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Research article

Modeling the effect of health education and individual participation on the increase of sports population and optimal design

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Abstract: Health education plays an important role in cultivating people's awareness of participating in physical exercise. In this paper, a new differential equation model is established to dynamically demonstrate the different impact of mass communication and interpersonal communication in health education on people's participation in physical exercise. Theoretical analysis shows that health education does not affect the system threshold, but individual participation does. The combination of the two leads to different equilibria and affects the stability of equilibria. When mass communication, interpersonal communication and individual participation satisfy different conditions, the system will obtain different positive equilibrium with different number of sports population. If the interpersonal transmission rate of information is bigger, there is a positive equilibrium with a large number of sports population in the system. Sensitivity and optimal design analysis show some interesting results. First, increasing interpersonal communication and mass communication can both increase the number of conscious non-sports population and sports population. For increasing the number of conscious non-sports population, the effect of mass communication is better than that of interpersonal communication. For increasing the number of sports population, the effect of interpersonal communication is better than that of mass communication. However, individual participation has the best effect on increasing the sports population. Second, increasing the daily fixed amount of new information will be more helpful for media information dissemination. Finally, the three control measures need to be implemented simultaneously for a period of time at first, and then health education and participation of sports people need to be implemented periodically in order to maximize the sports population.

Keywords: sports population; mathematical model; periodic health education; individual participation

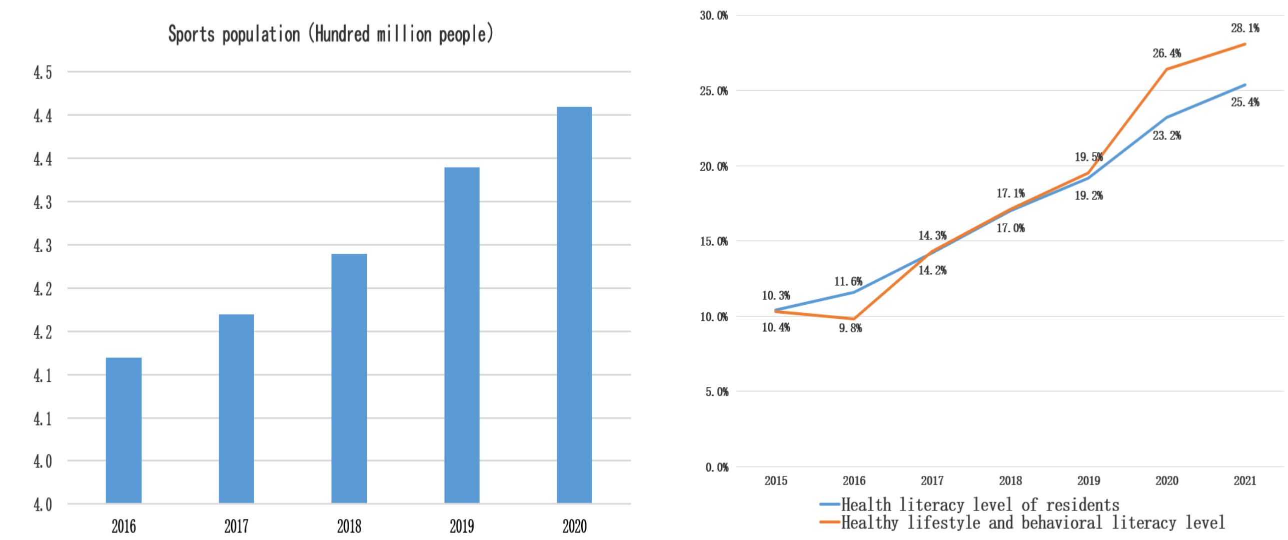
1. Introduction

A large number of studies have clarified the relationship between physical exercise and health. In 2013, the World Health Organization (WHO) estimated that physical inactivity causes an average of 3.2 million deaths each year [1]. Since the late 1980s, WHO has cooperated extensively with international sports organizations such as the International Olympic Committee in the field of mass sports. The World Health Organization's efforts to promote mass participation in physical activity and combat risks to human health reflect that health is not only the work of the sports sector alone, but also the common work of the whole society.

The development of sports population is highly dependent on health information dissemination and individual participation. With the extensive development of the internet and mass media, all kinds of health education have had a profound impact on people's life and behavior, because they can provide public health information that affects risk cognition and health behavior. Therefore, the study of health education and individual participation on the increase of sports population is of great significance.

Health education is an information dissemination and exchange activity carried out by human society around health problems. As a branch of communication, health education has attracted extensive attention because of its close relationship with personal life and great social influence. Wakefield et al. [2] believe that mass media campaigns can produce positive changes or prevent negative changes in health-related behaviors in a wide range of people, but some longer-term and more adequately funded media activities are needed to enable people to have full access to media information. Abroms et al. [3] studied the impact of mass media on the change of public behavior from the perspective of ecology, and believed that the intervention of mass media has direct and indirect effects. Based on social cognitive theory, Bandura [4] found that the belief in self-efficacy can affect the basic process of individual behavior change, including whether people consider changing their health habits, and how to maintain the changed behavior habits. To sum up, people who acquire the awareness of physical exercise through health education, coupled with the drive of sports people, will increase their possibility to become sports people. At the same time, the provision of health information should ensure sustainability.

In 2015, the fifth plenary session of the 18th central committee of China elevated healthy China into a national strategy. Around this strategy, the Chinese government issued a series of policy documents, and the mass media carried out extensive and in-depth publicity of relevant information. In recent years, with the continuous deepening of China's sports publicity, the general public's awareness of sports and fitness has been enhanced. The number of people who regularly participate in physical exercise in China has been increasing year by year, and the number of people who participate in physical exercise has increased from 410 million to 440 million during 2016 to 2020 (Figure 1(a)). At the same time, it also promotes the mastery of health knowledge and improves the level of health literacy (Figure 1(b)). With the enhancement of health awareness, people are no longer satisfied with the "disease-free" state, and are more willing to make efforts for health and participate in physical exercise. Here, the criteria for determining the sports population are: (1) frequency of physical activity more than 3 times per week (including 3 times); (2) more than 30 minutes of physical activity each time; (3) each physical activity intensity above medium.



(a) (b)

Figure 1. (a) Changes in the number of people who regularly take part in physical exercise from 2016 to 2020. (b) Health literacy of Chinese residents from 2015 to 2021. Data source: Public data, compiled by the People's Data Research Institute [5].

Many scholars have also discussed how to improve the influence of mass participation in physical exercise and health through communication. [6] showed that people who used mass media more frequently were more inclined to take physical exercise to maintain a healthy lifestyle, which in turn directly improved the health level of the population. [7] believed that Chinese sports should give full play to the leading role of the government in the process of sports publicity in the process of becoming a sports power. [8] proposed to take improving scientific fitness literacy as the goal, comprehensively apply mass communication, organizational communication, interpersonal communication and other methods, and carried out various forms of health communication activities by building national fitness demonstration area. However, it is worth studying how to use mass communication and interpersonal communication in the process of taking various measures to increase the sports population.

Many human behaviors and phenomena, such as the spread of information among people, the imitation and learning of each other's behaviors, are not static and independent, and there are often complex dynamic mechanisms behind them. There are already a number of research methods. For example, computational social science researchers have proposed various computational models to explain the mechanisms and possible influencing factors of these phenomena through the modeling and simulation method [9]. In additional, social dynamics also can be studied using statistical physics. Based on the kinetic theory of active particles, statistical physics has proven to be a fruitful framework to describe phenomena outside the realm of traditional physics [10–17]. Using mathematical model has also become a powerful method to analyze and solve practical problems [18–21]. Many scholars have done in-depth research in the field of the dynamics of the impact of health education on disease transmission. They establish the dynamic model of the impact of media information on disease transmission from different angles. In 2008, Cui et al. [18] found that when the threshold is greater than 1 and the influence of media is strong enough, multiple positive equilibria appear in the dynamic model. Agaba et al. [22] and Samanta et al. [23] divided the susceptible into two categories with different levels of consciousness and showed that the speed of implementing the awareness plan had a substantial impact on the system. Xiao et al. [21] proposed a classic mathematical model with media coverage and found that the media impact although does not affect the threshold, but media effect does not destabilize the positive steady-state. In the reference [24], the growth rate of awareness programs impacting the population is assumed to be proportional to the number of infective individuals. The model analysis shows that the spread of an infectious disease can be controlled by using awareness programs but the disease remains positive due to immigration. Almost none of these models consider both mass and interpersonal communication.

As we know, there are many modes of health education, mainly mass communication and interpersonal communication. Jin et al. [25] studied the impact of different health education modes on the health literacy of infectious diseases of different populations in China. The results showed that health education can significantly improve the health literacy of infectious diseases of different populations. Urban people are suitable for mass health communication methods such as health knowledge lectures, while rural people are more suitable for face-to-face interpersonal health communication methods such as group discussion and learning. Hu et al. [26] found that different ways of communication had significant difference in the awareness rate and behavioral formation rate. Those who adopt information late were more affected by interpersonal communication than mass communication [27]. Mass communication is easy to make people believe the news far from themselves, and interpersonal communication is easy to make people believe the news close to themselves [28].

Although most differential equation models are currently used in the study of epidemic transmission, more and more other professions are using this differential model to carry out research work, including some disciplines in the field of sociology. For example, there are many references in physical [29–32]. In information communication, there are also studies on establishing mathematical models [33, 34]. In the reference [32], authors established a mathematical model to analysis how to improve the participation of college students in physical exercise by maximizing the number of students in the third categories. The results showed that it is important to strengthen students' awareness of physical exercise and encourage those who often participate in physical exercise to actively participate in and lead those who do not often participate in physical exercise. However their work didn't consider health education. Hence, we want to examine the role of mass communication, interpersonal dissemination of health information and individual participation in the growth of the sports population using a mathematical model. Although information dissemination is similar in physical exercise as it is in disease transmission, the modes of communication considered in the existing disease models either only consider mass communication or interpersonal communication without comprehensive consideration. Therefore, this paper establishes a new mathematical model with health information as the medium, considering interpersonal communication, mass communication and individual participation simultaneously.

The paper is organized as follows. In Section 2 we establish a new model with health education and individual participation. The dynamical analysis for the model is studied in Section 3. This section includes threshold condition, existence and stability of the equilibria. In Section 4, sensitivity analysis of parameters is presented. Section 5 is optimal design. Some numerical simulations and discussions are presented in Section 6 and Section 7.

2. Modeling

In this model, the local people are divided into those who regularly take part in physical exercise (i.e. sports population) recorded as $P(t)$ and those who do not (i.e. non-sports population), and those who do not regularly take part in physical exercise are further divided into conscious non-sports population and unconscious non-sports population population, which are recorded as $S_m(t)$ and $S(t)$ respectively. Due to various reasons some sports people may remove to the other region and become $R(t)$. Because conscious non-sports population $S_m(t)$ have the consciousness to take part in physical exercise, they will occasionally take part in physical exercise, but the frequency is very small. The unconscious non-sports people hardly exercise, and they are the main people the sports people want to pull together. The amount of media information is recorded as $M(t)$. As we know, the dissemination of information about physical exercise can be divided into mass communication and interpersonal communication. Since interpersonal communication is linear [28], $hS(t)S_m(t)$ represents unconscious non-sports people to become conscious non-sports people because of information dissemination between conscious non-sports population and unconscious non-sports population. $cS(t)M(t)$ represents mass communication of information for unconscious non-sports population. Of course, conscious non-sports population can also become unconscious non-sports population due to forgetting information. This part is denoted as $qS_m(t)$. Conversely, sports population $P(t)$ can affect not only unconscious non-sports population $S(t)$ but also conscious non-sports population $S_m(t)$ to take part in physical exercise. Led by sports population, the conversion rate of unconscious non-sports population to sports population is β. Because conscious non-sports population are already aware of physical exercise, there is relatively little exposure to them by sports population. Then the conversion rate of conscious non-sports population to sports population is $\theta\beta$ with $0 < \theta < 1$. Death rate of people is μ and removal rate of sports population is γ because of the lack of local sports equipment. The removed portion of sports people will not help non sports people. The new increment of information includes daily routine health publicity reports M_0 and the amount of media publicity information proportional to the number of sports population $mP(t)$. The dissipation rate of information is recorded as d. Then the model is as the following

$$\begin{cases} \dfrac{dS}{dt} = B - \beta S(t)P(t) - cS(t)M(t) + qS_m(t) - hS(t)S_m(t) - \mu S(t), \\ \dfrac{dS_m}{dt} = cS(t)M(t) + hS(t)S_m(t) - \theta\beta S_m(t)P(t) - qS_m(t) - \mu S_m(t), \\ \dfrac{dP}{dt} = \beta S(t)P(t) + \theta\beta S_m(t)P(t) - \gamma P(t) - \mu P(t), \\ \dfrac{dM}{dt} = M_0 + mP(t) - dM(t), \\ \dfrac{dR}{dt} = \gamma P(t). \end{cases} \tag{2.1}$$

All the parameters are listed in Table 1.

Table 1. Parameters.

Parameters	Description
B	Recruitment rate of human
β	Conversion rate
θ	Discount on the conversion rate
γ	Removal rate of sports population
μ	Death rate of people
c	Acceptance rate of mass communication
q	Disappearance rate of consciousness
h	Acceptance rate of interpersonal communication
d	Dissipation rate of information
m	Recruitment rate of information
M_0	Daily routine publicity and reporting information

Our aim was to study different effects of mass communication (parameter c), interpersonal communication (parameter h) and individual participation (parameter β) on the dynamic behavior of disease transmission with health education.

3. Dynamics of the model

Because the variable $R(t)$ has no impact on the variables in other compartments, in the qualitative analysis process of the model, we simplified the model as follows:

$$\begin{cases} \dfrac{dS}{dt} = B - \beta S(t)P(t) - cS(t)M(t) + qS_m(t) - hS(t)S_m(t) - \mu S(t), \\ \dfrac{dS_m}{dt} = cS(t)M(t) + hS(t)S_m(t) - \theta\beta S_m(t)P(t) - qS_m(t) - \mu S_m(t), \\ \dfrac{dP}{dt} = \beta S(t)P(t) + \theta\beta S_m(t)P(t) - \gamma P(t) - \mu P(t), \\ \dfrac{dM}{dt} = M_0 + mP(t) - dM(t). \end{cases} \tag{3.1}$$

It is easy to see that for system (3.1) all trajectories in the positive cone enter or stay inside the region

$$\Omega = \{(S(t), S_m(t), P(t), M(t)) \mid S(t), S_m(t), P(t), M(t) \geq 0, S(t), S_m(t), P(t) \leq \frac{B}{\mu}, \frac{M_0}{d} \leq M(t) \leq \frac{mB}{d\mu}\}.$$

That means that Ω is a positively invariant set of system (3.1).

According to the biological significance, it is easy to obtain two thresholds for the two types of non-sports population:

$$R_0 = \frac{\beta B}{\mu(\mu + \gamma)}, \quad R_{0\theta} = \frac{\theta\beta B}{\mu(\mu + \gamma)}.$$

Obviously, $R_{0\theta} < R_0$ because $0 < \theta < 1$. This means the threshold of conversion in conscious people class is less than that in unconscious people class.

3.1. Existence of equilibria

First, let the right hand of the third equation of (3.1) be equal to 0, one can get

$$S(t) + \theta S_m(t) = \frac{\mu+\gamma}{\beta} \text{ or } P(t) = 0.$$

If $P(t) = 0$, by equating the right-hand side of the forth equation of (3.1) to zero, we can obtain $M^0 = \frac{M_0}{d}$. Combining the first three equations, we can get $S_m(t) = \frac{B}{\mu} - S(t)$ and $S(t)$ is the solution of the following quadratic equation:

$$f(S) \triangleq hS^2 - (q + \mu + cM^0 + \frac{hB}{\mu})S + (q+\mu)\frac{B}{\mu} = 0.$$

Note that $f(0) > 0$ and $f(\frac{B}{\mu}) = -cM^0\frac{B}{\mu} < 0$, It is known from the intermediate value theorem that there is a unique positive solution $S^0 = \frac{q+\mu+cM^0+\frac{hB}{\mu}-\sqrt{\Delta_1}}{2h}$ that conforms to the meaning of the problem. Here, $\Delta_1 = (q+\mu+cM^0+\frac{hB}{\mu})^2 - \frac{4hB(q+\mu)}{\mu} > 0$ and it is easy to verify that $S^0 < \frac{B}{\mu}$. Hence, the system (3.1) always has a boundary equilibrium $E^0 = (S^0, S_m^0, 0, M^0)$ with

$$\begin{aligned} S_m^0 &= \frac{B}{\mu} - S^0, \\ S^0 &= \frac{q+\mu+cM^0+\frac{hB}{\mu} - \sqrt{\Delta_1}}{2h}, \\ M^0 &= \frac{M_0}{d}. \end{aligned}$$

If $S(t) + \theta S_m(t) = \frac{\mu+\gamma}{\beta}$, we can obtain that $M(t) = \frac{m}{d}P(t) + \frac{M_0}{d}$ and

$$P(t) = \frac{1}{\mu+\gamma}[B - \frac{\mu(\mu+\gamma)}{\beta} - \mu(1-\theta)S_m(t)].$$

To make these variables meaningful, they must meet the following condition

(H): $0 < S_m(t) < \frac{\mu+\gamma}{\theta\beta}$ and $0 < S_m(t) < \frac{B-\frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}$.

This further requires $B - \frac{\mu(\mu+\gamma)}{\beta} > 0$, which happens to be the condition $R_0 > 1$.

Through simplification, it can be obtained that $S_m(t)$ is the solution satisfying the following quadratic equation:

$$F(S_m) \triangleq A_1 S_m^2 + A_2 S_m + A_3 = 0.$$

Here

$$\begin{aligned} A_1 &= \theta[h - (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}], \\ A_2 &= \theta(\beta + \frac{cm}{d})\frac{1}{\mu+\gamma}[B - \frac{\mu(\mu+\gamma)}{\beta}] + (q + \mu - \frac{h(\mu+\gamma)}{\beta} + \theta\frac{cM_0}{d}) + \frac{cm\mu(1-\theta)}{d\beta}, \\ A_3 &= -\frac{cm[B - \frac{\mu(\mu+\gamma)}{\beta}]}{d\beta} - \frac{cM_0}{d}\frac{\mu+\gamma}{\beta}. \end{aligned}$$

Under the condition $R_0 > 1$, $A_3 < 0$.

Next, we discuss the existence of positive equilibrium in three cases under the condition $R_0 > 1$.

Case I: $h > (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$.

In this case, $A_1 > 0$. It is easy to get that $F(0) < 0$ and

$$F(\frac{\mu+\gamma}{\theta\beta}) = B + \frac{q(\mu+\gamma)}{\theta\beta} > 0.$$

Thus, there is a positive solution between 0 and $\frac{\mu+\gamma}{\theta\beta}$. Note that $R_{0\theta} \geq 1$ is equivalent to $\frac{\mu+\gamma}{\theta\beta} \leq \frac{B-\frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}$ and there must be $R_0 > 1$. Hence, if $R_{0\theta} \geq 1$ there has a positive solution $S_m^1 = \frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1}$ in $(0, \frac{\mu+\gamma}{\theta\beta})$ that conforms to the condition (H) from the intermediate value theorem.

If $R_{0\theta} < 1$ and $R_0 > 1$, $\frac{\mu+\gamma}{\theta\beta} > \frac{B-\frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}$. To make variable $S_m(t)$ meaningful, it needs to satisfy $F(\frac{B-\frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}) \geq 0$. Since

$$F(\frac{B-\frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}) = \frac{1}{1-\theta}\{\frac{\mu+\gamma}{\beta}(R_{0\theta}-1)(\frac{h[B-\frac{\mu(\mu+\gamma)}{\beta}]}{\mu(1-\theta)} + \frac{cM_0}{d}) + \frac{q+\mu}{\mu}[B - \frac{\mu(\mu+\gamma)}{\beta}]\},$$

$F(\frac{B-\frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}) \geq 0$ is equivalent to

$$\frac{h}{1-\theta}(R_{0\theta}-1)(R_0-1) + \frac{\beta cM_0}{(\mu+\gamma)d}(R_{0\theta}-1) + \frac{\beta(q+\mu)}{\mu+\gamma}(R_0-1) \geq 0,$$

i.e.,

$$R_{0\theta} \geq 1 - \frac{(q+\mu)(R_0-1)}{\frac{h(\mu+\gamma)}{\beta(1-\theta)}(R_0-1) + \frac{cM_0}{d}} \triangleq R_{0\theta}^*.$$

Thus, if $R_{0\theta}^* \leq R_{0\theta} < 1$ and $R_0 > 1$, there also has a positive solution $S_m^1 = \frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1}$ that conforms to the condition (H). If $R_{0\theta} < R_{0\theta}^*$ and $R_0 > 1$, there has not a positive solution. Hence, in the first case there exists a positive equilibrium $E^1 = (S^1, S_m^1, P^1, M^1)$ if $R_{0\theta} \geq R_{0\theta}^*$ and $R_0 > 1$. There is no positive equilibrium if $R_{0\theta} < R_{0\theta}^*$ and $R_0 > 1$. Here

$$\begin{aligned} S_m^1 &= \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}, \\ S^1 &= \frac{\mu+\gamma}{\beta} - \theta S_m^1, \\ P^1 &= \frac{1}{\mu+\gamma}[B - \frac{\mu(\mu+\gamma)}{\beta} - \mu(1-\theta)S_m^1], \\ M^1 &= \frac{m}{d}P^1 + \frac{M_0}{d}. \end{aligned}$$

Case II: $h < (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$.

In this case, $A_1 < 0$. These two formulas $F(0) < 0$ and $F(\frac{\mu+\gamma}{\theta\beta}) > 0$ also hold. Similar to the discussion of the first case, we can get the conclusion of the second case. There exists a positive equilibrium $E^2 = (S^2, S_m^2, P^2, M^2)$ if $R_{0\theta} \geq R_{0\theta}^*$ and $R_0 > 1$. There is no positive equilibrium if $R_{0\theta} < R_{0\theta}^*$ and $R_0 > 1$. Here

$$\begin{aligned} S_m^2 &= \frac{-A_2 - \sqrt{A_2^2 - 4A_1A_3}}{2A_1}, \\ S^2 &= \frac{\mu+\gamma}{\beta} - \theta S_m^2, \\ P^2 &= \frac{1}{\mu+\gamma}[B - \frac{\mu(\mu+\gamma)}{\beta} - \mu(1-\theta)S_m^2], \\ M^2 &= \frac{m}{d}P^2 + \frac{M_0}{d}. \end{aligned}$$

Case III: $h = (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$.

In this case, $A_1 = 0$ and $F(S_m) \triangleq A_2 S_m + A_3 = 0$. Note that $A_3 < 0$ under the condition $R_0 > 1$. Thus, to make variable $S_m(t)$ meaningful, it needs to satisfy $A_2 > 0$, i.e.

$$h < \frac{\beta}{\mu+\gamma}[\frac{\mu\theta}{\beta}(\beta + \frac{cm}{d})(R_0 - 1) + q + \mu + \theta\frac{cM_0}{d} + \frac{\mu(1-\theta)cm}{\beta d}].$$

Substituting $h = (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$ into the above formula,

$$R_{0\theta} > 1 - \frac{\beta}{\mu(\beta + \frac{cm}{d})}[q + \mu + \theta\frac{cM_0}{d} + \frac{\mu(1-\theta)cm}{\beta d}] \triangleq R_{0\theta}^{***}.$$

Next, we need to verify that $S_m = -\frac{A_3}{A_2} < \min(\frac{\mu+\gamma}{\theta\beta}, \frac{B - \frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)})$. First, the following inequality holds under the condition $R_0 > 1$,

$$\frac{\mu+\gamma}{\theta\beta} - (-\frac{A_3}{A_2}) = \theta\beta[B - \frac{\mu(\mu+\gamma)}{\beta}] + (\mu+\gamma)(q + \mu\theta) > 0,$$

which means $S_m = -\frac{A_3}{A_2} < \frac{\mu+\gamma}{\theta\beta}$.

If $R_{0\theta} \geq 1$, $\frac{\mu+\gamma}{\theta\beta} \leq \frac{B - \frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}$, and then $\min(\frac{\mu+\gamma}{\theta\beta}, \frac{B - \frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}) = \frac{\mu+\gamma}{\theta\beta}$. Thus, $S_m = -\frac{A_3}{A_2} < \min(\frac{\mu+\gamma}{\theta\beta}, \frac{B - \frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)})$ holds if $R_{0\theta} \geq 1$.

If $R_{0\theta} < 1$, we need to verify $S_m = -\frac{A_3}{A_2} < \frac{B - \frac{\mu(\mu+\gamma)}{\beta}}{\mu(1-\theta)}$, which is equivalent to

$$\frac{\mu(\beta + \frac{cm}{d})}{\beta}(R_{0\theta} - 1)(R_0 - 1) + (q + \mu)(R_0 - 1) + \frac{cM_0}{d}(R_{0\theta} - 1) > 0,$$

i.e.,

$$R_{0\theta} > 1 - \frac{(q+\mu)(R_0 - 1)}{\frac{\mu(\beta + \frac{cm}{d})}{\beta}(R_0 - 1) + \frac{cM_0}{d}} \triangleq R_{0\theta}^{**}.$$

In fact, $R_{0\theta}^* = R_{0\theta}^{**}$ when $h = (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$. Note that

$$R_{0\theta}^{**} - R_{0\theta}^{***} = [\theta\frac{cM_0}{d} + \frac{\mu(1-\theta)cm}{\beta d}](R_0 - 1) + \frac{\beta}{\mu(\beta + \frac{cm}{d})}[q + \mu + \theta\frac{cM_0}{d} + \frac{\mu(1-\theta)cm}{\beta d}]\frac{cm}{d}.$$

$R_0 > 1$ ensures that $R_{0\theta}^{**} > R_{0\theta}^{***}$ is established. Hence, in the third case there exists a positive equilibrium $E^3 = (S^3, S_m^3, P^3, M^3)$ if $R_{0\theta} > R_{0\theta}^{**}$ and $R_0 > 1$. There is no positive equilibrium if $R_{0\theta} \le R_{0\theta}^{**}$ and $R_0 > 1$. Here

$$\begin{aligned}
S_m^3 &= -\frac{A_3}{A_2}, \\
S^3 &= \frac{\mu+\gamma}{\beta} - \theta S_m^3, \\
P^3 &= \frac{1}{\mu+\gamma}[B - \frac{\mu(\mu+\gamma)}{\beta} - \mu(1-\theta)S_m^3], \\
M^3 &= \frac{m}{d}P^3 + \frac{M_0}{d}.
\end{aligned}$$

In summary, the result about equilibrium existence of the system (3.1) is in Theorem 3.1.

Theorem 3.1. *For the system* (3.1)*, there always exists a boundary equilibrium* $E^0 = (S^0, S_m^0, 0, M^0)$*. If* $R_0 \le 1$*, there is no positive equilibria. If* $R_0 > 1$*, positive equilibria are as follows:*
(1) When $h > (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$*, there exists a positive equilibrium* $E^1 = (S^1, S_m^1, P^1, M^1)$ *if and only if* $R_{0\theta} \ge R_{0\theta}^*$*.*
(2) When $h < (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$*, there exists a positive equilibrium* $E^2 = (S^2, S_m^2, P^2, M^2)$ *if and only if* $R_{0\theta} \ge R_{0\theta}^*$*.*
(3) When $h = (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$*, there exists a positive equilibrium* $E^3 = (S^3, S_m^3, P^3, M^3)$ *if and only if* $R_{0\theta} > R_{0\theta}^{**}$*.*

Remark 3.2. *From the above three cases it can be seen that*
(1) The existence of positive equilibrium is related to both R_0 *and* $R_{0\theta}$*.*
(2) The number of sports population is different in the above three cases, which depends on the parameters h, c *and* β*. This shows that the impact of mass communication, interpersonal communication and individual participation on the increase of sports population is important.*
(3) Even if $R_0 > 1$*, the value of* $R_{0\theta}$ *can be very small when the value of* θ *is very small. To increase* $R_{0\theta}$ *we can increase* θ*. This means that we need to increase the role of sports people in promoting conscious non-sports people.*

3.2. Stability of equilibria

3.2.1. Local stability

Now we study the stability of equilibria. It is easy to calculate that the characteristic roots about E^0 are $\lambda_1 = -d$, $\lambda_2 = -\mu$, $\lambda_3 = -(cM^0 + hS_m^0 - hS^0 + q + \mu)$ and $\lambda_4 = \beta S^0 + \theta\beta S_m^0 - \mu - \gamma$. From $S_m^0 = \frac{B}{\mu} - S^0$, $\lambda_3 = -(cM^0 + \frac{hB}{\mu} - 2hS^0 + q + \mu)$. Substituting $S^0 = \frac{q+\mu+cM^0+\frac{hB}{\mu}-\sqrt{\Delta_1}}{2h}$, $\lambda_3 = -\sqrt{\Delta_1} < 0$.
Through simplification,

$$\lambda_4 = \beta S^0 + \theta\beta S_m^0 - \mu - \gamma = (1-\theta)\beta S^0 + (R_{0\theta} - 1)(\mu + \gamma).$$

It is easy to see that $\lambda_4 > 0$ if $R_{0\theta} > 1$. If $R_{0\theta} < 1$, we know $1 - R_{0\theta} > 0$ and can rewrite

$$\lambda_4 = (1-\theta)\beta S^0 - (1 - R_{0\theta})(\mu + \gamma).$$

Thus, $\lambda_4 < 0$ if and only if $R_{0\theta} < 1 - \frac{(1-\theta)\beta S^0}{\mu+\gamma} \triangleq R^s_{0\theta}$, Then, the real parts of all eigenvalues of E^0 are negative if and only if $R_{0\theta} < R^s_{0\theta}$. Hence, the local stability of the boundary equilibrium E^0 is following:

Theorem 3.3. *The boundary equilibrium E^0 of the system* (3.1) *is locally asymptotically stable if $R_{0\theta} < R^s_{0\theta}$ and unstable if $R_{0\theta} > R^s_{0\theta}$.*

Next the local stability of positive equilibrium $E^i (i = 1, 2, 3)$ is carried under the condition of the existence.

Theorem 3.4. *The positive equilibrium E^i of the system* (3.1) *is locally asymptotically stability if $h \leq \frac{q\beta}{\mu+\gamma}$.*

Proof. The Jacobian matrix corresponding to the positive equilibrium E^i of the system (3.1) is

$$J(E^i) = \begin{pmatrix} -\beta P^i - cM^i - hS^i_m - \mu & q - hS^i & -\beta S^i & -cS^i \\ cM^i + hS^i_m & hS^i - \theta\beta P^i - q - \mu & -\theta\beta S^i_m & cS^i \\ \beta P^i & \theta\beta P^i & \beta S^i + \theta\beta S^i_m - \gamma - \mu & 0 \\ 0 & 0 & m & -d \end{pmatrix}.$$

The corresponding characteristic equation is

$$\lambda^4 + B_1\lambda^3 + B_2\lambda^2 + B_3\lambda + B_4 = 0,$$

where

$$\begin{aligned} B_1 &= (\theta\beta P^i + q + \mu - hS^i) + (\beta P^i + cM^i + hS^i_m + \mu) + d, \\ B_2 &= 2(\beta P^i + cM^i + hS^i_m + \mu + d)(\theta\beta P^i + q + \mu - hS^i) + d(\beta P^i + cM^i + hS^i_m + \mu) \\ &\quad + \beta P^i\beta S^i + \theta\beta P^i\theta\beta S^i_m - (cM^i + hS^i_m)(-q + hS^i), \\ B_3 &= 2d(\beta P^i + cM^i + hS^i_m + \mu)(\theta\beta P^i + q + \mu - hS^i) + \theta\beta P^i\beta S^i(cM^i + hS^i_m) \\ &\quad - \beta P^i\theta\beta S^i_m(-q + hS^i) + \beta P^i\beta S^i(\theta\beta P^i + q + \mu - hS^i) + d\beta P^i\beta S^i \\ &\quad + \theta\beta P^i\theta\beta S^i_m(\beta P^i + cM^i + hS^i_m + \mu) + d\theta\beta P^i\theta\beta S^i_m - d(cM^i + hS^i_m)(-q + hS^i), \\ B_4 &= d\theta\beta P^i\beta S^i(cM^i + hS^i_m) - d\beta P^i\theta\beta S^i_m(-q + hS^i) + \theta\beta P^i\theta\beta S^i_m(\beta P^i + cM^i + hS^i_m + \mu). \end{aligned}$$

From $S^i + \theta S^i_m = \frac{\mu+\gamma}{\beta}$, $S^i \leq \frac{\mu+\gamma}{\beta}$. If $h \leq \frac{q\beta}{\mu+\gamma}$, it can lead to $q - hS^i \geq 0$. Then $B_1 > 0, B_2 > 0, B_3 > 0, B_4 > 0$.

Let

$$H_1 = B_1, H_2 = \begin{vmatrix} B_1 & B_3 \\ 1 & B_2 \end{vmatrix}, H_3 = \begin{vmatrix} B_1 & B_3 & 0 \\ 1 & B_2 & B_4 \\ 0 & B_1 & B_3 \end{vmatrix}, H_4 = B_4H_3.$$

Under the condition $h \leq \frac{q\beta}{\mu+\gamma}$, we can calculate

$$\begin{aligned}
H_1 &= B_1 = (\theta\beta P^i + q + \mu - hS^i) + (\beta P^i + cM^i + hS_m^i + \mu) + d > 0, \\
H_2 &= B_1B_2 - B_3 = [(\theta\beta P^i + q + \mu - hS^i) + (\beta P^i + cM^i + hS_m^i + \mu) + d] \\
&\quad [2d(\theta\beta P^i + q + \mu - hS^i) + \theta\beta P^i\theta\beta S_m^i + (cM^i + hS_m^i)(q - hS^i)] \\
&\quad +\beta P^i\beta S^i[2(\theta\beta P^i + \mu) + (q - hS^i) + d] + \theta\beta P^i\theta\beta S_m^i[(\beta P^i + \mu) + d] \\
&\quad +d(cM^i + hS_m^i)(q - hS^i) > 0, \\
H_3 &= B_3H_2 - B_1^2B_4 = [(\theta\beta P^i + q + \mu - hS^i) + (\beta P^i + cM^i + hS_m^i + \mu) + d] \\
&\quad [2d(\beta P^i + cM^i + hS_m^i + \mu)(\theta\beta P^i + q + \mu - hS^i) + d(\beta P^i\beta S^i + q - hS^i) \\
&\quad +d(\theta\beta P^i\theta\beta S_m^i + (cM^i + hS_m^i)(q - hS^i))][\theta\beta P^i\theta\beta S_m^i + (cM^i + hS_m^i)(q - hS^i)] \\
&\quad +\beta P^i\beta S^i(\theta\beta P^i + \mu) + \theta\beta P^i\theta\beta S_m^i[(\theta\beta P^i + \mu) + d] + d(cM^i + hS_m^i)(q - hS^i) > 0, \\
H_4 &= B_4H_3 > 0.
\end{aligned}$$

According to the Routh-Hurtwitz criterion, the real parts of all eigenvalues of E^i are negative. Then, E^i is locally asymptotically stable if $h \leq \frac{q\beta}{\mu+\gamma}$ under the condition of the existence. □

3.2.2. Global stability

Next we prove global stability of the boundary equilibrium E^0.

Theorem 3.5. *If $R_0 < \frac{1}{1+\theta}$, the boundary equilibrium E^0 of the system* (3.1) *is globally asymptotically stable.*

Proof. Let us construct the following Lyapunov function

$$V(S, S_m, P, M) = P(t).$$

Then, the total derivative of the function $V(S, S_m, P, M)$ along the solution of the system (3.1) can be solved as follow

$$\frac{dV(S, S_m, P, M)}{dt} = \beta S(t)P(t) + \theta\beta S_m(t)P(t) - \gamma P(t) - \mu P(t).$$

From $S(t) < \frac{B}{\mu}$ and $S_m(t) < \frac{B}{\mu}$, it is easy to get

$$\begin{aligned}
\frac{dV(S,S_m,P,M)}{dt} &< (\beta\frac{B}{\mu} + \theta\beta\frac{B}{\mu} - \gamma - \mu)P(t) \\
&= [\frac{(1+\theta)\beta B}{\mu} - \gamma - \mu]P(t) \\
&= \frac{P(t)}{\mu+\gamma}[(1 + \theta)R_0 - 1].
\end{aligned}$$

Then, we have $\frac{dV(S,S_m,P,M)}{dt} < 0$ if $R_0 < \frac{1}{1+\theta}$. In accordance with the LaSalle invariant set principle, the disease-free equilibrium E^0 of the system (3.1) is globally asymptotically stable if $R_0 < \frac{1}{1+\theta}$. □

For the global stability of positive equilibrium $E^i (i = 1, 2, 3)$, we have the following theorem.

Theorem 3.6. *If $h \leq \frac{q\beta}{\mu+\gamma}$, the positive equilibrium E^i of the system* (3.1) *is globally asymptotically stable.*

Proof. Let us construct the following Lyapunov function

$$V(S,S_m,P,M)=\frac{1}{2}(S-S^i+S_m-S_m^i+P-P^i)^2+a(P-P^i-P^i ln\frac{P}{P^i}),$$

here a is an undetermined coefficient. Then, the total derivative of the function $V(S,S_m,P,M)$ along the solution of the system (3.1) can be solved as follow

$$\begin{aligned}\frac{dV(S,S_m,P,M)}{dt} &= (S-S^i+S_m-S_m^i+P-P^i)(\dot{S}+\dot{S_m}+\dot{P})+a(1-\frac{P^i}{P})\dot{P}\\ &= (S-S^i+S_m-S_m^i+P-P^i)[B-\mu S-\mu S_m-(\mu+\gamma)P]\\ &\quad +a(1-\frac{P^i}{P})[\beta SP-\beta S^iP^i+\theta\beta S_mP-\theta\beta S_m^iP^i-(\mu+\gamma)P+(\mu+\gamma)P^i].\end{aligned}$$

From

$$\begin{aligned}B-\beta S^iP^i-cS^iM^i+qS_m^i-hS^iS_m^i-\mu S^i&=0,\\ cS^iM^i+hS^iS_m^i-\theta\beta S_m^iP^i-qS_m^i-\mu S_m^i&=0,\\ \beta S^iP^i+\theta\beta S_m^iP^i-(\mu+\gamma)P^i&=0,\end{aligned}$$

we can get

$$\begin{aligned}\frac{dV(S,S_m,P,M)}{dt} &= (S-S^i+S_m-S_m^i+P-P^i)[-\mu(S-S^i)-\mu(S_m-S_m^i)-(\mu+\gamma)(P-P^i)]\\ &\quad -a\frac{(\mu+\gamma)(P-P^i)^2}{P}+a\beta(S-S^i)(P-P^i)+a\theta\beta(S_m-S_m^i)(P-P^i)\\ &\quad +a\frac{\beta S^i}{P}(P-P^i)^2+a\frac{\theta\beta S_m^i}{P}(P-P^i)^2\\ &= -\mu(S-S^i)^2-\mu(S_m-S_m^i)^2-(\mu+\gamma)(P-P^i)^2-2\mu(S-S^i)(S_m-S_m^i)\\ &\quad -(2\mu+\gamma)(S-S^i)(P-P^i)-(2\mu+\gamma)(S_m-S_m^i)(P-P^i)\\ &\quad -a\frac{(\mu+\gamma)(P-P^i)^2}{P}+a\beta(S-S^i)(P-P^i)+a\theta\beta(S_m-S_m^i)(P-P^i)\\ &\quad +a\frac{\beta S^i}{P}(P-P^i)^2+a\frac{\theta\beta S_m^i}{P}(P-P^i)^2\\ &< -\mu[(S-S^i)+(S_m-S_m^i)]^2-(\mu+\gamma)(P-P^i)^2\\ &\quad +[a\beta-(2\mu+\gamma)](S-S^i)(P-P^i)+[a\beta-(2\mu+\gamma)](S_m-S_m^i)(P-P^i)\\ &\quad -a[(\mu+\gamma)-\beta S^i-\theta\beta S_m^i]\frac{(P-P^i)^2}{P}.\end{aligned}$$

Set $a\beta-(2\mu+\gamma)=0$, one has $a=\frac{2\mu+\gamma}{\beta}$ and since $S^i+\theta S_m^i=\frac{\mu+\gamma}{\beta}$, we have

$$\begin{aligned}\frac{dV(S,S_m,P,M)}{dt} &< -\mu[(S-S^i)+(S_m-S_m^i)]^2-(\mu+\gamma)(P-P^i)^2\\ &\quad -\frac{2\mu+\gamma}{\beta}[(\mu+\gamma)-\beta S^i-\theta\beta S_m^i]\frac{(P-P^i)^2}{P}\\ &< -\mu[(S-S^i)+(S_m-S_m^i)]^2-(\mu+\gamma)(P-P^i)^2<0.\end{aligned}$$

Then, we have $\dot{V}<0$ under the existence condition of E^i. Furthermore, $\frac{dV}{dt}=0$ if and only if $S=S^i, S_m=S_m^i, P=P^i$. According to the LaSalle invariant set principle, the positive equilibrium E^i of the system (3.1) is globally asymptotically stable. □

4. Sensitivity analysis

The sensitivity index can help us understand the sensitive parameters of the system. These indexes can be positive or negative. The absolute value of the index indicates the strength of the relationship, and the positive and negative properties of the index indicate positive and negative correlation. Now, we will use PRCC method to investigate the sensitivity of parameters on the positive equilibrium, and the threshold $R_0, R_{0\theta}$ of the system. Table 2 lists the range of model parameters. Table 3 provides the PRCC values of different parameters on R_0, $R_{0\theta}$ and various state variables of the positive equilibrium of the system.

Table 2. Ranges for parameters of model (3.1).

Parameter	Description	Range	Sources
B	Recruitment rate of human	1–300	[35]
β	Conversion rate	0.0001–0.1	[32]
c	Acceptance rate of mass communication	0–0.1	[36]
q	Disappearance rate of consciousness of non-sports population	0.5–0.67	[37]
h	Acceptance rate of interpersonal communication	0–0.1	[36]
μ	Death rate of people	0.003–0.009	[35]
θ	Discount on the conversion rate	0–0.8	/
γ	Removal rate of sports population	0.01–0.3	[32, 38]
M_0	Daily routine publicity and reporting information	1–100	[39]
m	Response parameters of media information to sports population	0-0.1	[39]
d	Dissipation rate of information	0–0.1	[39]

Table 3. Sensitivity to R_0, $R_{0\theta}$ and the positive equilibria of the system.

Parameter	Sensitivity(R_0)	Sensitivity($R_{0\theta}$)	Sensitivity(S_m)	Sensitivity(P)	Sensitivity(M)
B	0.3255	0.3401	−0.1321	0.3213	0.0949
β	0.8865	0.8850	-0.1953	0.8542	0.1773
c	\	\	0.1660	−0.0143	−0.0400
q	\	\	−0.1237	0.1110	0.0401
h	\	\	0.0218	−0.0565	−0.0428
μ	−0.4038	−0.4503	−0.0177	−0.1442	−0.1625
θ	\	0.1152	−0.2106	0.0705	−0.1960
γ	−0.8873	−0.8988	−0.2554	−0.5554	−0.0057
M_0	\	\	0.1665	0.0323	0.9938
m	\	\	0.1062	0.1062	−0.0251
d	\	\	−0.2708	0.1471	−0.1992

Table 3 and Figure 2(a) suggest that the magnitude of R_0 and $R_{0\theta}$ increase with increase in the values of parameters B and β as these parameters possess positive indices with R_0 and $R_{0\theta}$. β reflects the degree of participation of all individuals in physical exercise. To increase R_0 and $R_{0\theta}$, we need to increase individual participation. Meanwhile, θ has a positive correlation with $R_{0\theta}$. This means that the greater the influence of sports people on conscious non-sports people, the greater the value of $R_{0\theta}$, there will be positive equilibria with sports population. This is consistent with our theoretical analysis results.. Similarly, the parameters having negative correlation with R_0 and $R_{0\theta}$ are μ and γ. We can't control death. However we can improve local sports facilities and reduce the movement of sports people out.

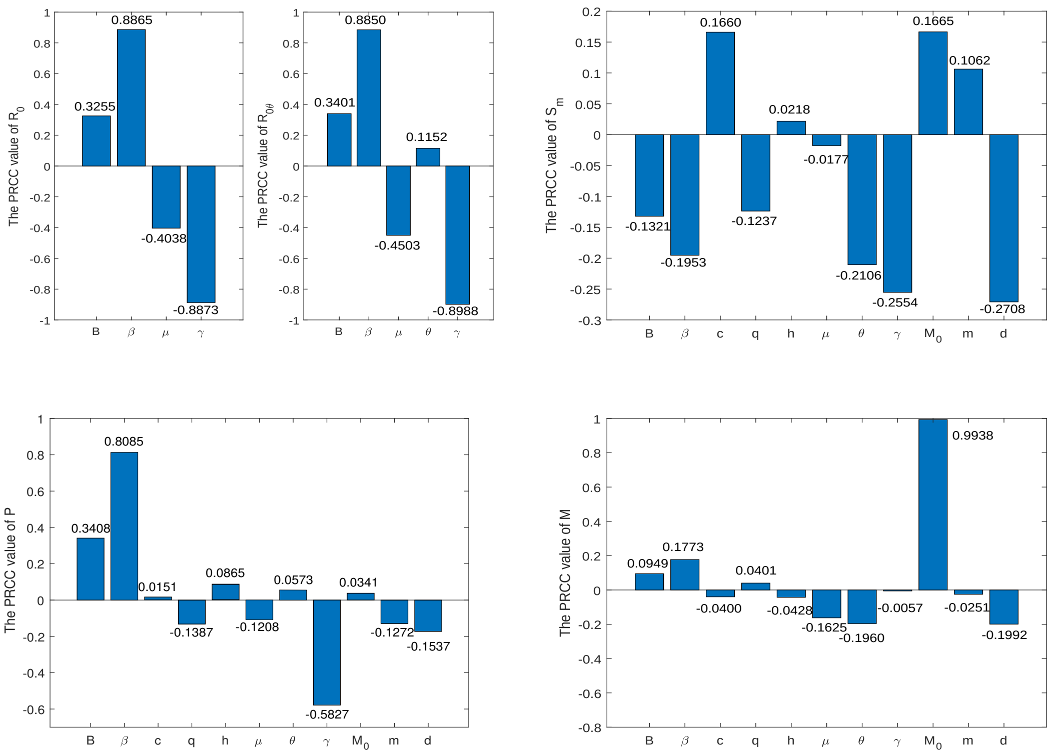


Figure 2. The significance analysis diagram of parameters to (a) $R_0, R_{0\theta}$ (b) S_m, (c) P and (d) M.

In Table 3 and Figure 2(b), we can find d and γ have the strong relationship to the number of conscious non-sports population S_m. B, β, q, θ and μ have different degrees of negative relationship to the number of conscious non-sports population. Among them, the negative correlation of β, θ confirms that with the increase of individual participation, there will be more conscious non-sports people transformed into sports people. The correlation of acceptance rate of interpersonal communication h is positive because the awareness of self-protection is gradually cultivated when the unconscious group communicates with the conscious group, so that the conscious group continues to increase. Meanwhile, the correlation of acceptance rate of mass communication c is positive because unconscious non-sports population develop into conscious non-sports population when they receive the opinions of mass communication information. Then the negative correlation of q is due to the fact that there is a rate of disappearance of consciousness, which makes conscious non-sports population to unconscious non-sports population. The negative correlation of d means that when the dissipation rate of media messages is too high, this will lead to a reduction in the number of media messages and, in turn, a corresponding reduction in the number of conscious non-sports population. The negative correlation of γ also indicates that the migration of sports people is disadvantageous to non-sports people.

In Table 3 and Figure 2(c), it is easy to see that β, B, c, h, θ and M_0 have different degrees of positive relationship to the number of sports population P, and the rest of the parameters are negatively correlated. These shows that increasing mass communication, interpersonal communication and individual participation can all increase the number of sports population. Increasing the dissemination of information about daily physical activity can also increase the physical population. The migration of sports people is the same disadvantageous to sports population.

In Table 3 and Figure 2(d), the parameters B, β, q, m and M_0 have the positive relationship to the amount of media information M. Additionally, the parameters c, h, μ, θ, γ and d have the negative relationship to the amount of media information M. Among them, for the positive correlation interpretation of M_0 and m, this is because daily routine publicity and reporting information and the response of media to the number of sports population are the main reasons for the generation of media information.

According to the analysis of the above sensitivity results, we have discovered the strong importance of parameters c, h and β for the increase of sports population. What measures should people take to achieve optimum effect in actual operation? In order to study this problem, the next section will study the optimal design.

5. Optimal design

In this section, we use an optimal control approach to study sports population taking into account the effect of health education. Assume that the total population is denoted by $N(t) = S(t) + S_m(t) + P(t)$. In order to reduce the cost of implementing control technology and to achieve the lowest cost, it is necessary to find time-dependent control strategies. Most of control strategies used in daily life are considering continuous control strategies. This problem is a typical optimal control problem. In fact, sports institutions need to maintain a high level of strategies in order to increase sports population, which has a high economic cost, so we need to find a time-dependent control strategy. The measures about health education we take are: (a) to increase the publicity of mass communication (parameter c), (b) to carry out active interpersonal communication (parameter h), (c) to strengthen the impact of sports population on conscious non-sports population (parameter β). Therefore, we introduce three time-dependent control functions u_1, u_2, u_3. Considering the above assumptions, the control problem of the system (3.1) with health education effect is given by the following equation.

$$\begin{cases} \frac{dS(t)}{dt} = B - [1 + u_3(t)]\beta S(t)P(t) - [1 + u_1(t)]cS(t)M(t) + qS_m(t) - [1 + u_2(t)]hS(t)S_m(t) - \mu S(t), \\ \frac{dS_m(t)}{dt} = [1 + u_1(t)]cS(t)M(t) + [1 + u_2(t)]hS(t)S_m(t) - [1 + u_3(t)]\theta\beta S_m(t)P(t) - qS_m(t) - \mu S_m(t), \\ \frac{dP(t)}{dt} = [1 + u_3(t)]\beta S(t)P(t) + [1 + u_3(t)]\theta\beta S_m(t)P(t) - \gamma P(t) - \mu P(t), \\ \frac{dM(t)}{dt} = M_0 + mP(t) - dM(t). \end{cases} \tag{5.1}$$

The parameter descriptions are as described previously. Assume that the set of control variables is

$$U = \{(u_1(t), u_2(t), u_3(t)) : [0, t_{end}] \to \mathbb{R}^3 | u_i(t) \ is\ a\ Lebesgue\ measure\ on\ [0, 1], i = 1, 2, 3\}.$$

This means that all control variables are bounded and Lebesgue measurable. Here, $u_1(t)$ represents the increase in mass communication publicity coverage that leads unconsciously non-sports individuals to value media messages, $u_2(t)$ represents the active interpersonal communication campaign that promotes communication among people to make the unconscious non-sports people more receptive to take part in physical exercise, and $u_3(t)$ represents the increase about driving effect of sports people on non-sports people.

The objective of our optimal control problem is to maximum the number of sports population and minimum the cost of implementing a control strategy by using optimal control variables. Therefore, we use bounded and Lebesgue measurable control variables and define the objective function as follows:

$$J(u_1(t), u_2(t), u_3(t)) = \min\{\int_0^{t_{end}} (-z_1 P(t) + z_2 N(t) + z_3 M(t) + \sum_{i=1}^{3} c_i u_i^2(t))dt\}, \tag{5.2}$$

where $z_1, z_2, z_3, c_1, c_2, c_3$ are all positive constants. Among these constants, z_1 represents the weight of sports population, z_2 represents the cost of national investment in physical exercise, and z_3 represents the cost of media information campaigns of health education. c_1,c_2,c_3 denote the weight constants of increasing the number of media messages in mass communication, active interpersonal communication campaigns in society, and increasing the enthusiasm of individuals to participate in physical exercise driven by sports people, respectively. Meanwhile, we assume that the cost is proportional to the quadratic form of the three control functions. The objective of the optimal control problem is to find the optimal control variables $(u_1^*(t), u_2^*(t), u_3^*(t))$ such that

$$J(u_1^*(t), u_2^*(t), u_3^*(t)) = \min_{(u_1(t),u_2(t),u_3(t))\in U} \{J(u_1(t), u_2(t), u_3(t))|(u_1(t), u_2(t), u_3(t)) \in U\}.$$

The existence of optimal control in system (3.1) can be obtained.

Theorem 5.1. *There exists an optimal control of the system $u^*(t) = (u_1^*(t), u_2^*(t), u_3^*(t)) \in U$, such that*

$$J(u_1^*(t), u_2^*(t), u_3^*(t)) = \min_{(u_1(t),u_2(t),u_3(t))\in U} \{J(u_1(t), u_2(t), u_3(t))|(u_1(t), u_2(t), u_3(t)) \in U\},$$

subject to the control system (5.1).

Proof. By the results in the above theorem, we prove the existence of optimal control with the control and state variables are both non-negative. In this minimization problem, the necessary convexity of the objective function in $u_1(t), u_2(t), u_3(t)$ is satisfied. Meanwhile, $u_1(t), u_2(t), u_3(t)$ all belong to the control set U. The optimal control system is bounded, which determines the compactness required for the existence of the optimal control. Moreover, for the objective function (5.2) the product function $-z_1 P(t) + z_2 N(t) + z_3 M(t) + \sum_{i=1}^{3} c_i u_i^2(t)$ is convex on the control set U. Furthermore, we can obtain that there exists a constant $\rho > 1$ and positive numbers ω_1, ω_2 such that

$$J(u_1(t), u_2(t), u_3(t)) \geq \omega_1(|u_1(t)|^2 + |u_2(t)|^2 + |u_3(t)|^2)^{\frac{\rho}{2}} - \omega_2.$$

Since the state variables are bounded, this completes the proof of the existence of optimal control. □

In order to find the optimal solution, we use Pontryagin's Maximum Principle. First, to simplify the above notation, we set

$$X(t) = (S(t), S_m(t), P(t), M(t))^T, u(t) = (u_1(t), u_2(t), u_3(t))^T, \lambda(t) = (\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)).$$

Then, the Lagrangian function is

$$L = -z_1 P(t) + z_2 N(t) + z_3 M(t) + c_1 u_1^2(t) + c_2 u_2^2(t) + c_3 u_3^2(t),$$

and we define the Hamiltonian function for this control problem $H(t, X(t), u(t), \lambda(t))$ as follows:

$$H(t, X(t), u(t), \lambda(t)) = -z_1 P(t) + z_2 N(t) + z_3 M(t) + \sum_{i=1}^{3} c_i u_i^2(t) + \lambda_1 \frac{dS}{dt} + \lambda_2 \frac{dS_m}{dt} + \lambda_3 \frac{dP}{dt} + \lambda_4 \frac{dM}{dt} \tag{5.3}$$

$$\begin{aligned}
&= -z_1 P(t) + z_2 N(t) + z_3 M(t) + c_1 u_1^2(t) + c_2 u_2^2(t) + c_3 u_3^2(t) \\
&+ \lambda_1\{B - [1 + u_3(t)]\beta S(t)P(t) - [1 + u_1(t)]cS(t)M(t) + qS_m(t) - [1 + u_2(t)]hS(t)S_m(t) - \mu S(t)\} \\
&+ \lambda_2\{[1 + u_1(t)]cS(t)M(t) + [1 + u_2(t)]hS(t)S_m(t) - [1 + u_3(t)]\theta\beta S_m(t)P(t) - qS_m(t) - \mu S_m(t)\} \\
&+ \lambda_3\{[1 + u_3(t)]\beta S(t)P(t) + [1 + u_3(t)]\theta\beta S_m(t)P(t) - \gamma P(t) - \mu P(t)\} + \lambda_4\{M_0 + mP(t) - dM(t)\}.
\end{aligned}$$

Therefore, we get
(1) State equations:

$$X'(t) = H_\lambda(t, X(t), u(t), \lambda(t)). \tag{5.4}$$

(2) Optimal conditions:

$$0 = H_u(t, X(t), u(t), \lambda(t)). \tag{5.5}$$

(3) Adjoint equations:

$$-\lambda'(t) = H_{X(t)}(t, X(t), u(t), \lambda(t)). \tag{5.6}$$

Now, we apply the above condition to the Hamiltonian function H of (5.3), we get

Theorem 5.2. *Let $(S^*(t), S_m^*(t), P^*(t), M^*(t))$ be the optimal state solutions of the optimal control problem* (5.1) *and* (5.2) *under the optimal control variables $u^*(t)$. Thus there exist adjoint variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ satisfying*

$$\begin{cases}
\frac{d\lambda_1(t)}{dt} = & -z_2 + \mu\lambda_1(t) + \{[1 + u_1^*(t)]cM^*(t) + [1 + u_2^*(t)hS_m^*(t)]\}[\lambda_1(t) - \lambda_2(t)] + [1 + u_3^*(t)]\beta P^*(t)[\lambda_1(t) + \lambda_3(t)], \\
\frac{d\lambda_2(t)}{dt} = & -z_2 + \mu\lambda_2(t) - \{q - [1 + u_2^*(t)]hS(t)\}[\lambda_1(t) - \lambda_2(t)] + [1 + u_3^*(t)]\theta\beta P^*(t)[-\lambda_2(t) + \lambda_3(t)], \\
\frac{d\lambda_3(t)}{dt} = & z_1 - z_2 + (\gamma + \mu)\lambda_3(t) + [1 + u_3^*(t)]\beta S^*(t)[\lambda_1(t) + \lambda_3(t)] + [1 + u_3^*(t)]\theta\beta S_m^*(t)[-\lambda_2(t) + \lambda_3(t)] - m\lambda_4(t), \\
\frac{d\lambda_4(t)}{dt} = & -z_3 + d\lambda_4(t) + [1 + u_1^*(t)]cS^*(t)[\lambda_1(t) - \lambda_2(t)],
\end{cases}$$

with transversality conditions (boundary conditions)

$$\lambda_1(t_{end}) = 0, \lambda_2(t_{end}) = 0, \lambda_3(t_{end}) = 0 \text{ and } \lambda_4(t_{end}) = 0.$$

In addition, the optimal control is given as follows:

$$\begin{cases}
u_1^*(t) = max\{0, min\{\frac{cS^*(t)M^*(t)[-\lambda_2(t)+\lambda_1(t)]}{2c_1}, 1\}\}, \\
u_2^*(t) = max\{0, min\{\frac{hS^*(t)S_m^*(t)[-\lambda_2(t)+\lambda_1(t)]}{2c_2}, 1\}\}, \\
u_3^*(t) = max\{0, min\{\frac{\theta\beta S_m^*(t)P^*(t)[-\lambda_3(t)+\lambda_2(t)]+\beta S^*(t)P^*(t)[-\lambda_3(t)+\lambda_1(t)]}{2c_3}, 1\}\}.
\end{cases}$$

Proof. To begin with, let $(S^*(t), S_m^*(t), P^*(t), M^*(t))$ be the optimal state solutions of the optimal control problem (5.1) and (5.2) under the optimal control variables $u^*(t)$. The following analysis is performed at $X^*(t) = (S^*(t), S_m^*(t), P^*(t), M^*(t))$. For the adjoint equation (5.6) and the Hamiltonian function (5.3), we can obtain the partial derivatives of H with respect to $S(t), S_m(t), P(t), M(t)$ respectively as follows:

$$\begin{cases} \frac{\partial H}{\partial S(t)} = & z_2 + \lambda_1(t)\{-[1+u_3^*(t)]\beta P^*(t) - [1+u_1^*(t)]cM^*(t) - [1+u_2^*(t)]hS_m^*(t) - \mu\} \\ & +\lambda_2(t)\{[1+u_1^*(t)]cM^*(t) + [1+u_2^*(t)]hS_m^*(t)\} + \lambda_3[1+u_3^*(t)]\beta P^*(t), \\ \frac{\partial H}{\partial S_m(t)} = & z_2 + \lambda_1(t)\{q - [1+u_2^*(t)]hS^*(t) - \mu\} + \lambda_2(t)\{[1+u_2^*(t)]hS^*(t) \\ & -[1-u_3^*(t)]\theta\beta P^*(t) - (q+\mu)\} + \lambda_3(t)[1+u_3^*(t)]\theta\beta P^*(t), \\ \frac{\partial H}{\partial P(t)} = & -z_1 + z_2 - \lambda_1(t)[1+u_3^*(t)]\beta S^*(t) - \lambda_2(t)[1+u_3^*(t)]\theta\beta S_m^*(t) \\ & +\lambda_3(t)\{[1+u_3^*(t)]\beta S^*(t) + [1+u_3^*(t)]\theta\beta S_m^*(t) - (\gamma+\mu)\} + \lambda_4(t)m, \\ \frac{\partial H}{\partial M(t)} = & z_3 - \lambda_1(t)[1+u_1^*(t)]cS^*(t) + \lambda_2(t)[1+u_1^*(t)]cS^*(t) - \lambda_4(t)d. \end{cases}$$

Then, according to the optimality condition (5.5) and the Hamiltonian function H (5.3), we can obtain

$$\begin{cases} \frac{\partial H}{\partial u_1(t)}|_{u(t)=u^*(t)} = 2c_1u_1^* + cS^*(t)M^*(t)[\lambda_2(t) - \lambda_1(t)] = 0, \\ \frac{\partial H}{\partial u_2(t)}|_{u(t)=u^*(t)} = 2c_2u_2^* + hS^*(t)S_m^*(t)[\lambda_2(t) - \lambda_1(t)] = 0, \\ \frac{\partial H}{\partial u_3(t)}|_{u(t)=u^*(t)} = 2c_3u_3^* + \theta\beta S_m^*(t)P^*(t)[-\lambda_2(t) + \lambda_3(t)] + \beta S^*(t)P^*(t)[-\lambda_1(t) + \lambda_3(t)] = 0. \end{cases}$$

This means that optimal control is obtained. The proof is complete. □

6. Numerical simulation

In this section, some numerical simulations are performed to verify the existence of equilibria, the local stability of the positive equilibrium. An investigation of system (3.1) with the coefficients above can be conducted via a numerical integration using the standard MATLAB algorithm.

Table 4. Numerical simulation results.

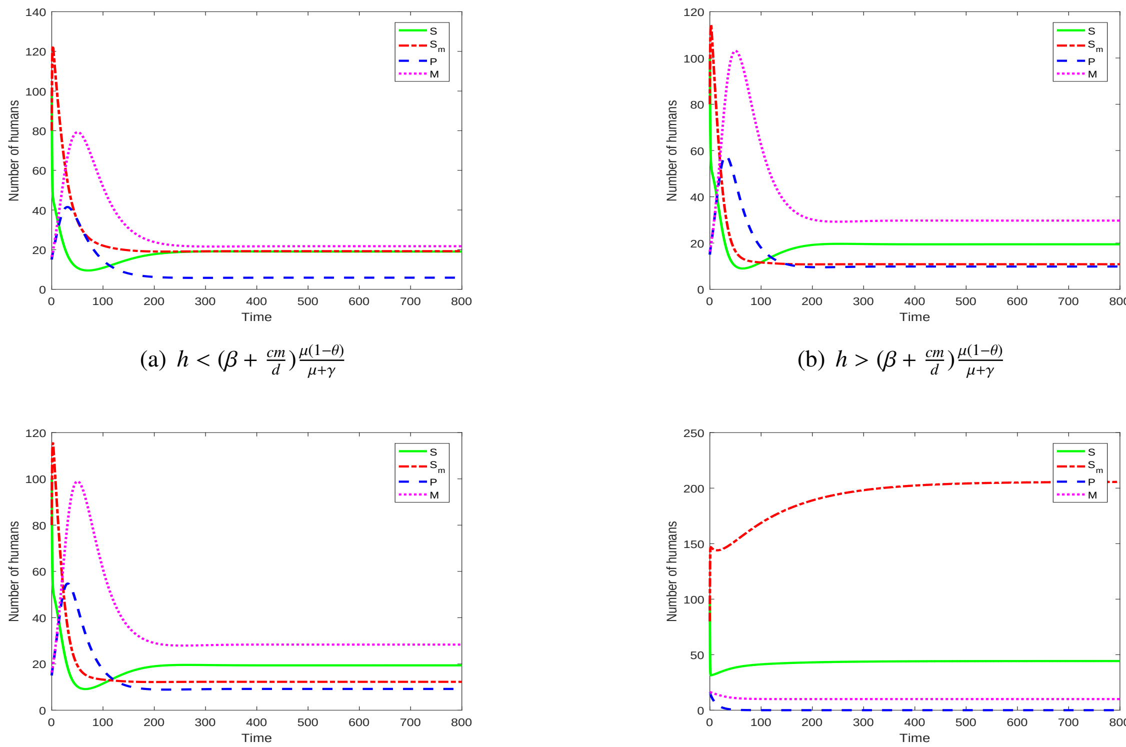
Parameter value	Theorem	Figure	Result
$B = 1, \beta = 0.002, c = 0.008, q = 0.6, h = 0.01, \mu = 0.02,$ $\theta = 0.05, \gamma = 0.02, M_0 = 0.5, m = 0.1, d = 0.05.$	Theorem 3.1(1)	Figure 3(a)	E^1 is LAS
$B = 1, \beta = 0.002, c = 0.01, q = 0.6, h = 0.01, \mu = 0.02,$ $\theta = 0.05, \gamma = 0.02, M_0 = 0.5, m = 0.1, d = 0.05.$	Theorem 3.1(2)	Figure 3(b)	E^2 is LAS
$B = 1, \beta = 0.002, c = 0.0095, q = 0.6, h = 0.01, \mu = 0.02,$ $\theta = 0.05, \gamma = 0.02, M_0 = 0.5, m = 0.1, d = 0.05.$	Theorem 3.1(3)	Figure 3(c)	E^3 is LAS
$B = 2, \beta = 0.002, c = 0.1, q = 0.65, h = 0.01, \mu = 0.008,$ $\theta = 0.5, \gamma = 0.15, M_0 = 0.5, m = 0.01116, d = 0.05.$	Theorem 3.3	Figure 3(d)	E^0 is LAS
$B = 2, \beta = 0.03, c = 0.05, q = 0.65, h = 0.02, \mu = 0.004,$ $\theta = 0.005, \gamma = 0.02, M_0 = 2, m = 0.05, d = 0.05.$	Theorem 5.1	Figure 4	Optimal control

In system (3.1), let the parameters satisfy the first set of parameter values in Table 4. At this time we get $h = 0.01 > (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma} = 0.0086$, $R_0 = 2.5 > 1$ and $R_{0\theta} = 0.1250 > R^*_{0\theta} = -1.3497$. Based on Theorem 3.1(1), it is easy to obtain that system (3.1) has a positive equilibrium $E^1 = (19.4615, 10.8386, 9.8499, 29.6999)$ which is locally asymptotically stable and be illustrated by Figure 3(a).

In system (3.1), let the parameters satisfy the second set of parameter values in Table 4. At this time we get $h = 0.01 < (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma} = 0.02$, $R_0 = 2.5 > 1$ and $R_{0\theta} = 0.1250 > R^*_{0\theta} = -0.8031$. Based on Theorem 3.1(2), it is easy to obtain that system (3.1) has a positive equilibrium $E^2 = (19.0397, 19.2311, 5.8646, 21.7293)$ which is locally asymptotically stable and be illustrated by Figure 3(b).

In system (3.1), let the parameters satisfy the third set of parameter values in Table 4. At this time we get $h = (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma} = 0.01$, $R_0 = 2.5 > 1$ and $R_{0\theta} = 0.1250 > R^{**}_{0\theta} = -0.1205$. Based on Theorem 3.1(3), it is easy to obtain that system (3.1) has a positive equilibrium $E^3 = (19.3883, 12.2600, 9.1759, 28.3518)$ which is locally asymptotically stable and be illustrated by Figure 3(c).

In system (3.1), let the parameters satisfy the fourth set of parameter values in Table 4. At this time we get $R_{0\theta} = 0.1582 < R^s_{0\theta} = 0.4676$. Based on Theorem 3.3, it is easy to obtain that system (3.1) has a boundary equilibrium $E^0 = (44.3125, 205.4817, 0, 10)$ which is locally asymptotically stable and be illustrated by Figure 3(d).



(a) $h < (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$

(b) $h > (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$

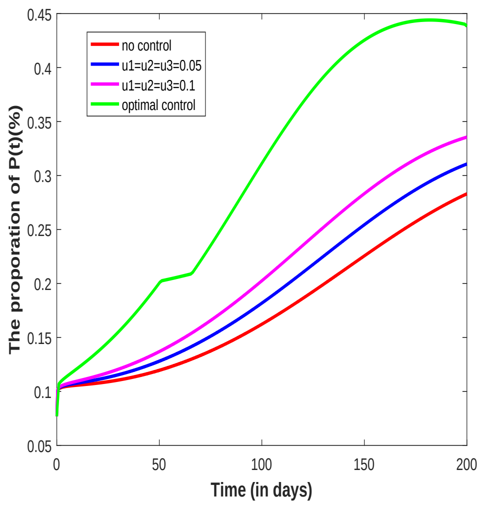
(c) $h = (\beta + \frac{cm}{d})\frac{\mu(1-\theta)}{\mu+\gamma}$

(d)

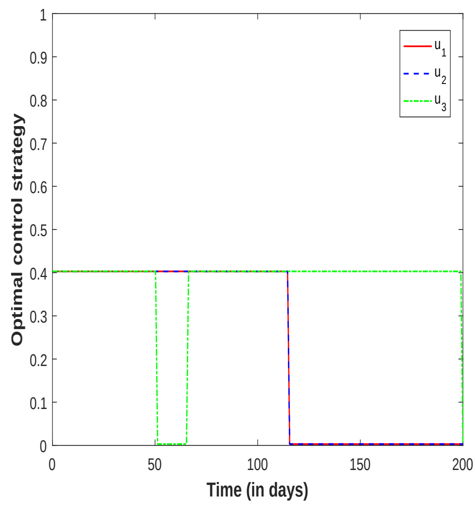
Figure 3. The temporal solution and phase portrait found by numerical integration of system (3.1), (a) E^1 is LAS, (b) E^2 is LAS, (c) E^3 is LAS, (d) E^0 is LAS.

In order to show the control measures more clearly, 200 days were selected for numerical simulation of optimal control. Figure 4(b) depicts the implementation intensity of the three control measures at different time periods. It is clear that the first 50 days, all three measures u_1, u_2, u_3 are to be carried out at the same time. We can see a huge increase in the sports population from Figure 4(a). Then the third measure u_3 is suspended for about 15 days, and it can be seen that the growth of sports population was flat at this time. After that the three control measures u_1, u_2, u_3 are continued for 50 days at the same time, and Figure 4(a) shows that the sports population surges during this period. Then the first and second two measures u_1, u_2 are suspended, and only the third measure u_3 is carried out for about 85 days. It can be seen from Figure 4(a) that the sports population grew slowly and showed a downward trend during this period. At this point, we return to the beginning of the cycle, suspend the third measure u_3 again, and enter the second cycle.

These results suggest that at the very beginning, not only the mass and interpersonal communication of health education should be implemented, but also the people who regularly participate in physical exercise should be encouraged to actively encourage non-physical exercise people to participate in physical exercise. When some of them become sports workers, we can alternately implement health education and sports promotion measures.



(a)



(b)

Figure 4. (a)The changes in the proportion of sports population under different control measures, respectively. Red line means no control measures, blue line means constant control measures $u_1 = u_2 = u_3 = 0.05$, purple line means constant control measures $u_1 = u_2 = u_3 = 0.1$, green line means optimal control. (b)The optimal control strategy, the solid red line represents measure u_1, the dashed blue line represents measure u_2 and the dotted green line represents measure u_3.

7. Discussion

In this paper, the influence of health education with two different forms and individual participation on physical exercise is mainly reflected in the existence and stability of the equilibrium in a differential equation model. Through theoretical analysis, it can be seen that only the threshold can not determine the existence of positive equilibrium, nor can it determine the number of sports population. The existence and stability of positive equilibrium is related to mass communication, interpersonal communication, the increase of physical information and individual participation. These shows health education and individual participation play very important roles and should be strengthened.

In addition to some traditional qualitative theoretical analysis results, we have obtained some new interesting results in the following through sensitivity and optimal control analysis. First, increasing interpersonal communication and mass communication can both increase the number of conscious non-sports population and sports population. For increasing the number of conscious non-sports population, the effect of mass communication is better than that of interpersonal communication. For increasing the number of sports population, the effect of interpersonal communication is better than that of mass communication. However, individual participation has the best effect on increasing the sports population. Second, increasing the daily fixed amount of new information will be more helpful for media information dissemination. Finally, the three control measures need to be implemented simultaneously for a period of time at first, and then health education and participation of sports people need to be implemented periodically in order to maximize the sports population. This conclusion is also different from previous research results.

In recent years, statistical physics has been proven to be a fruitful framework for describing phenomena outside the traditional field of physics. Physicists attempt to study collective phenomena arising from the interaction of individuals as fundamental units in social structure. Summarized a series of themes, from perspectives, cultural and linguistic dynamics to crowd behavior, hierarchical formation, human dynamics, and social communication. The connection between these issues and other more traditional topics in statistical physics has been emphasized. The comparison of model results with empirical data from social systems was also emphasized. The combination of differential equations and statistical physics will be our future research direction.

In this model, we only consider the information transmission between non-sports population. In fact, it is possible for an individual in $S(t)$ to enter $P(t)$ directly under the effect of mass media $M(t)$ or after communicating with an individual in $S_m(t)$. Additionally, if the reason for lack of sports equipment is too many sports population with limited equipment then it might be better to use $-\gamma P^2$ instead of $-\gamma P$ which is similar to intraspecific competition in ecology. All of these will be our future research work, with richer research results.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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