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Research article

Biodegradable waste to renewable energy conversion under a sustainable energy supply chain management

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Abstract: Effective surveillance during smart cities' sustainable development allows their cleanliness to be maintained by reusing waste to produce renewable energy. This study quantifies the biodegradable waste generated in specific regions of several cities and presents ways to convert it into renewable energy. This energy can then be used to partially support regional energy demands. This study explores ways of reducing carbon emissions for biodegradable waste collection processes in regional centers, ultimately sending the biodegradable waste to the energy conversion center. The smart production system allows for the flexible production and autonomation of rates of conversion; green technology depends on each regional center's research management, which is a decision variable for reducing carbon emissions. The major contribution of this study is to consider an energy supply chain management with flexibility of energy conversion under the reduction of carbon emissions, which leads to a sustainable ESCM with the global maximum profit. This study uses mathematical modeling to decrease biodegradable waste with conversion of energy through a classical optimization technique. The solution to this mathematical model yielded significant results, providing insight into waste reduction, reduced carbon emissions and the conversion of biodegradable waste to energy. The model is examined using numerical experiments, and its conclusion supports the model with the fundamental assumptions. Results of sensitivity analysis provide insight into the reduction and re-utilization of wastes, carbon emission reduction, and the benefits of using renewable energy.

Keywords: renewable energy; sustainability; waste reduction; flexible production; carbon emission

1. Introduction

Industrial production, transportation, households, and consumption-based businesses produce a variety of different types of waste. Globally, one of the most significant sustainability goals is to reduce garbage, but this process requires several processes to reuse types of waste for different purposes. Although this is an area of recent scientific interest, there are a few studies analyzing renewable energy production from waste. There are several studies on deterioration [1], but they did not convert those wastes to renewable energy. Another critical issue is the collection of waste and the excessive carbon emissions (CE) produced by trucks transporting them to the waste collection center (WCC) [2]. Some studies only consider reducing waste production [3] but donot consider any investments or profits that could be made from this industry. Other studies considered smart technologies [4] but ignored the energy and waste supply chain. Since reducing CE has become an area of increasing environmental concern, recent studies have begun to address CE reduction, specifically from the transportation industry [5]. This study considers energy supply chain management (ESCM) from biodegradable waste to renewable energy for a hypothetical smart city. The demands of renewable energy necessitate waste transportation, making the resulting CE largely unavoidable; transportation from the source to collection centers is the most important step of the process. Despite having control over how much energy conversion is needed, several research gaps exist within this field.

1.1. Research gaps

The following research gaps exist in this field of study:

- Several studies focus on converting waste to energy but rarely address the amount of energy conversion and its corresponding fulfillment of human demands. Most studies aim to solve environmental issues, but there is inadequate research into reducing CE. Existing studies prove that CE is reduced by using biodegradable waste as renewable energy. Since quantification of CE is yet to be attempted due to increased waste transportation, this study proposes a strategy to address this research gap.
- The ESCM has two players: one who collects biodegradable wastes and transports them to collection centers, and the other who converts that waste into energy. Since the former has no control over the quantity of waste, the entire amount is supplied to the latter. The second player cannot convert massive quantities of waste into energy at one time, thus making it necessary for renewable energy to be based on the demand of each city. Renewable energy production must be flexible; the research gap here is in the direction of variable/flexible production and the corresponding CE reduction. This study addresses the flexibility of production to achieve sustainable development goals (SDGs).
- If this model is compared with Sarkar and Seo [6], then it converges over the existing one in the direction of
    1). Sustainability
    2). Green technology
    3). Variable investment in GT for collection center
    4). Variable investment in GT for energy conversion center
    5). The variable selling price of energy.

This model addresses several new dimensions based on the real-life situation, which the existing study does not contain. Therefore, it has several novel contributions.

1.2. Novelty, importance, and significance of this study

The following concerns have been addressed in this study:

- Within ESCM several studies are considered with either comparative studies with data or with deterministic models. There were very few models which considered the random demand of energy. Still, those models with random demand of energy did not consider the energy demand of a smart city. This research considered the random demand of a smart city, and the demand can be partially fulfilled through renewable energy generated from biodegradable waste. This does not exist in the literature.
- Each of the industries nowadays tries to reach the sustainable development goal by maintaining any of the seventeen goals, decided by the United Nations. This study also considers the sustainable development goal through the reduction of the CE. CE happens due to transportation during the biodegradable waste transport. An investment has been utilized for the reduction of CE. This CE reduction is not considered yet for ESCM from biodegradable waste with random demand of energy.
- To tackle the random demand, it is important to consider the flexibility of the production rate. Flexible production is considered for energy conversion from biodegradable wastes. Due to flexible production, the excess production of energy or shortages of energy can be controlled easily.

These are the novelty, importance, and significance of this study.

1.3. Layout of this study

This study is arranged as follows: A literature review is explained in Section 2 and assumptions of this study, notation, and the problem are described in Section 3. Section 4 illustrates the model mathematically and the solution methodology used in this study. Numerical examples, parameter sensitivity and some insights are explained in Section 5, while Section 6 explains the conclusions from this study.

2. Literature review

Sustainable supply chain management (SCM) has become an urgent necessity in recent years. To support long-lasting businesses, future development should concentrate on making basic production processes more sustainable; the three pillars of such processes are economic, environmental, and social sustainability. Despite all three pillars being equally important, most industries focus solely on economic sustainability since this is more profitable than the other two pillars. The basic business strategy of any industry is to minimize cost and/or maximize profit; this concept can be adapted to meet sustainability goals by reusing biodegradable waste materials. There is tremendous scope for further research into reusing biodegradable waste to fulfill the energy demand. Since most energy is currently produced from non-renewable sources, using renewable energy can be beneficial to both society and industry. First-generation biofuel, second-generation biofuel, and fuel from animal fats can be utilized as alternative sources of renewable energy. The main benefit of converting waste to energy is that the purchasing cost of the raw material, i.e., waste, is almost negligible. Despite this advantage, very few studies tackle the management of renewable energy supply chains using green technologies.

2.1. Renewable energy supply chain management

A supply chain (SC) is a network where several players are interlinked, each performing different processes that keep the business running. Effective management of renewable ESCM is essential to meet the energy demands of any population. Wee et al. [7] proposed a renewable ESCM as an alternative to traditional energy production, focusing on sources of renewable energy, the performance of the ESCM, and obstacles to the development of ESCM. Another sustainable SCM was proposed by Wangsa et al. [8], who emphasized power propagation, transmission, and distribution system for electrical supply chains; they showed that customers' electricity demands are selling-price dependent.

Fernando et al. [9] proved the necessity of energy management in renewable ESCM, showing that knowledge and awareness about energy should be the basic criteria for any manufacturing system. Another approach for cross-docking was proposed by Mukherjee et al. [10]; they considered a logistic strategy to estimate the maximum flow of products. Sarkar et al. [11] developed a layout for warehouses for inventories to improve the manufacturing system. Their model considered several technological costs but did not address energy supply chain management.

Recently, Hoang et al. [12] made a review on how to handle with municipal solid waste (MSW) during converting to energy. Howerer, they did not think about the energy supply chain management. A system dynamic approach was used by Alam et al. [13] for generating energy along with revenue from MSW. The waste was minimized by them. Varjani et al. [14] described the technologies to convert energy from MSW. They proved that bio-electrochemical is the best technology for the conversion of energy. However, those studies did not consider any supply chain for energy from MSW.

2.2. Waste to energy: flexible conversion plant

With a fast-growing human population, municipal waste generation has increased exponentially all around the world. Effective disposal of this waste is a serious challenge, one that could be solved by reusing municipal waste. Simultaneously, more energy is needed to maintain modern technology-based lifestyles; if waste can be converted to energy, both these problems can be solved. Due to the limitations of using fossil fuels as non-renewable energy sources, Habib et al. [15] suggested producing biodiesel from animal fats while taking into account the uncertainty around the animal fat collection. The researchers collected animal fat from different slaughterhouses and sent it to collection centers for storage, from where it was prepossessed and transported to biorefineries. The biodiesel produced at these biorefineries (with a constant rate of production) was later sent to markets. While Habib et al. [15] considered the cost of CE, they did not suggest any policy changes to reduce CE.

Malav et al. [16] studied the conversion of municipal solid waste (MSW) to energy, showing the increasing amount of MSW in urban areas. This study aimed to convert biodegradable waste to electricity, heat, and fuel. Mohammadi and Harjunkoski [17] proposed a supply chain model to produce fuel and energy from waste feedstock by optimizing technology selection for the conversion of waste and maximizing the energy and fuel generation from the waste feedstock. Zhao et al. [18]

aimed to make Beijing (nearly) a zero-waste city. They proposed to decrease the amount of waste for landfilling and improve the rate of energy conversion waste. They also gave importance to the separation policy of waste and greenhouse gas emissions. The above studies did not explain the production system.

Sarkar and Chung [19] and Kugele et al. [20] sought to improve the production system. They considered the flexible production rate of manufacturing systems. By applying flexibility in production systems, they were able to reduce defective production and machine breakdown. Sarkar et al. [21] utilized an autonomation policy in the manufacturing system. With the help of an autonomation policy, the efforts, time and labor costs of the manufacturer can be reduced. Mondal et al. [22] developed a closed-loop supply chain model with a license-sharing strategy. They gave importance to remanufacturing of products. Hota et al. [23] used smart technologies in supply chain management, but they did not think about the waste to the energy supply chain. A flexible and automated production system is more useful to run any production system smoothly ([24]). Another important thing that should be considered for any business is CE. The government of each country has put forth regulations on CO_2 emission for each industry, known as the carbon-cap policy. Any industry can earn more through cap-and-trade policy if they adhere to the reduction policy of CO_2 emissions.

2.3. *Carbon emission reduction*

Čuček et al. [25] introduced a biomass-to-energy (BE) conversion SC. They divided their model into four layers: biomass from agricultural regions, preprocessed biomass, processing layer, and distribution layer. The profits of this supply chain were maximized, and the industry's carbon footprint was minimized. CE reduction for a coordinated supply chain was explored by Bai et al. [26]. They developed a vendor-managed inventory model for deteriorated waste products. The authors invested in green technologies to reduce CE from the system but did not address energy formation from waste. Yang et al. [27] discussed a closed-loop supply chain model (CLSCM) where heat is recovered from mobile waste; they concentrated on the greening level with the profit measure of the whole supply chain.

Yadav et al. [2] explained a sustainable SCM that reduced waste from the system. CE was considered while setting up the production system, during transportation, and while conducting inventories for both manufacturers and retailers. The researchers showed that using preservation technology reduced the rate of waste deterioration. Kumar et al. [28] established a reverse logistics model for two retailer warehouses; their model considered CE but did not address energy consumption or sources of energy. Controllable CE was considered by Sarkar et al. [29] for substitutable products, while Oryani et al. [30] developed a resource optimization model through ARDL approach; CE reduction along with the reduction of waste was considered in their model. Sarkar et al. [31] considered a three-echelon supply chain model for biodegradable products and reduction of CE to save the environment.

3. Problem description, notation, and assumptions

3.1. Problem description

A sustainable renewable ESCM is essential for the conversion of biodegradable waste to renewable energy (see Figure 1) in a smart city. The collection center gathers waste from different parts of the city, which is then transported and unloaded at the collection center. The collection center uses an autonomation policy to inspect all collected wastes. After inspection, hazardous wastes are discarded, and the remaining wastes are sent to a renewable energy conversion plant (ECP). During the transportation and storage of wastes at the collection center, unavoidable CE occurs. This CE is reduced using green technology (GT) to make this waste-to-energy system sustainable. ECP uses a combustion process for converting biodegradable wastes to renewable energy. The renewable energy thus produced is stored and supplied to the city on demand. Significant CEs occur during the process of converting biodegradable waste to renewable energy, as well as during the storage of energy. Investment into reducing CE is applied to the renewable ECP. A part of the ECP's revenue is shared with the WCC to balance the profit of both players and to make the supply chain more sustainable. This study aims to maintain a sustainable energy supply chain with reduced waste. The mathematical model is formed to show the maximum profit of sustainable energy supply chain management with the reduction of CE and waste.

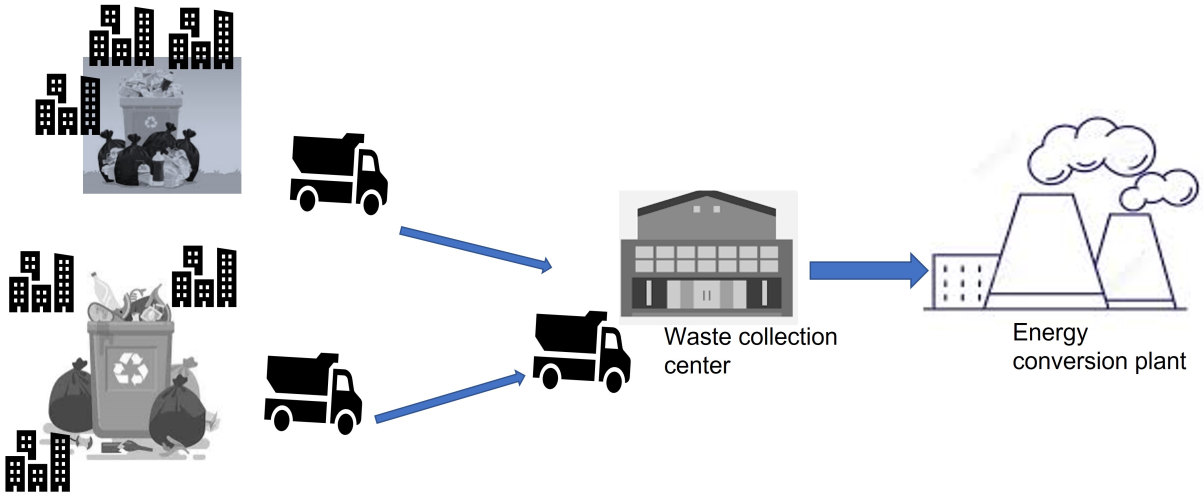


Figure 1. Waste to renewable energy conversion chain.

3.2. Notation

The following notation has been used in this model.

Decision	**variables**
Q	energy delivered from conversion plant (MWh)
P	energy conversion rate from waste (MWh)
ξ	investment into GT for collection center (\$)
$\xi^{'}$	investment into GT for energy conversion center (\$)
p_e	energy selling price (\$/MWh)
Parameters	
ω_c	collected wastes (ton)
p_c	price of waste (\$/ton)
C_c	collection cost of WCC (\$)
U_c	unloading cost of waste at collection center (\$/ton)
S_c	storage cost of waste at collection center (\$/ton)
ϵ_1	CE for storage at collection center without GT
ν_1	reduced CE in storing wastes at the collection center
ϵ_2	CE for transportation at collection center without GT
ν_2	reduced CE in transporting at the collection center
ϵ_3	CE during production at the ECP without GT
ν_3	reduced CE in producing energy at ECP
ϵ_4	CE for storing energy at the ECP without GT
ν_4	reduced CE in storing energy at energy conversion center
τ_c	shortage cost for collection center (\$/ton)
I_c	inspection cost of waste per unit quantity at collection center (\$/ton)
W_c	waste transfer cost from collection center to conversion plant (\$/ton)
γ_c	disposal cost per unit quantity for collection center (\$/ton)
R_C	regional setup cost from the place of waste collection (\$/setup)
T_{r_c}	transportation cost for collection center (\$/ton)
C_{AR1}	CE cost for collection center (\$/ton)
C_{S_E}	setup cost at ECP (\$/setup)
C_{D_E}	development cost of ECP (\$)
C_{T_E}	tool/die cost of ECP (\$/MWh)
η	scalling parameter for GT of collection center
δ	percentage of scrap waste of collection center
α	conversion rate to energy from waste (MWh/ton)
β	scalling parameter for GT of ECP
θ_1	revenue sharing of the energy plant to the WCC during holding of products (%)
θ_2	revenue sharing of the energy plant to the WCC during shortages of products (%)
R_{S_E}	per unit storage cost of energy (\$/MWh)
C_{H_E}	per unit shortage cost of energy (\$/MWh)
C_{AR2}	CE cost of ECP (\$/ton)

Random	**variables**
d	demand of wastes for energy conversion (ton)
D	energy demand of the city (MWh)
Expressions	**waste collection center**
CLC_c	total waste collection cost
TRP_c	total transportation cost
ULD_c	total waste unloading cost
STR_c	total waste storage and shortage cost
INP_c	total inspection cost
DIS_c	total disposal cost
BUF_c	total waste transfer cost
STC_c	total regional setup cost
AMU_c	total carbon emitted
CAB_c	total carbon emissions cost
INV_c	total investment for GT
REV_c	revenue of WCC
$E[\Gamma_c]$	expected profit of WCC
Expressions	**energy conversion plant**
STP_E	initial setup cost
COV_E	energy conversion cost
STR_E	total energy storage and shortage cost
AMU_E	total carbon emitted at ECP
CAB_E	total carbon emissions cost for ECP
INV_E	total investment for GT of ECP
REV_E	revenue of ECP
$E[\Gamma_E]$	expected profit of ECP
$E[\Gamma]$	expected profit of ESCM

3.3. Assumptions

The assumptions for this model are explained as follows:

1) Energy supply chain management (ESCM) consists of a WCC and a renewable ECP. Demands for both the renewable ECP and the WCC are random; a stochastic model is used to model ESCM.

2) An autonomation inspection policy is used to remove hazardous wastes from the collection center. The collection center disposes of $\delta\%$ of the collected waste from the systems and the other $(1-\delta)\%$ is sent to the renewable energy conversion center.

3) Due to the random energy demand (RED), the energy conversion rate is not fixed, and the conversion plant uses a flexible production rate. The production cost is found in the form of $(\frac{C_{D_E}}{P} + C_{T_E}P)$ (see, for reference, Sarkar and Chung [19]).

4) The energy demands of a smart city are denoted by D, which remains random. The random waste demand for energy conversion is d, and the relation between two types of demand is $d = \zeta D$, $\zeta > 1$. Using this relation, demand d is transferred to ζD. The RED has no specific distribution, and a distribution-free approach (DFA) is used to meet the expected energy demand. (μ),

the mean, and variance (σ) are found based on pre-existing data.

5) CE reduction for both the WCC and the ECP is achieved using GT, which requires investments from both industries.

6) To make the energy supply chain sustainable and long-lasting, the aforementioned "players" make a contract to share revenue. The conversion plant shares $\theta_1\%$ of total revenue when there are no shortages of energy and shares $\theta_2\%$ of revenue during shortages.

4. Mathematical model

A mathematical model of a renewable ESCM is established with detailed explanations of each cost parameter for both players. The expected total profit is calculated, and optimum decision variables are obtained using the classical optimization method. The total distributed renewable energy in the city is Q. At the collection center, ω_c units of waste are received from different regions of a city. This waste is then inspected by automated inspection policies, and $\delta\%$ of waste is received from different parts of the city. This waste is then inspected by automated inspection policies, and $(1-\delta)\omega_c$ units of waste are sent to the conversion plant for energy conversion. The rate of energy conversion from wastes of the flexible production system is α. Thus, the renewable energy produced from the plant is $(1-\delta)\alpha\omega_c$ units. This converted renewable energy is supplied to the city to fulfil the energy demand. The total supplied energy of the city is $(1-\delta)\alpha\omega_c = Q$. The demand of energy is a random variable, whose distribution is not known, but the mean and standard deviation are known from the longtime data.

4.1. Costs of collection center

From different parts of a city, waste is collected for sending to the ECP. To collect, hold and supply those wastes, the collection center incurs different types of expenditures which are explained below.

4.1.1. Collection cost (CLC_c)

The WCC is responsible for collecting all waste from different parts of the city. To do so, it incurs some expenditure to gather the waste from different parts of the city properly. The collection cost (CLC_C) of the center can be expressed as

$$CLC_c = C_c. \tag{4.1}$$

4.1.2. Transportation cost (TRP_c)

Waste is transported from different regions of the city by trucks, suitable for carrying trash. The cost per unit for transporting waste is T_{r_c}. ω_c is the total waste, and then, the transportation cost (TRP_c) can be written as

$$TRP_c = \omega_c T_{r_c}. \tag{4.2}$$

4.1.3. Unloading cost (ULD_c)

After reaching the truck with wastes at the collection center, it needs to unload there. The unloading cost per unit of waste is U_c. Thus, the cost for unloading (ULD_c) of wastes is

$$ULD_c = U_c\omega_c. \tag{4.3}$$

4.1.4. Storage and shortage cost (STR_c)

The storage or shortage of waste depends on the demand for waste from the conversion center. This demand is considered random. If $d \leq \omega_c$, then the collection center must store wastes. The per unit storage cost is S_c, and the per unit shortage cost is τ_c which happens if $\omega_c < d$. Thus, the storage and shortage cost of the collection center can be written as

$$STR_c = \begin{cases} S_c(\omega_c - d)^+; \ d \leq \omega_c, \\ \tau_c(d - \omega_c)^+; \ \omega_c < d. \end{cases} \tag{4.4}$$

4.1.5. Waste inspection cost (INP_c)

The collected wastes at the collection center should be inspected before sending to the energy conversion center. An automated inspection policy is used to inspect those wastes. The inspection cost per unit of waste is I_c. The inspection cost can be expressed as

$$INP_c = I_c\omega_c. \tag{4.5}$$

4.1.6. Disposal cost (DIS_c)

After inspecting the waste, $\delta\%$ of waste cannot be used to produce renewable energy. Hence, $\delta\omega_c$ units of waste are disposed of after inspection. The remaining $(1 - \delta)\omega_c$ units of waste are sent to the conversion plant for renewable energy production. Per unit amount of waste disposal cost is C_1, and the total disposal cost is

$$DIS_c = \gamma_c\delta\omega_c. \tag{4.6}$$

4.1.7. Waste transfer cost (BUF_c)

The ECP is situated near the WCC because both are required to be built far from populated areas to avoid additional pollution. Thus, a buffer is used to transport biodegradable waste from the WCC to the conversion plant. From collection center, $(1 - \delta)\omega_c$ of the biodegradable waste is transported from the collection center, and the buffer cost per unit is W_c. Therefore, the total buffer cost necessary to transfer the usable biodegradable waste to the conversion plant is

$$BUF_c = W_c(1 - \delta)\omega_c. \tag{4.7}$$

4.1.8. Setup cost (STC_c)

To arrange all the processes needed to collect waste and send it to the conversion center, a setup cost is needed. A fixed setup cost is considered as

$$STC_c = R_c. \tag{4.8}$$

4.1.9. Carbon emissions cost (CAB_c)

The most important thing to save the environment is the reduction of CE from any industry. For the WCC, CEs happen mainly during transportation and holding wastes. GT is being used to reduce those

CEs (see Bai et al. [26]). The emitted carbons before using GT due to holding and transportation of waste are ϵ_1 and ϵ_2. After using GT, the total amount of carbon emitted due to holding and transporting biodegradable waste becomes

$$AMU_c = (\epsilon_1 - \nu_1\xi)(\omega_c - d)^+ + (\epsilon_2 - \nu_2\xi)\omega_c, \tag{4.9}$$

where ξ is the level of GT used for the collection center, and ν_1 and ν_2 are the GT parameters for reducing CEs during holding and transportation. The CE cost per unit emission is C_{AR1} for the collection center. Thus, the carbon emissions cost after using GT of the collection center is

$$CAB_c = C_{AR1}\left[(\epsilon_1 - \nu_1\xi)(\omega_c - d)^+ + (\epsilon_2 - \nu_2\xi)\omega_c\right]. \tag{4.10}$$

4.1.10. Investment into green technology (INV_c)

To apply GT the collection center needs to invest in reducing CEs. Based on the literature, it is found that there are two types of investments: discrete (Sarkar and Chung [19]) and continuous (Bachar et al. [32]). For energy conversion, the investment must be continuous. Recently, Sarkar and Bhuniya [24] used green investment cost for environmental protection. Therefore, the continuous investment for GT can be written as

$$INV_c = \frac{\eta\xi^2}{2}. \tag{4.11}$$

4.1.11. Revenue for the WCC (REV_c)

The selling price of the collection center is p_c. The collection center and energy conversion center have an agreement that the conversion center shares θ_1 percentage of revenue when $d \le \omega_c$ and θ_2 percentage of revenue when $\omega_c < d$. Thus, the total revenue for the WCC is

$$REV_c = \begin{cases} (p_c d + \theta_1 p_e D);\ d \le \omega_c \\ (p_c + \theta_2 p_e(1-\delta)\alpha)\omega_c;\ \omega_c < d \end{cases} \tag{4.12}$$

4.1.12. Profit of the WCC

The expected profit of the WCC is

$$\begin{aligned} E[\Gamma_c] = {} & E[min\{(p_c\zeta + \theta_1 p_e)D, (p_c + \theta_2 p_e(1-\delta)\alpha)\omega_c\}] - C_c - U_c\omega_c \\ & -S_c E[\omega_c - \zeta D]^+ - I_c\omega_c - W_c(1-\delta)\omega_c - \gamma_c\delta\omega_c - R_c \\ & -T_{r_c}\omega_c - C_{AR1}\Big[(\epsilon_1 - \nu_1\xi)E[\omega_c - \zeta D]^+ + (\epsilon_2 - \nu_2\xi)T_{r_c}\omega_c\Big] \\ & -\frac{\eta\xi^2}{2} - \tau_c E[\zeta D - \omega_c]^+. \end{aligned} \tag{4.13}$$

4.2. ECP's model

The ECP receives $(1-\delta)\omega_c$ of the biodegradable waste for converting to energy at a rate of α. The converted renewable energy is supplied to the city for use.

4.2.1. Setup cost (STP_E)

Every manufacturing system needs an initial setting, and to do so a fixed cost is needed. The setup cost of the conversion center can be written as

$$STP_E = C_{S_E}. \tag{4.14}$$

4.2.2. Energy conversion cost (COV_E)

The rate of renewable energy production is considered flexible for the conversion plant and is denoted as P. The system development cost is considered as C_{D_E} and the tool/die cost as C_{T_E} for the renewable ECP. The energy conversion cost is expressed as

$$COV_E = \left(\frac{C_{D_E}}{P} + C_{T_E}P\right)(1-\delta)\alpha\omega_c. \tag{4.15}$$

4.2.3. Energy storage and shortage cost (STR_E)

After converting the biodegradable waste to energy, it needs to be stored for supply. The per unit storage cost is denoted as R_{S_E}. The storage happens when $D \leq (1-\delta)\alpha s$. However, if $(1-\delta)\alpha s < D$, then the shortage happens for the conversion plant, and per unit shortage cost is C_{H_E}. Thus, the storage and shortage cost of energy can be denoted by

$$STR_E = \begin{cases} R_{S_E}((1-\delta)\alpha\omega_c - D)^+; \; D \leq (1-\delta)\alpha\omega_c, \\ C_{H_E}(D-(1-\delta)\alpha\omega_c)^+; \; (1-\delta)\alpha\omega_c < D. \end{cases} \tag{4.16}$$

4.2.4. Carbon emission cost (CAB_E)

Most carbon emissions happen from different industries. During energy conversion and storage of energy, carbon is also emitted which should be reduced to decrease global warming. By using green technologies, CE can be reduced. ϵ_3 is the CE during production, and ϵ_4 is the carbon emission during energy storage. ν_3 and ν_4 are GT parameters for energy production and supply. The carbon emission cost for the conversion plant is C_{AR2}. Thus, the carbon emission cost after using GT is

$$\begin{aligned} CAB_E &= C_{AR2}AMU_E \\ &= C_{AR2}\Big[(\epsilon_3 - \nu_3\xi^{'})(1-\delta)\alpha\omega_c + (\epsilon_4 - \nu_4\xi^{'})((1-\delta)\alpha\omega_c - D)^+\Big]. \end{aligned} \tag{4.17}$$

4.2.5. Investment for green technology (INV_E)

To implement GT the ECP needs to invest, which can be expressed in the form of

$$INV_E = \frac{\beta\xi^{'2}}{2}. \tag{4.18}$$

4.2.6. Energy supply cost (EPC_E)

$(1-\delta)\alpha\omega_c = Q$ of energy is produced from an amount ω_c of waste. This waste is supplied to the city to fulfil its energy demands. The supply cost per unit of energy is S_E. Hence, the energy supply cost can be written as

$$EPC_E = S_E(1-\delta)\alpha\omega_c. \tag{4.19}$$

4.2.7. Revenue for the ECP (REV_E)

Two players are in a mutual contract of revenue sharing. The conversion plant shares its revenue with the collection center. The selling price per unit of energy is p_e. Thus, the revenue of the ECP can be written as

$$REV_E = \begin{cases} (1-\theta_1)p_e D; D \leq (1-\delta)\alpha\omega_c \\ (1-\theta_2)p_e(1-\delta)\alpha\omega_c; (1-\delta)\alpha\omega_c < D. \end{cases} \tag{4.20}$$

4.2.8. Expected profit of ECPs

The expected profit of the ECP is

$$\begin{aligned} &E[\Gamma_E] = (1-\theta_1)p_e E[D] - C_{S_E} - \left(\frac{C_{D_E}}{P} + C_{T_E}P\right)(1-\delta)\alpha\omega_c - R_{S_E}E[(1-\delta)\alpha\omega_c - D]^+ \\ &-C_{AR2}\Big[(\epsilon_3 - \nu_3\xi^{'})(1-\delta)\alpha\omega_c + (\epsilon_4 - \nu_4\xi^{'})E[(1-\delta)\alpha\omega_c - D]^+\Big] - \frac{\beta\xi^{'2}}{2} - S_E(1-\delta)\alpha\omega_c \\ &+(1-\theta_2)p_e(1-\delta)\alpha\omega_c - C_{H_E}E[D-(1-\delta)\alpha\omega_c]^+. \end{aligned} \tag{4.21}$$

Therefore, the expected total profit of the ESCM is

$$\begin{aligned} E[\Gamma] \;=\; & (p_c E[\zeta D] + \theta_1 p_e E[D]) - C_c - U_c\omega_c - S_c E[\omega_c - \zeta D]^+ - I_c\omega_c \\ & -W_c(1-\delta)\omega_c - \gamma_c\delta\omega_c - R_c - T_{r_c}\omega_c \\ & -C_{AR1}\Big[(\epsilon_1 - \nu_1\xi)E[\omega_c - \zeta D]^+ + (\epsilon_2 - \nu_2\xi)T_{r_c}\omega_c\Big] \\ & -\frac{\eta\xi^2}{2} + (p_c + \theta_2 p_e(1-\delta)\alpha)\omega_c - \tau_c E[\zeta D - \omega_c]^+ \\ & +(1-\theta_1)p_e E[D] - C_{S_E} - \left(\frac{C_{D_E}}{P} + C_{T_E}P\right)(1-\delta)\alpha\omega_c \\ & -R_{S_E}E[(1-\delta)\alpha\omega_c - D]^+ - C_{AR2}\Big[(\epsilon_3 - \nu_3\xi^{'})(1-\delta)\alpha\omega_c \\ & +(\epsilon_4 - \nu_4\xi^{'})E[(1-\delta)\alpha\omega_c - D]^+\Big] - \frac{\beta\xi^{'2}}{2} - S_E(1-\delta)\alpha\omega_c \\ & +(1-\theta_2)p_e(1-\delta)\alpha\omega_c - C_{H_E}E[D-(1-\delta)\alpha\omega_c]^+. \end{aligned} \tag{4.22}$$

4.3. Solution methodology

The demand for waste and energy is considered random. Both demands have no specific distribution. A distribution-free approach (Gallego and Moon [33]) is used for these random demands of the model.

$$E[\Gamma(\xi, \xi^{'}, P, p_e, \omega_c)]$$

$$
\begin{aligned}
&= (p_c\zeta + p_e\theta_1)\,\mu - \frac{1}{2}\Big(\sqrt{(p_c\zeta + p_e\theta_1)^{\,2}\sigma^2 - (\omega_c\,(p_c + p_e\alpha\,(1-\delta)\,\theta_2) - (p_c\zeta + p_e\theta_1)\,\mu)^{\,2}} \\
&-(\omega_c\,(p_c + p_e\alpha\,(1-\delta)\,\theta_2) - (p_c\zeta + p_e\theta_1)\,\mu)\Big) - S_{TC} - U_c\omega_c - R_c - I_c\omega_c \\
&-\frac{1}{2}R_{TC}\Big(\sqrt{\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^{\,2}} - (\zeta\mu - \omega_c)\Big) - \frac{T_{r_c}\omega_c}{C_T} - S_{TE} - \frac{\beta\xi^{'2}}{2} - \frac{\eta\xi^2}{2} \\
&-W_c\omega_c\,(1-\delta) - \left(\frac{C_{D_E}}{P} + C_{T_E}P\right)\omega_c\alpha\,(1-\delta) - \sum_{i=1}^{n}\omega_c S_E\alpha\,(1-\delta) - \gamma_c\omega_c\delta \\
&-\frac{1}{2}\tau_c\Big(\sqrt{\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^{\,2}} - (\omega_c - \zeta\mu)\Big) \\
&-C_{AR_1}\left((\epsilon_1 - \xi\nu_1)\,\frac{1}{2}\Big(\sqrt{\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^{\,2}} - (\zeta\mu - \omega_c)\Big) + (\epsilon_2 - \xi\nu_2)\,\frac{T_{r_c}\omega_c}{C_T}\right) \\
&+p_e\,(1-\theta_1)\,\mu - \frac{1}{2}R_{S_E}\Big(\sqrt{\sigma^2 + (\omega_c\alpha\,(1-\delta) - \mu)^{\,2}} - (\mu - \omega_c\alpha\,(1-\delta))\Big) \\
&-\frac{1}{2}\Big(\sqrt{p_e^2\,(1-\theta_1)^{\,2}\sigma^2 + (p_e\omega_c\alpha\,(1-\delta)\,(1-\theta_2) - p_e\,(1-\theta_1)\,\mu)^{\,2}} \\
&-(p_e\omega_c\alpha\,(1-\delta)\,(1-\theta_2) - p_e\,(1-\theta_1)\,\mu)\Big) \\
&-\frac{1}{2}C_{H_E}\Big(\sqrt{\sigma^2 + (\omega_c\alpha\,(1-\delta) - \mu)^{\,2}} - (\omega_c\alpha\,(1-\delta) - \mu)\Big) - C_{AR2}\Big(\Big(\epsilon_3 - \xi^{'}\nu_3\Big)\,\omega_c\alpha\,(1-\delta) \\
&+\Big(\epsilon_4 - \xi^{'}\nu_4\Big)\,\frac{1}{2}\Big(\sqrt{\sigma^2 + (\omega_c\alpha\,(1-\delta) - \mu)^{\,2}} - (\omega_c\alpha\,(1-\delta) - \mu)\Big)\Big)
\end{aligned}
\tag{4.23}
$$

The model is solved using the classical optimization method.

Theorem $E(\Gamma)$ can reach the global optimum profit when the GT investment for WCC is $\xi = \frac{C_{AR1}}{\eta}\left(\nu_2\frac{\omega_c T_{r_c}}{C_T} + \nu_1\frac{1}{2}\left(-\zeta\mu + \sqrt{\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^{\,2}} + \omega_c\right)\right)$, the GT investment for ECP is $\xi^{'} = \frac{C_{AR2}}{\beta}\left[\frac{1}{2}\nu_4\left(\mu + \sqrt{(\mu + \alpha\,(\delta-1)\,\omega_c)^{\,2} + \sigma^2} + \alpha\omega_c\,(\delta-1)\right) - \nu_3\alpha\,(\delta-1)\,\omega_c\right]$, the production rate is $P = \sqrt{\frac{C_{D_E}}{B}}$, the energy selling price is $p_e = \frac{\Delta_2\sqrt{\sigma^2(p_c\zeta+\theta_1 p_e)^2-(p_c(\omega_c-\zeta\mu)-p_e(\theta_1\mu+\theta_2\alpha(\delta-1)\omega_c))^2}}{\theta_1^2\sigma^2} - \frac{p_c\zeta}{\theta_1}$, and the amount of waste is $\omega_c = (\zeta\mu + \sqrt{\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^{\,2}}) + \frac{\Delta_1\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2}}{\tau_c}$ (see Appendix A for the values of Δ_1 and Δ_2).

Proof The optimum values of decision variables can be obtained by setting the first order derivatives of Eq (4.23) corresponding to $\xi, \xi^{'}, P, p_e$ and ω_c equal to zero.
The derivatives can be written as

$$
\frac{\partial\Gamma}{\partial\xi} = C_{AR1}\left(\nu_2\frac{\omega_c T_{r_c}}{C_T} + \nu_1\frac{1}{2}\left(-\zeta\mu + \sqrt{\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^{\,2}} + \omega_c\right)\right) - \eta\xi \tag{4.24}
$$

$$
\frac{\partial\Gamma}{\partial\xi^{'}} = \frac{1}{2}\nu_4 C_{AR2}\left(\mu + \sqrt{(\mu + \alpha\,(\delta-1)\,\omega_c)^{\,2} + \sigma^2} + \alpha\omega_c\,(\delta-1)\right) - \nu_3 C_{AR2}\alpha\,(\delta-1)\,\omega_c - \beta\xi^{'} \tag{4.25}
$$

$$
\frac{\partial\Gamma}{\partial P} = \alpha\,(1-\delta)\,\omega_c\left(\frac{C_{D_E}}{P^2} - C_{T_E}\right) \tag{4.26}
$$

$$
\begin{aligned}
\frac{\partial\Gamma}{\partial p_e} &= \frac{1}{2}\Big(\theta_1\mu + (\theta_2-1)\,\alpha\,(\delta-1)\,\omega_c - \theta_2\alpha\,(\delta-1)\,\omega_c \\
&+\frac{(\theta_2\alpha\,(1-\delta)\,\omega_c-\theta_1\mu)\,(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\alpha\,(\delta-1)\,\omega_c))-\theta_1\sigma^2\,(p_c\zeta+\theta_1 p_e)}{\sqrt{\sigma^2\,(p_c\zeta+\theta_1 p_e)^2-(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\alpha\,(\delta-1)\,\omega_c))^2}} \\
&-\sqrt{((\theta_1-1)^2\sigma^2+((\theta_1-1)\,\mu+(\theta_2-1)\,\alpha\,(\delta-1)\,\omega_c)^2)}-(\theta_1-1)\,\mu\Big)
\end{aligned}
\tag{4.27}
$$

$$
\begin{aligned}
\frac{\partial\Gamma}{\partial\omega_c} &= \frac{1}{2}\Bigg(\frac{2\alpha\,(\delta-1)\left(C_{D_E}+C_{T_E}P^2\right)}{P}-C_{AR2}\alpha\Bigg((1-\delta)\left(\epsilon_4-\xi^{'}\nu_4\right)\Bigg(\frac{\alpha\,(\omega_c-\omega_c\delta)-\mu}{\sqrt{(\mu+\omega_c\alpha\,(\delta-1))^2+\sigma^2}}-1\Bigg) \\
&-2\,(\delta-1)\left(\epsilon_3-\xi^{'}\nu_3\right)\Bigg)-2C_{AR1}\Bigg(\frac{T_{r_c}\,(\epsilon_2-\nu_2\xi)}{C_T}+\frac{1}{2}\,(\epsilon_1-\nu_1\xi)\Bigg(\frac{\omega_c-\zeta\mu}{\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2}}+1\Bigg)\Bigg) \\
&+2\,(\delta-1)\,W_c+\frac{(p_c-\theta_2 p_e\alpha\,(\delta-1))\,(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\omega_c\alpha\,(\delta-1)))}{\sqrt{\sigma^2\,(p_c\zeta+\theta_1 p_e)^2-(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\omega_c\alpha\,(\delta-1)))^2}} \\
&-\tau_c\Bigg(\frac{\omega_c-\zeta\mu}{\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2}}-1\Bigg)-2\delta\gamma_c-\frac{2T_{r_c}}{C_T}-2c+p_c+(\theta_2-1)\,p_e\alpha\,(\delta-1) \\
&-\theta_2 p_e\alpha\,(\delta-1)-\frac{(\theta_2-1)\,p_e^2\alpha\,(\delta-1)\,((\theta_1-1)\,\mu+(\theta_2-1)\,\omega_c\alpha\,(\delta-1))}{\sqrt{p_e^2\,((\theta_1-1)^2\sigma^2+((\theta_1-1)\,\mu+(\theta_2-1)\,\omega_c\alpha\,(\delta-1))^2)}} \\
&-C_{H_E}\alpha\,(1-\delta)\Bigg(\frac{\alpha\,(\omega_c-\omega_c\delta)-\mu}{\sqrt{(\mu+\omega_c\alpha\,(\delta-1))^2+\sigma^2}}-1\Bigg)+2\alpha\,(\delta-1)\,S_E-2U_c \\
&-\alpha\,(1-\delta)\,R_{S_E}\Bigg(\frac{\alpha\,(\omega_c-\omega_c\delta)-\mu}{\sqrt{(\mu+\omega_c\alpha\,(\delta-1))^2+\sigma^2}}+1\Bigg)-R_{TC}\Bigg(\frac{\omega_c-\zeta\mu}{\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2}}+1\Bigg)\Bigg)
\end{aligned}
\tag{4.28}
$$

Setting Eqs (4.24)–(4.28) equal to zero, the optimum values of decision variables can be obtained as

$$
\xi^* = \frac{C_{AR1}}{\eta}\left(\nu_2\frac{\omega_c T_{r_c}}{C_T}+\nu_1\frac{1}{2}\left(-\zeta\mu+\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2}+\omega_c\right)\right) \tag{4.29}
$$

$$
\xi^{'*} = \frac{C_{AR2}}{\beta}\left[\frac{1}{2}\nu_4\left(\mu+\sqrt{(\mu+\alpha\,(\delta-1)\,\omega_c)^2+\sigma^2}+\alpha\omega_c\,(\delta-1)\right)-\nu_3\alpha\,(\delta-1)\,\omega_c\right] \tag{4.30}
$$

$$
P^* = \sqrt{\frac{C_{D_E}}{C_{T_E}}} \tag{4.31}
$$

$$
p_e^* = \frac{\Delta_2\sqrt{\sigma^2\,(p_c\zeta+\theta_1 p_e)^2-(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\alpha\,(\delta-1)\,\omega_c))^2}}{\theta_1^2\sigma^2}-\frac{p_c\zeta}{\theta_1} \tag{4.32}
$$

$$
\omega_c^* = (\zeta\mu+\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2})+\frac{\Delta_1\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2}}{\tau_c} \tag{4.33}
$$

(See Appendix A for the values of Δ_1 and Δ_2).

The optimum values of decision variables can be said to be the global optimum values if sufficient conditions of optimality can be satisfied.

The optimum values $\xi^*, {\xi'}^*, P^*, p_e^*$ and ω_c^* are said to be the global optimum if the principal minors $H_{11}, H_{22}, H_{33}, H_{44}$ and H_{55} are alternative in sign.

The principal minors can be found by the Hessian matrix, and the principal minors are as follows:

The first principal minor is $H_{11} = \frac{\partial^2 E[\Gamma]}{\partial \xi^2} = -\eta < 0$.

The second principal minor is $H_{22} = \frac{\partial^2 E[\Gamma]}{\partial \xi^2}\frac{\partial^2 E[\Gamma]}{\partial {\xi'}^2} - \left(\frac{\partial^2 E[\Gamma]}{\partial \xi \partial \xi'}\right)^2 = \eta\beta > 0$.

The third principal minor is

$$\begin{aligned} H_{33} &= \begin{bmatrix} \frac{\partial^2 E[\Gamma]}{\partial \xi^2} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial P} \\ \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial {\xi'}^2} & \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial P} \\ \frac{\partial^2 E[\Gamma]}{\partial P \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial P \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial P^2} \end{bmatrix} \\ &= -\frac{\alpha\beta\eta(1-\delta)\omega_c C_{D_E}}{P^3} < 0 \end{aligned}$$

The fourth principal minor is

$$\begin{aligned} H_{44} &= \begin{bmatrix} \frac{\partial^2 E[\Gamma]}{\partial \xi^2} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial P} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial p_e} \\ \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial {\xi'}^2} & \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial P} & \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial p_e} \\ \frac{\partial^2 E[\Gamma]}{\partial P \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial P \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial P^2} & \frac{\partial^2 E[\Gamma]}{\partial P \partial p_e} \\ \frac{\partial^2 E[\Gamma]}{\partial p_e \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial p_e \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial p_e \partial P} & \frac{\partial^2 E[\Gamma]}{\partial p_e^2} \end{bmatrix} \\ &= \frac{\alpha\beta\eta(1-\delta)\omega_c C_{D_E}[\Delta_3((\theta_2\alpha(1-\delta)\omega_c - \theta_1\mu)^2 - \theta_1^2\sigma^2) - \Delta_4\Delta_5]}{2P^3\Delta_3^{3/2}} > 0 \end{aligned}$$

The fifth principal minor is

$$\begin{aligned} H_{55} &= \begin{bmatrix} \frac{\partial^2 E[\Gamma]}{\partial \xi^2} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial P} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial p_e} & \frac{\partial^2 E[\Gamma]}{\partial \xi \partial \omega_c} \\ \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial {\xi'}^2} & \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial P} & \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial p_e} & \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial \omega_c} \\ \frac{\partial^2 E[\Gamma]}{\partial P \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial P \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial P^2} & \frac{\partial^2 E[\Gamma]}{\partial P \partial p_e} & \frac{\partial^2 E[\Gamma]}{\partial P \partial \omega_c} \\ \frac{\partial^2 E[\Gamma]}{\partial p_e \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial p_e \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial p_e \partial P} & \frac{\partial^2 E[\Gamma]}{\partial p_e^2} & \frac{\partial^2 E[\Gamma]}{\partial p_e \partial \omega_c} \\ \frac{\partial^2 E[\Gamma]}{\partial \omega_c \partial \xi} & \frac{\partial^2 E[\Gamma]}{\partial \omega_c \partial \xi'} & \frac{\partial^2 E[\Gamma]}{\partial \omega_c \partial P} & \frac{\partial^2 E[\Gamma]}{\partial \omega_c \partial p_e} & \frac{\partial^2 E[\Gamma]}{\partial \omega_c^2} \end{bmatrix} \\ &= -\frac{2\alpha\eta(1-\delta)\omega_c C_{D_E}}{P^3}\left[\beta\left(\{\Omega_1\Omega_2 - \Omega_3^2\} + \Omega_1\left(\alpha(1-\delta)\left(\frac{C_{D_E}}{P^2} - C_{T_E}\right)\right)^2 + \Omega_5^2\right) - \Omega_1\Omega_4^2\right] < 0 \end{aligned}$$

All second order derivatives and values of $\Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \Delta_{11}, \Delta_{12}$, and Δ_{13}; and $\Omega_1, \Omega_2, \Omega_3, \Omega_4$, and Ω_5 are given in Appendix B.

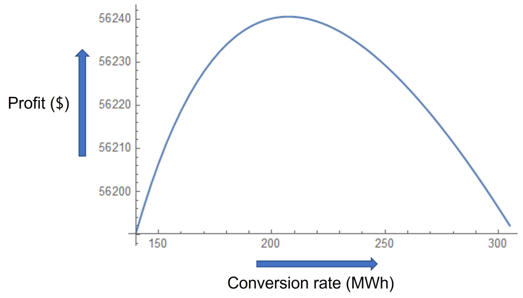
5. Numerical examples

This section illustrates the numerical experiments, results, and insights from the experiments. Numerical experiments were done using Mathematica 11.3. The data taken from Sarkar et al. [1]

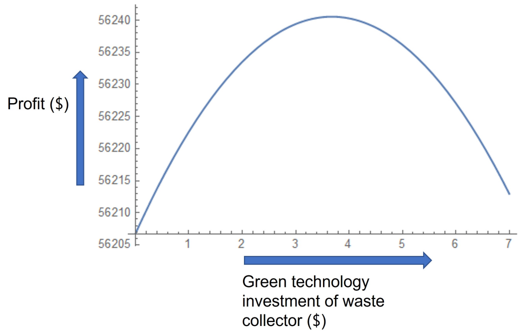
and [3] are given below.

Table 1. Input data used for numerical experiments.

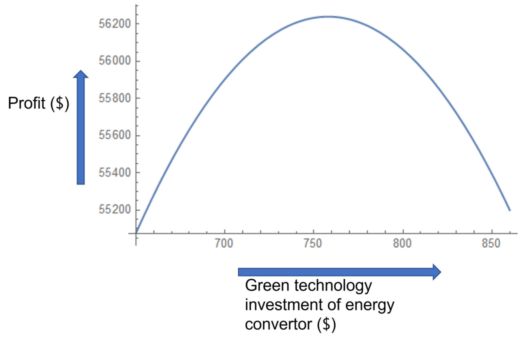
P_c = \$50/ton	$\theta_1 = 30\%$	$\zeta = 1.51$	μ = 7 ton
$\theta_2 = 16\%$	$\sigma = 11$	C_c = \$50/setup	U_c = \$0.20/ton
S_c = \$0.032/ton	I_c = \$3/ton	γ_c = \$0.01/ton	$\delta = 0.02$
R_c = \$0.15/setup	T_{r_c} = \$0.15/ton	C_{AR1} = \$6/ton	ϵ_1 = 3 ton
$\nu_1 = 0.4$	ϵ_2 = 4 ton	$\nu_2 = 0.3$	$\eta = 5$
τ_c = \$0.06/ton	C_{S_E} = \$100/setup	C_{D_E} = \$17200	C_{T_E} = \$0.4/MWh
α = 0.54 MWh/ton	R_{S_E} = \$0.3/MWh	C_{AR2} = \$5/ton	ϵ_3 = 7 ton
$\nu_3 = 2$	ϵ_4 = 4 ton	$\nu_4 = 3.1$	S_E = \$2.8/MWh
C_{H_E} = \$0.03/MWh	$\beta = 0.2$	W_c = \$3.4/ton	



(a) Profit versus production rate.



(b) Profit versus investment for green technology of waste collector.



(c) Profit versus investment for green technology of energy converter.

Figure 2. The graphical representations of optimum decision variables.

The optimum amount of waste is found as $\omega_c^* = 7.32$ ton. Thus, the optimum amount of energy can be obtained as $Q^* = (1-\delta)\alpha\omega_c^* = 3.87$ MWh. The optimum conversion rate is $P^* = 207.36$ MWh, the optimum investment for GT used in the collection center is ξ = \$3.68, and the optimum investment of GT for energy conversion center is ξ' = \$757.96. The optimum price of energy is obtained as p_e = \$144.56 MWh. The optimum profit for the ESCM is \$56240.60. The optimum graphical representations for production rate, GT investment for the waste collector and GT investment for energy converter are shown in Figure 2a–c.

5.1. Special cases

In this section, some special cases with their numerical results are explained.

5.1.1. Case I: Constant green technology investment for the collection center

The investment for GT is considered variable. If this investment is considered as constant, for different amounts of the investment, the expected profit of the ESCM is represented by Table 2. The profit is maximum at the optimum value of ξ = \$3.67 when it is considered variable. Other constant values of ξ give a lower profit than the profit when ξ is variable. There are also two limit points after which the solutions are infeasible. Thus, it can be said that the variable GT investment for WCC is more suitable than the constant.

Table 2. Constant investment for green technology of the collection center.

ξ(\$)	Profit of ESCM (\$)
2.9	infeasible
3	56239.4
3.5	56240.5
3.67	**56240.6**
3.8	56240.5
3.9	56240.4
4	infeasible

5.1.2. Case II: Constant green technology investment for the conversion plant

Profits of the ESCM are represented in Table 3 for different values of constant GT investment for the conversion plant. The maximum profit is found at ξ'^* = \$757.96. If the investment increases or decreases, the profit decreases. From the different experimental results, it is proved that the GT investment for the conversion plant should be variable.

5.1.3. Case III: No green technology investment for the collection center

The ESCM faces loss when there is no investment for GT for both the players. Investments in variable green technologies, maximize the profit of the ESCM. Thus, variable investments for GT are essential to gain more profit along with environmental issues.

5.1.4. Case IV: Constant production rate for the conversion plant

When the production rate is taken as a constant, the profits at different values of production rate are listed in Table 4. For variable production rate P = 207.36 MWh the maximum profit of the ESCM is \$56240.60. Variable production rate can reduce the excess holding or products in shortage.

Table 3. Constant investment for green technology of the conversion plant.

ξ' (\$)	Profit of ESCM (\$)
700	infeasible
730	56162.40
740	56208.30
757.96	**56240.60**
760	56240.10
780	56192
800	56063.80
850	55393.50
900	54223.20
1000	50382.70
1500	infeasible

Table 4. Constant production rate for the conversion plant.

P (MWh)	Profit of ESCM (\$)
150	infeasible
190	56238.10
200	56240.10
207.36	**56240.60**
230	56237.10
250	56229.30
300	infeasible

5.2. Sensitivity analysis

The changes in profit of the sustainable supply chain corresponding to key cost parameters in the range $[-50, +50]$ are discussed in this section with Table 5.

- The CE cost for the ECP is the most sensitive parameter among all cost parameters. CE has a huge effect on the profit of the sustainable ESCM. The profit reduces 76% with the reduction of 50% of the carbon emission cost. If the CE cost is reduced by 25%, the profit is reduced by almost 45%. Meanwhile, if the CE cost increases 50%, the profit increases 127%; and for 25% changes in a positive direction, the profit becomes about 58% higher. From this analysis, it can be said that the CE cost is very much important for the ECP.
- The next sensitive parameters are the development cost and tool/die cost of the ECP. It can be seen that the profit of the supply chain changes the same for both cost parameters. The profit increases with decreasing of both development cost and tool/die cost. The reverse case happens for increasing both costs.
- The profit of ECP changes in the opposite direction for setup cost changes within the range $[-50, +50]$. The profit fluctuates by almost 0.2 in both directions.

- Another pair of sensitive cost parameters is the setup cost of the collection center and the selling price of waste. The setup cost for the collection center is inversely proportional to the profit changes, but the waste selling price is directly proportional. Both changes are almost equivalent in both directions.
- The rest of the cost parameters are much less sensitive. Still, the changes in supply chain profit are equivalent in both directions for those cost parameters.

Table 5. Green firm's profit changes in response to changes in key parameters.

Parameters	Variations (%)	Variations of Π_S (%)	Parameters	Variations (%)	Variations of Π_S (%)
p_c	−50	−0.03	C_c	−50	+0.04
	−25	−0.02		−25	+0.02
	+25	+0.02		+25	−0.02
	+50	+0.04		+50	−0.04
U_c	−50	+0.001	R_c	−50	+0.0002
	−25	+0.0007		−25	+0.0001
	+25	−0.0007		+25	−0.0001
	+50	−0.001		+50	−0.0002
I_c	−50	+0.02	C_B	−50	+0.02
	−25	+0.01		−25	+0.01
	+25	−0.01		+25	−0.01
	+50	−0.02		+50	−0.02
T_{r_c}	−50	+0.02	C_{AR1}	−50	+0.09
	−25	+0.01		−25	+0.04
	+25	−0.01		+25	−0.03
	+50	−0.02		+50	−0.06
C_{S_E}	−50	+0.09	C_{D_E}	−50	+0.34
	−25	+0.04		−25	+0.15
	+25	−0.04		+25	−0.13
	+50	−0.09		+50	−0.26
C_{T_E}	−50	+0.36	C_{AR2}	−50	−76.36
	−25	+0.15		−25	−44.57
	+25	−0.13		+25	+57.34
	+50	−0.26		+50	+127.41
C_{S_E}	−50	+0.006	C_{H_E}	−50	+0.0002
	−25	+0.003		−25	+0.0001
	+25	−0.003		+25	−0.0001
	+50	−0.006		+50	−0.0002

5.3. Managerial insights

Based on the numerical results, some insights can be drawn from this study.

- The industry manager can decide the amount of carbon emissions because of the Government's regulations to control it. The findings indicate that it influences the profit of the whole ESCM. Therefore, both players have to be cautious about gaining the maximum profit. CE reduction has not yet been studied the way this study has. The findings of this study can allow the industry manager to decide properly.
- Investments for the reduction of CEs play an important role. It was found the investment needed is very small for the waste collector. Thus, if all transportation trucks are provided by the waste collector, then the industry can save a huge amount of investment without using any smart technology. Therefore, the industry managers can decide by themselves that the transportation sector's costs, and others should be taken care of by the waste converter.
- Because of the flexibility of the production system, it is found that the production amount of energy is rather high. It indicates that a huge part of the smart city is covered by renewable energy rather than traditional energy. However, because of the flexibility of energy production, the ESCM can obtain the maximum profit rather than any loss. The manager can continue the flexibility of the production system with the maximum profit.
- The random demand for renewable energy makes it a difficult decision to decide the amount of energy conversion along with the amount of energy storage. However, the industry players are well-behaved with each other, and the waste converter offers revenue sharing to the waste collector for the SDG and continues the business forever. This way, the industry manager can run the business for a long time by fulfilling the SDG.

6. Conclusions

This study was conducted for a sustainable development goal through waste to energy conversion. This study proved that by using a flexible production system, CEs can be reduced for ESCM. By sensitivity analysis, it was found that CE is the most sensitive parameter among all the parameters. The profit has a direct relation with the increase or decrease of carbon emission cost under the effect of GT. It was obtained that the effect of GT was very important and effective for both the collection center and the conversion plant. Pre-existing studies aimed to save the environment while only considering waste-to-energy conversion. It was proved numerically that the flexibility of the production process converges over the constant production rate. It helped to control the extra or reduced production to face the random demand. This study proved the reduction of emissions during this conversion through a flexible production system, even though there was a random demand for renewable energy. Through mathematical derivations, the total profit of the ESCM was globally maximized for the optimum amount of renewable energy and the corresponding selling price. There were two separate investments for carbon emissions reduction from both players' points of view. Numerically, the investment was found to be very small for the waste collector and the energy converter. The optimum production rate is quite high to maintain the smart city's demand. Through the numerical comparative study, if a constant production rate was used, the amount of shortages increased by a huge amount. Thus, the flexibility in production for the ESCM is highly effective. As a business strategy, revenue sharing was offered from the energy converter to the waste collector. Total profit varied with huge changes in CEs. This study did not address the way that renewable energy is stored; it can be done through a circular economy direction. The conversion rate and quality of waste are areas of potential future research. The data-driven approach can be utilized rather than the distribution-free approach to obtain better results if the data is available.

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Conflict of interest

The authors declare no conflict of interest.

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Appendixes

Appendix A

$$
\begin{aligned}
\Delta_1 \quad = \quad & \Bigg(\frac{2\alpha\,(\delta-1)\left(C_{D_E}+C_{T_E}P^2\right)}{P} - C_{AR2}\alpha \Bigg((1-\delta)\,(\epsilon_4-g\nu_4) \Bigg(\frac{\alpha\,(\omega_c-\omega_c\delta)-\mu}{\sqrt{(\mu+\omega_c\alpha\,(\delta-1))^{\,2}+\sigma^2}} - 1 \Bigg) \\
& -2\,(\delta-1)\,(\epsilon_3-g\nu_3)\Big) - 2C_{AR1} \Bigg(\frac{T_{r_c}\,(\epsilon_2-\nu_2\xi)}{C_T} + \frac{1}{2}\,(\epsilon_1-\nu_1\xi) \Bigg(\frac{\omega_c-\zeta\mu}{\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^{\,2}}} + 1 \Bigg)\Bigg) \\
& +2\,(\delta-1)\,W_c + \frac{(p_c-\theta_2 p_e\alpha\,(\delta-1))\,(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\omega_c\alpha\,(\delta-1)))}{\sqrt{\sigma^2\,(p_c\zeta+\theta_1 p_e)^{\,2}-(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\omega_c\alpha\,(\delta-1)))^{\,2}}} \\
& -2\delta\gamma_c - \frac{2T_{r_c}}{C_T} - 2c + p_c + (\theta_2-1)\,p_e\alpha\,(\delta-1) \\
& -\theta_2 p_e\alpha\,(\delta-1) - \frac{(\theta_2-1)\,p_e^2\alpha\,(\delta-1)\,((\theta_1-1)\,\mu+(\theta_2-1)\,\omega_c\alpha\,(\delta-1))}{\sqrt{p_e^2\,((\theta_1-1)^{\,2}\sigma^2+((\theta_1-1)\,\mu+(\theta_2-1)\,\omega_c\alpha\,(\delta-1))^{\,2})}} \\
& -C_{H_E}\alpha\,(1-\delta) \Bigg(\frac{\alpha\,(\omega_c-\omega_c\delta)-\mu}{\sqrt{(\mu+\omega_c\alpha\,(\delta-1))^{\,2}+\sigma^2}} - 1 \Bigg) + 2\alpha\,(\delta-1)\,S_E - 2U_c \\
& -\alpha\,(1-\delta)\,R_{S_E} \Bigg(\frac{\alpha\,(\omega_c-\omega_c\delta)-\mu}{\sqrt{(\mu+\omega_c\alpha\,(\delta-1))^{\,2}+\sigma^2}} + 1 \Bigg) - R_{TC} \Bigg(\frac{\omega_c-\zeta\mu}{\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^{\,2}}} + 1 \Bigg)\Bigg) \\
\Delta_2 \quad = \quad & (\theta_1\mu+(\theta_2-1)\,\alpha\,(\delta-1)\,\omega_c-\theta_2\alpha\,(\delta-1)\,\omega_c \\
& + \frac{(\theta_2\alpha\,(1-\delta)\,\omega_c-\theta_1\mu)\,(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\alpha\,(\delta-1)\,\omega_c))}{\sqrt{\sigma^2\,(p_c\zeta+\theta_1 p_e)^{\,2}-(p_c\,(\omega_c-\zeta\mu)-p_e\,(\theta_1\mu+\theta_2\alpha\,(\delta-1)\,\omega_c))^{\,2}}} \\
& - \sqrt{((\theta_1-1)^{\,2}\sigma^2+((\theta_1-1)\,\mu+(\theta_2-1)\,\alpha\,(\delta-1)\,\omega_c)^{\,2})} - (\theta_1-1)\,\mu\Big)
\end{aligned}
$$

Appendix B

$$
\begin{aligned}
\frac{\partial^2 E[\Gamma]}{\partial \xi^2} \quad &= \quad -\eta \\
\frac{\partial^2 E[\Gamma]}{\partial \xi \partial \xi'} \quad &= \quad \frac{\partial^2 E[\Gamma]}{\partial \xi \partial P} = \frac{\partial^2 E[\Gamma]}{\partial \xi \partial p_e} = 0 \\
\frac{\partial^2 E[\Gamma]}{\partial \xi \partial \omega_c} \quad &= \quad C_{AR1}\left[\nu_2 T_{r_c} + \frac{\nu_1}{2}\left(1+\frac{\omega_c-\zeta\mu}{\sqrt{\zeta^2\sigma^2+(\omega_c-\zeta\mu)^2}}\right)\right] \\
\frac{\partial^2 E[\Gamma]}{\partial \xi'^{\,2}} \quad &= \quad -\beta \\
\frac{\partial^2 E[\Gamma]}{\partial \xi' \partial P} \quad &= \quad \frac{\partial^2 E[\Gamma]}{\partial \xi' \partial p_e} = 0 \\
\frac{\partial^2 E[\Gamma]}{\partial \xi' \partial \omega_c} \quad &= \quad C_{AR2}\alpha(1-\delta)\left[\nu_3 - \frac{\nu_4}{2} - \frac{\mu-\alpha(1-\delta)\omega_c}{\sqrt{\sigma^2+(\mu-\alpha(1-\delta)\omega_c)^2}}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E[\Gamma]}{\partial P^2} &= -\frac{2\alpha(1-\delta)\omega_c C_{D_E}}{P^3} \\
\frac{\partial^2 E[\Gamma]}{\partial P \partial p_e} &= 0 \\
\frac{\partial^2 E[\Gamma]}{\partial P \partial \omega_c} &= \alpha\,(1-\delta)\left(\frac{C_{D_E}}{P^2} - C_{T_E}\right) \\
\frac{\partial^2 E[\Gamma]}{\partial p_e^2} &= -\frac{\Delta_3\{(\theta_2\alpha(1-\delta)\omega_c - \theta_1\mu)^2 - {\theta_1}^2\sigma^2\} - \Delta_4\Delta_5}{2\Delta_3^{3/2}} \\
\frac{\partial^2 E[\Gamma]}{\partial p_e \partial \omega_c} &= \frac{1}{2}\left[\frac{\Delta_9}{\sqrt{\Delta_6}} + \alpha(1-\delta) + \frac{2\Delta_7\Delta_3 - \Delta_8\Delta_4}{2\Delta_3^{3/2}}\right] \\
\frac{\partial^2 E[\Gamma]}{\partial \omega_c^2} &= -\frac{1}{2}\left[\frac{(p_c + p_e\theta_2\alpha(1-\delta))\,\{(p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c))\Delta_{11} + \Delta_{10}\}}{\Delta_{10}^{3/2}}\right. \\
&\quad + \frac{2\alpha(\theta_2 - 1)(1-\delta)\Delta_{12} - p_e\alpha(1-\delta)(\theta_2 - 1)\Delta_{13}}{2\Delta_{12}^{3/2}} - \frac{C_{AR2}\alpha^2\sigma^2(1-\delta)^2}{(\sigma^2 + (\mu - \omega_c\alpha(1-\delta))^2)^{3/2}} \\
&\quad \left. - \frac{(C_{AR1}(\epsilon_1 - \nu_1\xi) + \tau_c + S_c)\zeta^2\sigma^2}{(\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^2)^{3/2}} - \frac{(C_{H_E} + R_{S_E})\alpha^2\sigma^2(1-\delta)^2}{(\sigma^2 + (\mu - \alpha(1-\delta)\omega_c)^2)^{3/2}}\right] \\
\Delta_3 &= \sigma^2(p_c\zeta + \theta_1 p_e)^2 - (p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c))^2 \\
\Delta_4 &= (\theta_2\alpha\,(1-\delta)\,\omega_c - \theta_1\mu)\,(p_c\,(\omega_c - \zeta\mu) - p_e\,(\theta_1\mu - \theta_2\alpha\,(1-\delta)\,\omega_c)) - \theta_1\sigma^2\,(p_c\zeta + \theta_1 p_e) \\
\Delta_5 &= \sigma^2\theta_1(p_c\omega_c + \theta_1 p_e) - \Big(p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c)\Big)(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c) \\
\Delta_6 &= (\theta_1 - 1)^2\sigma^2 + ((\theta_1 - 1)\mu - (\theta_2 - 1)\alpha(1-\delta)\omega_c)^2 \\
\Delta_7 &= (\theta_2\alpha(1-\delta)\omega_c - \theta_1\mu)(p_c + \theta_2\alpha(1-\delta)) + \theta_2\alpha(1-\delta)(p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c)) \\
\Delta_8 &= 2(p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c))p_c + p_e\theta_2\alpha(1-\delta) \\
\Delta_9 &= ((\theta_1 - 1)\mu + (\theta_2 - 1)\alpha(\delta - 1)\omega_c)(\theta_2 - 1)\alpha(\delta - 1) \\
\Delta_{10} &= \sigma^2(p_c\zeta + \theta_1 p_e)^2 - (p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\omega_c\alpha(1-\delta)))^2 \\
\Delta_{11} &= (p_c + \theta_2 p_e\alpha(1-\delta))(p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\omega_c\alpha(1-\delta))) \\
\Delta_{12} &= p_c^2((\theta_1 - 1)^2\sigma^2 + ((\theta_1 - 1)\mu - (\theta_2 - 1)\omega_c\alpha(1-\delta))^2) \\
\Delta_{13} &= (\theta_2 - 1)p_e^2\alpha(\delta - 1)((\theta_1 - 1)\mu - (\theta_2 - 1)\omega_c\alpha(1-\delta)) \\
\Omega_1 &= \frac{\Delta_3\{(\theta_2\alpha(1-\delta)\omega_c - \theta_1\mu)^2 - {\theta_1}^2\sigma^2\} - \Delta_4\Delta_5}{2\Delta_3^{3/2}} \\
\Omega_2 &= \frac{1}{2}\left[\frac{(p_c + p_e\theta_2\alpha(1-\delta))\,\{(p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c))\Delta_{11} + \Delta_{10}\}}{\Delta_{10}^{3/2}}\right. \\
&\quad + \frac{2\alpha(\theta_2 - 1)(1-\delta)\Delta_{12} - p_e\alpha(1-\delta)(\theta_2 - 1)\Delta_{13}}{2\Delta_{12}^{3/2}} - \frac{C_{AR2}\alpha^2\sigma^2(1-\delta)^2}{(\sigma^2 + (\mu - \omega_c\alpha(1-\delta))^2)^{3/2}} \\
&\quad \left. - \frac{(C_{AR1}(\epsilon_1 - \nu_1\xi) + \tau_c + S_c)\zeta^2\sigma^2}{(\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^2)^{3/2}} - \frac{(C_{H_E} + R_{S_E})\alpha^2\sigma^2(1-\delta)^2}{(\sigma^2 + (\mu - \alpha(1-\delta)\omega_c)^2)^{3/2}}\right]
\end{aligned}
$$

$$
\begin{aligned}
\Omega_3 &= \frac{1}{2}\left[\frac{(p_c + p_e\theta_2\alpha(1-\delta))\left\{(p_c(\omega_c - \zeta\mu) - p_e(\theta_1\mu - \theta_2\alpha(1-\delta)\omega_c))\Delta_{11} + \Delta_{10}\right\}}{\Delta_{10}^{3/2}}\right. \\
&\quad + \frac{2\alpha(\theta_2 - 1)(1-\delta)\Delta_{12} - p_e\alpha(1-\delta)(\theta_2 - 1)\Delta_{13}}{2\Delta_{12}^{3/2}} - \frac{C_{AR2}\alpha^2\sigma^2(1-\delta)^2}{(\sigma^2 + (\mu - \omega_c\alpha(1-\delta))^2)^{3/2}} \\
&\quad \left. - \frac{(C_{AR1}(\epsilon_1 - \nu_1\xi) + \tau_c + S_c)\zeta^2\sigma^2}{(\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^2)^{3/2}} - \frac{(C_{H_E} + R_{S_E})\alpha^2\sigma^2(1-\delta)^2}{(\sigma^2 + (\mu - \alpha(1-\delta)\omega_c)^2)^{3/2}}\right] \\
\Omega_4 &= C_{AR2}\alpha(1-\delta)\left[\nu_3 - \frac{\nu_4}{2} - \frac{\mu - \alpha(1-\delta)\omega_c}{\sqrt{\sigma^2 + (\mu - \alpha(1-\delta)\omega_c)^2}}\right] \\
\Omega_5 &= \left(C_{AR1}\left[\nu_2 T_{r_c} + \frac{\nu_1}{2}\left(1 + \frac{\omega_c - \zeta\mu}{\sqrt{\zeta^2\sigma^2 + (\omega_c - \zeta\mu)^2}}\right)\right]\right)
\end{aligned}
$$

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