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Research article

Bipartite consensus for multi-agent networks of fractional diffusion PDEs via aperiodically intermittent boundary control

Xinxin Zhang[1,*] **and Huaiqin Wu**[2]

[1] School of Information Engineering, Tianjin University of Commerce, Tianjin, China
[2] School of Science, Yanshan University, Qinhuangdao 066001, China

* **Correspondence:** Email: zhangxinxin0723@163.com.

Abstract: In this paper, the exponential bipartite consensus issue is investigated for multi-agent networks, whose dynamic is characterized by fractional diffusion partial differential equations (PDEs). The main contribution is that a novel exponential convergence principle is proposed for networks of fractional PDEs via aperiodically intermittent control scheme. First, under the aperiodically intermittent control strategy, an exponential convergence principle is developed for continuously differentiable function. Second, on the basis of the proposed convergence principle and the designed intermittent boundary control protocol, the exponential bipartite consensus condition is addressed in the form of linear matrix inequalities (LMIs). Compared with the existing works, the result of the exponential intermittent consensus presented in this paper is applied to the networks of PDEs. Finally, the high-speed aerospace vehicle model is applied to verify the effectiveness of the control protocol.

Keywords: multi-agent system; networks; fractional PDEs; bipartite consensus; intermittent boundary control

1. Introduction

Multi-agent systems (MASs) evolve from the distributed artificial intelligence, which aim to solve large-scale, complex and uncertain realistic problems. MASs can undertake more complex tasks that a single agent can not finish. Therefore, the application of MASs has attracted extensive attention in various fields, such as the air traffic management, the control of unmanned aerial vehicles, the synchronization of team satellites and other engineering fields, see [1–3] and references therein. As a basic problem for MASs, consensus has attracted considerable attention from scholars in the different fields, and some results can be found in [4–6].

However, in practical engineering problems, there are situations with antagonism information between agents, such as the networks in [7, 8]. To solve the consensus with antagonism information, the concept of bipartite consensus is proposed in [9]. At the same time, many researchers pay attention to the bipartite consensus of MASs. In [10], the bipartite consensus is considered for networks of MASs via event-triggered control protocol. The exponential bipartite consensus is researched for fractional nonlinear MASs with the switching directed signed networks in [11]. In [12], the bipartite consensus issue is investigated for fractional nonlinear MASs.

It is worth noting that, in the literature mentioned above [4–12], MASs are described by ordinary differential equations (ODEs). However, in the real world, the state of MASs is not only related to the time, but also the space position, such as the moving beam, the service function chain for telecom operator and surface of the aerospace vehicle, see [13–16]. Therefore, many efforts are devoted to the consensus problem for MASs modeled by PDEs in [17–20]. In [17], the consensus issue was investigated for networks of diffusion PDEs. In [18], the consensus tracking was solved for networks of parabolic PDEs with unknown disturbances. In [19], the bipartite consensus is studied for MASs of wave PDEs. [20] studied finite and fixed-time bipartite consensus for diffusion PDEs Multi-agent system.

Compared with integer-order systems, the application of fractional systems is more popular in physics, engineering, biology and so on, because of its hereditary and memory. In recent years, large numbers of works are devoted to solve the consensus issue of multi-agent networks depicted by fractional ODEs, see [21–26]. Simultaneously, networks of fractional PDEs have attracted the extensive attention of many scholars. For example, in [27], the boundary fractional derivative control was researched for MASs of wave equations. In [28], authors considered the boundary control issue for MASs of fractional diffusion PDEs with the time-varying input disturbance. In [29], boundary feedback stabilisation problem was investigated for MASs of fractional anomalous diffusion PDEs. It should be point out that, the works in [27–29] are concerned with the asymptotical consensus for the networks of fractional PDEs, where the convergence rate can not be estimated.

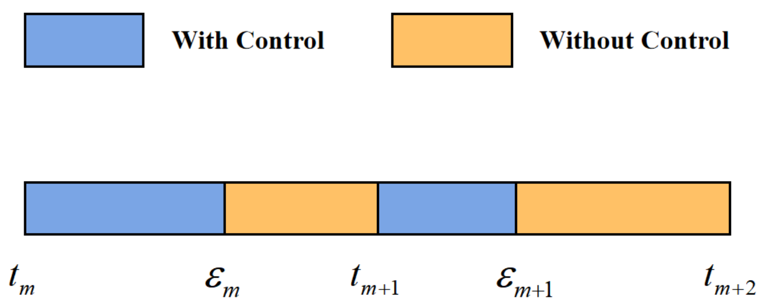

Figure 1. The working mechanism of aperiodically intermittent control protocol.

To realize the consensus for MASs, some control mechanisms are designed, such as feedback control, impulsive control, adaptive control, sliding mode control and event-triggered control etc., see [30–35]. It is worth noting that, the controllers mentioned above are continuous, which cannot save communication resources. Therefore, it becomes very meaningful to study intermittent control approach. So there are many works on intermittent control for PDEs networks, see [36–38]. Meanwhile, the working mechanism of aperiodically intermittent control protocol is shown in Figure 1. As far as we know, few scholars have studied the consensus issue for networks of fractional PDEs via aperiodically intermittent control.

Obviously, the control strategies aforementioned in [30–32], which requires to assign actuators to each position, brings the waste of space resources. In order to overcome this shortcoming, boundary control strategy is presented. Under this control method, the actuator only needs to be placed on the boundary, which reduces the space cost. Clearly, the intermittent boundary control strategy can not only save the space cost, but also the time cost. Thus, there are some works about intermittent boundary control for networks of PDEs, see [39–41]. Unfortunately, little work has been devoted to aperiodically intermittent boundary control of fractional PDEs system. Therefore, how to deploy aperiodically intermittent control in PDEs Multi-agent system is the main challenge. In addition, how to design the control protocol to make the PDEs system reach the exponential bipartite consensus is another difficulty.

Based on the above analysis, the bipartite consensus problem of fractional PDEs networks via aperiodically intermittent boundary control method has not been studied. Therefore, our object is to study the exponential bipartite consensus issue for multi-agent networks of fractional diffusion PDEs via aperiodically intermittent boundary control strategy. The contributions are briefly summarized.

1) Under the aperiodically intermittent control method, an exponential convergence principle is developed for the continuously differentiable functions, see Lemma 5. Moreover, Compared with the ideal results for ODEs systems in [42,43], in this paper, the consensus issue is investigated for PDEs system. Particularly, compared with the results of [42] with the intermittent communication, this paper is devoted to the intermittent control for the networks of PDEs.

2) Under the designed intermittent boundary control strategy and the presented convergence principle, the bipartite consensus condition can be derived in terms of LMIs. In addition, different from the asymptotical consensus of PDEs systems in [27–29], this paper focuses on the exponential bipartite consensus for PDEs system via aperiodically intermittent boundary control.

3) The high-speed aerospace vehicle model is applied to verify the effectiveness of the theoretical results;

The remaining components of this paper is organized as follows: Section 2 introduces the related knowledge with regard to graph theory, fractional calculus and aperiodically intermittent control method. In addition, a new exponential convergence principle is developed, under the aperiodically intermittent control strategy. In Section 3, the multi-agent network of fractional diffusion PDEs is modeled. In Section 4, the exponential bipartite consensus is considered for multi-agent networks of fractional diffusion PDEs. Section 5 provides an application example of the high-speed aerospace vehicle model to verify the effectiveness of the control strategy. Finally, the final decision is drawn in Section 6.

Notation: see Table 1. Moreover, the symbol $*$ denotes that

$$\begin{bmatrix} E+F+* & H \\ * & J \end{bmatrix} \triangleq \begin{bmatrix} E+F+F^T+E^T & H \\ H^T & J \end{bmatrix}.$$

Table 1. Notations

Symbol	Stand for
R	Real numbers set
R^n	n-dimensional real vector set
R^+	Positive real numbers set
$\tilde{R}$	Nonnegative real numbers set
I	Identify matrix
A^T	Transpose of A
$A > 0$	Positive definite matrix A
$\lambda_{\min}(\cdot)$	The minimum eigenvalue of matrix
$\vert * \vert$	Absolute value of $*$
$\Vert x_i \Vert_2$	$(\sum_{i=1}^n \vert x_i \vert^2)^{\frac{1}{2}}$, $x_i \in R^n$
$\Vert f(x,t) \Vert$	$(\int_0^l f^T(x,t)f(x,t)dx)^{\frac{1}{2}}$: the standard Euclidean norm
$C([0,l],R)$	The family of continuous function from $[0,l]$ to R

2. Preliminaries

2.1. Graph theory

Let $G = (\upsilon, \epsilon, A)$ be the directed signed graph, which represents the information among N dynamical nodes, where $\upsilon = \{1, 2, \ldots, N\}$ represents the set of nodes; $\epsilon \subseteq (\upsilon \times \upsilon)$ denotes the set of edges; and the adjacency matrix of G is $A = (a_{ij})_{N\times N}$, which stands for signed weights among nodes. If $(i, j) \in \epsilon$, then $a_{ij} \neq 0$, otherwise, $a_{ij} = 0$. In addition, $a_{ii} = 0$ for $i \in \upsilon$.

Divide υ into two non-empty subsets υ_1 and υ_2, i.e., $\upsilon_1 \cup \upsilon_2 = \upsilon$, $\upsilon_1 \cap \upsilon_2 = \varnothing$, if satisfy,

$$\begin{cases} a_{ij} \geq 0 \text{ for } i, j \in \upsilon_p, p = 1, 2, \\ a_{ij} \leq 0 \text{ for } i \in \upsilon_p \text{ and } j \in \upsilon_q, p \neq q, \end{cases}$$

then the directed signed graph G is said to be structurally balanced, otherwise, G is structurally unbalanced. If $(j, i) \in \epsilon$, then j is addressed as the neighbor of i. The neighbor set of agent i is $\mathcal{N}_i = \{(j, i) \in \epsilon, j \neq i\}$.

For the directed signed graph G, corresponding Laplacian matrix $L = (l_{ij})_{N\times N}$, is represented as

$$L = \text{diag}(\sum_{k\in\mathcal{N}_1} |a_{1k}|, \sum_{k\in\mathcal{N}_2} |a_{2k}|, \ldots, \sum_{k\in\mathcal{N}_N} |a_{Nk}|) - A.$$

2.2. Fractional calculus

Definition 1: For the integrable function $f : [t_0, +\infty) \longrightarrow R$, $t_0 \geq 0$, the Riemann-Liouville fractional integral of order $0 < \alpha \leq 1$ is defined by,

$${}_{t_0}I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau,$$

where $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$ is the Gamma function.

Definition 2: For the continuously differentiable function $f : [t_0, +\infty) \longrightarrow R$, $t_0 \geq 0$, the Caputo fractional derivative of order $0 < \alpha \leq 1$ is defined as,

$$ {}_{t_0}^{c}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{\frac{df(\tau)}{d\tau}}{(t-\tau)^{\alpha}} d\tau, & 0 < \alpha < 1, \\ \frac{df(t)}{dt}, & \alpha = 1. \end{cases} $$

Let $f(x, t) : [0, l] \times [t_0, +\infty) \to R$ be continuously differentiable with respect to t. Then, the Caputo partial fractional derivative and Riemann-Liouville partial fractional-order integral on t, respectively, are given by,

$$ \frac{\partial^{\alpha} f(x,t)}{\partial t^{\alpha}} = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{\partial f(x,\tau)}{\partial \tau} \frac{1}{(t-\tau)^{\alpha}} d\tau, & 0 < \alpha < 1, \\ \frac{\partial f(x,t)}{\partial t}, & \alpha = 1, \end{cases} $$

and ${}_{t_0}\mathcal{I}_t^{\alpha} f(x, t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(x,\tau)}{(t-\tau)^{1-\alpha}} d\tau$, for $0 < \alpha \leq 1$.

Lemma 1: [44] Let $g(x, t) : R \times \tilde{R} \to R$ be a continuously differentiable function about t, then,

$$ \frac{\partial^{\alpha}(g^T(x, t)g(x, t))}{\partial t^{\alpha}} \leq 2g^T(x, t) \frac{\partial^{\alpha} g(x, t)}{\partial t^{\alpha}}. $$

Lemma 2: [45] (Wirtinger's inequality) If $\xi : R \to R$ is a continuously differentiable function, which satisfies $\xi(0) = 0$ or $\xi(l) = 0$, and $P > 0$. Then,

$$ \int_0^l \xi^T(\mathfrak{s}) P \xi(\mathfrak{s}) d\mathfrak{s} \leq \frac{4l^2}{\pi^2} \int_0^l \left(\frac{d\xi(\mathfrak{s})}{d\mathfrak{s}} \right)^T P \frac{d\xi(\mathfrak{s})}{d\mathfrak{s}} d\mathfrak{s}. $$

Lemma 3: [46] (Gronwall-Bellman Integral Inequality) If there exists a function $z(t)$ satisfies $z(t) \leq \int_{t_0}^{t} a(s)z(s)ds + b(t)$, with the real function $a(t)$ and differential real function $b(t)$. Then it holds that,

$$ z(t) \leq b(t_0) \exp\left(\int_{t_0}^{t} a(s)ds \right) + \int_{t_0}^{t} b(s) \exp\left(\int_{s}^{t} a(\tau)d\tau \right) ds. $$

Specifically, if $b(t) = b$, it follows $z(t) \leq b \exp\left(\int_{t_0}^{t} a(s)ds \right)$.

Definition 3: If $\vartheta : R^n \to \tilde{R}$ satisfy the following conditions: *i)* continuously differentiable; *ii)* positive definite; *iii)* radially unbounded, then function ϑ is called as C_P-function.

Lemma 4: For continuously differentiable function $x : [t_0, +\infty) \to R$ and C_P-function $\vartheta : R^n \to \tilde{R}$. Set $V(t) = \vartheta(x(t))$, for $0 < \alpha \leq 1$, If there exists constant $\tilde{k}$, such that

$$ {}_{t_0}^{c}D_t^{\alpha} V(t) \leq \tilde{k} V(t), \tag{2.1} $$

then we can get $V(t) \leq V(t_0) \exp\left(\frac{\tilde{k}(t-t_0)^{\alpha}}{\Gamma(1+\alpha)} \right)$.

Proof. On the basis of (2.1), it holds that

$$ {}_{t_0}^{c}D_t^{\alpha} V(t) = \tilde{k} V(t) - \zeta(t), \tag{2.2} $$

then taking the fractional integral ${}_{t_0}I_t^{\alpha}$ on the both sides of (2.2), it obtains that,

$$ V(t) = V(t_0) + \int_{t_0}^{t} \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \left(\tilde{k} V(s) - \zeta(s) \right) ds \leq V(t_0) + \tilde{k} \int_{t_0}^{t} \frac{(t-s)^{\alpha-1} V(s)}{\Gamma(\alpha)} ds. $$

Then according to Lemma 3, it holds that $V(t) \leq V(t_0) \exp\left(\int_{t_0}^{t} \tilde{k} \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} ds \right)$, then from $\int_{t_0}^{t} \tilde{k} \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} ds = \tilde{k} \frac{(t-t_0)^{\alpha}}{\Gamma(\alpha+1)}$, we can further get, $V(t) \leq V(t_0) \exp\left(\frac{\tilde{k}(t-t_0)^{\alpha}}{\Gamma(1+\alpha)} \right)$, the proof is completed.

2.3. Basic knowledge of aperiodically intermittent control

As for the aperiodically intermittent control protocol, some corresponding knowledge are given in this subsection, which in order to make the networks of fractional diffusion equations PDEs achieve exponential consensus.

Assumption 1: Regarding aperiodically intermittent control protocol, there exists two constants θ and η, satisfy $0 < \theta < \eta < +\infty$, for $m = 0, 1, 2, \ldots$, such that,

$$\begin{cases} \inf_m(\varepsilon_m - t_m) = \theta, \\ \sup_m(t_{m+1} - t_m) = \eta, \end{cases}$$

which means that the rest width no more than $\eta - \theta$.

Lemma 5: For continuously differentiable function $x : [t_0, +\infty) \to R$ and C_P-function $\vartheta : R^n \to \tilde{R}$. Set $V(t) = \vartheta(x(t))$ and $0 < \alpha \le 1$. If there exist constants $a > 0$ and $b > 0$, satisfy the following conditions for $m = 0, 1, 2, \ldots$,

$$i)\begin{cases} {}^{c}_{t_m}D^{\alpha}_{t}V(t) \le -aV(t), & t_m \le t < \varepsilon_m, \\ {}^{c}_{\varepsilon_m}D^{\alpha}_{t}V(t) \le bV(t), & \varepsilon_m \le t < t_{m+1}; \end{cases}$$

ii) $0 < \varrho = \exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right)\exp\left(\frac{b\eta^\alpha}{\Gamma(1+\alpha)}\right) < 1$ (i.e., $a\theta^\alpha > b\eta^\alpha$), then $V(t) \le V(0)\exp\left(\frac{-a\theta^\alpha + b\eta^\alpha}{\Gamma(1+\alpha)}\right)$.

Proof. 1) For $0 = t_0 \le t < \varepsilon_0$, on the basis of Lemma 4, it follows that,

$$V(t) \le V(0)\exp\left(\frac{-at^\alpha}{\Gamma(1+\alpha)}\right), \tag{2.3}$$

then, we can further get,

$$V(\varepsilon_0) \le V(0)\exp\left(\frac{-a\varepsilon_0^\alpha}{\Gamma(1+\alpha)}\right) \le V(0)\exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right). \tag{2.4}$$

2) If $\varepsilon_0 \le t < t_1$, combine Lemma 4, (2.3) and (2.4), it holds,

$$V(t) \le V(\varepsilon_0)\exp\left(\frac{b(t-\varepsilon_0)^\alpha}{\Gamma(1+\alpha)}\right) \le V(0)\exp\left(\frac{-a\varepsilon_0^\alpha}{\Gamma(1+\alpha)}\right)\exp\left(\frac{b(t-\varepsilon_0)^\alpha}{\Gamma(1+\alpha)}\right), \tag{2.5}$$

by (2.5), we can easily get,

$$V(t_1) \le V(0)\exp\left(\frac{-a\varepsilon_0^\alpha}{\Gamma(1+\alpha)}\right)\exp\left(\frac{b(t_1-\varepsilon_0)^\alpha}{\Gamma(1+\alpha)}\right) \le V(0)\exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right)\exp\left(\frac{b\eta^\alpha}{\Gamma(1+\alpha)}\right) \le V(0)\varrho. \tag{2.6}$$

3) When $t_1 \le t < \varepsilon_1$, it follows from (2.6) that,

$$V(t) \le V(t_1)\exp\left(\frac{-a(t-t_1)^\alpha}{\Gamma(1+\alpha)}\right) \le V(0)\varrho\exp\left(\frac{-a(t-t_1)^\alpha}{\Gamma(1+\alpha)}\right), \tag{2.7}$$

next, it obtains that,

$$V(\varepsilon_1) \le V(0)\varrho\exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right).$$

4) If $\varepsilon_1 \le t < t_2$, one has,

$$V(t) \le V(\varepsilon_1) \exp\left(\frac{b(t-\varepsilon_1)^\alpha}{\Gamma(1+\alpha)}\right) \le V(0)\varrho \exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right)\exp\left(\frac{b(t-\varepsilon_1)^\alpha}{\Gamma(1+\alpha)}\right), \tag{2.8}$$

then on the basis of (2.8), yields,

$$V(t_2) \le V(0)\varrho^2.$$

Then according to mathematical induction, it follows that, for $t_m \le t < \varepsilon_m$,

$$V(t) \le V(0)\varrho^m \exp\left(\frac{-a(t-t_m)^\alpha}{\Gamma(1+\alpha)}\right), \tag{2.9}$$

meanwhile, by (2.9), we have, for $\varepsilon_m \le t < t_{m+1}$

$$V(t) \le V(0)\varrho^m \exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right)\exp\left(\frac{b(t-\varepsilon_m)^\alpha}{\Gamma(1+\alpha)}\right). \tag{2.10}$$

Assuming that (2.9) and (2.10) hold, then for $m = k+1$, it is easy to obtain that,

$$V(t_{k+1}) \le V(0)\varrho^{k+1}.$$

Therefore, if $t_{k+1} \le t < \varepsilon_{k+1}$,

$$V(t) \le V(0)\varrho^{k+1} \exp\left(\frac{-a(t-t_{k+1})^\alpha}{\Gamma(1+\alpha)}\right),$$

so $V(\varepsilon_{k+1}) \le V(0)\varrho^{k+1}\exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right)$.

And for $\varepsilon_{k+1} \le t < t_{k+2}$,

$$V(t) \le V(0)\varrho^{k+1} \exp\left(\frac{-a\theta^\alpha}{\Gamma(1+\alpha)}\right)\exp\left(\frac{b(t-\varepsilon_{k+1})^\alpha}{\Gamma(1+\alpha)}\right),$$

thus, $V(t_{k+2}) \le V(0)\varrho^{k+2}$.

Above all, (2.9) and (2.10) hold for $m = k+1$. Therefore, for any $t_m \le t < \varepsilon_{m+1}$, we have $V(t) \le V(0)\varrho^{m+2}$, i.e., $V(t) \le V(0)\left(\exp\left(\frac{-a\theta^\alpha + b\eta^\alpha}{\Gamma(1+\alpha)}\right)\right)^{m+2} \le V(0)\exp\left(\frac{-a\theta^\alpha + b\eta^\alpha}{\Gamma(1+\alpha)}\right)$, the proof of this Lemma is completed.

3. System model description

3.1. System model

The networks of fractional diffusion PDEs, which consist of N subnetworks are presented in this subsection. The dynamic of *i-th* subnetwork is described by,

$$\begin{cases} \frac{\partial^\alpha \omega_i(x,t)}{\partial t^\alpha} = \rho \frac{\partial^2 \omega_i(x,t)}{\partial x^2} + f(\omega_i(x,t)), t \ge 0, x \in [0,l], \\ \frac{\partial \omega_i(x,t)}{\partial x} \mid_{x=0} = 0, \quad \frac{\partial \omega_i(x,t)}{\partial x} \mid_{x=l} = u_i(t), \\ \omega_i(x,t) \mid_{t=0} = \omega_{i0}(x), \end{cases} \tag{3.1}$$

where $\omega_i(x,t) \in R$, denotes the state of the *i-th* subnetwork at the position x with the time t; $\rho \in R$ is the diffusivity parameter; $u_i(t) \in R$ stands for the boundary control input for the *i-th* agent; $\omega_{i0}(x) \in C([0,l],R)$, is the initial value, $i \in \upsilon$; $f: R \to R$ is the nonlinear function about $\omega_i(x,t)$.

Set the error is $e_{ij}(x,t) = \omega_i(x,t) - sgn(a_{ij})\omega_j(x,t)$, then the error system can be described as

$$\begin{cases} \frac{\partial^\alpha e_{ij}(x,t)}{\partial t^\alpha} = \rho\frac{\partial^2 \omega_i(x,t)}{\partial x^2} + \hat{f}(e_{ij}(x,t)), t \geq 0, x \in [0,l], \\ \frac{\partial e_{ij}(x,t)}{\partial x} \mid_{x=0} = \frac{\partial e_{ij}(x,t)}{\partial x} \mid_{x=l} = \tilde{u}(t), \\ e_{ij}(x,t) \mid_{t=0} = e_{ij0}(x), \end{cases} \tag{3.2}$$

where $e_{ij0}(x) \in C([0,l],R)$, is the initial value of $e_{ij}(x,t)$. ρ is the same as that in (3.1). Meanwhile, $\hat{f}(e_{ij}(x,t)) = f(\omega_{ij}(x,t)) - sgn(a_{ij})f(\omega_{ij}(x,t))$, $\tilde{u}(t) = u_i(t) - sgn(a_{ij})u_j(t)$ and $e_{ij0}(x) = \omega_{i0}(x) - sgn(a_{ij})\omega_{j0}(x)$, for $i, j \in \upsilon$. Set $e_i(x,t) = \sum_{j\in\mathcal{N}_i} e_{ij}(x,t) = \sum_{j\in\mathcal{N}_i}\Big(\omega_i(x,t) - sgn(a_{ij})\omega_j(x,t)\Big)$.

Throughout this paper, for the nonlinear function f, the following assumption is satisfied.

Assumption 2: For function f, the Lipschitz condition is satisfied, i.e., $\forall\ \delta, \eta \in R$, there exists $\mu > 0$, such that,

$$|f(\delta) - f(\eta)| \leq \mu|\delta - \eta|.$$

Under Assumption 2, the well-posedness of networks (3.1) and (3.2) can be easily get by [47].

Definition 4: (Exponential Bipartite Consensus), If there exists a constant ι, for any $x \in (0,l)$,

$$\lim_{t\to\infty} \|e_i(x,t)\| \leq \|e_{i0}(x,t)\| \exp\{-\iota\}, \quad \forall i, j \in \upsilon,$$

then networks of diffusion PDEs are said to achieve the exponential bipartite consensus with the rate ι, under control protocol $u_i(t)$.

On the basis of Definition 4, the exponential bipartite consensus issue is investigated, which can be transformed to discuss the exponential stability of error system (3.2).

4. Exponential bipartite consensus for networks of fractional diffusion PDEs

In this section, the exponential bipartite consensus for MASs of fractional diffusion PDEs is considered on a directed graph via intermittent boundary control protocol.

4.1. Exponential bipartite consensus via aperiodically intermittent boundary control

Consider the designed aperiodically intermittent control protocol as following:

$$u_i(t) = \begin{cases} -k\sum_{j\in\mathcal{N}_i} a_{ij}\Big(\omega_i(x,t) - sgn(a_{ij})\omega_j(x,t)\Big), & t_m \leq t \leq \varepsilon_m, \\ 0, & \varepsilon_m \leq t \leq t_{m+1}, \end{cases} \tag{4.1}$$

where $i \in \upsilon$, and $k > 0$, is called the control gain which to be determined later.

Theorem 1: Under Assumptions 1 and 2, if the following inequalities hold,

1) $m_1\theta^\alpha \geq m_2\eta^\alpha$;

2) if there exists constant $k > 0$, such that,

$$\Psi = \begin{bmatrix} \Psi_{11} & -\rho kL \\ * & \Psi_{22} \end{bmatrix} < 0;$$

where $\Psi_{11} = -\frac{\pi}{4l^2}(\rho I_N + *)$, $\Psi_{22} = -2\rho kL + (\mu^2+1)I_N$, $m_1 = \lambda_{\min}(-\Psi)$ and $m_2 = \mu^2 + 1$, then networks of diffusion PDEs can achieve the exponential bipartite consensus via intermittent control protocol (4.1).

Proof. Construct the following Lyapunov function

$$V(t) = \sum_{i=1}^{N} V_i(t), \quad V_i(t) = \int_0^l e_i^T(t)e_i(t)dx,$$

for $t \in [t_m, t_{m+1}]$, by Lemma 1, take the fractional derivative of order α with respect to $V_i(t)$, derives,

$$\begin{aligned}
{}_{t_m}^{c}D_t^{\alpha} V_i(t) \leq 2\int_0^l e_i^T(x,t)\frac{\partial^{\alpha} e_i(x,t)}{\partial t^{\alpha}}dx =& 2\int_0^l \sum_{j\in\mathcal{N}_i} e_{ij}^T(x,t)\sum_{j\in\mathcal{N}_i}\frac{\partial^{\alpha} e_{ij}(x,t)}{\partial t^{\alpha}}dx \\
=& 2\int_0^l \sum_{j\in\mathcal{N}_i} e_{ij}^T(x,t)\sum_{j\in\mathcal{N}_i}\left(\rho\frac{\partial^2 e_{ij}(x,t)}{\partial x^2} + f\left(e_{ij}(x,t)\right)\right)dx.
\end{aligned}$$

Next we divide the time into two sections for discussion.

Step 1: For $t \in [t_m, \varepsilon_m)$, applying Lemmas 2 and 3, we obtain,

$$\begin{aligned}
&2\int_0^l \sum_{j\in\mathcal{N}_i} e_{ij}^T(x,t)\sum_{j\in\mathcal{N}_i}\left(\rho\frac{\partial^2 e_{ij}(x,t)}{\partial x^2}\right)dx \\
=& 2\rho\sum_{j\in\mathcal{N}_i} e_{ij}^T(l,t)\sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}(l,t)}{\partial x} - 2\rho\int_0^l \sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}^T(x,t)}{\partial x}\sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}(x,t)}{\partial x}dx \\
=& -2\rho k\int_0^l \sum_{j\in\mathcal{N}_i} e_{ij}^T(l,t)\sum_{j\in\mathcal{N}_i} l_{ij}e_{ij}(x,t)dx - 2\rho\int_0^l \sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}^T(x,t)}{\partial x}\sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}(x,t)}{\partial x}dx \\
\leq& -2\rho k l_i\int_0^l e_i^T(l,t)e_i(x,t)dx - 2\rho\int_0^l \frac{\partial e_i^T(x,t)}{\partial x}\frac{\partial e_i(x,t)}{\partial x}dx \\
\leq& -2\rho k l_i\int_0^l \left(e_i^T(x,t) + \tilde{e}_i^T(x,t)\right)e_i(x,t)dx - \frac{\pi\rho}{2l^2}\int_0^l \tilde{e}_i^T(x,t)\tilde{e}_i(x,t)dx
\end{aligned}$$

where $\tilde{e}_i(x,t) = e_i(l,t) - e_i(x,t)$. Then on the basis of Assumption 2, we have,

$$\begin{aligned}
&2\int_0^l e_i^T(x,t)f(e_i(x,t))dx \\
\leq& \int_0^l e_i^T(x,t)e_i(x,t)dx + \int_0^l f(e_i^T(x,t))f(e_i(x,t))dx \leq (\mu^2+1)\int_0^l e_i^T(x,t)e_i(x,t)dx.
\end{aligned} \tag{4.2}$$

Combining the above three formulas, we can easily get the following results,

$$\begin{aligned}
{}_{t_m}^{c}D_t^{\alpha} V_i(t) \leq& -2\rho k l_i\int_0^l \left(e_i^T(x,t) + \tilde{e}_i^T(x,t)\right)e_i(x,t)dx \\
&- \frac{\pi\rho}{2l^2}\int_0^l \tilde{e}_i^T(x,t)\tilde{e}_i(x,t)dx + (\mu^2+1)\int_0^l e_i^T(x,t)e_i(x,t)dx
\end{aligned}$$

where $\bar{e}_i^T(x,t) = \left(\tilde{e}_i^T(x,t), e_i^T(x,t)\right)^T$. Hence, we get,

$$\begin{aligned}
{}_{t_m}^{c}D_t^{\alpha}V(t) \leq & -2\rho kL \int_0^l \left(e^T(x,t) + \tilde{e}^T(x,t)\right) e(x,t)dx \\
& - \frac{\pi}{4l^2}\int_0^l \tilde{e}^T(x,t)(\rho I_N + *)\tilde{e}(x,t)dx + (\mu^2+1)I_N \int_0^l e^T(x,t)e(x,t)dx \\
= & \int_0^l \bar{e}^T(x,t)\Psi\bar{e}(x,t)dx,
\end{aligned}$$

where $\bar{e}^T(x,t) = \left(\tilde{e}^T(x,t), e^T(x,t)\right)^T$, set $m_1 = \lambda_{\min}(-\Psi)$, we can further get,

$$ {}_{t_m}^{c}D_t^{\alpha}V(t) \leq -m_1 V(t). $$

Step 2: For $t \in [\varepsilon_m, t_{m+1})$, apply Lemma 2, we obtains

$$\begin{aligned}
& 2\int_0^l \sum_{j\in\mathcal{N}_i} e_{ij}^T(x,t) \sum_{j\in\mathcal{N}_i}\left(\rho\frac{\partial^2 e_{ij}(x,t)}{\partial x^2}\right)dx \\
=& 2\rho \sum_{j\in\mathcal{N}_i} e_{ij}^T(l,t)\sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}(l,t)}{\partial x} - 2\rho\int_0^l \sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}^T(x,t)}{\partial x}\sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}(x,t)}{\partial x}dx \\
=& -2\rho\int_0^l \sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}^T(x,t)}{\partial x}\sum_{j\in\mathcal{N}_i}\frac{\partial e_{ij}(x,t)}{\partial x}dx = -2\rho\int_0^l \frac{\partial e_i^T(x,t)}{\partial x}\frac{\partial e_i(x,t)}{\partial x}dx \\
\leq & -\frac{\pi\rho}{2l^2}\int_0^l \tilde{e}_i^T(x,t)\tilde{e}_i(x,t)dx,
\end{aligned}$$

then combine with (4.2), the following inequality is established,

$$ {}_{\varepsilon_m}^{c}D_t^{\alpha}V_i(t) \leq -\frac{\pi\rho}{2l^2}\int_0^l \tilde{e}_i^T(x,t)\tilde{e}_i(x,t)dx + (\mu^2+1)\int_0^l e_i^T(x,t)e_i(x,t)dx \leq (\mu^2+1)V_i(t). $$

Furthermore, set $m_2 = (\mu^2+1)$, we can get ${}_{\varepsilon_m}^{c}D_t^{\alpha}V(t) \leq m_2 V(t)$. Above all, it holds that

$$\begin{cases}
{}_{t_m}^{c}D_t^{\alpha}V_i(t) \leq -m_1 V_i(t), & t_m \leq t < \varepsilon_m, \\
{}_{\varepsilon_m}^{c}D_t^{\alpha}V_i(t) \leq m_2 V_i(t), & \varepsilon_m \leq t < t_{m+1};
\end{cases}$$

By Lemma 5, it follows that, $V_i(t) \leq V_i(t_0)\exp(\frac{-m_1\theta^{\alpha}+m_2\eta^{\alpha}}{\Gamma(1+\alpha)})$. By applying $= V_i(t) = \int_0^l e_i^T(t)e_i(t)dx$, it holds that $\|\sum_{j\in\mathcal{N}_i} e_{ij}(x,t)\|^2 \leq \|\sum_{j\in\mathcal{N}_i} e_{ij0}(x,t)\|^2\exp(\frac{-m_1\theta^{\alpha}+m_2\eta^{\alpha}}{\Gamma(1+\alpha)})$, we can easily get $\lim_{t\to\infty}\|e_i(x,t)\| \leq \|e_{i0}(x,t)\|\exp(\frac{-m_1\theta^{\alpha}+m_2\eta^{\alpha}}{\Gamma(1+\alpha)})$. Then by means of Definition 4, which shows that, the exponential bipartite consensus for networks of diffusion PDEs can be achieved, under the control protocol (4.1).

5. An application for high-speed aerospace vehicle

The high-speed aerospace vehicle, which displayed in Figure 2, is modeled to show that the theoretical results are effective.

Figure 2. The high-speed aerospace vehicle model (from the internet).

The surface of high-speed aerospace vehicle is shown in Figure 3, and it holds that, $E_x + E_{gen} = E_{conv} + E_{rad} + E_{x+\Delta x} + E_{chg}$. More details of the modeling process can be found in [48]. By analogy with [46], we can further establish the high-speed aerospace vehicle model via fractional calculus. Next we research the bipartite consensus issue of high-speed aerospace vehicles via aperiodically intermittent boundary control strategy, by applying the theoretical results we presented.

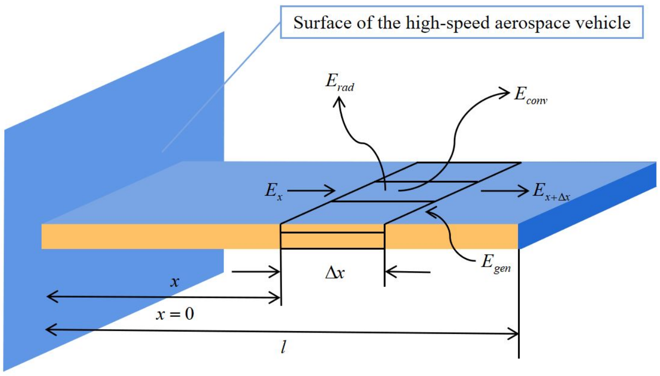

Figure 3. The topologies among high-speed aerospace vehicles.

Consider the high-speed aerospace vehicles modeled by networks of diffusion PDEs (3.1) with $N = 20$. Number each high-speed aerospace vehicles, the topological graph among them is shown in Figure 4. Next, some suitable parameters are given. $f(\omega(x,t))$ is chosen as $\cos(\omega(x,t))$, which satisfies Lipschitz condition with $\mu = 1$. Set $\alpha = 0.3$, $l = 5$, $\rho = 6$, $\theta = 0.08$ and $\eta = 0.03$. Furthermore, the validity of theorem is verified by simulation results. The initial values of the aerospace vehicles states are selected as $\omega_{i0}(x) = \sin(0.5ix)$, for $i \in \upsilon$. The state trajectories of aerospace vehicles are described in Figure 5. The communication information among followers is stored in $A = (a_{ij})_{N\times N}$ and the corresponding $L = (l_{ij})_{N\times N}$.

By LMI toolbox, it holds that, $k = 0.076$. The bipartite consensus of high-speed aerospace vehicles modeled by (3.1) can achieve, which displays in Figures 6 and 7. Therefore, the goal of this brief shown can achieve, and the theoretical result is verified.

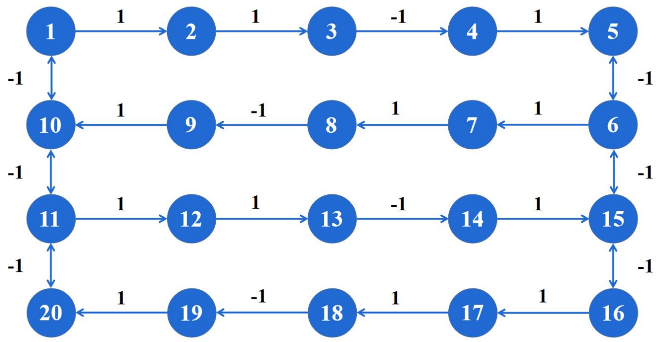

Figure 4. The topologies among high-speed aerospace vehicles.

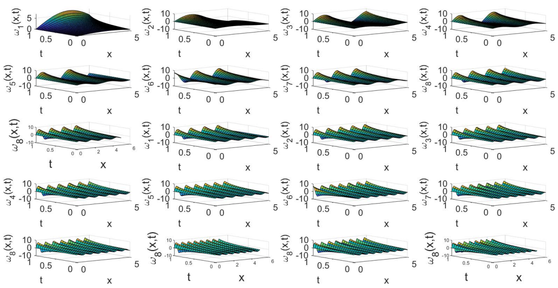

Figure 5. The The evolution of t high-speed aerospace vehicles over position x and time t.

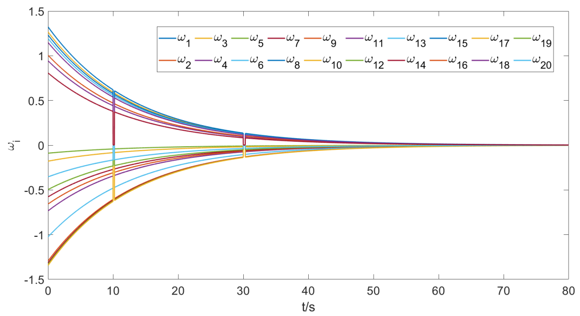

Figure 6. The state trajectories of high-speed aerospace vehicles under boundary control.

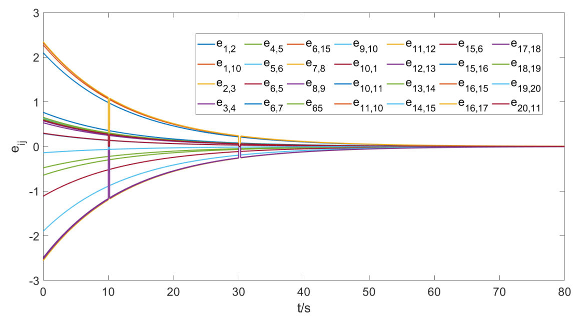

Figure 7. The error state trajectories of high-speed aerospace vehicles.

6. Conclusions

In this paper, the bipartite consensus has been investigated for the multi-agent networks of fractional diffusion PDEs via aperiodically intermittent control protocol. To achieve the exponential goal, a new aperiodically intermittent boundary control strategy was designed for PDEs system. Under the designed aperiodically intermittent boundary control mechanism, the exponential convergence principle is developed for continuously differential function. In addition, the exponential consensus conditions have been established in terms of LMIs.

In the future, the research direction will focus on the consensus for networks of fractional/variable-order PDEs systems

1) control protocol design: sliding mode control, event-triggered control and so on;

2) topology: switching topology, stochastic topology, fuzzy topology and so on;

3) application: viscoelasticity, transport processes and control, as well as the biological interaction seen in nature.

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Conflict of interest

The authors declare no conflicts of interest.

References

1. C. Tomlin, G. J. Pappas, S. Sastry, Conflict resolution for air traffic management: a study in multiagent hybrid systems, *IEEE Trans. Autom. Control*, **43** (1998), 509–521. https://doi.org/10.1109/9.664154

2. Y. Zou, H. J. Zhang, W He, Adaptive coordinated formation control of heterogeneous vertical takeoff and landing UAVs subject to parametric uncertainties, *IEEE Trans. Cybern.*, **52** (2022), 3184–3195. https://doi.org/10.1109/TCYB.2020.3009404

3. L. Cao, Y. Q. Chen, Z. D. Zhang, H. N. Li, A. K. Misra, Predictive smooth variable structure filter for attitude synchronization estimation during satellite formation flying, *IEEE Trans. Aerosp. Electron. Syst.*, **53** (2017), 1375–1383. https://doi.org/10.1109/TAES.2017.2671118

4. R. Olfati-Saber, R. Murray, Consensus problems in networks of agents with switching topology and time-delays, *IEEE Trans. Autom. Control*, **49** (2004), 1520–1533. https://doi.org/10.1109/TAC.2004.834113

5. G. H. Wen, W. X. Zheng, On constructing multiple lyapunov functions for tracking control of multiple agents with switching topologies, *IEEE Trans. Autom. Control*, **64** (2019), 3796–3803. https://doi.org/10.1109/TAC.2018.2885079

6. H. Q. Li, X. F. Liao, T. W. Huang, W. Zhu, Event-triggering sampling based leader-following consensus in second-order multi-agent systems, *IEEE Trans. Autom. Control*, **60** (2015), 1998–2003. https://doi.org/10.1109/TAC.2014.2365073

7. S. Wasserman, K. Faust, *Social Network Analysis: Methods and Applications*, Cambridge University Press, 1994.

8. C. Altafini, G. Lini, Predictable dynamics of opinion forming for networks with antagonistic interactions, *IEEE Trans. Autom. Control*, **60** (2015), 342–357. https://doi.org/10.1109/TAC.2014.2343371

9. C. Altafini, Consensus problems on networks with antagonistic interactions, *IEEE Trans. Autom. Control*, **58** (2013), 935–946. https://doi.org/10.1109/TAC.2012.2224251

10. A. H. Hu, Y. Y. Wang, J. D. Cao, A. Alsaedi, Event-triggered bipartite consensus of multi-agent systems with switching partial couplings and topologies, *Inf. Sci.*, **521** (2020), 1–13. https://doi.org/10.1016/j.ins.2020.02.038

11. P. Gong, Exponential bipartite consensus of fractional-order non-linear multi-agent systems in switching directed signed networks, *IET Control Theory Appl.*, **14** (2020), 2582–2591. https://doi.org/10.1049/iet-cta.2019.1241

12. M. Shahvali, A. Azarbahram, M. Naghibi-Sistani, J. Askari, Bipartite consensus control for fractional-order nonlinear multi-agent systems: An output constraint approach, *Neurocomputing*, **397** (2020), 212–223. https://doi.org/10.1016/j.neucom.2020.02.036

13. Y. Cheng, Y. H. Wu, B. Z. Guo, Absolute boundary stabilization for an axially moving Kirchhoff beam, *Automatica*, **129** (2021), 109667. https://doi.org/10.1016/j.automatica.2021.109667

14. S. T. Le, Y. H. Wu, Y. Q. Guo, C. D. Vecchio, Game theoretic approach for a service function chain routing in NFV with coupled constraints, *IEEE Trans. Circuits Syst. II Express Briefs*, **68** (2021), 3557–3561. https://doi.org/10.1109/TCSII.2021.3070025

15. Y. Cheng, Y. H. Wu, B. Z. Guo, Boundary stability criterion for a nonlinear axially moving beam, *IEEE Trans. Autom. Control*, **67** (2022), 5714–5729. https://doi.org/10.1109/TAC.2021.3124754

16. X. N. Song, Q. Y. Zhang, M. Wang, S. Song, Distributed estimation for nonlinear PDE systems using space-sampling approach: applications to high-speed aerospace vehicle, *Nonlinear Dyn.*, **106** (2021), 3183–3198. https://doi.org/10.1007/s11071-021-06725-4

17. A. Pilloni, A. Pisano, Y. Orlov, E. Usai, Consensus-based control for a network of diffusion PDEs with boundary local interaction, *IEEE Trans. Autom. Control*, **61** (2016), 2708–2713. https://doi.org/10.1109/TAC.2015.2506990

18. P. He, Consensus of uncertain parabolic PDE agents via adaptive unit-vector control scheme, *IET Control Theory Appl.*, **12** (2018), 2488–2494. https://doi.org/10.1049/iet-cta.2018.5202

19. Y. N. Chen, Z. Q. Zuo, Y. J. Wang, Bipartite consensus for a network of wave PDEs over a signed directed graph, *Automatica*, **129** (2021), 109640. https://doi.org/10.1016/j.automatica.2021.109640

20. L. R. Zhao, H. Q. Wu, J. D. Cao, Finite/fixed-time bipartite consensus for networks of diffusion PDEs via event-triggered control, *Inf. Sci.*, **609** (2022), 1435–1450. https://doi.org/10.1016/j.ins.2022.07.151

21.X. H. Wang, H. Q. Wu, J. D. Cao, Global leader-following consensus in finite time for fractional-order multi-agent systems with discontinuous inherent dynamics subject to nonlinear growth, *Nonlinear Anal. Hybrid Syst*, **37** (2020), 100888. https://doi.org/10.1016/j.nahs.2020.100888

22.Y. Q. Zhang, H. Q. Wu, J. D. Cao, Global mittag-leffler consensus for fractional singularly perturbed multi-agent systems with discontinuous inherent dynamics via event-triggered control strategy, *J. Franklin Inst.*, **358** (2021), 2086–2114. https://doi.org/10.1016/j.jfranklin.2020.12.033

23.Y. Q. Zhang, H. Q. Wu, J. D. Cao, Group consensus in finite time for fractional multiagent systems with discontinuous inherent dynamics subject to hölder growth, *IEEE Trans. Cybern.*, **52** (2022), 4161–4172. https://doi.org/10.1109/TCYB.2020.3023704

24.X. N. Li, H. Q. Wu, J. D. Cao, Prescribed-time synchronization in networks of piecewise smooth systems via a nonlinear dynamic event-triggered control strategy, *Math. Comput. Simul.*, **203** (2023), 647–668. https://doi.org/10.1016/j.matcom.2022.07.010

25.H. G. Zhang, Z. Y. Gao, Y. C. Wang, Y. L. Cai, Leader-following exponential consensus of fractional-order descriptor multiagent systems with distributed event-triggered strategy, *IEEE Trans. Syst. Man Cybern.: Syst.*, **52** (2022), 3967–3979. https://doi.org/10.1109/TSMC.2021.3082549

26.P. Gong, Exponential bipartite consensus of fractional-order non-linear multi-agent systems in switching directed signed networks, *IET Control Theory Appl.*, **14** (2020), 2582–2591. https://doi.org/10.1049/iet-cta.2019.1241

27.B. Mbodje, G. Montseny, Boundary fractional derivative control of the wave equation, *IEEE Trans. Autom. Control*, **40** (1995), 378–382. https://doi.org/10.1109/9.341815

28.F. D. Ge, Y. Q. Chen, Event-driven boundary control for time fractional diffusion systems under time-varying input disturbance, in *2018 Annual American Control Conference (ACC)*, (2018), 140–145. https://doi.org/10.23919/ACC.2018.8431000

29.F. D. Ge, Y. Q. Chen, C. H. Kou, Boundary feedback stabilisation for the time fractional-order anomalous diffusion system, *IET Control Theory Appl.*, **10** (2018), 1250–1257. https://doi.org/10.1049/iet-cta.2015.0882

30.Y. Cao, Y. G. Kao, J. H. Park, H. B. Bao, Global Mittag-Leffler stability of the delayed fractional-coupled reaction-diffusion system on networks without strong connectedness, *IEEE Trans. Neural Networks Learn. Syst.*, **33** (2021), 6473–6483. https://doi.org/10.1109/TNNLS.2021.3080830

31.J. D. Cao, G. Stamov, I. Stamova, S. Simeonov, Almost periodicity in impulsive fractional-order reaction-diffusion neural networks with time-varying delays, *IEEE Trans. Cybern.*, **51** (2021), 151–161. https://doi.org/10.1109/TCYB.2020.2967625

32.J. H. Qin, G. S. Zhang, W. X. Zheng, Y. Kang, Adaptive sliding mode consensus tracking for second-order nonlinear multiagent systems with actuator faults, *IEEE Trans. Cybern.*, **49** (2019), 1605–1615. https://doi.org/10.1109/TCYB.2018.2805167

33.J. Sun, C. Guo, L. Liu, Q. H. Shan, Adaptive consensus control of second-order nonlinear multi-agent systems with event-dependent intermittent communications, *J. Franklin Inst.*, **360** (2023), 2289–2306. https://doi.org/10.1016/j.jfranklin.2022.10.045

34.J. Wang, M. Krstic, Output-feedback boundary control of a heat PDE sandwiched between two ODEs, *IEEE Trans. Autom. Control*, **64** (2019), 4653–4660. https://doi.org/10.1109/TAC.2019.2901704

35.J. Sun, Z. S. Wang, Event-triggered consensus control of high-order multi-agent systems with arbitrary switching topologies via model partitioning approach, *Neurocomputing*, **413** (2020), 14–22. https://doi.org/10.1016/j.neucom.2020.06.058

36.X. Z. Liu, K. N. Wu, Z. T. Li, Exponential stabilization of reaction-diffusion systems via intermittent boundary control, *IEEE Trans. Autom. Control*, **67** (2022), 3036–3042. https://doi.org/10.1109/TAC.2021.3100289

37.X. Z. Liu, K. N. Wu, W. H. Zhang, Intermittent boundary stabilization of stochastic reaction-diffusion Cohen -Grossberg neural networks, *Neural Networks*, **131** (2020), 1–13. https://doi.org/10.1016/j.neunet.2020.07.019

38.X. Y. Li, Q. L. Fan, X. Z. Liu, K. N. Wu, Boundary intermittent stabilization for delay reaction-diffusion cellular neural networks, *Neural Comput. Appl.*, **34** (2022), 18561–18577. https://doi.org/10.1007/s00521-022-07457-1

39.N. Espitia, A. Polyakov, D. Efimov, W. Perruquetti, Boundary time-varying feedbacks for fixed-time stabilization of constant-parameter reaction-diffusion systems, *Automatica*, **103** (2019), 398–407. https://doi.org/10.1016/j.automatica.2019.02.013

40.T. Hashimoto, M. Krstic, Stabilization of reaction-diffusion equations with state delay using boundary control input, *IEEE Trans. Autom. Control*, **61** (2016), 4041–4047. https://doi.org/10.1109/TAC.2016.2539001

41.C. Prieur, E. Trélat, Feedback stabilization of a 1-D linear reaction-diffusion equation with delay boundary control, *IEEE Trans. Autom. Control*, **64** (2019), 1415–1425. https://doi.org/10.1109/TAC.2018.2849560

42.J. Sun, Z. S. Wang, Consensus of multi-agent systems with intermittent communications via sampling time unit approach, *Neurocomputing*, **397** (2020), 149–159. https://doi.org/10.1016/j.neucom.2020.02.055

43.Z. S. Wang, J. Sun, H. G. Zhang, Stability analysis of T-S fuzzy control system with sampled-dropouts based on time-varying Lyapunov function method, *IEEE Trans. Syst. Man Cybern.: Syst.*, **50** (2020), 2566–2577. https://doi.org/10.1109/TSMC.2018.2822482

44.Z. B. Wang, H. Q. Wu, Stabilization in finite time for fractional-order hyperchaotic electromechanical gyrostat systems, *Mech. Syst. Signal Proc.*, **111** (2018), 628–642. https://doi.org/10.1016/j.ymssp.2018.04.009

45.C. D. Yang, T. W. Huang, A. C. Zhang, J. L. Qiu, J. D. Cao, F. E. Alsaadi, Output consensus of multiagent systems based on PDEs with input constraint: A boundary control approach, *IEEE Trans. Syst. Man Cybern.: Syst.*, **51** (2021), 370–377. https://doi.org/10.1109/TSMC.2018.2871615

46.P. Gong, Q. L. Han, W. Y. Lan, Finite-time consensus tracking for incommensurate fractional-order nonlinear multiagent systems with directed switching topologies, *IEEE Trans. Cybern.*, **52** (2022), 65–76. https://doi.org/10.1109/TCYB.2020.2977169

47. S. Q. Zhang, Monotone method for initial value problem for fractional diffusion equation, *Sci. China Ser. A: Math.*, **49** (2006), 1223–1230. https://doi.org/10.1007/s11425-006-2020-6

48. V. Yadav, R. Padhi, S. N. Balakrishnan, Robust/optimal temperature profile control of a high-speed aerospace vehicle using neural networks, *IEEE Trans. Neural Networks* , **18** (2007), 1115–1128. https://doi.org/10.1109/TNN.2007.899229