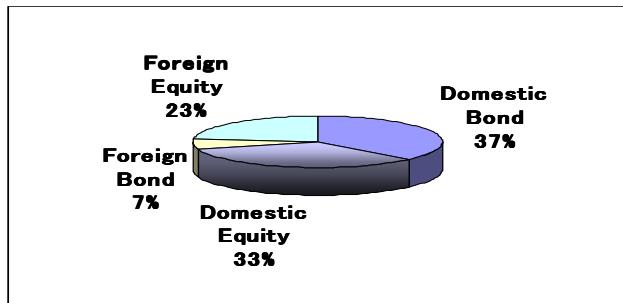


Figure 1. Pension Fund Association's Basic Portfolio

In this paper, we will demonstrate that the use of integrated approach can lead to a significantly better portfolio than two-stage asset allocation strategies and benchmarks.

We use the stock-bond integrated model proposed by one of the authors in 1997[1] and later applied to internationally diversified investment[3,4], where the universe is 710 stocks and 5 bonds of 6 countries. The main feature of the present paper is to derive a more convincing result of the advantage of integrated approach using much larger universe. As far as we know, this is the largest computer simulation ever reported on the internationally diversified investment.

In Section 2, we will describe the basic model presented in [1,4]. Section 3 will be devoted to the outline of the simulation and results of simulation will be presented in Section 4. Finally in Section 5, we summarize the result and discuss the future direction of research.

## 2. Description of The Model.

**2.1. MV model and MAD model.** Let us assume that an investor is interested in investing into  $n$  assets  $S_j (j = 1, 2, \dots, n)$ . Let  $R_j$  be the random variable representing the rate of return of  $S_j$  and let  $x_j$  be the proportion of the fund to be invested in  $S_j$ . Then the rate of return  $R(\mathbf{x})$  of the portfolio  $\mathbf{x} = (x_1, x_2, \dots, x_n)$  is given by

$$R(\mathbf{x}) = \sum_{j=1}^n R_j x_j. \quad (1)$$

Let  $E[R(\mathbf{x})]$  and  $V[R(\mathbf{x})]$  be, respectively the expected value and the variance of  $R(\mathbf{x})$ . Then the mean variance(MV) model is represented as follows:

$$\left| \begin{array}{ll} \text{minimize} & V[R(\mathbf{x})] \\ \text{subject to} & E[R(\mathbf{x})] = \rho \\ & \sum_{j=1}^n x_j = 1, \quad 0 \leq x_j \leq \alpha, \quad j = 1, 2, \dots, n. \\ & \sum_{j=1}^n a_{ij} x_j \geq b_i, i = 1, 2, \dots, m, \end{array} \right. \quad (2)$$

where the last linear constraints are various constraints imposed by fund managers.

Unfortunately, we could not solve a practical MV model consisting of more than 1,000 assets until mid 1980's since this problem leads to a dense convex quadratic programming problem. Even today, it is still not easy to solve a large scale dense quadratic programming problem with over 10,000 assets.

Mean-absolute deviation(MAD) model is a variant of the MV model in which the measure of risk is replaced by the absolute deviation:

$$W[R(\mathbf{x})] = E[|R(\mathbf{x}) - E[R(\mathbf{x})]|]. \quad (3)$$

The MAD model is defined as follows:[5]

$$\left| \begin{array}{ll} \text{minimize} & W[R(\mathbf{x})] \\ \text{subject to} & E[R(\mathbf{x})] = \rho \\ & \sum_{j=1}^n x_j = 1, \quad 0 \leq x_j \leq \alpha, \quad j = 1, 2, \dots, n. \\ & \sum_{j=1}^n a_{ij} x_j \geq b_i, i = 1, 2, \dots, m. \end{array} \right. \quad (4)$$

When the return of each asset follows a discrete distribution, this model can be reduced to a linear programming problem. Let us note that the absolute deviation is an authentic measure of risk. In fact, it is a more attractive measure of risk both computationally[5] and theoretically[7]. In addition, when the stock data follows multi-dimensional normal distribution, MV model and MAD model generates the same portfolio (See [2] for details).

## 2.2. Formulation of Integrated Internationally Diversified Investment Model.

Let us assume that a resident of country 1 is interested in investing the fund into stocks and bonds of  $I$  different countries. Let  $S_{ij}(j = 1, \dots, n_i)$  and  $B_{ik}(k = 1, \dots, m_i)$  be, respectively the  $j$  th stock and  $k$  th bond of country  $i$ . Let  $x_{ij}$  and  $y_{ik}$  be, respectively the proportion of the fund to be allocated into  $S_{ij}$  and  $B_{ik}$ . Also, let  $R_{ij}$  and  $H_{ik}$  be the rate of return of  $S_{ij}$  and  $H_{ik}$  measured by the currency of country  $i$ . Further let  $Q_i$  be the rate of return of the currency of country  $i$  measured by the currency of country 1. Then the rate of return of portfolio  $R(\mathbf{x}, \mathbf{y})$  is represented as follows:

$$\begin{aligned} & \sum_{i=1}^I \left[ \sum_{j=1}^{n_i} \{(1 + R_{ij})(1 + Q_i) - 1\} x_{ij} + \sum_{k=1}^{m_i} \{(1 + H_{ik})(1 + Q_i) - 1\} y_{ik} \right] \\ &= \sum_{i=1}^I \left\{ \sum_{j=1}^{n_i} (R_{ij} + Q_i + R_{ij}Q_i) x_{ij} + \sum_{k=1}^{m_i} (H_{ik} + Q_i + H_{ik}Q_i) y_{ik} \right\}. \end{aligned} \quad (5)$$

The rate of return  $R(\mathbf{x}, \mathbf{y})$  is affected by the return of assets and the return of currencies. Also, let us assume that we are allowed to purchase forward contract on the currency of country  $i$ . Let  $u_i$  and  $\alpha_i$  be, respectively the hedge proportion of the currency of country  $i$  and the return associated with purchasing the forward contract. Then we have to add the following term to  $R(\mathbf{x}, \mathbf{y})$ :

$$\sum_{i=1}^I \alpha_i u_i z_i,$$

where

$$z_i = \sum_{j=1}^{n_i} x_{ij} + \sum_{k=1}^{m_i} y_{ik}. \quad (6)$$

The rate of return of the portfolio is given as follows:

$$R(\mathbf{x}, \mathbf{y}, \mathbf{u}) = \sum_{i=1}^I \left\{ \sum_{j=1}^{n_i} (R_{ij} + Q_i + R_{ij}Q_i) x_{ij} + \sum_{k=1}^{m_i} (H_{ik} + Q_i + H_{ik}Q_i) y_{ik} + \alpha_i u_i z_i \right\}. \quad (7)$$

Further let  $R_{ijt}, H_{ikt}, Q_{it}$  ( $t = 1, \dots, T$ ) be, respectively the realization of the random variable  $R_{ij}, H_{ik}, Q_i$ . Also let  $\tilde{R}_{ij}, \tilde{H}_{ik}, \tilde{Q}_i, \mu_{ij}, \lambda_{ik}$  be respectively the expected value of  $R_{ij}, H_{ik}, Q_i, R_{ij}Q_i, H_{ik}Q_i$ . We use average of  $R_{ijt}$  for  $\tilde{R}_{ij}$ . Also, the risk-free rate of country  $i$  is used for the expected value of  $H_{ik}$  [1,11]

The absolute deviation is

$$\begin{aligned} W[R(\mathbf{x}, \mathbf{y}, \mathbf{u})] &= E[|R(\mathbf{x}, \mathbf{y}, \mathbf{u}) - E[R(\mathbf{x}, \mathbf{y}, \mathbf{u})]|] \\ &= \frac{1}{T} \sum_{t=1}^T \left| \sum_{i=1}^I \left[ \sum_{j=1}^{n_i} (R_{ijt} - \tilde{R}_{ij} + Q_{it} - \tilde{Q}_i + R_{ijt}Q_{it} - \mu_{ij}) x_{ij} \right. \right. \\ &\quad \left. \left. + \sum_{k=1}^{m_i} (H_{ikt} - \tilde{H}_{ikt} + Q_{it} - \tilde{Q}_i + H_{ikt}Q_{it} - \lambda_{ik}) y_{ik} \right] \right|, \end{aligned} \quad (8)$$

where  $\tilde{H}_{ikt}$  is the risk-free rate of country  $i$  during period  $t$ . The MAD model is therefore given by:

$$\begin{aligned} &\text{minimize} \quad W[R(\mathbf{x}, \mathbf{y}, \mathbf{u})] \\ &\text{subject to} \quad \sum_{i=1}^I \left\{ \sum_{j=1}^{n_i} (\tilde{R}_{ij} + \tilde{Q}_i + \mu_{ij}) x_{ij} + \sum_{k=1}^{m_i} (\tilde{H}_{ik} + \tilde{Q}_i + \lambda_{ik}) y_{ik} + \alpha_i u_i z_i \right\} = \rho \\ &\quad x_{ij} \geq 0; y_{ik} \geq 0, \quad j = 1, \dots, n_i; k = 1, \dots, m_i; i = 1, \dots, I \\ &\quad z_i = \sum_{j=1}^{n_i} x_{ij} + \sum_{k=1}^{m_i} y_{ik}, \quad i = 1, \dots, I \\ &\quad \sum_{i=1}^I z_i = 1 \\ &\quad \sum_{i=1}^I \left( \sum_{j=1}^{n_i} a_{lij} x_{ij} + \sum_{k=1}^{m_i} a'_{lij} y_{ik} \right) \geq b_l, \quad l = 1, \dots, L. \end{aligned} \quad (9)$$

The difficulty associated with this formulation is the existence of the product term  $u_i z_i$ . The objective function looks to be nonconvex. Fortunately however, as discussed in [2,3], this problem can be converted to a linear programming problem by introducing a new set of variables:

$$w_i = u_i z_i, \quad i = 1, \dots, I. \quad (10)$$

For details, the readers are referred to [3].

### 3. Numerical Simulation.

**3.1. Data Set.** To compare the integrated model and asset allocation, we collected historical data of indexes and individual stocks of 46 countries. Indexes are those 46 indexes included Morgan Stanley Capital Inc.(MSCI) Index Sets. Also 3500 individual assets of 46 countries, which is the largest available sets at the time of simulation. Among 46 countries, 23 are developed countries and the rest are so-called “emerging” countries.

Contrary to stocks, the available data of bonds were quite limited, i.e., 5 indexes of risk free bonds for each 20 developed countries. These indexes can be classified according to their maturities. These indexes are provided by Citigroup Global Markets Inc.

Models to be compared are therefore the following four:

A-1 Model : One stock index and one bond index of 23 developed countries

A-2 Model : One stock index and one bond index of 46 countries

I-1 Model : 1800 individual stocks and four bond indexes of 20 countries out of 23 developed countries

I-2 Model : 3500 individual stocks and four bond indexes of 20 countries out of 46 countries

For A-models, we use a single bond index for each developed countries. For I-models, we use 4 bond indexes of each 20 developed countries with different maturities. Figure 2 shows the structure of four data sets used for simulation.

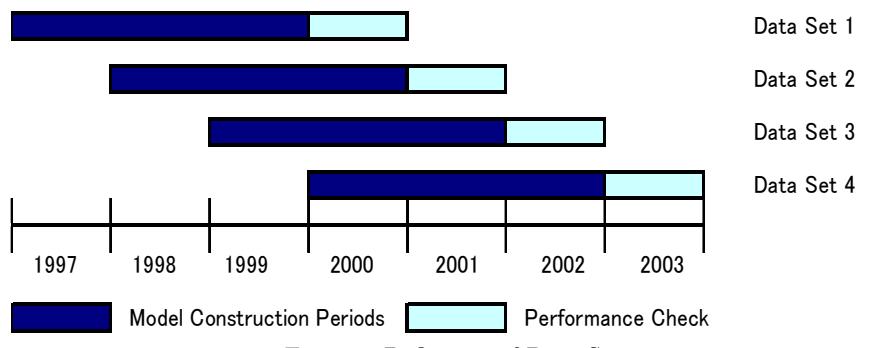


Figure 2. Definition of Data Sets

We imposed the following constraints:

1. At most 70 percent of the total fund can be allocated to both stocks and bonds.
2. At most 20 percent of the fund can be allocated to individual countries. Also, at most 30 percent of the fund can be allocated to domestic assets.
3. At most 2 percent and 10 percent of the fund can be allocated to individual stocks and bond indexes.

4. Hedge ratio  $u_i$  belongs to the interval  $[0,1]$  for all  $i$ .

#### 4. Result of Simulation.

##### 4.1. Ex-Ante Performance. (a) Efficient Frontier

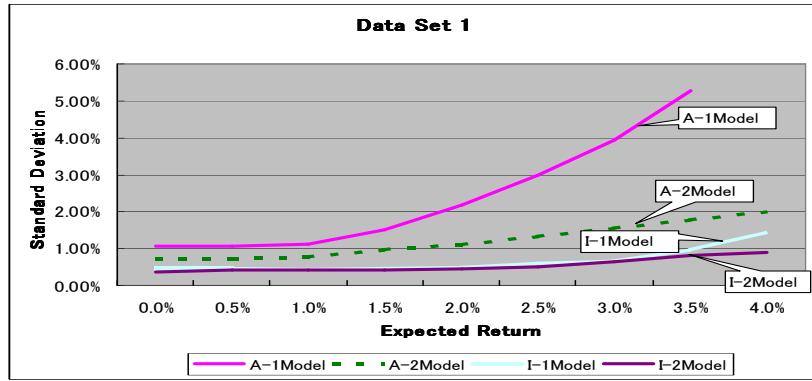


Figure3. Efficient Frontier

Figure 3 shows the efficient frontiers of A-models and I-models. As expected, the expansion of universe leads to a significant improvement of risk-return structure. In addition, risk increases very mildly as we increase the level of expected return except A-1 model. Monthly standard deviation of I-model is less than 1.5 percent even when  $\rho$  is 4 percent. When we choose  $\rho = 0.6$  percent/mo. (7.2 percent/annum) which is the level of monthly return of MSCI index, the annual standard deviation is less than 1.7 percent. This means that we can construct a stable portfolio in terms of ex-ante distribution using I-models.

##### (b) Organization of Portfolios

Table 1 shows the organization of the portfolio of each model for Data Set 1 and the portfolio of Pension Fund Associates of Japan(PFA). We see from this table that the weight of investment into domestic assets of our models is much less than that of PFA fund. Also the proportion of the fund invested into bonds is larger than 60 percent for our models, while that of PFA fund is only 44 percent. Hedge ratios are widely different between A-models and I-models. The reason of this difference is due to the fact that we can find individual stocks whose correlation with currency is low. It is rational then to reduce the currency risk by including more stocks in the portfolio. This result is rather surprising since the optimal hedge ratio for Japan-US integrated model reported in[3] is over 80 percent.

Finally, there is a big difference in the number of countries to be included in each portfolio. For asset allocation model, we purchase assets of only 10 to 13 countries among 46 countries, while we purchase assets of 23 and 38 countries in I-models. Therefore I-models are more compatible with the notion of “international diversification”.

Characteristics of Portfolio	A-1Model	A-2Model	I-1Model	I-2Model	PFA
# of assets in portfolio	11	13	51	51	43
# of countries included in portfolio	10	13	23	38	23
proportion of fund invested into Japanese assets	4.9%	5.7%	7.0%	5.4%	70.0%
proportion of fund invested into developed countries	30.0%	11.0%	36.0%	25.1%	56.0%
proportion of fund invested into emerging countries	0.0%	19.0%	0.0%	14.0%	0.0%
proportion of fund invested into bond	70.0%	70.0%	64.0%	60.9%	44.0%
total amount of currency hedging	48.0%	52.3%	20.3%	11.8%	0.0%

Table1. Organization of Portfolios

#### 4.2. Ex-post performance.

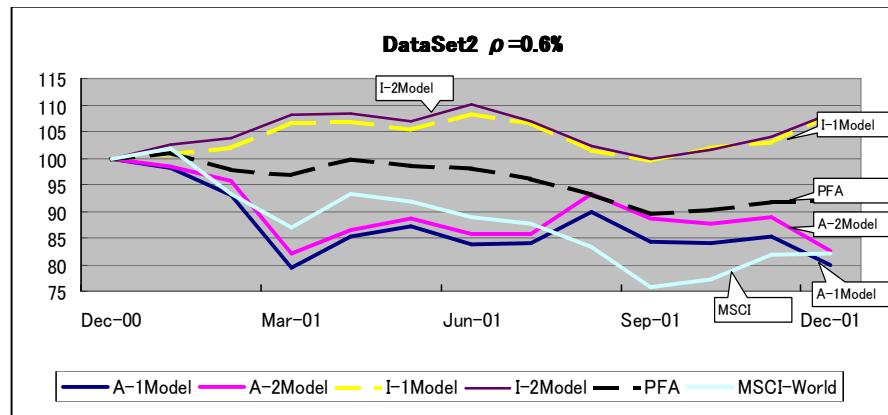


Figure4. Ex-Post Performance for Data Set 2

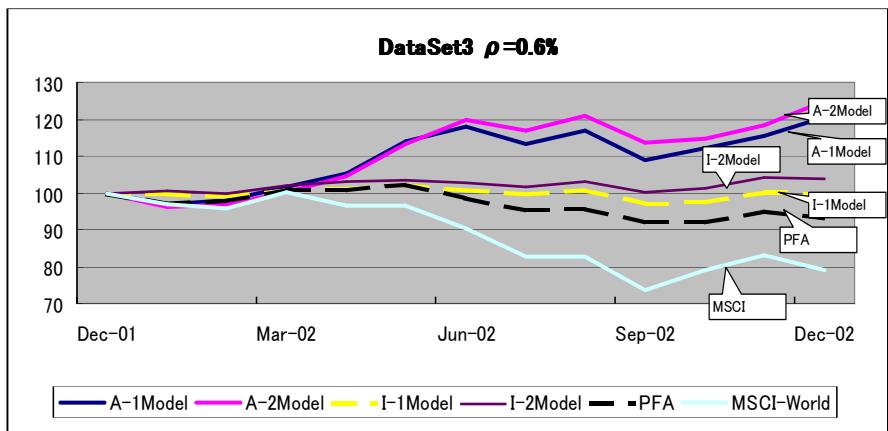


Figure5. Ex-Post Performance for Data Set 3

DataSet 1	A-1Model	A-2Model	I-1Model	I-2Model	MSCI-World	PFA
Return ( $\mu$ )	-16.89%	-18.80%	38.39%	11.39%	-13.21%	-5.04%
Standard Deviation ( $\sigma$ )	16.86%	16.94%	20.04%	6.60%	14.44%	6.80%
$\mu / \sigma$	-1.00	-1.11	1.92	1.73	-0.92	-0.74

DataSet 2	A-1Model	A-2Model	I-1Model	I-2Model	MSCI-World	PFA
Return ( $\mu$ )	-18.48%	-15.68%	8.70%	8.81%	-16.54%	-7.67%
Standard Deviation ( $\sigma$ )	21.18%	20.87%	9.70%	9.88%	18.20%	7.43%
$\mu / \sigma$	-0.87	-0.75	0.90	0.89	-0.91	-1.03

DataSet 3	A-1Model	A-2Model	I-1Model	I-2Model	MSCI-World	PFA
Return ( $\mu$ )	21.54%	26.09%	-0.17%	3.89%	-19.64%	-6.59%
Standard Deviation ( $\sigma$ )	14.60%	14.71%	5.61%	5.51%	19.30%	8.60%
$\mu / \sigma$	1.48	1.77	-0.03	0.71	-1.02	-0.77

DataSet 4	A-1Model	A-2Model	I-1Model	I-2Model	MSCI-World	PFA
Return ( $\mu$ )	49.92%	43.74%	20.78%	20.09%	31.67%	15.91%
Standard Deviation ( $\sigma$ )	8.27%	6.92%	5.00%	4.76%	12.26%	6.68%
$\mu / \sigma$	6.03	6.32	4.16	4.22	2.58	2.38

Table2. Performance Comparison

To compare the ex-post performance, we added one more benchmark. It is the MSCI-World Index, which is a popular benchmark of the international investment.

Figure 4 and 5 show the performance of two benchmarks and 4 portfolios generated by A-1, A-2, I-1, I-2 models. Table 2 shows the annual rate of return  $\mu$ , annualized standard deviation  $\sigma$  and  $\mu/\sigma$  of each portfolio during the next 12 months after portfolio construction. The reason why we use  $\mu/\sigma$  instead of the well used Sharpe's ratio  $(\mu - r_0)/\sigma$  is that there is no appropriate risk-free rate  $r_0$  in the world market.

We see from Figure 4 that I-models result in stable performance even when two benchmarks are declining. In fact, we see from Table 2 that I-models outperform benchmarks in terms of  $\mu/\sigma$  for all data sets. In addition, I-2 model achieves positive return for all data sets. This can be considered as a strong evidence of the superiority of I-2 model over benchmarks.

Let us now proceed to the comparison of A-models and I-models. Viewing at Table 2, I-models outperform A-models for Data Sets 1 and 2, while A-models outperform I-models for Data Sets 3 and 4. One may conclude that there is no clear evidence that I-models are better than A-models. However, I-models leads to portfolios with much more stable portfolios than A-models. In fact, A-models achieve large positive return in 2 data sets, while they suffer very large losses in 2 data sets.

On the other hand, I-2 model always leads to positive return for all 4 data sets. This remarkable stability of I-2 model endorses our claim that I-2 model is the best among all six models. Also, associated with A-model is nonnegligible transaction cost for purchasing individual assets to track the indexes. As a result, we conclude that I-model is the best model for investors who want to make 0.6 percent of monthly return constantly.

This result is mainly due to the diversification effect. I-1 model invests into 51 shares in all developed countries and I-2 model invests into 51 shares in 38 of 46

countries. A large universe enables us to use various assets with different risk-return structure.

On the other hand, we observe that there is a relatively small effect of the inclusion of the assets of emerging countries. At the same time, we see from these results that the inclusion of emerging countries is not a dangerous strategy either.

Let us add that the results are more or less the same when we vary the level of  $\rho$  as long as it stays in the range of 0.6 percent/mo to 2 percent/mo.

**5. Conclusion and Future Direction of Research.** We showed in this paper that we can achieve stable performance by using a stock-bond integrated model when applied to internationally diversified investment. We can construct this portfolio without difficulty due to the progress of the market infrastructure all over the world. In fact, a number of security companies and investment banks support the transaction of execution of assets of the over fifty countries. Also, they hold offices in various part of the world and provide services with low service costs. The difference of fund management costs associated with domestic assets and those of emerging countries is less than 1.0 percent. Therefore, we are now in a position to manage fund in an internationally diversified framework.

Therefore, the results presented in this paper should be of interest to many financial institutions including pension fund and various asset management business. However it may not be convincing enough for those who feel comfortable with standard asset allocation strategies. We need to conduct more extensive simulation to persuade them to switch to the integrated approach.

Let us emphasize here that asset allocation strategy is subject to a significant amount of service/ transaction cost to purchase indexes or to construct a portfolio simulating indexes by index tracking strategies. This is another drawback of asset allocation strategy.

We are now planning a simulation to further demonstrate the advantage of the integrated approach by using historical data for the past 15 years, whose result will be reported elsewhere.

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