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Review

Using Lyapunov's method for analysing of chaotic behaviour on financial time series data: a case study on Tehran stock exchange

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Abstract: In the last decade there is a constantly growing interest in application of mathematics methods and econophysics methods to solve various problems concerning finance, economics, etc. Chaos and its application are importance for most of the current financial and economic phenomena. Financial markets can potentially provide financial long-term series which can be used in analysing and forecasting. Most recent studies, shows the existence of long and short-range correlations in the financial market and economic phenomena. For testing the existence of chaotic behaviour, Lyapunov's method is one of the best methods. In the current study time-series tests of Lyapunov's method were applied, among listed companies on Tehran stock exchange over a period from 2005 to 2015. The obtained findings prove the existence multifractality process in the evolution of time series stock price.

Keywords: chaotic behaviour; financial time series data; Lyapunov's method; Tehran stock exchange

JEL Codes: C01, C22, C58

1. Introduction

The chaos theory in recent decades has been the subject of scientific research in various disciplines such as physics and mathematics. The basis of the theory of chaos can be stated as follows: although there is a kind of disorder in the events of the world and as a result, they seem unpredictable, but there is still a kind of order in the phenomena of the universe. For example, dynamic nonlinear systems exhibit a variety of behaviors that can be used to justify many economic phenomena that occur randomly or very complex time series, such as the prices of financial markets, are usually random and, as a result, their changes are assumed to be unpredictable, while these series may be the produce of a certain dynamic (chaotic) nonlinear process and therefore predictable in the short term.

In economics, monetary and financial markets are one of the most appropriate ways to apply the chaos theory, because theories in the financial and monetary economy suggest that monetary variables, such as exchange rate and stock prices, are randomized therefore due to of changes of these variables, these variables are unpredictable.

In this regard, chaos theory can be used as one of the justifications of the business cycle and also long-term imbalance without the need to interfere exogenous shocks in macroeconomic models. Analyzing and forecasting financial asset prices has always been a topic of interest, both in the field of science and in practice, various methods have been developed to analyze financial assets. The latest ideas presented in this area include the theory of chaos and fractal analysis. Therefore, in this study, we have been analyzed the behavior of the time series stock price using chaos theory.

Forecasting the stock prices and timing to buy/sell decisions becomes an extremely challenging task, due to the nonlinear nature of stock price changes. Therefore, forecasting of financial variables, such as stock prices, stock market index, and stock returns are hard (Podsiadlo and Rybinski, 2016; Nair et al., 2017; Zhong and Enke, 2017).

A good application for demonstrating the value of time series forecasting in financial market is forecasting stock prices. In essence, modern finance and economic activities are dynamic and change frequently and most studies shows that the behavior of financial time series show nonlinearity and non stationary feature. Because stock markets are affected by many other economic and political factors, thus, it is difficult to accurately forecasting the variations of stock prices (Shynkevich et al., 2017; Wei, 2016; Zahedi and Rounaghi, 2015).

In analysing of financial and economic markets, efficient market hypothesis is important concept that relies on the fact that in an efficient market relevant information is fully for the investors. In recent years, many efforts have been made to accurately modelling non-linear time series. On the other hand, analysts of stock markets strived to find ways which can increase the profit from the stock market (Rounaghi and Nassir Zadeh, 2016; Niu and Wang, 2013a; Niu and Wang, 2013b; Li and Wang, 2017; Ibarra-Valdez et al., 2016; Dai et al., 2016; Moradi et al., 2019; Mastroeni et al., 2019; Mastroeni et al., 2018; Salvino and Cawley, 1994; Wayland et al., 1993; Kaplan and Glass, 1992).

Given that relevant information of the financial market, helps the investors to make proper decision, so, finding the efficient market is important topic in financial market (Li et al., 2014). On the other hand, in fractal markets hypothesis (FMH), financial markets are described as: market is stable and has sufficient liquidity when it comprises with different time horizons, although the available information may not be reflected in the market prices, and the market prices trend indicates the changes in unexpected earnings (Mantegna et al., 1999; Münnix et al., 2012; Mantegna and Stanley, 1996).

In empirical economics models try to verify the qualitative forecasting of theoretical models and convert these predictions to proper, valuable outcomes. Nowadays, mathematics methods and econophysics methods such as GARCH-type models and ARIMA models has already made a number of important empirical contributions to our understanding of the social and economic world. These fall mainly into the areas of finance and economics, where in each case there is a large amount of reasonably well-defined data (Foley, 2016; De Luca and Loperfido, 2004; De Luca and Loperfido, 2015; Engle, 1982; Rydber, 2000).

2. Chaos theory and financial markets

Time series have played an important role in financial and economic research in recent times. Time series are the collection of variable data over a period of time that are predict or used to examine a series of specific statistical objectives. This data is contrasted with cross-sectional data that is collected for a large number of variables at a specific time. The main goal in analyzing a time series about a phenomenon is to create a statistical model for time-dependent data based on past information about that phenomenon. The creation and use of statistical and random models in the form of time series analysis has become very widespread today with the help of high-speed computers. Applications of time series analysis can be seen in various fields such as business, finance, economic, engineering, and so on.

This paper focuses on the application of chaotic dynamic systems in financial markets. According to the efficient market hypothesis, very complex time series, such as stock market prices, are usually considered random and therefore their changes are unpredictable. The possibility of turmoil in the financial markets has raised countless questions for economic and financial researchers. In this regard, with a brief overview of the basic concepts of chaos theory, we examine the background and features of this theory and finally introduce tests that can detect chaotic processes.

Complex time series, such as stock market prices, are more likely to be random and therefore change them unpredictably. The forecasting of chaotic time series has been tested in finance, geological environment, weather forecasting. In financial markets, there are various definitions of asset volatility that are not directly visible to the stock return series. As a result of this uncertainty, statistical theory and its methods play an important role in financial time series.

In general, time series model applied in investigating and forecasting financial markets can be branched into single variable model and multivariate model according to the selection of variables, according to the structure of the model and can be branched into linear and nonlinear model. Linear stochastic models, in special the class of ARMA models, have been examined a practical tool for financial analysis and forecasting but they suffer from a number of serious drawback for studying financial fluctuations.

Connecting nonlinear analysis methods with flexible prediction techniques can bring useful information about the finding of the best solution. Applying techniques like correlation dimension that measures how complex is the system, which created the time series data, or largest Lyapunov exponent that measures exponential divergence of close by trajectories, should help group similar complexity predictability signals. This grouping should help to select, which of the time series could be predicted or even select model's parameters.

There are different methods that can be applied to test chaos. One of these methods is based in the analysis of the permutation entropy and permutation complexity of time series. It is not only useful method to test chaos but also to discriminate different stochastic dynamics.

In economy and finance, measuring volatility in the stock markets is one of the important factors in empirical finance to manage risk, such as portfolio allocation and to derivative assets pricing (Takaishi, 2017; Madaleno and Vieira, 2018). Most financial time series data have shown that the conditional distribution of returns series shows several stylized features such as excess price jumps, leverage effects, negative skewness, and time-varying volatility (Gyamerah, 2019). On the other hand, in recent years many research has shown that stock prices do not follow a "Random Walk" (Rosini and Shenai, 2020; Fernandez and Swanson, 2017; Rounaghi et al., 2015).

A nonlinear system is chaotic when it is characterized by regular motion and unpredictability. Chaos tests have been widely applied to find presence of chaos behaviour in numerous fields of engineering and science. A chaotic system has random-like behaviour and very sensitivity to initial conditions and it assumes that the system under study follows a nonlinear and deterministic process. Measuring the degree of divergence of trajectories can be achieved by estimating Lyapunov exponents (Jahanshahi et al., 2019; Gupta et al., 2017; Su and Cheng, 2016; Lahmiri, 2017a; Lahmiri, 2017b; Peinke et al., 1992; Lahmiri et al., 2017).

The chaos theory offers a new way of looking at the trend of changes in monetary and financial markets, and it can reveal hidden trends and patterns in financial data that are not available with conventional models. One of the most important methods for detecting chaos is the presence of small positive Lyapunov's impressions and the absorbent dimension test. The positive value of the Lyapunov's view reflects the local average rate of divergence (unstable) and chaotic process (Ola et al., 2014; Ghadiri Moghadam et al., 2014).

In recent years, the importance of chaos and its application have been shown in many studies. Many tests were proposed when chaos theory was discussed in the field of economics, before specific models of stock market were designed. Recently, many tests were proposed, when chaos theory was discussed in the field of finance and economics. By comparing different models, economists have shown increasing interest in univariate models of time series in the field of forecasting. Also, chaotic behaviors are a reflection of internal behaviors in the time history of one (or more) of system variables, normally referred to time series, which may therefore bear external signals for indicating their behaviours.

So far, there has been the view that economic timescales, especially those of economic groups of the series of monetary and financial markets follow a random process and, consequently, their variation can be not predicted significant progress has been made in computing tools. In the traditional texts of economics and econometrics, more economic variables are considered to be random behavior. The result of such hypothesis is that the changes these variables are not predictable. In fact, the chaos theory offers the possibility of pattern and order the complex behavior governing these variables is discovered and used to predict the future trend.

This is very important issue to shows chaos behaviour in financial market and help to Investor's to better understand the behaviour of price, returns, and volatilities in times of boom and bust. Indeed, analysing chaos in volatility series and understanding the intrinsic nonlinear dynamics in financial and economical time series is vital for portfolio optimization, forecasting and pricing.

3. Lyapunov's method

From the point of view of chaos theory, complex systems have a purely chaotic appearance and, as a result, appear irregular and accidental, while they may be subject to a certain current with a specific mathematical formula. In economics, monetary and financial markets are one of the most suitable for the use of chaos theory, because the existing theories in economics of money and financial markets indicate that monetary variables such as exchange rate and stock price are accidental. According to the theory of chaos, if the process of determining monetary variables follows a certain nonlinear process, their changes can be predicted.

Noisy data and non-stationary data are the two fundamental aspects in financial time series forecasting. On the other hand, financial time series forecasting is, definitely, the excellent option of computational economic for finance researchers from both academia and financial industry due to its expansive application areas and substantial impact. Also, financial time series data shows cluttered and irregular trend.

In chaos theory, a phenomenon that seems completely random and unpredictable on a local scale may be stable and predictable on a larger scale. In this method, there is an emphasis on dependence or sensitivity to the initial conditions, which means that very small changes in the initial values of a process can lead to significant differences in the fate of the process. The turbulent process is the data of a dynamic nonlinear system that is not random but seems random and repetitive. Economic fluctuations are a kind of dynamic nonlinear behavior and are the basis of all turbulent processes and even dynamic economic systems of positive and negative feedback loops that cause exponential growth or decline in the system.

Often, due to the non-linear and chaotic nature of financial markets, classical prediction models do not perform well, and the information in the data disappears quickly over time, so it will not be useful to use them in the long run. Chaos theory offers a new way to look at the trend of change in the monetary and financial markets, and it can reveal hidden trends and patterns in financial data that are not available with conventional models. The Liapanov test can be used to detect a turbulent process. Liapanov's test can be explained by the characteristic of the turbulent series in which the adjacent points in these series are separated over time and diverge from each other. In fact, this method calculates the average velocity at which the two-point transition paths that were initially close to each other deviate exponentially.

Financial time series forecasting is applied with theory and practice of asset valuation over time. It is a deeply experimental discipline. There is, notwithstanding, an essential aspect that distinguishes financial time series analysis from other time series analysis. In financial time series, financial approach and its experimental time series contain a factor of uncertainty. For illustration, there are exist different aspects of asset volatility, and for a stock return series, the volatility is not directly observable. Therefore, the added ambiguity, statistical theory and methods play a key role in financial time series forecasting.

Liapanov's test was used by the Russian mathematician Liapanov in 1892 to control the stability of nonlinear differential equations. This method makes it possible to study the stability of differential equations without solving them. Liapanov's facade method is one of the best tools for examining the range of stability and transient behavior of dynamic systems.

To describe the attractor, some methods have been examined such as dimensions and Lyapunov exponents. These quantities are stable under the evolution operator of the structure and therefore are independent of diversity in the initial conditions of the orbit, and both are independent of the coordinate structure in which the attractor is identified.

A useful method for testing the stability of nonlinear systems is Lyapunov exponents method. The average divergence is measured by Lyapunov exponents. A positive largest Lyapunov exponent shows chaos.

The Lyapunov test is based on this feature of the chaotic series, with the adjacent points in these series being reviewed. Time is divided and divergent. The Lyapunov view of this divergence is represented by an exponential function measures. If the largest calculated value of Lyapunov has a positive value, the system has a chaotic behavior and the Lyapunov view can also be presented as follows (Sun et al., 2011; Arnold, 1997; Gomez, 2017):

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \log \left| \frac{df^n(x_0)}{dx_0} \right| \tag{1}$$

The Lyapunov exponent prepare a more useful characterisation of chaotic systems because of the adverse the correlation dimension, which evaluation the complexity of a nonlinear system, and it shows a system's level of chaos.

4. Local polynomial approximation model

In this paper we recommend to using local polynomial method that takes account exceptionally of constraints between the coefficients of the polynomials at neighbouring frequencies. This newly alternative give a new and significant reduction in the mean square error of the nonparametric estimates. The local polynomial method is a newly advanced program for nonparametric estimation of the nonlinear system. In comparison with other nonparametric estimators based nonparametric techniques, it has proved to be surprisingly efficient in decreasing the leakage errors created by the application of nonparametric techniques to non stable data.

A local search theory has come into presence in the last few years. It about both the aspects linked to the complexity of local search algorithms and the analysis of their performance from an approximation point of view. A polynomial approximation model is an arrangement Ap of polynomial algorithms guaranteeing the (classical) ratio $1-1/p$ for a maximization problem and $1+1/p$ for a minimization problem. A differential polynomial approximation model is an arrangement Ap of polynomial algorithms that guarantees the (differential) ratio $1-1/p$.

The specification local search algorithm involves of starting from an initial solution (which can be achieved randomly for example), moving to route a superior neighbouring solution, until to embracing a local optimum.

Presume that the state vector of time T is:

$$\mathbf{x}_\mathrm{T} = \left(\mathbf{x}_\mathrm{T}, \mathbf{x}_{\mathrm{T}-\tau}, \dots, \mathbf{x}_{\mathrm{T}-(\mathrm{m}-1)\tau}\right) \tag{2}$$

$$\mathrm{f} = (\mathrm{X}^\mathrm{T}\mathrm{X})^{-1}\mathrm{X}^\mathrm{T}\mathrm{y} \tag{3}$$

Given that the intricacy of each iteration is polynomial, and the execution time of a local search algorithm, build upon on the number of iterations carried out before reaching a local optimum.

Notice that a specification local search algorithm is pseudo-polynomial. This means that its intricacy, in the worst case, build upon on the numbers (weights) that exist in the instance being dealt with.

The local polynomial approximation (LPA) of noisy input data is designed with the advanced adaptive scheme for fluctuating bandwidth selection. The algorithm is simple to appliance and nearly optimal within in N factor in the point-wise risk for estimating the function and its derivatives (Ola et al., 2014).

5. Results

In relation with other economic time series, the financial time series have some characteristic feature and shapes given by the structure of the financial market. The elemental aspect of the financial time series is a high frequency of these data.

For testing the accuracy of the models, there are two principles in selecting the proper mathematical representation: mathematical reliability and empirical verifiability. Contrary empirical economics, economic time series are not reproducible in nature. On the other hand, conventional tests in econometric analysis have limited capability in studies of an empirical economy containing deterministic factors.

The present study, investigates the presence and changes in long memory features in monthly time series stock price of Tehran stock exchange by using Lyapunov's method over a period from 2005 to 2015. The obtained findings prove the existence multifractality process in the evolution of monthly time series stock price.

Table 1 demonstrates the descriptive statistics of the monthly time series stock price on Tehran stock exchange.

Table 1. Descriptive statistics of the monthly time series stock price (Iranian rial rates).

Year	Mean ± SD
2005	11186.40 ± 8206.81
2006	8334.08 ± 9602.74
2007	7899.24 ± 9097.09
2008	6298.30 ± 6710.90
2009	4940.13 ± 4284.08
2010	4406.42 ± 3958.65
2011	5879.24 ± 6138.52
2012	5814.55 ± 5281.58
2013	5120.45 ± 4875.89
2014	9384.12 ± 9329.08
2015	8351.84 ± 8922.41

In the section 5.1, the result of using Lyapunov's method for chaos process testing has been shown and in the section 5.2, we forecasted the monthly stock price index for Tehran Stock Exchange by using local polynomial approximation model. Also, in this section the results of our model have been shown.

5.1. Chaos process testing (fractal market analysis) on Tehran stock exchange by using Lyapunov's method

The achieved results showed at error level of 1% the existence of chaotic process in time series stock price on Tehran stock exchange. Furthermore, predictability, martingale process and non-linearity of the series were confirmed.

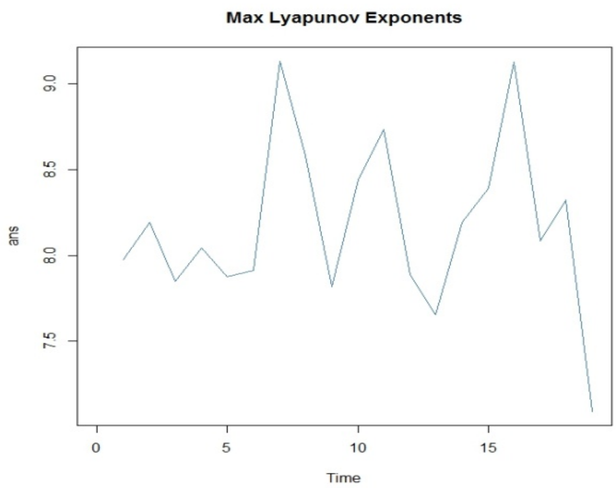

Figure 1. Maximal Lyapunov exponential.

5.2. Monthly forecasting of time series stock price on Tehran stock exchange by using local polynomial approximation model

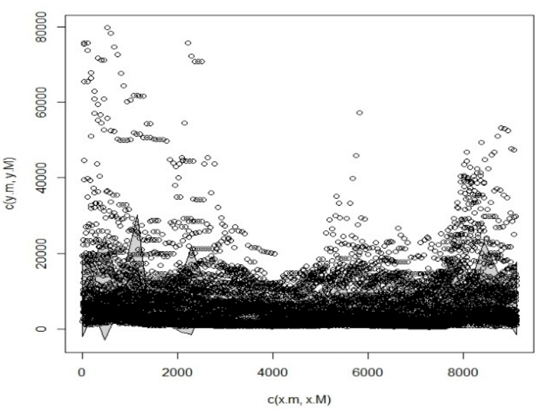

Figure 2. local polynomial approximation model.

Given that the RMSE and MAE for monthly stock price index for Tehran stock exchange by using local polynomial approximation model are 8137.633, 5404.928 respectively, so we can find this market has multifractality process in the evolution of time series stock price and the obtained findings indicate that long term forecasting of time series are confirmed in Tehran Stock Exchange at the error level of 1%.

6. Conclusion

If a dynamic system behaves erratically, it will be nonlinear. The key to understanding chaos is the concept of nonlinearity. Some sudden and dramatic changes in nonlinear systems may cause a complex behavior called chaos. The word "chaotic" is used to describe the behavior of a system when its behavior is rare (never completely repeated) and is "apparently" accidental or noisy. Behind this chaotic coincidence, there is an order that is determined by the system's equations. We define a system in which the subsequent behaviors of the system can be fully determined, that is, a system in which subsequent states follow or are determined by the previous ones. Such a system is the opposite of a "random system" in which future states are not distinguished from previous states. If a system is definite, it does not necessarily mean that its subsequent states can be predicted by knowing the previous ones. In this way, the chaos is similar to a random system.

Scientists in finance and economics usually use time series data to study stock price behavior. It is important for brokers, stock market investors, and financial experts and economists to predict the stock price trend even for a day.

In econometrics and computational methods in order to reveal the chaotic process, it is still not possible to claim that these methods are well able to distinguish a linear process with random disorders from a nonlinear process (Chaos).

In this study time-series tests of Lyapunov's method were applied in the current study, and with a high level of confidence confirmed the chaotic process among listed companies on Tehran stock exchange over a period from 2005 to 2015. The obtained findings prove the existence multifractality process in the evolution of time series stock price. The obtained findings indicate that long term forecasting of time series is confirmed in Tehran stock exchange at the error level of 1%.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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