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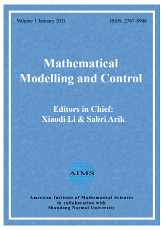

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Research article

A novel reduced-order model reference adaptive control approach for output tracking of SISO systems with unknown high-frequency gain

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Abstract: In this paper, the output tracking control problem for single-input single-output (SISO) systems with unknown high-frequency gain is investigated. For the situation where the system matrix has unknown parameters and the high-frequency gain cannot be measured, based on the output feedback control strategy, a novel reduced-order model reference adaptive control (MRAC) approach that relies solely on the sign of the gain is proposed. This approach only requires updating a scalar function, which substantially alleviates the computational burden of the designed controller. Furthermore, the control strategy ensures closed-loop stabilization of the system and enables asymptotic tracking of the system's output state. The effectiveness of the proposed method is finally validated through two simulation examples.

Keywords: reduced-order MRAC; computational burden; scalar update law; asymptotic output tracking

1. Introduction

Parameter uncertainties are prevalent in real engineering systems and usually adversely affect the systems' performance. These uncertainties may arise from measurement errors, incomplete data, or deviations between the theoretical model and the actual physical behavior, which can lead to deviations from the expected control effect or even instability of the system. Therefore, the design of control strategies that can effectively cope with these uncertainties has become a challenging and valuable research topic in the field of control.

Currently, researchers have proposed various control strategies. Among them, methods such as sliding mode control (SMC) [1, 2], robust control (RC) [3, 4], and adaptive control [5–8], have gained widespread application due to their ability to effectively handle systems' parameter uncertainties. SMC effectively handles uncertainties and disturbances, but it may cause chattering, leading to wear and energy loss. Although robust against parameter changes, it requires accurate system modeling. While RC ensures stability under uncertainties, it requires complex design and may involve trade-offs in performance. Unlike these, adaptive control does not rely on a precise system model and can compensate for system uncertainties by dynamically adjusting the control parameters, thus making it more adaptable. It avoids chattering and performs well in complex, uncertain systems, making it a promising solution for uncertain systems. Various adaptive control technologies have been developed for dynamic systems with uncertainties, such as adaptive sliding mode control [9, 10], adaptive fuzzy fault-tolerant control [11–14], adaptive optimal control [15, 16], and model reference adaptive control (MRAC) [17–19], etc. The choice of the most suitable control approach is crucial and should be based on the specific requirements of the application. If the system exhibits significant uncertainties and potential faults, adaptive fuzzy fault-tolerant control may be more appropriate. On the other hand, for systems that require precise tracking of a reference model, MRAC may offer the desired performance.

MRAC, initially presented by [17, 19], is one of the more mature major branches in various adaptive control methodologies. In particular, by designing feedback controller frameworks and adaptive update laws for controlled systems, the adaptive parameters can be adjusted online to achieve closed-loop bounded and asymptotic output or state tracking of a pre-designed stable model reference system in the presence of uncertainty of system parameters; see [20–22].

The MRAC scheme can usually be constructed using either state feedback (SF) or output feedback (OF). When the system's full state signals are available for measurement, the state feedback-based model reference adaptive control (SFMRAC) strategy is widely adopted due to the simplicity of its controller structure. For different control objectives, SFMRAC can enable the system to achieve output tracking or state tracking; see [20, 23]. It is worth noting that the SF controller for state tracking has restricted matching conditions, which largely constrain its application to practical control problems. Compared with the SF-based state tracking controller, the SF-based output tracking controller is not subject to matching conditions while maintaining the simple controller construction; see [7, 24, 25]. Therefore, in practical applications such as flight control, the SF-based output tracking controller has strong suitability when the model matching conditions are hard to satisfy owing to the system's parameter uncertainties; see [26, 27].

It should be noted that in some cases, the full state information of the system is not readily available, thus limiting the utility of the SFMRAC strategy in real application systems. Therefore, the output feedback-based model reference adaptive control (OFMRAC) strategy has been investigated. The OFMRAC strategy has received attention from researchers in recent years due to the fact that it does not require the system's state information when solving the tracking control problems. Research on OFMRAC for output tracking has been published in the literature, such as [28–31]. Among them, the classical OF controller with standard adaptive update laws proposed by [31] allows the controlled system output to asymptotically track the desired reference model output. The authors of [32, 33] further applied the OFMRAC strategy to the fractional-order (FO) systems. Reference [34] also proposes a mixed OFMRAC control strategy based on a fractional-order adaptive law for integer-order single-input single-output (SISO) systems, significantly enriching the research on OFMRAC for output tracking. Moreover, the OFMRAC method has also been applied to solve practical control problems, such as the longitudinal movement control of pitch angle [35]. Regrettably, the aforementioned control strategies typically necessitate the online estimation of multiple unknown controller parameters. Moreover, the order of the controller (the number of parameters requiring online updates) escalates with an increase in the number of system states. This escalation substantially augments the computational load and consumes excessive system resources. Consequently, devising methods to reduce the order of the OF model reference adaptive controller, while maintaining the desired tracking performance, represents a critical and pressing challenge.

Recently, [36] proposed a novel OF adaptive control strategy to effectively solve the existing computational burden problem in OFMRAC for the case where the high-frequency gain of the controlled system is known. However, this assumption has its limitations. In practice, there may be uncertainty in the amplitude of the high-frequency gain, making it challenging to determine its exact value. Nevertheless, the sign of the gain can generally be determined more easily. Therefore, it is worth investigating further whether relaxing this assumption, and considering scenarios where only the sign of the high-frequency gain is known, could lead to more robust results.

Building upon the preceding research endeavors, this paper introduces a new reduced-order OFMRAC approach for output tracking control of SISO systems with unknown parameters and a known sign of high-frequency gain. By applying the Lyapunov stability theorem, this strategy ensures both the closed-loop stability of the system and the asymptotic tracking performance of the output. The control strategy has the following advantages: (1) The high-frequency gain of the system is known only in the sign; (2) it depends only on the input and output information of the system; and (3) only one scalar function needs to be updated online, which alleviates the pressure of the traditional OF adaptive controller, which requires a large number of parameter updates. Finally, two simulation examples are conducted to illustrate the effectiveness of the proposed method.

2. Preliminaries and problem statement

2.1. Preliminaries

The notation used in this paper adheres to conventional standards. Let $\mathbb{R}$ represent the set of real numbers, and $\mathbb{R}^+$ the set of non-negative real numbers. The set of non positive complex numbers is denoted by $\mathbb{C}^-$. The notation $\mathbb{R}^n$ refers to the set of $n \times 1$ real column vectors, while $\mathbb{R}^{n\times m}$ indicates the set of $n \times m$ real matrices. The symbols $(\cdot)^T$, $(\cdot)^{-1}$, and $tr(\cdot)$ denote the transpose, inverse, and trace operator, respectively. The notation $\lambda_{max}(M)$ (respectively, $\lambda_{min}(M)$) are used to represent the maximum and minimum eigenvalues of a Hermitian matrix M, respectively. A matrix P is said to be symmetric if $P = P^T$. Additionally, $\| \,.\, \|_2$ signifies the L_2 norm.

2.2. Problem statement

Consider a SISO linear time-invariant system, denoted as

$$\begin{cases} \dot{x}_n(t) = A_n x_n(t) + b_n u(t), \\ y_n(t) = h_n^T x_n(t), \end{cases} \tag{2.1}$$

where $u(t) \in \mathbb{R}$ denotes the input signal, $y_n \in \mathbb{R}$ represents the output signal, and $x_n : \mathbb{R}^+ \to \mathbb{R}^n$ denotes the state vector. Additionally, $A_n \in \mathbb{R}^{n\times n}$, b_n, and $h_n^T \in \mathbb{R}^n$ have parameter uncertainties, (A_n, b_n) is controllable and (A_n, h_n^T) is observable. In this case, the full state of the system is not available and only the system's output signal can be measured. The corresponding input output description of the system (2.1) is formulated as follows:

$$y_n(t) = W_n(s)[u](t) = k_p \frac{Z_p(s)}{R_p(s)}[u](t), \tag{2.2}$$

where $Z_p(s)$ and $R_p(s)$ are monic polynomials of degrees $n-1$ and n, respectively. The high-frequency gain of the controlled system is denoted by k_p.

The following model reference system is selected as:

$$y_m(t) = W_m(s)[r](t) = k_m \frac{Z_m(s)}{R_m(s)}[r](t), \tag{2.3}$$

where $y_m(t) \in \mathbb{R}$ represents the model reference output signal, and $r(t) \in \mathbb{R}$ denotes a piecewise-continuous and uniformly bounded reference input signal. Furthermore, $Z_m(s)$ and $R_m(s)$ are defined as monic Hurwitz polynomials of orders $n-1$ and n, respectively.

The control goal of OFMRAC for output tracking is to derive an OF control law for $u(t)$ such that all signals in the closed-loop system remain bounded, and the system's output $y_n(t)$ asymptotically follows the model reference output $y_m(t)$, on the basis of the following general assumptions:

(1) The sign of k_p is known.
(2) The polynomials $Z_p(s)$ and $R_p(s)$ are coprime.
(3) All zeros of the monic polynomial $Z_p(s)$ are located in $\mathbb{C}^-$.
(4) The transfer function $W_m(s)$ is strictly positive real (SPR).

3. Controller structure

According to Narendra et al. [31], the OF controller structure is designed as shown in Figure 1. In particular, it contains a gain $k_0(t)$, a feedback control loop with the parameter vector $\theta_1(t)$, and a feedback controller with the parameters $\theta_2(t)$ and $\theta_0(t)$. Two auxiliary filters are required in this controller, and their state equations are expressed as

$$\begin{cases} \dot{w}_1(t) = \Lambda w_1(t) + \ell u(t), \\ \dot{w}_2(t) = \Lambda w_2(t) + \ell y_n(t), \end{cases}$$

where $\Lambda \in \mathbb{R}^{(n-1)\times(n-1)}$ represents an asymptotically stable matrix, and the pair (Λ, ℓ) is controllable. Then, the controller can be presented as follows:

$$\begin{cases} u(t) = \theta(t)^T w(t), \\ w(t) \triangleq [r(t), w_1^T(t), y_n(t), w_2^T(t)]^T, \\ \theta(t) \triangleq [k_0(t), \theta_1^T(t), \theta_0(t), \theta_2^T(t)]^T, \end{cases} \tag{3.1}$$

where $\theta(t) \in \mathbb{R}^{2n}$ corresponds to the adaptive parameter vector to be adjusted online, and $w(t) \in \mathbb{R}^{2n}$ represents auxiliary signals vector.

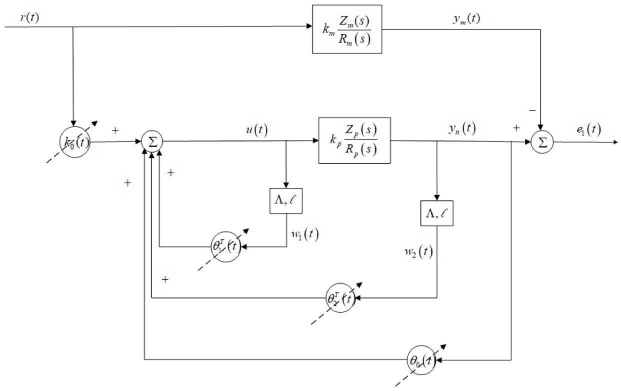

Figure 1. The controller structure.

The transfer functions of the feedforward controller and the feedback controller can be expressed, respectively, as

$$W_{1p}(s) = k_0 \frac{M(s)}{M(s) - C(s)}$$

where

$$\frac{C(s)}{M(s)} \triangleq \theta_1^T (sI - \Lambda)^{-1} \ell;$$

and

$$W_{2p}(s) = \frac{D(s)}{M(s)} + \theta_0$$

where

$$\frac{D(s)}{M(s)} \triangleq \theta_2^T (sI - \Lambda)^{-1} \ell,$$

respectively. Then the transfer function of the entire controlled system can be characterized as follows:

$$W(s) = \frac{k_0 k_p Z_p(s) M(s)}{(M(s) - C(s)) R_p(s) - k_p Z_p(s)[\theta_0 M(s) + D(s)]} \tag{3.2}$$

where $C(s)$ and $D(s)$ are polynomials of order $n-2$, and $M(s)$ is a monic Hurwitz polynomial of order $n-1$.

Let the nominal controller parameters θ^* be defined as

$$\theta^{*T} \triangleq [k_0^*, \theta_1^{*T}, \theta_0^*, \theta_2^{*T}],$$

such that

$$\begin{aligned} &k_0^* = \tfrac{k_m}{k_p}, \\ &\theta_1^{*T} (sI - \Lambda)^{-1} \ell = \tfrac{C^*(s)}{M(s)}, \\ &\theta_2^{*T} (sI - \Lambda)^{-1} \ell = \tfrac{D^*(s)}{M(s)}, \\ &M(s) - C^*(s) = Z_p(s), \\ &R_p(s) - k_p[\theta_0^* M(s) + D^*(s)] = R_m(s). \end{aligned}$$

Furthermore, assume $M(s) = Z_m(s)$. If we choose $\theta(t) \equiv \theta^*$, then the transfer function $W(s)$ turns out to be

$$W(s) = \frac{k_m Z_p(s) Z_m(s)}{Z_p(s) R_m(s)} = W_m(s).$$

Define the parameter estimation errors $\varepsilon(t) = \theta(t) - \theta^* \in R^{2n}$ as follows:

$$\varepsilon(t)^T \triangleq [\check{k}_0(t), \check{\theta}_1^T(t), \check{\theta}_0(t), \check{\theta}_2^T(t)]$$

where $\check{k}_0(t) \triangleq k_0(t) - k_0^*$, $\check{\theta}_0(t) \triangleq \theta_0(t) - \theta_0^*$, $\check{\theta}_1(t) \triangleq \theta_1(t) - \theta_1^*$, and $\check{\theta}_2(t) \triangleq \theta_2(t) - \theta_2^*$. Then the controlled system (2.1) combined with the controller for this adaptive case is rewritten as

$$\begin{aligned} &\dot{x}(t) = A_c x(t) + b_c(\varepsilon^T(t) w(t) + k_0^* r(t)), \\ &y_n(t) = h_c^T x(t), \end{aligned} \tag{3.3}$$

where

$$A_c = \begin{bmatrix} A_p + \theta_0^* b_n h_n^T & b_n \theta_1^{*T} & b_n \theta_2^{*T} \\ \theta_0^* \ell h_n^T & \Lambda + \ell \theta_1^{*T} & \ell \theta_2^{*T} \\ \ell h_n^T & 0 & \Lambda \end{bmatrix},$$

$$b_c = \begin{bmatrix} b_n & \ell & 0 \end{bmatrix}^T,$$

$$h_c = \begin{bmatrix} h_n^T & 0 & 0 \end{bmatrix}^T, \; x = \begin{bmatrix} x_n^T & w_1^T & w_2^T \end{bmatrix}^T. \tag{3.4}$$

Since $W(s) \equiv W_m(s)$ can be produced at $\varepsilon(t) = 0$, the model reference system is redescribed as

$$\begin{cases} \dot{x}_{mc}(t) = A_c x_{mc}(t) + b_c k_0^* r(t), \\ y_m(t) = h_c^T x_{mc}(t), \end{cases} \tag{3.5}$$

where

$$\begin{cases} x_{mc} = [x_m^T, w_{m1}^T, w_{m2}^T]^T, \\ h_c^T (sI - A_c)^{-1} b_c = \frac{1}{k_0^*} W_m(s). \end{cases} \tag{3.6}$$

Based on the above Eqs (3.3)–(3.6), the error dynamics are derived as follows:

$$\begin{cases} \dot{e}(t) = A_c e(t) + \bar{b}_c \tau^* \varepsilon(t)^T w(t), \\ e_1(t) = h_c^T e(t), \end{cases} \tag{3.7}$$

where $e(t) = x(t) - x_{mc}(t)$ represents the unmeasured state error and $e_1(t) = y_n(t) - y_m(t)$ denotes the available output error, $\bar{b}_c = b_c k_0^*$, $\tau^* = \frac{1}{k_0^*}$, $e_1(t) = W_m(s)\tau^*\varepsilon(t)^T w(t)$. In the context of the error dynamics in Eq (3.7), we have

$$W_e(s) = h_c^T(sI - A_c)^{-1}\bar{b}_c = W_m(s), \tag{3.8}$$

which is SPR.

The classical OF adaptive controller, as introduced by [31], uses a standard parameter update law defined by

$$\dot{\theta}(t) = -\Gamma_s w(t) e_1(t) sgn(k_p) \tag{3.9}$$

where Γ_s represents a symmetric positive definite matrix. This adaptive control strategy ensures that the output error dynamics asymptotically converge to zero. However, the complexity of the controller structure and the computational demand can limit its practical application. In detail, $\theta(t)$, the adaptive parameter vector, adheres to $2n$ update laws, where n represents the number of system states. Consequently, as the system's order increases, the number of updated parameters also increase, which will definitively increase the computational effort and the complexity involved in implementing the adaptive controller.

4. The reduced-order MRAC design and asymptotically stability analysis

On the basis of the abovementioned analysis, we further investigate a MRAC scheme, which reduces the number of adaptive update laws. To do this, we use the controller given by (3.1), and set the parameter estimation error as $\varepsilon(t) = \delta\varphi(t)$, where $\varphi(t) \in \mathbb{R}$ is a scalar function to be updated and $\delta = (\delta_i) \in \mathbb{R}^{2n}$ is an arbitrarily selected parameter vector satisfying $\delta_i \neq 0$ for some $i \in (1, ..., s)$. The adaptive law for updating parameter vector $\theta(t)$ is set to

$$\dot{\theta}(t) = \delta\dot{\varphi}(t), \tag{4.1}$$

with the scalar update law

$$\dot{\varphi}(t) = -\frac{e_1^T(t)\delta^T w(t) sgn(\tau^*)}{tr(\delta^T\Gamma\delta)}, \tag{4.2}$$

where $\Gamma = \Gamma^T > 0$, $\tau^* = \frac{1}{k_0^*}$.

In the presented OFMRAC framework, the adaptive update law for the parameter vector $\theta(t) \in \mathbb{R}^{2n}$ can be represented by the update law of the scalar function $\varphi(t) \in \mathbb{R}$. Using this control scheme significantly reduces the complexity, as it requires the online update of only one parameter. Therefore, the reduced-order MRAC scheme proposed in this paper has lower computational complexity than the classical scheme. The long-standing computational burden problem in the current OFMRAC can then be greatly reduced. For a visual representation of the proposed reduced-order MRAC architecture, kindly refer to Figure 2.

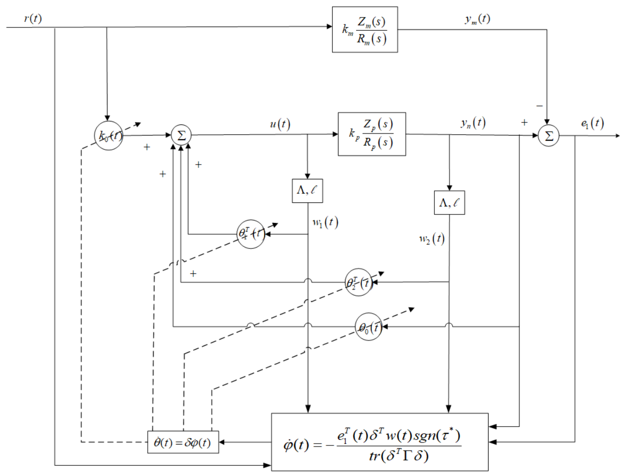

Figure 2. Visual representation of the proposed reduced-order MRAC architecture.

Following the controller design outlined above, the system error model can be described as

$$\begin{cases} \dot{e}(t) = A_c e(t) + \bar{b}_c\tau^*\delta^T\varphi(t)w(t), \\ e_1(t) = h_c^T e(t). \end{cases} \tag{4.3}$$

Then, the asymptotic stability analysis of the reduced-order MRAC approach is presented in the following theorem.

Theorem 4.1. *Consider the dynamic system described by* (2.1)*, controlled by* (3.1)*, and the model reference system given by* (2.3) *under the abovementioned assumptions. Furthermore, let the adaptive update law be set as* (4.1) *with the scalar update law* (4.2)*. Then, all the signals in the adaptive system remain bounded, and the output error* e_1 *is found to be asymptotically Lyapunov stable.*

Proof. Let us select the following Lyapunov function:

$$V(e(t), \varphi(t)) = e^T(t)P_c e(t) + tr[(\delta^T\varphi(t))\Gamma(\delta\varphi(t))|\tau^*|], \tag{4.4}$$

where Γ and P_c are constant positive definite and symmetric matrices.

By considering the derivative of $V(e, \varphi)$, one has

$$\dot{V}(e(t), \varphi(t)) = 2e^T(t)P_c\dot{e}(t) + 2tr[(\delta^T\varphi(t))\Gamma(\delta\dot{\varphi}(t))|\tau^*|]. \tag{4.5}$$

Substituting Eq (4.3) in (4.5) and using the properties of the transpose, one can see

$$\begin{aligned}\dot{V}(e(t), \varphi(t)) =& e^T(t)(P_cA_c + A_c^TP_c)e(t) \\ &+ 2e^T(t)P_c\bar{b}_c\tau^*\delta^T\varphi(t)w(t) \\ &+ 2tr[(\delta^T\varphi(t))\Gamma(\delta\dot{\varphi}(t))|\tau^*|] \\ =& e^T(t)(P_cA_c + A_c^TP_c)e(t) \\ &+ 2e^T(t)P_c\bar{b}_c\tau^*\delta^T\varphi(t)w(t) \\ &+ 2\varphi(t)tr(\delta^T\Gamma\delta)\dot{\varphi}(t)|\tau^*|.\end{aligned} \tag{4.6}$$

Since $W_e(s) = h_c^T(sI - A_c)^{-1}\bar{b}_c$ is SPR, then by applying the Meyer-Kalman-Yakubovich lemma [37], we can insure that given a positive definite and symmetric matrix J, a $P_c = P_c^T > 0$ exists such that

$$\begin{aligned}&P_cA_c + A_c^TP_c = -J \\ &\bar{b}_c^TP_c = h_c^T.\end{aligned} \tag{4.7}$$

Combining (4.7) with (4.6), we can see that

$$\begin{aligned}\dot{V}(e(t), \varphi(t)) =& -e^T(t)Je(t) + 2e^T(t)h_c\tau^*\delta^T\varphi(t)w(t) \\ &+ 2\varphi(t)tr(\delta^T\Gamma\delta)\dot{\varphi}(t)|\tau^*| \\ =& -e^T(t)Je(t) + 2e_1^T(t)\tau^*\delta^T\varphi(t)w(t) \\ &+ 2\varphi(t)tr(\delta^T\Gamma\delta)\dot{\varphi}(t)|\tau^*|.\end{aligned} \tag{4.8}$$

Using the scalar update law (4.2), then Eq (4.8) can be written as

$$\dot{V}(e(t), \varphi(t)) = -e^T(t)Je(t). \tag{4.9}$$

Accordingly

$$\dot{V}(e(t), \varphi(t)) \le -\lambda_{min}(J)\|e(t)\|_2^2 \le 0. \tag{4.10}$$

Since $\dot{V}(e, \varphi)$ is negative semidefinite, e and φ remain bounded. Then

$$\lim_{t\to\infty} V(e(t), \varphi(t)) = V(e_0, \varphi_0) - \lambda_{min}(J)\|e(t)\|_2^2, \tag{4.11}$$

which means that $V(e, \varphi)$ has a finite limit as $t \to \infty$.

Furthermore, $\dot{V}(e, \varphi)$ can be shown to be uniformly continuous by examining the boundedness of its derivative, where

$$\begin{aligned}\ddot{V}(e(t), \varphi(t)) =& -\dot{e}^T(t)Je(t) - e^T(t)J\dot{e}(t) \\ =& -2e^T(t)J(A_ce(t) + \bar{b}_c\tau^*\delta^T\varphi(t)w(t)).\end{aligned} \tag{4.12}$$

Since $e(t)$ and $\varphi(t)$ are bounded by the virtue of $\dot{V}(e(t), \varphi(t)) \le 0$, and $w(t)$ is bounded by assumption, then $\ddot{V}(e(t), \varphi(t))$ is bounded. Therefore, $\dot{V}(e(t), \varphi(t))$ is uniformly continuous. Utilizing the Lyapunov-like Lemma [38], it can be deduced that $\lim_{t\to\infty} \dot{V}(e(t), \varphi(t)) = 0$. Combined with $e_1(t) = h_c^Te(t)$, it follows that $\lim_{t\to\infty} e_1(t) = 0$. As a result, the dynamics of the output error exhibit asymptotic Lyapunov stability, thereby concluding the proof. □

5. Simulation

Two numerical examples are provided in this section to evaluate the efficacy of the proposed reduced-order MRAC approach.

5.1. Example 1

The transfer function forms of the controlled system and the model reference system are given by

$$W_n(s) = \frac{s+1}{s^2 - 3s + 2},$$

and

$$W_m(s) = \frac{s+2}{s^2 + 3s + 6},$$

respectively, with the sign of $k_p > 0$.

For demonstrating the tracking performance of the reduced-order MRAC approach, we apply the presented controller design to the adaptive parameter vector $\theta(t)$, which is given by (4.1). We set $\ell = 1$ and $\Lambda = -2$ in (3.1) and initialize $x_m(0) = [0, 0]^T$, $x_n(0) = [0.2, 0.2]^T$, $\delta = [-1, 3, -1, 3]$, and $\Gamma = I_2$. And the other initial conditions are set to zero. Furthermore, we select the reference input as $r(t) = sin(t)$. Subsequently, the simulation results presented in Figure 3 are obtained.

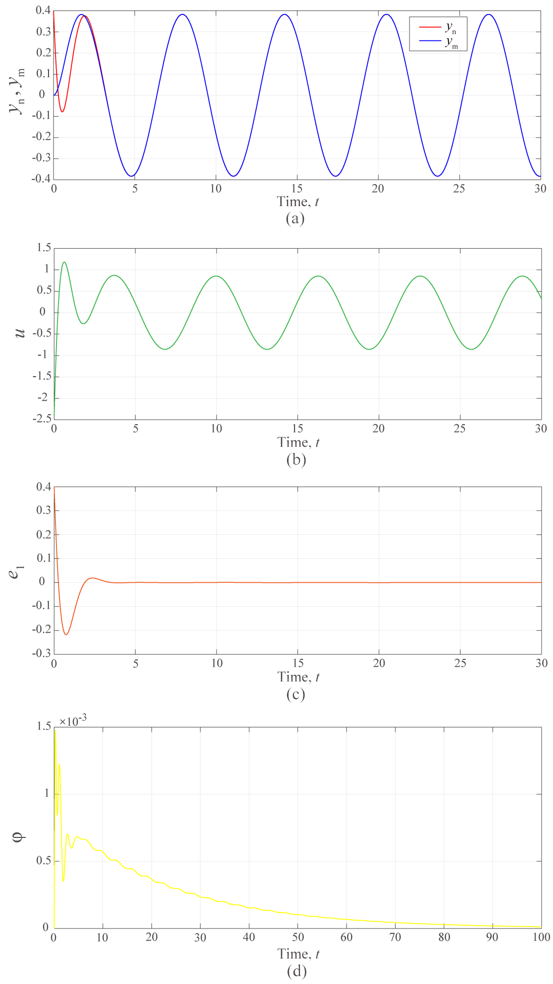

Figure 3. The system's response with the reduced-order MRAC scheme.

Figure 3(a) displays the time evolution of the model reference output y_m and the controlled system output y_n. We can clearly see that the response of the controlled system quickly tracks the reference trajectory. The time evolution of the controller u is shown in Figure 3(b), confirming the boundedness of the control signal. Furthermore, Figure 3(c),(d) presents the time responses of the system output error e_1 and scalar function φ, respectively. We can observe that the output error converges asymptotically to zero and the scalar function remains bounded by adjusting the adaptive parameters online according to the proposed MRAC strategy.

Moreover, we can deduce from (3.9) and (4.1) that the number of parameter update laws to be calculated in the classical OFMRAC and the work in this paper is 4 and 1, respectively. In addition, under the same conditions, we compare the system's response adjustment time using these two adaptive controllers, and we find that it takes 0.0740 s and 0.0370 s, respectively, to complete the calculation. This underscores the computational advantage offered by our proposed approach over traditional method.

In addition, under the same conditions, we compare the numerical results of the system's output y_n and tracking error e_1 with the classical OFMRAC method. We select $\Gamma_s = I_2$ in the standard parameter update law. We then get the simulation results in Figure 4.

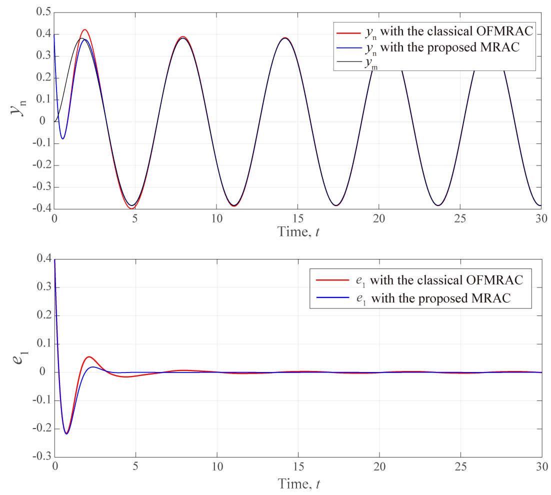

Figure 4. Time evolution of the system's output y_n and the tracking error $e_1(t)$ with the classical OFMRAC and the proposed MRAC.

Figure 4 illustrates that both the proposed MRAC scheme and the classical OFMRAC scheme enable the output error to converge asymptotically to zero such that $y_n(t)$ tracks $y_m(t)$. In this case, we can also observe that the proposed controller maintains good control performance in terms of tracking accuracy and speed compared to the classical adaptive controller.

On the basis of the numerical findings above, it can be deduced that the suggested MRAC approach is capable of preserving the asymptotic stability of the output error dynamics as well as drastically reducing the computational burden and effectively saving a large number of control resources.

5.2. Example 2

Consider the second-order plant characterized by the transfer function presented below:

$$y_n(t) = -\frac{s+1}{s^2 - 5s + 6}[u](t).$$

We choose the model reference system given by the following equation:

$$y_m(t) = \frac{s+2}{s^2 + 3s + 6}[r](t),$$

where the sign of $k_p < 0$.

We set

$$w_1(t) = \frac{1}{s+2}[u](t),$$

$$w_2(t) = \frac{1}{s+2}[y_n](t).$$

Further, we set $\delta = [-1, 1, 4, 2]$, $\Gamma = I_2$. To study the tracking efficacy of the reduced-order MRAC approach, we choose the initial condition of the output variable in the uncertain dynamical system to be $y_n(0) = [0.2, 0.2]^T$, and the other initial conditions are set to zero. The reference signal $r(t)$ is chosen as $20 * cos(6 * t) + 30 * sin(8 * t)$. The simulation results are illustrated in Figure 5.

From Figure 5, it is observed that the proposed adaptive controller ensures the asymptotic stability of the output error system, and the closed-loop signals remain bounded, aligning with the theoretical results. Specifically, Figure 5(a) presents the time response of the system's output error e_1, where we observe that the output error asymptotically converges to zero. Figure 5(b) shows the time evolution of the model reference output y_m and the controlled system's output y_n. It is clear that the response of the controlled system quickly tracks the reference trajectory. The time evolution of the control signal u is depicted in Figure 5(c), which confirms the boundedness of the control signal. Finally, Figure 5(d) illustrates the time response of the scalar function φ, demonstrating that it remains bounded. These behaviors are achieved through online adjustment of the adaptive parameters, in accordance with the proposed MRAC strategy.

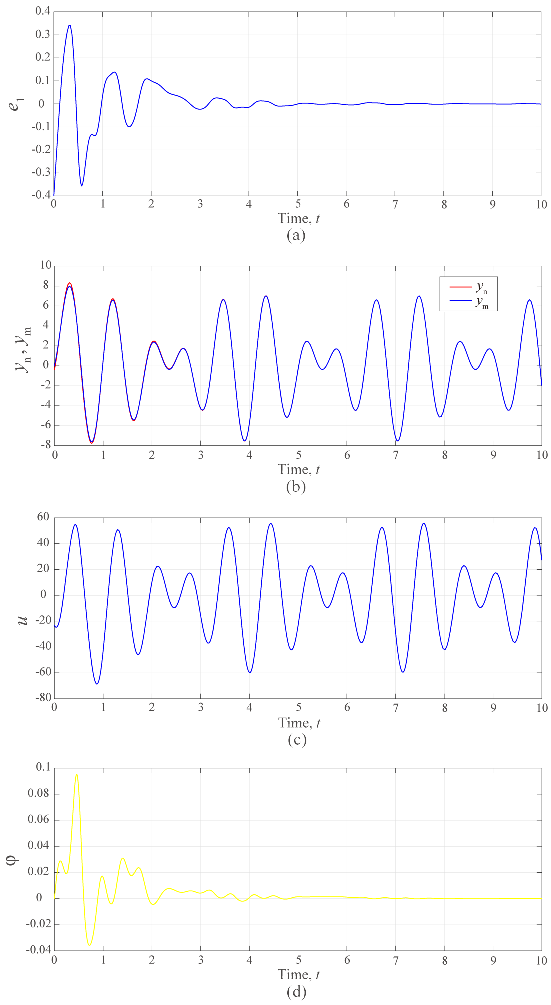

Figure 5. The system's response with the reduced-order MRAC scheme.

In addition, from the analysis of Eqs (3.9) and (4.1), it is deduced that the classical OFMRAC method requires the computation of four parameter update laws, whereas our method requires only one. Furthermore, a comparative analysis of the system's running time with these two control schemes reveals that the classical method necessitates 0.2120 s to complete computations, while our method requires only 0.0570 s. Consequently, our control scheme not only reduces the number of parameter updates but also decreases the computational time required, thereby lessening the computational burden while ensuring the asymptotic stability of the system's error dynamics.

6. Conclusions

This study has focused on the output tracking control of SISO systems with unknown parameters and a known sign of high-frequency gain by an OFMRAC scheme. By developing a scalar update law-based control method, the computational burden caused by numerous online updates of controller parameters is lessened. It has been proven that the reduced-order control method is able to guarantee that the closed-loop signals remain bounded and that asymptotic output tracking is achieved. While the proposed method performs well under nominal conditions, it is important to recognize that external disturbances are often encountered in practical systems. Under such circumstances, the current approach may not fully maintain the desired performance, as the system's robustness to disturbances is not fully addressed. Future work will focus on enhancing the OFMRAC scheme with an adaptive disturbance compensator to estimate and mitigate the impact of such disturbances. This will improve the system's robustness, ensuring stable performance in real-world environments with varying and unpredictable uncertainties.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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