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Research article

A revision of Altman Z-Score model and a comparative analysis of Turkish companies' financial distress prediction

Zeynep Cındık* and Ismail H. Armutlulu

Department of Quantitative Methods, Faculty of Business Administration, Marmara University, Istanbul, Turkey

* Correspondence: Email: zeynep_cindik@hotmail.com.

Abstract: This paper aims to predict companies' financial distress situation with the use of four different models; Altman Z score, Revised Altman Z Score (Linear Discriminant Analysis) and Quadratic Discriminant Analysis, Random Forest Machine Learning Model with the use of same variables suggested by Altman. Also a review of Altman Z score model for Turkey case is assessed whether it is applicable to Turkish companies and how accurate the results are. This study differentiates itself from the previous studies by the content of the data; it includes both publicly traded companies and private companies. Additionally, there are only a few studies are applied for random forest model in bankruptcy prediction in Turkey. For this reason, it aims to fill the gap from both a rare used model and originality of the data. The data consisted of the 80 firms' financial ratio analysis between the years 2013 to 2018. There are 44 firms are listed in BIST; remaining 36 firms are private firms with the size of small and micro enterprises. Random forest model with use of Altman variables has shown 95% performance and surpassed the other three models. Moreover, the classification results for publicly traded companies was 100% for Random Forest whereas other models have shown greater performance for private firms than publicly traded firms.

Keywords: financial distress; financial ratios; discriminant analysis; revised Altman score; MDA; random forest; bankruptcy

1.   Introduction

Future development of the companies is highly dependent on the evaluation and analysis of financial situation as well as taken steps at present. To interpret the indicators correctly for the future position of the companies is crucial regardless of the size, the type of operational activities, or any other characteristics that companies have (Svabova et al., 2020).

Financial distress is a term that is used to describe the condition of a firm that is experiencing financial difficulties. This sometimes occurs as being unable to pay their dividends, not meeting the obligations and even more coming to an end in all operational areas (Beaver, 1966). The prediction of the companies' financial distress position is crucial for rating agencies, managers, investors, bankers and also for the shareholders of the company itself and even more the countries' economy at large (Alaka et al., 2018).

The early stage signs are tried to be found out for the distressed companies; the earlier the distress is detected the better the solution can be applied in order to overcome the situation. So there are many methods developed and still being in process regarding prediction of the distress of the companies. Depending of the size of company; a distressed company may have great impact on the lenders, shareholders and even on the industry and economy. Hence, this topic has become one of the main concerns of the analysts and they are still having difficulties finding a reliable tool for the risks and threats of failure of the companies (Svabova et al., 2020).

The origin of the financial situation theory relies on 20s'–30s' however early proposed models are appeared in the literature in 60s'. Business financial distress prediction calculation through ratio analysis has started with Beaver (1966). Beaver has applied univariate analysis which is a traditional method for interpretation of financial ratios (Beaver, 1966). Altman (1968) has developed a new model by using a statistical tool, multivariate discriminant analysis (MDA), using five financial ratios (Altman, 1968). This new model was named as Altman Z-score model and has become the most popular method since then in the accounting and finance research field. The shortcoming of the model was the coefficients calculated via multiple discriminant analysis were highly dependent on the economic environment as well as company's operating industry (Georgiev & Petrova, 2015). Further, this model was developed only using listed US companies as their sample. The applied studies have shown that in different countries' models lose their prediction power for the reason differences in economic conditions of each country (Karas & Srbová, 2019). With this understanding, this paper aims to review and revise Altman Z-score model from Turkey case and to test whether it is applicable to Turkish publicly traded and non-traded companies simultaneously. Also, the coefficients will be reviewed and the most accurate parameters will be estimated with the use of MDA model, additionally the quadratic discriminant analysis and as one of the most popular machine learning models, Random Forest, is applied with the variables suggested by Altman.

This paper is organized as follows; second part contains the literature related to this field also includes the financial distress indicators. In the third part, the methodology for four models are

proposed and financial ratios are explained. Addition to that, in data analysis part, the data is introduced with the main characteristics; how it is selected and the criteria in selection of sampling explained. In the next part, the empirical results are discussed in detail. Finally, in the conclusion part the whole paper is concluded.

2.   Literature review

The prediction of financial distress through financial ratios started in 1932 by Fitzpatrick (Colak, 2019). Fitzpatrick's study has remarked that accounting ratios could be used as financial distress indicators. In the following years, Beaver (1966) has used univariate analysis for prediction by using some selected ratios. He has used five ratios and reached as a result "net cash flow to Total Liabilities" was founded as the most significant variable to explain the distress position of companies (Affes & Hentati-Kaffel, 2019).

Altman (1968) has applied multiple discriminant analysis (MDA) by using five ratios to assess potential failure of companies and this method became one of the pionering and the most applied model for prediction of financial distress of companies. The quantity of companies chosen for this study was 66 and they were all manufacturing companies. These companies were classified as bankrupt and non-bankrupt and all bankrupt companies were filed bankruptcy petition between the years 1946–1965. In the beginning, 22 variables have chosen however with further analysis the discriminant analysis has formed with five variables (see Equation (1)). These are all shown in the below model;

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5 \tag{1}$$

where
X_1 = Working Capital/Total Assets
X_2 = Retained Earnings/Total Assets
X_3 = Earnings Before Interest and Taxes/Total Assets
X_4 = Market Value of Equity/Book Value of Total Debt
X_5 = Sales/Total Assets
Z = Overall Index
Source: Altman (1968).

According to the overall index; when Z value is greater than 2.99 firm is classified as non-bankrupt, when index value ranges from 1.81 to 2.99 firm is classified in the gray area, and index value is less than 1.81 represents that firm is in difficult situation and in that case it is classified as bankrupt (For this study; because of the differences of the bankruptcy process in Turkey than many other European countries and USA; the term financial distress will be used as an alternative to bankruptcy). The outcomes of Altman's study has shown that the model has very high accuracy. The model's accuracy was 95% prior to a year before bankruptcy and 72% two years prior to bankruptcy.

As an improvement of this model another study represented in 1977 by Altman, Haldeman and Narayanan and they proposed zeta model with seven variables and accuracy of the model was 96% (Affes & Hentati-Kaffel, 2019).

Altman Z score model has been updated by Altman (1983) for privately held firms and it was called as Z' model, in that model variable X_4 with market value replaced by book value of equity. The

coefficients for the variables in this model is slightly different than the Z-Score model (see Equation (2)) (Altman, 1983).

$$Z' = 0.717\text{X}_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.998X_5 \tag{2}$$

X_1 = Working Capital/Total Assets
X_2 = Retained Earnings/Total Assets
X_3 = Earnings Before Interest and Taxes/Total Assets
X_4 = Book Value of Equity/Total Liabilities
X_5 = Sales/Total Assets
Z' = Overall Index
Source: Altman (1983).

According to the overall index; if the Z' value is greater than 2.90 they are classified as non-bankrupt firms, if their index value ranges from 1.23 to 2.90 they are classified as in the gray area, and the if index values are less than 1.23 represents that companies are in difficult situation and classified as in the high risk of bankruptcy. The classification accuracy of this model for bankrupt firms was 90.9% and for non bankrupt firms it was 97.0% (Altman, 1983).

Another revision is made over original Z score model and a new model called Z'' which is developed for both manufacturing and non-manufacturing companies and private and public firms (see Equation (3)) (Altman et al., 2017).

$$Z'' = 6.56\text{X}_1 + 3.26X_2 + 6.72X_3 + 1.05X_4 \tag{3}$$

X_1 = Working Capital/Total Assets
X_2 = Retained Earnings/Total Assets
X_3 = Earnings Before Interest and Taxes/Total Assets
X_4 = Book Value of Equity/Total Liabilities
Z''= Overall Index
Source: Altman (1983).

In this formula X_5 variable has been removed in order to minimize the potential industry impact. Also as in Z', book value of equity has been used for X_4 variable. If the Z'' value is greater than 2.60 they are classified as non-bankrupt firms, if their index value ranges between 1.10 to 2.60 they are classified as in the gray area, and if index values are less than 1.10 companies are in difficult situation and and classified in the high risk of bankruptcy. The accuracy of this model was 97% for non bankrupt firms and 90.9% for bankrupt firms (Altman, 1983).

There are many other theoretical, statistical and artificial intelligence related methods are used to predict financial distress beside Altman Z Score. One of these is cased- based reasoning model applied in 1993 by Kolodner, one other entropy theory (Theil, 1969; Lev, 1973) also credit risk theories (Merton, 1974), rough sets are examined in 1982 by Pawlack (Dimitras et al., 1999), genetic algorithms (Varetto, 1998); neural networks (Odom & Sharda, 1990), logit (Ohlson, 1980) and probit analysis in 1984 applied firstly in bankruptcy analysis by Zmijewski (Balcaen & Ooghe, 2006) and random forests (Breiman, 2001; Wang et al., 2017). These are the studies which generally consist of the first developers of the models.

More recent studies; Gordini (2014) applied genetic algorithm 3100 Small to Medium Enterprises in Italy, according to the results 84.4% total predictive performance is achieved (Gordini, 2014). Another study was also applied including two-step classification method based on genetic algorithm in 2017, it included 912 Russian company values and the accuracy of the model was 93.4% (Zelenkov et al., 2017). Also, in 2017 eight different models compared in a study which was conducted in North America. They have compared machine learning models with statistical models; the models were bagging, boosting, random forest, logistic regression, support vector machines (linear and radial basis), discriminant analysis and artificial neural networks. Over these models; the highest accuracy was found as bagging, boosting and random forest models, they were outperformed the other models (Barboza et al., 2017). Another study has applied with an Artificial Neural Network hybrid model; Magnetic Optimization Algorithm and Particle Swarm Optimization, and the accuracy of the model has detected as 99.7% (Ansari et al., 2020) All methods have their own strengths and weaknesses. Depending on the data; some methods give superior accuracy among others (Huang & Yen, 2019).

From machine learning perspective, there were various studies were conducted for bankruptcy prediction. Popular machine learning models are support vector machines, decision trees, random forests, Gradient Boosting, XGBoosting, Bagging and hybrid models that bring different models together. Random Forest model was developed by Breiman (2001). It is an ensemble learning method used for both regression and classification. It is a decision tree algorithm; each decision tree is built with a random selection of each node in order to determine the split. The same distribution is applied for all trees in forest and each tree is independently sampled with a random vector. Following the construction of each tree, the classified sample is inputted, and each tree provides a classification and then all classifiers give the results according to a majority rule, this becomes the classification results (Breiman, 2001).

The revision of the Altman Z Score has been applied in many different countries therefore country specific studies have various results. They have recalculated the coefficients or just checked the validation of the model in a specific country. One of the studies was applied by Diakomihalis (2012) to see the effectiveness of the model in hotel industry and the outcome of this study has approved that Altman Z Score model is still valid in determination of failed companies one year prior to bankruptcy. Altman Z score model has widely applied all over the world; for instance, for Greece; (Grammatikos & Gloubos, 1984) for Pakistan; (Abbas & Ahmad, 2012) for China; (Wang & Campbell, 2010), for USA (Barreda et al., 2017), for India; (Singh & Singla, 2019); for Indonesia (Prabowo, 2019). These are only a few examples; hundreds of studies can be found the applications of the model in various countries. Some resulted in favour of Altman Z Score model's validity some others revised the model or offered more state of art technology added methods.

In Turkey some studies previously reviewed Altman Z score in order to analyse the coefficients and performance rates of MDA. One of them sampled 70 listed firms in Borsa İstanbul and 35 of them was assigned as non-distressed and 35 assigned as distressed (Muzir & Caglar, 2009). According to that study of Muzir and Caglar (2009) the coefficients related to X_3 and X_5 are found negative compare to Altman score. Also the accuracy of the model was 73.3%. Another study was applied by Yilmaz and Yildiran (2015) and they have applied the model over 36 firms, half of them was distressed and the other half was non-distressed. They have found out the coefficients related to X_1 was negative compare to Altman Z score also the accuracy of the model has been calculated as 62.5% (Yilmaz & Yildiran, 2015). Another recent study has been applied by Colak (2019) that he applied Altman Z score

model to 54 companies half of them was distressed and the other half was non-distressed. He found that Altman Z scores' accuracy rate was 79% (Colak, 2019).

There are only a few studies are applied for Turkish companies in Random Forest machine learning field. One of these studies was conducted by Yıldız and Ağdeniz (2019). Their study consisted of 216 companies traded in Borsa Istanbul and they compared 16 machine learning algorithms' performance. The models with the highest accuracy for their study were Decision Tables, however Random Forest Model has also had third highest accuracy over 16 models (Yıldız and Ağdeniz, 2019). Beside that there is not any study applied in Turkey related to bankruptcy and financial distress by applying Random Forest Model. A few conducted for bank failure prediction nevertheless there is not any non-financial companies' failure/bankruptcy prediction is applied in this field.

Summing up all these results; this study shows a difference than all other studies in Turkey; the reason for this is all other studies applied Altman model to listed companies however this study allows us to analyse if it is valid for both listed and non-listed companies at the same time. Also Random Forest Model for bankruptcy model is only applied in a few studies and they are all applied with publicly traded companies or banks' failure prediction. Therefore, with this unique data, Random Forest Model is evaluated with the same variables suggested by Altman differentiates this study from the previous ones in this field.

3. Methods

Multivariate Discriminant analysis is one of the best distinguishing analysis methods for two or more groups with similar number of members from each other with derivation of index score (Balcaen & Ooghe, 2006). This analysis is used for classification of observations depending on the characteristics of the observations into one for several a priori groupings. This model can be described as below as a linear combination of discriminatory variables (See Equation (4));

$$Z = \alpha + \beta_1 X_1 + \beta_2 X_2 \ldots \beta_n X_n \ldots \qquad (4)$$

Z value stands for the dependent variable score (value) to assess the classification of the group it belongs to, β values are the coefficients of the discriminant, X values are the discriminatory independent variables. In this study's case the Z score is the multivariate discriminant score of a firm which gets value attribute value of for a firm (Balcaen & Ooghe, 2006).

In the application of the MDA analysis there are some assumptions described for the model;
- Variables are independent to each other.
- The groups are mutually exclusive.
- Number of the independent variables is not supposed to be more than two less of the sample size.
- In the significance testing, multivariate normal distribution has to be followed by independent variables.
- Errors have to be randomly distributed.
- Variance-covariance structure of the independent variables is similar within each group of the dependent variable. (Tatsuoka, 1971; Stevens, 2002).

In this study, all firms are assigned whether distressed or non-distressed according to their discriminatory scores. The coefficients of the model are calculated with maximization of the

between- group variances and minimization of them within-groups. There is a cut-off point calculated and according to the index score, the values higher than cut off value are defined as non-distressed or financially successful companies, below than cut off value, companies are defined as distressed or failed companies.

In the classification process Z scores achieved via model goes to the closest group which is earlier defined with some notion of distance between each group centroids and the case. The metrics accuracy, sensitivity, precision and specificity are all calculated via confusion matrix and cells in the table represent the classification.

Table 1. Confusion matrix.

		Predicted	
		Positive	*Negative*
Actual	*Positive*	True Positive	False Negative
	Negative	False Positive	True Negative

Calculation of accuracy and some other metrics are shown in the following page. According to accuracy formula this ratio reflects the rate of companies classified correctly over the whole data. When accuracy rate is higher the prediction power of the model is stronger.

$$\text{Accuracy} = \frac{True\ Pos. + True\ Neg.}{\text{Total Observation}} \qquad \text{Sensitivity} = \frac{True\ Pos.}{\text{Observed Positive}}$$

$$\text{Specificity} = \frac{True\ Neg.}{\text{Observed Negative}} \qquad \text{Precision} = \frac{True\ Pos.}{\text{Predictive Pos.}}$$

Another performance metric is sensitivity which is used to determine the rate of how many of the actual positives are predicted correctly as positive. In this data set; it will be an answer for the rate of the distressed firms are correctly classified as distressed. Therefore, this metric is another important measure in the evaluation of the model performance. Specificity is another measure checks true negative rates over actual negative values. In this data set, it will be an answer for the ratio of how many of the non-distressed firms are classified as non-distressed.

The final metric for confusion matrix is used for this study is precision. It is the ratio of true positive values over all predicted positive values. Again for this study, it is interpreted as; the ratio gives an answer to in the predicted distressed firms how many of them are actually distressed (Luque et al., 2019).

In this study, with a comparative perspective; in order to choose the most appropriate model for this data set three models will be evaluated with these metrics. Altman Z score model is revised whether it is still an effective model in Turkish market. While doing that there are some hypotheses are tested to evaluate the results. Hypotheses for this study are;

Hypothesis 1:

H_0: *The dependent variable does not depend on any of the independent variables.*

H_1: *The dependent variable depends on at least one of the independent variables.*

Or in mathematical form:

- *H_0: $\beta i = 0$, for $I = 1, 2,..., 5$*
- *H_1: $\beta i \neq 0$ for at least one I*

Hypothesis 2:

H_0: The model is useful for predicting Y

H_1: The model is not useful for predicting Y

- *H_0: $\beta_1 = \beta_2 = ... = \beta_i = 0$, for $I = 1, 2,..., 5$*
- *H_1: $\beta_1 \neq \beta_2... \beta_i \neq 0$, (for at least one i)*

There are two hypotheses are tested within the model, one is related to each independent variables' significance over the model. The second is whether the model is useful to classify the distressed and non-distressed companies.

The chosen ratios are the same used in the Altman Z'' Score model which are recommended both for public and private firms at the same time manufacturers and non-manufacturing firms. These ratios are the variables used for these models. The first variable is X_1 stands for Working Capital to Total Assets; working capital is calculated as the difference between current assets and current liabilities. This is a liquidity ratio and directly related to assess if the company's operating loss is increasing because if so the ratio will decrease accordingly. This ratio is considered as one of the best indicator for discontinuance of firms (Altman, 1983).

Second variable X_2 stands for Retained Earnings to Total Assets; This ratio combines the firms' entire life reinvested earnings and losses. This ratio may not be in favour of relatively young companies because they have not had enough time to reinvest their earnings. Nevertheless, it is still a quite valid ratio because the failure rates of the firms generally occur as 54% in their first five years (Altman, 1983).

Third variable X_3 stands for Earnings Before Interest and Taxes to Total Assets; this ratio is related to how productively company's assets are used and turned into income without taking into account any taxes and interests. The existence of firms relies on the earning power of their assets. The last variable X_4 stands for Book Value of Equity to Total Liabilities; it questions before the firms become insolvent, how much the assets can decrease in value before assets are exceeded by liabilities (Altman, 1983). In the X_4 ratio market value dimension is not added due to having both private and public firms in the dataset. Therefore, the calculation will be based on only book value.

4. Data Set

This paper consisted of the 80 firms' financial ratio analysis between the years 2013 to 2018. The information is gathered via balance sheet and income statement of each firm with their year-end reports.

Firstly, the data related to private companies are collected via public accountants with the consent of the firms' owners. There are 4 different accountants have shared all these companies' financial year-end statements. These firms are all classified as small and micro sized enterprises, which have less than 25 million Turkish Liras sales per year and have employees less than 50 people. These limits have been standardized by "Turkey Accounting and Auditing Standards Authority". All companies are classified according to these standards.
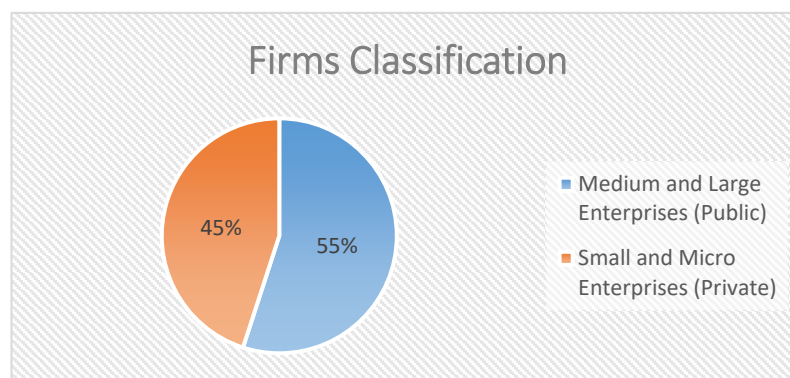
Figure 1. Firms classification.

The rest of the companies have chosen from Borsa Istanbul Stock Exchange (BIST) the similar/same industries with the non-listed companies. Financial statement of these firms are all checked by the authorities as well as independent auditors. Remaining private firms are not listed however also these companies are checked by independent auditors and have reliable values on their financial statements. So the whole data set is checked in detail whether they have misguiding information or not. Also the companies which are in the banking and finance field have not included in this study due to having different operations and dynamics from other industries. In the below figure, the all firms' industries can be seen. As it has been recognized that data consisted of various industries. The ratio of listed companies is 55% and the ratio related to non-listed companies are 45% (see Figure 2).
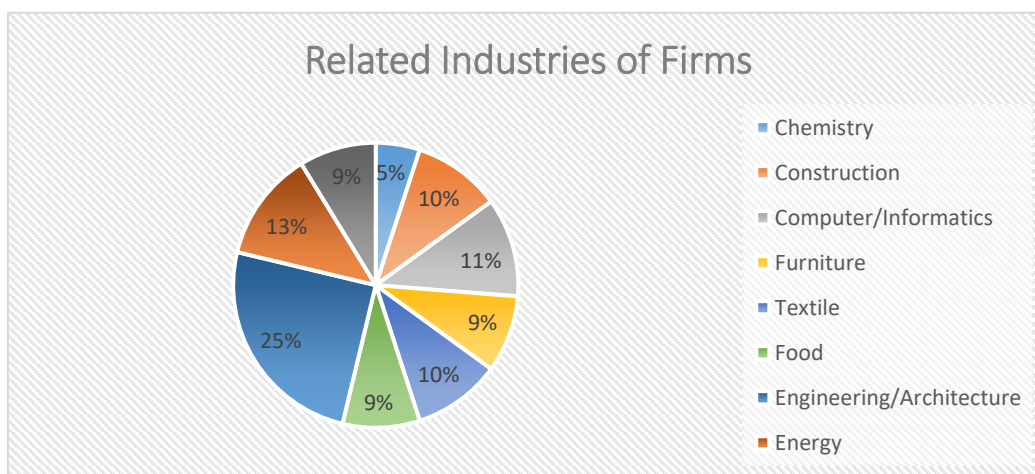

Figure 2. Data set-firms' related industries.

The heterogeneity is high however there are a few sectors are dominant in this data set including; engineering and architecture, textile, construction industry, computer and informatics.

In the consideration of all the features related to the data set used for this study; it could be said that it is a unique data which highlights both small and large companies, various industries and listed and unlisted companies at the same time.

In Turkey, related to insolvencies of the companies three major mechanisms are used, one is concordat, other one is bankruptcy/liquidation and the last one is reconstructing the companies. The legal procedure for Turkish Companies related to bankruptcy is defined by Turkish Commercial Code and Code of Compulsory Enforcement and Bankruptcy (CCEB); bankruptcy only occurs with a request from commercial court by creditors or board of directors of the company and with the application and judgement of the commercial court liquidation starts. For the companies which has not requested to commercial code nevertheless they are in financial distress; they must firstly address that they are in financial distress and accordingly there are some actions are defined with these laws must be taken. (Goksu, 2020). Financial distress has been defined in different ways by these laws, one is used when 2/3 of the capital has been lost according to the last balance sheet. In this case board of directors must take an action to replenish the company's loss. For this reason, one of the criteria for this study is chosen as having negative equity which means that company has lost all its capital (Goksu, 2020).

Another criteria is chosen as applying to Trade Registry for liquidation instead of going to court, this is also an option to close down the company. In this case, if there are any payables to creditors then court process starts. Some of the companies in data set had payables only "due to shareholders" when the shareholders give up for receivables. Those companies have closed down and liquidated without going to court within only six months. Hence, all distressed company criteria have chosen in the light of laws and applications in Turkey and revision of the previous studies in this field.

In the analysis of classification of the distressed companies the criteria involved going to bankruptcy, making loss for three years in a sequence, having negative net value of equity, being excluded from the quota on the stock exchange or closing down the company with different methods such as liquidation. In the whole data set 40 firms are classified as non-distressed and 40 firms are classified as distressed.

In the ratio selection; the used ratios are the same for each model. According to the book of Altman (1983) the most appropriate model for this data is Altman Z'' Score model being suitable for both public and private and manufacturing and non-manufacturing companies at the same time. For this reason the model is designed over four ratios shown in Equation (3).

Finally the software package used for this study is R, for MDA analysis including revised Altman Z score, Quadratic Discriminant Analysis and Random Forest, R is applied to get summary of the calculations as well as representation of them.

5. Empirical results

In this study financial distressed companies are predicted with the use of different models and accuracies of each model is elaborated as well as a getting a comprehensive review for Altman z score model is aimed as an outcome for this study. First model evaluation Altman z score is detailed with classification results in the below Table 2. The coefficients defined by Altman is used and index scores companies are calculated accordingly.

Table 2. Confusion matrix for Altman Z'' score.

		Distressed	Non-Distressed	Total
Actual	Distressed	31	9	40
		78%	23%	100%
	Non- Distressed	10	30	40
		25%	75%	100%
	Total	41	39	80
	Total Altman Z'' score performance			76.25%

The predictive power of the model is calculated through accuracy values. According to the calculation accuracy of the model is 76.25% for this data. It has low predicted power, another model (Linear Discriminant Analysis or Revised Altman z Score) is designed with MDA analysis however, this time the coefficients for the model is generated via MDA instead of the coefficients readily given by Altman.

Before applying and revising Altman score there is a need occurred to detect the outliers in the data set. Before than that, the summary of the variables is checked and it has seen that min and max values of the variables have a huge range, which shows that there are outliers exist.

Table 3. Summary of variables.

	WC/TA = x1	RE/TA = x2	EBIT/TA = x3	NW/TL = x4
Min	−61.990	−11.236	−3.723	−0.859
1st Qu.	−0.077	−0.2665	−0.1155	0.1405
Median	0.154	−0.0115	−0.0075	0.4925
Mean	−0.05042	−0.4748	−0.1802	2.211
3rd Qu.	0.4788	0.1042	0.05325	1.9215
Max	1.902	0.647	0.477	43.946

To deal with the outliers winsorization technique is adopted. The idea behind is based on replacing the original outlier value by the nearest value of an observation not seriously suspected as being an outlier as itself (Dixon, 1980).

Table 4. Summary of variables after winsorization.

	WC/TA = x1	RE/TA = x2	EBIT/TA = x3	NW/TL = x4
Min	−0.9106	−0.82263	−0.36862	−0.859
1st Qu.	−0.077	−0.2665	−0.1155	0.1405
Median	0.154	−0.0115	−0.0075	0.4925
Mean	0.154	−0.09971	−0.04781	1.207
3rd Qu.	0.4788	0.1042	0.05325	1.9215
Max	1.3124	0.647	0.30638	4.593

The outliers were dealt with winsorization technique with use od R package and now the data is ready to be applied after this process.
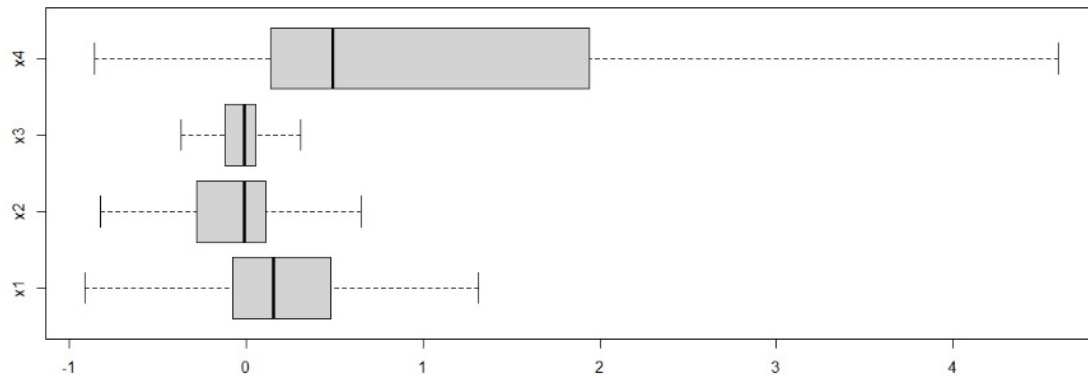
Figure 3. Box Plot after Winsorization technique applied for dataset.

The box plot gives a better understanding how the outliers are dealt with. The values; such as -61.990 for X_1 variable or 43.946 for x4 variable are all reshaped with winsoriation technique.

The changes can be seen especially in min and max values; ranges and min-max values of variables have changed in the case they had outliers.

In the application process of the linear discriminant analysis (LDA), correlation analysis is applied between the variables in order to detect whether the model suits with the dataset. A pairwise correlation test has run in order to see whether collinearity is an issue for this dataset. According to figure 4 there is a strong correlation between X_1 and X_2, also a less strong correlation between X_1 and X_3 and also between X_2 and X_3 variables exist.
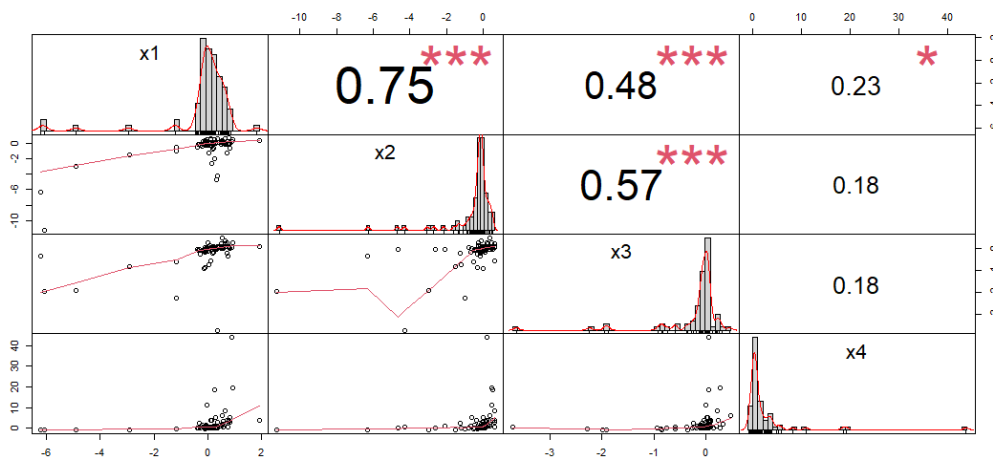
Figure 4. Correlation between the variables.

One of the assumptions for LDA was not collinearity that variables are independent to each other or in other words, they are not significantly correlated; for this case they are not completely independent. When the high correlation is seen between the variables, it means some of the variables should be discarded due to the reason of being redundant. One of the other assumptions is related to the linear

discriminant analysis is the covariance- variance matrices for each group have to be equal. For this reason, the boxM and Bartlett tests have been used to determine whether LDA is appropriate to use.

Table 5. Testing the homogeneity of covariance matrices of each group.

Box's M-test for Homogeneity of Covariance Matrices			Bartlett test of homogeneity of variances		
Chi-Sq. (Approx)	df	p-value	Chi-Sq. (Approx)	df	p-value
28.149	10	<0.001708	372.72	3	<0.00000000000000022

Both test statistic uses a chi-square approximation method.

H_0: *The variance−Covariance Matrices are equal*

H_1: *The variance−Covariance Matrices are not equa*l

P values obtained via these tests are smaller than 0.05; with the confidence interval 95% null hypothesis (the covariance-variances matrices for each group is equal) rejected. Variances are unequal for at least two groups, in other words variances across groups are not equal. After the analysis of LDA, quadratic discriminant analysis is used due to giving more flexibility to LDA and also it does not require the homogeneity of variance across groups. Linear Discriminant Analysis is defined earlier as a revision of Altman Z score. According to LDA/Revised Altman Z Score model new coefficients are calculated for this data and discriminant equation is gathered for analysis. The new coefficients with the model is represented as (See Equation (3));

$$Revised\ Z\ Score = 0.519X_1 + 1.788X_2 + 3.874X_3 − 0.018X_4 \tag{5}$$

According to the equation, only X_4 variable has a negative impact on the score, and X_3 has the highest impact on the determination of the Z score.

Table 6. Statistics related Revised-Altman Z score.

	Correlation Ratio	Wilks Lambda	F Statistics	p-value
x1	0.231756	0.768245	23.53018	0.0000061660
x2	0.358252	0.641748	43.54303	0.0000000045
x3	0.364274	0.635726	446.94429	0.0000000031
x4	0.180239	0.819761	17.14973	0.0000867794

In Table 6, with the revised Altman Z score there are some statistical values are represented. According to it; all variables are tested whether they have a significant contribution to the model. They all have significant contribution they should all stay in the model. The correlation ratio has the highest value with X_3 variable that is calculated as "Earnings before Interest and Taxes(EBIT)/Total Assets". The F value which is used to test the null hypothesis the correlation is equal to zero shows the rejection of the null hypothesis. Wilks Lambda value is also used to define the discriminating power of the model; the smaller value means that the variable has more discriminating power. In this case X_3 yet again has the smallest value which illustrates that the variable has the highest discriminatory power over other variables on the model. Lastly, p-value with 95% confidence interval; the values have less than 0.05 has significant contribution to the model. With this understanding; all variables are

significant at 95% confidence level. They are explanatory variables for this model hence they should remain in the model.

Finally with the revised Altman Z score confusion matrix is designed and the performance evaluation of the model is represented in Table 7.

Table 7. Confusion matrix for Revised-Altman Z score.

		Predicted		
		Distressed	Non-Distressed	Total
Actual	Distressed	31	9	40
		78%	23%	
	Non-Distressed	3	37	40
		7.5%	92.5%	
		34	46	80
	Total Revised Altman Z'' score performance			**85.00%**

Revised Altman Z score's overall performance rate is calculated as 85.00%. Distressed 31 firms are classified correctly out of 40 firms, also 37 non-distressed firms are classified as correctly in 40 firms which equals to 92.5% of accuracy. It is very clear to see that the model is predicting non-distressed firms with a very high performance also, in the prediction of the distress firms it meets the expectations with a lower prediction rate 78%. According to the Type II error values 9 distressed firms are predicted as non-distressed which is not a desired result even though non-distressed firms have excessively high rate performance with 92.5%. QDA confusion matrix is represented in Table 8.

Table 8. Confusion matrix for quadratic discriminant anaylsis.

		Predicted		
		Distressed	Non-Distressed	Total
Actual	Distressed	30	10	40
		75%	25%	
	Non- Distressed	3	37	40
		7.5%	92.5%	
	Total	33	47	80
	Total QDA score performance			**83.75%**

Quadratic Discriminant Analysis's performance is calculated as 83.75%. Out of 40 distressed firms 30 firms are predicted correctly (75%) also in non-distressed out of 40 firms 37 firms (92.5%) are classified in the right class.

After the prediction of these two models cross validation test is applied for both models. The used method for cross validation of predicted groups is "leave one-out method" (hold out method). It is performed by leaving one sample out of observation and then classification function is repeated for each observation with the omitted observations groups membership. In this data set with the use of N-1 observation; Revised Z score model and quadratic discriminant analysis are both evaluated with cross validation results. They both resulted as same for each model therefore shown in Table 9 together.

Table 9. Confusion matrix for revised Altman Z score and QDA model after cross validation.

		Predicted		
		Distressed	**Non-Distressed**	**Total**
Actual	**Distressed**	30	10	40
		75%	25%	
	Non-Distressed	4	36	40
		10%	90%	
	Total	34	46	80
	Total performance			**83.75%**

After the cross validation results came out same which means that Revised Altman Z score model performs superior because of having greater accuracy over two models. Another evaluation method is used for choosing the most appropriate model for this study; confusion matrices' metrics are compared for all models.

As the final model, Random Forest is applied with the same variables used for Altman evaluation. However as being a machine learning model the data has been divided as training and test set. The division is made with the ratio ¾ which means that 60 out of 80 companies are chosen as training set and the rest 20 companies as test set. After application of the tuning and all other enhancement related to random forest model the results were surpassing other three models. Only one company has been misclassified out of 20 companies therefore accuracy was 95%.

Table 10. Confusion matrix for random forest.

		Predicted		
		Distressed	**Non-Distressed**	**Total**
Actual	**Distressed**	10	0	10
		100%	0%	
	Non-Distressed	1	9	10
		10%	90%	
	Total	11	9	20
	Total performance			**95%**

The model had a great performance over other models. The final comparison is made upon all four models. The results are represented in Table 11.

Table 11. Comparison of four models.

	Altman Z'' Score	**Revised-Altman Z'' Score**	**Quadratic DA**	**Random Forest Model**
Accuracy	76.25%	85%	83.75%	95%
Sensitivity	78%	78%	75%	100%
Specificity	75%	92.5%	92.5%	90%
Precision	75.61%	91.18%	90.9%	90.9%

In the comparison of the all three models it has seen that the accuracy has the highest value in Random Forest machine learning model. The model predicted non-distressed companies quite well and predicted distressed companies with a great performance. Sensitivity of the model is 1 also in all metrics beside specificity it performed superior than others. In the comparison of discriminant analysis models; The Revised Altman Z'' Model has performed superior than original Altman Z'' score model represented by Altman (1983) in consideration of accuracy values. In the consideration of all four models the most accurate model is Random Forest.

Table 12. Accuracy comparison of the models (Private vs. Publicly Traded).

	Altman Z''-Score	Revised Altman Z'' Score	QDA	Random Forest Model
Private Firms	88.89%	94.44%	97.22%	85.71%
Publicly Traded Firms	65.91%	75.00%	72.73%	100%

As the final point prediction, accuracy of the model according to being listed on Borsa Istanbul Stock Exchange market are shown in Table 12. The same calculation is also valid for small and micro enterprises (non- traded companies) and medium and large enterprises (listed on BIST). In the evaluation of the four models only in Random Forest listed companies had perfect result with 100% accuracy. However other three models have shown greater performance for private companies. Especially the performance of quadratic discriminant analysis is greater than others also Revised Altman Z'' has shown a significant performance on the accuracy of the private firms.

6. Conclusions

The main idea for this study was to evaluate the efficiency of Altman Z score model and modify it for Turkish firms to get more accurate prediction power. The study has composed of some parts; after the introduction part the literature has been represented related to this study. In the literature review the main criteria for the financial distress is evaluated both from universal perspective as well as in Turkey case. Turkey has some different regulations related to bankruptcy process; for this reason, the criteria is differed from the most of the studies related in the international field.

In the literature review, the used models are discussed and the Altman Z score model has been reviewed with other discriminant analysis techniques. Even though there are many studies are implemented for Altman Z Score, very little studies are tested the validity of the model none of the studies tested Turkish firms' efficiency with a data set including both public and non-listed companies simultaneously. Thus this study has a revision of the model as well as questioning whether the model is still valid for small and large companies or listed and non-listed companies at the same time.

Following, the methodology has consisted of the methods used in this study; Multiple Discriminant Analysis is represented and in particular the limitations of Linear Discriminant Analysis, which is claimed as the Revision of Altman Z score model, therefore as an alternative Quadratic Discriminant Analysis and as a newer method compare to discriminant analyses; one of the Machine Learning models Random Forest is applied. Confusion matrix metrics are stated how they are

interpreted and used in the determination of the most effective model. As the last point, in the methodology part some characteristics of data set are represented. The data set has shown that the data illustrates a wide range of sectors and sized Turkish companies.

In the empirical result section original Altman Z score's accuracy has been calculated and it has 76.25% prediction power to classify companies as distressed and non-distressed. Then, revised model has been calculated and it has seen that the accuracy was higher than original model. Especially in the prediction power of non-distressed firms was 92.5%; out of 40 non-distressed firms only 3 of them misclassified. Another model quadratic discriminant analysis is finally applied in this model to provide an alternative to revised Altman Z score model. Nevertheless, its prediction performance resulted lower than Revised Model. Finally, Random Forest evaluated and it had 95% accuracy over three models. The model had superiority almost in all metrics of confusion matrix. The validity of the models has been reviewed and as a Random Forest outperformed other three models with consideration of confusion matrices metrics.

A final discussion was made on the effectiveness of prediction power of models according to being private and publicly traded companies. Beside Random Forest, all models perform greater with the classification of private firms' financial distress situation Nevertheless Random forest had 100% accuracy for the classification of publicly traded firms.

Conflict of interest

The authors declare no conflicts of interest in this paper.

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