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Research article

Threat assessment of aerial targets based on improved GRA-TOPSIS method and three-way decisions

Yongfeng Yin[1], **Routing Zhang**[2] **and Qingran Su**[3,*]

[1] School of Software, Beihang University, Beijing 100089, China
[2] School of Reliability and Systems Engineering, Beihang University, Beijing 100089, China
[3] School of Computer Science and Engineering, Beihang University, Beijing 100089, China

* **Correspondence:** Email: suqingran@buaa.edu.cn.

Abstract: Target threat assessment is a critical aspect of information warfare and can offer valuable auxiliary support to combat command decision-making. Aiming to address the shortcomings of three decision-making methods in air combat target assessment, such as the inability to effectively handle uncertain situation information and quantitatively rank the decision-making targets according to their importance, a dynamic intuitionistic fuzzy decision model based on the improved GRA-TOPSIS method and three-way decisions is proposed. First, the target attribute weight is obtained by cosine intuitionistic fuzzy entropy algorithm; then, a novel intuitionistic fuzzy distance measure is introduced, and grey incidence analysis and TOPSIS are used to build the conditional probability for three-way decisions that fully utilize the existing information and reflect the consistency of dynamic change trend; finally, the comprehensive loss function matrix is constructed and the threat classification results are obtained using the decision rules. The example analysis shows that the proposed method can not only effectively handle complex battlefield situations and dynamic uncertain information, but it can also classify targets, improving the effectiveness and rationality of decision-making and providing a reference basis for scientific command decision-making.

Keywords: intuitionistic fuzzy set; distance measure; threat assessment; three-way decision

1. Introduction

In modern combat, large-scale, intelligent weapons equipment and systems are becoming increasingly vital, and intelligent decision-making has become the core of the battle. Threat assessment (TA) is a critical element in military decisions. With the advancement of battle theory and enhanced combat equipment, troops' combat effectiveness has dramatically improved, defining modern warfare as adversarial and timely [1]. As a result, it establishes more stringent criteria for enemy target threat assessment procedures. Threat assessment forecasts the threat level of enemy targets based on information fusion and aids in battlefield command and decision-making [2]. Therefore, intelligent threat assessment in air combat has significant research implications as well as vast application prospects.

Recently, threat assessment methods have mainly included multi-attribute decision theory [3, 4], intuitionistic fuzzy sets (IFS) [5–7], grey theory [8–10], Bayesian network [11, 12], technique for ordering preference by similarity to an ideal solution (TOPSIS) [13, 14], adaptive network based fuzzy inference inference system (ANFIS) [15,16] and so on. A threat assessment model with respect to small data sets based on a Bayesian network was proposed [11]. The model was obtained by the modeling method based on the improved Bayesian Information Criterion (BIC) score. The intelligent situation awareness (SA) modeling method was based on the Fuzzy Grey Cognitive Map (FGCM) [17], which can better deal with complex battlefield situations and handle uncertain information. A threat assessment model of attribute reduction and back propagation (BP) neural network [18] was established to assess and judge threat degree of the target. The assessment method based on Grey Relational Analysis (GRA)-TOPSIS was proposed in [19]. An improved adaptive network was proposed in [1] based on fuzzy inference system model. Although the above methods can effectively carry out threat assessment, they still have the following problems: Bayesian network and neural network methods need a lot of prior information, which makes it difficult to meet the dynamic and real-time characteristics of threat assessment; the fuzzy cognitive map model cannot solve the uncertainty problem of experts in the initial setting of the model; grey theory relies on expert experience and there are many subjective factors.

With the uncertainty and variability of battlefield situation information, threat assessment needs to effectively handle dynamic battlefield environments. Many scholars have studied threat assessment in a fuzzy decision-making environment. Intuitive fuzzy sets [20] can more accurately represent uncertain information. The method based on IFS and multi-attribute decision theory has been widely used in threat assessment problems [21–24]. However, this method still has limitations: Traditional threat assessment methods often only rank threat targets, requiring decision-makers to further judge threat levels and choose priority combat targets, making it difficult to deal with complex and ever-changing environments. Moreover, a target is either accepted or rejected in two-way decision-making method, which can easily result in erroneous judgments. Yao et al. [25,26] proposed three-way decisions, which can give reasonable explanations to three regions of rough sets and linked them with three decision actions. Applying three-way decision-making (TWD) to multi-attribute decision-making (MADM) problems can effectively handle uncertain information and better classify targets.

To fully utilize uncertain situation information and obtain more objective and reasonable evaluation results, we propose a dynamic threat assessment method based on the improved GRA-TOPSIS and three-way decisions. There are three main contributions of our work:

1) The cosine intuitionistic fuzzy entropy is introduced to calculate weights of target attributes, combined with a dynamic data fusion method to generate time series weights for dynamic matching of attribute parameters and weights.

2) A new intuitionistic fuzzy distance measure is used to calculate the distance between intuitionistic fuzzy numbers, and grey incidence analysis and TOPSIS are used to build the conditional probability for TWD.

3) Three-way decisions under the MADM model are used to classify threats against aerial targets.

2. Related works

2.1. Three-way decisions

TWD have been extensively studied to solve realistic MADM problems since they can minimize decision risks by adding the non-commitment option [27,28]. The improved TOPSIS method, combined with entropy weights, was used for optimal granularity selection [29]. A novel intuitionistic fuzzy (IF) large-scale group decision-making (LSGDM) method was constructed based on adjustable multi-granularity (MG)-IF probabilistic rough sets (PRSs) and the evidence reasoning (ER) method [30]. A new TWD model based on prospect theory (PT) on multi-scale information systems (MS-ISs) was revealed for pursuing MADM problems [31]. The intuitionistic fuzzy TOPSIS method [32] was used to obtain the conditional probability for the target and intuitionistic fuzzy evaluation value to construct the decision threshold for each target. However, the above threat assessment methods cannot make full use of uncertain situation information, the determination of conditional probability obtained has certain subjectivity and limitations, and it is difficult to get perfect results when dealing with the problem of "poor information".

2.2. Intuitionistic fuzzy multi-attribute decision making

IFMADM has been widely used as an extension of MADM under intuitionistic fuzzy information [21]. For the assessment of target attribute values with unknown intervals and weights, a Quantum Bee Group Threat Assessment Method for Intuitive Vague Multi-Attribute Decision (IFMADM) with Optimized Attribute Weighting was proposed [22]. To address the uncertainty and imprecision of experts' opinions, an improved hierarchical fuzzy TOPSIS method was used to aggregate the factors that affect the exposure rates of buildings in the two different scenarios [23]. A new intuitionistic fuzzy decision-making model was developed based on decision field theory [24]. The model stresses the contrasts regarding competition and influence of each attribute in different schemes in order to provide a dynamic evolution of preferences for various schemes and obtain optimum results.

2.3. Target threat assessment

Threat assessment is an essential part of modern warfare. With battlefield environments and situation information becoming more dynamic and uncertain, many scholars have studied threat assessment in a fuzzy intuitionistic environment. Jin et al. [33] proposed an intuitionistic fuzzy TOPSIS model and a multi-criteria optimization compromise decision-making (VIKOR) model with variable weights for static attribute and dynamic attribute threat assessment, but only the current situation was considered, and the dynamic threat assessment could not be conducted. The TOPSIS ranking method [34] based on improved relative entropy was proposed to dynamically evaluate incoming targets, but the method can only rank threats and cannot achieve threat classification. Thus, we study threat assessment in the IFMADM environment based on the current trend.

3. Preliminaries of intuitionistic fuzzy set and three-way decision

Definition 1 [35] Let X be a finite universal set. An intuitionistic fuzzy set (IFS) A in X can be mathematically described as

$$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{3.1}$$

where $\mu_A(x), \nu_A(x) : X \to [0,1]$ are the membership and non-membership degrees of element x in X belonging to A, and conditions of $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $x \in X$ are met; $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, which shows the hesitation degree and uncertainty of element x, and the condition of $\pi_A(x) \in [0,1]$ is met.

Definition 2 [25, 26] Based on minimum risk Bayesian theory, Yao proposed decision-theoretic rough sets which are composed of 2 states and 3 actions. Given two states $\Omega = \{C, \neg C\}$, which mean a target belongs to C or not. Let $A = \{a_p, a_B, a_N\}$ be actions set,and a_p, a_B, a_N describe $x \in POS(C), x \in BND(X), x \in NEG(C)$ respectively. For the above two states and three actions, Table 1 shows the loss functions of TWD. The $\{\lambda_{PP}, \lambda_{BP}, \lambda_{NP}\}$ and $\{\lambda_{PN}, \lambda_{BN}, \lambda_{NN}\}$ describe the loss function of a_p, a_B, a_N respectively, when the target belongs to C and $\neg C$. In addition, $Pr(C|[x])$ and $Pr(\neg C|[x])$ represent conditional probabilities when targets belong to C and $\neg C$ respectively.

Table 1. The relative loss functions of three-way decisions.

	C	$\neg C$
a_p	λ_{pp}	λ_{PN}
a_B	λ_{BP}	λ_{BN}
a_N	λ_{NP}	λ_{NN}

4. Threat assessment method

The method can fully utilize uncertain battlefield information and apply three-way decisions to multi-target dynamic threat assessment. Figure 1 shows the assessment process of the improved GRA-TOPSIS method and three-way decisions. A specific description of the method is given in the pseudocode below.

4.1. Dynamic data fusion

The battlefield situation is dynamic. Gathering multi-time information is a vital step in conducting a comprehensive and reasonable threat assessment of aerial targets. The closer to the current moment, the more significant the target information is. Therefore, we can calculate the series weight vector $\eta = (\eta_1, \eta_2, ..., \eta_k)$ of k times by using the inverse form of the Poisson distribution. Through the weighted aggregation of threat assessment information at each time by the intuitionistic fuzzy weighted average (IFWA) operator, a comprehensive intuitionistic fuzzy decision matrix R can be obtained:

$$R = \left(\left\langle \mu_{ij}, \nu_{ij} \right\rangle\right)_{m \times n} \tag{4.1}$$

where $\left\langle \mu_{ij}, \nu_{ij} \right\rangle = \left\langle 1 - \prod_{k=1}^{K} \left(1 - \mu_{ij}(t_k)\right)^{\eta_k}, \prod_{k=1}^{K} \nu_{ij}(t_k)^{\eta_k} \right\rangle$ and $t = \{t_1, t_2, ..., t_k\}$ is a set of moments.

Algorithm 1: The multi-target dynamic threat assessment algorithm

input : The multi-time information data D
output: The ranking and classification of targets

Calculate weighted dynamic decision matrix Z and attribute weights W by Eqs (4.1)– (4.5);
Calculate positive and negative ideal solutions Z^+, Z^- by Eqs (4.6) and (4.7);
for $z_i \in Z$ **do**
    Calculate the normalized distance between the target and positive, negative ideal solutions d_i^+, d_i^- by Eqs (4.8)– (4.10) and (4.13);
    Calculate the normalized positive and negative grey correlation degrees $\varepsilon_i^+, \varepsilon_i^-$ by Eqs (4.11), (4.12) and (4.14);
    Calculate the positive and negative comprehensive similarities $cs(z_i, z^+)$, $cs(z_i, z^-)$ by Eqs (4.15) and (4.16);
    Calculate the conditional probability of target $\Pr(C|[x])$ by Eqs (4.17) and (4.18);
end
for $z_i \in Z$ **do**
    Calculate the comprehensive loss function matrix λ_i by Eqs (4.19) and (4.20);
    Calculate decision thresholds α, β by Eqs (4.21) and (4.22);
    if $Pr(C|[x]) \geq \alpha$ **then** $X \in POS(C)$;
    if $\Pr(C|[x]) \leq \beta$ **then** $X \in NEG(C)$;
    else $X \in BND(C)$;
end
Rank the targets according to $\Pr(C|[x])$ and classify targets ;

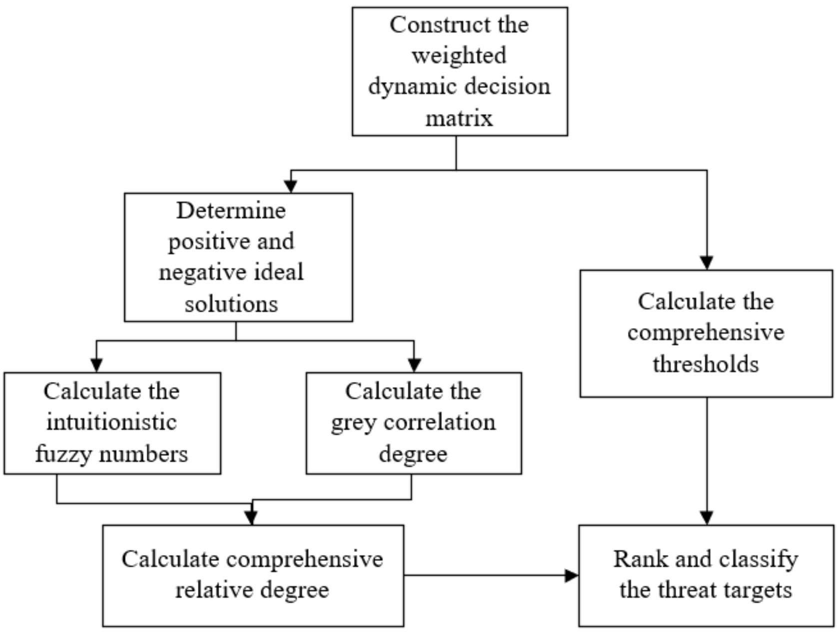

Figure 1. The assessment process of the proposed method.

4.2. Attribute weight determination based on cosine intuitionistic fuzzy entropy

The calculation of objective attribute weights is a key problem in MADM. The larger the intuitionistic fuzzy entropy, the fuzzier the information given by attributes for judgment; the lower the effectiveness of the evaluation information and the lower the corresponding attribute weight. Thus,weights of target attributes can be calculated according to cosine intuitionistic fuzzy entropy. The specific steps are as follows:

1) Calculate the cosine intuitionistic fuzzy entropy [36] E_j of the target attribute:

$$E_j = \frac{1}{n}\sum_{i=1}^{n}\cos\frac{\left(\mu_{ij}-\nu_{ij}\right)\left(1-\pi_{ij}\right)}{2}\pi \tag{4.2}$$

where π describes the hesitation degree and uncertainty of the element.

2) A nonlinear programming model for target attribute weights is established by:

$$\begin{cases} min\sum_{j=1}^{m}\left(w_j\right)^2\times E_j \\ s.t.\sum_{j=1}^{m}w_j = 1 \end{cases} \tag{4.3}$$

where W_j describes the weight of attribute j.

3) Calculate the target attribute weights. The Lagrange function is expressed as:

$$L(w,\lambda) = \sum_{j=1}^{m}\left(w_j\right)^2\cdot E_j + 2\lambda\left(\sum_{j=1}^{m}w_j - 1\right) \tag{4.4}$$

where λ is Lagrange factor;

Finally, the target attribute weights are obtained by:

$$w_j = \frac{\left(E_j\right)^{-1}}{\sum_{j=1}^{m}\left(E_j\right)^{-1}} \tag{4.5}$$

4.3. Conditional probability estimation based on the improved GRA-TOPSIS method

The intuitionistic fuzzy TOPSIS method [32] is used to calculate the conditional probabilities of targets, but it is difficult to get perfect results when dealing with the problem of "poor information". Since the grey correlation analysis can not only make full use of the existing information but also reflect the consistency of dynamic change trend [37], and effectively deal with "poor information", we can use the improved GRA-TOPSIS method to calculate conditional probabilities of threat targets. There are main calculation steps:

1) Determining the positive and negative ideal solutions

$$Z^+ = (Z_1^+, Z_2^+, ..., Z_n^+) = (\langle\mu_1^+,\nu_1^+\rangle, \langle\mu_2^+,\nu_2^+\rangle, ..., \langle\mu_n^+,\nu_n^+\rangle) \tag{4.6}$$

$$Z^{-}=\left(Z_1^{-},Z_2^{-},...,Z_n^{-}\right)=\left(\left\langle \mu_1^{-},\nu_1^{-}\right\rangle,\left\langle \mu_2^{-},\nu_2^{-}\right\rangle,...,\left\langle \mu_n^{-},\nu_n^{-}\right\rangle\right) \tag{4.7}$$

2) Calculating the distance of intuitionistic fuzzy numbers

We introduce a novel distance measure method to obtain the distance of intuitionistic fuzzy numbers to effectively reflect the characteristics of intuitionistic fuzzy information. The distance measure [38] of two intuitionistic fuzzy numbers $\alpha=(\mu_\alpha,\nu_\alpha),\beta=\left(\mu_\beta,\nu_\beta\right)$ is expressed as:

$$d(\alpha,\beta)=\sqrt{\frac{\left(\tilde{\mu}_\alpha-\tilde{\mu}_\beta\right)+\left(\tilde{\nu}_\alpha-\tilde{\nu}_\beta\right)}{2}} \tag{4.8}$$

where $\tilde{\mu}_\alpha=\mu_\alpha\left(1+\frac{2}{3}\pi_\alpha(1+\pi_\alpha)\right),\tilde{\nu}_\alpha=\nu_\alpha\left(1+\frac{2}{3}\pi_\alpha(1+\pi_\alpha)\right),\tilde{\mu}_\beta=\mu_\beta\left(1+\frac{2}{3}\pi_\beta\left(1+\pi_\beta\right)\right),\tilde{\nu}_\beta=\nu_\beta\left(1+\frac{2}{3}\pi_\beta\left(1+\pi_\beta\right)\right)$.

The distances between targets and positive,negative ideal solutions are calculated by:

$$d\left(z_i,z^{+}\right)=\sum_{j=1}^{n}w_j d\left(z_{ij},z^{+}\right) \tag{4.9}$$

$$d\left(z_i,z^{-}\right)=\sum_{j=1}^{n}w_j d\left(z_{ij},z^{-}\right) \tag{4.10}$$

3) Calculating the grey correlation degree

The distance measure only represents the relationship between the target and the ideal solution in position and cannot reflect the difference in trend. So, we can use the positive and negative grey correlation degrees $\varepsilon(z_i,z^{+})$, $\varepsilon(z_i,z^{-})$ to represent change trend of the indicator series. We can calculate the grey correlation degree:

$$\varepsilon\left(z_i,z^{+}\right)=\sum_{j=1}^{n}w_j\frac{\min_i\min_j d\left(z_{ij},z^{+}\right)+\varepsilon\max_i\max_j d\left(z_{ij},z^{+}\right)}{d\left(z_{ij},z^{+}\right)+\varepsilon\max_i\max_j d\left(z_{ij},z^{+}\right)} \tag{4.11}$$

$$\varepsilon\left(z_i,z^{-}\right)=\sum_{j=1}^{n}w_j\frac{\min_i\min_j d\left(z_{ij},z^{-}\right)+\varepsilon\max_i\max_j d\left(z_{ij},z^{-}\right)}{d\left(z_{ij},z^{-}\right)+\varepsilon\max_i\max_j d\left(z_{ij},z^{-}\right)} \tag{4.12}$$

where ε is the discrimination coefficient, $\varepsilon\in[0,1]$, generally taken as $\varepsilon=0.5$.

4) Calculating the conditional probability

The distance and grey correlation degree are normalized as follows:

$$d_i^{+}=\frac{d\left(z_i,z^{+}\right)}{\sqrt{\sum_{i=1}^{n}\left(d\left(z_i,z^{+}\right)\right)^2}},d_i^{-}=\frac{d\left(z_i,z^{-}\right)}{\sqrt{\sum_{i=1}^{n}\left(d\left(z_i,z^{-}\right)\right)^2}} \tag{4.13}$$

$$\varepsilon_i^{+}=\frac{\varepsilon\left(z_i,z^{+}\right)}{\sqrt{\sum_{i=1}^{n}\left(\varepsilon\left(z_i,z^{+}\right)\right)^2}},\varepsilon_i^{-}=\frac{\varepsilon\left(z_i,z^{-}\right)}{\sqrt{\sum_{i=1}^{n}\left(\varepsilon\left(z_i,z^{-}\right)\right)^2}} \tag{4.14}$$

The bigger d_i^- and ε_i^+, the closer the target is to the positive ideal solution. Conversely, the bigger d_i^+ and ε_i^-, the closer it is to the negative one. So, the following hybrid formulas are established by combining the distance measure and grey correlation degree to obtain the positive and negative comprehensive similarities $cs\,(z_i, z^+)$, $cs\,(z_i, z^-)$.

$$cs\,(z_i, z^-) = \eta d_i^- + (1-\eta)\varepsilon_i^+ \tag{4.15}$$

$$cs\,(z_i, z^+) = \eta d_i^+ + (1-\eta)\varepsilon_i^- \tag{4.16}$$

where $\eta \in [0, 1]$ is the preference coefficient for distance and grey incidence degree in decision-making. According to the TOPSIS decision method, comprehensive relative degree of position and shape similarity between the threat target and we can obtain the positive ideal solution:

$$R_c\,(T_i) = \frac{cs\,(z_i, z^-)}{cs\,(z_i, z^+) + cs\,(z_i, z^-)} \tag{4.17}$$

Noticeably, $R_c\,(T_i)$ describes the probability that the target T_i belongs to a positive ideal solution state and $1 - R_c\,(T_i)$ describes the probability that the target T_i belongs to a negative one. Thus, the conditional probability of TWD can be expressed as:

$$Pr\,(C|\,[x]) = R_c\,(T_i) \tag{4.18}$$

4.4. Threat classification based on three-way decisions

1) Constructing the loss function matrix

The target attributes are intuitionistic fuzzy numbers. So, the loss function matrix of each target based on each attribute can be expressed as:

$$\lambda\left(z_{ij}\right) = \begin{pmatrix} \lambda_{PP}^{ij} & \lambda_{PN}^{ij} \\ \lambda_{BP}^{ij} & \lambda_{BN}^{ij} \\ \lambda_{NP}^{ij} & \lambda_{NN}^{ij} \end{pmatrix} = \begin{pmatrix} 0 & d\left(z_{ij}, z_{max}^{j}\right) \\ \sigma d\left(z_{ij}, z_{min}^{j}\right) & \sigma d\left(z_{ij}, z_{max}^{j}\right) \\ d\left(z_{ij}, z_{min}^{j}\right) & 0 \end{pmatrix} \tag{4.19}$$

where σ is the risk avoidance coefficient and $0 \leq \sigma < 0.5$. The more fully the battlefield situation information is perceived, the greater the value.

2) Determining the comprehensive loss function matrix

We can gather multiple attributes of the target T_i and build the comprehensive loss function matrix as follows:

$$\lambda_i = \begin{pmatrix} \sum\limits_j w_j\lambda_{PP}^{ij} & \sum\limits_j w_j\lambda_{PN}^{ij} \\ \sum\limits_j w_j\lambda_{BP}^{ij} & \sum\limits_j w_j\lambda_{BN}^{ij} \\ \sum\limits_j w_j\lambda_{NP}^{ij} & \sum\limits_j w_j\lambda_{NN}^{ij} \end{pmatrix} \tag{4.20}$$

3) Calculating the decision thresholds

According to comprehensive loss function matrix, the decision thresholds can be obtained by:

$$\alpha_i = \frac{(1-\sigma)\sum_j w_j d\left(z_{ij}, z_{\max}^j\right)}{(1-\sigma)\sum_j w_j d\left(z_{ij}, z_{\max}^j\right) + \sum_j \sigma w_j d\left(z_{ij}, z_{\min}^j\right)} \tag{4.21}$$

$$\beta_i = \frac{\sum_j \sigma w_j d\left(z_{ij}, z_{\max}^j\right)}{\sum_j \sigma w_j d\left(z_{ij}, z_{\max}^j\right) + (1-\sigma)\sum_j w_j d\left(z_{ij}, z_{\min}^j\right)} \tag{4.22}$$

4) Classifying the targets

After obtaining the categories of threat level, the corresponding tactic for each level can be provided, as shown in Table 2.

If $Pr(C|[x]) \geq \alpha$, $X \in POS(C)$, which indicates that the target is a necessary strike target; if $\beta < \Pr(C|[x]) < \alpha$, $X \in BND(C)$, which indicates that the necessity of strike cannot be determined and further observation is needed for decision analysis; in addition, $x \in NEG(C)$, which indicates that the target is an unnecessary strike target.

Table 2. Three rules for threat assessment.

Conditional probability	Categories	Tactic
$Pr(C\|[x]) \geq \alpha$	POS(C): threat level	attacking
$\beta < \Pr(C\|[x]) < \alpha$	BND(C): potential threat level	further observation
$\beta \geq \Pr(C\|[x])$	NEG(C): non-threat level	no attacking

5. Example of air target threat assessment

We assume that there are four enemy targets $T = \{T_1, T_2, T_3, T_4\}$. $A = \{A_1, A_2, A_3, A_4\}$ denotes four threat factors: combat capability, speed, distance and angle, of which the first two are benefit attributes and the last two are cost attributes. Three consecutive moments $t = \{t_1, t_2, t_3\}$ are selected for threat assessment, of which t_3 is the current moment. Table 3 shows the multi-time information data of the target [32].

According to Eq (4.1), we can fuse dynamic data to obtain weighted dynamic decision matrix:

$$Z = \begin{pmatrix} (0.7655, 0.1150)(0.6834, 0.1620)(0.6628, 0.2127)(0.7747, 0.1522) \\ (0.8357, 0.1084)(0.7164, 0.1620)(0.7117, 0.1280)(0.8617, 0.0691) \\ (0.7563, 0.2127)(0.7540, 0.1458)(0.7550, 0.2068)(0.7188, 0.1598) \\ (0.7193, 0.1835)(0.6994, 0.2026)(0.6935, 0.1899)(0.6928, 0.1644) \end{pmatrix} \tag{5.1}$$

According to Eqs (4.2)–(4.5), we can obtain the attribute weights of each time and comprehensive objective:

$$\begin{aligned} w(t_1) &= (0.2484, 0.2351, 0.2005, 0.3161), \\ w(t_2) &= (0.2592, 0.2215, 0.2431, 0.2762), \\ w(t_3) &= (0.2821, 0.2348, 0.2297, 0.2535), \\ w &= (0.2703, 0.2321, 0.2268, 0.2708). \end{aligned} \tag{5.2}$$

Table 3. Threat information of targets at different times.

t	T	A_1	A_2	A_3	A_4
	T_1	(0.76,0.10)	(0.68,0.15)	(0.75,0.15)	(0.80,0.15)
t_1	T_2	(0.80,0.15)	(0.70,0.15)	(0.60,0.20)	(0.90,0.10)
	T_3	(0.78,0.15)	(0.72,0.13)	(0.65,0.15)	(0.75,0.15)
	T_4	(0.65,0.28)	(0.80,0.20)	(0.55,0.30)	(0.80,0.11)
	T_1	(0.78,0.10)	(0.65,0.20)	(0.70,0.20)	(0.80,0.11)
t_2	T_2	(0.83,0.10)	(0.65,0.20)	(0.70,0.15)	(0.85,0.10)
	T_3	(0.75,0.20)	(0.85,0.15)	(0.82,0.18)	(0.73,0.19)
	T_4	(0.70,0.20)	(0.64,0.21)	(0.76,0.15)	(0.75,0.15)
	T_1	(0.76,0.13)	(0.70,0.15)	(0.60,0.25)	(0.75,0.18)
t_3	T_2	(0.85,0.10)	(0.75,0.15)	(0.75,0.10)	(0.85,0.05)
	T_3	(0.75,0.25)	(0.70,0.15)	(0.75,0.25)	(0.70,0.15)
	T_4	(0.75,0.15)	(0.68,0.20)	(0.70,0.18)	(0.60,0.20)

According to Eqs (4.6) and (4.7), we can obtain positive and negative ideal solutions of target set:

$$\begin{aligned} R^+ &= (< 0.8357, 0.1084 >< 0.7540, 0.1458 >< 0.6628, 0.2127 >< 0.6928, 0.1644 >), \\ R^- &= (< 0.7193, 0.2127 >< 0.6834, 0.2026 >< 0.7550, 0.1280 >< 0.8617, 0.0691 >). \end{aligned} \tag{5.3}$$

According to Eqs (4.8)–(4.10) and (4.13), we can obtain the normalized distance from targets to positive and negative ideal solutions:

$$d^+ = (0.2926, 0.6653, 0.4518, 0.5173), d^- = (0.6298, 0.3561, 0.5154, 0.4592). \tag{5.4}$$

According to Eqs (4.11), (4.12) and (4.14), we can obtain the normalized positive and negative grey correlation degree of targets:

$$\varepsilon^+ = (0.5396, 0.4596, 0.5154, 0.4816), \varepsilon^- = (0.3990, 0.5902, 0.4677, 0.5231). \tag{5.5}$$

According to Eqs (4.15)–(4.18), we can obtain the conditional probabilities: [0.6284 0.3938 0.5285 0.4749].

Obviously, the threat ranking of targets is $T_1 > T_3 > T_4 > T_2$. Based on decision rules, the classification results of three-way decisions can be obtained: $POS(A) = \{T_1\}, BND(A) = \{T_3, T_4\}, NEG(A) = \{T_2\}$.

Table 4. Conditional probabilities and synthetic thresholds of TWD.

threshold	T_1	T_2	T_3	T_4
α	0.5314	0.5643	0.5456	0.5519
β	0.4315	0.4644	0.4456	0.4519
$\Pr(A\|T)$	0.6284	0.3938	0.5285	0.4749

According to Eqs (4.19)–(4.22), we can calculate the thresholds of TWD. Combining the conditional probabilities of targets, the results are shown in Table 4. Hence, we should give priority to attacking or intervening in target T_1, not attack T_2 first and obtain more information for T_3, T_4.

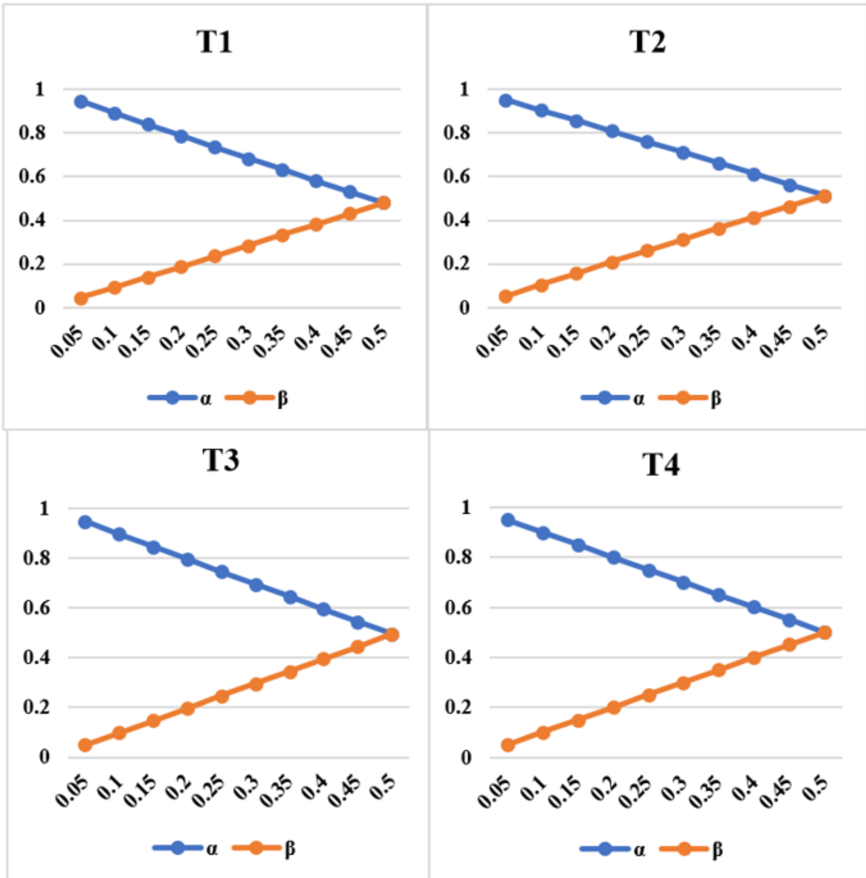

Figure 2. Influence of risk avoidance coefficient on the comprehensive threshold.

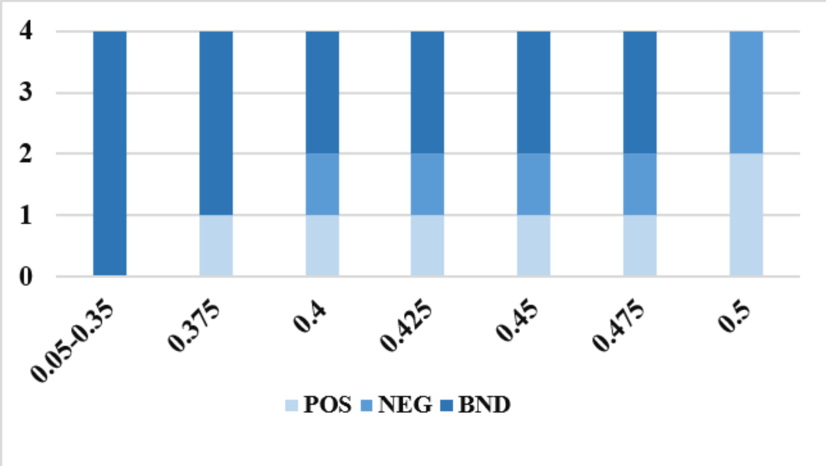

Figure 3. Influence of risk avoidance coefficient on threat classification.

We set up $\delta = 0.05$–0.5. The influence of the risk avoidance coefficient on the comprehensive threshold is shown in Figure 2 and the impact on threat classification is shown in Figure 3. It can be seen that with δ increasing, α gradually reduces, β gradually increases, the positive and negative regions become larger, the boundary regions gradually shrink or even become empty, and the classification results become clearer. Compared with the method in reference [32] and the GRA method, the effectiveness and advantages of the proposed method can be described. Figure 4 shows the result.

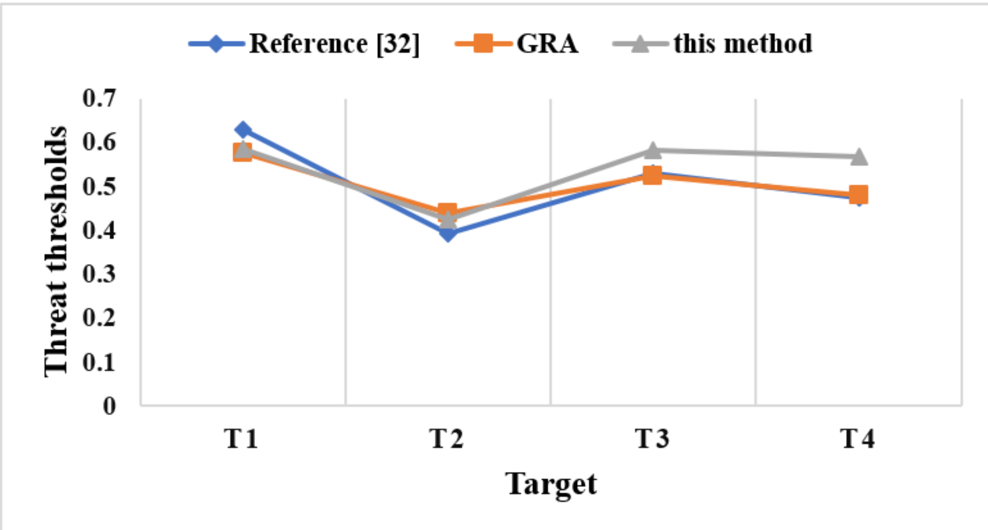

Figure 4. The threat assessment results of three methods.

As we can see, the threat values of targets are [0.5856 0.4249 0.5821 0.5685] and the threat ranking result is $T_1 > T_3 > T_4 > T_2$ based on the method in [32]. The threat values of targets are [0.5749,0.4378,0.5243,0.4793] based on the GRA method. We get the same ranking results, which verifies the effectiveness of our method. The TOPSIS method only focuses on the difference in absolute distance among targets. The proposed method considers both the difference in distance and the difference in sequence curve shape, which can produce more reasonable ranking results. So, the proposed method has the advantages of these two methods.

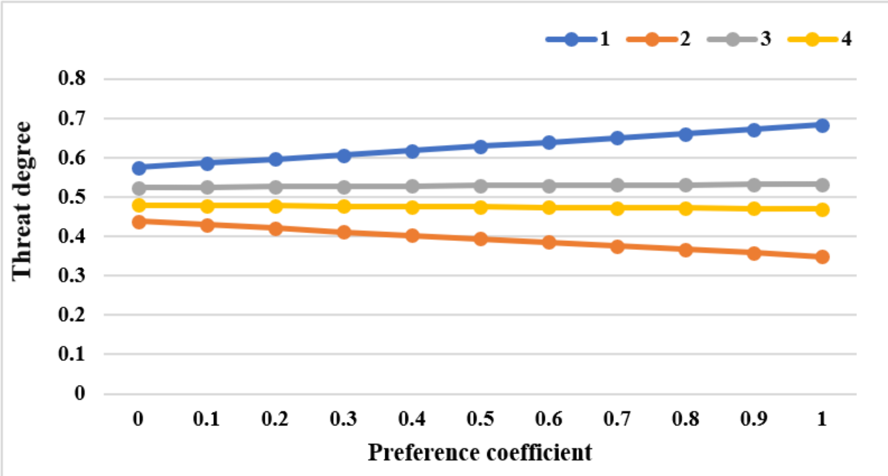

Figure 5. Preference coefficient analysis.

As an important parameter, the preference coefficient reflects the preference for distance and grey correlation, that is, the preference of decision-makers for absolute and trend differences. So, the preference coefficient can influence conditional probabilities. Assuming that the preference coefficient changes from 0.00 to 1.00 and the step size is 0.10, Figure 5 shows the change result of the target threat degree from 0 to 1.

With the increase of the preference coefficient, the threat degrees of target 1 and target 2 gradually increase, and threat degrees of target 2 and target 4 gradually decrease. The comprehensive closeness of target 1 and target 3 is directly proportional to the preference of location, and these two targets tend to the maximum rational solution in distance; the comprehensive closeness of target 2 and target 4 is in inverse proportion to the preference of location, and these two targets are closer to the positive ideal solution in shape than in distance. The proposed model can reflect different decision-making schemes by changing the preference coefficient according to the subjective tendencies of decision-makers. If the absolute difference is more obvious, a larger preference coefficient can be used; if the trend is more obvious, a smaller preference coefficient can be used. The battlefield situation is changing rapidly. The difference between different target threat degrees in a method mainly determines the accuracy of the method. The more obvious the difference, the more conducive it is to decision-making, what's more, the stronger superiority of the method [39]. In order to better reflect the target differentiation, the superiority of the target i over j is expressed as:

$$SD_{\mathrm{ij}} = \left(\frac{\zeta_i - \zeta_j}{\zeta_i}\right) \times 100 \tag{5.6}$$

where ζ_i and $\zeta_j (i = 1, 2, ..., m; j = 1, 2, ..., m; i \neq j)$ are threat values of targets.

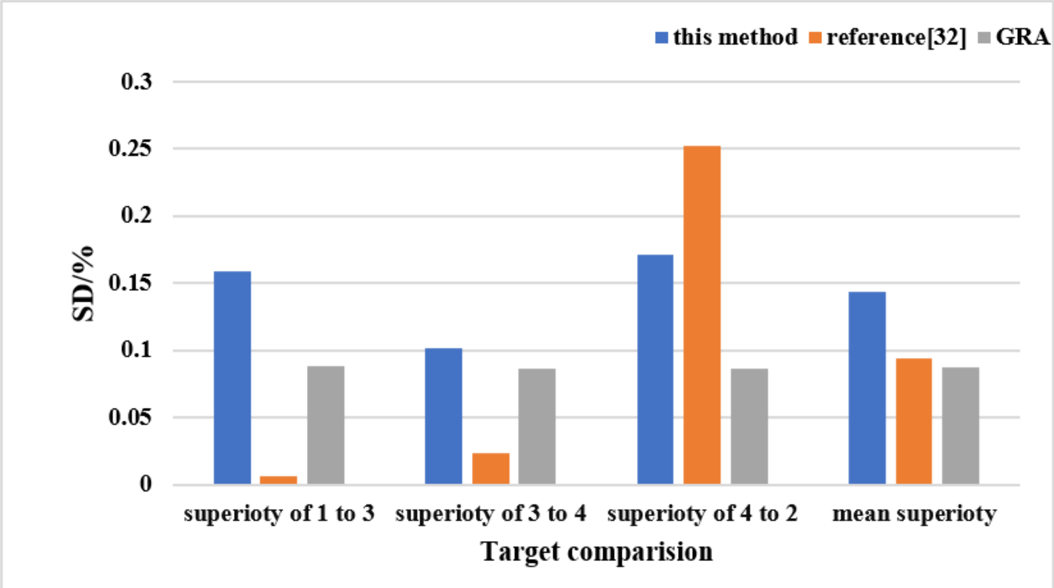

Figure 6. Comparison of the superiorities.

As shown in Figure 6, the average superiority of the method proposed is 1.529 times that of the method in reference [32] and 1.656 times that of the GRA method. In the process of threat assessment, the larger the superiority difference, the better the target threat can be distinguished and the threat ranking and decision-making are more reasonable. So, the method proposed in this article can better distinguish targets for decision-making.

To further illustrate the dynamic assessment ability of this method in the battlefield situation, the above simulation example is used to calculate only the data at the current time t_3 and the result is [0.6459 0.3965 0.4384 0.5428], which is ranked as $T_1 > T_4 > T_3 > T_2$. At time t_2, if the distance of target T_3 changes from $< 0.85, 0.15 >$ to $< 0.10, 0.15 >$, the threat ranking of each target obtained by the dynamic method considering multi-time information is $T_1 > T_4 > T_3 > T_2$, and the threat degree of T_3 becomes smaller. However, the result obtained only at the current time remains unchanged, which cannot reflect the impact of distance change. It shows that the method proposed can reflect the dynamic changes of the battlefield situation and get more reasonable and effective ranking results.

6. Conclusions

To handle the uncertain battlefield situation information, a new threat assessment method based on TWD is proposed. The multi-time threat assessment data is aggregated, and a cosine intuitionistic fuzzy entropy is established to calculate attribute weights. The improved intuitionistic fuzzy TOPSIS method and grey correlation analysis are used to obtain the conditional probability of TWD, with both the development trend difference of the data and the location difference being considered. The decision thresholds are obtained based on the comprehensive loss function matrix, and a TWD model is established for objective classification of targets. The results obtained are closer to the reality of the battlefield. Our next research work is mainly to explore advanced optimization algorithms to optimize target dynamic threat assessment.

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Conflict of interest

The authors declare there is no conflict of interest.

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