



Research article

# Global boundedness and large time behavior in a forager-exploiter model of parabolic-parabolic-elliptic type

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**Abstract:** This paper deals with the parabolic–parabolic–elliptic forager–exploiter model under homogeneous Neumann boundary conditions. It is shown that if the taxis effects of exploiters are suitably weak, its classical solution is globally bounded in arbitrary dimensions. Moreover, the foragers and exploiters will approach spatially homogeneous distributions in the large time limit.

**Keywords:** forager-exploiter model; boundedness; large time behavior

## 1. Introduction

Prey-taxis describes the nonrandom foraging behavior of predators, which move toward regions of higher prey density. It is known that prey-taxis stabilizes the predator–prey model and does not give rise to any pattern formation [1]. However, it is important to note that this conclusion may not apply in the context of multispecies coexistence. In [2], Tania et al. proposed a foraging model involving a mixedspecies group, known as a forager–exploiter model, and demonstrated that taxis within forager–exploiter groups can result in pattern formation. The model reads as follows:

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla w), \\ v_t = \Delta v - \xi \nabla \cdot (v \nabla u), \\ w_t = \Delta w - \lambda(u + v)w - \mu w + r. \end{cases} \quad (1.1)$$

Here,  $u$  is the population density of foragers,  $v$  stands for the population density of exploiters, and  $w$  denotes the concentration of food. The constants  $\chi$ ,  $\xi$ ,  $\lambda$ , and  $\mu$  are assumed to be positive. The nonnegative function  $r$  represents a renewable resource.

For model (1.1), we collect some relevant results here. In the one-dimensional setting, globally bounded classical solutions of model (1.1) are established, which stabilize to the spatially uniform

state [3]. Subsequently, for the multidimensional cases, the global boundedness of classical solutions of (1.1) is obtained under the smallness conditions on the initial data and production rate (or weak taxis effect) [4,5]. Under some explicit conditions linking  $w_0$  and  $r$ , Winkler [6] investigated the global existence and large time behavior of generalized solutions. In addition, through modifications to (1.1), there are many research findings, such as model (1.1), with the logistic source [5, 7–13], nonlinear resource consumption [14–18], nonlinear diffusion [19–21], singular sensitivities [22], gradient-dependent flux limitation [23, 24], limited saturation [25], volume-filling [26], or competitive kinetics [27]. We also note that Tao and Winkler [28] established the global boundedness and large time behavior of classical solutions for the elliptic–parabolic–parabolic version of model (1.1).

When the  $v$ -species is disregarded (i.e.,  $v \equiv 0$ ), model (1.1) simplifies to a prey(nutrient)-taxis model,

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla w), \\ w_t = \Delta w - \lambda u w - \mu w + r. \end{cases} \quad (1.2)$$

In the case  $\mu = r = 0$ , for one-dimensional and two-dimensional spaces, the model (1.2) admits globally bounded classical solutions, whereas in three dimensions, it has global weak solutions that will become classical solutions after a waiting time [29]. For higher dimensions, the global boundedness of its classical solutions can be established by adding a logistic source [30], nonlinear chemotaxis function [31], sufficiently small initial value  $\|w_0\|_{L^\infty}$  [32,33], and so on. Moreover, Tao and Winkler [34] considered the parabolic–elliptic version of (1.2), and they showed that the classical solution exists globally and is also uniformly bounded in arbitrary dimensions in the absence of any constraints on the initial data or taxis coefficient.

Motivated by the parabolic–elliptic version of (1.2) in the work of Tao et al. [34], we consider the parabolic–parabolic–elliptic version of the forager–exploiter model

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla w), & x \in \Omega, t > 0, \\ v_t = \Delta v - \xi \nabla \cdot (v \nabla u), & x \in \Omega, t > 0, \\ 0 = \Delta w - \lambda(u + v)w - \mu w + r(x, t), & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x), v(x, 0) = v_0(x), & x \in \Omega, \end{cases} \quad (1.3)$$

where  $\Omega \subset \mathbb{R}^n (n \geq 1)$  is a bounded domain with smooth boundary  $\partial\Omega$ . The initial data satisfy

$$(u_0, v_0) \in [W^{2,\infty}(\Omega)]^2, u_0, v_0 \geq 0, \neq 0, \quad (1.4)$$

and  $r(x, t) \geq 0$  is supposed to satisfy

$$r(x, t) \in C^1(\bar{\Omega} \times [0, \infty)) \cap L^\infty(\Omega \times (0, \infty)) \quad \text{and} \quad r_* := \|r(x, t)\|_{L^\infty(\Omega \times (0, \infty))}. \quad (1.5)$$

Next, we outline the main challenges and ideas for the present paper. To begin, we provide the following abbreviations for conciseness:  $\int_\Omega \psi = \int_\Omega \psi(x) dx$ ,  $\int_{t_1}^{t_2} \int_\Omega \psi = \int_{t_1}^{t_2} \int_\Omega \psi(x, s) dx ds$  and  $\|\psi\|_{L^p} = \|\psi\|_{L^p(\Omega)}$ .

Compared with the model (1.2), the taxis mechanism in model (1.3) is cascaded, which complicates the analysis and leads to challenges in proving the global boundedness of solutions via energy function

construction in higher dimensions. Inspired by research [5], based on the local existence theory, we suppose that  $\|v\|_{L^\infty}$  is locally bounded. By means of energy estimates and Moser iteration, we obtain the global estimate of  $\|u\|_{L^\infty}$ . Then, leveraging the regularity theory of elliptic equations and the maximal Sobolev regularity theory, we derive higher-order estimates for  $w$ , thereby establishing the global boundedness of solutions for sufficiently small  $\xi$  through a series of energy estimates alongside the Neumann heat semigroup theory. Moreover, the large time behavior of solutions to (1.3) is proved for suitably small  $\chi$  and  $\xi$ . Our main results are stated as follows.

**Theorem 1.1.** *Let  $\Omega \subset \mathbb{R}^n (n \geq 1)$  be a bounded domain with a smooth boundary, and let  $\chi, \xi, \lambda, \mu$  be positive constants. Suppose that (1.4) and (1.5) hold. If*

$$\xi < \frac{C}{(\|u_0\|_{W^{2,p}} + \|v_0\|_{L^\infty} + 1)^{p+1} \left( (\|u_0\|_{W^{2,p}} + \|v_0\|_{L^\infty} + 1)^p + 2^p \|v_0\|_{L^\infty}^p + 1 \right)} \quad \text{with } p > n, \tag{1.6}$$

where  $C := C(\chi, \lambda, \mu, r_*, n, p, \Omega) > 0$ , then the model (1.3) has a unique global classical solution

$$(u, v, w) \in [C^0(\bar{\Omega} \times [0, \infty)) \cap C^{2,1}(\bar{\Omega} \times (0, \infty))]^3$$

satisfying  $u, v, w > 0$  for all  $t > 0$ . Moreover, there exists a constant  $C > 0$  independent of  $t$  such that

$$\|u(\cdot, t)\|_{W^{1,\infty}} + \|v(\cdot, t)\|_{W^{1,\infty}} + \|w(\cdot, t)\|_{W^{1,\infty}} \leq C. \tag{1.7}$$

**Remark 1.1.** In the analysis of large time behavior of solutions, the upper bounds of  $\|u\|_{L^\infty}$ ,  $\|v\|_{L^\infty}$ , and  $\|w\|_{L^\infty}$  play a pivotal role. In fact, we point out that

$$\|u\|_{L^\infty} \leq C$$

and

$$\|v\|_{L^\infty} \leq 2\|v_0\|_{L^\infty}$$

as well as

$$\|w\|_{L^\infty} \leq M = \frac{r_*}{\mu},$$

where  $C := C(\|u_0\|_{W^{2,p}}, \|v_0\|_{L^\infty}, \chi, \lambda, \mu, r_*, n, p, \Omega) > 0$  with  $p > n$  (see Lemmas 3.1 and 3.3 and Eq (3.19)).

**Theorem 1.2.** *Assume the assumptions in Theorem 1.1 hold, and let  $r(x, t) \equiv r_0$  be a positive constant. If  $\chi, \xi$  satisfy*

$$\chi < \frac{r_0}{\lambda \|v\|_{L^\infty} \|w\|_{L^\infty}^2} \tag{1.8}$$

and

$$\xi^2 < \frac{3}{\|u\|_{L^\infty} \|v\|_{L^\infty}}, \tag{1.9}$$

then one can find  $C > 0$  and  $\alpha > 0$  independent of  $t$ , fulfilling

$$\|u(\cdot, t) - \bar{u}_0\|_{L^\infty} + \|v(\cdot, t) - \bar{v}_0\|_{L^\infty} + \|w(\cdot, t) - w_*\|_{L^\infty} \leq Ce^{-\alpha t} \text{ for all } t > 0,$$

where

$$(\bar{u}_0, \bar{v}_0, w_*) = \left( \frac{1}{|\Omega|} \int_{\Omega} u_0, \frac{1}{|\Omega|} \int_{\Omega} v_0, \frac{r_0}{\lambda(\bar{u}_0 + \bar{v}_0) + \mu} \right). \tag{1.10}$$

The remainder of this paper is organized as follows. In the next section, we collect some useful lemmas to prepare for proving the main results. In Section 3, we prove the global boundedness of the solution to model (1.3) when  $\xi$  is sufficiently small (Theorem 1.1), and Section 4 is dedicated to constructing the required Lyapunov functional, thereby completing the proof of Theorem 1.2.

## 2. Local existence and a priori estimates

In what follows, we shall use  $C$  and  $C_i$  ( $i = 1, 2, \dots$ ) to denote generic positive constants independent of  $t$ , which may vary in context. The existence and uniqueness of local solutions of (1.3) can be derived by the standard fixed point theory [34]. The positivity of solutions can be shown by the strong maximum principle.

**Lemma 2.1.** *Let  $\Omega \subset \mathbb{R}^n$  ( $n \geq 1$ ) be a bounded domain, and let  $\chi, \xi, \lambda, \mu > 0$ . Assume that (1.4) and (1.5) hold. Then, there exists  $T_{\max} \in (0, \infty]$  such that (1.3) has a classical solution*

$$(u, v, w) \in [C^0(\bar{\Omega} \times [0, T_{\max})) \cap C^{2,1}(\bar{\Omega} \times (0, T_{\max}))]^3$$

fulfilling  $u, v, w > 0$  in  $\bar{\Omega} \times (0, \infty)$ . Moreover, if  $T_{\max} < \infty$ , then

$$\limsup_{t \nearrow T_{\max}} (\|u(\cdot, t)\|_{W^{1,\infty}} + \|v(\cdot, t)\|_{W^{1,\infty}} + \|w(\cdot, t)\|_{W^{1,\infty}}) = \infty.$$

For subsequent use, we present several well-known  $L^p$ - $L^q$  estimates for the Neumann heat semigroup.

**Lemma 2.2.** ([35]) Let  $(e^{t\Delta})_{t \geq 0}$  be the Neumann heat semigroup in  $\Omega$ , and let  $\lambda_1 > 0$  denote the first nonzero eigenvalue of  $-\Delta$  in  $\Omega$  under Neumann boundary conditions. Then, for all  $t > 0$ , there exist some constants  $\gamma_i$  ( $i = 1, 2, 3, 4$ ) depending only on  $\Omega$  such that

(i) If  $1 \leq q \leq p \leq \infty$ , then

$$\|e^{t\Delta} z\|_{L^p} \leq \gamma_1 \left(1 + t^{-\frac{n}{2}(\frac{1}{q} - \frac{1}{p})}\right) \|z\|_{L^q}$$

for all  $z \in L^q(\Omega)$ .

(ii) If  $1 \leq q \leq p \leq \infty$ , then

$$\|\nabla e^{t\Delta} z\|_{L^p} \leq \gamma_2 \left(1 + t^{-\frac{1}{2} - \frac{n}{2}(\frac{1}{q} - \frac{1}{p})}\right) e^{-\lambda_1 t} \|z\|_{L^q}$$

for all  $z \in L^q(\Omega)$ .

(iii) If  $2 \leq p < \infty$ , then

$$\|\nabla e^{t\Delta} z\|_{L^p} \leq \gamma_3 e^{-\lambda_1 t} \|\nabla z\|_{L^p}$$

for all  $z \in W^{1,p}(\Omega)$ .

(iv) If  $1 < q \leq p \leq \infty$ , then

$$\|e^{t\Delta} \nabla \cdot z\|_{L^p} \leq \gamma_4 \left(1 + t^{-\frac{1}{2} - \frac{n}{2}(\frac{1}{q} - \frac{1}{p})}\right) e^{-\lambda_1 t} \|z\|_{L^q}$$

for all  $z \in (C_0^\infty(\Omega))^n$ .

The following is an auxiliary result to be utilized in subsequent sections.

**Lemma 2.3.** ([36]) Let  $T > 0$ ,  $\tau \in (0, T)$ ,  $a > 0$ , and  $b > 0$ , and assume that  $y : [0, T) \rightarrow [0, \infty)$  is continuous such that

$$y'(t) + ay(t) \leq h(t) \quad \text{for all } t \in (0, T)$$

with some nonnegative function  $h \in L^1_{loc}([0, T))$  fulfilling

$$\int_t^{t+\tau} h(s) \leq b \quad \text{for all } t \in [0, T - \tau).$$

Then,

$$y(t) \leq y(0) + 2b + \frac{b}{a} \quad \text{for all } t \in (0, T).$$

### 3. Global boundedness

Throughout this section, we introduce the following notations:

$$A = 2\|v_0\|_{L^\infty}, \quad M = \frac{r_*}{\mu}, \quad \Phi = \{\chi, \lambda, \mu, r_*, n, p, \Omega\},$$

$$I_0 = \|u_0\|_{W^{2,p}} + \|v_0\|_{L^\infty} + 1, \quad J_0 = I_0^p(M^{2p} + 1)(I_0^p + A^p + 1) \quad \text{with } p = 2q > n.$$

In addition, we define

$$\tilde{T} := \sup\{T \in (0, T_{\max}) : \|v(\cdot, t)\|_{L^\infty} \leq A \text{ for all } t \in (0, T)\}.$$

By virtue of the continuity of the solution, it is clear that  $\tilde{T} \in (0, T_{\max}]$  and

$$\sup_{t \in (0, \tilde{T})} \|v(\cdot, t)\|_{L^\infty} \leq A. \quad (3.1)$$

This is a bootstrap hypothesis to be closed in Lemma 3.6. For the later use, we denote  $\tau := \min\{\frac{\tilde{T}}{2}, 1\}$ .

**Lemma 3.1.** Let  $(u, v, w)$  be the solution of (1.3) obtained in Lemma 2.1. Then, for all  $t \in (0, \tilde{T})$ , we have

$$\|w\|_{L^\infty} \leq M. \quad (3.2)$$

*Proof.* It is easy to check that  $\underline{w} \equiv 0$  is a lower solution to the third equation in (1.3). Define  $\bar{w} \equiv M = r_*/\mu$ . Alongside the positivity of  $u$  and  $v$ , one has

$$\Delta \bar{w} - (\lambda(u + v) + \mu)\bar{w} + r(x, t) = -(\lambda(u + v) + \mu)\frac{r_*}{\mu} + r(x, t) \leq -r_* + r(x, t) \leq 0.$$

Then,  $M$  is an upper solution for the equation. Applying the comparison principle, we obtain  $0 \leq w \leq M$ , which implies (3.2).

**Lemma 3.2.** Let  $(u, v, w)$  be the solution of (1.3) obtained in Lemma 2.1. Then, there exists  $C := C(\Phi) > 0$  such that

$$\|u\|_{L^1} \leq CI_0 \quad \text{for all } t \in (0, \tilde{T}). \quad (3.3)$$

*Proof.* Integrating the first equation in (1.3) directly yields (3.3).

Next, we derive the  $L^\infty$ -estimate for  $u$ .

**Lemma 3.3.** *Let  $(u, v, w)$  be the solution of (1.3) obtained in Lemma 2.1. There exists  $C = C(\Phi) > 0$  such that*

$$\|u\|_{L^\infty} \leq CI_0 \quad \text{for all } t \in (0, \tilde{T}). \tag{3.4}$$

*Proof.* From the first equation of (1.3), we can see that

$$\begin{aligned} \frac{1}{p} \frac{d}{dt} \int_{\Omega} u^p &= \int_{\Omega} u^{p-1} u_t \\ &= \int_{\Omega} u^{p-1} (\Delta u - \chi \nabla \cdot (u \nabla w)) \\ &= -(p-1) \int_{\Omega} u^{p-2} |\nabla u|^2 + \chi(p-1) \int_{\Omega} u^{p-1} \nabla u \cdot \nabla w \\ &= -(p-1) \int_{\Omega} u^{p-2} |\nabla u|^2 + \frac{\chi(p-1)}{p} \int_{\Omega} \nabla u^p \cdot \nabla w \\ &= -(p-1) \int_{\Omega} u^{p-2} |\nabla u|^2 - \frac{\chi(p-1)}{p} \int_{\Omega} u^p \Delta w \\ &\leq -(p-1) \int_{\Omega} u^{p-2} |\nabla u|^2 + \frac{\chi(p-1)}{p} \int_{\Omega} r u^p, \end{aligned}$$

which means

$$\frac{d}{dt} \int_{\Omega} u^p + \frac{4(p-1)}{p} \int_{\Omega} |\nabla u^{\frac{p}{2}}|^2 + \int_{\Omega} u^p \leq (\chi(p-1)r_* + 1) \int_{\Omega} u^p. \tag{3.5}$$

With the help of the Gagliardo–Nirenberg inequality, Young’s inequality, and (3.3), it follows that

$$\begin{aligned} (\chi(p-1)r_* + 1) \int_{\Omega} u^p &= (\chi(p-1)r_* + 1) \left\| u^{\frac{p}{2}} \right\|_{L^2}^2 \\ &\leq C_1 \left( \|\nabla u^{\frac{p}{2}}\|_{L^2}^{2\theta} \|u^{\frac{p}{2}}\|_{L^{\frac{p}{2}}}^{2(1-\theta)} + \|u^{\frac{p}{2}}\|_{L^{\frac{p}{2}}}^2 \right) \\ &= C_1 \left( \|\nabla u^{\frac{p}{2}}\|_{L^2}^{2\theta} \|u\|_{L^1}^{p(1-\theta)} + \|u\|_{L^1}^p \right) \\ &\leq \frac{2(p-1)}{p} \|\nabla u^{\frac{p}{2}}\|_{L^2}^2 + C_2 I_0^p, \end{aligned} \tag{3.6}$$

where  $C_1, C_2 > 0$  are constants depending on  $\Phi$  and  $\theta = (np - n)/(2 + np - n) \in (0, 1)$ . Substituting (3.6) into (3.5), we obtain that

$$\frac{d}{dt} \int_{\Omega} u^p + \frac{2(p-1)}{p} \int_{\Omega} |\nabla u^{\frac{p}{2}}|^2 + \int_{\Omega} u^p \leq C_2 I_0^p,$$

which, in conjunction with Grönwall’s inequality and the standard Moser iteration method [37], entails (3.4).

**Lemma 3.4.** *Let  $(u, v, w)$  be the solution of (1.3) obtained in Lemma 2.1. Then, there exists  $C = C(\Phi) > 0$  such that*

$$\|w\|_{W^{2,p}} \leq CJ_0^{\frac{1}{p}} \quad \text{for all } t \in (0, \tilde{T}) \tag{3.7}$$

and

$$\|w\|_{W^{1,\infty}} \leq C J_0^{\frac{1}{p}} \quad \text{for all } t \in (0, \tilde{T}) \quad (3.8)$$

as well as

$$\int_t^{t+\tau} \|\Delta u\|_{L^p}^p \leq C J_0^3 \quad \text{for all } t \in (0, \tilde{T} - \tau). \quad (3.9)$$

*Proof.* The standard elliptic regularity argument [38] implies that

$$\|w\|_{W^{2,p}}^p \leq C_1 \left( \|-\lambda(u+v)w - \mu w + r(x,t)\|_{L^p}^p + \|w\|_{L^p}^p \right),$$

which, together with (3.1), (3.2), and (3.4) yields

$$\begin{aligned} \|w\|_{W^{2,p}}^p &\leq C_2 M^p I_0^p + C_2 M^p A^p + C_2 M^p + C_2 \\ &\leq C_3 (M^p + 1)(I_0^p + A^p + 1), \end{aligned}$$

where  $C_i (i = 1, 2, 3) > 0$  depending on  $\Phi$ . This shows (3.7). Thanks to the Sobolev embedding theorem  $W^{2,p} \hookrightarrow W^{1,\infty}$  with  $p > n$  and (3.7), one can find  $C_4, C_5 > 0$  depending on  $\Phi$  to derive

$$\|w\|_{W^{1,\infty}} \leq C_4 \|w\|_{W^{2,p}} \leq C_5 J_0^{\frac{1}{p}},$$

which leads to (3.8). By leveraging the maximal Sobolev regularity of the Neumann heat semigroup  $(e^{t\Delta})_{t \geq 0}$  [39], we can use the ideas of Lemma 2.5 in [8] to get  $C_6 := C_6(\Phi) > 0$  such that

$$\int_t^{t+\tau} \|u\|_{W^{2,p}}^p \leq C_6 \|u_0\|_{W^{2,p}}^p + C_6 \chi \int_t^{t+\tau} \int_{\Omega} |\nabla \cdot (u \nabla w)|^p. \quad (3.10)$$

Using Hölder's inequality alongside (3.4), (3.7), and (3.8), one can obtain

$$\begin{aligned} C_6 \chi \int_{\Omega} |\nabla \cdot (u \nabla w)|^p &= C_6 \chi \int_{\Omega} |\nabla u \cdot \nabla w + u \Delta w|^p \\ &\leq C_7 \int_{\Omega} |\nabla u \cdot \nabla w|^p + C_7 \int_{\Omega} |u \Delta w|^p \\ &\leq C_7 \|\nabla u\|_{L^{2p}}^p \|\nabla w\|_{L^{2p}}^p + C_7 \|u\|_{L^\infty}^p \|\Delta w\|_{L^p}^p \\ &\leq C_8 J_0 \|\nabla u\|_{L^{2p}}^p + C_8 I_0^p J_0, \end{aligned} \quad (3.11)$$

where  $C_7, C_8$  are positive constants depending on  $\Phi$ . We rely on the Gagliardo–Nirenberg inequality, the Young inequality, and (3.4) to find  $C_9, C_{10}, C_{11} > 0$  depending on  $\Phi$  such that

$$\begin{aligned} C_8 J_0 \|\nabla u\|_{L^{2p}}^p &\leq C_9 J_0 \left( \|\Delta u\|_{L^p}^{p\theta} \|u\|_{L^\infty}^{p(1-\theta)} + \|u\|_{L^\infty}^p \right) \\ &= C_9 J_0 \left( \|\Delta u\|_{L^p}^{\frac{p}{2}} \|u\|_{L^\infty}^{\frac{p}{2}} + \|u\|_{L^\infty}^p \right) \\ &\leq C_{10} I_0^{\frac{p}{2}} J_0 \|\Delta u\|_{L^p}^{\frac{p}{2}} + C_{10} I_0^p J_0 \\ &\leq \frac{1}{2} \|\Delta u\|_{L^p}^p + C_{11} J_0^3, \end{aligned} \quad (3.12)$$

where  $\theta = \frac{1}{2} \in (0, 1)$ . The combination of (3.10)–(3.12) indicates that

$$\int_t^{t+\tau} \|u\|_{W^{2,p}}^p \leq C_6 \|u_0\|_{W^{2,p}}^p + \frac{1}{2} \int_t^{t+\tau} \|\Delta u\|_{L^p}^p + C_{12} J_0^3,$$

which gives

$$\int_t^{t+\tau} \|u\|_{W^{2,p}}^p \leq C_{13} J_0^3,$$

where  $C_{12}, C_{13} > 0$  are constants depending on  $\Phi$ . This means (3.9).

Using Lemma 3.4 together with heat semigroup theory, we can derive the following lemma.

**Lemma 3.5.** *Let  $(u, v, w)$  be the solution of (1.3) obtained in Lemma 2.1. Then, there exists  $C = C(\Phi) > 0$  such that*

$$\|\nabla u\|_{L^\infty} \leq C I_0 (J_0^3 + 1) \quad \text{for all } t \in (0, \tilde{T}). \quad (3.13)$$

*Proof.* From the proof of Lemma 3.3 in [5] together with (3.4), (3.7), and (3.8), it follows that

$$\begin{aligned} \frac{1}{2q} \frac{d}{dt} \int_{\Omega} |\nabla u|^{2q} + \int_{\Omega} |\nabla u|^{2q} &\leq C_1 I_0^{2q} \int_{\Omega} |\nabla w|^{2(q+1)} + C_1 I_0^{2q} \int_{\Omega} |\Delta w|^{q+1} + C_1 I_0^{2q} \\ &\leq C_2 I_0^{2q} J_0^{\frac{2(q+1)}{2q}} + C_2 I_0^{2q} J_0^{\frac{q+1}{2q}} + C_2 I_0^{2q} \\ &\leq C_3 I_0^{2q} \left( J_0^{\frac{2(q+1)}{2q}} + 1 \right), \end{aligned}$$

where  $C_i (i = 1, 2, 3) > 0$  are constants depending on  $\Phi$ . This, alongside Lemma 2.3, entails

$$\|\nabla u\|_{L^{2q}} \leq C_4 I_0 (J_0^2 + 1), \quad (3.14)$$

where  $C_4 := C_4(\Phi) > 0$ . Next, thanks to a variation-of-constants representation, there holds

$$\nabla u(\cdot, t) = \nabla e^{t\Delta} u_0 - \chi \int_0^t \nabla e^{(t-s)\Delta} \nabla \cdot (u \nabla w) ds,$$

which, in conjunction with Lemma 2.2, the Hölder inequality, and the facts (3.4), (3.7), (3.8), and (3.14), indicates that

$$\begin{aligned} \|\nabla u\|_{L^\infty} &\leq \gamma_3 \|u_0\|_{W^{1,\infty}} + \chi \gamma_2 \int_0^t \left( 1 + (t-s)^{-\frac{1}{2} - \frac{n}{2p}} \right) e^{-\lambda_1(t-s)} \|\nabla u \cdot \nabla w\|_{L^p} ds \\ &\quad + \chi \gamma_2 \int_0^t \left( 1 + (t-s)^{-\frac{1}{2} - \frac{n}{2p}} \right) e^{-\lambda_1(t-s)} \|u \Delta w\|_{L^p} ds \\ &\leq \gamma_3 \|u_0\|_{W^{1,\infty}} + \frac{C_5 I_0 (J_0^3 + 1) \chi \gamma_2}{\lambda_1} \left( 1 + \lambda_1^{\frac{1}{2} + \frac{n}{2p}} \Gamma \left( \frac{1}{2} - \frac{n}{2p} \right) \right) \\ &\quad + \frac{C_5 I_0 (J_0 + 1) \chi \gamma_2}{\lambda_1} \left( 1 + \lambda_1^{\frac{1}{2} + \frac{n}{2p}} \Gamma \left( \frac{1}{2} - \frac{n}{2p} \right) \right) \\ &\leq C_6 I_0 (J_0^3 + 1), \end{aligned}$$

from which (3.13) follows.

We can now establish the  $L^\infty$ -estimate of  $v$  over the interval  $(0, \tilde{T})$ .

**Lemma 3.6.** *Let  $(u, v, w)$  be the solution of (1.3) obtained in Lemma 2.1. Then, there exists  $C = C(\Phi) > 0$  such that*

$$\|v\|_{L^\infty} \leq \|v_0\|_{L^\infty} + C\xi AI_0(J_0^3 + 1) \quad \text{for all } t \in (0, \tilde{T}). \tag{3.15}$$

*Proof.* Applying the variation-of-constants formula to the second equation of model (1.3), we can show that

$$v(\cdot, t) = e^{t\Delta}v_0 - \xi \int_0^t e^{(t-s)\Delta} \nabla \cdot (v\nabla u) ds,$$

which, combined with Lemma 2.2, leads to

$$\begin{aligned} \|v\|_{L^\infty} &\leq \|e^{t\Delta}v_0\|_{L^\infty} + \xi \int_0^t \|e^{(t-s)\Delta} \nabla \cdot (v\nabla u)\|_{L^\infty} ds \\ &\leq \|v_0\|_{L^\infty} + \xi\gamma_4 \int_0^t \left(1 + (t-s)^{-\frac{1}{2} - \frac{n}{2p}}\right) e^{-\lambda_1(t-s)} \|v\nabla u\|_{L^p} ds. \end{aligned} \tag{3.16}$$

Thanks to Hölder’s inequality, in conjunction with (3.1) and (3.13), one can find  $C_1 := C_1(\Phi) > 0$  satisfying

$$\|v\nabla u\|_{L^p} \leq \|v\|_{L^\infty} \|\nabla u\|_{L^\infty} |\Omega|^{\frac{1}{p}} \leq C_1 AI_0(J_0^3 + 1). \tag{3.17}$$

Inserting (3.17) into (3.16), together with the fact that  $0 < 1/2 + n/2p < 1$  due to  $p > n$ , we can find a positive constant  $C_2 := C_2(\Phi)$  such that

$$\|v\|_{L^\infty} \leq \|v_0\|_{L^\infty} + C_2\xi AI_0(J_0^3 + 1),$$

which yields (3.15).

**Proof of Theorem 1.1.** From Lemma 3.6, it follows that there exists a positive constant  $C = C(\Phi) > 0$  such that for any  $\xi > 0$ ,

$$\|v(\cdot, t)\|_{L^\infty} \leq \|v_0\|_{L^\infty} + C(\Phi)\xi AI_0(J_0^3 + 1) \quad \text{for all } t \in (0, \tilde{T}). \tag{3.18}$$

It should be noted that  $C(\Phi)$  is independent of  $\xi$ . Consequently, when  $\xi \leq \|v_0\|_{L^\infty} / 2C(\Phi)AI_0(J_0^3 + 1)$ , that is,  $\xi$  satisfies (1.6), (3.18) implies that  $\|v(\cdot, t)\|_{L^\infty} \leq 3\|v_0\|_{L^\infty} / 2 < A$  for all  $t \in (0, \tilde{T})$ . By virtue of the definition of  $\tilde{T}$ , we thus conclude that  $\tilde{T} = T_{\max}$ , and

$$\|v(\cdot, t)\|_{L^\infty} \leq A \quad \text{for all } t \in (0, T_{\max}). \tag{3.19}$$

Furthermore, Lemmas 3.1–3.6 hold with  $\tilde{T}$  being replaced by  $T_{\max}$ . Similar to Lemma 3.5, using (3.9) and (3.13), we can show that  $\|\nabla v\|_{L^\infty} < C(\Phi, \xi, A, I_0, J_0, \|v_0\|_{W^{1,p}})$  for all  $t \in (0, T_{\max})$ . Combining this with (3.2), (3.4), (3.8), (3.13), and (3.19), we can derive the following inequality:

$$\|u(\cdot, t)\|_{W^{1,\infty}} + \|v(\cdot, t)\|_{W^{1,\infty}} + \|w(\cdot, t)\|_{W^{1,\infty}} \leq C \quad \text{for all } t \in (0, T_{\max}),$$

where  $C$  is a positive constant independent of  $t$ . By virtue of Lemma 2.1, it follows that  $T_{\max} = \infty$ . The proof is completed.

### 4. Large time behavior

In this section, we analyze the large time behavior of solutions by constructing appropriate Lyapunov functionals together with Barbălat’s lemma as stated below.

**Lemma 4.1.** ([40]) If  $f(t)$  ( $t \in (1, \infty)$ ) is a uniformly continuous nonnegative function and satisfies

$$\int_1^\infty f(t)dt < \infty,$$

then  $f(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

In addition, higher regularity properties of the solution are needed, which are stated as follows.

**Lemma 4.2.** If  $(u, v, w)$  is a global bounded classical solution of (1.3), and the initial data  $(u_0, v_0)$  satisfies (1.4), then there exist  $\theta \in (0, 1)$  and  $C > 0$  independent of  $t$  such that

$$\|u\|_{C^{2+\theta, 1+\frac{\theta}{2}}(\bar{\Omega} \times [t, t+1])} + \|v\|_{C^{2+\theta, 1+\frac{\theta}{2}}(\bar{\Omega} \times [t, t+1])} + \|w\|_{C^{2+\theta, \frac{\theta}{2}}(\bar{\Omega} \times [t, t+1])} \leq C$$

for all  $t > 1$ .

*Proof.* The conclusion is a consequence of Theorem 1.1, and the claim can be obtained via standard parabolic Schauder estimates [41] applied on sliding time intervals, using the uniform  $W^{1,\infty}$  bounds (see Theorem 1.1).

**Lemma 4.3.** Let the assumptions of Theorem 1.1 hold, and suppose that  $r(x, t) \equiv r_0$  is a positive constant. Assume (1.8) and (1.9) hold, and  $(\bar{u}_0, \bar{v}_0, w_*)$  is given by (1.10). Then, there exists  $\delta > 0$  such that the functions  $\mathcal{E}_1(t)$  and  $\mathcal{E}_2(t)$  defined by

$$\mathcal{E}_1(t) := \int_\Omega u \ln \frac{u}{\bar{u}_0} + \int_\Omega v \ln \frac{v}{\bar{v}_0}$$

and

$$\mathcal{E}_2(t) := \int_\Omega \frac{|\nabla u|^2}{u} + \int_\Omega \frac{|\nabla v|^2}{v}$$

satisfy

$$\mathcal{E}_1(t) \geq 0 \tag{4.1}$$

as well as

$$\frac{d}{dt} \mathcal{E}_1(t) + \delta \mathcal{E}_2(t) \leq 0 \tag{4.2}$$

for all  $t > 0$ .

*Proof.* Using Taylor’s formula, we can derive (4.1). By direct calculation, one has

$$\begin{aligned} \frac{d}{dt} \mathcal{E}_1(t) &= \frac{d}{dt} \int_\Omega u \ln \frac{u}{\bar{u}_0} + \frac{d}{dt} \int_\Omega v \ln \frac{v}{\bar{v}_0} \\ &= \frac{d}{dt} \int_\Omega u \ln u + \frac{d}{dt} \int_\Omega v \ln v \\ &= - \int_\Omega \frac{|\nabla u|^2}{u} + \chi \int_\Omega \nabla u \cdot \nabla w - \int_\Omega \frac{|\nabla v|^2}{v} + \xi \int_\Omega \nabla u \cdot \nabla v. \end{aligned} \tag{4.3}$$

Multiplying both sides of the third equation in (1.3) by  $\Delta w/w$  and integrating by parts, we have

$$0 = \int_{\Omega} \frac{|\Delta w|^2}{w} - \lambda \int_{\Omega} u \Delta w - \lambda \int_{\Omega} v \Delta w + r_0 \int_{\Omega} \frac{|\nabla w|^2}{w^2},$$

which gives

$$\frac{\chi}{\lambda} \int_{\Omega} \frac{|\Delta w|^2}{w} + \frac{r_0 \chi}{\lambda} \int_{\Omega} \frac{|\nabla w|^2}{w^2} = -\chi \int_{\Omega} \nabla u \cdot \nabla w - \chi \int_{\Omega} \nabla v \cdot \nabla w. \quad (4.4)$$

Using Young's inequality, it follows that

$$\begin{aligned} -\chi \int_{\Omega} \nabla v \cdot \nabla w &\leq \frac{1}{4} \int_{\Omega} \frac{|\nabla v|^2}{v} + \chi^2 \int_{\Omega} v |\nabla w|^2 \\ &\leq \frac{1}{4} \int_{\Omega} \frac{|\nabla v|^2}{v} + \chi^2 \|v\|_{L^\infty} \|w\|_{L^\infty}^2 \int_{\Omega} \frac{|\nabla w|^2}{w^2}. \end{aligned} \quad (4.5)$$

Upon combining (4.3)–(4.5) and using (1.8), we therefore obtain that

$$\begin{aligned} \frac{d}{dt} \mathcal{E}_1(t) &\leq - \int_{\Omega} \frac{|\nabla u|^2}{u} - \frac{3}{4} \int_{\Omega} \frac{|\nabla u|^2}{u} + \xi \int_{\Omega} \frac{\nabla u \cdot \nabla v}{\sqrt{uv}} \sqrt{uv} \\ &:= - \int_{\Omega} X P X^T, \end{aligned} \quad (4.6)$$

function  $X$  is defined by

$$X := \begin{pmatrix} \frac{\nabla u}{\sqrt{u}} & \frac{\nabla v}{\sqrt{v}} \end{pmatrix},$$

and matrix  $P$  is given by

$$P := \begin{pmatrix} 1 & -\frac{\xi \sqrt{uv}}{2} \\ -\frac{\xi \sqrt{uv}}{2} & \frac{3}{4} \end{pmatrix}.$$

Because of (1.9), we have

$$|P| = \frac{3}{4} - \frac{\xi^2 uv}{4} \geq \frac{3}{4} - \frac{\xi^2 \|u\|_{L^\infty} \|v\|_{L^\infty}}{4} > 0;$$

thus,  $P$  is positive definite. For some  $\delta > 0$ , one knows that

$$X P X^T \geq \delta |X|^2.$$

This, together with (4.6), leads to (4.2).

Next, we analyze the convergence rate of the solution.

**Lemma 4.4.** *Let the assumptions of Theorem 1.1 hold, and suppose that  $r(x, t) \equiv r_0$  is a positive constant. Assume (1.8) and (1.9) hold, and  $(\bar{u}_0, \bar{v}_0, w_*)$  is given by (1.10). Then, there exist  $C > 0$  and  $\alpha > 0$  such that*

$$\|u(\cdot, t) - \bar{u}_0\|_{L^\infty} + \|v(\cdot, t) - \bar{v}_0\|_{L^\infty} + \|w(\cdot, t) - w_*\|_{L^\infty} \leq C e^{-\alpha t}, \text{ for all } t > 0. \quad (4.7)$$

*Proof.* From Lemma 4.3, we conclude

$$\frac{d}{dt}\mathcal{E}_1(t) + \delta\mathcal{E}_2(t) \leq 0.$$

Considering that  $\mathcal{E}_1(t)$  is nonnegative, integrating the above inequality over the interval  $(1, \infty)$  yields

$$\int_1^\infty \mathcal{E}_2(t)dt \leq \frac{\mathcal{E}_1(1)}{\delta} < \infty.$$

By the regularity of  $(u, v)$  (see Lemma 4.2), it follows that  $\mathcal{E}_2(t)$  is uniformly continuous in  $(1, \infty)$ . Using Lemma 4.1, we see that

$$\mathcal{E}_2(t) \rightarrow 0 \text{ as } t \rightarrow \infty.$$

By direct computation, one obtains that

$$\int_\Omega (|\nabla u|^2 + |\nabla v|^2) = \int_\Omega \left( \frac{|\nabla u|^2}{u} u + \frac{|\nabla v|^2}{v} v \right) \leq \max\{\|u\|_{L^\infty}, \|v\|_{L^\infty}\} \mathcal{E}_2(t),$$

which gives

$$\int_\Omega (|\nabla u|^2 + |\nabla v|^2) \rightarrow 0 \text{ as } t \rightarrow \infty.$$

Thanks to Poincaré's inequality, we can find  $C_1 > 0$  such that

$$\int_\Omega |u - \bar{u}_0|^2 + \int_\Omega |v - \bar{v}_0|^2 \leq C_1 \left( \int_\Omega |\nabla u|^2 + \int_\Omega |\nabla v|^2 \right)$$

and then

$$\|u - \bar{u}_0\|_{L^2} + \|v - \bar{v}_0\|_{L^2} \rightarrow 0 \text{ as } t \rightarrow \infty. \quad (4.8)$$

We integrate by parts in the third equation from (1.3) to infer that

$$\begin{aligned} 0 &= \int_\Omega (w - w_*)(\Delta w - \lambda(u + v)w - \mu w + r_0) \\ &= - \int_\Omega |\nabla w|^2 + \int_\Omega (w - w_*)(-\lambda(\bar{u}_0 + \bar{v}_0)w - \mu w + r_0) \\ &\quad - \lambda \int_\Omega w(u - \bar{u}_0)(w - w_*) - \lambda \int_\Omega w(v - \bar{v}_0)(w - w_*), \end{aligned}$$

where by the fact that  $r_0 = (\lambda\bar{u}_0 + \lambda\bar{v}_0 + \mu)w_*$  yields

$$\begin{aligned} 0 &= - \int_\Omega |\nabla w|^2 - (\lambda\bar{u}_0 + \lambda\bar{v}_0 + \mu) \int_\Omega (w - w_*)^2 \\ &\quad - \lambda \int_\Omega w(u - \bar{u}_0)(w - w_*) - \lambda \int_\Omega w(v - \bar{v}_0)(w - w_*) \\ &\leq - \int_\Omega |\nabla w|^2 - \frac{\lambda\bar{u}_0 + \lambda\bar{v}_0 + \mu}{2} \int_\Omega (w - w_*)^2 \\ &\quad + \frac{\lambda^2 \|w\|_{L^\infty}}{\lambda\bar{u}_0 + \lambda\bar{v}_0 + \mu} \int_\Omega (u - \bar{u}_0)^2 + \frac{\lambda^2 \|w\|_{L^\infty}}{\lambda\bar{u}_0 + \lambda\bar{v}_0 + \mu} \int_\Omega (v - \bar{v}_0)^2. \end{aligned}$$

Therefore, we have

$$\frac{\lambda\bar{u}_0 + \lambda\bar{v}_0 + \mu}{2} \int_{\Omega} (w - w_*)^2 \leq \frac{\lambda^2 \|w\|_{L^\infty}^2}{\lambda\bar{u}_0 + \lambda\bar{v}_0 + \mu} \int_{\Omega} ((u - \bar{u}_0)^2 + (v - \bar{v}_0)^2), \tag{4.9}$$

which, alongside (4.8), gives

$$\|w - w_*\|_{L^2} \rightarrow 0 \text{ as } t \rightarrow \infty. \tag{4.10}$$

The Gagliardo–Nirenberg inequality implies the existence of  $C_2, C_3, C_4 > 0$  such that

$$\|u - \bar{u}_0\|_{L^\infty} \leq C_2 \|u\|_{W^{1,\infty}}^{\frac{n}{n+2}} \|u - \bar{u}_0\|_{L^2}^{\frac{2}{n+2}}$$

and

$$\|v - \bar{v}_0\|_{L^\infty} \leq C_3 \|v\|_{W^{1,\infty}}^{\frac{n}{n+2}} \|v - \bar{v}_0\|_{L^2}^{\frac{2}{n+2}}$$

as well as

$$\|w - w_*\|_{L^\infty} \leq C_4 \|w\|_{W^{1,\infty}}^{\frac{n}{n+2}} \|w - w_*\|_{L^2}^{\frac{2}{n+2}}, \tag{4.11}$$

which, along with (1.7), (4.8), and (4.10) prove the following claim:

$$\|u - \bar{u}_0\|_{L^\infty} + \|v - \bar{v}_0\|_{L^\infty} + \|w - w_*\|_{L^\infty} \rightarrow 0 \text{ as } t \rightarrow \infty.$$

According to a logarithmic Sobolev inequality [42], there exists  $C_5 > 0$  satisfying

$$\int_{\Omega} u \ln \frac{u}{\bar{u}_0} + \int_{\Omega} v \ln \frac{v}{\bar{v}_0} \leq C_5 \left( \int_{\Omega} \frac{|\nabla u|^2}{u} + \int_{\Omega} \frac{|\nabla v|^2}{v} \right),$$

which, combined with (4.2), shows that

$$\frac{d}{dt} \mathcal{E}_1(t) \leq -\delta \mathcal{E}_2(t) \leq -\frac{\delta}{C_5} \mathcal{E}_1(t).$$

This means

$$\mathcal{E}_1(t) \leq \mathcal{E}_1(t_0) e^{-\frac{\delta(t-t_0)}{C_5}},$$

which, together with the Csiszár–Kullback inequality [43], entails

$$\|u - \bar{u}_0\|_{L^1}^2 + \|v - \bar{v}_0\|_{L^1}^2 \leq C_6 \mathcal{E}_1(t) \leq C_6 \mathcal{E}_1(t_0) e^{-\frac{\delta(t-t_0)}{C_5}} \tag{4.12}$$

with  $C_6 > 0$ . By means of the Gagliardo–Nirenberg inequality, we find  $C_7, C_8 > 0$  such that

$$\|u - \bar{u}_0\|_{L^\infty} \leq C_7 \|u\|_{W^{1,\infty}}^{\frac{n}{n+1}} \|u - \bar{u}_0\|_{L^1}^{\frac{1}{n+1}},$$

$$\|v - \bar{v}_0\|_{L^\infty} \leq C_8 \|v\|_{W^{1,\infty}}^{\frac{n}{n+1}} \|v - \bar{v}_0\|_{L^1}^{\frac{1}{n+1}}.$$

In view of (1.7) and (4.12), we therefore obtain

$$\|u - \bar{u}_0\|_{L^\infty} + \|v - \bar{v}_0\|_{L^\infty} \leq C_9 e^{-\rho t}, \tag{4.13}$$

which, in conjunction with (4.9) and (4.11), indicates that

$$\|w - w_*\|_{L^\infty} \leq C_{10} e^{-\rho t} \tag{4.14}$$

with some positive constants  $C_9, C_{10}$ , and  $\rho$ . The combination of (4.13) and (4.14) yields (4.7).

**Proof of Theorem 1.2.** Theorem 1.2 can be directly derived from Lemma 4.4.

## 5. Conclusions

This article aims to explore the dynamics of the parabolic–parabolic–elliptic forager–exploiter model (1.3). By means of local existence theory, energy estimates, and semigroup theory, we mainly establish that the solutions of (1.3) are globally bounded provided that the taxis coefficient of exploiters is sufficiently small. Moreover, by constructing a Lyapunov functional, we show that if the taxis coefficients of the foragers and exploiters are sufficiently small, then the solution  $(u, v, w)$  converges exponentially to the equilibrium point  $(\int_{\Omega} u_0/|\Omega|, \int_{\Omega} v_0/|\Omega|, r_0/(\lambda(\bar{u}_0 + \bar{v}_0) + \mu))$  in  $L^\infty$  as  $t \rightarrow \infty$ .

### Use of AI tools declaration

No generative AI tools were used in the preparation of this manuscript.

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### Conflict of interest

All authors declare no conflicts of interest in this paper.

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