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Research article

Dynamics of an Eco-epidemiological model with infected prey in fractional order

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Abstract: In this article, a model of a diseased prey-predator with fractional order has been studied. The model has been used as a functional response of Holling type II in a non-delayed model. The eigenvalues of a model are used to test its stability using critical points. Furthermore, the boundedness, uniqueness, existence, and positivity of the solutions have been studied. The locally asymptotically stable model has been analyzed using the critical points, and the globally asymptotically stable model has been examined using the Lyapunov function. The occurrence of Hopf bifurcation for fractional order has been examined. Finally, numerical simulations are presented to confirm the analytical solutions.

Keywords: fractional differential equations; Caputo-type derivatives; Holling type-II functional response; stability analysis; Hopf bifurcation

1. Introduction

The predator-prey models, developed by Lotka [1] and Volterra [2], are considered early developments in contemporary mathematical ecology in coupled systems of non-linear differential equations. Eco-epidemiological models are used to investigate the relationship between predator and prey infection in infected and diseased prey populations. Eco-epidemiology is a field that studies the transmission of diseases among interacting organisms with a significant environmental impact [3, 4]. Since Kermack-Mckendrick's pioneering work on SIRS, epidemiological models have attracted much interest, as functional response is among the most important factors in the prey-predator population [5, 6]. Mathematical models are crucial for understanding, studying, and investigating the spread and management of infectious diseases [7, 8]. Fractional calculus is a generalization of the classical differentiation and integration of arbitrary orders. Many researchers are interested in scientific and engineering fields, including biology, fluid dynamics, and medicine [9, 10]. Due to its numerous applications, fractional-order calculus has attracted the interest of researchers throughout the last two decades [11, 12]. Fractional-order biological models have recently attracted the interest of many authors [13, 14]. The main reason lies in the fact that memory-based systems, which exist in a large number of biological systems, are easily relatable to fractional-order models [15, 16]. The fractional-order derivative has the benefit that it allows you to remember the concept of numerical derivative calculation as well as important information about derivative values. Javidi studied the biological behaviors of a prey-predator model with fractional order [17, 18]. This article includes an investigation of the stability of a derivative of a fractional-order model of the mutualistic interaction between two species with infection. Alidousti studied how the capture of predators and scavengers was affected by a prey-predator model with fractional order [19]. Mukherjee et al. [20] studied the existence, uniqueness, and boundedness of solutions to a fractional-order prey-predator system in restricted space. Recently, fractional calculations have developed rapidly and displayed a wide range of possible applications [21, 22]. However, due to memory effects, fractional-order derivatives in the biological model are more sensible than integer-order derivatives. To change ordinary calculus to fractional calculus, it is important to use the Riemann-Liouville and Caputo fractional derivatives. One of the most important processes in any natural ecosystem is the predator-prey model. Caputo introduced the Caputo-type derivative at 1967 [23]. A system of fractional order with a Holling type-II functional response was investigated [24]. Routh-Hurwitz criteria is the condition for stability of a system in fractional order. A system with non-linear fractional order stability with the use of the Routh-Hurwitz criterion was investigated by Ahmed et al [25]. Garrappa studied how to solve fractional-order nonlinear differential equations [26]. In a prey-predator model with fractional order, Javidi and Nyamoradi investigated the effects of harvesting [27, 28]. Several mathematical models in fractional order can be used to solve real-world problems. The proceeding discussion gives the motivation to learn about the dynamic behavior on the fractional prey-predator model. The unique aspect of this work is to examine the prior history of the prey-predator model.

The novelty of this work is to investigate the stability analysis of the prey-predator model through fractional-order derivatives. The analysis demonstrates that fractional calculus is well suited to explain the memory and inherited features of several techniques and materials that are not taken into consideration by classical integer models.

The paper is organized as follows: A mathematical model is developed in Section 2. Section 3 examines the fractional-order dynamical system's preliminary dynamics. The proposed model's uniqueness and boundedness solution have been examined in Section 4. The stability analysis of the suggested model has been investigated in Section 5. The Hopf bifurcation of the system is studied in Section 6. Section 7 examines numerical simulations of the proposed model. Finally, the conclusion of the paper and the biological implications of our mathematical results are found in Section 8.

2. Mathematical model formation

The model has basically two types of population:

(i) Prey population and (ii) Predator population.

Melese [29] studied and discussed refuge and harvesting in the prey-predator model with the Holling type II functional response. Then the proposed model,

$$\left.\begin{aligned} \frac{dS}{dT} &= RS\left(1-\frac{S+I}{K}\right)-\lambda IS-\frac{\alpha_1 SP}{a_1+S},\\ \frac{dI}{dT} &= \lambda IP - D_1 I - \frac{b_1 IP}{a_1+I},\\ \frac{dP}{dT} &= -D_2 P + \frac{cb_1 IP}{a_1+I}+\frac{c\alpha_1 SP}{a_1+S}, \end{aligned}\right\} \tag{2.1}$$

subject to initial conditions $S(0) \geq 0$, $I(0) \geq 0$ and $P(0) \geq 0$. An example in real life: rabbit populations are afflicted with myxomatosis, a condition carried on by the myxoma virus. This disease may have an impact on the interactions between foxes and other predators in a region where rabbits are a major source of food for them. The predator-prey dynamic is disrupted because infected rabbits are more easily captured by predators.

To reduce the number of system parameters, one needs to non-dimensionalize the above model (2.1) by $s = \frac{S}{K}$, $i = \frac{I}{K}$, and $p = \frac{P}{K}$, and to take into account the dimension time $t = \lambda KT$. Now, we apply the following transformations:

$$r = \frac{R}{\lambda k}, \alpha = \frac{\alpha_1}{\lambda K}, a = \frac{a_1}{K}, d_1 = \frac{D_1}{\lambda K}, \theta = \frac{b_1}{\lambda k}, d_2 = \frac{D_2}{\lambda k}.$$

The Eq (2.1) can be rewritten in the following non-dimensional form using the above transformations.

$$\left.\begin{aligned} \frac{ds}{dt} &= rs(1-s-i)-is-\frac{\alpha sp}{a+s},\\ \frac{di}{dt} &= is - d_1 i - \frac{\theta ip}{a+i},\\ \frac{dp}{dt} &= -d_2 p + \frac{c\theta ip}{a+i}+\frac{c\alpha sp}{a+s}. \end{aligned}\right\} \tag{2.2}$$

In the system (2.2), we have taken the Caputo fractional-order derivative β to model, and then the model (2.2) is taken into the following form:

$$\left.\begin{aligned} \frac{d^\beta s}{dt^\beta} &= rs(1-s-i) - is - \frac{\alpha sp}{a+s}, \\ \frac{d^\beta i}{dt^\beta} &= is - d_1 i - \frac{\theta ip}{a+i}, \\ \frac{d^\beta p}{dt^\beta} &= -d_2 p + \frac{c\theta ip}{a+i} + \frac{c\alpha sp}{a+s}, \end{aligned}\right\} \tag{2.3}$$

subject to the initial conditions $s(0) \geq 0$, $i(0) \geq 0$, $p(0) \geq 0$. Table 1 shows the biological representation system (2.1) parameters [29].

Table 1. Biological representation of system (2.1) parameters.

Parameters	Environmental representation
S	Susceptible prey
I	Infected prey
P	Predator
r	Prey growth rate
K	Environmental carrying capacity
a_1	Constant of Half-saturation
α_1	Predation rate of Susceptible prey
b_1	Predation rate of Infected prey
c	Conversion rate of prey and predator
d_1	Infected prey death rate
d_2	Predator death rate
λ	Infection rate

3. Preliminaries

In this section, we provide basic definitions, significant results, and characteristics of fractional differential equations that are useful in the proof of theorems.

Definition 3.1. *The Caputo fractional derivative of order β is defined as*

$$^C D_t^\beta f(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t (t-s)^{-\beta} f'(s) ds$$

where $t \geq 0$, $f \in C^n([0,+\infty), R)$ and Γ is a Gamma function.

Lemma 3.1. *[30] Consider a system of fractional-order Caputo derivatives*

$$^C D_t^\beta x(t) = f(t, x(t)), t > 0, x(0) > 0, \beta \in (0,1],$$

where $f : (0,\infty) \times \Omega \to R^n$. If $f(t, x(t))$ satisfies the locally Lipschitz condition with respect to x, then the equation on $(0,\infty) \times \Omega$ has a unique solution.

Theorem 3.1. *[31] Consider the N-dimensional fractional differential equation system*

$$\frac{d^\beta(x)}{dt} = f(x); x(0) = x_0,$$

where A is the arbitrary constant $N \times N$ is the matrix and $\beta \in (0,1)$.
(i) The solution $x = 0$ is asymptotically stable if and only if all eigenvalues λ_{ij}, $j = 1,2,3,...N$ of A satisfy $|arg(\lambda_j)| > \frac{\beta\pi}{2}$.
(ii) The solution $x = 0$ is stable if and only if all the eigenvalues with $|arg(\lambda_j)| = \frac{\beta\pi}{2}$ have the same geometric multiplicity and algebraic multiplicity.

Theorem 3.2. *[27] Consider the fractional order system*

$$\frac{d^\beta(x)}{dt} = f(x), x(0) = x_0$$

with $x \in R^n$ and $\beta \in (0,1)$. The above system's equilibrium points are the solutions to the equation $f(x) = 0$. If all of the eigenvalues of the Jacobian matrix $J = \frac{df}{dx}$ evaluated at equilibrium satisfy $|arg(\lambda_j)| \geq \frac{\beta\pi}{2}$, then the equilibrium point is considered to be locally asymptotically stable.

Lemma 3.2. *[6] Let $x(t) \in C([0,+\infty))$.If $x(t)$ satisfies*

$$^C D_t^\beta x(t) + \lambda x(t) \leq \mu, x(0) = x_0,$$

where $\beta \in (0,1]$, $(\lambda,\mu) \in R^2$, and $\lambda \neq 0$, then $x(t) \leq (x_0 - \frac{\mu}{\lambda})E_\beta[-\lambda t^\beta] + \frac{\mu}{\lambda}$.

4. Existence and uniqueness of the solutions

In this section, boundedness of solution of the system (2.3) has been examined. The fractional-order system is as follows:

$$\frac{d^\beta X(t)}{dt^\beta} = f(t, X(t)), \quad \beta \in (0,1].$$

Theorem 4.1. *For the non-negative initial conditions, there is only one solution to the fractional-order system (2.3).*

Proof. A sufficient condition for the solutions of system (2.3) in the region $\chi \times (0, T]$.

Where,

$$\chi = \Big\{(s, i, p) \in R^3 : max(|s|, |i|, |p|) \leq \eta\Big\}.$$

Now, let us define a mapping $V(X) = (V_1(X), V_2(X), V_3(X))$.

Where

$$V_1(X) = rs(1 - s - i) - is - \frac{\alpha sp}{a + s},$$
$$V_2(X) = is - d_1 i - \frac{\theta ip}{a + i},$$
$$V_3(X) = -d_2 p + \frac{c\theta ip}{a + i} + \frac{c\alpha sp}{a + s}.$$

$$\begin{aligned}
&\|V(X) - V(\bar{X})\| \\
=&|V_1(X) - V_1(\bar{X})| + |V_2(X) - V_2(\bar{X})| + |V_3(X) - V_3(\bar{X})| \\
=&\left|rs(1 - s - i) - is - \frac{\alpha sp}{a + s} - r\bar{s}(1 - \bar{s} - \bar{i}) + \bar{i}\bar{s} + \frac{\alpha \bar{s}\bar{i}}{a + \bar{s}}\right| \\
&+ \left|is - d_1 i - \frac{\theta ip}{a + i} - \bar{s}\bar{i} + d_1\bar{i} + \frac{\theta \bar{i}\bar{p}}{a + \bar{i}}\right| \\
&+ \left| - d_2 p + \frac{c\theta ip}{a + i} + \frac{c\alpha sp}{a + s} + d_2\bar{p} - \frac{c\theta\bar{i}\bar{p}}{a + \bar{i}} - \frac{c\alpha\bar{s}\bar{p}}{a + \bar{s}}\right| \\
\leq&\Big\{r + 2r\eta + \eta + \alpha a\eta(1 + c) + \eta(\alpha + 1)\eta\Big\}\Big|s - \bar{s}\Big| \\
&\times \Big\{r\eta + 2\eta + d_1 + s\theta a\eta(1 + a) + \theta\eta + \eta(r + 2 + \theta) \\
&+ \theta a\eta(1 + a)\Big\}\Big|i - \bar{i}\Big| + \Big\{(1 + c)\alpha a\eta + (1 + c)\theta a\eta \\
&+ c\eta(\theta + \alpha + d_2)\Big\}\Big|p - \bar{p}\Big|.
\end{aligned}$$

Where,

$$\begin{aligned}
\mathcal{H} =&max\Big\{r + 2r\eta + \eta + \alpha a\eta(1 + c) + \eta(\alpha + 1), r\eta + 2\eta \\
&+ d_1 + \theta a\eta(1 + a) + \theta\eta + \eta(r + 2 + \theta) + \theta a\eta(1 + a), \\
&\frac{\alpha(1 + c)}{a} + \frac{\theta(1 + c)}{a} + d_2 + \alpha(1 - c)\eta + \theta(1 - c)\eta\Big\}.
\end{aligned}$$

Hence, the solution of the system (2.3) exists and is unique. □

Boundedness of the solutions

Theorem 4.2. *Each and every one of the system (2.3) solutions are non-negative and uniformly bounded.*

Proof. Construct a function

$$V(t) = s + i + \zeta p.$$

Then, for each $\eta > 0$, we obtain

$$\begin{aligned}
{}^C D_t^\beta + \eta V(t) =& \left(rs - is - \frac{\alpha sp}{a + s}\right) + \left(is - d_1 i - \frac{\theta ip}{a + i}\right) \\
&+ \eta\left(-d_2 p + \frac{c\theta ip}{a + i} + \frac{c\alpha sp}{a + s}\right) + \zeta\,(s + i + \zeta p) \\
=& (r + \zeta)s - rs^2 - rsi + c\theta\left(\zeta - \frac{1}{c}\right)\frac{ip}{a + i} \\
&+ \frac{c\alpha sp}{a + s}\left(\zeta - \frac{1}{c}\right) + (\zeta - d_1)i + \zeta(\zeta - d_2)p.
\end{aligned}$$

By choosing $\zeta < min\,\{d_1, d_2\}$ and $\zeta < min\left\{\frac{1}{c}\right\}$, we have

$$\begin{aligned}
{}^C D_t^\beta + \zeta V(t) \leq& (r + \zeta)\,s - rs^2 \\
=& (r + \zeta)s - rs^2 - \left(\frac{r + \zeta}{2}\right)^2 + \left(\frac{r + \zeta}{2}\right)^2 \\
\leq& \frac{(r + \zeta)^2}{4}.
\end{aligned}$$

Applying lemma 2, which gives,

$$V(t) \leq \left(V(0) - \frac{(r + \zeta)^2}{4\zeta}\right)E_\beta[-\zeta t^\beta] + \frac{(r + \zeta)^2}{4\zeta}.$$

Here, we know that $V(t)$ is convergent to $\frac{(r + \zeta)^2}{4\zeta}$ for $t \to \infty$.

Therefore, all the solutions of the system (2.3) with non-negative initial conditions are confined in the region Ω.

Where,

$$\Omega = \left\{(s, i, p) \in R^3_+ : V(t) \leq \frac{(r + \zeta)^2}{4\zeta} + \epsilon, \epsilon > 0\right\}.$$

□

5. Equilibrium points and stability analysis

In this section, the system (2.3) has the following possible equilibrium points:

(i) $E_0(0, 0, 0)$ is the trivial equilibirum point.

(ii) $E_1(1, 0, 0)$ is the infected-free and predator-free equilibrium point.

(iii) $E_2(\bar{s}, 0, \bar{p})$ is the infected prey-free equilibrium point. Where $\bar{s} = \frac{d_2 a}{d_2 - c\alpha}$, and $\bar{p} = \frac{ac((c\alpha - d_2)r - ard_2)}{(c\alpha - d_2)^2}$.

(iv) $E_3(\hat{s}, \hat{i}, 0)$ is the predator free equilibrium point, where $\hat{s} = d_1$ and $\hat{i} = \frac{r(1 - d_1)}{r + 1}$.

(v) The interior equilibrium point $E^*(s^*, i^*, p^*)$.

Where,

$$i^* = \frac{a(ad_2 + (d_2 - c\alpha)s^*)}{(c\alpha s^* + (c\theta - d_2)(a + s^*)},$$

$$p^* = \frac{ac(s^* - d_1)(a + s^*)}{(c\alpha s^* + (c\theta - d_1)(a + s^*))}$$

and s^* is the only positive root of the equation for a quadratic function.

$$As^2 + Bs + C = 0. \quad (5.1)$$

Where

$$A = r(c\alpha + c\theta - d_2),$$
$$B = (c\theta - d_2)(-r + ar) - r\alpha c + a(d_1 + (d_1 - c\alpha)r),$$
$$C = -a((r)(c\theta - d_2) + (c\alpha(d_1) - ad_2(1 + r)).$$

5.1. Stability analysis

Now, we want to calculate the Jacobian matrix for local stability analysis around different equilibrium points. The Jacobian matrix at an arbitrary point (l, m, n) is given by

$$J(l, m, n) = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix}.$$

Where,

$$n_{11} = r(1 - 2s) - i(r + 1) - \frac{\alpha ap}{(a + s)^2}, n_{12} = -s(r + 1),$$
$$n_{13} = -\frac{\alpha s}{a + s}, n_{21} = i, n_{22} = s - d_1 - \frac{a\theta p}{(a + i)^2}, n_{23} = \frac{\theta i}{a + i},$$
$$n_{31} = \frac{ac\alpha p}{(a + s)^2}, n_{32} = \frac{ac\theta p}{(a + i)^2}, n_{33} = -d_2 + \frac{c\theta i}{a + i} + \frac{\alpha cs}{a + s}.$$

Theorem 5.1. *The trivial equilibrium point $E_0(0, 0, 0)$, which is a saddle.*

Proof. At an equilibrium point E_0, the Jacobian matrix is given by

$$J(E_0) = \begin{pmatrix} r & 0 & 0 \\ 0 & -d_1 & 0 \\ 0 & 0 & -d_2 \end{pmatrix}.$$

The eigenvalues are $\lambda_1 = r$, $\lambda_2 = -d_1$ and $\lambda_3 = -d_2$.

Thus, $|arg(\lambda_1)| = 0 < \frac{\beta\pi}{2}$, $|arg(\lambda_2)| = \pi > \frac{\beta\pi}{2}$ and $|arg(\lambda_3)| = \pi > \frac{\beta\pi}{2}$.

Hence, the trivial equilibrium point $E_0(0, 0, 0)$ is unstable. □

Theorem 5.2. *The infected free and predator free equilibrium point $E_1(1, 0, 0)$ is unstable.*

Proof. At an equilibrium point E_1, the Jacobian matrix is given by

$$J(E_1) = \begin{pmatrix} -r & -(r + 1) & \frac{-\alpha}{a + s} \\ 0 & 1 - d_1 & 0 \\ 0 & 0 & -d_2 + \frac{c\alpha}{a + 1} \end{pmatrix}.$$

The eigenvalues are $\lambda_1 = -r, \lambda_2 = 1 - d_1 - \frac{\theta p}{a + 1}$ and $\lambda_3 = -d_2 + \frac{c\alpha}{a + 1}$.

Thus, $|arg(\lambda_1)| = 0 < \frac{\beta\pi}{2}$, $|arg(\lambda_2)| = \pi > \frac{\beta\pi}{2}$ and $|arg(\lambda_3)| = \pi > \frac{\beta\pi}{2}$.

Due to numerical simulation table values, $1 - d_1$ is positive.

The equilibrium point $E_1(1, 0, 0)$ is unstable. □

Theorem 5.3. *The infected-free equilibrium point $E_2(\bar{s}, 0, \bar{p})$ is locally asymptotically stable if P, R, $PQ - R$ are positive.*

Proof. At an equilibrium point E_2, the Jacobian matrix is given by

$$J(E_2) = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix}.$$

Where,

$$f_{11} = r - \frac{2ard_2}{c\alpha - s_2} - \frac{(c\alpha - d_2)^2\bar{p}}{a\alpha c^2}, f_{12} = -\frac{a(1 + r)d_2}{c - d_2},$$
$$f_{13} = -\frac{d_2}{c}, f_{21} = 0, f_{22} = -d_1 + \frac{ad_2}{c\alpha - d_2}, f_{23} = 0,$$
$$f_{31} = \frac{((c\alpha - d_2)^2\bar{p})}{ac\alpha}, f_{32} = \frac{c\theta\bar{p}}{a}, f_{33} = 0.$$

Here, the characteristic equation of the above Jacobian matrix is provided by

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0. \quad (5.2)$$

Where,

$$P = -f_{11} - f_{22},$$
$$Q = -f_{31}f_{13} + f_{22}f_{11},$$

$$R = f_{13}f_{12}f_{31}.$$

The Routh–Hurwitz criteria state that all of the roots in the equations mentioned above have negative real parts if and only if P, R, and $PQ - R$ are all positive.

Now,

$$PQ - R = -f_{11}f_{22}(f_{11} + f_{22}) + f_{11}f_{22}f_{33}.$$

Now the sufficient condition for f_{11} and f_{22} to be negative.

The infected-free equilibrium point $E_2((\bar{s}, 0, \bar{p})$ is locally asymptotically stable. □

Theorem 5.4. *The predator-free equilibrium point $E_3(\hat{s}, \hat{i}, 0)$ is locally asymptotically stable if $d_2 > c(\alpha + \theta)$.*

Proof. At an equilibrium point E_3, the Jacobian matrix is given by

$$J(E_3) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

Where,

$$\begin{aligned} a_{11} &= -d_1 r, a_{12} = (-1 - r)\bar{s}, a_{13} = \frac{-\alpha}{a + s}, \\ a_{21} &= i, a_{22} = 0, a_{23} = \frac{\theta\bar{i}}{a + \bar{i}}, \\ a_{31} &= 0, a_{32} = 0, a_{33} = \frac{c\alpha\bar{s}}{a + \bar{s}} - s_6 + \frac{c\theta\bar{i}}{a + i}. \end{aligned}$$

Here, the characteristic equation of the above Jacobian matrix is provided by

$$\lambda^3 + X\lambda^2 + Y\lambda + Z = 0. \tag{5.3}$$

Where,

$$\begin{aligned} X &= -a_{11} - a_{33}, \\ Y &= -a_{21}a_{12} + a_{33}a_{11}, \\ Z &= a_{12}a_{21}a_{33}. \end{aligned}$$

The Routh–Hurwitz criteria state that all of the roots in the equations mentioned above have negative real parts if and only if X, Z, and $XY - Z$ are all positive.

Now, $XY - Z = -a_{11}(-a_{12}a_{21} + a_{33} + a_{11})$.

Now, the sufficient conditions for a_{33} to be negative is $d_2 > c(\alpha + \theta)$.

Hence,the equilibrium point E_3 is locally asymptotically stable. □

Theorem 5.5. *The interior equilibrium point $E^*(s^*, i^*, p^*)$ is locally asymptotically stable.*

Proof. Here, the characteristic equation of the above Jacobian matrix is provided by

$$J(E^*) = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix}.$$

Where,

$$\begin{aligned} g_{11} &= \frac{-s^*(-r + ar + (1 + r)i^* + 2rs^*)}{a + s^*}, \\ g_{12} &= -s^*(r + 1), g_{13} = \frac{-\alpha s^*}{a + s^*}, \\ g_{21} &= i^*, g_{22} = \frac{a\theta p^* i^*}{(a + i^*)^2)}, g_{23} = \frac{\theta i^*}{a + i^*}, \\ g_{31} &= *\frac{ac\alpha p^*}{(a + s^*)^2}, g_{32} = \frac{ac\theta p^*}{(a + i^*)^2)}, g_{33} = 0. \end{aligned}$$

Here, the characteristic equation of the above Jacobian matrix is provided by

$$\lambda^3 + E\lambda^2 + F\lambda + G = 0. \tag{5.4}$$

Where

$$\begin{aligned} E &= -g_{11} - g_{33}, \\ F &= g_{21}g_{12} + g_{22}g_{11} - g_{13}g_{31} + g_{23}g_{32}, \\ G &= g_{13}(-g_{22}g_{31} + g_{21}g_{32}) + g_{23}(g_{12}g_{31} - g_{11}g_{32}). \end{aligned}$$

The Routh–Hurwitz criteria state that all of the roots in the equations mentioned above have negative real parts if and only if E, G and $EF - G$ are all positive.

Therefore, E^* is locally asymptotically stable. □

5.2. Global stability analysis

Theorem 5.6. *The interior equilibrium point E^* is globally asymptotically stable.*

Proof. Consider a Lyapunov function

$$V(s, i, p) = \left[s - s^* - s^* ln\frac{s}{s^*}\right] + e_1\left[i - i^* - i^* ln\frac{i}{i^*}\right] + e_2\left[p - p^* - p^* ln\frac{p}{p^*}\right].$$

Applying the Caputo fractional derivative, we obtain

$$
\begin{aligned}
&\le \left[\frac{s-s^*}{s}\right] {}^C D^\alpha s(t) + e_1 \left[\frac{i-i^*}{i}\right] {}^C D^\alpha i(t) \\
&+ e_2 \left[\frac{p-p^*}{p}\right] {}^C D^\alpha p(t) \\
&\le \left(\frac{s-s^*}{s}\right)\left[rs(1-s-i) - is - \frac{\alpha sp}{a+s}\right] \\
&+ e_1 \left(\frac{i-i^*}{i}\right)\left[is - d_1 i - \frac{\theta ip}{a+i}\right] \\
&+ e_2 \left(\frac{p-p^*}{p}\right)\left[-d_2 p + \frac{c\theta ip}{a+i} + \frac{c\alpha sp}{(a+s)}\right] \\
&\le -(s-s^*)\left[r\{(s+i)-(s^*+i^*)+(i-i^*)\}\right] \\
&+ \alpha \left[\frac{p}{a+s} - \frac{s^*}{a+s^*}\right] \\
&- e_1(i-i^*)\left[\left((s-s^*) - \frac{p}{a+i} - \frac{p^*}{a+i^*}\right)\right] \\
&- e_2(p-p^*)\left[c\theta\left(\frac{i(a+i^*)-i^*(a+i)}{(a+i)(a+i^*)}\right)\right] \\
&+ c\alpha\left[\frac{(a+s^*)s-(a+s)s^*}{(a+s)(a+s^*)}\right].
\end{aligned}
$$

Obviously, ${}^C D^\beta V(s,i,p) \le 0$.

We conclude that E^* is globally asymptotically stable. □

6. Hopf-Bifurcation analysis

In this section, we discuss the Hopf-bifurcation analysis of system (2.3).

Define a function, with respect to β by

$$m(\beta) = \frac{\beta\pi}{2} - \min_{1\le i\le 3} |arg(\lambda_i)|.$$

Theorem 6.1. *The fractional-order system (2.3) experiences a Hopf bifurcation at the endemic equilibrium point E^* when bifurcation parameter α passes through the critical value $\alpha^* \in (0,1)$, provided that the following conditions are satisfied:*

(i) The corresponding characteristic Eq (5.4) of system (2.3) has a pair of complex conjugates $\lambda_{1,2} = \chi + i\omega$ (where $\chi > 0$) and one negative real root λ_3.

(ii) $m(\beta^*) = \frac{\beta^*\pi}{2} - \min_{1\le i\le 3} |arg(\lambda_i)| = 0$.

(iii) $\frac{dm(\beta)}{d\beta}|_{\beta=\beta^*} \neq 0$.

Here, we give the conditions under which a Hopf bifurcation would exist as the derivative's order approaches a critical value at the interior equilibrium point E^*.

Proof. The following theorem, which takes the fear parameter as a variable, shows the existence of Hopf bifurcation.

The characteristic equation must be of the form

$$\lambda^3 + E\lambda^2 + F\lambda + G = 0.$$

The Hopf-bifurcation occurs at $\alpha^* = \alpha$, as demonstrated by the transversality condition for the roots of the above equations $\pm i\sqrt{F}$ and $-E$.

$$\frac{d}{d\alpha}\{Re(\lambda(f))\}|_{\alpha=\alpha^*} \neq 0.$$

For all f, the roots are generally in the form

$$
\begin{aligned}
\lambda_1(\alpha) &= r(\alpha) + is(\alpha), \\
\lambda_2(\alpha) &= r(\alpha) - is(\alpha), \\
\lambda_3(\alpha) &= -E.
\end{aligned}
$$

Now, we check the condition,

$$\frac{d}{d\alpha}\left\{Re(\lambda_j(\alpha))\right\}|_{\alpha=\alpha^*} \neq 0,\ j = 1,2.$$

Let $\lambda_1\alpha = r(\alpha) + is(\alpha)$ in equation, we obtain

$$A(\alpha) + iB(\alpha) = 0,$$

where

$$
\begin{aligned}
A(\alpha) &= r^3(\alpha) + r^2(\alpha)E - 3r(\alpha)s^2(\alpha) - s^2(\alpha)E + r(\alpha)F + EF, \\
B(\alpha) &= 3r^2ks(\alpha) + 2r(\alpha)s(\alpha)E - s^3(\alpha) + s(\alpha)F.
\end{aligned}
$$

In order to solve the problem, $A(\alpha) = 0$ and $B(\alpha) = 0$ must also be correct. After differentiating A and B with respect to α, we obtain

$$
\begin{aligned}
\frac{dA}{d\alpha} &= \phi_1(\alpha)r'(\alpha) - \phi_2(\alpha)s'(\alpha) + \phi_3(\alpha), \\
\frac{dB}{d\alpha} &= \phi_2(\alpha)r'(\alpha) + \phi_1(\alpha)s'(\alpha) + \phi_4(\alpha),
\end{aligned}
$$

where,

$$
\begin{aligned}
\phi_1(\alpha) =& 3r^2\alpha + 2r(\alpha)E - 3s^2(\alpha) + F, \\
\phi_2(\alpha) =& [6r(\alpha)s(\alpha) + 2s(\alpha)]E, \\
\phi_3(\alpha) =& r^2(\alpha)E'(\alpha) - s^2(\alpha)E'(\alpha) + r(\alpha)F'(\alpha) + EF' + FE', \\
\phi_4(\alpha) =& 2r(\alpha)s(\alpha)E'(\alpha) + s(\alpha)F'(k).
\end{aligned}
$$

On multiplying by $\phi_1(\alpha)$ and $\phi_2(\alpha)$, respectively, and then summing two equations, we have

$$r^{'}(\alpha) = \frac{-\phi_1(\alpha)\phi_3(\alpha) + \phi_2(\alpha)\phi_4(\alpha)}{\phi_1^2(\alpha) + \phi_2^2(\alpha)}.$$

Substituting $r(\alpha) = 0$ and $s(\alpha) = \sqrt{B(\alpha)}$ at $\alpha = (\alpha^*)$ on ϕ_1,ϕ_2,ϕ_3,ϕ_4, we obtain

$$\phi_1(\alpha^*) = -2B(\alpha^*), \phi_2(\alpha^*) = 2\sqrt{B(\alpha^*)A(\alpha^*)},$$
$$\phi_3(\alpha^*) = -B(\alpha^*)A^{'}(\alpha^*) + C^{'}(\alpha^*), \phi_4(\alpha^*) = \sqrt{B(\alpha^*)B^{'}(\alpha^*)}.$$

The equation implies

$$r^{'}(\alpha) = \frac{c^{'}(\alpha^*) - [A(\alpha^*)B^{'}(\alpha^*) + B(\alpha^*)A^{'}(\alpha^*)]}{2[B(\alpha^*) + A^2(\alpha^*)]}.$$

If $c^{'}(\alpha^*) - [A(\alpha^*)B^{'}(\alpha^*) + B(\alpha^*)A^{'}(\alpha^*)] \neq 0$, which implies that

$$\frac{d}{d\alpha}(Re(\lambda_3(\alpha)) = r^{'}(k) \neq 0, j = 1, 2,$$

and

$$\lambda_3(\alpha^*) = -A(\alpha^*) \neq 0.$$

Consequently, $c^{'}(\alpha^*) - [A(\alpha^*)B^{'}(\alpha^*) + B(\alpha^*)A^{'}(\alpha^*)] \neq 0$ holds.

Since it is guaranteed that the transversality requirement is satisfied, the model has entered Hopf-bifurcation at $\alpha = \alpha^*$. □

7. Numerical analysis

In this section, we present some numerical simulation results for Caputo-sense fractional-order eco-epidemic models. To accomplish this, we use Diethelm et al.'s [32], predictor-corrector approach to solve the defined model.

Predictor-Corrector approach:

$$D_t^{\beta} y(t) = f(t, y(t)), 0 \leq t \leq T, m-1 < \beta < m,$$
$$y^u(0) = y_0^u, u = 0, 1, 2, 3, ..., z-1, \text{ where } m = [\beta],$$

which is the same as the equation of Volterra integral

$$y(t) = \sum_{u=0}^{z-1} y_0^u \frac{t^u}{u!} + \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} f(s, y(s)) ds.$$

The fractional-order system (2.3) can be described in the way that follows using the predictor-corrector approach and the setting $h = \frac{T}{N}$, $t_q = qh$, $n = 0, 1, 2, ..., N \times Z^+$.

Corrector formula:

$$s_{q+1} = s_0 + \frac{h^{\beta}}{\Gamma(\beta+2)}\left[rs_{q+1}^w\left(1 - \frac{l_{q+1}^w}{\beta}\right) - \beta l_{q+1}^w i_{q+1}^w - \beta_1 l_{q+1}^w q_{q+1}^w\right]$$
$$+ \frac{h^{\beta}}{\Gamma(\beta+2)} \sum_{j=1}^{q} a_{j,q+1}\left[rs_j\left(1 - \frac{s_j}{\beta}\right) - \beta s_j i_j - \beta_1 s_j p_j\right],$$
$$i_{q+1} = i_0 + \frac{h^{\beta}}{\Gamma(\beta+2)}\left[\beta s_{q+1}^w i_{q+1}^w - \beta_2 i_{q+1}^w p_{q+1}^w - \mu_1 i_{q+1}^w\right]$$
$$+ \frac{h^{\beta}}{\Gamma(\beta+2)} \sum_{j=1}^{q} a_{j,q+1}[\beta s_j i_j - \beta_2 i_j p_j - \mu_1 i_j],$$
$$p_{q+1} = p_0 + \frac{h^{\beta}}{\Gamma(\beta+2)}\left[\epsilon\beta_1 s_{q+1}^w p_{q+1}^w + \epsilon\beta_2 i_{q+1}^w p_{q+1}^w - \mu_2 p_{q+1}^w\right]$$
$$+ \frac{h^{\beta}}{\Gamma(\beta+2)} \sum_{j=1}^{q} a_{j,q+1}[\epsilon\beta_1 d_j p_j + \epsilon\beta_2 y i_j p_j - \mu_2 p_j].$$

Predictor formula:

Where,

$$s_{q+1}^w = s_0 + \frac{1}{\Gamma(\beta)} \sum_{j=0}^{q} b_{j,q+1}\left[rs_j\left(1 - \frac{x_j}{\beta}\right) - \beta s_j i_j - \beta_1 s_j p_j\right],$$
$$i_{q+1}^w = i_0 + \frac{1}{\Gamma(\beta)} \sum_{j=0}^{q} b_{j,q+1}\left[\beta s_j i_j - \beta_2 i_j p_j - \mu_1 i_j\right],$$
$$p_{q+1}^w = p_0 + \frac{1}{\Gamma(\beta)} \sum_{j=0}^{q} b_{j,q+1}[\epsilon\beta_1 s_j p_j + \epsilon\beta_2 i_j p_j - \mu_2 p_j].$$

The simulations are executed with the following assumed parameter values:

$$r = 0.7, \alpha = 0.2, a = 0.3, d_1 = 0.4,$$
$$\theta = 0.4, d_2 = 0.1, c = 0.5.$$

(i) When $\beta = 1$, the equilibrium point $E_2(0.7, 0.01, 0.5)$ becomes unstable, as shown in Figures 1 and 2.

(ii) When $\beta = 0.94$, the equilibrium point $E_2(0.7, 0.01, 0.5)$ becomes locally asymptotically stable is shown in Figures 3 and 4.

Next, the parameter values are chosen as $r = 0.5$,$\alpha = 0.15$,$a = 0.2$,$d_1 = 0.1$,$\theta = 0.4$,$d_2 = 0.1$,$c = 0.5$.

(i) When $\beta = 1$, the interior equilibrium point $E_4(0.7, 0.04, 0.3)$ becomes unstable is shown in Figures 5 and 6.

(ii) When $\beta = 0.92, 0.84, 0.76, 0.64$, the interior equilibrium point $E_4(0.7, 0.04, 0.3)$ becomes locally asymptotically stable and shows the effectiveness of the fractional-order β is shown in Figures 7–9.

In this instance, we see that the density of the susceptible prey-predator species and the infected prey species start to oscillate; however, this high-amplitude species oscillation results in an extremely low population density, which may lead to the instability of the multispecies community and increase the probability that some species will become extinct. From Figures 7–9, we can conclude that the fractional-order derivative values decreased from 1 to $0.92, 0.84, 0.76, 0.64$ the equilibrium point is transformed into unstable to a stable state.

We have also plotted a bifurcation diagram of system (2.3), choosing predation rate α as a bifurcation parameter in Figures 10–12, while keeping all the parameter values the same as in Figures 7–9. We have seen that system (2.3) exhibits a local stable coexistence equilibrium for $\beta \in (0, 1)$, but while α crosses the critical value $\alpha^* = 0.41$, the system loses its stability and becomes stable. Hence, we conclude that the fractional-order model is more stable than the integer order model. From these Figures 10–12, we observed that β has a great impact on each population.

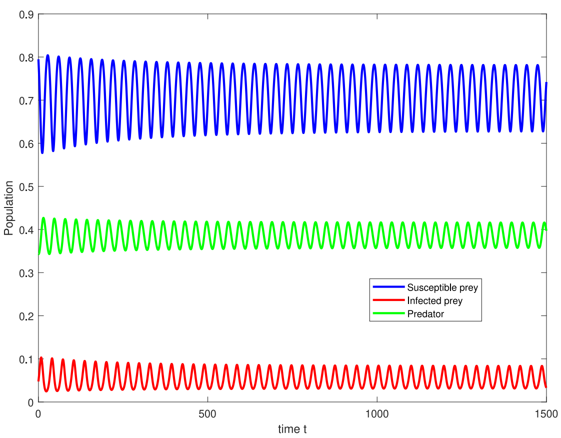

Figure 1. Time series solution for the equilibrium point E_2 of system (2.3) with $\beta = 1$.

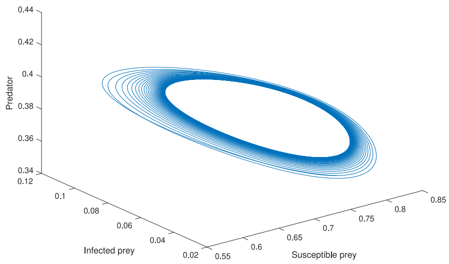

Figure 2. Phase portrait for the equilibrium point E_2 of system (2.3) with $\beta = 1$.

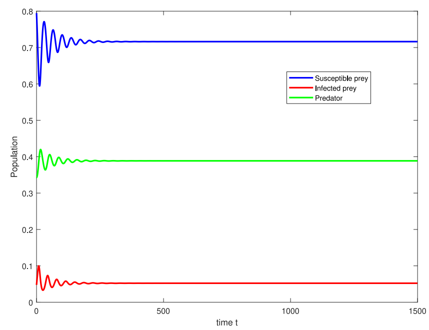

Figure 3. Time series solution for the equilibrium point E_2 of system (2.3) with $\beta = 0.94$.

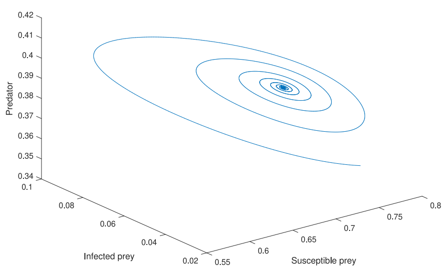

Figure 4. Phase portrait for the equilibrium point E_2 of system (2.3) with $\beta = 0.94$.

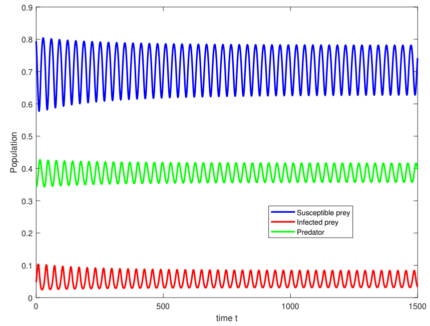

Figure 5. Time series solution for the equilibrium point E_4 of system (2.3) with $\beta = 1$.

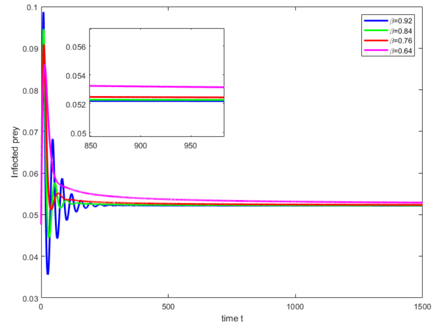

Figure 8. Effectiveness of fractional-order β on infected prey population for the equilibrium point E_4 of system (2.3).

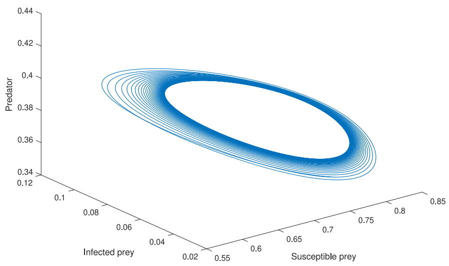

Figure 6. Phase portrait for the equilibrium point E_4 of system (2.3) with $\beta = 1$.

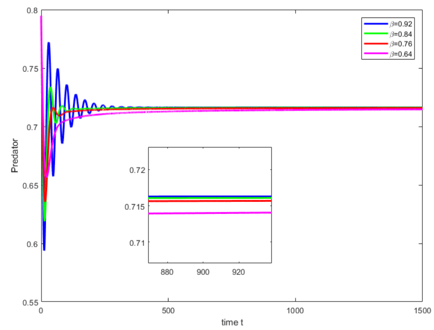

Figure 9. Effectiveness of fractional-order β on predator population for the equilibrium point E_4 of system (2.3).

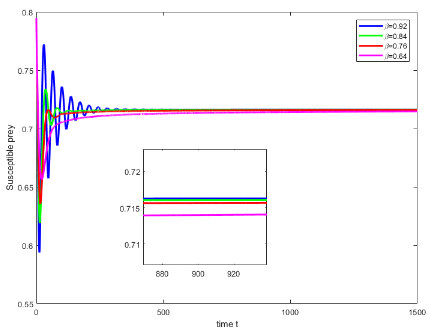

Figure 7. Effectiveness of fractional-order β on susceptible prey population for the equilibrium point E_4 of system (2.3).

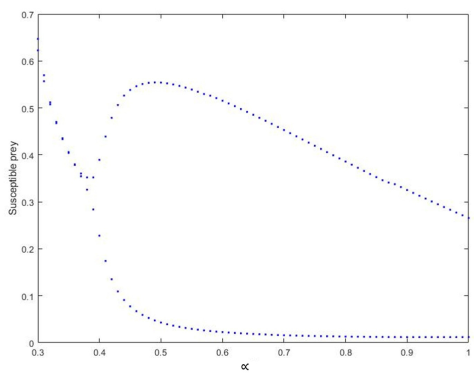

Figure 10. Bifurcation diagram for susceptible prey population of system (2.3) when $\beta = 0.92$.

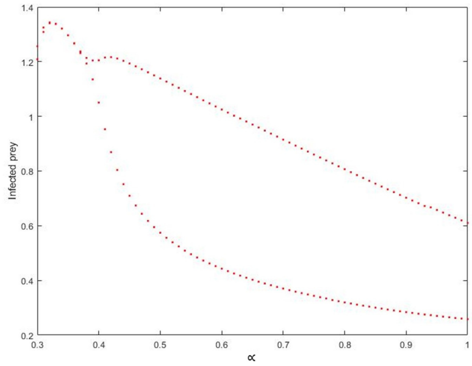

Figure 11. Bifurcation diagram for infected prey population of system (2.3) when $\beta = 0.92$.

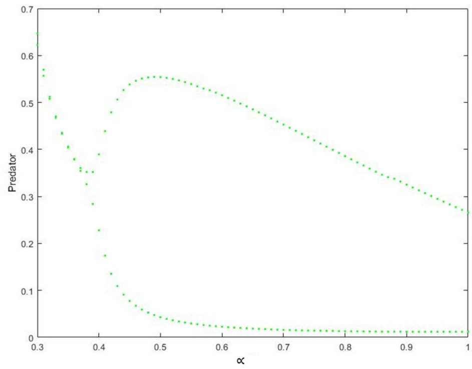

Figure 12. Bifurcation diagram for predator population of system (2.3) when $\beta = 0.92$.

8. Conclusions

In this study, we examined a three-species food web model based on fractional-order derivatives. In our proposed fractional-order system, the local stability of each equilibrium point has also been investigated.A number of biological systems that are highly dependent on historical events have been described using fractional-order mathematical models. These findings suggest that the mathematical model of fractional-order can be useful in explaining system dynamics with useful memory. It can be seen that the unstable system with integer-order $\beta = 1$ turns into a stable system for different values of β in the range $0 < \beta < 1$. As a result, the fractional-order derivative β provides in-depth detail of the dynamical behaviour of the proposed model. We observed that the model shows integer order system is unstable behavior and while changing into fractional-order the model shows stable behavior. We also have seen that the solution of our considered model is unstable in the integer order system but stable in the fractional order system.Thus, one can conclude that due to memory effect the fractional -order derivative can stabilize the system. The future work will extend the three-species food web eco-epidemiological model to a four-species eco-epidemiological model, which consists of two prey and two predators, which are susceptible and infected by both prey and predator populations.

Use of Generative-AI tools declaration

This article was not created using Artifical Intelligence (AI) tools.

Conflict of interest

All authors declare that they have no conflicts of interest.

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