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Research article

An improved multi-strategy beluga whale optimization for global optimization problems

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Abstract: This paper presents an improved beluga whale optimization (IBWO) algorithm, which is mainly used to solve global optimization problems and engineering problems. This improvement is proposed to solve the imbalance between exploration and exploitation and to solve the problem of insufficient convergence accuracy and speed of beluga whale optimization (BWO). In IBWO, we use a new group action strategy (GAS), which replaces the exploration phase in BWO. It was inspired by the group hunting behavior of beluga whales in nature. The GAS keeps individual belugas whales together, allowing them to hide together from the threat posed by their natural enemy, the tiger shark. It also enables the exchange of location information between individual belugas whales to enhance the balance between local and global lookups. On this basis, the dynamic pinhole imaging strategy (DPIS) and quadratic interpolation strategy (QIS) are added to improve the global optimization ability and search rate of IBWO and maintain diversity. In a comparison experiment, the performance of the optimization algorithm (IBWO) was tested by using CEC2017 and CEC2020 benchmark functions of different dimensions. Performance was analyzed by observing experimental data, convergence curves, and box graphs, and the results were tested using the Wilcoxon rank sum test. The results show that

IBWO has good optimization performance and robustness. Finally, the applicability of IBWO to practical engineering problems is verified by five engineering problems.

Keywords: beluga whale optimization; group action; dynamic pinhole imaging strategy; quadratic interpolation strategy; engineering problems

1. Introduction

Heuristic algorithms are a method of finding the best solution within an acceptable computational cost, but they may not necessarily guarantee obtaining feasible and optimal solutions. In most cases, heuristic algorithms cannot provide a detailed explanation of the approximation between the obtained solution and the optimal solution. The advantage of a heuristic algorithm is that in limited search space, the number of attempts is greatly reduced so that it can solve the problem quickly. However, failure is also possible, as this method can make wrong judgments. The meta-heuristic algorithm (MA) is an improvement of the heuristic algorithm. Its main structure is a local search algorithm and a random algorithm, which often contains random strategies. MA can perform global search to a certain extent and find the optimal or approximate optimal solution. The core of the meta-heuristic algorithm is exploration and exploitation. Continuously searching throughout the entire search space is called exploration. Because the optimal solution may appear randomly, each location is important and cannot be ignored. Exploitation refers to utilizing as much available and effective information as possible.

The MA is an iterative generation process that allows for as much balance between exploration and exploitation as possible through an intelligent combination of different concepts. In most cases, there is a certain correlation between the existing effective information and the optimal solution. The correlation between information is used to gradually adjust and search from the initial solution to the optimal solution. Meta-heuristic algorithms have been studied effectively in the real world in recent years. With the increasing complexity of optimization problems encountered in real life, the meta-heuristic algorithms stand out due to their powerful search capabilities. The meta-heuristic algorithms can find a reasonable and effective result in a relatively short time [1]. However, as the difficulty of the problem increases, the trend of using improvement strategies to improve the performance of existing algorithms is gradually increasing [2]. For example, Mohammad et al. [3] used grey wolf optimizer for engineering issues. Multi-trial vector-based differential evolution algorithm (MTDE) [4] is characterized by the use of a new multi-test vector method MTV. Three experimental vectors were designed to deal with different problems, and a winner allocation strategy was proposed. Zhao et al. [5] improved the marine predators algorithm through quasi position learning and Q-learning.

The beluga whale optimization (BWO) algorithm [6] is a recently proposed algorithm. The BWO simulates the natural habits of beluga whales. The inspiration for the exploration phase comes from the paired swimming behavior of beluga whales. The exploitation phase simulates the process of beluga whales preying on prey, and there is also a whale fall behavior used to balance the two stages. BWO is considered an effective meta heuristic algorithm. Therefore, it is considered feasible to improve the shortcomings of BWO. However, the BWO is unable to obtain better positions in the later stages of the algorithm and is plagued by local optima. In addition, there is a lack of population diversity and a lack of a well balanced relationship between exploration and exploitation. To address the shortcomings of the original BWO, we propose an improved beluga whale optimization algorithm

(IBWO) to address the aforementioned issues. The beluga whale is a social animal. Participating in predation is usually a coordinated action of several beluga whales. Therefore, we propose a group action strategy (GAS) to replace the formula of the BWO algorithm in the exploration stage. During the hunting process, some weak beluga whales may also become prey for their natural enemies, tiger sharks, so weak beluga whales will approach strong beluga whales for protection. This strategy enables the beluga whale to reduce the threat from its natural enemies and enables it to explore more unknown places. It improves the stability of the algorithm, enhances global search capabilities and effectively balances the relationship between exploration and exploitation. After that, by adding a dynamic pinhole imaging strategy (DPIS) and a quadratic interpolation strategy (QIS), the ability of the IBWO to jump out of local optima and its convergence ability are improved.

In the experimental section, we tested the specific performance of the IBWO by using different dimensions of CEC2017 and CEC2020 test functions. Then, we analyzed the specific data of the benchmark function, convergence curve and box plot, and we used the Wilcoxon rank sum test for testing. In addition, we also conducted specific research on the balance between exploration and exploitation of the IBWO. Finally, we selected five real-life common engineering problems to test the adaptability of the IBWO to practical problems.

This article has the following contributions:

- The IBWO has been improved using GAS, which enhances its global exploration ability and stability.
- The DPIS enhanced each individual beluga whale's searching ability and efficiency.
- Through the QIS, individual beluga whales are more likely to find positions close to the global optimal value, improving search accuracy and convergence performance.
- The IBWO was tested and compared with other excellent algorithms, proving that the IBWO has stronger performance.
- We selected five reliable engineering problems to test the IBWO to verify its reliability in engineering problems.
- A specialized analysis and discussion were conducted on the balance between exploration and exploitation in IBWO.

The organizational structure of this article is as follows: The second section introduces the relevant work. The third section briefly describes BWO. The fourth section describes the added strategies, GAS, DPIS and QIS, and it gives the algorithm flow chart and pseudo code. The specific performance of the IBWO was tested and analyzed from multiple aspects in Sections 5 and 6. The seventh section summarizes this article and briefly introduces future research directions.

2. Related work

MA are often inspired by animal populations, humans, physics, and evolution. So, we can divide MA into four categories: 1) algorithms based on animal populations, 2) algorithms based on humans, 3) algorithms based on physics and 4) algorithms based on evolution. This is shown in Figure 1. In the first category, the algorithm is mainly created by simulating the living habits of social animals. For example, particle swarm optimization (PSO) [7] is an imitation of the predatory behavior of birds, and gray wolf optimization (GWO) [8] simulates the social behavior of wolves and their prey. The whale optimization algorithm (WOA) [9] was inspired by the behavior of whale groups to catch food. Ant colony optimization (ACO) [10] seeks the path between the ant colony and food based on the ant's foraging behavior. In addition, there have been many excellent and novel swarm intelligence algorithms in recent years, such as the remote optimization algorithm (ROA) [11], moth flame optimization (MFO) [12], prairie dog optimization algorithm (PDO) [13] and sand cat swarm optimization (SCSO) [14]. In the second category are algorithms based on human behavior, such as human teaching behavior, learning behavior, social behavior, emotional behavior, management behavior, etc. According to these human behaviors, many scholars have proposed excellent algorithms. These include, teaching learning based optimization (TLBO) [15], colliding bodies optimization (CBO) [16], ideology algorithm (IA) [17], brain storm optimization algorithm (BSO) [18], imperialist competitive algorithm (ICA) [19], harmony search (HS) [20], group search optimizer (GSO) [21] and group teaching optimization algorithm (GTOA) [22]. The third kind of inspiration comes from the physical rules in the universe. Math formulas inspire sine cosine algorithm (SCA) [23]. Simulated annealing (SA) [24] is inspired by the principle of solid annealing, and it is a probabilistic algorithm. In addition, there are many excellent algorithms such as central force optimization (CFO) [25], multiple verse optimizer (MVO) [26], arithmetic optimization algorithm (AOA) [27], black hole algorithm (BH) [28], graphical search algorithm (GSA) [29] and small-world optimization algorithm (SWOA) [30]. In the last category, the evolution-based algorithms mainly realizes the overall progress of the population by simulating the evolutionary law of survival of the fittest (Darwin's law) in nature. Finally, it completes the finding of the optimal solution. Examples include, genetic programming (GP) [31], differential evolution (DE) [32], evolutionary programming (EP) [33], biogeography based optimizer (BBO) [34], genetic algorithm (GA) [35], evolutionary strategy (ES) [36] and virulence optimization algorithm (VOA) [37]. According to the NFL theorem [38], we can know that there is no single MA that can solve all problems. Therefore, improving the known MA has become a research hotspot at present. Because of the defects of the algorithms, different scholars have good solutions. For example, Wang et al. [39] used adaptive dynamic probability, sailfish optimizer (SFO) with Lévy flight and restart strategy to improve the algorithmic performance of ROA. Modified slime mould algorithm via levy flight (LF-SMA) [40] is also an excellent algorithm for using Lévy flight, solving the problem of SMA easily falling into local optima. Wu et al. [41] enabled the individual sand cat to seek a better position actively, enabling the algorithm to have stronger exploration ability. Multi-trial vector-based monkey king evolution algorithm (MMKE) [42] solves optimization problems in real life by adding the best historical test vector generator and random test vector generator. The same Multi-trial vector-based moth-flame optimization algorithm (MTV-MFO) [43] is also an excellent and novel algorithm for multi experimental vector problems. This algorithm uses the multi-trial vector (MTV) method to replace the motion strategy in MFO, which is an excellent and clever improvement and greatly improves the performance of the algorithm. In addition, a discrete moth-flame optimization algorithm for community detection (DMFO-CD) [44] is also an improved MFO algorithm for community detection, and the effect is also very significant. Yang et al. [45,46] improved the AOA algorithm and improved the exploration and exploitation ability of AOA. The improved optimization algorithm is also applicable to a number of fields: medicine [47], structural engineering [48] and clustering [49].

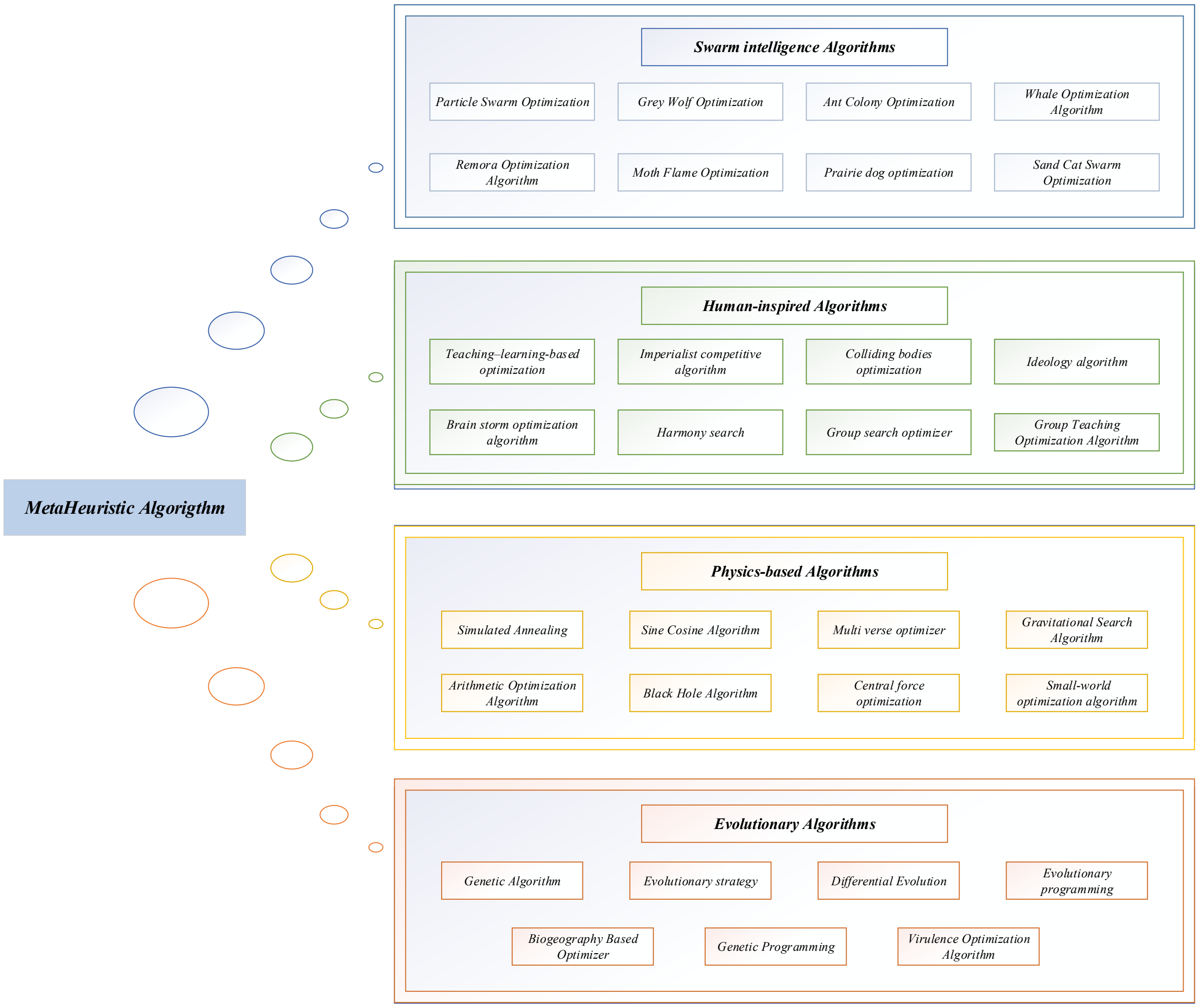


Figure 1. Classification of meta-heuristic optimization algorithms.

3. Beluga Whale optimization (BWO)

The beluga whale optimization algorithm is a meta-heuristic optimization algorithm proposed in 2022, which simulates the living habits of beluga whales in the ocean. Beluga whales are famous for the pure white of adult whales. They are highly social animals. They can gather in groups, with 2 to 25 members, with an average of 10 members. Like other MA methods, BWO includes exploration and exploitation stages. In addition, the algorithm also simulates the phenomenon of whale fall.

3.1. Initialization

According to the population mechanism of BWO, the beluga whale is regarded as a search agent, and the effect of moving in the search space is achieved by changing its position vector. The matrix of search agent location is:

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,dim} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,dim} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,dim} \end{bmatrix} \tag{1}$$

where, n is the number of search agent beluga whales, and dim is the dimension of the design variable. The fitness value of a beluga whale is expressed by Eq (2):

$$F_X = \begin{bmatrix} f(x_{1,1}, x_{1,2}, \ldots, x_{1,dim}) \\ f(x_{2,1}, x_{2,2}, \ldots, x_{2,dim}) \\ \vdots \\ f(x_{n,1}, x_{n,2}, \ldots, x_{n,dim}) \end{bmatrix} \tag{2}$$

The BWO algorithm can transition from the exploration stage to the exploitation stage smoothly. The balance factor, B_f, changes in each iteration:

$$B_f = B_0 \times (1 - \frac{t}{2T}) \tag{3}$$

where t is the current iteration, T is the maximum iteration number, and B_0 is a random number in the interval (0, 1). When the balance factor $B_f > 0.5$, it corresponds to the exploitation stage. The balance factor $B_f \leq 0.5$ corresponds to the exploitation stage. With the increasing number of iterations, the probability of the exploitation phase increases, while the probability of the exploration phase decreases.

3.2.1. Exploration phase

The swimming of the beluga whale determines the position update in the exploration phase of the BWO algorithm. The beluga's position update is as follows:

$$\begin{cases} X_{i,j}^{t+1} = X_{i,p_j}^t + (X_{r,pr}^t - X_{i,p_j}^t) \times (1 + r_1) \times sin(2\pi r_2), j = even \\ X_{i,j}^{t+1} = X_{i,p_j}^t + (X_{r,pr}^t - X_{i,p_j}^t) \times (1 + r_2) \times cos(2\pi r_2), j = odd \end{cases} \tag{4}$$

$X_{i,j}^{t+1}$ is the position of the i-th individual on the j-th dimension during the next iteration. P_j and P_r are randomly selected positive integers in [1, dim], and they are not equal. X_{i,p_j}^t and $X_{r,pr}^t$ represent the position of the i-th and r-th individuals under the current iteration, and r_1 and r_2 are random numbers in (0, 1). According to the dimension chosen by odd and even number, the updated position reflects the synchronous or mirror behaviors of beluga whale in swimming or diving.

3.2.2. Exploitation phase

Beluga whales can move and feed based on the location of nearby beluga whales. Beluga whales hunt by sharing information about their location with each other, so they need to consider how they relate to the best individual and other individuals. Assuming that beluga whales can use a Lévy flight strategy to capture prey, the specific formula is shown as:

$$X_i^{t+1} = r_3 X_{best}^t - r_4 X_i^t + C_1 \times L \times (X_r^t - X_i^t) \tag{5}$$

$$C_1 = 2r_4 \times (1 - \frac{t}{T}) \tag{6}$$

X_i^t and X_r^t represent the current positions of the i and r individuals in the current iteration. X_i^{t+1} is the new position of the i-th individual, X_{best}^t is the best position, and r_3 and r_4 are random numbers in (0, 1). C_1 is a random jump, which measures the intensity of the Lévy flight.

L is a random number consistent with the Lévy distribution, calculated by the following formula:

$$L = 0.05 \times \frac{u \times \sigma}{|v|^{1/\beta}} \tag{7}$$

$$\sigma = \left(\frac{\Gamma(1+\beta) \times sin(\pi\beta/2)}{\Gamma((1+\beta)/2) \times \beta \times 2^{(\beta-1)/2}}\right)^{1/\beta} \tag{8}$$

where u and v are random numbers obeying a normal distribution, and β is a constant set to 1.5.

3.2.3. *Whale fall*

The beluga whales either migrate elsewhere or die. In order to keep the population size constant, the positions of beluga whales and step size of whale fall are using to establish the updated position. The formula is as follows:

$$X_i^{t+1} = r_5 X_i^t - r_6 X_r^t + r_7 X_{step} \tag{9}$$

where r_5, r_6, and r_7 are random numbers in (0, 1), and X_{step} is the step size of whale fall established as:

$$X_{step} = (ub - lb)\, exp(-C_2 \frac{t}{T}) \tag{10}$$

C_2 is the step size factor, which is related to the probability of beluga whale fall and population size ($C_2 = 2W_f \times n$). ub and lb are the upper and lower bounds of variables, respectively. The probability of a beluga whale fall is calculated as a linear function, and the formula is as follows:

$$W_f = 0.1 - 0.05 \frac{t}{T} \tag{11}$$

The probability of whale fall decreases from the initial iteration to the end of the iteration, indicating that during the optimization process, when the beluga whale is closer to the food source, the risk of the beluga whale dying is lower.

3.3. The procedure of BWO

The pseudo-code of the BWO is given in Algorithm 1. The BWO mainly consists of three points: The exploration phase simulates beluga whale swimming, the exploitation phase simulates beluga whale hunting, and there is the final whale falling phase. During an iteration, the whale fall phase is performed when the exploration and exploitation phases are completed.

Algorithm 1. The pseudo-code of the BWO algorithm.

Initialization of parameters *T*, *Tmax*, *ub*, *lb*, *N*, *dim*.
Initialize population *X* according to Eq (1).
Calculate the fitness values of all individuals, and select the optimal solution G.
While T ≤ Tmax
  Obtain balance factor B_f by Eq (3) and probability of whale fall W_f by Eq (11).
    For $i = 1{:}N$
     If $B_f(i) > 0.5$
       The beluga whale exploration phase is achieved according to Eq (4).
     Else If $B_f(i) \leq 0.5$
       The beluga whale exploitation phase is achieved according to Eqs (5)–(8).
    End If
    For $i = 1{:}N$
      If $B_f(i) \leq W_f$
        The beluga whale fall is achieved according to Eqs (9)–(11).
        Determine whether the new location is within the boundary and calculate the fitness value based on its location.
      End If
    End For
  Find the current best solution *G*.
  $T = T + 1$
End While
Output the best solution *G*.

4. The proposed approach

4.1. Group action

Beluga whales are highly social animals. They hunt and migrate in groups. Beluga whale sounds are so diverse that they can be heard above water. They use many different clicks, chirps, and whistles (ranging from 3–9 kHz) to communicate with each other. But they also use a unique "bell tone", unique to this beluga whale. They can share their position with their unique voice and avoid killer whales. Because beluga whales eat a lot, they should not be too close to their companions when hunting. Otherwise, they will not get enough food. However, they should not be too far apart, or the natural enemy killer whale will threaten them.

In this paper, W_k is selected as the critical value for a beluga whale to meet its food needs and minimize the threat from natural enemies. The expression of W_k is shown in Eq (12). Meanwhile, as the number of iterations increases, individual beluga whales will gradually grow, which will also lead to an increase in W_k. W_t is the minimum distance between beluga whales, set as 0.35 in this paper. At the same time, we also assume that they can use a Lévy flight strategy to capture prey; the formula is as follows:

$$W_K = \frac{t}{5T} + W_t \tag{12}$$

$$X_i^{t+1} = X_i^t + L_F \cdot \left(X_{best}^t - W_K \cdot (X_r^t + X_i^t)\right) \tag{13}$$

Figures 2 and 3 are schematic diagrams of group action.

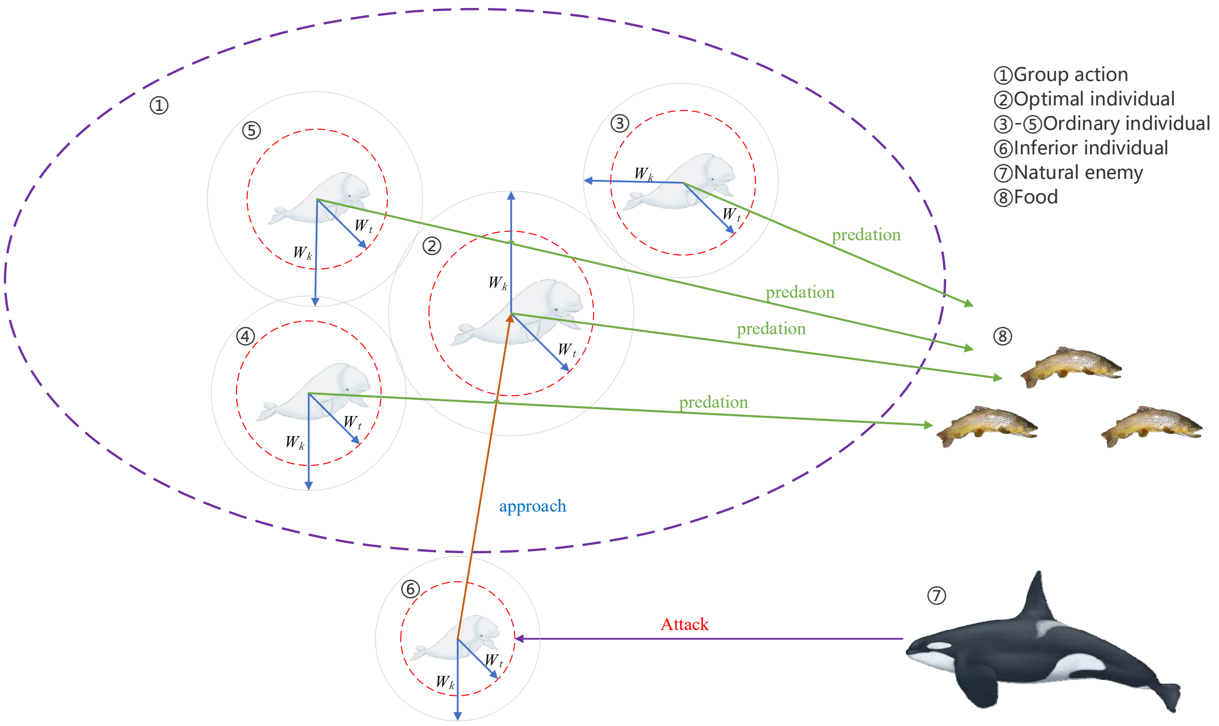


Figure 2. Before group action.

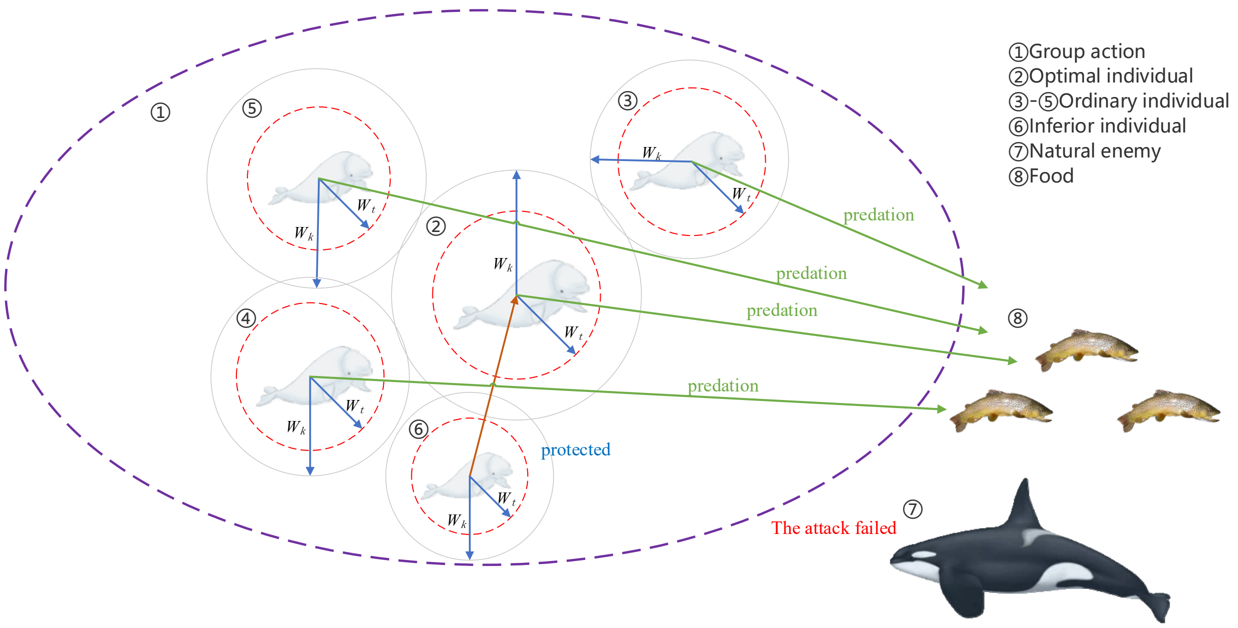


Figure 3. After group action.

4.2. Dynamic pinhole imaging strategy

In 2005, Tizhoosh proposed the opposition-based learning strategy [50], whose principle is to generate a reverse solution according to the known solution. Then, compare the reverse solution and choose a better solution. The dynamic pinhole imaging strategy [51] is a general strategy proposed by Li et al. in 2022 which imitates pinhole imaging theory in optics. Pinhole imaging theory is more accurate than the common opposition-based learning strategy and can produce more diversified corresponding points. Figure 4 shows a typical theoretical model of pinhole imaging. Applying it to the overall search space, the following mathematical model can be obtained:

$$\frac{X_{best}^{t}-(ub+lb)/2}{(ub+lb)/2-X_{i,j}^{t+1}}=\frac{L_p}{L_{-p}} \tag{14}$$

L_p and L_{-p} are the lengths of the virtual candle at the current optimal and the opposite positions, respectively. Notably, the location of the virtual candle in the ocean is also the location of the beluga whale. Still, the dots representing individual beluga whales do not have an effective length. Therefore, the ratio of two candles can be set to a variable K. From this, and we can get:

$$X_{i,j}^{t+1}=\frac{(K+1)\times(ub+lb)-2X_{best}^{t}}{2K} \tag{15}$$

When two virtual candles are the same length, this strategy morphs into a basic reverse learning strategy. Appropriately adjusting the value of K can change the position of the opposite points, giving individual beluga whales more opportunities to search. This paper uses the same value of 1.5×10^4 for K as in [51]. At each iteration, a new opposite point is generated near the center line of the search space. When the other side is in a better position, its position becomes a new boundary.

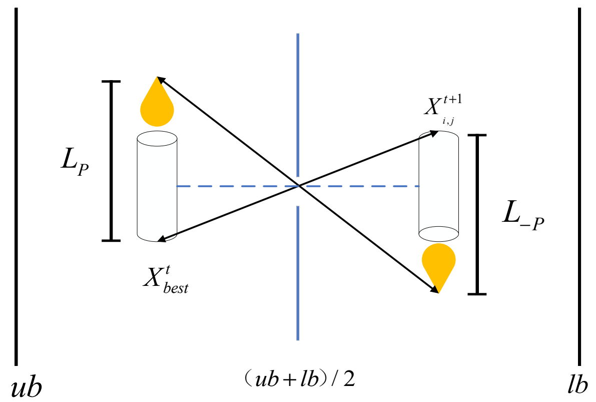


Figure 4. Dynamic pinhole imaging strategy.

4.3. Quadratic interpolation strategy

Quadratic interpolation [52] is a method used to search for extreme points in the determined initial interval and is a curve-fitting method. It uses the function information of multiple points on the objective function to form a low order polynomial similar to the objective function. Then, it uses this formula's optimal solution as the function's approximate optimal solution. With the gradual shortening of the interval, the distance between the optimal solution of the polynomial and the optimal advantage of the original function gradually decreases until it meets certain accuracy requirements. The principle is shown in Figure 5.

Use the function values of the objective function at three different points to form a quadratic polynomial $p(x)$ that is similar to the original function $f(x)$ and take the extreme point of the function $p(x)$ (i.e., the root of $p(x) = 0$) as the approximate extreme point of the objective function $f(x)$. Let the unimodal interval of the objective function be x_1, x_3, and the function values at x_1, x_2, x_3 are f_1, f_2 f_3, respectively.

$$\begin{cases} p(x_1) = a_0 + a_1 x_1 + a_2 x_1^2 = f_1 \\ p(x_2) = a_0 + a_1 x_2 + a_2 x_2^2 = f_2 \\ p(x_3) = a_0 + a_1 x_3 + a_2 x_3^2 = f_3 \end{cases} \tag{16}$$

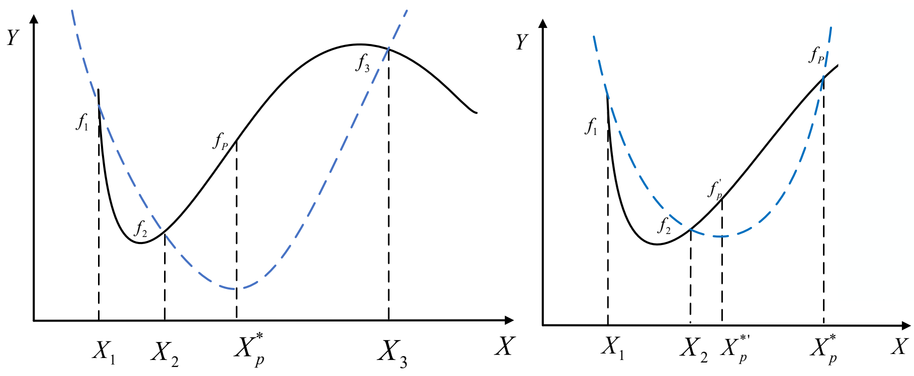
The extreme point of $p(x)$ is:

$$p(x)' = a_1 + 2a_2 x = 0 \tag{17}$$

Calculate the values of coefficients a1, a2, and a3 and substitute them into Eqs (16) and (17) to obtain.

$$x_p^* = \frac{(x_2^2-x_3^2)f_1+(x_3^2-x_1^2)f_2+(x_1^2-x_2^2)f_3}{2(x_2-x_3)f_1+(x_2-x_1)f_2+(x_1-x_2)f_3} \tag{18}$$

By substituting the result obtained from Eq (18) into the objective function $f(x)$, the function value is f_p. Suppose $f(x)$ itself is a quadratic function, and then the best advantage can be found by evaluating it according to Eq (18). If $f(x)$ is a function higher than quadratic or some other function, then we need to gradually reduce the interval. As shown in Figure 5, the black solid line is the function image of $f(x)$, while the blue dotted line is the function image of $p(x)$.



(a) First iteration. (b) Second iteration.

Figure 5. Quadratic interpolation strategy.

In this paper, when the position of the beluga whale is updated every time, the optimal individual X_{best}^t and two other random individuals X_{rl}^t and X_{rr}^t will be selected to form a low-order polynomial with a similar value to the objective function, and the solution $X_{i,j}^{t+1}$ will be obtained by using this formula. By substituting the above variables into Eq (18), the following Eq (19) can be obtained:

$$X_{i,j}^{t+1} = \frac{1}{2} \times \frac{[({X_{rl,j}^{t}}^{2} - X_{best}^{t}) f(X_{rr}^{t}) + (X_{best}^{t} - {X_{rr,j}^{t}}^{2}) f(X_{rl}^{t}) + ({X_{rr,j}^{t}}^{2} - {X_{rl,j}^{t}}^{2}) f(X_{best}^{t})]}{[(X_{rl,j}^{t} - X_{best}^{t}) f(X_{rr}^{t}) + (X_{best}^{t} - X_{rr,j}^{t}) f(X_{rl}^{t}) + (X_{rr,j}^{t} - X_{rl,j}^{t}) f(X_{best}^{t}) + eps]} \quad (19)$$

where, $f(X_{best}^{t})$, $f(X_{rl}^{t})$ and $f(X_{rr}^{t})$ represent the fitness values of X_{best}^{t}, X_{rl}^{t} and X_{rr}^{t} respectively. By using this method, the local search capability of the BWO is improved to a certain extent, and it also has a positive effect on the escape of the BWO from local optima in high dimensional space. It also enhances the search progress and search performance to a certain extent.

4.4. The proposed IBWO

Time complexity is also an important aspect of evaluating the algorithm's ability. The time complexity of the proposed algorithm depends on the number of beluga whales (*N*), the given dimension (*dim*), the number of iterations of the algorithm (*T*), the evaluation fitness value (*C*) and other factors. The specific analysis is as follows:

$$\begin{aligned} O(IBWO) = O(define\ parameters) + O(population\ initialization) + \\ O(function\ evaluation\ cost) + O(location\ update) \end{aligned} \quad (20)$$

1). It takes $O(1)$ to initialize the parameter.

2). Time of population initialization: $O(N \times dim)$.

3). Time required for position update of beluga whales after searching and preying on prey $O(T \times N \times dim)$, and function evaluation cost is $O(T \times N \times C)$.

4). Group action has replaced the original exploitation formula, so the time cost has not changed.

5). Dynamic pinhole imaging strategy location update time is $O(T \times N \times dim)$, and function evaluation cost is $O(T \times N \times C)$.

6). Quadratic interpolation strategy location update time is $O(T \times N \times dim)$, and function evaluation cost is $O(T \times N \times C)$.

7). Whale fall can be approximated as $O(0.1 \times T \times N \times dim)$, and function evaluation cost is $O(0.1 \times T \times N \times C)$.

8). The time cost of IBWO combined with the above introduction is

$$\begin{aligned} O(1 + N \times dim + T \times N \times dim + T \times N \times C + T \times N \times dim + T \times N \times C + T \times N \times dim + T \times \\ N \times C + 0.1 \times T \times N \times dim + 0.1 \times T \times N \times C) \end{aligned} \quad (21)$$

9). Since $1 << N \times dim$, $1 << T \times N \times dim$, $1 << T \times N \times C$, Eq (21) can be simplified to Eq (22).

$$O\left(N \times \left(dim \times \left(1 + 3.1 \times T \times \left(1 + \frac{C}{dim}\right)\right)\right)\right) \quad (22)$$

The time complexity of the original BWO is $O(N \times dim \times (1 + 1.1 \times T \times C))$. Although the complexity of IBWO is slightly increased compared with BWO algorithm, the optimization performance of IBWO is significantly improved compared with BWO algorithm.

4.5. Computational complexity of IBWO

By improving the exploration phase of BWO and combining two general strategies, an improved Beluga optimization algorithm (IBWO) is proposed. Through the above improvement strategies, IBWO can achieve a better balance between exploration and exploitation, and also improve the performance of the algorithm to a certain extent. Pseudo-code is shown in Algorithm 2. The flowchart is shown in Figure 6.

Algorithm 2. The pseudo-code of IBWO.

Initialization of parameters T, $Tmax$, ub, lb, N, dim, W_t.
Initialize population X according to Eq (1).
Calculate the fitness values of all individuals, and select the optimal solution G.
While $T \leq T_{max}$
  Obtain balance factor B_f by Eq (3) and probability of whale fall W_f by Eq (11).
    For $i = 1{:}N$
     If $B_f\,(i) > 0.5$
      The beluga Exploration phase is achieved according to Eq (4).
     Else If B_f (i) ≤ 0.5
      The beluga Exploitation phase is achieved according to Eqs (12) and (13).
    End If
    For $i = 1{:}N$
     If B_f (i) $\leq W_f$
        The beluga Whale fall is achieved according to Eqs (9)–(11).
        Determine whether the new location is within the boundary and calculate the fitness value based on its location.
      End If
    End For
    For $i = 1{:}N$
      Calculate with dynamic pinhole imaging strategy Eq (15)
    End For
    For $i = 1{:}N$
      Calculate with quadratic interpolation strategy Eq (19)
    End For
  Find the current best solution G
  $T = T + 1$
End While
Output the best solution G.

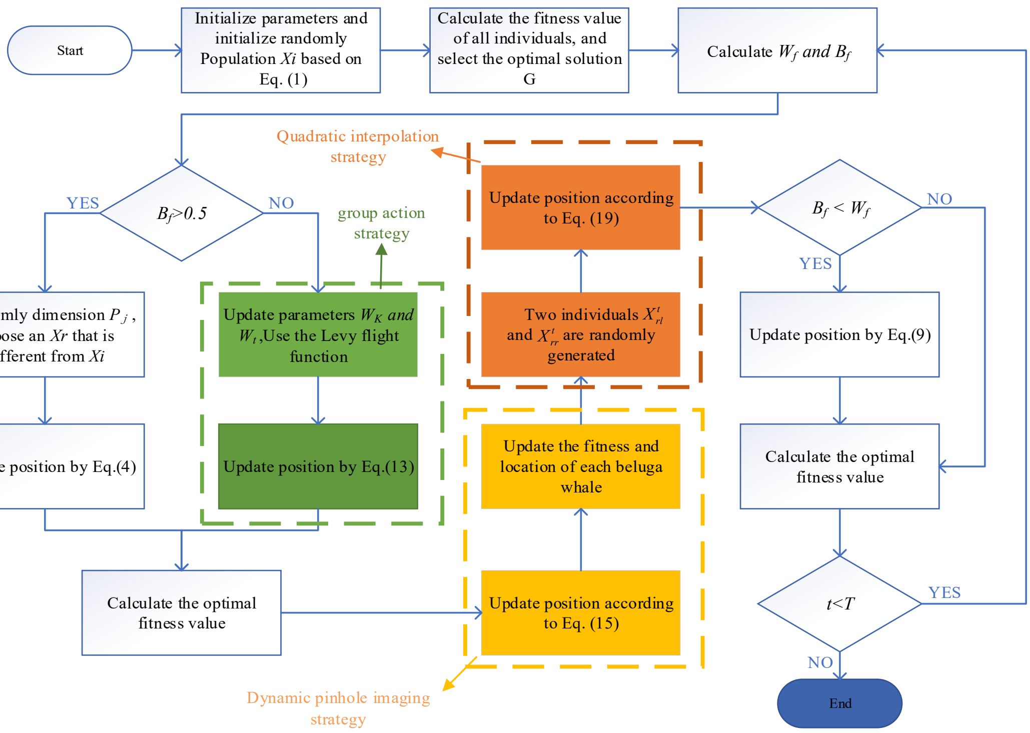


Figure 6. IBWO flowchart.

5. Analysis and discussion of experimental results

Table 1. Compare the specific parameter variables in the algorithm.

Algorithm	Parameter
IBWO	$W_t = 0.35$
	$K = 1.5*10^4$
	$W_f = [0.05,0.1]$
BWO [6]	$W_f = [0.05,0.1]$
ROA [11]	$C = 0.1$
AOA [27]	$MOP\_Max = 1$
	$MOP\_Min = 0.2$
	$A = 5$
	$Mu = 0.499$
HHO [53]	$\beta = 1.5$
BES [54]	$\alpha = [1.5, 2.0]$;
	$r = [0, 1]$
SCSO [14]	$SM = 2$
PDO [13]	$\rho = 0.1$
	$\varepsilon = 2.2204e\text{-}16$
	$\Delta = 0.005$

All the experiments in this paper were completed on a computer with an 11th Gen Intel(R) Core(TM) i7-11700 processor with a primary frequency of 2.50 GHz, 16 GB of memory, and the operating system 64-bit Windows 11 using MATLAB2021a. We used the CEC2017 benchmark function and CEC2020 benchmark function to test the actual performance of the algorithm. At the same time, 10 and 100 dimensions were used to test the specific performance of IBWO in different dimensions. Next, we used the BWO to compare with other six excellent algorithms and IBWO. The six algorithms are the simple and excellent remote optimization algorithm (ROA), analog mathematics arithmetic optimization algorithm (AOA), The famous harris hawks optimization (HHO) [53], bald eagle search optimization algorithm (BES) [54], sand cat optimization algorithm (SCSO) and prairie dog optimization algorithm (PDO), proposed in past two years. Table 1 shows the parameter design of these excellent comparison algorithms.

5.1. Experiments on the CEC2017 standard benchmark functions

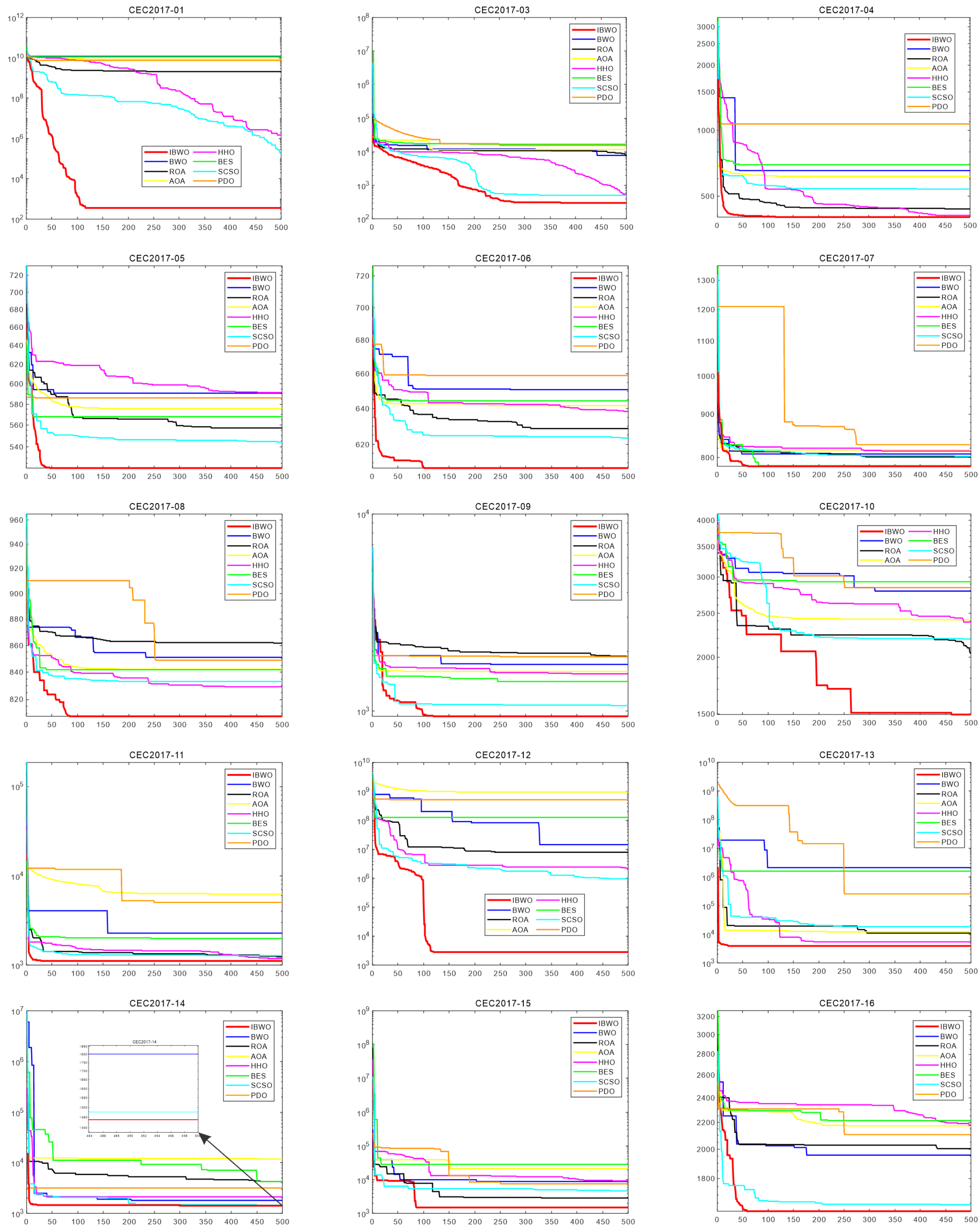
5.1.1. Detailed description of CEC2017 functions

IBWO was evaluated in this performance evaluation using 29 test functions from the CEC2017 benchmark [55] suite. It is worth noting that the test function CEC2017-02 did not participate in the test. The CEC2017 test function contains four types of test functions. Single-modal functions CEC2017-01 and CEC2017-03, multimodal functions (4–10), the relatively difficult mixing functions (11–20) and composition functions (21–30). The following experimental conclusions are based on the results obtained when dim = 10/100, N = 30, and T = 500 were run separately 30 times.

5.1.2. Analysis of the running results and convergence curves of CEC2017 for IBWO and comparison functions

The specific data of this experiment are shown in Table 2. On unimodal functions, the IBWO is excellent. Unimodal functions were mainly used to test the continuous convergence ability of the algorithm, and GAS can solve this problem well. In terms of multimodal functions, IBWO is still stronger than other comparison algorithms. This indicates that IBWO has good global performance, which depends on DPIS to provide many new and effective locations for IBWO. Compared with the difficult mixed functions and combination functions, IBWO can always find the global optimal value when dim = 10, and it is more stable and efficient than other comparison algorithms when dim = 100. Mixed functions and combined functions test whether the algorithm can maintain a balance between exploration and exploitation. According to the experimental results, it is clear that IBWO with GAS, DPIS and QIS strategies can maintain a good balance between them.

We also analyzed the convergence curves of IBWO. We observed that the IBWO has two performances on unimodal functions. The convergence curve is shown in Figures 7 and 8. On the unimodal function CEC2017-01, the effect of IBWO remains excellent with changes in dimensions. On CEC2017-03, the IBWO cannot solve this problem well with increasing dimensions. On multimodal functions, the IBWO performs differently in low and high dimensions. When the dimension is low, the IBWO can find the optimal value with very few iterations. Other comparative algorithms also perform well, and one cannot distinguish their differences well. When dim = 100, IBWO can continuously converge and optimize its performance far better than other comparative algorithms. This is because the QIS can provide effective and legal positions for the IBWO, enabling it to continuously approach the optimal solution. For difficult mixed and combined functions, IBWO can quickly find the optimal value when the dimension is low. When dim = 100, the IBWO can still converge well, which is not possessed by other algorithms. Also, other algorithms often encounter local optimization problems, and IBWO algorithm can effectively solve this problem. This is because the DPIS can provide new positions when the IBWO falls into local optima, allowing it to escape from local optima. In summary, we believe that the IBWO is reliable.



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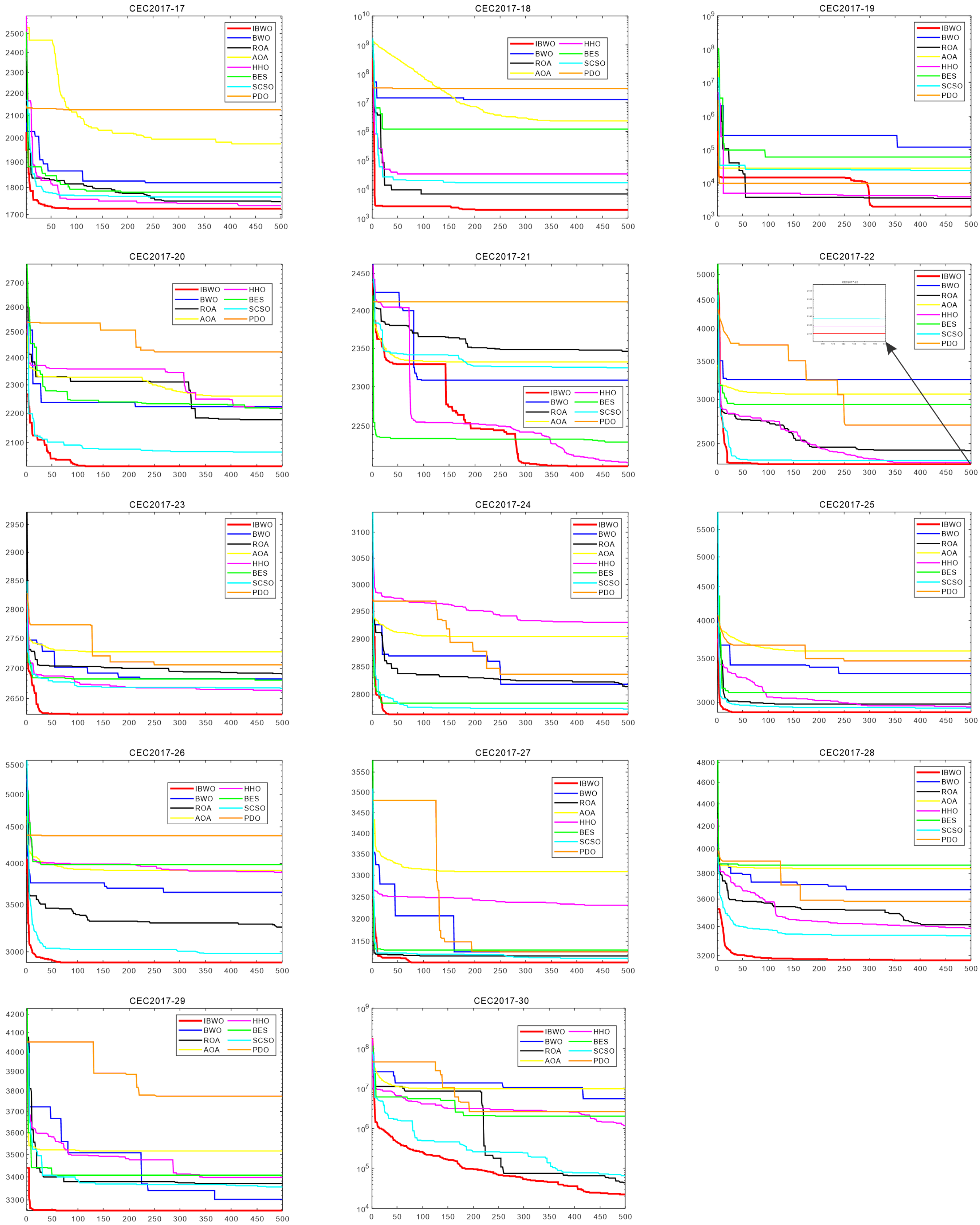
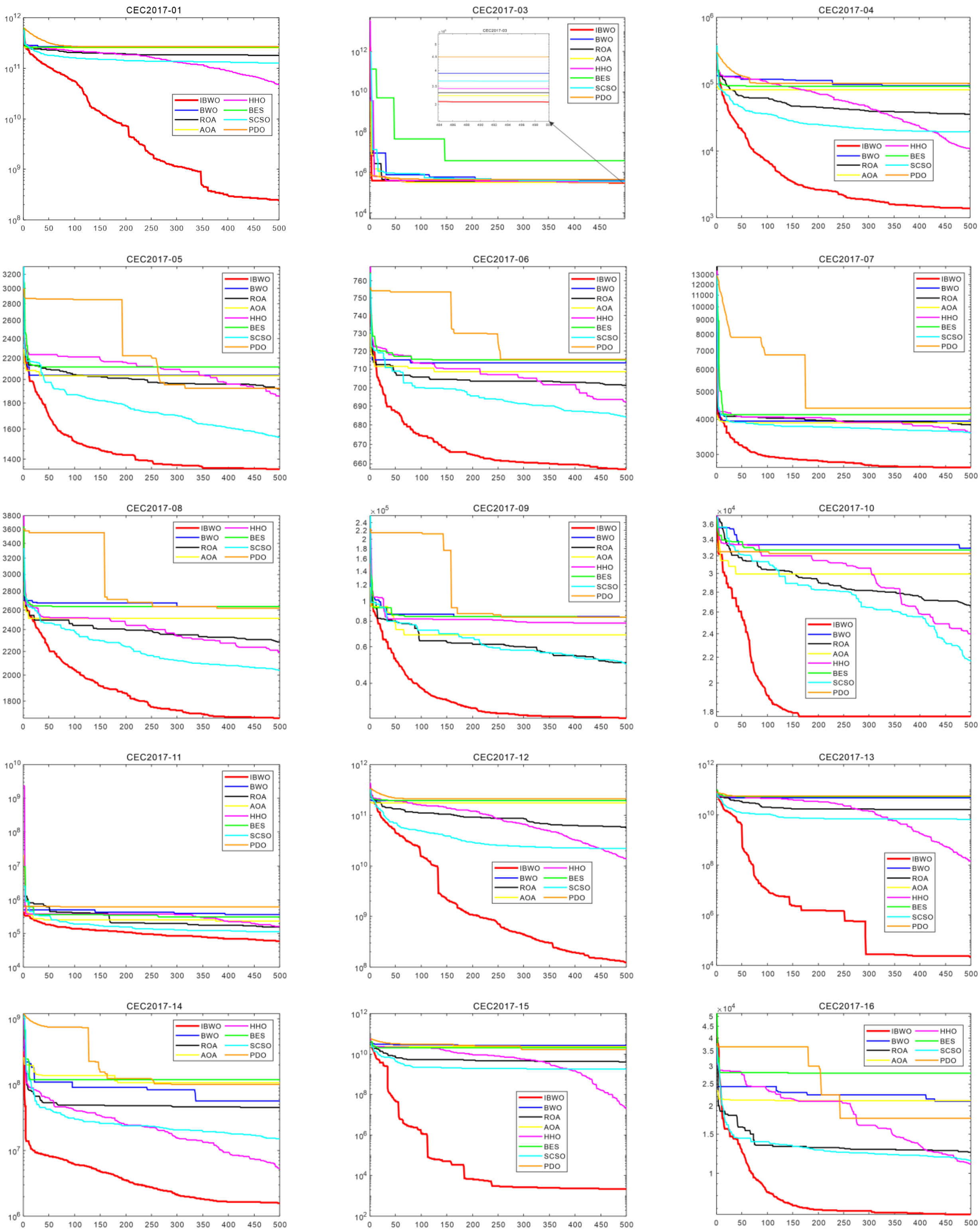


Figure 7. Convergence curves of IBWO and comparison algorithm on CEC2017 (dim = 10).



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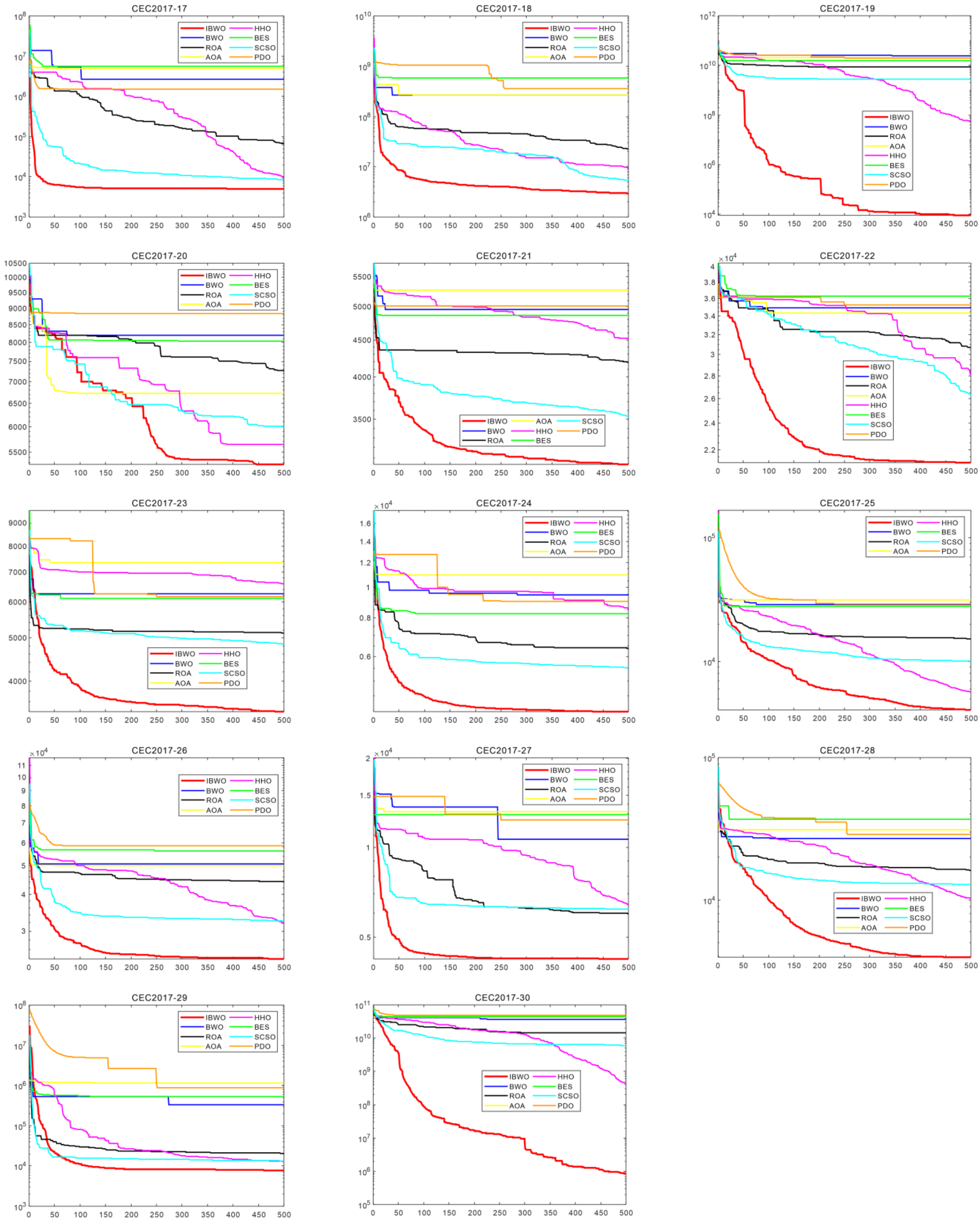

Figure 8. Convergence curves of IBWO and comparison algorithm on CEC2017 (dim = 100).

Table 2. The results of each algorithm benchmark test function in CEC2017.

CEC2017	dim	Metric	IBWO	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2017	10	min	**1.00E+02**	3.95E+09	1.88E+09	3.33E+09	3.52E+05	6.50E+08	2.32E+04	3.31E+09
_01		mean	**1.82E+03**	8.87E+09	5.96E+09	1.03E+10	2.22E+07	5.11E+09	2.43E+08	9.76E+09
		std	**1.74E+03**	2.73E+09	3.43E+09	4.93E+09	1.15E+08	3.73E+09	4.46E+08	3.79E+09
	100	min	**4.78E+08**	2.44E+11	1.42E+11	2.45E+11	3.68E+10	2.24E+11	6.91E+10	2.40E+11
		mean	**9.07E+09**	2.60E+11	1.74E+11	2.69E+11	4.92E+10	2.61E+11	1.04E+11	2.58E+11
		std	**5.83E+09**	6.49E+09	1.74E+10	1.19E+10	7.43E+09	1.75E+10	1.65E+10	7.97E+09
CEC2017	10	min	**3.00E+02**	7.20E+03	3.63E+03	7.85E+03	3.20E+02	1.10E+04	4.55E+02	7.45E+03
_03		mean	**3.00E+02**	9.65E+03	1.11E+04	1.28E+04	7.85E+02	2.41E+04	3.30E+03	1.32E+04
		std	**1.54E-03**	1.66E+03	4.90E+03	2.80E+03	5.51E+02	1.39E+04	2.68E+03	3.35E+03
	100	min	**2.84E+05**	3.05E+05	3.19E+05	3.29E+05	3.19E+05	3.61E+05	2.87E+05	4.15E+05
		mean	4.06E+05	4.32E+05	3.50E+05	3.76E+05	3.51E+05	6.25E+07	**3.32E+05**	7.31E+05
		std	8.20E+04	1.33E+05	**1.34E+04**	4.54E+04	2.86E+04	3.26E+08	9.12E+04	2.27E+05
CEC2017	10	min	**4.00E+02**	6.00E+02	4.81E+02	5.74E+02	4.01E+02	5.01E+02	4.03E+02	5.99E+02
_04		mean	**4.04E+02**	8.44E+02	8.03E+02	1.16E+03	4.38E+02	1.01E+03	4.34E+02	1.16E+03
		std	**2.55E+00**	1.44E+02	3.03E+02	7.42E+02	4.50E+01	4.50E+02	4.74E+01	4.88E+02
	100	min	**1.08E+03**	1.24E+03	2.63E+04	6.14E+04	6.27E+03	6.64E+04	8.12E+03	4.07E+04
		mean	**1.71E+03**	2.00E+03	3.83E+04	9.52E+04	1.01E+04	1.03E+05	1.58E+04	9.43E+04
		std	**3.46E+02**	1.16E+03	7.99E+03	2.03E+04	2.09E+03	1.78E+04	4.87E+03	1.67E+04
CEC2017	10	min	**5.04E+02**	5.77E+02	5.35E+02	5.37E+02	5.28E+02	5.48E+02	5.21E+02	5.63E+02
_05		mean	**5.22E+02**	5.93E+02	5.74E+02	5.68E+02	5.62E+02	5.82E+02	5.43E+02	5.93E+02
		std	**7.99E+00**	1.17E+01	2.66E+01	2.31E+01	2.52E+01	2.36E+01	1.55E+01	1.88E+01
	100	min	**1.08E+03**	2.09E+03	1.79E+03	1.95E+03	1.59E+03	2.06E+03	1.55E+03	1.84E+03
		mean	**1.28E+03**	2.13E+03	1.89E+03	2.07E+03	1.69E+03	2.13E+03	1.64E+03	2.15E+03
		std	6.98E+01	**2.28E+01**	5.66E+01	6.90E+01	5.58E+01	3.99E+01	6.93E+01	1.64E+02
CEC2017	10	min	**6.00E+02**	6.35E+02	6.20E+02	6.23E+02	6.23E+02	6.28E+02	6.02E+02	6.36E+02
_06		mean	**6.04E+02**	6.49E+02	6.45E+02	6.42E+02	6.42E+02	6.49E+02	6.22E+02	6.51E+02
		std	**3.45E+00**	7.80E+00	1.39E+01	1.00E+01	1.23E+01	1.10E+01	1.49E+01	9.69E+00
	100	min	**6.30E+02**	6.36E+02	6.90E+02	6.99E+02	6.84E+02	7.02E+02	6.75E+02	6.99E+02
		mean	**6.51E+02**	6.52E+02	6.99E+02	7.09E+02	6.92E+02	7.13E+02	6.86E+02	7.22E+02
		std	7.98E+00	9.72E+00	4.76E+00	4.77E+00	**4.63E+00**	4.70E+00	4.83E+00	1.09E+01
CEC2017	10	min	**7.17E+02**	7.89E+02	7.61E+02	7.78E+02	7.55E+02	7.75E+02	7.43E+02	7.82E+02
_07		mean	**7.63E+02**	8.06E+02	8.19E+02	8.05E+02	7.91E+02	8.11E+02	7.75E+02	8.18E+02
		std	2.40E+01	**8.42E+00**	2.75E+01	1.63E+01	2.12E+01	2.40E+01	2.18E+01	2.90E+01
	100	min	**1.64E+03**	3.76E+03	3.60E+03	3.87E+03	3.58E+03	3.74E+03	3.14E+03	3.62E+03
		mean	**2.48E+03**	3.90E+03	3.80E+03	4.00E+03	3.81E+03	3.99E+03	3.45E+03	4.00E+03
		std	4.11E+02	**6.45E+01**	8.85E+01	8.15E+01	1.20E+02	1.20E+02	1.53E+02	1.98E+02
CEC2017	10	min	**8.04E+02**	8.46E+02	8.28E+02	8.23E+02	8.16E+02	8.42E+02	8.08E+02	8.33E+02
_08		mean	**8.20E+02**	8.56E+02	8.58E+02	8.40E+02	8.32E+02	8.59E+02	8.31E+02	8.61E+02
		std	6.66E+00	**4.86E+00**	1.36E+01	1.18E+01	9.23E+00	9.47E+00	1.10E+01	1.40E+01
	100	min	**1.32E+03**	1.37E+03	2.18E+03	2.41E+03	2.06E+03	2.44E+03	1.93E+03	2.19E+03
		mean	**1.71E+03**	1.76E+03	2.34E+03	2.54E+03	2.17E+03	2.60E+03	2.12E+03	2.59E+03
		std	1.05E+02	1.14E+02	7.50E+01	8.00E+01	**5.54E+01**	6.54E+01	7.51E+01	1.90E+02

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CEC2017	dim	Metric	IBWO	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2017	10	min	**9.00E+02**	1.23E+03	1.21E+03	1.14E+03	1.02E+03	1.07E+03	9.65E+02	1.30E+03
_09		mean	**1.15E+03**	1.54E+03	1.64E+03	1.46E+03	1.58E+03	1.61E+03	1.17E+03	1.71E+03
		std	2.22E+02	**1.58E+02**	3.03E+02	2.31E+02	2.78E+02	3.48E+02	2.20E+02	3.35E+02
	100	min	**2.33E+04**	2.72E+04	4.91E+04	6.49E+04	6.00E+04	6.95E+04	3.75E+04	6.16E+04
		mean	**3.43E+04**	4.02E+04	6.16E+04	7.31E+04	6.92E+04	8.07E+04	4.98E+04	9.68E+04
		std	1.45E+04	1.76E+04	7.29E+03	5.64E+03	**5.12E+03**	5.32E+03	8.83E+03	1.69E+04
CEC2017	10	min	**1.00E+03**	2.20E+03	1.99E+03	1.86E+03	1.48E+03	1.76E+03	1.37E+03	2.17E+03
_10		mean	**1.42E+03**	2.58E+03	2.50E+03	2.35E+03	2.04E+03	2.60E+03	2.00E+03	2.60E+03
		std	**1.46E+02**	1.57E+02	2.89E+02	2.68E+02	3.36E+02	3.14E+02	3.99E+02	2.81E+02
	100	min	**1.36E+04**	1.52E+04	2.49E+04	2.91E+04	2.14E+04	3.02E+04	1.85E+04	3.06E+04
		mean	**1.95E+04**	2.08E+04	2.89E+04	3.16E+04	2.42E+04	3.25E+04	2.25E+04	3.28E+04
		std	5.80E+03	6.34E+03	2.20E+03	1.00E+03	1.52E+03	**9.41E+02**	1.65E+03	1.25E+03
CEC2017	10	min	**1.10E+03**	1.58E+03	1.21E+03	1.21E+03	1.12E+03	1.32E+03	1.12E+03	1.98E+03
_11		mean	**1.12E+03**	2.98E+03	2.60E+03	4.06E+03	1.22E+03	3.16E+03	1.20E+03	4.86E+03
		std	**1.22E+01**	1.01E+03	2.65E+03	3.03E+03	9.88E+01	3.32E+03	9.26E+01	2.83E+03
	100	min	**1.10E+04**	1.53E+04	1.24E+05	1.36E+05	7.91E+04	2.02E+05	6.51E+04	2.47E+05
		mean	**2.58E+04**	3.01E+04	1.66E+05	1.74E+05	1.37E+05	5.29E+05	9.53E+04	5.71E+05
		std	**7.46E+03**	1.12E+04	2.73E+04	2.10E+04	3.17E+04	3.51E+05	2.24E+04	1.87E+05
CEC2017	10	min	**1.26E+03**	8.94E+06	3.35E+06	1.55E+04	8.32E+04	1.54E+07	4.92E+04	4.38E+07
_12		mean	**8.19E+03**	9.08E+07	8.34E+07	3.96E+08	4.97E+06	1.95E+08	2.26E+06	3.22E+08
		std	**5.41E+03**	7.78E+07	2.08E+08	4.36E+08	4.86E+06	2.25E+08	2.49E+06	2.62E+08
	100	min	**1.04E+08**	2.40E+08	4.18E+10	1.39E+11	6.32E+09	1.38E+11	1.77E+10	1.56E+11
		mean	**1.07E+09**	1.60E+09	8.44E+10	1.85E+11	1.17E+10	1.93E+11	4.04E+10	2.00E+11
		std	**2.05E+09**	2.92E+09	2.23E+10	2.74E+10	3.96E+09	2.43E+10	1.46E+10	1.97E+10
CEC2017	10	min	**1.31E+03**	4.94E+04	2.99E+03	3.71E+03	3.35E+03	1.69E+04	3.44E+03	6.80E+04
_13		mean	**6.16E+03**	2.62E+06	1.97E+04	1.42E+04	1.88E+04	1.24E+07	1.59E+04	4.57E+06
		std	**4.66E+03**	2.86E+06	1.55E+04	1.08E+04	1.96E+04	3.29E+07	1.13E+04	5.92E+06
	100	min	**5.45E+03**	7.39E+03	9.24E+09	2.83E+10	9.46E+07	2.80E+10	1.35E+09	3.32E+10
		mean	**2.98E+06**	1.62E+08	1.90E+10	4.62E+10	3.13E+08	4.26E+10	5.80E+09	4.59E+10
		std	**1.10E+07**	6.47E+08	6.19E+09	7.49E+09	1.69E+08	8.45E+09	3.71E+09	5.01E+09
CEC2017	10	min	**1.40E+03**	1.56E+03	1.51E+03	1.51E+03	1.52E+03	1.59E+03	1.47E+03	1.67E+03
_14		mean	**1.48E+03**	1.79E+03	3.19E+03	9.57E+03	2.41E+03	3.71E+03	3.56E+03	1.00E+04
		std	**1.14E+02**	3.63E+02	1.60E+03	9.28E+03	1.47E+03	4.93E+03	1.92E+03	9.71E+03
	100	min	**2.52E+06**	4.18E+07	5.00E+06	3.29E+07	3.85E+06	2.64E+07	4.09E+06	5.56E+07
		mean	**4.86E+06**	8.26E+07	1.76E+07	1.12E+08	9.47E+06	9.50E+07	1.16E+07	2.14E+08
		std	**2.28E+06**	2.67E+07	7.96E+06	6.48E+07	3.20E+06	5.53E+07	6.13E+06	1.10E+08
CEC2017	10	min	**1.50E+03**	2.85E+03	1.78E+03	5.21E+03	2.01E+03	4.47E+03	1.64E+03	2.35E+03
_15		mean	**1.52E+03**	9.32E+03	1.11E+04	2.21E+04	8.18E+03	2.97E+04	4.99E+03	2.17E+04
		std	**1.55E+01**	3.12E+03	6.63E+03	1.06E+04	3.65E+03	5.05E+04	3.94E+03	2.78E+04
	100	min	**2.44E+03**	2.67E+03	8.96E+08	1.85E+10	5.71E+06	9.63E+09	6.13E+07	1.66E+10
		mean	**5.99E+05**	1.73E+08	7.69E+09	2.43E+10	4.50E+07	1.96E+10	1.54E+09	2.53E+10
		std	**3.18E+06**	9.24E+08	3.42E+09	4.74E+09	1.33E+08	5.96E+09	1.49E+09	4.59E+09
CEC2017	10	min	**1.60E+03**	1.82E+03	1.70E+03	1.75E+03	1.73E+03	1.83E+03	1.65E+03	1.96E+03
_16		mean	**1.76E+03**	2.09E+03	2.00E+03	2.05E+03	1.97E+03	2.07E+03	1.82E+03	2.17E+03

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CEC2017	dim	Metric	IBWO	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2017	10	std	**1.00E+02**	1.11E+02	1.63E+02	1.55E+02	1.72E+02	1.75E+02	1.24E+02	1.46E+02
_16	100	min	**4.48E+03**	1.92E+04	1.17E+04	1.51E+04	7.91E+03	1.30E+04	7.43E+03	1.43E+04
		mean	**6.21E+03**	2.30E+04	1.55E+04	2.08E+04	9.97E+03	2.19E+04	1.05E+04	2.40E+04
		std	**8.27E+02**	1.54E+03	2.11E+03	3.09E+03	1.07E+03	3.87E+03	1.63E+03	4.30E+03
CEC2017	10	min	**1.70E+03**	1.79E+03	1.76E+03	1.77E+03	1.74E+03	1.78E+03	1.75E+03	1.78E+03
_17		mean	**1.74E+03**	1.83E+03	1.83E+03	1.95E+03	1.79E+03	1.87E+03	1.78E+03	1.91E+03
		std	**1.88E+01**	2.70E+01	4.57E+01	1.24E+02	6.40E+01	8.59E+01	2.87E+01	1.11E+02
	100	min	**3.65E+03**	4.77E+03	9.75E+03	7.64E+05	6.12E+03	8.37E+04	6.62E+03	1.46E+06
		mean	**5.49E+03**	5.82E+03	2.50E+05	6.99E+06	8.49E+03	7.40E+06	2.16E+04	1.28E+07
		std	**8.89E+02**	1.24E+03	3.50E+05	6.54E+06	2.20E+03	7.88E+06	2.56E+04	9.85E+06
CEC2017	10	min	**1.80E+03**	1.75E+06	2.56E+03	3.98E+03	3.04E+03	3.63E+04	2.85E+03	1.14E+04
_18		mean	**2.81E+03**	9.91E+06	2.34E+04	1.54E+07	1.95E+04	3.09E+07	2.43E+04	5.39E+07
		std	**1.61E+03**	9.54E+06	1.61E+04	5.72E+07	1.35E+04	7.17E+07	1.72E+04	1.23E+08
	100	min	**8.27E+05**	1.68E+06	6.84E+06	5.81E+07	3.08E+06	4.47E+07	5.40E+06	1.34E+08
		mean	**3.77E+06**	4.89E+06	2.97E+07	1.75E+08	1.11E+07	2.05E+08	1.38E+07	3.76E+08
		std	**1.65E+06**	2.99E+06	2.20E+07	1.39E+08	6.64E+06	1.46E+08	8.47E+06	1.86E+08
CEC2017	10	min	**1.90E+03**	1.51E+04	2.71E+03	3.98E+03	2.67E+03	2.72E+03	1.97E+03	9.78E+03
_19		mean	**1.91E+03**	1.09E+05	2.15E+05	1.32E+05	2.34E+04	1.75E+05	1.68E+04	1.38E+06
		std	**6.34E+00**	9.64E+04	9.13E+05	8.98E+04	2.84E+04	4.06E+05	4.65E+04	2.49E+06
	100	min	**2.39E+03**	3.36E+03	1.61E+09	1.20E+10	1.32E+07	1.39E+10	1.07E+08	1.19E+10
		mean	**1.44E+04**	5.03E+06	6.16E+09	2.45E+10	4.52E+07	2.23E+10	1.43E+09	2.20E+10
		std	**2.67E+04**	2.75E+07	3.72E+09	5.83E+09	4.36E+07	4.83E+09	1.61E+09	5.10E+09
CEC2017	10	min	**2.00E+03**	2.14E+03	2.06E+03	2.06E+03	2.05E+03	2.06E+03	2.05E+03	2.12E+03
_20		mean	**2.06E+03**	2.24E+03	2.22E+03	2.17E+03	2.22E+03	2.25E+03	2.15E+03	2.31E+03
		std	5.07E+01	**4.84E+01**	8.95E+01	9.28E+01	9.43E+01	8.64E+01	6.73E+01	9.25E+01
	100	min	**3.26E+03**	7.24E+03	5.83E+03	6.95E+03	4.72E+03	7.22E+03	5.45E+03	6.51E+03
		mean	**6.07E+03**	7.79E+03	6.79E+03	7.55E+03	6.31E+03	7.99E+03	6.29E+03	8.38E+03
		std	1.16E+03	**3.01E+02**	5.04E+02	3.35E+02	6.73E+02	4.33E+02	5.04E+02	6.13E+02
CEC2017	10	min	**2.10E+03**	2.23E+03	2.23E+03	2.28E+03	2.21E+03	2.23E+03	2.20E+03	2.22E+03
_21		mean	**2.24E+03**	2.30E+03	2.33E+03	2.34E+03	2.35E+03	2.32E+03	2.31E+03	2.32E+03
		std	5.02E+01	4.10E+01	5.52E+01	**3.11E+01**	5.08E+01	6.35E+01	6.17E+01	6.65E+01
	100	min	**2.72E+03**	2.79E+03	4.11E+03	4.25E+03	4.02E+03	4.42E+03	3.55E+03	4.70E+03
		mean	**2.93E+03**	2.96E+03	4.34E+03	4.70E+03	4.43E+03	4.81E+03	3.78E+03	5.12E+03
		std	**1.12E+02**	1.42E+02	1.25E+02	2.32E+02	2.56E+02	2.37E+02	1.84E+02	1.88E+02
CEC2017	10	min	**2.22E+03**	2.37E+03	2.38E+03	2.60E+03	2.31E+03	2.41E+03	2.26E+03	2.66E+03
_22		mean	**2.30E+03**	2.79E+03	2.83E+03	3.14E+03	2.44E+03	2.81E+03	2.36E+03	3.00E+03
		std	**1.06E+00**	2.57E+02	4.99E+02	3.72E+02	3.79E+02	4.12E+02	2.15E+02	3.24E+02
	100	min	**2.51E+03**	3.38E+04	2.83E+04	3.04E+04	2.52E+04	3.31E+04	2.30E+04	3.34E+04
		mean	**1.77E+04**	3.49E+04	3.12E+04	3.40E+04	2.78E+04	3.52E+04	2.63E+04	3.50E+04
		std	9.47E+03	**5.72E+02**	1.89E+03	1.26E+03	1.83E+03	9.96E+02	1.78E+03	9.41E+02
CEC2017	10	min	**2.61E+03**	2.67E+03	2.63E+03	2.70E+03	2.63E+03	2.64E+03	2.62E+03	2.67E+03
_23		mean	**2.62E+03**	2.70E+03	2.69E+03	2.77E+03	2.68E+03	2.68E+03	2.64E+03	2.70E+03
		std	**8.23E+00**	1.63E+01	3.83E+01	4.81E+01	3.29E+01	2.54E+01	2.22E+01	1.59E+01

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CEC2017	dim	Metric	IBWO	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2017	100	min	**3.27E+03**	5.74E+03	4.92E+03	6.61E+03	5.03E+03	5.41E+03	4.11E+03	5.61E+03
_23		mean	**3.49E+03**	6.15E+03	5.30E+03	7.48E+03	5.93E+03	6.09E+03	4.50E+03	6.12E+03
		std	**1.36E+02**	1.91E+02	1.76E+02	5.83E+02	5.68E+02	3.35E+02	1.78E+02	1.50E+02
CEC2017	10	min	**2.50E+03**	2.60E+03	2.76E+03	2.75E+03	2.50E+03	2.77E+03	2.51E+03	2.72E+03
_24		mean	**2.71E+03**	2.75E+03	2.83E+03	2.87E+03	2.75E+03	2.80E+03	2.77E+03	2.83E+03
		std	9.91E+01	8.56E+01	5.46E+01	7.57E+01	1.37E+02	**2.42E+01**	5.22E+01	3.51E+01
	100	min	**3.91E+03**	8.55E+03	5.28E+03	1.02E+04	6.24E+03	7.51E+03	5.28E+03	7.94E+03
		mean	**4.13E+03**	9.44E+03	6.40E+03	1.18E+04	8.46E+03	8.79E+03	5.74E+03	8.67E+03
		std	**1.50E+02**	3.56E+02	5.07E+02	9.79E+02	7.42E+02	6.93E+02	2.96E+02	2.75E+02
CEC2017	10	min	**2.60E+03**	3.03E+03	2.95E+03	3.08E+03	2.90E+03	3.00E+03	2.91E+03	3.10E+03
_25		mean	**2.93E+03**	3.25E+03	3.21E+03	3.45E+03	2.95E+03	3.19E+03	2.96E+03	3.36E+03
		std	**2.34E+01**	1.06E+02	1.99E+02	2.50E+02	3.17E+01	1.69E+02	3.43E+01	2.10E+02
	100	min	**3.76E+03**	3.80E+03	1.19E+04	2.19E+04	6.04E+03	2.47E+04	8.32E+03	2.27E+04
		mean	**4.17E+03**	4.38E+03	1.69E+04	2.97E+04	6.94E+03	2.87E+04	1.05E+04	2.86E+04
		std	**1.83E+02**	4.06E+02	1.84E+03	2.86E+03	5.13E+02	2.46E+03	1.71E+03	2.49E+03
CEC2017	10	min	**2.60E+03**	3.44E+03	3.23E+03	3.46E+03	2.83E+03	3.14E+03	2.60E+03	3.13E+03
_26		mean	**3.03E+03**	3.84E+03	3.92E+03	4.17E+03	3.77E+03	3.70E+03	3.25E+03	3.98E+03
		std	**2.53E+02**	2.59E+02	4.96E+02	3.62E+02	5.92E+02	4.14E+02	4.18E+02	5.11E+02
	100	min	**7.92E+03**	8.07E+03	3.48E+04	4.38E+04	2.95E+04	4.83E+04	2.70E+04	4.60E+04
		mean	**2.23E+04**	2.26E+04	3.88E+04	5.25E+04	3.24E+04	5.30E+04	3.25E+04	5.63E+04
		std	4.48E+03	5.00E+03	2.30E+03	3.73E+03	**2.04E+03**	2.68E+03	2.60E+03	5.89E+03
CEC2017	10	min	**3.09E+03**	3.13E+03	3.11E+03	3.17E+03	3.11E+03	3.12E+03	3.09E+03	3.12E+03
_27		mean	**3.11E+03**	3.16E+03	3.18E+03	3.29E+03	3.20E+03	3.16E+03	3.12E+03	3.14E+03
		std	**1.06E+01**	2.73E+01	6.37E+01	6.58E+01	6.22E+01	4.49E+01	2.97E+01	1.42E+01
	100	min	**3.72E+03**	1.02E+04	4.77E+03	1.11E+04	5.28E+03	8.33E+03	4.83E+03	9.48E+03
		mean	**3.98E+03**	1.27E+04	5.90E+03	1.35E+04	7.03E+03	1.42E+04	5.60E+03	1.26E+04
		std	**1.89E+02**	8.39E+02	7.96E+02	1.25E+03	1.09E+03	2.31E+03	5.37E+02	1.49E+03
CEC2017	10	min	**2.80E+03**	3.41E+03	3.24E+03	3.36E+03	3.16E+03	3.27E+03	3.10E+03	3.41E+03
_28		mean	**3.39E+03**	3.65E+03	3.56E+03	3.80E+03	3.43E+03	3.51E+03	3.40E+03	3.62E+03
		std	**6.99E+01**	1.08E+02	2.12E+02	1.74E+02	1.57E+02	1.40E+02	1.51E+02	1.20E+02
	100	min	**3.90E+03**	2.55E+04	1.34E+04	2.64E+04	7.38E+03	3.10E+04	9.18E+03	2.83E+04
		mean	**4.50E+03**	2.78E+04	1.61E+04	3.49E+04	9.65E+03	4.66E+04	1.34E+04	3.47E+04
		std	**3.64E+02**	7.49E+02	1.70E+03	3.65E+03	9.75E+02	1.60E+04	2.29E+03	3.36E+03
CEC2017	10	min	**3.14E+03**	3.26E+03	3.22E+03	3.24E+03	3.24E+03	3.22E+03	3.18E+03	3.27E+03
_29		mean	**3.21E+03**	3.39E+03	3.37E+03	3.44E+03	3.41E+03	3.40E+03	3.29E+03	3.46E+03
		std	**4.50E+01**	6.98E+01	1.36E+02	1.63E+02	1.19E+02	1.19E+02	8.03E+01	1.04E+02
	100	min	**6.19E+03**	1.33E+05	1.75E+04	1.60E+05	1.08E+04	3.97E+04	1.12E+04	7.74E+04
		mean	**7.99E+03**	4.73E+05	5.01E+04	8.17E+05	1.29E+04	5.06E+05	1.62E+04	7.95E+05
		std	**8.99E+02**	2.04E+05	2.68E+04	7.18E+05	1.35E+03	5.72E+05	5.50E+03	7.45E+05
CEC2017	10	min	**3.89E+03**	8.86E+05	1.23E+05	1.17E+06	8.69E+04	8.53E+05	2.24E+04	8.85E+05
_30		mean	**6.58E+04**	3.79E+06	9.74E+06	3.90E+07	2.92E+06	1.27E+07	1.61E+06	1.34E+06
		std	**1.49E+05**	3.11E+06	1.55E+07	3.09E+07	3.80E+06	1.64E+07	2.11E+06	1.56E+06
	100	min	**2.57E+05**	5.51E+05	4.04E+09	2.61E+10	3.29E+08	2.23E+10	1.30E+09	3.12E+10
		mean	**2.95E+07**	1.31E+08	1.58E+10	4.08E+10	7.16E+08	3.89E+10	5.01E+09	4.34E+10
		std	**6.08E+07**	4.02E+08	5.48E+09	8.49E+09	3.76E+08	7.05E+09	3.74E+09	5.38E+09

5.1.3. Analysis of the Wilcoxon rank sum test results

In order to further compare the differences between IBWO and other algorithms, we use the Wilcoxon rank sum test. The Wilcoxon rank sum test is to compare the function results, and its value is p. We calculate the p-value to determine the difference between two algorithms. If the p-value is less than 5%, there is a significant difference between the two algorithms. Table 3 shows the experimental results of IBWO and other seven different algorithms running 30 times with CEC2017 benchmark functions (dim = 10/100). In combination with the table, we can see that IBWO is significantly different from the other seven algorithms. The IBWO generally achieves good results in the Wilcoxon rank sum test.

Table 3. Wilcoxon rank sum test results of CEC2017 with dim = 10 and dim = 100.

CEC2017	dim	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2017_01	10	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	1.16E-03	6.10E-05
CEC2017_03	10	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	100	**8.90E-01**	3.36E-03	**7.30E-02**	5.37E-03	1.53E-03	8.36E-03	1.83E-04
CEC2017_04	10	1.73E-06	2.60E-06	1.73E-06	2.26E-03	1.73E-06	2.22E-04	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	**8.04E-01**	6.10E-05
CEC2017_05	10	1.73E-06	1.24E-05	1.73E-06	4.29E-06	1.73E-06	6.89E-05	1.73E-06
	100	6.10E-05	4.79E-02	6.10E-05	6.10E-05	6.10E-05	6.10E-05	8.54E-04
CEC2017_06	10	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.41E-05	1.73E-06
	100	6.10E-05	**9.78E-01**	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2017_07	10	4.07E-05	**9.78E-02**	4.86E-05	1.24E-05	1.02E-05	**7.19E-02**	1.64E-05
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	3.05E-04	6.10E-05
CEC2017_08	10	1.73E-06	8.92E-05	3.88E-06	5.31E-05	1.73E-06	1.97E-05	1.73E-06
	100	6.10E-05	6.10E-04	6.10E-05	6.10E-05	6.10E-05	6.10E-05	1.22E-04
CEC2017_09	10	1.29E-03	**1.92E-01**	1.96E-02	2.61E-04	1.96E-03	1.40E-02	3.06E-04
	100	6.10E-05	**2.08E-01**	1.22E-04	4.27E-03	6.10E-05	1.83E-04	6.10E-05
CEC2017_10	10	1.73E-06	2.13E-06	1.92E-06	1.02E-05	1.73E-06	4.29E-06	1.73E-06
	100	1.22E-04	**4.89E-01**	8.36E-03	1.16E-03	6.10E-05	6.10E-05	6.10E-05
CEC2017_11	10	1.73E-06	3.52E-06	1.73E-06	2.88E-06	1.73E-06	7.69E-06	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	2.01E-03	6.10E-05	6.10E-05
CEC2017_12	10	1.73E-06	1.73E-06	2.35E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	1.22E-04	6.10E-05	6.10E-05	6.10E-05
CEC2017_13	10	1.73E-06	5.71E-04	4.72E-02	6.42E-03	2.13E-06	**2.54E-01**	2.35E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	3.05E-04	6.10E-05
CEC2017_14	10	2.70E-02	**1.25E-01**	3.52E-06	3.33E-02	2.41E-04	3.32E-04	4.73E-06
	100	6.10E-05	**6.37E-02**	6.10E-05	8.54E-04	6.10E-05	2.01E-03	6.10E-05
CEC2017_15	10	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	3.36E-03	6.10E-05
CEC2017_16	10	1.92E-06	1.20E-03	4.29E-06	1.74E-04	2.84E-05	**7.86E-02**	1.73E-06
	100	6.10E-05	1.16E-03	6.10E-05	6.10E-05	6.10E-05	8.36E-03	6.10E-05

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CEC2017	dim	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2017_17	10	1.73E-06	1.80E-05	2.13E-06	3.32E-04	1.73E-06	1.97E-05	1.73E-06
	100	6.10E-05	1.22E-04	6.10E-05	1.81E-02	6.10E-05	2.15E-02	6.10E-05
CEC2017_18	10	1.73E-06	3.18E-06	7.69E-06	2.22E-04	1.73E-06	1.49E-05	1.73E-06
	100	6.10E-05	**2.77E-01**	6.10E-05	3.36E-03	6.10E-05	**5.61E-01**	6.10E-05
CEC2017_19	10	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.92E-06	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	2.62E-03	6.10E-05
CEC2017_20	10	2.60E-06	1.83E-03	1.89E-04	1.89E-04	6.34E-06	2.22E-04	4.73E-06
	100	4.79E-02	6.10E-05	3.02E-02	6.10E-05	1.81E-02	6.10E-05	1.53E-03
CEC2017_21	10	1.17E-02	3.38E-03	3.18E-06	3.06E-04	6.98E-06	4.49E-02	3.41E-05
	100	6.10E-05	6.10E-05	6.10E-05	1.22E-04	6.10E-05	3.05E-04	6.10E-05
CEC2017_22	10	1.73E-06	1.73E-06	1.73E-06	2.16E-05	1.92E-06	1.60E-04	1.73E-06
	100	6.10E-05	2.01E-03	2.01E-03	6.10E-05	4.27E-04	6.10E-05	1.22E-04
CEC2017_23	10	1.73E-06	1.83E-03	1.73E-06	3.18E-06	2.13E-06	8.31E-04	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	2.56E-02	6.10E-05
CEC2017_24	10	1.40E-02	3.68E-02	6.32E-05	3.06E-04	2.88E-06	2.85E-02	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2017_25	10	1.73E-06	7.69E-06	1.73E-06	**5.45E-02**	1.73E-06	3.00E-02	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	**7.62E-01**	6.10E-05
CEC2017_26	10	3.72E-05	2.70E-02	5.75E-06	1.29E-03	2.60E-05	**1.99E-01**	2.35E-06
	100	6.10E-05	6.10E-05	6.10E-05	3.05E-04	6.10E-05	6.71E-03	6.10E-05
CEC2017_27	10	1.73E-06	**7.66E-01**	1.73E-06	6.34E-06	3.52E-06	**7.19E-01**	6.98E-06
	100	6.10E-05	1.22E-04	6.10E-05	6.10E-05	6.10E-05	8.54E-04	6.10E-05
CEC2017_28	10	1.73E-06	**1.78E-01**	1.73E-06	3.33E-02	2.60E-06	**4.05E-01**	5.75E-06
	100	6.10E-05	8.36E-03	6.10E-05	6.10E-05	6.10E-05	**8.90E-01**	6.10E-05
CEC2017_29	10	1.73E-06	8.19E-05	3.52E-06	2.35E-06	3.88E-06	3.16E-02	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	**4.21E-01**	6.10E-05	8.36E-03	6.10E-05
CEC2017_30	10	1.73E-06	5.75E-06	1.73E-06	1.24E-05	1.73E-06	2.84E-05	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	3.05E-04	6.10E-05	1.22E-04	6.10E-05

5.2. Experiments on the CEC2020 benchmark functions

5.2.1. Detailed description of CEC2020 functions

Using only CEC2017 is not comprehensive enough. In order to further verify the excellent performance of the IBWO, CEC2020 was used for testing, as described in this section. The information on the test functions of CEC2020 comes from [56]. In this comparative experiment, we used CEC2020 with different dimensions (10 and 100 dimensions), the number of iterations was 500, N = 30. Table 4 shows the results of CEC2020 with 10 and 100 dimensions for each algorithm running 30 times.

Table 4. Test results of various algorithms on CEC2020.

CEC2020	dim	Metric	IBWO	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2020	10	min	**1.01E+02**	5.37E+09	2.32E+07	6.14E+09	3.55E+05	5.06E+08	1.22E+04	3.70E+09
_01		mean	**2.49E+03**	8.86E+09	1.16E+09	1.08E+10	1.11E+07	4.95E+09	1.53E+08	9.18E+09
		std	**3.38E+03**	2.03E+09	1.81E+09	2.86E+09	3.98E+07	3.74E+09	2.54E+08	3.24E+09
	100	min	**6.22E+08**	2.40E+09	1.66E+11	2.61E+11	4.33E+10	2.56E+11	1.07E+11	2.53E+11
		mean	**4.72E+09**	1.70E+10	1.84E+11	2.75E+11	5.13E+10	2.72E+11	1.21E+11	2.65E+11
		std	**2.04E+09**	1.79E+10	1.76E+10	1.19E+10	7.95E+09	1.29E+10	1.57E+10	1.05E+10
CEC2020	10	min	**1.10E+03**	2.16E+03	1.45E+03	1.85E+03	1.62E+03	2.21E+03	1.64E+03	2.32E+03
_02		mean	**1.54E+03**	2.63E+03	2.18E+03	2.27E+03	2.12E+03	2.68E+03	2.01E+03	2.88E+03
		std	**2.00E+02**	2.17E+02	3.37E+02	2.84E+02	2.75E+02	2.68E+02	2.74E+02	3.13E+02
	100	min	**1.37E+04**	1.99E+04	2.77E+04	3.12E+04	2.27E+04	3.25E+04	2.04E+04	3.18E+04
		mean	3.18E+04	2.96E+04	3.01E+04	3.21E+04	2.60E+04	3.36E+04	**2.36E+04**	3.27E+04
		std	**3.56E+02**	5.44E+03	2.28E+03	1.01E+03	3.76E+03	8.11E+02	2.88E+03	1.14E+03
CEC2020	10	min	**7.18E+02**	7.94E+02	7.63E+02	7.72E+02	7.54E+02	7.79E+02	7.39E+02	7.87E+02
_03		mean	**7.58E+02**	8.03E+02	7.95E+02	8.01E+02	7.91E+02	8.11E+02	7.67E+02	8.22E+02
		std	2.37E+01	**1.07E+01**	2.01E+01	1.73E+01	2.46E+01	2.14E+01	2.36E+01	3.27E+01
	100	min	**1.69E+03**	2.10E+03	3.51E+03	3.88E+03	3.72E+03	4.00E+03	3.37E+03	3.77E+03
		mean	**2.33E+03**	2.60E+03	3.79E+03	4.00E+03	3.85E+03	4.09E+03	3.55E+03	4.03E+03
		std	2.43E+02	6.45E+02	1.62E+02	8.75E+01	**8.44E+01**	8.69E+01	1.59E+02	2.06E+02
CEC2020	10	min	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**
_04		mean	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	1.90E+03	**1.90E+03**	**1.90E+03**
		std	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	5.29E-01	**0.00E+00**	**0.00E+00**
	100	min	**1.90E+03**	**1.90E+03**	**1.90E+03**	1.90E+03	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**
		mean	**1.90E+03**	**1.90E+03**	**1.90E+03**	1.90E+03	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**
		std	**0.00E+00**	**0.00E+00**	**0.00E+00**	6.14E-05	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
CEC2020	10	min	**1.87E+03**	1.65E+04	3.22E+03	3.97E+04	2.38E+03	2.03E+04	2.90E+03	7.38E+04
_05		mean	**1.06E+04**	2.71E+05	7.70E+04	3.97E+05	7.23E+04	1.63E+06	3.13E+04	5.03E+05
		std	**8.65E+03**	1.31E+05	1.18E+05	2.11E+05	5.52E+04	3.07E+06	9.15E+04	3.12E+05
	100	min	**1.76E+07**	2.64E+07	2.25E+08	9.63E+08	1.21E+08	7.92E+08	1.08E+08	9.50E+08
		mean	**2.94E+07**	5.48E+07	3.31E+08	1.37E+09	1.84E+08	1.60E+09	1.36E+08	1.62E+09
		std	**6.44E+06**	2.00E+07	1.22E+08	5.55E+08	5.13E+07	9.17E+08	4.24E+07	7.26E+08
CEC2020	10	min	**1.60E+03**	1.80E+03	1.63E+03	1.75E+03	1.67E+03	1.80E+03	1.69E+03	1.92E+03
_06		mean	**1.71E+03**	2.01E+03	1.86E+03	2.05E+03	1.86E+03	2.03E+03	1.82E+03	2.12E+03
		std	**8.78E+01**	9.58E+01	1.14E+02	1.92E+02	1.53E+02	1.34E+02	9.69E+01	1.45E+02
	100	min	**4.49E+03**	6.88E+03	1.55E+04	1.89E+04	1.23E+04	2.18E+04	1.22E+04	2.49E+04
		mean	**6.03E+03**	8.27E+03	1.82E+04	2.76E+04	1.43E+04	2.89E+04	1.37E+04	3.53E+04
		std	**5.07E+02**	2.17E+03	1.96E+03	6.96E+03	1.65E+03	8.55E+03	1.71E+03	8.97E+03
CEC2020	10	min	**2.12E+03**	1.09E+04	3.19E+03	3.95E+03	2.89E+03	3.33E+03	2.77E+03	1.59E+04
_07		mean	**4.07E+03**	9.74E+04	1.20E+04	8.07E+05	5.70E+04	2.22E+05	9.53E+03	1.27E+06
		std	**4.55E+03**	8.39E+04	9.33E+03	1.81E+06	1.65E+05	6.20E+05	4.86E+03	1.45E+06
	100	min	**4.57E+06**	8.71E+06	7.95E+07	2.88E+08	3.85E+07	2.66E+08	1.10E+07	3.69E+08
		mean	**6.64E+06**	1.00E+07	1.22E+08	4.43E+08	5.74E+07	4.08E+08	5.27E+07	7.01E+08
		std	**7.75E+05**	2.59E+06	4.48E+07	1.78E+08	2.22E+07	1.19E+08	4.70E+07	2.92E+08

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CEC2020	dim	Metric	IBWO	BWO	ROA	AOA	HHO	BES	SCSO	PDO
CEC2020	10	min	**2.30E+03**	2.36E+03	2.31E+03	2.79E+03	2.31E+03	2.42E+03	2.30E+03	2.58E+03
_08		mean	**2.30E+03**	2.80E+03	2.42E+03	3.06E+03	2.45E+03	2.80E+03	2.33E+03	3.17E+03
		std	**1.56E+00**	2.12E+02	1.05E+02	3.41E+02	4.24E+02	4.28E+02	4.55E+01	4.53E+02
	100	min	**2.59E+03**	2.07E+04	2.92E+04	3.35E+04	2.65E+04	3.33E+04	2.47E+04	3.50E+04
		mean	**6.21E+03**	2.65E+04	3.12E+04	3.45E+04	2.85E+04	3.50E+04	2.61E+04	3.58E+04
		std	7.28E+03	6.11E+03	1.72E+03	1.08E+03	1.73E+03	1.44E+03	1.92E+03	**6.93E+02**
CEC2020	10	min	**2.50E+03**	2.64E+03	2.75E+03	2.78E+03	2.74E+03	2.77E+03	2.55E+03	2.82E+03
_09		mean	**2.71E+03**	2.79E+03	2.76E+03	2.89E+03	2.83E+03	2.81E+03	2.75E+03	2.85E+03
		std	9.76E+01	8.50E+01	7.95E+01	7.43E+01	1.02E+02	3.61E+01	6.66E+01	**3.30E+01**
	100	min	**3.86E+03**	4.16E+03	6.05E+03	1.18E+04	7.17E+03	8.63E+03	5.69E+03	8.65E+03
		mean	**4.09E+03**	4.39E+03	6.73E+03	1.20E+04	8.63E+03	9.42E+03	5.87E+03	8.91E+03
		std	**7.44E+01**	3.84E+02	6.06E+02	2.31E+02	1.13E+03	7.02E+02	2.20E+02	2.71E+02
CEC2020	10	min	**2.60E+03**	3.14E+03	2.92E+03	3.08E+03	2.90E+03	2.97E+03	2.91E+03	3.12E+03
_10		mean	**2.93E+03**	3.27E+03	3.04E+03	3.43E+03	2.94E+03	3.26E+03	2.96E+03	3.34E+03
		std	**2.36E+01**	7.21E+01	1.04E+02	2.54E+02	2.61E+01	2.72E+02	3.89E+01	2.12E+02
	100	min	**3.74E+03**	4.12E+03	1.35E+04	2.89E+04	5.50E+03	2.50E+04	9.27E+03	2.54E+04
		mean	**4.07E+03**	4.99E+03	1.76E+04	3.14E+04	6.51E+03	2.95E+04	1.14E+04	2.87E+04
		std	**1.39E+02**	5.81E+02	3.32E+03	2.32E+03	9.78E+02	3.12E+03	2.23E+03	2.58E+03

5.2.2 Analysis of the running results and convergence curves for CEC2020 with IBWO and comparison functions

The data in Table 4 indicates that the IBWO also performs well on the CEC2020 benchmark functions in 10 and 100 dimensions. On the unimodal function CEC1, we compared IBWO with BWO, and it is not difficult to see that the optimization performance of BWO on unimodal functions is not as good as that of IBWO. This is because the GAS can enhance the search ability of beluga individuals in local space and enable them to continuously search for the best fitness value. Therefore, we can conclude that IBWO has more exploration capabilities compared to other comparative algorithms. IBWO also exhibits excellent competitiveness in basic multimodal functions. This is thanks to the DPIS, which generates a candidate position for each beluga whale and ultimately selects a better position. They are thus able to explore better in the search space. Mixing and combining functions are challenging. The QIS can help IBWO continuously find the optimal location, and IBWO also has a good ability to balance exploration and exploitation. The experimental data can effectively verify that the IBWO achieves a certain degree of local optimum avoidance, indicating its applicability.

We evaluated the convergence accuracy and speed of IBWO on the convergence curve and compared it with the other seven algorithms. In the optimization process, IBWO showed two kinds of convergence behaviors for different characteristics of convergence curves. In the first case, the optimal value can be found when the number of iterations is no more than 100. In the second case, as the number of iterations increases, the curve of IBWO starts to accelerate in its convergence and finally gets the optimal iteration. It can be inferred that IBWO can better strike a balance between exploration and exploitation than other comparison algorithms.

The specific convergence images are shown in Figures 9 and 10. As can be seen from the unimodal function, IBWO has better optimization effect and the fastest convergence speed. This is because IBWO uses the QIS to improve utilization. IBWO can also fully explore multimodal functions and constantly search for optimal values. This is due to the introduction of the DPIS, which generates new locations and makes the search more extensive. When faced with more complex combined and mixed functions, the IBWO is still excellent. The GAS maintains a good balance between exploration and exploitation and performs well when faced with complex scenarios.

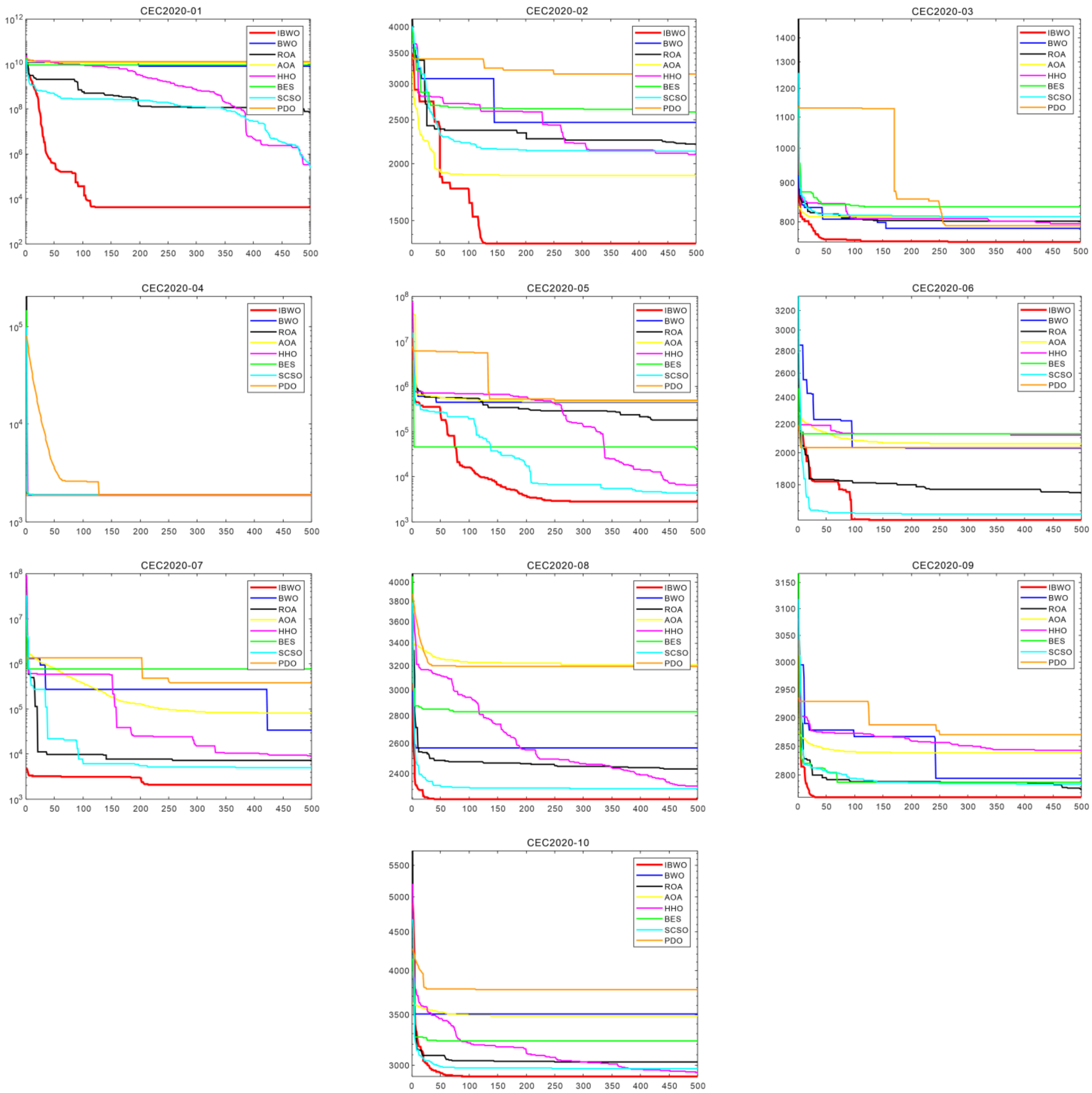


Figure 9. Convergence curves of IBWO and comparison algorithms on CEC2020 (dim = 10).

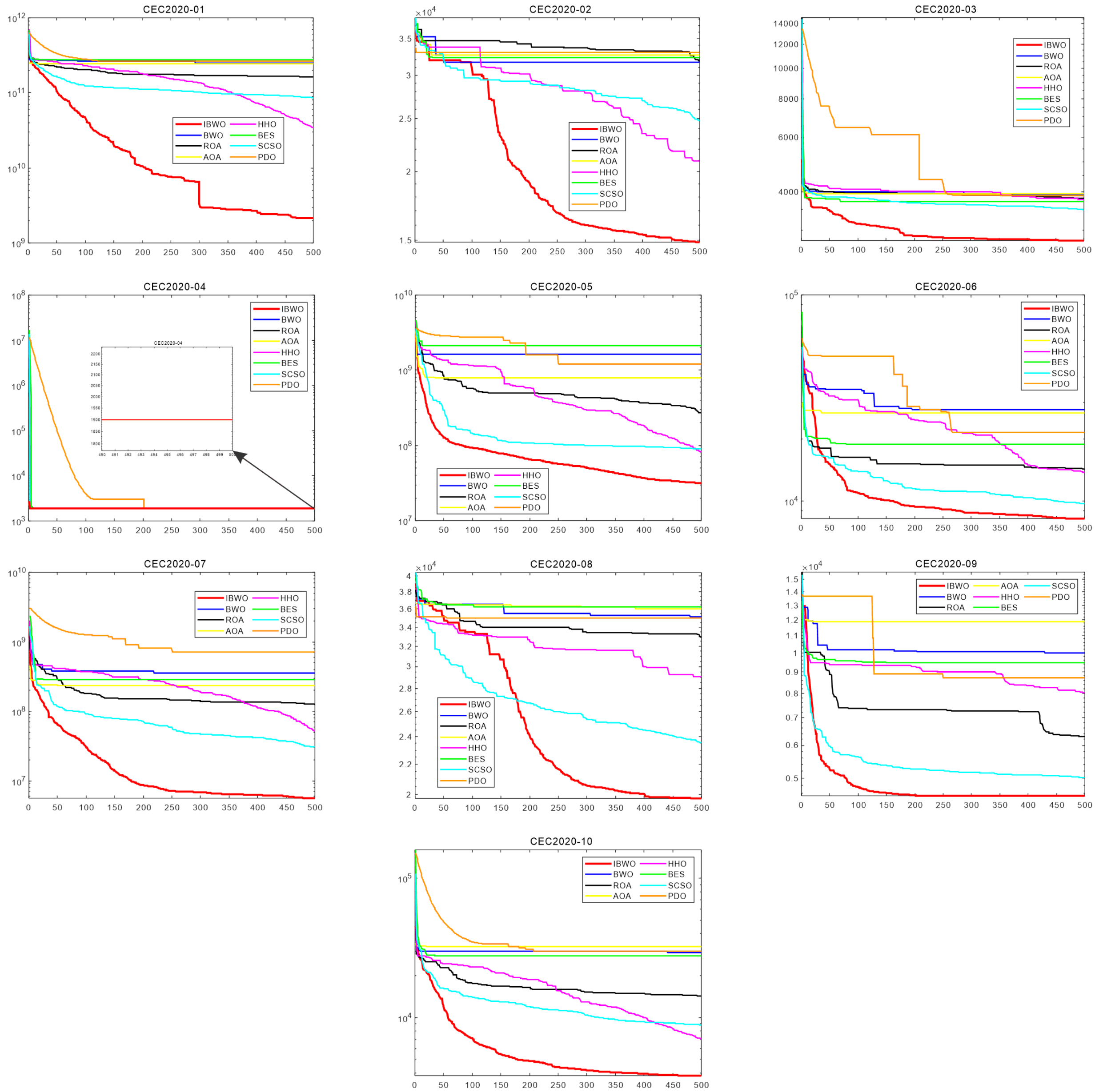


Figure 10. Convergence curves of IBWO and comparison algorithms on CEC2020 (dim = 100).

5.2.3. Wilcoxon rank sum test experimental results

Table 5 shows the results of IBWO and seven other algorithms for Wilcoxon rank sum detection in different dimensions. In most cases, the p value is less than 0.05, indicating that IBWO is significantly different from the comparison algorithms in most cases. It can be found that the p values for CEC2020-04 functions are all greater than 5%. Combining Table 4 and Figures 9 and 10, it is not difficult to see that all algorithms perform well on CEC2020-04, and most algorithms can find the optimal fitness value. The results show that the excellence of IBWO is statistically significant compared with the other algorithms.

Table 5. Experimental results of Wilcoxon rank sum test with dim = 10 and dim = 100.

CEC2020	dim	IBWO vs BWO	IBWO vs ROA	IBWO vs AOA	IBWO vs HHO	IBWO vs BES	IBWO vs SCSO	IBWO vs PDO
CEC2020_01	10	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2020_02	10	1.73E-06	1.73E-06	1.92E-06	3.18E-06	1.73E-06	7.69E-06	1.73E-06
	100	6.10E-05	4.79E-02	6.10E-05	4.13E-02	6.10E-05	8.36E-03	6.10E-05
CEC2020_03	10	9.32E-06	2.43E-02	1.48E-04	6.42E-03	2.16E-05	**6.56E-02**	3.52E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2020_04	10	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**2.50E-01**	**1.00E+00**	**1.00E+00**
	100	**1.00E+00**	**1.00E+00**	6.10E-05	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**
CEC2020_05	10	1.73E-06	1.24E-05	1.73E-06	2.35E-06	1.73E-06	1.74E-04	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2020_06	10	1.73E-06	9.27E-03	1.92E-06	1.48E-04	5.75E-06	5.29E-04	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2020_07	10	1.73E-06	5.22E-06	1.73E-06	1.73E-06	2.13E-06	1.02E-05	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2020_08	10	1.73E-06	1.49E-05	1.73E-06	1.73E-06	1.73E-06	2.41E-04	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	4.27E-03	6.10E-05	3.53E-02	6.10E-05
CEC2020_09	10	2.61E-04	5.71E-04	2.35E-06	1.74E-04	2.60E-06	4.99E-03	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
CEC2020_10	10	1.73E-06	7.69E-06	1.73E-06	3.87E-02	1.73E-06	2.18E-02	1.73E-06
	100	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05

5.2.4. Analysis of the results of IBWO and other algorithms on box chart

A box chart is a standard statistical chart that can visually show the outliers of a data the dispersion of data distribution, and the symmetry of data. It shows five kinds of statistical data: The maximum is the black line segment at the top of the box, the upper quartile is the blue line segment under the black line segment, the median is the red line segment in the box, the lower quartile is the blue line segment under the red line segment, and the minimum is the black line segment at the bottom of the box graph. Outliers are represented by a red "+". Figure 11 is the image obtained after 30 runs of IBWO and the comparison algorithms. It is not difficult to see that the box obtained by the IBWO is generally narrow and at the lowest point. Compared with BWO, IBWO's box is lower and generally shorter, indicating that compared with BWO, the IBWO improves the optimization ability and enhances the stability. Compared with other genetic algorithms, IBWO also performs well. In general, the IBWO box shows a good effect.

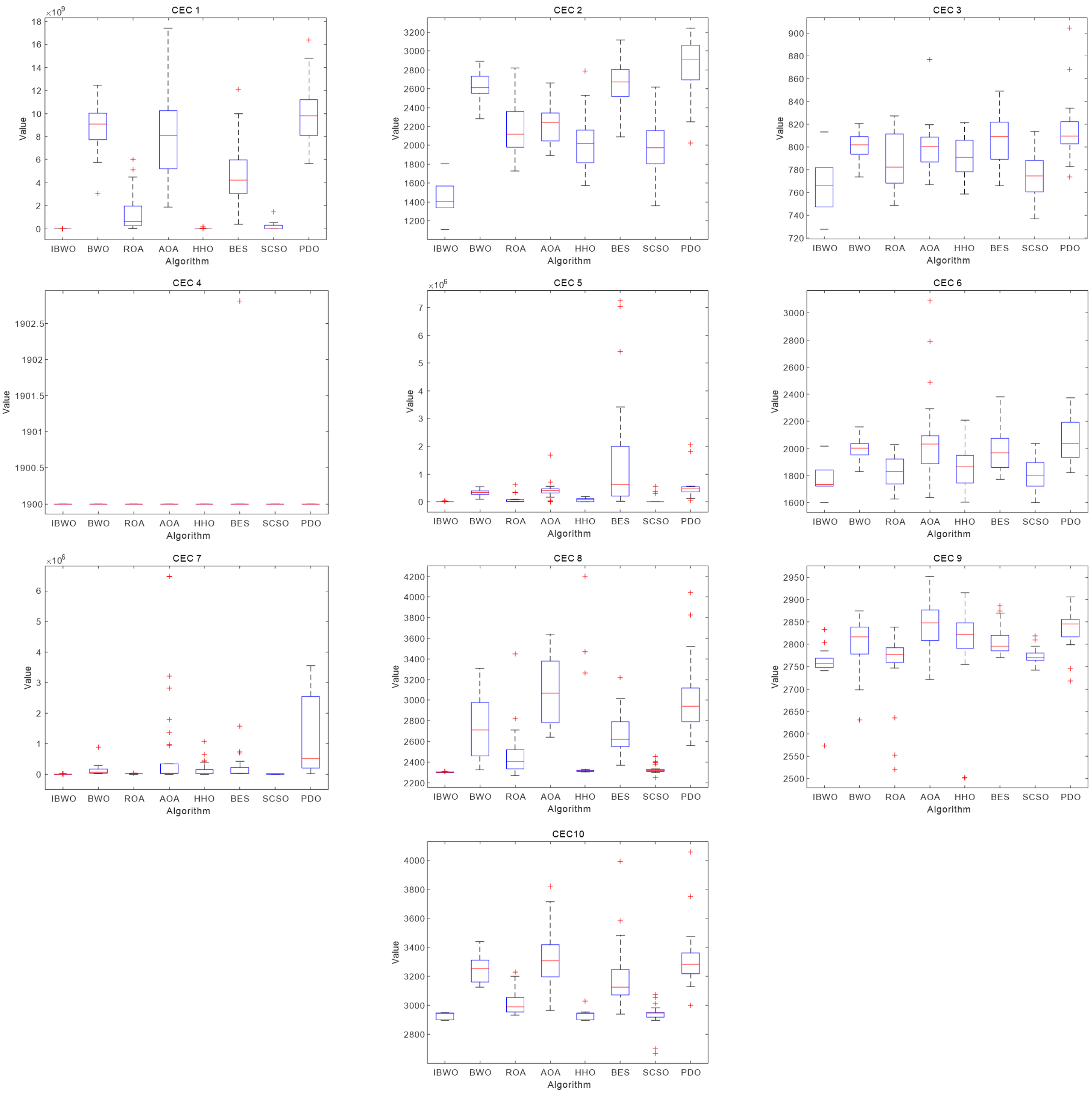
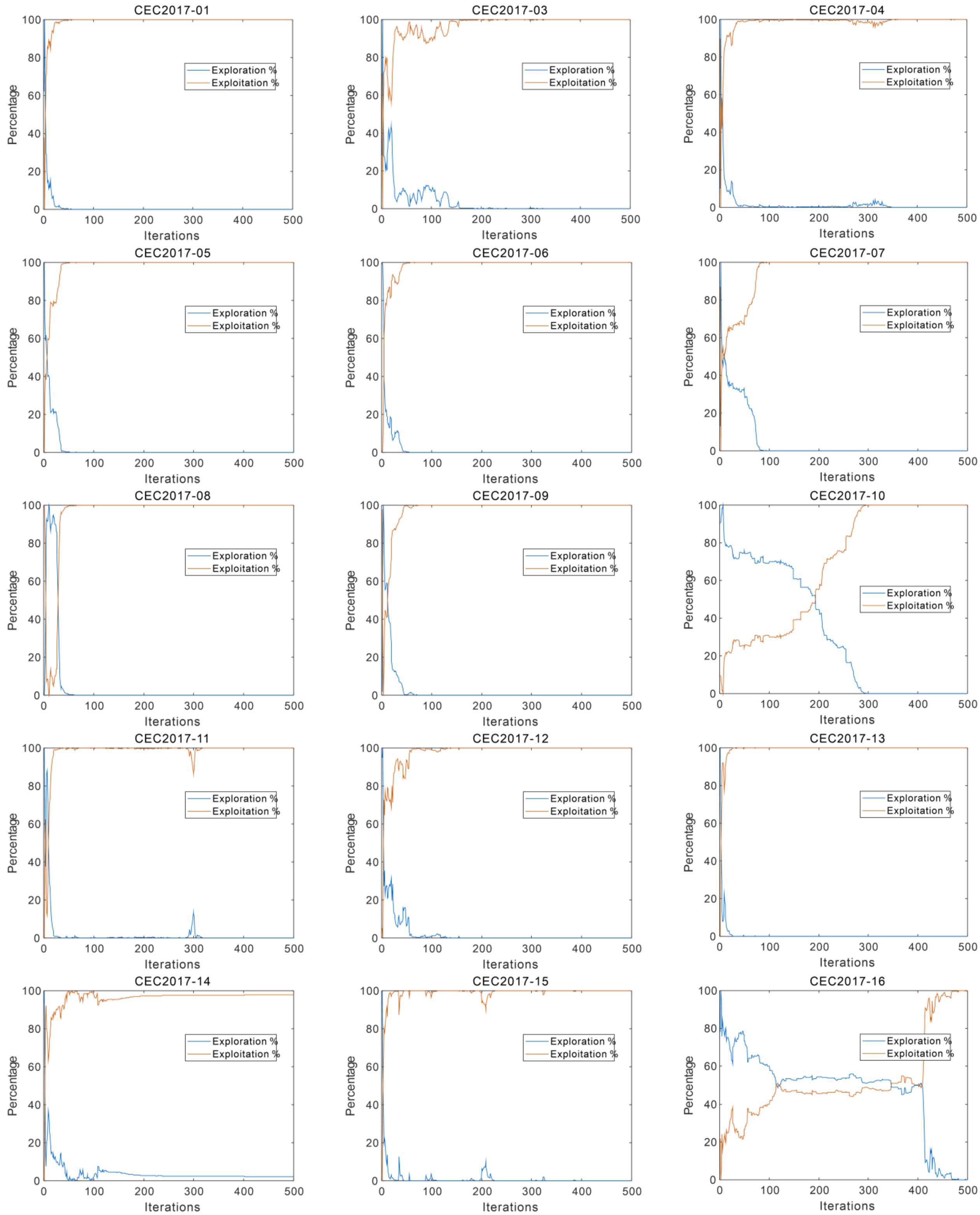


Figure 11. Box plots obtained from CEC2020.

5.3. Analysis of exploration and exploitation

As we all know, whether the algorithm can achieve a balance between exploration and exploitation is also one of the criteria to test the performance of the algorithm. Therefore, in this section, we conduct balance detection of IBWO on CEC2017 and CEC2020. We used the testing method in [57].



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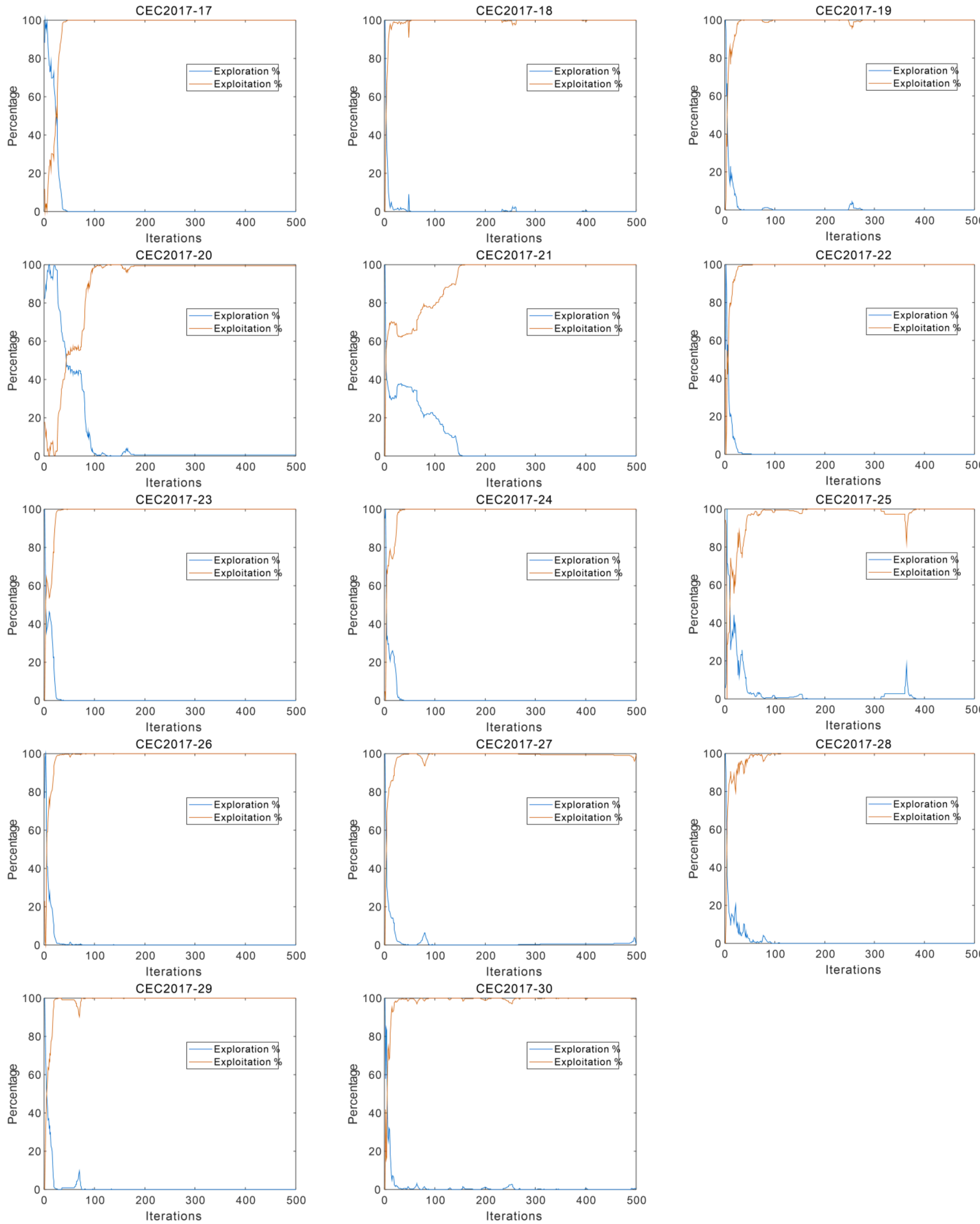

Figure 12. Exploration and exploitation balance detection based on CEC2017 test function.

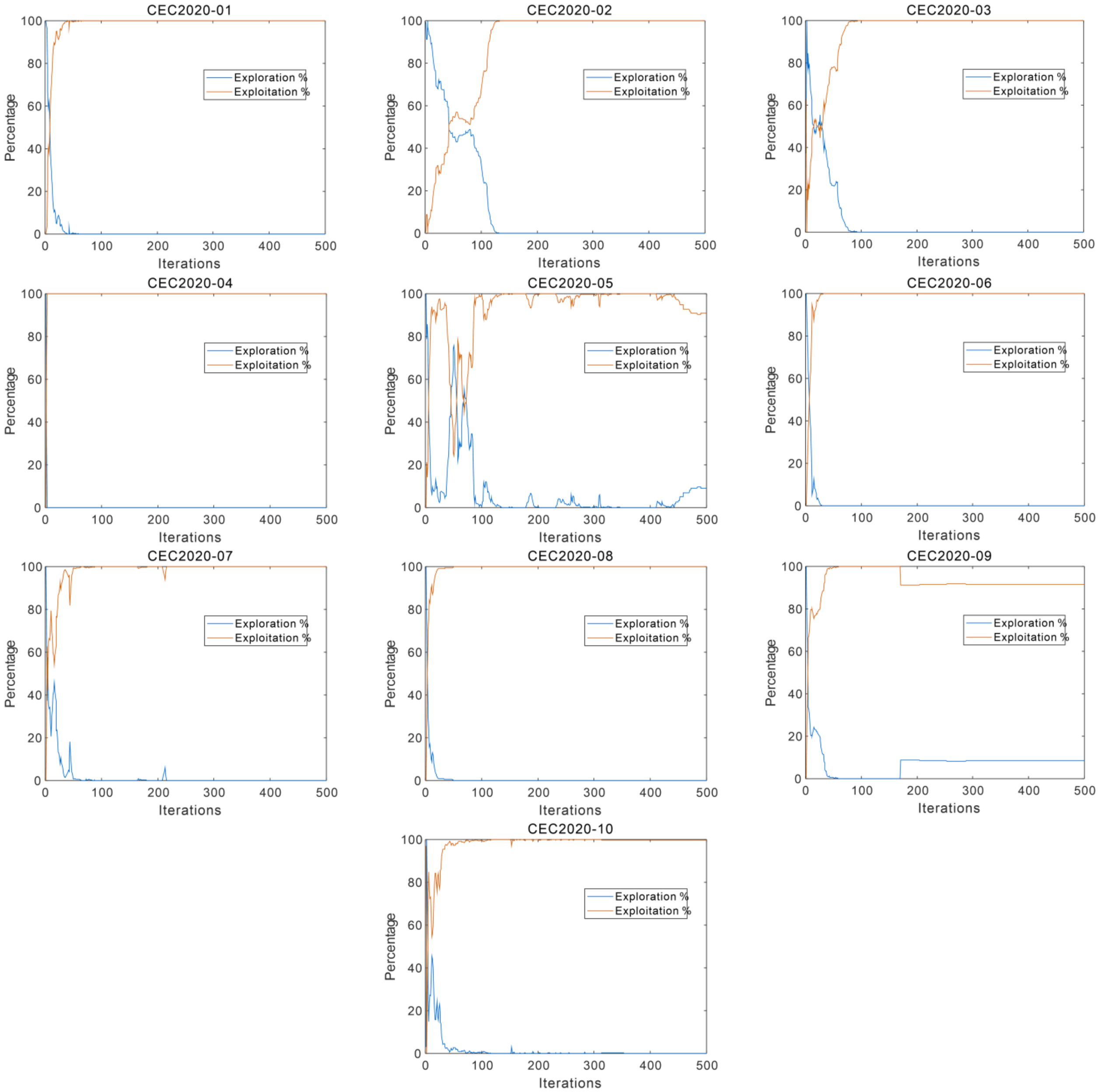

Figure 13. Exploration and exploitation balance detection based on CEC2020 test function.

$$Div_j = \frac{1}{n}\sum_{i=1}^{n} |median(x^j) - x_i^j| \quad (23)$$

$$Div = \frac{1}{m}\sum_{j=1}^{m} Div_j \quad (24)$$

Median(x^j) is the median of the complete population with a dimension of j, $x_i^{\,j}$ represents the j dimension of the i-th solution, and m is the number of decision variables in the optimization problem. The diversity of Div_j in each dimension is represented as the length between the median of the j dimension and that dimension, and the average value is taken. Calculate the complete population *Div* using Div_j for each dimension. It is worth mentioning that these two values will be calculated for each generation. Next, we can use this data to evaluate the relationship between exploration and exploitation.

$$XPL\% = \left(\frac{Div}{Div_{max}}\right) \times 100 \quad (25)$$

$$XPT\% = \left(\frac{|Div - Div_{max}|}{Div_{max}}\right) \times 100 \quad (26)$$

$Div_{\max}$ is the maximum diversity value during full optimization. *XPL*% is the percentage of exploration, and the percentage of exploitation is *XPT*%. They are complementary. In this experiment, dim = 10, T = 500, N = 30. In Figures 12 and 13, we can see that the optimal value of most functions can be found at the early stage of iteration, and a relatively good balance could be achieved. In Figure 13 we can see that for the relatively simple CEC2020-04 benchmark function, it takes only a few iterations to find a nice equilibrium position. The optimal values for CEC2020-05 and CEC2020-09 are obtained using 90% exploitation and 10% exploration. In the CEC2017 and CEC2020 test functions, the balance of the IBWO is dynamic, which means that the IBWO can continuously change between exploration and exploitation without being troubled by local optimality. Therefore, IBWO can achieve a relative balance between exploration and exploitation, so that the search scope can be wide, and the search speed can be fast.

5.4. Sensitivity analysis of W_t and constant on IBWO

W_t and constant are important parameters in IBWO, which affect the stability of the algorithm and significantly affect the performance of the algorithm. In order to conform to the actual situation, W_t = 0.05–0.5 and constant = 1–10 were selected for the experiment in this paper, and the results of 30 runs in CEC2020 were recorded. The specific data are shown in Tables 6 and 7. It is not difficult to see that when W_t = 0.35 and constant = 5, the algorithm has the best performance.

Table 6. Parameter analysis on W_t.

CEC2020	W_t	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
CEC2020-01	min	1.65E+02	1.56E+02	1.06E+02	1.17E+02	1.05E+02	1.09E+02	**1.00E+02**	1.05E+02	1.00E+02	1.01E+02
	mean	3.37E+03	2.97E+03	2.82E+03	3.22E+03	3.08E+03	2.87E+03	**2.00E+03**	3.25E+03	2.72E+03	2.92E+03
	avg	3.41E+03	2.93E+03	3.29E+03	2.78E+03	3.45E+03	3.29E+03	**1.63E+03**	2.75E+03	2.89E+03	3.69E+03
CEC2020-02	min	1.11E+03	1.36E+03	1.41E+03	1.42E+03	1.12E+03	1.35E+03	**1.10E+03**	1.14E+03	1.53E+03	1.33E+03
	mean	2.01E+03	2.57E+03	2.44E+03	2.60E+03	2.51E+03	2.66E+03	**1.69E+03**	2.58E+03	2.57E+03	2.55E+03
	avg	3.80E+02	5.13E+02	5.22E+02	4.55E+02	4.48E+02	3.72E+02	**2.94E+02**	5.01E+02	3.65E+02	4.82E+02
CEC2020-03	min	7.27E+02	7.37E+02	7.35E+02	7.39E+02	7.32E+02	7.24E+02	**7.14E+02**	7.36E+02	7.33E+02	7.37E+02
	mean	7.61E+02	7.65E+02	7.62E+02	7.63E+02	7.65E+02	7.65E+02	**7.53E+02**	7.64E+02	7.63E+02	7.62E+02
	avg	2.74E+01	2.42E+01	**1.92E+01**	2.00E+01	2.26E+01	2.90E+01	2.48E+01	2.26E+01	2.45E+01	2.04E+01
CEC2020-04	min	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**
	mean	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**
	avg	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
CEC2020-05	min	2.24E+03	2.53E+03	2.77E+03	8.53E+03	4.90E+03	3.54E+03	**1.96E+03**	4.93E+03	3.29E+03	5.35E+03
	mean	4.33E+03	6.52E+04	7.05E+04	5.90E+04	8.93E+04	9.01E+04	**3.74E+03**	8.30E+04	8.28E+04	6.91E+04
	avg	3.95E+03	6.28E+04	7.44E+04	5.55E+04	9.23E+04	9.22E+04	**1.65E+03**	8.00E+04	6.94E+04	6.87E+04
CEC2020-06	min	1.60E+03	1.60E+03	1.60E+03	1.60E+03	1.60E+03	1.60E+03	**1.60E+03**	1.60E+03	1.60E+03	1.60E+03
	mean	1.75E+03	1.79E+03	1.79E+03	1.80E+03	1.79E+03	1.80E+03	**1.71E+03**	1.80E+03	1.78E+03	1.79E+03
	avg	8.89E+01	1.19E+02	1.20E+02	1.22E+02	1.08E+02	9.47E+01	**7.72E+01**	1.27E+02	1.15E+02	1.28E+02
CEC2020-07	min	2.10E+03	2.10E+03	2.10E+03	2.10E+03	2.12E+03	2.10E+03	**2.10E+03**	2.11E+03	2.12E+03	2.13E+03
	mean	5.63E+03	4.04E+03	3.79E+03	4.59E+03	4.38E+03	4.42E+03	**2.82E+03**	3.78E+03	3.57E+03	4.06E+03
	avg	5.01E+03	4.04E+03	4.93E+03	5.47E+03	4.82E+03	7.99E+03	**1.45E+03**	4.23E+03	3.12E+03	5.36E+03

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CEC2020	W_t	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
CEC2020-08	min	2.24E+03	2.30E+03	2.30E+03	2.30E+03	2.30E+03	2.30E+03	**2.21E+03**	2.30E+03	2.30E+03	2.30E+03
	mean	2.31E+03	2.32E+03	2.33E+03	2.32E+03	2.32E+03	2.32E+03	**2.30E+03**	2.32E+03	2.32E+03	2.31E+03
	avg	1.58E+01	2.15E+01	6.86E+01	2.53E+01	3.50E+01	2.52E+01	1.79E+01	3.18E+01	2.48E+01	**8.32E+00**
CEC2020-09	min	2.50E+03	2.50E+03	2.68E+03	2.50E+03	2.64E+03	2.69E+03	**2.40E+03**	2.73E+03	2.50E+03	2.50E+03
	mean	**2.72E+03**	2.75E+03	2.75E+03	2.75E+03	2.75E+03	2.75E+03	2.72E+03	2.76E+03	2.73E+03	2.74E+03
	avg	6.90E+01	4.95E+01	2.41E+01	4.87E+01	2.77E+01	1.83E+01	6.92E+01	**1.71E+01**	6.43E+01	6.47E+01
CEC2020-10	min	2.90E+03	2.90E+03	2.90E+03	2.90E+03	2.90E+03	2.90E+03	**2.90E+03**	2.90E+03	2.90E+03	2.90E+03
	mean	2.93E+03	2.94E+03	2.94E+03	2.95E+03	2.95E+03	2.94E+03	**2.93E+03**	2.93E+03	2.94E+03	2.95E+03
	avg	2.43E+01	2.81E+01	2.89E+01	2.92E+01	3.08E+01	2.33E+01	**2.30E+01**	2.90E+01	2.78E+01	3.15E+01

Table 7. Parameter analysis on constant.

CEC2020	constant	1	2	3	4	5	6	7	8	9	10
CEC2020-01	min	1.06E+02	1.02E+02	1.35E+02	1.09E+02	**1.00E+02**	1.02E+02	1.01E+02	1.08E+02	1.13E+02	1.05E+02
	mean	2.85E+03	2.36E+03	2.06E+03	2.06E+03	**1.87E+03**	2.95E+03	3.18E+03	2.88E+03	2.71E+03	2.78E+03
	avg	2.89E+03	3.26E+03	2.81E+03	2.46E+03	**1.86E+03**	2.80E+03	2.82E+03	2.99E+03	3.18E+03	2.82E+03
CEC2020-02	min	1.14E+03	1.44E+03	1.59E+03	1.26E+03	**1.10E+03**	1.54E+03	1.12E+03	1.22E+03	1.41E+03	1.45E+03
	mean	1.78E+03	2.60E+03	2.63E+03	2.44E+03	**1.75E+03**	2.60E+03	2.49E+03	2.56E+03	2.56E+03	2.48E+03
	avg	3.59E+02	4.86E+02	3.66E+02	5.18E+02	**3.47E+02**	4.14E+02	5.84E+02	5.00E+02	4.25E+02	4.50E+02
CEC2020-03	min	7.24E+02	7.29E+02	7.32E+02	7.35E+02	**7.17E+02**	7.38E+02	7.25E+02	7.25E+02	7.31E+02	7.28E+02
	mean	7.66E+02	7.59E+02	7.64E+02	7.63E+02	**7.49E+02**	7.64E+02	7.64E+02	7.63E+02	7.71E+02	7.69E+02
	avg	3.03E+01	2.28E+01	2.61E+01	2.41E+01	**2.05E+01**	2.25E+01	2.59E+01	2.71E+01	2.53E+01	2.64E+01

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CEC2020	W_t	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
CEC2020-04	min	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**
	mean	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**
	avg	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
CEC2020-05	min	2.37E+03	2.39E+03	3.75E+03	2.40E+03	**2.05E+03**	7.20E+03	2.88E+03	4.00E+03	4.58E+03	3.84E+03
	mean	4.56E+03	5.81E+04	7.30E+04	7.13E+04	**3.71E+03**	6.97E+04	5.99E+04	7.46E+04	8.04E+04	8.88E+04
	avg	2.34E+03	5.39E+04	1.15E+05	6.94E+04	**1.60E+03**	5.77E+04	5.62E+04	7.63E+04	8.86E+04	6.99E+04
CEC2020-06	min	1.60E+03	1.60E+03	1.72E+03	1.60E+03	**1.60E+03**	1.60E+03	1.60E+03	1.60E+03	1.60E+03	1.60E+03
	mean	1.73E+03	1.77E+03	1.80E+03	1.79E+03	**1.73E+03**	1.78E+03	1.79E+03	1.78E+03	1.80E+03	1.79E+03
	avg	9.58E+01	1.37E+02	1.10E+02	1.32E+02	**6.90E+01**	1.08E+02	1.32E+02	1.29E+02	1.25E+02	1.15E+02
CEC2020-07	min	2.12E+03	2.12E+03	2.13E+03	2.10E+03	**2.10E+03**	2.11E+03	2.14E+03	2.15E+03	2.15E+03	2.12E+03
	mean	3.98E+03	3.29E+03	3.62E+03	3.93E+03	**2.82E+03**	4.95E+03	5.19E+03	3.81E+03	2.83E+03	3.60E+03
	avg	3.71E+03	2.93E+03	4.99E+03	3.93E+03	**1.39E+03**	4.98E+03	6.42E+03	4.50E+03	1.65E+03	2.75E+03
CEC2020-08	min	2.23E+03	2.30E+03	2.30E+03	2.30E+03	**2.21E+03**	2.30E+03	2.30E+03	2.30E+03	2.30E+03	2.30E+03
	mean	2.31E+03	2.32E+03	2.32E+03	2.32E+03	**2.30E+03**	2.32E+03	2.32E+03	2.32E+03	2.32E+03	2.32E+03
	avg	5.30E+01	1.68E+01	4.79E+01	5.03E+01	**9.21E+00**	2.51E+01	2.78E+01	2.70E+01	3.88E+01	3.62E+01
CEC2020-09	min	2.50E+03	2.50E+03	2.54E+03	2.73E+03	**2.50E+03**	2.73E+03	2.73E+03	2.50E+03	2.50E+03	2.50E+03
	mean	**2.73E+03**	2.74E+03	2.74E+03	2.76E+03	2.73E+03	2.76E+03	2.76E+03	2.75E+03	2.74E+03	2.73E+03
	avg	6.95E+01	4.85E+01	3.96E+01	**1.63E+01**	5.32E+01	1.64E+01	2.00E+01	4.83E+01	6.85E+01	6.95E+01
CEC2020-10	min	**2.60E+03**	2.90E+03	2.90E+03	2.90E+03	2.90E+03	2.90E+03	2.90E+03	2.90E+03	2.90E+03	2.60E+03
	mean	**2.92E+03**	2.94E+03	2.94E+03	2.95E+03	2.93E+03	2.95E+03	2.94E+03	2.94E+03	2.94E+03	2.92E+03
	avg	6.48E+01	2.82E+01	2.74E+01	3.30E+01	2.34E+01	**2.25E+01**	2.94E+01	2.75E+01	3.00E+01	6.54E+01

6. Constrained engineering design problems

The essence of studying optimization algorithms is to solve practical problems. Therefore, in this section, we will test the application of the IBWO in practical engineering problems. This section selects five classic practical engineering problems [41,56,58] and compares them with other optimization algorithms to verify the applicability of IBWO in engineering problems. In each engineering problem, we set the population size to 30 and the maximum number of iterations to 500. The average value obtained by running the IBWO for an average of 30 times is used as the experimental result.

6.1 *Car crashworthiness design problem*

Car collision are a very common problem in life. With the increasing number of traffic accidents in recent years, we have also noticed that cars with good safety performance can protect passengers from being hurt to a certain extent. In the problem of this study, we study taking the minimum of 10 variables while having 11 constraints. The schematic diagram of the vehicle collision problem is shown in Figure 14.

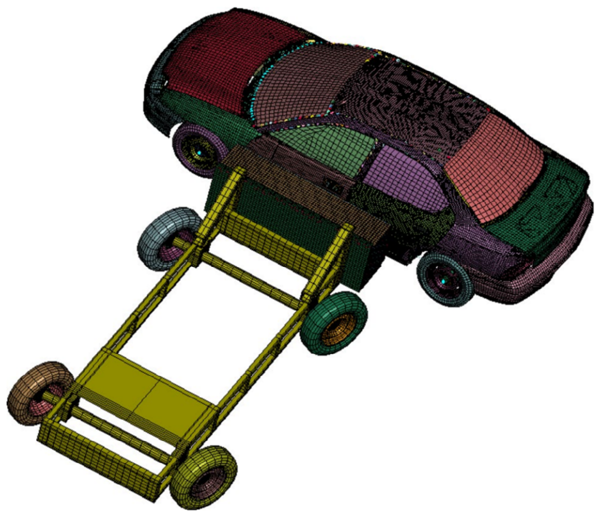

Figure 14. Car crashworthiness design.

The mathematical formulation of this problem is shown below:
Minimize

$$f(\vec{x}) = \text{Weight}, \tag{27}$$

Subject to

$$g_1(\vec{x}) = F_a(\text{load in abdomen}) \leq 1 \text{ kN}, \tag{28}$$

$$g_2(\vec{x}) = V \times Cu(\text{dummy upper chest}) \leq 0.32 \text{ m/s}, \tag{29}$$

$$g_3(\vec{x}) = V \times Cm(\text{dummy middle chest}) \leq 0.32 \text{ m/s}, \tag{30}$$

$$g_4(\vec{x}) = V \times Cl(\text{dummy lower chest}) \leq 0.32 \text{ m/s}, \tag{31}$$

$$g_5(\vec{x}) = \Delta_{\mathrm{ur}}(\text{upper rib deflection}) \leq 32\ \text{mm}, \tag{32}$$

$$g_6(\vec{x}) = \Delta_{\mathrm{mr}}(\text{middle rib deflection}) \leq 32\ \text{mm}, \tag{33}$$

$$g_7(\vec{x}) = \Delta_{\mathrm{lr}}(\text{lower rib deflection}) \leq 32\ \text{mm}, \tag{34}$$

$$g_8(\vec{x}) = F(\text{Public force})_p \leq 4\ \text{kN}, \tag{35}$$

$$g_9(\vec{x}) = V_{\mathrm{MBP}}(Velocity\ of\ V - \text{Pillar}\ \ at\ middle\ point) \leq 9.9\ \text{mm/ms}, \tag{36}$$

$$g_{10}(\vec{x}) = V_{\mathrm{FD}}(Velocity\ of\ front\ door\ at\ V - Pillar) \leq 15.7\ \frac{\text{mm}}{\text{ms}}. \tag{37}$$

Variable range

$$0.5 \leq x_1 - x_7 \leq 1.5,\ \ x_8, x_9 \in (0.192, 0.345), -30 \leq x_{10}, x_{11} \leq 30, \tag{38}$$

Table 8 shows the final results. In complex car collision problems, the IBWO still performs well. x_3, x_5, and x_7 all reached the optimal value of 0.5, while x_1 also approached the optimal value. Overall, the results were also relatively good.

Table 8. Experimental results of car crashworthiness design.

Algorithm	IBWO	GTOA [22]	MALO [59]	ROLGWO [60]	WOA [9]	HHOCM [61]	ROA [11]
x_1	0.506259762	0.662833	0.5	0.501255	0.8521	0.500164	0.5
x_2	1.229381122	1.217247	1.2281	1.245551	1.2136	1.248612	1.22942
x_3	0.5	0.734238	0.5	0.500046	0.6604	0.659558	0.5
x_4	1.201402384	1.11266	1.2126	1.180254	1.1156	1.098515	1.21197
x_5	0.5	0.613197	0.5	0.500035	0.5	0.757989	0.5
x_6	1.136778751	0.670197	1.308	1.16588	1.195	0.767268	1.37798
x_7	0.5	0.615694	0.5	0.500088	0.5898	0.500055	0.50005
x_8	0.345	0.271734	0.3449	0.344895	0.2711	0.343105	0.34489
x_9	0.192	0.23194	0.2804	0.299583	0.2769	0.192032	0.19263
x_{10}	1.076816248	0.174933	0.4242	3.59508	4.3437	2.898805	0.62239
x_{11}	0.134091706	0.462294	4.6565	2.29018	2.2352	-	-
Best Weight	23.22327984	25.70607	23.2294	23.22243	25.83657	24.48358	23.23544

6.2. Cantilever beam design problem

The experimental model in this section is a cantilever beam connected with five hollow bricks, as shown in Figure 15. The purpose of this experiment is to make the width X_i (i = 1, 2, …, 5) of five different bricks in the cantilever beam meet the actual demand while minimizing the weight.

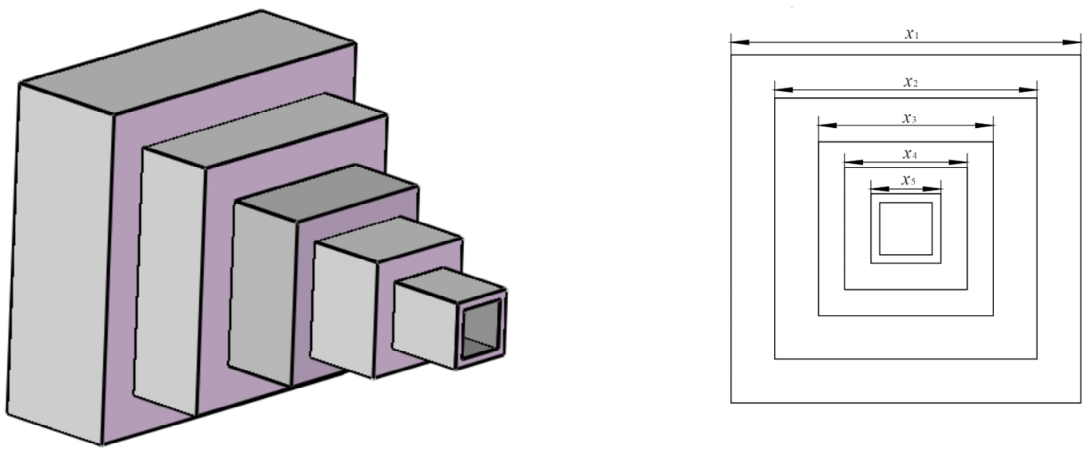

Figure 15. Model of cantilever beam design.

The mathematical formulation of this problem is shown below.
Consider

$$x = [x_1\ x_2\ x_3\ x_4\ x_5] \tag{39}$$

Objective function:

$$f(x) = 0.0624(x_1 + x_2 + x_3 + x_4 + x_5) \tag{40}$$

Subject to

$$g(x) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \le 0 \tag{41}$$

Boundaries

$$0.01 \le x_i \le 100\ (i = 1, 2,\ \cdots 5) \tag{42}$$

See Table 9 for specific data on cantilever beam engineering design. According to Figure 15, we can observe that variable X_i (i = 1, 2, ⋯, 5) is decreasing. IBWO, MSCSO and GSA meet this practical need, while WOA and GSA do not. The results show that the IBWO performs well while meeting the actual needs.

Table 9. Experimental results of cantilever beam design.

Algorithm	Optimal values for variables					Optimum weight
	x_1	x_2	x_3	x_4	x_5	
IBWO	6.015626591	5.3098906	4.4940231	3.497323751	2.156820232	1.339957942
MSCSO [41]	6.01265	5.315452	4.492016	3.501096	2.152481	1.33995853466334
WOA [9]	5.1261	5.6188	5.0952	3.9329	2.3219	1.37873150673956
PSO [7]	6.0040	5.2950	4.4915	3.5125	2.1710	1.33998298081255
GSA [29]	5.6052	4.9553	5.6619	3.1959	3.2026	1.41155753917296

6.3. Tension/Compression spring design problem

The model for the tension/pressure spring design problem is shown in Figure 16. The purpose of this experiment is to reduce the weight of the spring under four constraints. The constraint conditions are minimum deviation (g_1), shear stress (g_2), impact frequency (g_3), and outer diameter limit (g_4). The corresponding variables include coil diameter d, average coil diameter D, and effective coil number N. $f(x)$ is the weight of the spring.

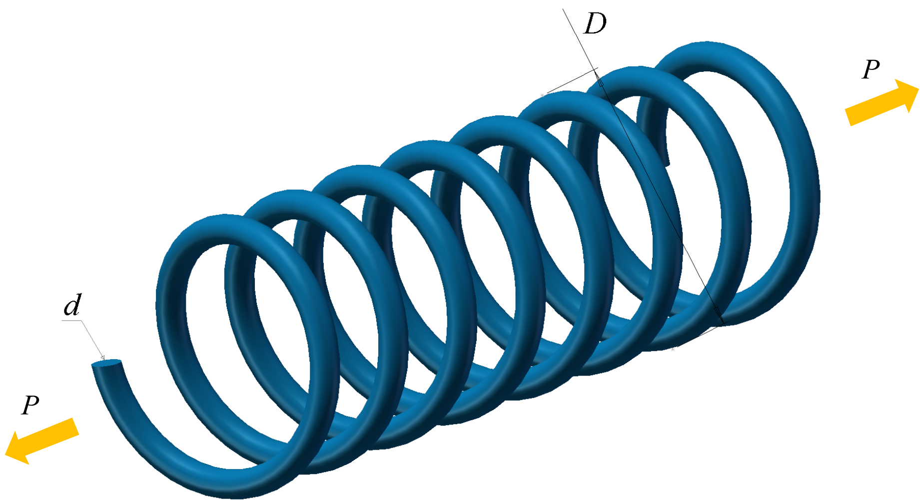

Figure 16. Tension/Compression spring design.

The mathematical formulation of this problem is shown below:
Consider

$$x = [x_1 x_2 x_3]=[d\ D\ N] \tag{43}$$

Minimize

$$f(x) = (x_3 + 2) \times x_2 \times x_1^2 \tag{44}$$

Subject to

$$g_1(x) = 1 - \frac{x_3 \times x_2^3}{71785 \times x_1^4} \leq 0 \tag{45}$$

$$g_2(x) = \frac{4 \times x_2^2 - x_1 \times x_2}{12566 \times x_1^4} + \frac{1}{5108 \times x_1^2} - 1 \leq 0 \tag{46}$$

$$g_3(x) = 1 - \frac{140.45 \times x_1}{x_2^2 \times x_3} \leq 0 \tag{47}$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0 \tag{48}$$

Variable range

$$0.05 \leq x_1 \leq 2.0; 0.25 \leq x_2 \leq 1.3; 2.0 \leq x_3 \leq 15.0 \tag{49}$$

In this experiment, we can see that the IBWO performs better than other algorithms, and the wire diameter d takes the optimal value, while D and N are near the optimal value. See Table 10 for calculation results. Combined with the data in the table, when d = 0.05, D = 0.374432881, N = 8.546568594, the optimal value of 0.009872455 can be obtained. The results show that the IBWO has stronger competitiveness for this engineering problem.

Table 10. Experimental results of tension/compression spring design.

Algorithm	*d*	*D*	*N*	Best weight
IBWO	0.05	0.374432881	8.546568594	0.009872455
BWO [6]	0.0517	0.3568	11.3132	0.012703
PSO [7]	0.051728	0.357644	11.24454	0.012675
HHO [54]	0.051796	0.359305	11.13886	0.012665
SSA [62]	0.051207	0.345215	12.00403	0.012676
GA [35]	0.05148	0.351661	11.6322	0.012705
MVO [26]	0.05251	0.37602	10.33513	0.01279
DE [32]	0.051609	0.354714	11.41083	0.01267

6.4. Multiple disc clutch brake problem

Multi-disc clutch brake is a kind of clutch and brake commonly used in industrial production and automobile equipment. This clutch can provide high torque and braking force, so it is widely used in industrial fields. The multi-disc clutch brake studied in this section has five relevant parameters, which are the number of friction surfaces Z, driving force F, inner radius r_i, outer radius r_o and disk thickness t. In addition, it has eight constraint variables. The diagram is shown in Figure 17.

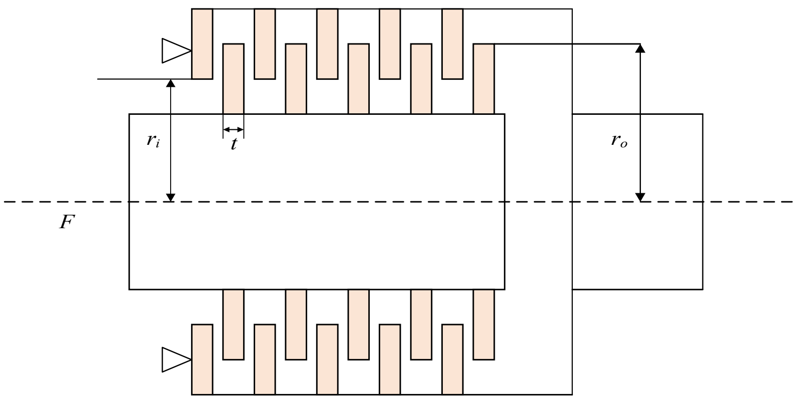

Figure 17. Model of multiple disc clutch brake.

The mathematical model of multiple disc clutch brake design is as follows:

Consider:

$$\vec{x} = [x_1 x_2 x_3 x_4 x_5] = [r_i r_o t F Z] \tag{50}$$

Objective function:

$$f(x) = II(r_o^2 - r_i^2)t(Z+1)\rho \ \ (\rho = 0.0000078) \tag{51}$$

Subject to:

$$g_1(x) = r_o - r_i - \Delta r \geq 0 \tag{52}$$

$$g_2(x) = l_{max} - (Z+1)(t+\delta) \geq 0 \tag{53}$$

$$g_3(x) = P_{max} - P_{rz} \geq 0 \tag{54}$$

$$g_4(x) = P_{max}v_{srmax} - P_{rz}v_{sr} \geq 0 \tag{55}$$

$$g_5(x) = v_{srmax} - v_{sr} \geq 0 \tag{56}$$

$$g_6(x) = T_{max} - T \geq 0 \tag{57}$$

$$g_7(x) = M_{\hbar} - sM_s \geq 0 \tag{58}$$

$$g_8(x) = T \geq 0 \tag{59}$$

Variable range:

$$60 \leq x_1 \leq 80, 90 \leq x_2 \leq 110, 1 \leq x_3 \leq 3, 600 \leq x_4 \leq 1000, 2 \leq x_5 \leq 9 \tag{60}$$

Other parameters:

$$M_{\hbar} = \frac{2}{3}\mu FZ\frac{r_o^3 - r_i^2}{r_o^2 - r_i^3}, P_{rz} = \frac{F}{II(r_o^2 - r_i^2)}, \tag{61}$$

$$v_{rz} = \frac{2II(r_o^3 - r_i^3)}{90(r_o^2 - r_i^2)}, T = \frac{I_z IIn}{30\left(M_{\hbar} + M_f\right)} \tag{62}$$

$$\Delta r = 20\text{ mm}, I_z = 55\text{ kgmm}^2, Pmax_{max} \tag{63}$$

$$T_{max} = 15\text{ s},\ \mu = 0.5,\ s = 1.5,\ M_s = 40\text{ Nm},\ M_f = 3\text{ Nm} \tag{64}$$

$$n = 250\text{ rpm},\ v_{sr\,max} = 10\text{ m/s},\ l_{max} = 30\text{ mm} \tag{65}$$

We present the results in Table 11. Observing the table data, we can find that when $x_1 = 69.99997848$, $x_2 = 90$, $x_3 = 1$, $x_4 = 781.7974$, $x_5 = 2$, the optimal value of 0.235242679 can be obtained. Obviously, the IBWO is effective for this engineering problem.

Table 11. Concrete data of multiple disc clutch brake design problem.

Algorithm	Optimal values for variables					Optimum weight
	x_1	x_2	x_3	x_4	x_5	
IBWO	69.99997848	90	1	781.7974	2	0.235242679
CMVO [63]	70	90	1	910	3	0.313656
RSA [64]	70.0347	90.0349	1	801.7285	2.974	0.31176
MVO [26]	70	90	1	910	3	0.313656
TLBO [15]	70	90	1	810	3	0.313656611
MFO [12]	70	90	1	910	3	0.313656
WCA [65]	70	90	1	910	3	0.313656

6.5. Speed reducer design problem

The speed reducer design problem in this study is essentially a problem of solving the minimum mass. The problem has four design constraints: the bending stress of the tooth, the covering stress, the shaft's lateral deflection and the stress in the shaft. The model is shown in Figure 18.

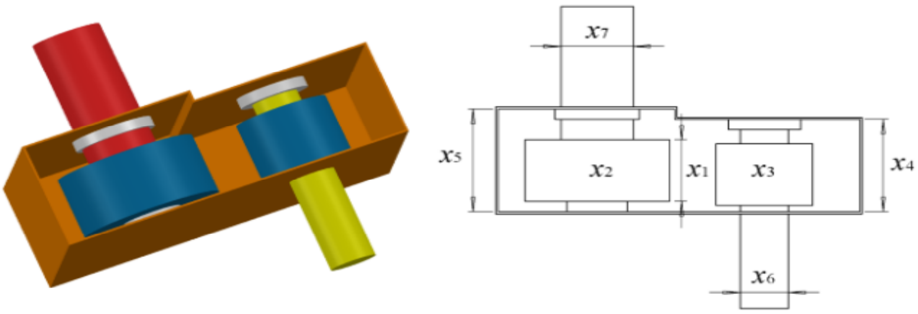

Figure 18. Model of speed reducer design.

The mathematical model of reducer design is as follows:
Objective function:

$$f(\vec{x}) = 07854 \times x_1 \times x_2{}^2 \times (3.3333 \times x_3{}^2 + 14.9334 \times x_3 - 43.0934) - 1.508 \times x_1 \times (x_6{}^2 + x_7{}^2) + 7.4777 \times x_6{}^3 + x_7{}^3 + 0.7854 \times x_4 \times x_6{}^2 + x_5 \times x_7{}^2 \tag{66}$$

Subject to:

$$g_1(\vec{x}) = \frac{27}{x_1 \times x_2{}^2 \times x_3} - 1 \leq 0 \tag{67}$$

$$g_2(\vec{x}) = \frac{397.5}{x_1 \times x_2{}^2 \times x_3{}^2} - 1 \leq 0 \tag{68}$$

$$g_3(\vec{x}) = \frac{1.93 \times x_4{}^3}{x_2 \times x_3 \times x_6{}^4} - 1 \leq 0 \tag{69}$$

$$g_4(\vec{x}) = \frac{1.93 \times x_5{}^3}{x_2 \times x_3 \times x_7{}^4} - 1 \leq 0 \tag{70}$$

$$g_5(\vec{x}) = \frac{1}{110 \times x_6{}^3} \times \sqrt{(\frac{745 \times x_4}{x_2 \times x_3})^2 + 16.9 \times 10^6} - 1 \leq 0 \tag{71}$$

$$g_6(\vec{x}) = \frac{1}{85 \times x_7{}^3} \times \sqrt{(\frac{745 \times x_5}{x_2 \times x_3})^2 + 16.9 \times 10^6} - 1 \leq 0 \tag{72}$$

$$g_7(\vec{x}) = \frac{x_2 \times x_3}{40} - 1 \leq 0 \tag{73}$$

$$g_8(\vec{x}) = \frac{5 \times x_2}{x_1} - 1 \leq 0 \tag{74}$$

$$g_9(\vec{x}) = \frac{x_1}{12\times x_2} - 1 \leq 0 \tag{75}$$

$$g_{10}(\vec{x}) = \frac{1.5\times x_6+1.9}{x_4} - 1 \leq 0 \tag{76}$$

$$g_{11}(\vec{x}) = \frac{1.1\times x_7+1.9}{x_5} - 1 \leq 0 \tag{77}$$

Boundaries:

$$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.3 \leq x_5 \leq 8.3, 2.9 \leq x_6 \leq 3.9, 5 \leq x_7 \leq 5.5 \tag{78}$$

As can be seen from Table 12, when x = [3.49760, 0.7, 17, 7.3, 7.8, 3.3500564, 5.2855316], the minimum weight obtained by IBWO is 2995.437366, which is the first in comparison to the other algorithm.

Table 12. Speed reducer design problem.

Algorithm	Optimal values for variables							Optimal weight
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
IBWO	3.49760	0.7	17	7.3	7.8	3.3500564	5.2855316	2995.437366
AOA [27]	3.50384	0.7	17	7.3	7.72933	3.35649	5.2867	2997.9157
HS [20]	3.520124	0.7	17	8.37	7.8	3.36697	5.288719	3029.002
CS [66]	3.5015	0.7	17	7.605	7.8181	3.352	5.2875	3000.981
MFO [12]	3.497455	0.7	17	7.82775	7.712457	3.351787	5.286352	2998.94083
WSA [67]	3.5	0.7	17	7.3	7.8	3.350215	5.286683	2996.348225
AAO [68]	3.499	0.6999	17	7.3	7.8	3.3502	5.2872	2996.783
FA [69]	3.507495	0.7001	17	7.719674	8.080854	3.351512	5.287051	3010.137492
RSA [64]	3.50279	0.7	17	7.30812	7.74715	3.35067	5.28675	2996.5157

7. Conclusions

The BWO is a kind of optimization algorithm which is easy to implement and has strong searching ability. However, the effect of BWO in the exploration stage is not clear, and when faced with complex functions, it often fails to achieve good results, resulting in premature convergence or algorithm stagnation and other problems. We developed an enhanced variant of the BWO, IBWO, to address the above weaknesses. In IBWO, the exploration phase was replaced with GAS, inspired by the fact that beluga whales hunt in groups in nature. Beluga whales are vulnerable to threats in nature from their natural enemy, the tiger shark. Weak beluga whales cannot protect themselves and need to seek the protection of strong individuals. The GAS is clearly in line with natural law. At the same time, we can also see its excellent performance in the experiment. In addition, we use two search strategies, DPIS and QIS, to generate new valid locations. The collaborative use of these strategies improves the population diversity of the IBWO and better maintains the balance between exploration and exploitation. Moreover, we used the Wilcoxon rank sum test to test the significant differences between the IBWO and other algorithms. At the same time, the parameters of GAS are analyzed effectively. Finally, five engineering experiments are carried out, and good solutions were proposed by IBWO.

From experimental performance, functional images, statistical analysis, and engineering problems, we can derive a conclusion:

• The group action strategy (GAS) dynamic pinhole imaging strategy (DPIS) and quantitative interpolation strategy (QIS) in this article can effectively enhance the ability of algorithm exploration and exploitation, enabling the IBWO to break the stagnation of the algorithm.

• By comparing and analyzing the results of CEC2017 and CEC2020 test functions of different dimensions, the actual performance of IBWO is verified

• We find that IBWO can successfully solve nonlinear, complex and constrained engineering problems.

In future work, we will investigate the binary version of IBWO and use it to solve binary problems, such as feature selection problems. At the same time, it will improve the performance of IBWO in engineering problems, so that it can meet the requirements of most real-world engineering problems

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Conflict of interest

We declare that we have no conflict of interest.

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