Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/mbe

MBE, 20(3): 4609–4642.
DOI: 10.3934/mbe.2023214
Received: 21 August 2022
Revised: 06 October 2022
Accepted: 11 October 2022
Published: 27 December 2022

Research article

Methods to find strength of job competition among candidates under single-valued neutrosophic soft model

Sundas Shahzadi[1], **Areen Rasool**[1] **and Gustavo Santos-García**[2,*]

[1] Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan

[2] IME, Universidad de Salamanca, 37007 Salamanca, Spain

* **Correspondence:** Email: santos@usal.es.

Abstract: Neutrosophic soft set theory is one of the most developed interdisciplinary research areas, with multiple applications in various fields such as computational intelligence, applied mathematics, social networks, and decision science. In this research article, we introduce the powerful framework of single-valued neutrosophic soft competition graphs by integrating the powerful technique of single-valued neutrosophic soft set with competition graph. For dealing with different levels of competitive relationships among objects in the presence of parametrization, the novel concepts are defined which include single-valued neutrosophic soft k-competition graphs and p-competition single-valued neutrosophic soft graphs. Several energetic consequences are presented to obtain strong edges of the above-referred graphs. The significance of these novel concepts is investigated through application in professional competition and also an algorithm is developed to address this decision-making problem.

Keywords: single-valued neutrosophic set; soft set; competition graphs; single-valued neutrosophic digraph; decision-making

1. Introduction

Graph theory is a remarkable approach introduced by Euler in 1973. This alluring technique is effectively used in the mainstream of mathematics, primarily due to its applications in social networking, communication networks, information technology, and neural networks. Digraphs are powerful mathematical structures to depict point-to-point relationships among objects connected in a directed network. In 1968, Cohen [1] introduced the idea of competition graphs of digraphs that arose in competition between species in an ecosystem. It has a remarkable origin to identify the explicit behaviors of objects and especially used in ecological networks. Competition graphs have been applicable in different research areas of scientific and technical knowledge. After this captivating scheme, several researchers studied competition graphs in different forms regarding double competition graphs of digraphs [2], $\boldsymbol{p}$-competition graphs [3], tolerance competition graphs [4], and m-step competition graphs [5].

The notion of fuzzy set theory, which generalizes the crisp theory, was initiated by Zadeh [6] to interact with the phenomena of uncertainty and approximate reasoning in real-life problems. Fuzzy set theory paved a different way of dealing with ambiguities in various domains of scientific and technical knowledge. Kaufmann [7] defined the powerful and most implemented technique of a fuzzy graph that is based on Zadeh [8] fuzzy relations. Rosenfeld [9] introduced another refined definition of a fuzzy graph and discuss the framework of fuzzy relations between sets. Moreover, some remarkable results on fuzzy graphs were introduced by Bhattacharya [10], Mordeson and Nair [11] studied certain operations on fuzzy graphs. The theory of competition graphs whose idea was initiated by Cohen was inadequate to describe all real-world competitions involving predator-prey relations, powerful communities on a social networks, rivalries in the business market, and signal influence of wireless tools. To overcome this difficulty, Samanta and Pal [12] introduced the idea of fuzzy competition graphs and also gave the generalizations of these graphs as fuzzy $\boldsymbol{k}$-competition graphs and $\boldsymbol{p}$-competition fuzzy graphs. Sarwar and Akram [13] presented novel concepts of bipolar fuzzy (BF) competition graphs. Sahu et al. [14] introduced picture fuzzy set and rough set-based approaches to help the student to choose an appropriate subject and consequently provide a good service or contribution to society particularly in that domain. Ashraf et al. [15] studied Maclaurin symmetric mean in the framework of interval-valued picture fuzzy sets and discussed their application in picking the most suitable company benefit plan using interval-valued picture fuzzy data. Shahzadi et al. [16] also proposed BF competition hypergraph and its application in decision-making problems.

Atanassov [17] gave the idea of intuitionistic fuzzy sets as an extension of Zadeh's fuzzy set. In fuzzy sets, the uncertainty was considered only in one direction but in an intuitionistic fuzzy set, we deal with membership (truth-membership) function and a non-membership (falsity-membership) function at a time the only requirement is that the sum of both values is not more than one. In addition, Sahoo and Pal [18] discussed the concept of the intuitionistic fuzzy competition graph.

In many real physical problems, fuzzy sets and intuitionistic fuzzy sets cannot handle all kinds of uncertainties even though the fuzzy set has been generalized and developed. Both theories only deal with incomplete information, not inconsistent information and indeterminate information which occur usually in belief systems. Smarandache [19] introduced the idea of the neutrosophic set for dealing with problems involving imprecise, indeterminacy, and inconsistent data. The concept of a neutrosophic set is a more general framework to overcome the mentioned issues. Further several new concepts on neutrosophic graphs with their applications were discussed in [20, 21]. Akram and Siddique introduced the idea of neutrosophic competition graphs [22] and presented an application of single-valued neutrosophic (SVN) competition graphs in decision-making.

However, the idea of fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets are not sufficient to cope with parametrization tools. Karamasa et al. [23] discussed the weighting factors affecting logistics outsourcing. In 1999, Molodtsov [24] proposed the idea of a soft set that has the ability to deal with this difficulty. The idea of fuzzy soft (FS) sets, intuitionistic FS sets, and neutrosophic soft sets were proposed by Maji et al. [25] and some properties of FS sets were discussed by Ahmad and Kharal [26]. Hybrid models: rough soft sets, soft rough sets, and soft rough fuzzy sets were constructed by Feng et al. [27]. Deli and Broumi [28, 29] introduced several concepts including neutrosophic soft relations, neutrosophic soft matrices, and neutrosophic soft multi-set theory.

Akram et al. [30] proposed decision-making methods based on FS competition hypergraphs. Nawaz and Akram [31] presented an application on FS economic competition graphs that shows diverse competition among the wireless internet service providers of Malaysia. Nawaz et al. [32] proposed the idea of Pythagorean fuzzy hypergraphs and also construct an algorithm to compute the strength of competing interactions in the Bering Sea. Akram and Shahzadi [33] first introduced the concept of neutrosophic soft graphs. Akram and Nawaz [34, 35] proposed algorithms for the computation of regular single-valued neutrosophic soft hypergraphs applied to supranational Asian bodies and implementation of single-valued neutrosophic soft hypergraphs on human nervous system. A list of the involvement of authors in competition theory is presented in Table 1.

Table 1. Contribution of authors in competition graphs.

Authors	Year	Contributions
Cohen	1968	Defined competition graph
Samanta and Pal	2013	Introduced fuzzy competition graph
Samanta et al.	2015	Defined m-step fuzzy competition graph
Sahoo and Pal	2016	Introduced intuitionistic fuzzy competition graph
Sarwar and Akram	2017	Extended the idea of BF competition graph
Akram and Siddique	2017	Defined neutrosophic competition graph
Akram and Nasir	2017	Defined intuitionistic neutrosophic competition graphs
Sarwar et al.	2018	Presented fuzzy competition hypergraph
Habib et al.	2019	Introduced q-rung orthopair fuzzy competition graph
Akram et al.	2020	q-rung picture fuzzy competition graphs
Nawaz and Akram	2021	Introduced FS competition graph
Akram et al.	2021	Complex fuzzy competition graphs
Shahzadi et al.	This paper	Introduced neutrosophic soft competition graph

1.1. Motivations and objectives

The motivation of this article is as follows:

1) The single-valued neutrosophic soft (SVNS) set is a hybrid technique of SVN set and soft set which has merged the characteristics and features of both these theories. The SVNS graphs are able to depict all those correlations that have SVN membership grades and depend on various factors.
2) The initiation of SVNS competition graphs and two of its generalizations are substantial to interpret the different levels of competition in the ecological system. The most important feature of this technique is that it inherits the properties of existing models.

The main contribution of this article is as follows:

1) The idea of SVNS competition graphs is initiated by merging the framework of SVNS sets with competition graphs. Two generalizations of this proposed approach with numerical examples are introduced in this article. We also discuss some interesting results relating to strong independent edges of all the above-mentioned terminologies.

2) We apply the notion of SVNS competition graphs in a decision-making problem and design an algorithm to calculate the strength of job competition among applicants. Moreover, we prove the effectiveness and validity of our proposed model by comparing with existing methods.

1.2. Structure of the article

The structure of this paper is as follows.

1) Section 2 presents some crucial concepts related to this article and provide the new ideas of SVNS competition graphs, two generalizations of this proposed technique, offers certain consequences related to strong independent edges, and add some captivating constructions of proposed method.
2) Section 3 offers the decision-making approach of our theoretical framework which not only increase the significance of our article but also give us a new direction that how to solve real-life problems in decision support system by utilizing our proposed model.
3) Section 4 allocates the comparison of SVNS competition graphs with some existing approaches which prove the effectiveness and productivity of our planned article.
4) Section 5 tackles the conclusion and further direction.

2. Single-valued neutrosophic soft competition graphs

First, we review some fundamental notions relating to neutrosophic and SVNS sets. We also define the SVNS competition graphs and some arithmetic operations on SVNS competition graphs.

Definition 2.1. [19] Let $\mathscr{X}$ be a universe of discourse. A neutrosophic set $\mathscr{N}$ on $\mathscr{X}$ is characterized by a truth membership function $T_{\mathscr{N}}$, an indeterminacy membership function $I_{\mathscr{N}}$, and a falsity membership function $F_{\mathscr{N}}$, where $T_{\mathscr{N}}, I_{\mathscr{N}}, F_{\mathscr{N}} : \mathscr{X} \longrightarrow [0^-, 1^+]$ are real standard or non-standard subsets of $[0^-, 1^+]$. It is represented as $\mathscr{N} = \{x(T_{\mathscr{N}}(x), I_{\mathscr{N}}(x), F_{\mathscr{N}}(x)) : x \in \mathscr{X}\}$, where the sum of $T_{\mathscr{N}}(x)$, $I_{\mathscr{N}}(x)$, and $F_{\mathscr{N}}(x)$ has no restriction, so $0^- \leq T_{\mathscr{N}}(x) + I_{\mathscr{N}}(x) + F_{\mathscr{N}}(x) \leq 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $[0^-, 1^+]$. In real life applications in scientific and engineering problems, it is difficult to use neutrosophic set with value from real standard or non-standard subsets of $[0^-, 1^+]$. So, for technical applications, we have to take the standard unit interval $[0, 1]$ instead of $[0^-, 1^+]$.

Definition 2.2. [36] A SVN set $\mathscr{N}$ on $\mathscr{X}$ is characterized by a truth membership function $T_{\mathscr{N}}$, an indeterminacy membership function $I_{\mathscr{N}}$, and a falsity membership function $F_{\mathscr{N}}$, where $T_{\mathscr{N}}, I_{\mathscr{N}}, F_{\mathscr{N}} : \mathscr{X} \longrightarrow [0, 1]$. It is written as $\mathscr{N} = \{x(T_{\mathscr{N}}(x), I_{\mathscr{N}}(x), F_{\mathscr{N}}(x)) : x \in X\}$, where $0 \leq T_{\mathscr{N}}(x) + I_{\mathscr{N}}(x) + F_{\mathscr{N}}(x) \leq 3$.

Definition 2.3. [25] Let $\mathscr{X}$ be a universe of discourse and $\mathscr{P}$ represents the set of all parameters referring to the objects in $\mathscr{X}$. Suppose $P(\mathscr{X})$ denotes the set of all SVN sets over $\mathscr{X}$ and $\mathscr{K} \subseteq \mathscr{P}$. The collection $(\mathscr{U}, \mathscr{K})$ is termed to be the SVNS set over $\mathscr{X}$, where $\mathscr{U}$ is a mapping given by $\mathscr{U} : \mathscr{K} \longrightarrow P(\mathscr{X})$. It is represented as $(\mathscr{U}, \mathscr{K}) = \{k, \mathscr{U}(k) : k \in \mathscr{K}\}$, where $\mathscr{U}(k) = \{x(T_{\mathscr{U}(k)}(x), I_{\mathscr{U}(k)}(x), F_{\mathscr{U}(k)}(x)) : x \in \mathscr{X}\}$ is a SVN set corresponding to a parameter k.

Definition 2.4. [33] A SVNS digraph on $\mathscr{X}$ is a 3-tuple $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ where

(i) $(\mathscr{U}, \mathscr{K})$ is a SVNS set over $\mathscr{X}$.

(ii) $(\overrightarrow{\mathscr{V}}, \mathscr{K})$ is a SVNS set over $E \subseteq \mathscr{X} \times \mathscr{X}$.

(iii) For each $k \in \mathscr{K}$, $(\mathscr{U}(k), \overrightarrow{\mathscr{V}}(k))$ is a SVN digraph. That is

$$T_{\overrightarrow{\mathscr{V}}(k)}(x,y) \leq T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y),\ I_{\overrightarrow{\mathscr{V}}(k)}(x,y) \leq I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y),\ F_{\overrightarrow{\mathscr{V}}(k)}(x,y) \leq F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y),$$

such that

$$0 \leq T_{\overrightarrow{\mathscr{V}}(k)}(x,y) + I_{\overrightarrow{\mathscr{V}}(k)}(x,y) + F_{\overrightarrow{\mathscr{V}}(k)}(x,y) \leq 3, \text{ for all } x, y \in \mathscr{X}.$$

It is denoted as $\overrightarrow{\mathscr{G}}(k) = (\mathscr{U}(k), \overrightarrow{\mathscr{V}}(k))$, where $k \in \mathscr{K}$. Hence the set of all SVN digraphs corresponding to parameters $k \in \mathscr{K}$ is called a SVNS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k) : k \in \mathscr{X}\}$.

Definition 2.5. The height of a SVNS set $(\mathscr{U}, \mathscr{K})$ is defined as

$$h(\mathscr{U}, \mathscr{K}) \quad = \quad (\sup_{k \in \mathscr{K}} h_1(\mathscr{U}(k)), \sup_{k \in \mathscr{K}} h_2(\mathscr{U}(k)), \inf_{k \in \mathscr{K}} h_3(\mathscr{U}(k))),$$

where $h_1(\mathscr{U}(k)) = \sup_{x \in \mathscr{X}}(T_{\mathscr{U}(k)}(x))$, $h_2(\mathscr{U}(k)) = \sup_{x \in \mathscr{X}}(I_{\mathscr{U}(k)}(x))$, and $h_3(\mathscr{U}(k)) = \inf_{x \in \mathscr{X}}(F_{\mathscr{U}(k)}(x))$ for each $k \in \mathscr{K}$.

Definition 2.6. The cardinality of a SVNS set $(\mathscr{U}, \mathscr{K})$ is defined as follows:

$$|(\mathscr{U}, \mathscr{K})| \quad = \quad (\sum_{k \in \mathscr{K}} |T_{\mathscr{U}(k)}|, \sum_{k \in \mathscr{K}} |I_{\mathscr{U}(k)}|, \sum_{k \in \mathscr{K}} |F_{\mathscr{U}(k)}|),$$

where $|T_{\mathscr{U}(k)}| = \sum_{x \in \mathscr{X}} T_{\mathscr{U}(k)}(x)$, $|I_{\mathscr{U}(k)}| = \sum_{x \in \mathscr{X}} I_{\mathscr{U}(k)}(x)$, $|F_{\mathscr{U}(k)}| = \sum_{x \in \mathscr{X}} F_{\mathscr{U}(k)}(x)$ for each $k \in \mathscr{K}$.

Definition 2.7. The support of a SVNS set $(\mathscr{U}, \mathscr{K})$ is defined as follows:

$$Supp(\mathscr{U}, \mathscr{K}) \quad = \quad \{(k, supp(\mathscr{U}(k))) : k \in \mathscr{K}\},$$

where $\text{supp}(\mathscr{U}(k)) = \{x \in \mathscr{X} : T_{\mathscr{U}(k)}(x) > 0, I_{\mathscr{U}(k)}(x) > 0, F_{\mathscr{U}(k)}(x) > 0\}$.

Before moving towards the main objective of this paper, first we define the terminologies like SVNS out neighborhood and SVNS in neighborhood of a vertex in a SVNS digraph, we introduce the concept of SVNS competition graph.

Definition 2.8. The SVNS out neighborhood of a vertex x in a SVNS diagraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ is a SVNS set

$$(\mathscr{N}^+(x), \mathscr{K}) = \{k, \mathscr{N}^+(x)(k) \ : k \ \in \ \mathscr{K}\},$$

where

$$\mathscr{N}^+(x)(k) \quad = \quad \{y(T_{\overrightarrow{\mathscr{V}}(k)}(x,y), I_{\overrightarrow{\mathscr{V}}(k)}(x,y), F_{\overrightarrow{\mathscr{V}}(k)}(x,y)) \mid (T_{\overrightarrow{\mathscr{V}}(k)}(x,y) > 0, I_{\overrightarrow{\mathscr{V}}(k)}(x,y) > 0, \mathit{and}\ F_{\overrightarrow{\mathscr{V}}(k)}(x,y) > 0\}.$$

Definition 2.9. The SVNS in neighborhood of a vertex x in a SVNS diagraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ is a SVNS set

$$(\mathscr{N}^-(x), \mathscr{K}) = \{k, \mathscr{N}^-(x)(k) \ : k \ \in \ \mathscr{K}\},$$

where

$$\mathscr{N}^{-}(x)(k) = \{y(T_{\overrightarrow{\mathscr{V}}(k)}(y,x), I_{\overrightarrow{\mathscr{V}}(k)}(y,x), F_{\overrightarrow{\mathscr{V}}(k)}(y,x)) \mid T_{\overrightarrow{\mathscr{V}}(k)}(y,x) > 0, I_{\overrightarrow{\mathscr{V}}(k)}(y,x) > 0, \mathit{and}\ F_{\overrightarrow{\mathscr{V}}(k)}(y,x) > 0\}.$$

Definition 2.10. The SVNS competition graph $C(\overrightarrow{\mathscr{G}}) = (\mathscr{U}, \mathscr{R}, \mathscr{K})$ of a SVNS digraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ having the SVNS vertex set same as in $\overrightarrow{\mathscr{G}}$ and for all $k \in \mathscr{K}$, there exists a SVN edge (x,y) in $C(\overrightarrow{\mathscr{G}}(k)) = (\mathscr{U}(k), \mathscr{R}(k))$ if and only if $\mathscr{N}^{+}(x)(k) \cap \mathscr{N}^{+}(y)(k) \neq \emptyset$. The truth, indeterminacy, and falsity membership values of the edge (x,y) in $C(\overrightarrow{\mathscr{G}}(k))$ corresponding to the parameter $k \in \mathscr{K}$ are defined as:

$$\begin{aligned} T_{\mathscr{R}(k)}(x,y) &= [T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)]h_1(\mathscr{N}^{+}(x)(k) \cap \mathscr{N}^{+}(y)(k)), \\ I_{\mathscr{R}(k)}(x,y) &= [I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)]h_2(\mathscr{N}^{+}(x)(k) \cap \mathscr{N}^{+}(y)(k)), \\ F_{\mathscr{R}(k)}(x,y) &= [F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)]h_3(\mathscr{N}^{+}(x)(k) \cap \mathscr{N}^{+}(y)(k)). \end{aligned}$$

Example 2.1. Suppose $\mathscr{X} = \{x_1, y_1, z_1, w_1\}$ be a non-empty set and $\mathscr{K} = \{k_1, k_2\}$ be a set of parameter on $\mathscr{X}$. Suppose $(\mathscr{U}, \mathscr{K})$ be a SVNS set over $\mathscr{X}$ and $(\overrightarrow{\mathscr{V}}, \mathscr{K})$ be a SVNS set over $E \subseteq \mathscr{X} \times \mathscr{X}$ which are represented in Tables 2 and 3, respectively. The SVNS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k_1), \overrightarrow{\mathscr{G}}(k_2)\}$ is shown in Figure 1.

Table 2. SVNS set on $\mathscr{X}$.

$x \in \mathscr{X}$	$\mathscr{U}(k_1)$	$\mathscr{U}(k_2)$
x_1	$(0.4, 0.7, 0.2)$	$(0.1, 0.7, 0.4)$
y_1	$(0.9, 0.1, 0.4)$	$(0.4, 0.6, 0.9)$
z_1	$(0.2, 0.1, 0.6)$	$(0.7, 0.8, 0.3)$
w_1	$(0.1, 0.8, 0.7)$	$(0.5, 0.5, 0.6)$

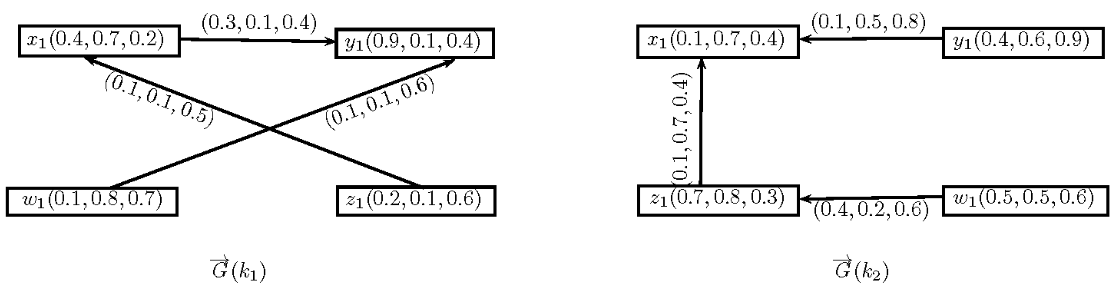

Figure 1. SVNS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k_1), \overrightarrow{\mathscr{G}}(k_2)\}$.

Table 3. SVNS set on $E \subseteq \mathscr{X} \times \mathscr{X}$.

E	$\overrightarrow{\mathscr{V}}(k_1)$	E	$\overrightarrow{\mathscr{V}}(k_2)$
$(\overrightarrow{x_1, y_1})$	$(0.3, 0.1, 0.4)$	$(\overrightarrow{x_1, y_1})$	$(0.0, 0.0, 0.0)$
$(\overrightarrow{y_1, x_1})$	$(0.0, 0.0, 0.0)$	$(\overrightarrow{y_1, x_1})$	$(0.1, 0.5, 0.8)$
$(\overrightarrow{z_1, x_1})$	$(0.1, 0.1, 0.5)$	$(\overrightarrow{z_1, x_1})$	$(0.1, 0.7, 0.4)$
$(\overrightarrow{w_1, y_1})$	$(0.1, 0.1, 0.6)$	$(\overrightarrow{z_1, w_1})$	$(0.0, 0.0, 0.0)$
$(\overrightarrow{w_1, z_1})$	$(0.0, 0.0, 0.0)$	$(\overrightarrow{w_1, z_1})$	$(0.4, 0.2, 0.6)$

The SVNS out neighborhoods of all vertices in $\overrightarrow{\mathscr{G}}$ are specified in Table 4.

Table 4. SVNS out neighborhoods of $\overrightarrow{\mathscr{G}}$.

$x \in \mathscr{X}$	$\mathscr{N}^+(x)(k_1)$	$\mathscr{N}^+(x)(k_2)$	$(\mathscr{N}^+(x), \mathscr{K})$
x_1	$\{y_1(0.3, 0.1, 0.4)\}$	ϕ	$\{\frac{y_1(0.3,0.1,0.4)}{k_1}, \frac{\phi}{k_2}\}$
y_1	ϕ	$\{x_1(0.1, 0.5, 0.8)\}$	$\{\frac{\phi}{k_1}, \frac{x_1(0.1,0.5,0.8)}{k_2}\}$
z_1	$\{x_1(0.1, 0.1, 0.5)\}$	$\{x_1(0.1, 0.7, 0.4)\}$	$\{\frac{x_1(0.1,0.1,0.5)}{k_1}, \frac{x_1(0.1,0.7,0.4)}{k_2}\}$
w_1	$\{y_1(0.1, 0.1, 0.6)\}$	$\{z_1(0.4, 0.2, 0.6)\}$	$\{\frac{y_1(0.1,0.1,0.6)}{k_1}, \frac{z_1(0.4,0.2,0.6)}{k_2}\}$

Since

$$\mathscr{N}^+(x_1)(k_1) \cap \mathscr{N}^+(w_1)(k_1) = \{y_1(0.1, 0.1, 0.6)\}.$$

So, by Definition 2.10, there is a SVN edge between x_1 and w_1 in $C(\overrightarrow{\mathscr{G}}(k_1)) = (\mathscr{U}(k_1), \mathscr{R}(k_1))$. We calculate the membership grade of the edge (x_1, w_1) corresponding to a parameter $k_1 \in \mathscr{K}$.

$$\begin{aligned}
T_{\mathscr{R}(k_1)}(x_1, w_1) &= [T_{\mathscr{U}(k_1)}(x_1) \wedge T_{\mathscr{U}(k_1)}(w_1)]h_1(\mathscr{N}^+(x_1)(k_1) \cap \mathscr{N}^+(w_1)(k_1)) = (0.1)(0.1) = 0.01, \\
I_{\mathscr{R}(k_1)}(x_1, w_1) &= [I_{\mathscr{U}(k_1)}(x_1) \wedge I_{\mathscr{U}(k_1)}(w_1)]h_2(\mathscr{N}^+(x_1)(k_1) \cap \mathscr{N}^+(w_1)(k_1)) = (0.7)(0.1) = 0.07, \\
F_{\mathscr{R}(k_1)}(x_1, w_1) &= [F_{\mathscr{U}(k_1)}(x_1) \vee F_{\mathscr{U}(k_1)}(w_1)]h_3(\mathscr{N}^+(x_1)(k_1) \cap \mathscr{N}^+(w_1)(k_1)) = (0.7)(0.6) = 0.42.
\end{aligned}$$

Similarly, there is a SVN edge between y_1 and z_1 in $C(\overrightarrow{\mathscr{G}}(k_2)) = (\mathscr{U}(k_2), \mathscr{R}(k_2))$ corresponding to a parameter $k_2 \in \mathscr{K}$ whose membership grades are $T_{\mathscr{R}(k_2)}(y_1, z_1) = 0.04$, $I_{\mathscr{R}(k_2)}(y_1, z_1) = 0.3$, and $F_{\mathscr{R}(k_2)}(y_1, z_1) = 0.72$. The SVNS competition graph $C(\overrightarrow{\mathscr{G}}) = \{C(\overrightarrow{\mathscr{G}}(k_1)), C(\overrightarrow{\mathscr{G}}(k_2))\}$ is given in Figure 2.

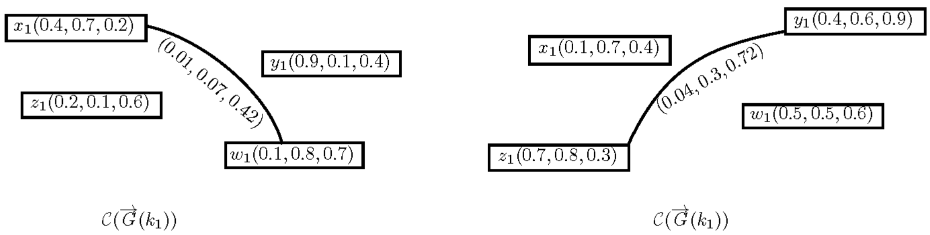

Figure 2. SVNS competition graph $C(\overrightarrow{\mathscr{G}}) = \{C(\overrightarrow{\mathscr{G}}(k_1)), C(\overrightarrow{\mathscr{G}}(k_2))\}$.

Now we define the SVNS $\boldsymbol{k}$-competition graph which is an extension of SVNS competition graph.

Definition 2.11. Let $\boldsymbol{k}$ be a non-negative number. The SVNS $\boldsymbol{k}$-competition graph $C_{\boldsymbol{k}}(\overrightarrow{\mathscr{G}}) = (\mathscr{U}, \mathscr{S}, \mathscr{K})$ of a SVNS digraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ having the SVNS vertex set same as in $\overrightarrow{\mathscr{G}}$ and for all $k \in \mathscr{K}$, there is a SVN edge between x and y in $C_{\boldsymbol{k}}(\overrightarrow{\mathscr{G}}(k)) = (\mathscr{U}(k), \mathscr{S}(k))$ if and only if $|T_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > \boldsymbol{k}$, $|I_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > \boldsymbol{k}$, and $|F_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > \boldsymbol{k}$. The membership values of the edge (x, y) in $C_{\boldsymbol{k}}(\overrightarrow{\mathscr{G}}(k))$ corresponding to a parameter $k \in \mathscr{K}$ are:

$$\begin{aligned} T_{\mathscr{S}(k)}(x,y) &= \frac{k_1-\boldsymbol{k}}{k_1}[T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)]h_1(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)), \\ I_{\mathscr{S}(k)}(x,y) &= \frac{k_2-\boldsymbol{k}}{k_2}[I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)]h_2(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)), \\ F_{\mathscr{S}(k)}(x,y) &= \frac{k_3-\boldsymbol{k}}{k_3}[F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)]h_3(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)), \end{aligned}$$

where $|\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)| = (k_1, k_2, k_3)$.

So, SVNS $\boldsymbol{k}$-competition graph is simply SVNS competition graph when $\boldsymbol{k} = 0$. Following we give an example of SVNS 0.1-competition graph.

Example 2.2. Consider a SVNS digraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ as shown in Figure 3.

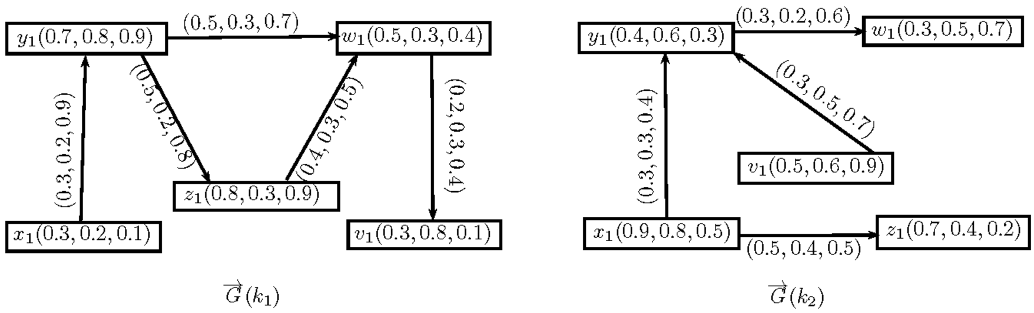

Figure 3. SVNS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k_1), \overrightarrow{\mathscr{G}}(k_2)\}$.

Since

$$\begin{gathered} \mathscr{N}^+(y_1)(k_1) \cap \mathscr{N}^+(z_1)(k_1) = \{w_1(0.4,0.3,0.7)\}, \\ |\mathscr{N}^+(y_1)(k_1) \cap \mathscr{N}^+(z_1)(k_1)| = (k_1,k_2,k_3) = (0.4,0.3,0.7). \end{gathered}$$

Therefore, $|T_{\mathscr{N}^+(y_1)(k_1)\cap\mathscr{N}^+(z_1)(k_1)}| = 0.4 > 0.1$, $|I_{\mathscr{N}^+(y_1)(k_1)\cap\mathscr{N}^+(z_1)(k_1)}| = 0.3 > 0.1$, and $|F_{\mathscr{N}^+(y_1)(k_1)\cap\mathscr{N}^+(z_1)(k_1)}| = 0.7 > 0.1$. So, by Definition 2.11, there exist a SVN edge (y_1, z_1) in $C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}(k_1)) = (\mathscr{U}(k_1), \mathscr{S}(k_1))$. The membership values of the edge (y_1, z_1) corresponding to a parameter $k_1 \in \mathscr{K}$ are computed as follows.

$$\begin{aligned} T_{\mathscr{S}(k_1)}(y_1,z_1) &= \frac{0.4-0.1}{0.4}[T_{\mathscr{U}(k_1)}(y_1) \wedge T_{\mathscr{U}(k_1)}(z_1)]h_1(\mathscr{N}^+(y_1)(k_1) \cap \mathscr{N}^+(z_1)(k_1)) = 0.21, \\ I_{\mathscr{S}(k_1)}(y_1,z_1) &= \frac{0.3-0.1}{0.3}[I_{\mathscr{U}(k_1)}(y_1) \wedge I_{\mathscr{U}(k_1)}(z_1)]h_2(\mathscr{N}^+(y_1)(k_1) \cap \mathscr{N}^+(z_1)(k_1)) = 0.059, \end{aligned}$$

$$F_{\mathscr{S}(k_1)}(y_1, z_1) = \frac{0.7 - 0.1}{0.7}[F_{\mathscr{U}(k_1)}(y_1) \vee F_{\mathscr{U}(k_1)}(z_1)]h_3(\mathscr{N}^+(y_1)(k_1) \cap \mathscr{N}^+(z_1)(k_1)) = 0.539.$$

Similarly, there is an edge between x_1 and v_1 in $C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}(k_2)) = (\mathscr{U}(k_2), \mathscr{S}(k_2))$. The membership value of the edge (x_1, v_1) corresponding to a parameter $k_2 \in \mathscr{K}$ are $T_{\mathscr{S}(k_2)}(x_1, v_1) = 0.099$, $I_{\mathscr{S}(k_2)}(x_1, v_1) = 0.119$, and $F_{\mathscr{S}(k_2)}(x_1, v_1) = 0.539$. The SVNS ***0.1***-competition graph $C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}) = \{C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}(k_1)), C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}(k_2))\}$ is shown in Figure 4.

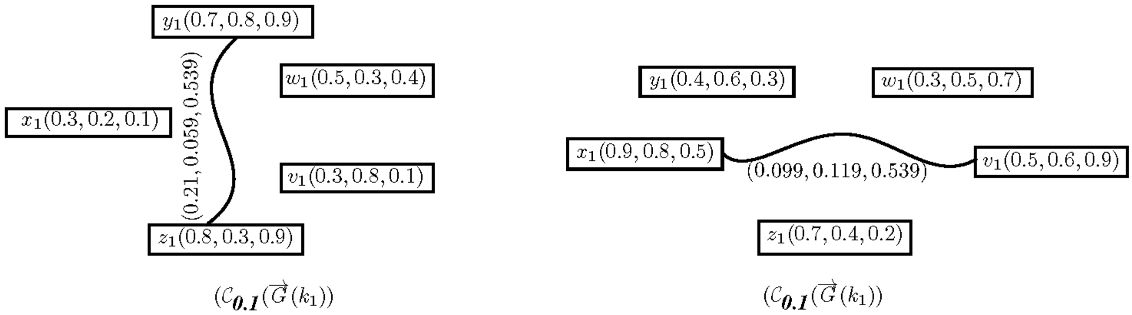

Figure 4. SVNS ***0.1***-competition graph $C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}) = \{C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}(k_1)), C_{\boldsymbol{0.1}}(\overrightarrow{\mathscr{G}}(k_2))\}$.

Definition 2.12. Let $\boldsymbol{p}$ be a positive integer. The $\boldsymbol{p}$-competition SVNS graph $C^{\boldsymbol{p}}(\overrightarrow{\mathscr{G}}) = (\mathscr{U}, \mathscr{H}, \mathscr{K})$ of a SVNS digraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ having the SVNS vertex set same as in $\overrightarrow{\mathscr{G}}$ and for all $k \in \mathscr{K}$, there is a SVN edge between x and y in $C^{\boldsymbol{p}}(\overrightarrow{\mathscr{G}}(k)) = (\mathscr{U}(k), \mathscr{H}(k))$ if and only if $|Supp(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k))| \geq \boldsymbol{p}$. The membership values of the edge (x, y) in $C^{\boldsymbol{p}}(\overrightarrow{\mathscr{G}}(k))$ corresponding to a parameter $k \in \mathscr{K}$ are defined as:

$$\begin{aligned}
T_{\mathscr{H}(k)}(x, y) &= \frac{(n - \boldsymbol{p}) + 1}{n}[T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)]h_1(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)),\\
I_{\mathscr{H}(k)}(x, y) &= \frac{(n - \boldsymbol{p}) + 1}{n}[I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)]h_2(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)),\\
F_{\mathscr{H}(k)}(x, y) &= \frac{(n - \boldsymbol{p}) + 1}{n}[F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)]h_3(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)),
\end{aligned}$$

where $n = |Supp(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k))|$.

Following example illustrates the ***2***-competition SVNS graph.

Example 2.3. Consider a SVNS digraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ as shown in Figure 5.

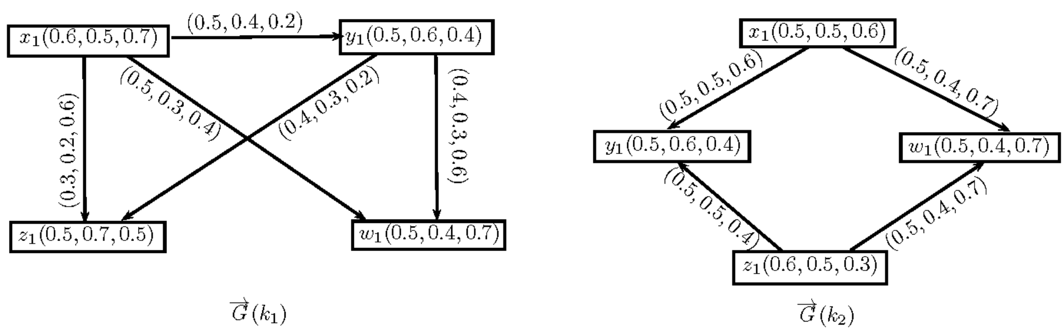

Figure 5. SVNS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k_1), \overrightarrow{\mathscr{G}}(k_2)\}$.

Here, $\mathcal{N}^+(x_1)(k_1) = \{z_1(0.3, 0.2, 0.6), w_1(0.5, 0.3, 0.4), y_1(0.5, 0.4, 0.2)\}$ and $\mathcal{N}^+(y_1)(k_1) = \{z_1(0.4, 0.3, 0.2), w_1(0.4, 0.3, 0.6)\}$. Hence

$$\mathcal{N}^+(x_1)(k_1) \cap \mathcal{N}^+(y_1)(k_1) = \{z_1(0.3, 0.2, 0.6), w_1(0.4, 0.3, 0.6)\},$$
$$|Supp(\mathcal{N}^+(x_1)(k_1) \cap \mathcal{N}^+(y_1)(k_1))| = 2.$$

Thus, by Definition 2.12, there is a SVN edge between x_1 and y_1. The membership values of the edge (x_1, y_1) corresponding to parameter $k_1 \in \mathcal{K}$ are:

$$\begin{aligned}
T_{\mathscr{H}(k_1)}(x_1, y_1) &= \frac{2-2+1}{2}[T_{\mathscr{U}(k_1)}(x_1) \wedge T_{\mathscr{U}(k_1)}(y_1)]h_1(\mathcal{N}^+(x_1)(k_1) \cap \mathcal{N}^+(y_1)(k_1)) = 0.1,\\
I_{\mathscr{H}(k_1)}(x_1, y_1) &= \frac{2-2+1}{2}[I_{\mathscr{U}(k_1)}(x_1) \wedge I_{\mathscr{U}(k_1)}(y_1)]h_2(\mathcal{N}^+(x_1)(k_1) \cap \mathcal{N}^+(y_1)(k_1)) = 0.075,\\
F_{\mathscr{H}(k_1)}(x_1, y_1) &= \frac{2-2+1}{2}[F_{\mathscr{U}(k_1)}(x_1) \vee F_{\mathscr{U}(k_1)}(y_1)]h_3(\mathcal{N}^+(x_1)(k_1) \cap \mathcal{N}^+(y_1)(k_1)) = 0.21.
\end{aligned}$$

Similarly, there is an edge between x_1 and z_1 in $C^2(\overrightarrow{\mathscr{G}}(k_2)) = (\mathscr{U}(k_2), \mathscr{H}(k_2))$. The membership value of the edge (x_1, z_1) corresponding to parameter $k_2 \in \mathcal{K}$ are $T_{\mathscr{H}(k_2)}(x_1, z_1) = 0.125$, $I_{\mathscr{H}(k_2)}(x_1, z_1) = 0.125$, and $F_{\mathscr{H}(k_2)}(x_1, z_1) = 0.18$. The **2**-competition SVNS graph $C^2(\overrightarrow{\mathscr{G}}) = \{C^2(\overrightarrow{\mathscr{G}}(k_1)), C^2(\overrightarrow{\mathscr{G}}(k_2))\}$ is shown in Figure 6.

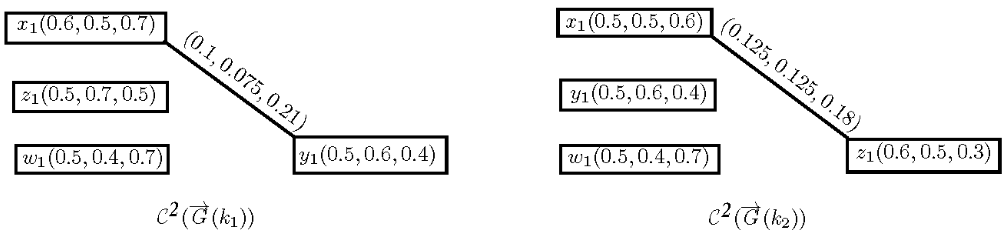

Figure 6. **2**-competition SVNS graph $C^2(\overrightarrow{\mathscr{G}}) = \{C^2(\overrightarrow{\mathscr{G}}(k_1)), C^2(\overrightarrow{\mathscr{G}}(k_2))\}$.

Definition 2.13. Let $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathcal{K})$ be a SVNS digraph on $\mathscr{X}$. An edge $(\overrightarrow{x, y})$ of a SVNS digraph is said to be a strong independent edge if for each $k \in \mathcal{K}$ the following conditions hold:

$$\begin{aligned}
\frac{1}{2}[T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)] &< T_{\overrightarrow{\mathscr{V}}(k)}(x, y),\\
\frac{1}{2}[I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)] &< I_{\overrightarrow{\mathscr{V}}(k)}(x, y),\\
\frac{1}{2}[F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)] &> F_{\overrightarrow{\mathscr{V}}(k)}(x, y).
\end{aligned}$$

Otherwise, it will be a weak edge.

Theorem 2.1. *Let $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathcal{K})$ be a SVNS digraph on a non-void set $\mathscr{X}$. If for each $k \in \mathcal{K}$, $\mathcal{N}^+(x)(k) \cap \mathcal{N}^+(y)(k)$ contains only one SVN vertex, then the SVNS edge (x, y) of $C(\overrightarrow{\mathscr{G}})$ is strong independent if and only if $|T_{\mathcal{N}^+(x)(k)\cap\mathcal{N}^+(y)(k)}| > 0.5$, $|I_{\mathcal{N}^+(x)(k)\cap\mathcal{N}^+(y)(k)}| > 0.5$, and $|F_{\mathcal{N}^+(x)(k)\cap\mathcal{N}^+(y)(k)}| < 0.5$ for all $k \in \mathcal{K}$.*

Proof. Let $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ be a SVNS digraph and $C(\overrightarrow{\mathscr{G}}) = (\mathscr{U}, \mathscr{R}, \mathscr{K})$ be a SVNS competition graph of $\overrightarrow{\mathscr{G}}$. Suppose for each $k \in \mathscr{K}$, $\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k) = \{a(m_T, m_I, m_F)\}$, where m_T, m_I and m_F are the truth, indeterminacy, and falsity membership values of a. Here $h_1(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)) = m_T = |T_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}|$, $h_2(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)) = m_I = |I_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}|$, and $h_3(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)) = m_F = |F_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}|$. Therefore,

$$\begin{aligned}
T_{\mathscr{R}(k)}(x, y) &= (T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)) \times m_T, \\
I_{\mathscr{R}(k)}(x, y) &= (I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)) \times m_I, \\
F_{\mathscr{R}(k)}(x, y) &= (F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)) \times m_F.
\end{aligned}$$

Hence, the SVNS edge (x, y) of a SVNS competition graph $C(\overrightarrow{\mathscr{G}})$ is strong independent if and only if $|T_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > 0.5$, $|I_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > 0.5$, and $|F_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| < 0.5$ for all $k \in \mathscr{K}$.

Theorem 2.2. *Let $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ be a SVNS digraph. If for each $k \in \mathscr{K}$, $h_1(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)) = 1$, $h_2(\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)) = 1$, $h_3(\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)) = 1$, $|T_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > 2\boldsymbol{k}$, $|I_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > 2\boldsymbol{k}$, and $|F_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| < 2\boldsymbol{k}$ then the edge (x, y) is strong independent in $C_{\boldsymbol{k}}(\overrightarrow{\mathscr{G}})$.*

Proof. Let $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$ be a SVNS digraph and $C_{\boldsymbol{k}}(\overrightarrow{\mathscr{G}}) = (\mathscr{U}, \mathscr{S}, \mathscr{K})$ be the SVNS $\boldsymbol{k}$-competition graph of $\overrightarrow{\mathscr{G}}$. Suppose for each $k \in \mathscr{K}$, $h_1(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)) = 1$ and $|T_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > 2\boldsymbol{k}$ then

$$\begin{aligned}
T_{\mathscr{S}(k)}(x, y) &= \frac{k_1 - \boldsymbol{k}}{k_1}(T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y))h_1(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)), \\
T_{\mathscr{S}(k)}(x, y) &= \frac{k_1 - \boldsymbol{k}}{k_1}(T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)), \\
\frac{T_{\mathscr{S}(k)}(x, y)}{[T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)]} &= \frac{k_1 - \boldsymbol{k}}{k_1} > \frac{1}{2}, \\
\frac{1}{2}[T_{\mathscr{U}(k)}(x) \wedge T_{\mathscr{U}(k)}(y)] &< T_{\mathscr{S}(k)}(x, y).
\end{aligned}$$

If $h_2(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)) = 1$ and $|I_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| > 2\boldsymbol{k}$ then

$$\begin{aligned}
I_{\mathscr{S}(k)}(x, y) &= \frac{k_2 - \boldsymbol{k}}{k_2}(I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y))h_2(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)), \\
I_{\mathscr{S}(k)}(x, y) &= \frac{k_2 - \boldsymbol{k}}{k_2}(I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)), \\
\frac{I_{\mathscr{S}(k)}(x, y)}{[I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)]} &= \frac{k_2 - \boldsymbol{k}}{k_2} > \frac{1}{2}, \\
\frac{1}{2}[I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)] &< I_{\mathscr{S}(k)}(x, y).
\end{aligned}$$

If $h_3(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)) = 1$ and $|F_{\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)}| < 2\boldsymbol{k}$ then

$$F_{\mathscr{S}(k)}(x, y) = \frac{k_3 - \boldsymbol{k}}{k_3}(F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y))h_3(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)),$$

$$
\begin{aligned}
F_{\mathscr{S}(k)}(x,y) &= \frac{k_3-\boldsymbol{k}}{k_3}(F_{\mathscr{U}(k)}(x)\vee F_{\mathscr{U}(k)}(y)),\\
\frac{F_{\mathscr{S}(k)}(x,y)}{[F_{\mathscr{U}(k)}(x)\vee F_{\mathscr{U}(k)}(y)]} &= \frac{k_3-\boldsymbol{k}}{k_3}<\frac{1}{2},\\
\frac{1}{2}[F_{\mathscr{U}(k)}(x)\vee F_{\mathscr{U}(k)}(y)] &> F_{\mathscr{S}(k)}(x,y).
\end{aligned}
$$

Hence the SVNS edge (x,y) is strong independent in $C_{\boldsymbol{k}}(\overrightarrow{\mathscr{G}})$.

Theorem 2.3. *If all the edges of a SVNS digraph $\overrightarrow{\mathscr{G}}=(\mathscr{U},\overrightarrow{\mathscr{V}},\mathscr{K})$ are independent strong then $\frac{T_{\mathscr{R}(k)}(x,y)}{(T_{\mathscr{U}(k)}(x)\wedge T_{\mathscr{U}(k)}(y))^2}>\frac{1}{2}$, $\frac{I_{\mathscr{R}(k)}(x,y)}{(I_{\mathscr{U}(k)}(x)\wedge I_{\mathscr{U}(k)}(y))^2}>\frac{1}{2}$, and $\frac{F_{\mathscr{R}(k)}(x,y)}{(F_{\mathscr{U}(k)}(x)\vee F_{\mathscr{U}(k)}(y))^2}<\frac{1}{2}$ for all $k\in\mathscr{K}$ and edges (x,y) in $C(\overrightarrow{\mathscr{G}})$.*

Proof. Suppose all edges of a SVNS digraph $\overrightarrow{\mathscr{G}}=(\mathscr{U},\overrightarrow{\mathscr{V}},\mathscr{K})$ are independent strong and $C(\overrightarrow{\mathscr{G}})$ be the SVNS competition graph corresponding to $\overrightarrow{\mathscr{G}}$.

Case (i): Let $\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)=\phi$ for all $x,y\in\mathscr{X}$ and $k\in\mathscr{K}$. Then there does not exist any SVNS edge in $C(\overrightarrow{\mathscr{G}})$ between x and y. In this case, there is nothing to prove.

Case (ii): Assume that for each $k\in\mathscr{K}$, $\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)\neq\phi$. Let $\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)=\{a_1(T_1,I_1,F_1),a_2(T_2,I_2,F_2),\cdots,a_n(T_n,I_n,F_n)\}$, where T_i,I_i, and F_i are the truth, indeterminacy, and falsity membership values of either $(\overrightarrow{x,a_i})$ or $(\overrightarrow{y,a_i})$ for $i=1,2,\cdots,n$, respectively. So $T_i=[T_{\overrightarrow{\mathscr{V}}(k)}(\overrightarrow{x,a_i})\wedge T_{\overrightarrow{\mathscr{V}}(k)}(\overrightarrow{y,a_i})]$, $I_i=[I_{\overrightarrow{\mathscr{V}}(k)}(\overrightarrow{x,a_i})\wedge I_{\overrightarrow{\mathscr{V}}(k)}(\overrightarrow{y,a_i})]$, and $F_i=[F_{\overrightarrow{\mathscr{V}}(k)}(\overrightarrow{x,a_i})\vee F_{\overrightarrow{\mathscr{V}}(k)}(\overrightarrow{y,a_i})]$ for $i=1,2,\cdots,n$. Let

$$
\begin{aligned}
h_1(\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)) &= max\{T_i,i=1,2,\cdots,n\}=T_{max},\\
h_2(\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)) &= max\{I_i,i=1,2,\cdots,n\}=I_{max},\\
h_3(\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)) &= min\{F_i,i=1,2,\cdots,n\}=F_{min}.
\end{aligned}
$$

Obviously, $T_{max}>T_{\overrightarrow{\mathscr{V}}(k)}(x,y)$, $I_{max}>I_{\overrightarrow{\mathscr{V}}(k)}(x,y)$, and $F_{min}<F_{\overrightarrow{\mathscr{V}}(k)}(x,y)$ for all $k\in\mathscr{K}$ and edges (x,y) shows that

$$
\begin{aligned}
\frac{T_{max}}{T_{\mathscr{U}(k)}(x)\wedge T_{\mathscr{U}(k)}(y)} &> \frac{T_{\overrightarrow{\mathscr{V}}(k)}(x,y)}{T_{\mathscr{U}(k)}(x)\wedge T_{\mathscr{U}(k)}(y)}>0.5,\\
\frac{I_{max}}{I_{\mathscr{U}(k)}(x)\wedge I_{\mathscr{U}(k)}(y)} &> \frac{I_{\overrightarrow{\mathscr{V}}(k)}(x,y)}{I_{\mathscr{U}(k)}(x)\wedge I_{\mathscr{U}(k)}(y)}>0.5,\\
\frac{F_{min}}{F_{\mathscr{U}(k)}(x)\vee F_{\mathscr{U}(k)}(y)} &< \frac{F_{\overrightarrow{\mathscr{V}}(k)}(x,y)}{F_{\mathscr{U}(k)}(x)\vee F_{\mathscr{U}(k)}(y)}<0.5.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
T_{\mathscr{R}(k)}(x,y) &= [T_{\mathscr{U}(k)}(x)\wedge T_{\mathscr{U}(k)}(y)]h_1(\mathscr{N}^+(x)(k)\cap\mathscr{N}^+(y)(k)),\\
&= [T_{\mathscr{U}(k)}(x)\wedge T_{\mathscr{U}(k)}(y)]T_{max},\\
\frac{T_{\mathscr{R}(k)}(x,y)}{(T_{\mathscr{U}(k)}(x)\wedge T_{\mathscr{U}(k)}(y))^2} &= \frac{T_{max}}{T_{\mathscr{U}(k)}(x)\wedge T_{\mathscr{U}(k)}(y)}>0.5,
\end{aligned}
$$

$$
\begin{aligned}
I_{\mathscr{R}(k)}(x,y) &= [I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)]h_2(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)),\\
&= [I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)]I_{max},\\
\frac{I_{\mathscr{R}(k)}(x,y)}{(I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y))^2} &= \frac{I_{max}}{I_{\mathscr{U}(k)}(x) \wedge I_{\mathscr{U}(k)}(y)} > 0.5,\\
F_{\mathscr{R}(k)}(x,y) &= [F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)]h_3(\mathscr{N}^+(x)(k) \cap \mathscr{N}^+(y)(k)),\\
&= [F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)]F_{min},\\
\frac{F_{\mathscr{R}(k)}(x,y)}{(F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y))^2} &= \frac{F_{min}}{F_{\mathscr{U}(k)}(x) \vee F_{\mathscr{U}(k)}(y)} < 0.5.
\end{aligned}
$$

Now, we present a method of construction of SVNS competition graph of the Cartesian product of SVNS digraph in a Theorem.

Theorem 2.4. *Let $C(\overrightarrow{\mathscr{G}}_1) = (\mathscr{U}_1, \mathscr{R}_1, \mathscr{K})$ and $C(\overrightarrow{\mathscr{G}}_2) = (\mathscr{U}_2, \mathscr{R}_2, \mathscr{L})$ be two SVNS competition graphs of SVNS digraphs $\overrightarrow{\mathscr{G}}_1 = (\mathscr{U}_1, \overrightarrow{\mathscr{V}}_1, \mathscr{K})$ and $\overrightarrow{\mathscr{G}}_2 = (\mathscr{U}_2, \overrightarrow{\mathscr{V}}_2, \mathscr{L})$, respectively. Then for each $(k,l) \in \mathscr{K} \times \mathscr{L}$, $C(\overrightarrow{\mathscr{G}}_1(k)\square\overrightarrow{\mathscr{G}}_2(l)) = \mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k))^*\square C(\overrightarrow{\mathscr{G}}_2(l))^*} \cup \mathscr{G}^\square(k,l)$, where $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k))^*\square C(\overrightarrow{\mathscr{G}}_2(l))^*}$ is a SVN graph on the crisp graph $(\mathscr{X}_1 \times \mathscr{X}_2, \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*})$, $C(\overrightarrow{\mathscr{G}}_1(k))^*$ and $C(\overrightarrow{\mathscr{G}}_2(l))^*$ are the crisp competition graphs of $\overrightarrow{\mathscr{G}}_1(k)$ and $\overrightarrow{\mathscr{G}}_2(l)$, respectively. $\mathscr{G}^\square(k,l)$ is a SVN graph on $(\mathscr{X}_1 \times \mathscr{X}_2, \mathscr{E}^\square(k,l))$ such that:*

1) $\mathscr{E}^\square(k,l) = \{(x_1,x_2)(z_1,z_2) : z_1 \in \mathscr{N}^-(x_1)(k)^*, z_2 \in \mathscr{N}^+(x_2)(l)^*\}$.
2) $\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*} = \{(x_1,x_2)(x_1,z_2) : x_1 \in \mathscr{X}_1, x_2z_2 \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}\} \cup \{(x_1,x_2)(z_1,x_2) : x_2 \in \mathscr{X}_2, x_1z_1 \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\}$.
3) $T_{\mathscr{U}_1(k)}\square T_{\mathscr{U}_2(l)}(x_1,x_2) = T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_2(l)}(x_2)$, $I_{\mathscr{U}_1(k)}\square I_{\mathscr{U}_2(l)}(x_1,x_2) = I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_2(l)}(x_2)$, *and* $F_{\mathscr{U}_1(k)}\square F_{\mathscr{U}_2(l)}(x_1,x_2) = F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_2(l)}(x_2)$, *for all* $(x_1,x_2) \in \mathscr{X}_1 \times \mathscr{X}_2$.
4) $T_{\mathscr{R}(k,l)}((x_1,x_2)(x_1,z_2)) = [T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_2(l)}(x_2) \wedge T_{\mathscr{U}_2(l)}(z_2)] \times \vee_{a_2}\{T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\overrightarrow{\mathscr{V}}_2(l)}(x_2a_2) \wedge T_{\overrightarrow{\mathscr{V}}_2(l)}(z_2a_2)\}$, $(x_1,x_2)(x_1,z_2) \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}$, $a_2 \in (\mathcal{N}^+(x_2)(l) \cap \mathcal{N}^+(z_2)(l))^*$.
5) $I_{\mathscr{R}(k,l)}((x_1,x_2)(x_1,z_2)) = [I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_2(l)}(x_2) \wedge I_{\mathscr{U}_2(l)}(z_2)] \times \vee_{a_2}\{I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\overrightarrow{\mathscr{V}}_2(l)}(x_2a_2) \wedge I_{\overrightarrow{\mathscr{V}}_2(l)}(z_2a_2)\}$, $(x_1,x_2)(x_1,z_2) \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}$, $a_2 \in (\mathcal{N}^+(x_2)(l) \cap \mathcal{N}^+(z_2)(l))^*$.
6) $F_{\mathscr{R}(k,l)}((x_1,x_2)(x_1,z_2)) = [F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_2(l)}(x_2) \vee F_{\mathscr{U}_2(l)}(z_2)] \times \vee_{a_2}\{F_{\mathscr{U}_1(k)}(x_1) \vee F_{\overrightarrow{\mathscr{V}}_2(l)}(x_2a_2) \vee F_{\overrightarrow{\mathscr{V}}_2(l)}(z_2a_2)\}$, $(x_1,x_2)(x_1,z_2) \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}$, $a_2 \in (\mathcal{N}^+(x_2)(l) \cap \mathcal{N}^+(z_2)(l))^*$.
7) $T_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,x_2)) = [T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_1(k)}(z_1) \wedge T_{\mathscr{U}_2(l)}(x_2)] \times \vee_{a_1}\{T_{\mathscr{U}_2(l)}(x_2) \wedge T_{\overrightarrow{\mathscr{V}}_1(k)}(x_1a_1) \wedge T_{\overrightarrow{\mathscr{V}}_1(k)}(z_1a_1)\}$, $(x_1,x_2)(z_1,x_2) \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}$, $a_1 \in (\mathcal{N}^+(x_1)(k) \cap \mathcal{N}^+(z_1)(k))^*$.
8) $I_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,x_2)) = [I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_1(k)}(z_1) \wedge I_{\mathscr{U}_2(l)}(x_2)] \times \vee_{a_1}\{I_{\mathscr{U}_2(l)}(x_2) \wedge I_{\overrightarrow{\mathscr{V}}_1(k)}(x_1a_1) \wedge I_{\overrightarrow{\mathscr{V}}_1(k)}(z_1a_1)\}$, $(x_1,x_2)(z_1,x_2) \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}$, $a_1 \in (\mathcal{N}^+(x_1)(k) \cap \mathcal{N}^+(z_1)(k))^*$.
9) $F_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,x_2)) = [F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_1(k)}(z_1) \vee F_{\mathscr{U}_2(l)}(x_2)] \times \vee_{a_1}\{F_{\mathscr{U}_2(l)}(x_2) \vee F_{\overrightarrow{\mathscr{V}}_1(k)}(x_1a_1) \vee F_{\overrightarrow{\mathscr{V}}_1(k)}(z_1a_1)\}$, $(x_1,x_2)(z_1,x_2) \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\square\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}$, $a_1 \in (\mathcal{N}^+(x_1)(k) \cap \mathcal{N}^+(z_1)(k))^*$.
10) $T_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,z_2)) = [T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_1(k)}(z_1) \wedge T_{\mathscr{U}_2(l)}(x_2) \wedge T_{\mathscr{U}_2(l)}(z_2)] \times [T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\overrightarrow{\mathscr{V}}_1(k)}(z_1x_1) \wedge T_{\mathscr{U}_2(l)}(z_2) \wedge T_{\overrightarrow{\mathscr{V}}_2(l)}(x_2z_2)]$, $(x_1,x_2)(z_1,z_2) \in \mathscr{E}^\square(k,l)$.
11) $I_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,z_2)) = [I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_1(k)}(z_1) \wedge I_{\mathscr{U}_2(l)}(x_2) \wedge I_{\mathscr{U}_2(l)}(z_2)] \times [I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\overrightarrow{\mathscr{V}}_1(k)}(z_1x_1) \wedge I_{\mathscr{U}_2(l)}(z_2) \wedge I_{\overrightarrow{\mathscr{V}}_2(l)}(x_2z_2)]$, $(x_1,x_2)(z_1,z_2) \in \mathscr{E}^\square(k,l)$.
12) $F_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,z_2)) = [F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_1(k)}(z_1) \vee F_{\mathscr{U}_2(l)}(x_2) \vee F_{\mathscr{U}_2(l)}(z_2)] \times [F_{\mathscr{U}_1(k)}(x_1) \vee F_{\overrightarrow{\mathscr{V}}_1(k)}(z_1x_1) \vee F_{\mathscr{U}_2(l)}(z_2) \vee F_{\overrightarrow{\mathscr{V}}_2(l)}(x_2z_2)]$, $(x_1,x_2)(z_1,z_2) \in \mathscr{E}^\square(k,l)$.

Proof. This theorem can be proved by utilizing the analogous arguments as in Theorem 3.3 [13].

Example 2.4. Let $\overrightarrow{\mathscr{G}}_1 = (\mathscr{U}_1, \overrightarrow{\mathscr{V}}_1, \mathscr{K})$ and $\overrightarrow{\mathscr{G}}_2 = (\mathscr{U}_2, \overrightarrow{\mathscr{V}}_2, \mathscr{L})$ be two SVNS digraphs of the crisp digraphs $\overrightarrow{\mathscr{G}}^*_1 = (\mathscr{X}_1, \overrightarrow{\mathscr{E}}_1)$ and $\overrightarrow{\mathscr{G}}^*_2 = (\mathscr{X}_2, \overrightarrow{\mathscr{E}}_2)$ as shown in Figures 7 and 8, respectively.

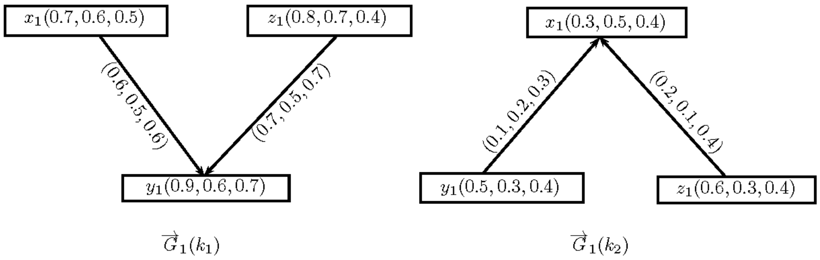

Figure 7. SVNS digraph $\overrightarrow{\mathscr{G}_1} = (\mathscr{U}_1, \overrightarrow{\mathscr{V}}_1, \mathscr{K})$.

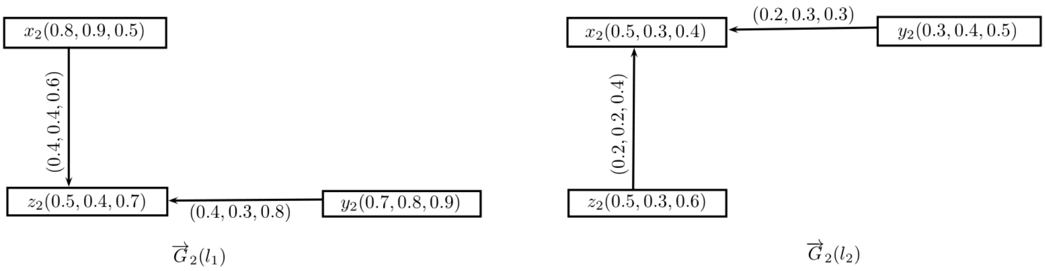

Figure 8. SVNS digraph $\overrightarrow{\mathscr{G}_2} = (\mathscr{U}_2, \overrightarrow{\mathscr{V}}_2, \mathscr{L})$.

The SVNS out neighborhoods of both $\overrightarrow{\mathscr{G}}_1$ and $\overrightarrow{\mathscr{G}}_2$ are given in Tables 5 and 6, respectively. The SVNS in neighborhood of $\overrightarrow{\mathscr{G}}_1$ and $\overrightarrow{\mathscr{G}}_2$ are given in Tables 7 and 8, respectively.

Table 5. SVNS out neighborhood of vertices in $\overrightarrow{\mathscr{G}}_1$.

$x \in \mathscr{X}_1$	$\mathscr{N}^+(x)(k_1)$	$\mathscr{N}^+(x)(k_2)$	$(\mathscr{N}^+(x), \mathscr{K})$
x_1	$\{y_1(0.6, 0.5, 0.6)\}$	ϕ	$\{\frac{y_1(0.6,0.5,0.6)}{k_1}, \frac{\phi}{k_2}\}$
y_1	ϕ	$\{x_1(0.1, 0.2, 0.3)\}$	$\{\frac{\phi}{k_1}, \frac{x_1(0.1,0.2,0.3)}{k_2}\}$
z_1	$\{y_1(0.7, 0.5, 0.7)\}$	$\{x_1(0.2, 0.1, 0.4)\}$	$\{\frac{y_1(0.7,0.5,0.7)}{k_1}, \frac{x_1(0.2,0.1,0.4)}{k_2}\}$

Table 6. SVNS out neighborhood of vertices in $\overrightarrow{\mathscr{G}}_2$.

$x \in \mathscr{X}_2$	$\mathcal{N}^+(x)(l_1)$	$\mathcal{N}^+(x)(l_2)$	$(\mathcal{N}^+(x), \mathscr{L})$
x_2	$\{z_2(0.4, 0.4, 0.6)\}$	ϕ	$\{\frac{z_2(0.4,0.4,0.6)}{l_1}, \frac{\phi}{l_2}\}$
y_2	$\{z_2(0.4, 0.3, 0.8)\}$	$\{x_2(0.2, 0.3, 0.3)\}$	$\{\frac{z_2(0.4,0.3,0.8)}{l_1}, \frac{x_2(0.2,0.3,0.3)}{l_2}\}$
z_2	ϕ	$\{x_2(0.2, 0.2, 0.4)\}$	$\{\frac{\phi}{l_1}, \frac{x_2(0.2,0.2,0.4)}{l_2}\}$

Table 7. SVNS in neighborhood of vertices in $\overrightarrow{\mathscr{G}}_1$.

$x \in \mathscr{X}_1$	$\mathscr{N}^-(x)(k_1)$	$\mathscr{N}^-(x)(k_2)$	$(\mathscr{N}^-(x), \mathscr{K})$
x_1	ϕ	$\{y_1(0.1, 0.2, 0.3), z_1(0.2, 0.1, 0.4)\}$	$\{\frac{\phi}{k_1}, \frac{y_1(0.1,0.2,0.3), z_1(0.2,0.1,0.4)}{k_2}\}$
y_1	$\{x_1(0.6, 0.5, 0.6), z_1(0.7, 0.5, 0.7)\}$	ϕ	$\{\frac{x_1(0.6,0.5,0.6), z_1(0.7,0.5,0.7)}{k_1}, \frac{\phi}{k_2}\}$
z_1	ϕ	ϕ	$\{\frac{\phi}{k_1}, \frac{\phi}{k_2}\}$

Table 8. SVNS in neighborhood of vertices in $\overrightarrow{\mathscr{G}}_2$.

$x \in \mathscr{X}_2$	$\mathcal{N}^-(x)(l_1)$	$\mathcal{N}^-(x)(l_2)$	$(\mathcal{N}^-(x), \mathscr{L})$
x_2	ϕ	$\{y_2(0.2, 0.3, 0.3), z_2(0.2, 0.2, 0.4)\}$	$\{\frac{\phi}{l_1}, \frac{y_2(0.2,0.3,0.3), z_2(0.2,0.2,0.4)}{l_2}\}$
y_2	ϕ	ϕ	$\{\frac{\phi}{l_1}, \frac{\phi}{l_2}\}$
z_2	$\{x_2(0.4, 0.4.0.6), y_2(0.4, 0.3, 0.8)\}$	ϕ	$\{\frac{x_2(0.4,0.4.0.6), y_2(0.4,0.3,0.8)}{l_1}, \frac{\phi}{l_2}\}$

The SVNS competition graphs $C(\overrightarrow{\mathscr{G}}_1)$ and $C(\overrightarrow{\mathscr{G}}_2)$ of $\overrightarrow{\mathscr{G}}_1$ and $\overrightarrow{\mathscr{G}}_2$ are specified in Figures 9 and 10, respectively.

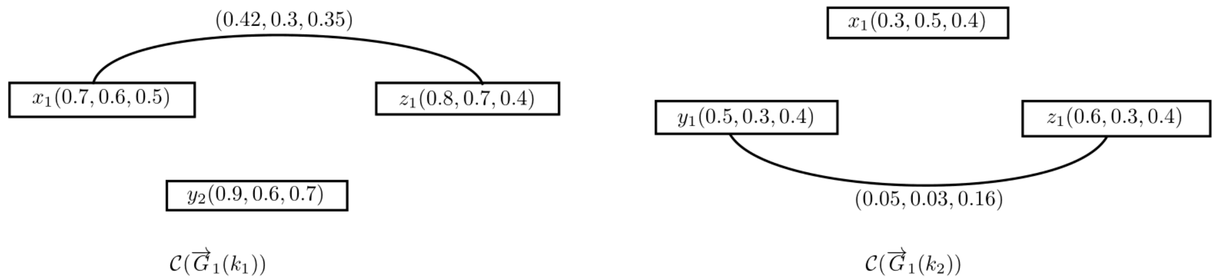

Figure 9. SVNS competition graph $C(\overrightarrow{\mathscr{G}}_1) = (\mathscr{U}_1, \mathscr{R}_1, \mathscr{K})$.

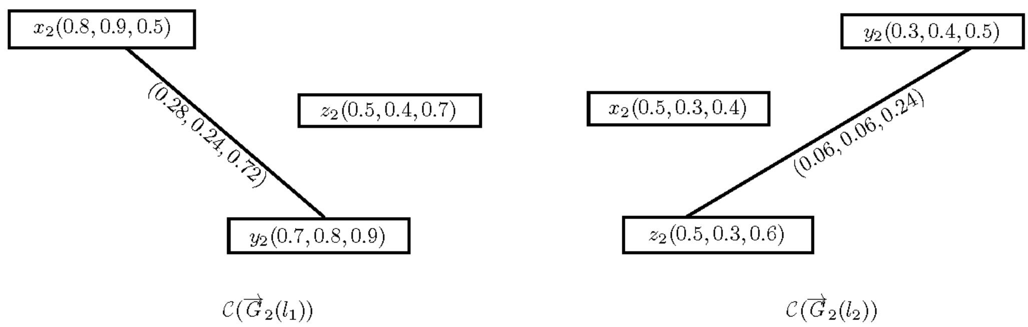

Figure 10. SVNS competition graph $C(\overrightarrow{\mathscr{G}}_2) = (\mathscr{U}_2, \mathscr{R}_2, \mathscr{L})$.

Now for $(k_1, l_1) \in \mathscr{K} \times \mathscr{L}$, we construct the SVN competition graph $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k_1))^* \square C(\overrightarrow{\mathscr{G}}_2(l_1))^*} \cup \mathscr{G}^{\square}(k_1, l_1) = (\mathscr{U}(k_1, l_1), \mathscr{R}(k_1, l_1))$, where $\mathscr{U}(k_1, l_1) = (T_{\mathscr{U}(k_1,l_1)}, I_{\mathscr{U}(k_1,l_1)}, F_{\mathscr{U}(k_1,l_1)})$ and $\mathscr{R}(k_1, l_1) =$

$(T_{\mathscr{R}(k_1,l_1)}, I_{\mathscr{R}(k_1,l_1)}, F_{\mathscr{R}(k_1,l_1)})$ from $C(\overrightarrow{\mathscr{G}}_1(k_1))^*$ and $C(\overrightarrow{\mathscr{G}}_2(l_1))^*$ by using Theorem 2.4. According to conditions (1) and (2), the two set of edges corresponding to parameter $(k_1, l_1) \in \mathscr{K} \times \mathscr{L}$ are

$$\begin{aligned}
\mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k_1))^*} \Box \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l_1))^*} &= \{(x_1, x_2)(x_1, y_2), (y_1, x_2)(y_1, y_2), (z_1, x_2)(z_1, y_2), (x_1, x_2)(z_1, x_2), \\
&\quad (x_1, y_2)(z_1, y_2), (x_1, z_2)(z_1, z_2)\}. \\
\mathscr{E}^{\Box}(k_1, l_1) &= \{(y_1, x_2)(x_1, z_2), (y_1, y_2)(x_1, z_2), (y_1, x_2)(z_1, z_2), (y_1, y_2)(z_1, z_2)\}.
\end{aligned}$$

According to conditions (4)–(12), the degrees of truth membership, indeterminacy membership, and falsity membership of the edges can be calculated as

$$\begin{aligned}
\mathscr{R}(k_1, l_1)((x_1, x_2)(x_1, y_2)) &= (T_{\mathscr{U}_1(k_1)}(x_1) \wedge T_{\mathscr{U}_2(l_1)}(x_2) \wedge T_{\mathscr{U}_2(l_1)}(y_2), \\
&\quad I_{\mathscr{U}_1(k_1)}(x_1) \wedge I_{\mathscr{U}_2(l_1)}(x_2) \wedge I_{\mathscr{U}_2(l_1)}(y_2), \\
&\quad F_{\mathscr{U}_1(k_1)}(x_1) \vee F_{\mathscr{U}_2(l_1)}(x_2) \vee F_{\mathscr{U}_2(l_1)}(y_2)) \\
&\quad \times (T_{\mathscr{U}_1(k_1)}(x_1) \wedge T_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(x_2 z_2) \wedge T_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(y_2 z_2), \\
&\quad I_{\mathscr{U}_1(k_1)}(x_1) \wedge I_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(x_2 z_2) \wedge I_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(y_2 z_2), \\
&\quad F_{\mathscr{U}_1(k_1)}(x_1) \vee F_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(x_2 z_2) \vee F_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(y_2 z_2)). \\
&= (0.7, 0.6, 0.9) \times (0.4, 0.3, 0.8), \\
&= (0.28, 0.18, 0.72). \\
\mathscr{R}(k_1, l_1)((y_1, x_2)(x_1, z_2)) &= (T_{\mathscr{U}_1(k_1)}(y_1) \wedge T_{\mathscr{U}_1(k_1)}(x_1) \wedge T_{\mathscr{U}_2(l_1)}(x_2) \wedge T_{\mathscr{U}_2(l_1)}(z_2), \\
&\quad I_{\mathscr{U}_1(k_1)}(y_1) \wedge I_{\mathscr{U}_1(k_1)}(x_1) \wedge I_{\mathscr{U}_2(l_1)}(x_2) \wedge I_{\mathscr{U}_2(l_1)}(z_2), \\
&\quad F_{\mathscr{U}_1(k_1)}(y_1) \vee F_{\mathscr{U}_1(k_1)}(x_1) \vee F_{\mathscr{U}_2(l_1)}(x_2) \vee F_{\mathscr{U}_2(l_1)}(z_2)) \\
&\quad \times (T_{\mathscr{U}_1(k_1)}(y_1) \wedge T_{\overrightarrow{\mathscr{V}}_{1(k_1)}}(x_1 y_1) \wedge T_{\mathscr{U}_2(l_1)}(z_2) \wedge T_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(x_2 z_2), \\
&\quad I_{\mathscr{U}_1(k_1)}(y_1) \wedge I_{\overrightarrow{\mathscr{V}}_{1(k_1)}}(x_1 y_1) \wedge I_{\mathscr{U}_2(l_1)}(z_2) \wedge I_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(x_2 z_2), \\
&\quad F_{\mathscr{U}_1(k_1)}(y_1) \vee F_{\overrightarrow{\mathscr{V}}_{1(k_1)}}(x_1 y_1) \vee T_{\mathscr{U}_2(l_1)}(z_2) \vee F_{\overrightarrow{\mathscr{V}}_{2(l_1)}}(x_2 z_2)). \\
&= (0.5, 0.4, 0.7) \times (0.4, 0.4, 0.7), \\
&= (0.20, 0.16, 0.49).
\end{aligned}$$

Table 9. Adjacency edges of $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k_1))^* \Box C(\overrightarrow{\mathscr{G}}_2(l_1))^*} \cup \mathscr{G}^{\Box}(k_1, l_1)$.

$(a_1, a_2)(b_1, b_2)$	$\mathscr{R}(k_1, l_1)((a_1, a_2)(b_1, b_2))$
$(y_1, x_2)(x_1, z_2)$	$(0.2, 0.16, 0.49)$
$(y_1, y_2)(x_1, z_2)$	$(0.2, 0.12, 0.72)$
$(y_1, x_2)(z_1, z_2)$	$(0.2, 0.16, 0.49)$
$(y_1, y_2)(z_1, z_2)$	$(0.2, 0.12, 0.72)$
$(x_1, x_2)(x_1, y_2)$	$(0.28, 0.18, 0.72)$
$(y_1, x_2)(y_1, y_2)$	$(0.28, 0.18, 0.72)$
$(x_1, x_2)(z_1, y_2)$	$(0.28, 0.21, 0.72)$
$(x_1, x_2)(z_1, x_2)$	$(0.42, 0.3, 0.35)$
$(x_1, y_2)(z_1, y_2)$	$(0.42, 0.3, 0.81)$
$(x_1, z_2)(z_1, z_2)$	$(0.25, 0.16, 0.49)$

The truth membership, indeterminacy membership, and falsity membership degrees of all the adjacent edges of $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_{1(k_1)})^* \square C(\overrightarrow{\mathscr{G}}_{2(l_1)})^*} \cup \mathscr{G}^\square(k_1, l_1)$ are given in Table 9.

The SVN competition graph corresponding to parameter $(k_1, l_1) \in \mathscr{K} \times \mathscr{L}$ obtained using this method is given in Figure 11, where the solid lines indicate the part of SVN competition graph obtained from $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_{1(k_1)})^* \square C(\overrightarrow{\mathscr{G}}_{2(l_1)})^*}$, the dotted lines represent the part $\mathscr{G}^\square(k_1, l_1)$.

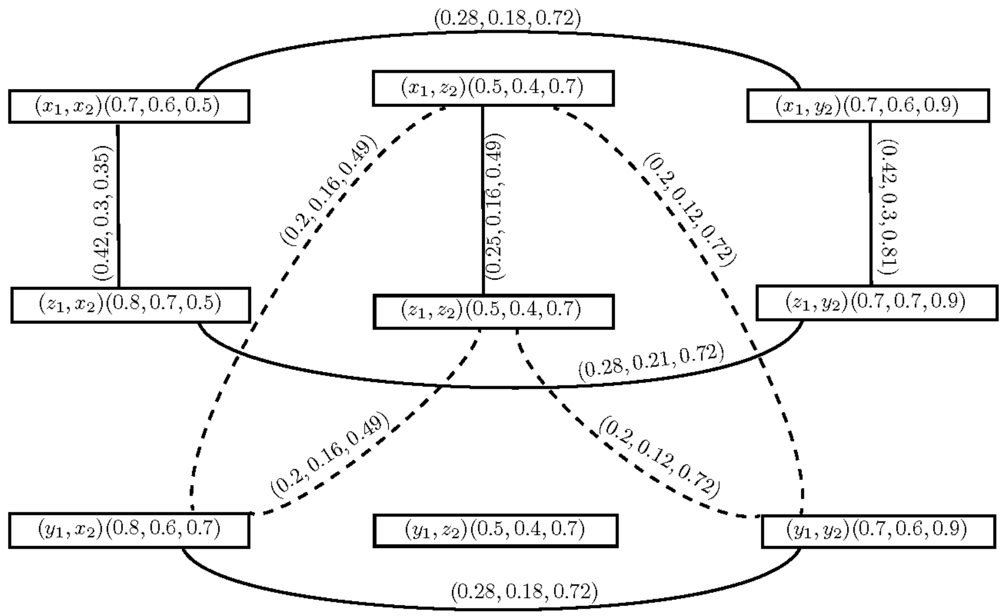

Figure 11. $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_{1(k_1)})^* \square C(\overrightarrow{\mathscr{G}}_{2(l_1)})^*} \cup \mathscr{G}^\square(k_1, l_1)$.

The Cartesian product $\overrightarrow{\mathscr{G}}_1(k_1) \square \overrightarrow{\mathscr{G}}_2(l_1)$ of SVN digraphs $\overrightarrow{\mathscr{G}}_1(k_1)$ and $\overrightarrow{\mathscr{G}}_2(l_1)$ is shown in Figure 12.

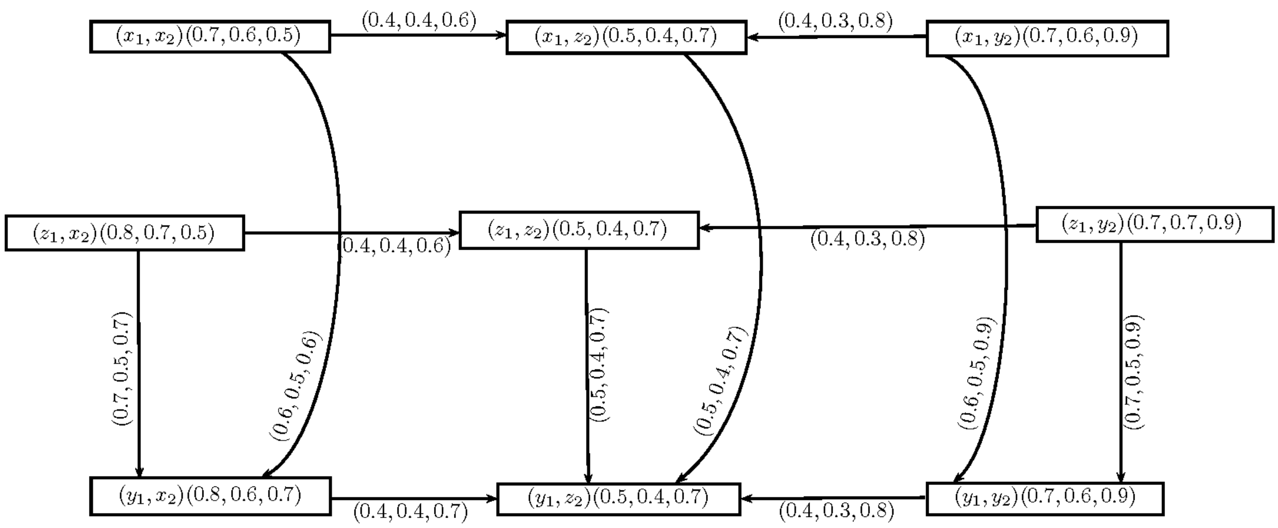

Figure 12. $\overrightarrow{\mathscr{G}}_1(k_1) \square \overrightarrow{\mathscr{G}}_2(l_1)$.

The SVN out neighborhood of vertices in $\overrightarrow{\mathscr{G}}_1(k_1) \square \overrightarrow{\mathscr{G}}_2(l_1)$ are calculated in Table 10.

Table 10. SVN out neighborhood of vertices in $\overrightarrow{\mathscr{G}}_1(k_1)\square\overrightarrow{\mathscr{G}}_2(l_1)$.

$(x_1, x_2) \in \mathscr{X} \times \mathscr{X}$	$\mathscr{N}^+(x_1, x_2)(k_1, l_1)$
(x_1, x_2)	$\{(x_1, z_2)(0.4, 0.4, 0.6), (y_1, x_2)(0.6, 0.5, 0.6)\}$
(x_1, y_2)	$\{(x_1, z_2)(0.4, 0.3, 0.8), (y_1, y_2)(0.6, 0.5, 0.9)\}$
(x_1, z_2)	$\{(y_1, z_2)(0.5, 0.4, 0.7)\}$
(y_1, x_2)	$\{(y_1, z_2)(0.4, 0.4, 0.7)\}$
(y_1, y_2)	$\{(y_1, z_2)(0.4, 0.3, 0.8)\}$
(y_1, z_2)	ϕ
(z_1, x_2)	$\{(y_1, x_2)(0.7, 0.5, 0.7), (z_1, z_2)(0.4, 0.4, 0.6)\}$
(z_1, y_2)	$\{(y_1, y_2)(0.7, 0.5, 0.9), (z_1, z_2)(0.4, 0.3, 0.8)\}$
(z_1, z_2)	$\{(y_1, z_2)(0.5, 0.4, 0.7)\}$

The SVN competition graph of $\overrightarrow{\mathscr{G}}_1(k_1)\square\overrightarrow{\mathscr{G}}_2(l_1)$ is shown in Figure 13.

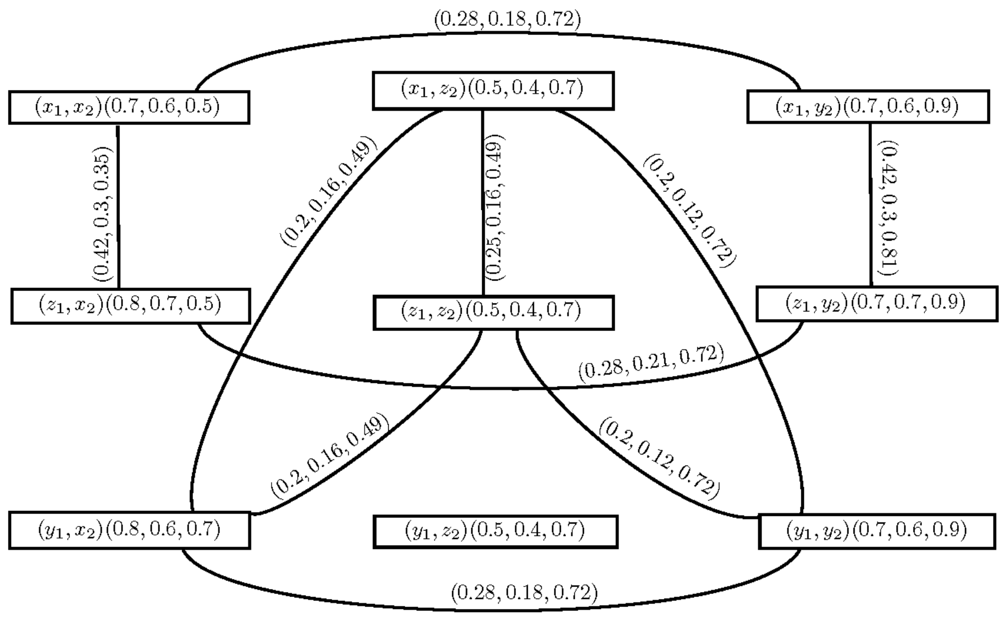

Figure 13. SVN competition graph $C(\overrightarrow{\mathscr{G}}_1(k_1)\square\overrightarrow{\mathscr{G}}_2(l_1))$.

It is clear from Figures 11 and 13 that $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k_1))^*\square C(\overrightarrow{\mathscr{G}}_2(l_1))^*} \cup \mathscr{G}^\square(k_1, l_1) \cong C(\overrightarrow{\mathscr{G}}_1(k_1)\square\overrightarrow{\mathscr{G}}_2(l_1))$. By a similar procedure, we can also obtain the $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k_1))^*\square C(\overrightarrow{\mathscr{G}}_2(l_2))^*} \cup \mathscr{G}^\square(k_1, l_2) \cong C(\overrightarrow{\mathscr{G}}_1(k_1)\square\overrightarrow{\mathscr{G}}_2(l_2))$, $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k_2))^*\square C(\overrightarrow{\mathscr{G}}_2(l_1))^*} \cup \mathscr{G}^\square(k_2, l_1) \cong C(\overrightarrow{\mathscr{G}}_1(k_2)\square\overrightarrow{\mathscr{G}}_2(l_1))$, and $\mathscr{G}_{C(\overrightarrow{\mathscr{G}}_1(k_2))^*\square C(\overrightarrow{\mathscr{G}}_2(l_2))^*} \cup \mathscr{G}^\square(k_2, l_2) \cong C(\overrightarrow{\mathscr{G}}_1(k_2)\square\overrightarrow{\mathscr{G}}_2(l_2))$.

Definition 2.14. Let $\mathscr{G}_1 = (\mathscr{U}_1, \mathscr{V}_1, \mathscr{K})$ and $\mathscr{G}_2 = (\mathscr{U}_2, \mathscr{V}_2, \mathscr{L})$ be two SVNS graphs of $\mathscr{G}_1^* = (\mathscr{X}_1, \mathscr{E}_1)$ and $\mathscr{G}_2^* = (\mathscr{X}_2, \mathscr{E}_2)$, respectively. The direct product of $\mathscr{G}_1$ and $\mathscr{G}_2$ is denoted by $\mathscr{G}_1 \times \mathscr{G}_2$ and defined as $(\mathscr{U}, \mathscr{V}, \mathscr{K} \times \mathscr{L})$, where $\mathscr{U} = (\mathscr{U}_1 \times \mathscr{U}_2, \mathscr{K} \times \mathscr{L})$ is a SVNS set on $\mathscr{X} = \mathscr{X} \times \mathscr{X}$ and $\mathscr{V} = (\mathscr{V}_1 \times \mathscr{V}_2, \mathscr{K} \times \mathscr{L})$ is a SVNS relation on $\mathscr{X} = \mathscr{X} \times \mathscr{X}$ such that

(i) $T_{\mathscr{U}(k,l)}(x_1,x_2) = T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_2(l)}(x_2)$, $I_{\mathscr{U}(k,l)}(x_1,x_2) = I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_2(l)}(x_2)$, and $F_{\mathscr{U}(k,l)}(x_1,x_2) = F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_2(l)}(x_2)$, for all $(x_1,x_2) \in \mathscr{X} \times \mathscr{X}$ and $(k,l) \in \mathscr{K} \times \mathscr{L}$.

(ii) $T_{\mathscr{V}(k,l)}((x_1,x_2)(z_1,z_2)) = T_{\mathscr{V}_1(k)}(x_1,z_1) \wedge T_{\mathscr{V}_2(l)}(x_2,z_2)$, $I_{\mathscr{V}(k,l)}((x_1,x_2)(z_1,z_2)) = I_{\mathscr{V}_1(k)}(x_1,z_1) \wedge I_{\mathscr{V}_2(l)}(x_2,z_2)$, and $F_{\mathscr{V}(k,l)}((x_1,x_2)(z_1,z_2)) = F_{\mathscr{V}_1(k)}(x_1,z_1) \vee F_{\mathscr{V}_2(l)}(x_2,z_2)$,

for all $x_1z_1 \in \mathscr{E}_1$ and $x_2z_2 \in \mathscr{E}_2$.

Now we present the method of SVNS competition graph of the direct product of SVNS digraphs from respective SVNS competition graphs of the SVNS digraphs in a theorem.

Theorem 2.5. *Let $C(\overrightarrow{\mathscr{G}}_1) = (\mathscr{U}_1, \mathscr{R}_1, \mathscr{K})$ and $C(\overrightarrow{\mathscr{G}}_2) = (\mathscr{U}_2, \mathscr{R}_2, \mathscr{L})$ be two SVNS competition graphs of SVNS digraphs $\overrightarrow{\mathscr{G}}_1 = (\mathscr{U}_1, \overrightarrow{\mathscr{V}}_1, \mathscr{K})$ and $\overrightarrow{\mathscr{G}}_2 = (\mathscr{U}_2, \overrightarrow{\mathscr{V}}_2, \mathscr{L})$, respectively, without isolated vertices such that neither is a SVNS empty graph. Then for each $(k,l) \in \mathscr{K} \times \mathscr{L}$, $C(\overrightarrow{\mathscr{G}}_1(k) \times \overrightarrow{\mathscr{G}}_2(l)) = [C(\overrightarrow{\mathscr{G}}_1(k)) \times C(\overrightarrow{\mathscr{G}}_2(l))] \cup \mathscr{G}^{\times}(k,l)$, where $\mathscr{G}^{\times}(k,l) = (\mathscr{U}(k,l), \mathscr{R}(k,l))$ is a SVN graph on the crisp graph $(\mathscr{X}_1 \times \mathscr{X}_2, \mathscr{E}^{\times}(k,l))$ such that*

1) $T_{\mathscr{U}(k,l)}(x_1,x_2) = T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_2(l)}(x_2)$, $I_{\mathscr{U}(k,l)}(x_1,x_2) = I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_2(l)}(x_2)$, *and* $F_{\mathscr{U}(k,l)}(x_1,x_2) = F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_2(l)}(x_2)$, *for all* $(x_1,x_2) \in \mathscr{X}_1 \times \mathscr{X}_2$.

2) $\mathscr{E}^{\times}(k,l) = \{(x_1,x_2)(x_1,z_2) | x_1 \in \mathscr{X}_1, x_2, z_2 \in \mathscr{X}_2, \mathscr{N}^+(x_1)(k) \neq \phi, x_2z_2 \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_2(l))^*}\} \cup \{(x_1,x_2)(z_1,x_2) | x_1, z_1 \in \mathscr{X}_1, x_2 \in \mathscr{X}_2, \mathscr{N}^+(x_2)(l) \neq \phi, x_1z_1 \in \mathscr{E}_{C(\overrightarrow{\mathscr{G}}_1(k))^*}\}$.

3) $T_{\mathscr{R}(k,l)}((x_1,x_2)(x_1,z_2)) = [T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_2(l)}(x_2) \wedge T_{\mathscr{U}_2(l)}(z_2)] \times \vee_{a_1 \in \mathscr{X}_1, a_2 \in \mathscr{X}_2}\{T_{\overrightarrow{\mathscr{V}}_{1(k)}}(x_1a_1) \wedge T_{\overrightarrow{\mathscr{V}}_{2(l)}}(x_2a_2) \wedge T_{\overrightarrow{\mathscr{V}}_{2(l)}}(z_2a_2) | a_1 \in \mathscr{N}^+(x_1)(k)^*, a_2 \in \mathscr{N}^+(x_2)(l)^* \cap \mathscr{N}^+(z_2)(l)^*\}$, $(x_1,x_2)(x_1,z_2) \in \mathscr{E}^{\times}(k,l)$.

4) $I_{\mathscr{R}(k,l)}((x_1,x_2)(x_1,z_2)) = [I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_2(l)}(x_2) \wedge I_{\mathscr{U}_2(l)}(z_2)] \times \vee_{a_1 \in \mathscr{X}_1, a_2 \in \mathscr{X}_2}\{I_{\overrightarrow{\mathscr{V}}_{1(k)}}(x_1a_1) \wedge I_{\overrightarrow{\mathscr{V}}_{2(l)}}(x_2a_2) \wedge I_{\overrightarrow{\mathscr{V}}_{2(l)}}(z_2a_2) | a_1 \in \mathscr{N}^+(x_1)(k)^*, a_2 \in \mathscr{N}^+(x_2)(l)^* \cap \mathscr{N}^+(z_2)(l)^*\}$, $(x_1,x_2)(x_1,z_2) \in \mathscr{E}^{\times}(k,l)$.

5) $F_{\mathscr{R}(k,l)}((x_1,x_2)(x_1,z_2)) = [F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_2(l)}(x_2) \vee F_{\mathscr{U}_2(l)}(z_2)] \times \vee_{a_1 \in \mathscr{X}_1, a_2 \in \mathscr{X}_2}\{F_{\overrightarrow{\mathscr{V}}_{1(k)}}(x_1a_1) \vee F_{\overrightarrow{\mathscr{V}}_{2(l)}}(x_2a_2) \vee F_{\overrightarrow{\mathscr{V}}_{2(l)}}(z_2a_2) | a_1 \in \mathscr{N}^+(x_1)(k)^*, a_2 \in \mathscr{N}^+(x_2)(l)^* \cap \mathscr{N}^+(z_2)(l)^*\}$, $(x_1,x_2)(x_1,z_2) \in \mathscr{E}^{\times}(k,l)$.

6) $T_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,x_2)) = [T_{\mathscr{U}_1(k)}(x_1) \wedge T_{\mathscr{U}_1(k)}(z_1) \wedge T_{\mathscr{U}_2(l)}(x_2)] \times \vee_{a_1 \in \mathscr{X}_1, a_2 \in \mathscr{X}_2}\{T_{\overrightarrow{\mathscr{V}}_{1(k)}}(x_1a_1) \wedge T_{\overrightarrow{\mathscr{V}}_{1(k)}}(z_1a_1) \wedge T_{\overrightarrow{\mathscr{V}}_{2(l)}}(x_2a_2) | a_2 \in \mathscr{N}^+(x_2)(l)^*, a_1 \in \mathscr{N}^+(x_1)(k)^* \cap \mathscr{N}^+(z_1)(k)^*\}$, $(x_1,x_2)(z_1,x_2) \in \mathscr{E}^{\times}(k,l)$.

7) $I_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,x_2)) = [I_{\mathscr{U}_1(k)}(x_1) \wedge I_{\mathscr{U}_1(k)}(z_1) \wedge I_{\mathscr{U}_2(l)}(x_2)] \times \vee_{a_1 \in \mathscr{X}_1, a_2 \in \mathscr{X}_2}\{I_{\overrightarrow{\mathscr{V}}_{1(k)}}(x_1a_1) \wedge I_{\overrightarrow{\mathscr{V}}_{1(k)}}(z_1a_1) \wedge I_{\overrightarrow{\mathscr{V}}_{2(l)}}(x_2a_2) | a_2 \in \mathscr{N}^+(x_2)(l)^*, a_1 \in \mathscr{N}^+(x_1)(k)^* \cap \mathscr{N}^+(z_1)(k)^*\}$, $(x_1,x_2)(z_1,x_2) \in \mathscr{E}^{\times}(k,l)$.

8) $F_{\mathscr{R}(k,l)}((x_1,x_2)(z_1,x_2)) = [F_{\mathscr{U}_1(k)}(x_1) \vee F_{\mathscr{U}_1(k)}(z_1) \vee F_{\mathscr{U}_2(l)}(x_2)] \times \vee_{a_1 \in \mathscr{X}_1, a_2 \in \mathscr{X}_2}\{F_{\overrightarrow{\mathscr{V}}_{1(k)}}(x_1a_1) \vee F_{\overrightarrow{\mathscr{V}}_{1(k)}}(z_1a_1) \vee F_{\overrightarrow{\mathscr{V}}_{2(l)}}(x_2a_2) | a_2 \in \mathscr{N}^+(x_2)(l)^*, a_1 \in \mathscr{N}^+(x_1)(k)^* \cap \mathscr{N}^+(z_1)(k)^*\}$, $(x_1,x_2)(z_1,x_2) \in \mathscr{E}^{\times}(k,l)$.

Proof. This can be proved by using the similar arguments as in Theorem 3.7 [13].

Now we construct the SVNS competition graph of the direct product of SVNS digraphs from respective SVNS competition graphs of the SVNS digraphs in a theorem.

Example 2.5. Let $\overrightarrow{\mathscr{G}}_1 = (\mathscr{U}_1, \overrightarrow{\mathscr{V}}_1, \mathscr{K})$ and $\overrightarrow{\mathscr{G}}_2 = (\mathscr{U}_2, \overrightarrow{\mathscr{V}}_2, \mathscr{L})$ be two SVNS digraphs of the crisp digraphs $\overrightarrow{\mathscr{G}}^*_1 = (\mathscr{X}_1, \overrightarrow{\mathscr{E}}_1)$ and $\overrightarrow{\mathscr{G}}^*_2 = (\mathscr{X}_2, \overrightarrow{\mathscr{E}}_2)$ as shown in Figures 14 and 15, respectively.

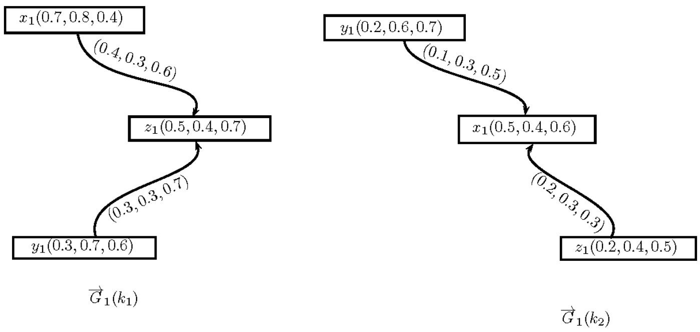

Figure 14. SVNS digraph $\overrightarrow{\mathscr{G}_1} = (\mathscr{U}_1, \overrightarrow{\mathscr{V}}_1, \mathscr{K})$.

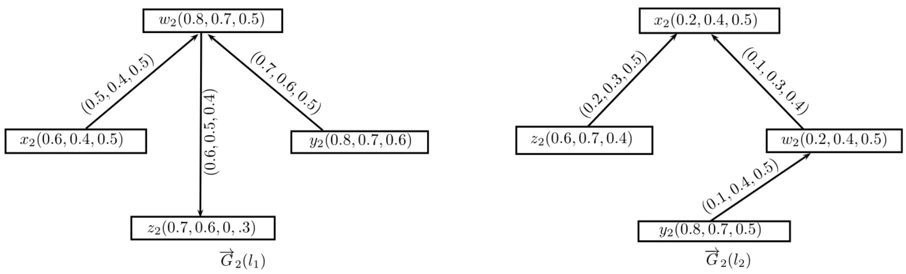

Figure 15. SVNS digraph $\overrightarrow{\mathscr{G}_2} = (\mathscr{U}_2, \overrightarrow{\mathscr{V}}_2, \mathscr{L})$.

The SVNS out neighborhoods of both $\overrightarrow{\mathscr{G}}_1$ and $\overrightarrow{\mathscr{G}}_2$ are given in Tables 11 and 12, respectively.

Table 11. SVNS out neighborhood of vertices in $\overrightarrow{\mathscr{G}}_1$.

$x \in \mathscr{X}_1$	$\mathscr{N}^+(x)(k_1)$	$\mathscr{N}^+(x)(k_2)$	$(\mathscr{N}^+(x), \mathscr{K})$
x_1	$\{z_1(0.4, 0.3, 0.6)\}$	ϕ	$\{\frac{z_1(0.4,0.3,0.6)}{k_1}, \frac{\phi}{k_2}\}$
y_1	$\{z_1(0.3, 0.3, 0.7)\}$	$\{x_1(0.1, 0.3, 0.5)\}$	$\{\frac{z_1(0.3,0.3,0.7)}{k_1}, \frac{x_1(0.1,0.3,0.5)}{k_2}\}$
z_1	ϕ	$\{x_1(0.2, 0.3, 0.3)\}$	$\{\frac{\phi}{k_1}, \frac{x_1(0.2,0.3,0.3)}{k_2}\}$

Table 12. SVNS out neighborhood of vertices in $\overrightarrow{\mathscr{G}}_2$.

$x \in \mathscr{X}_2$	$\mathcal{N}^+(x)(l_1)$	$\mathcal{N}^+(x)(l_2)$	$(\mathcal{N}^+(x), \mathscr{L})$
x_2	$\{w_2(0.5, 0.4, 0.5)\}$	ϕ	$\{\frac{w_2(0.5,0.4,0.5)}{l_1}, \frac{\phi}{l_2}\}$
y_2	$\{w_2(0.7, 0.6, 0.5)\}$	$\{w_2(0.1, 0.4, 0.5)\}$	$\{\frac{w_2(0.7,0.6,0.5)}{l_1}, \frac{(w_2,0.1,0.4,0.5)}{l_2}\}$
z_2	ϕ	$\{x_2(0.2, 0.3, 0.5)\}$	$\{\frac{\phi}{l_1}, \frac{x_2(0.2,0.3,0.5)}{l_2}\}$
w_2	$\{z_2(0.6, 0.5, 0.4)\}$	$\{x_2(0.1, 0.3, 0.3)\}$	$\{\frac{z_2(0.6,0.5,0.4)}{l_1}, \frac{x_2(0.1,0.3,0.4)}{l_2}\}$

The SVNS competition graphs $C(\overrightarrow{\mathscr{G}}_1)$ and $C(\overrightarrow{\mathscr{G}}_2)$ of $\overrightarrow{\mathscr{G}}_1$ and $\overrightarrow{\mathscr{G}}_2$ are specified in Figures 16 and 17, respectively.

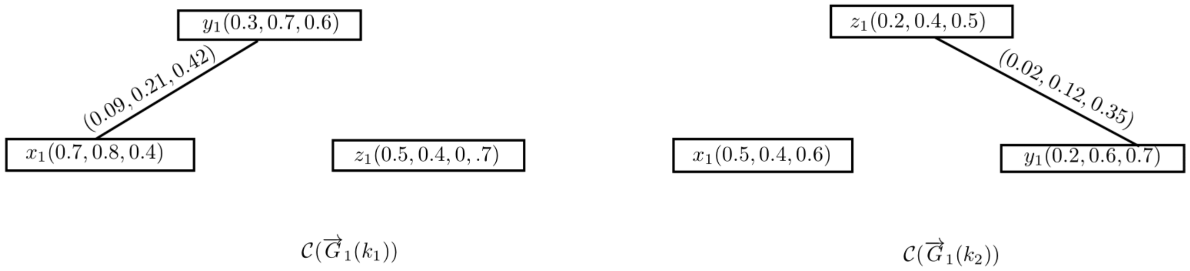

Figure 16. SVNS competition graph $C(\overrightarrow{\mathscr{G}}_1) = (\mathscr{U}_1, \mathscr{R}_1, \mathscr{K})$.

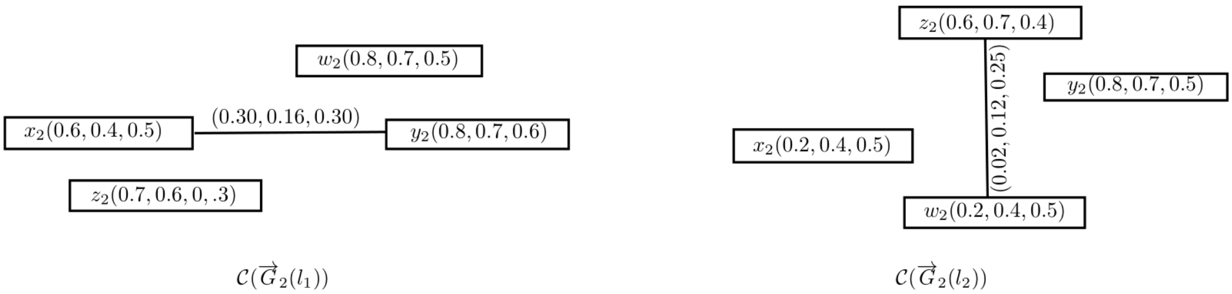

Figure 17. SVNS competition graph $C(\overrightarrow{\mathscr{G}}_2) = (\mathscr{U}_2, \mathscr{R}_2, \mathscr{L})$.

For $(k_1, l_1) \in \mathscr{K} \times \mathscr{L}$, we will show that $C(\overrightarrow{\mathscr{G}}_1(k_1) \times \overrightarrow{\mathscr{G}}_2(l_1)) = [C(\overrightarrow{\mathscr{G}}_1(k_1)) \times C(\overrightarrow{\mathscr{G}}_2(l_1))] \cup \mathscr{G}^{\times}(k_1, l_1)$, where $\mathscr{G}^{\times}(k_1, l_1)$ is a SVNS digraph on $(\mathscr{X}_1 \times \mathscr{X}_2, \mathscr{E}^{\times}(k_1, l_1))$. We construct the edge set $\mathscr{E}^{\times}(k_1, l_1)$ by using condition (2) in Theorem 2.5.

$$\mathscr{E}^{\times}(k_1, l_1) = \{(x_1, x_2)(x_1, y_2), (y_1, x_2)(y_1, y_2), (x_1, w_2)(y_1, w_2), (x_1, x_2)(y_1, x_2), (x_1, y_2)(y_1, y_2)\}.$$

Using conditions (3) and (8), the degrees of truth membership, indeterminacy membership, and falsity membership of all the edges from $\mathscr{E}^{\times}(k_1, l_1)$ are calculated in Table 13.

Table 13. Adjacency edges of $[C(\overrightarrow{\mathscr{G}}_1(k_1)) \times C(\overrightarrow{\mathscr{G}}_2(l_1))] \cup \mathscr{G}^{\times}(k_1, l_1)$.

$(a_1, a_2)(b_1, b_2)$	$\mathscr{R}(k_1, l_1)((a_1, a_2)(b_1, b_2))$
$(x_1, x_2)(x_1, y_2)$	$(0.24, 0.12, 0.36)$
$(x_1, x_2)(y_1, x_2)$	$(0.09, 0.12, 0.42)$
$(y_1, x_2)(y_1, y_2)$	$(0.09, 0.12, 0.42)$
$(x_1, y_2)(y_1, y_2)$	$(0.09, 0.21, 0.42)$
$(x_1, w_2)(y_1, w_2)$	$(0.09, 0.21, 0.42)$

The SVN graph obtained using Theorem 2.5 is shown in Figure 18. The dashed lines represent the part of SVN graph obtained from $C(\overrightarrow{\mathscr{G}}_1(k_1)) \times C(\overrightarrow{\mathscr{G}}_2(l_1))$ and solid lines indicate the part of SVN graph obtained from $\mathscr{G}^{\times}(k_1, l_1)$. The direct product of $\overrightarrow{\mathscr{G}}_1(k_1)$ and $\overrightarrow{\mathscr{G}}_2(l_1)$ is presented in Figure 19 and its

SVN competition graph in Figure 20 which is similar as Figure 18. By repeating the same procedure, the remaining SVN graphs corresponding to each parameter can be drawn.

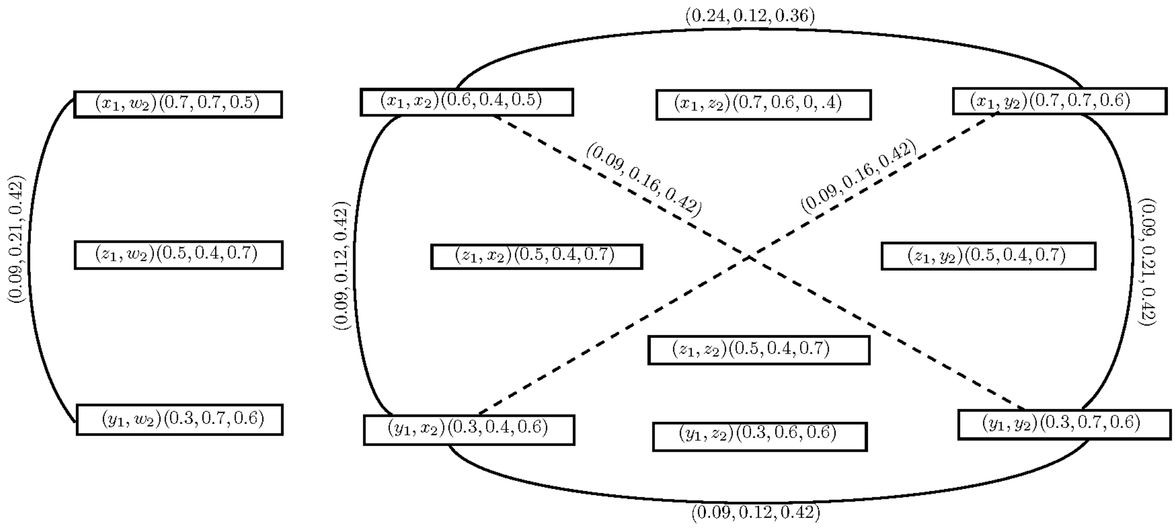

Figure 18. $[C(\overrightarrow{\mathscr{G}}_1(k_1)) \times C(\overrightarrow{\mathscr{G}}_2(l_1))] \cup \mathscr{G}^{\times}(k_1, l_1)$.

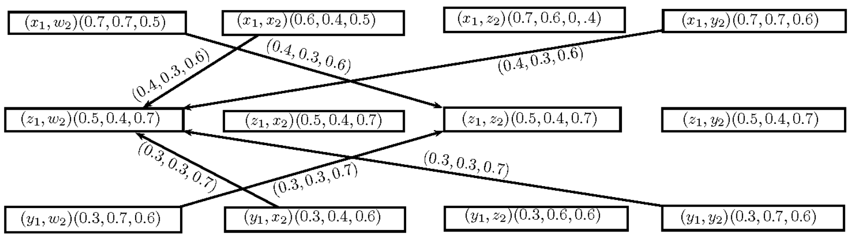

Figure 19. $\overrightarrow{\mathscr{G}}_1(k_1) \times \overrightarrow{\mathscr{G}}_2(l_1)$.

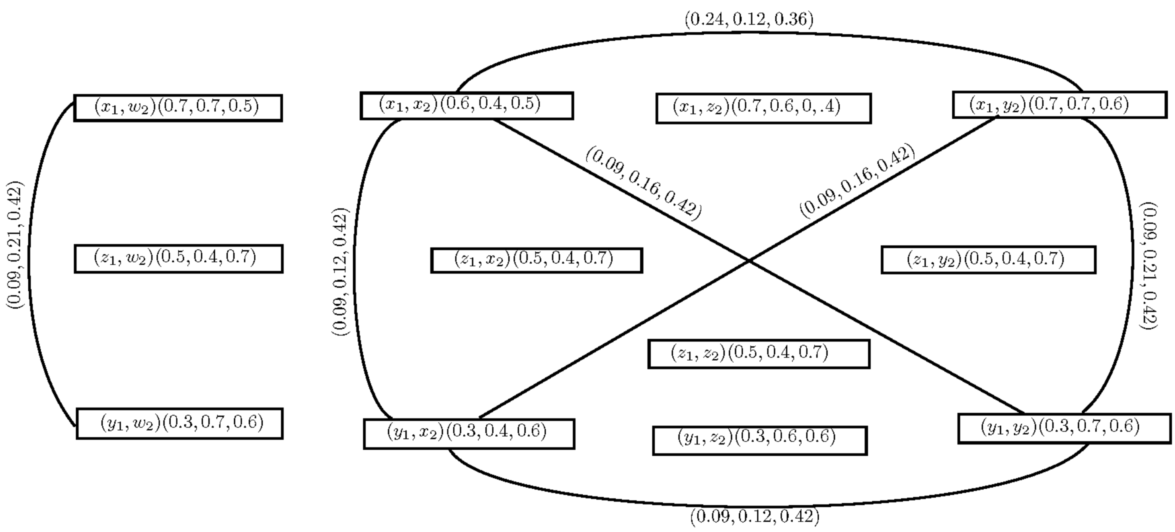

Figure 20. $C(\overrightarrow{\mathscr{G}}_1(k_1) \times \overrightarrow{\mathscr{G}}_2(l_1))$.

3. Decision-making approach

Competition graph theory play a vital role to present the competition among different objects. But the existing competition graphical models in literature do not have the ability to examine all real-world competitions among objects in the presence of parametrization. Following we present an application of SVNS competition graph to examine the strength of job competition among candidates.

Consider James, Henry, David, Charlotte, Olivia, and Casper are five candidates that are competing for different jobs including nutritionist, dentist, and psychologist. Each candidate competes with one another on the basis of their qualities, e.g., honesty, adaptable, and confidence to get a job. To study the strength of job competition among candidates corresponding to the above-mentioned qualities that act as parameters, we use SVNS competition graph. Following Algorithm 3.1 tell us the method to calculate the strength of job competition among candidates in the presence of above parameters k_1, k_2, and k_3, where k_1 represents honesty, k_2 represents the adaptable, and k_3 represents the confidence.

Algorithm 3.1. *Method to evaluate the strength of job competition among candidates*

1. Input the most suitable set of parameters $\mathscr{K}$ on n candidates according to their r jobs.
2. Define the SVNS set $(\mathscr{U}, \mathscr{K})$ on n candidates and r jobs, also define SVNS relation $(\overrightarrow{\mathscr{V}}, \mathscr{K})$ between n candidates and r jobs corresponding to each parameter.
3. Give a graphical representation of the above information, we get a SVNS digraph $\overrightarrow{\mathscr{G}} = (\mathscr{U}, \overrightarrow{\mathscr{V}}, \mathscr{K})$.
4. Calculate the SVNS out neighborhood of each candidate.
5. Construct the SVNS competition graph $C(\overrightarrow{\mathscr{G}}) = (\mathscr{U}, \mathscr{R}, \mathscr{K})$ by using Definition 2.10.
6. Compute the resultant SVN graph $\mathscr{G}(k) = \bigcap_i \mathscr{G}(k_i)$ for $k = \bigwedge_i k_i \ \forall i$.
7. If $x, c_1, c_2, c_3, \cdots, c_n$ are the candidates that are competing for job a, then the strength of competition $\mathscr{W}(x, a) = (T_{\mathscr{W}}(x, a), I_{\mathscr{W}}(x, a), F_{\mathscr{W}}(x, a))$ of each candidate x for a particular job a is:

$$\begin{aligned} T_{\mathscr{W}}(x,a) &= \frac{T_{\mathscr{V}(k)}(xc_1) + T_{\mathscr{V}(k)}(xc_2) + \cdots + T_{\mathscr{V}(k)}(xc_n)}{n}, \\ I_{\mathscr{W}}(x,a) &= \frac{I_{\mathscr{V}(k)}(xc_1) + I_{\mathscr{V}(k)}(xc_2) + \cdots + I_{\mathscr{V}(k)}(xc_n)}{n}, \\ F_{\mathscr{W}}(x,a) &= \frac{F_{\mathscr{V}(k)}(xc_1) + F_{\mathscr{V}(k)}(xc_2) + \cdots + F_{\mathscr{V}(k)}(xc_n)}{n}. \end{aligned}$$

Evaluate $\mathscr{S}(x, a)$ the strength of competition of each applicant x that are competing for a particular job a by utilizing the following formula:

$$\mathscr{S}(x,a) = T_{\mathscr{W}}(x,a) - (I_{\mathscr{W}}(x,a) + F_{\mathscr{W}}(x,a)) + 1.$$

Graphical model of candidates that are competing for above-mentioned jobs with respect to considered three parameters is shown in Figure 21.

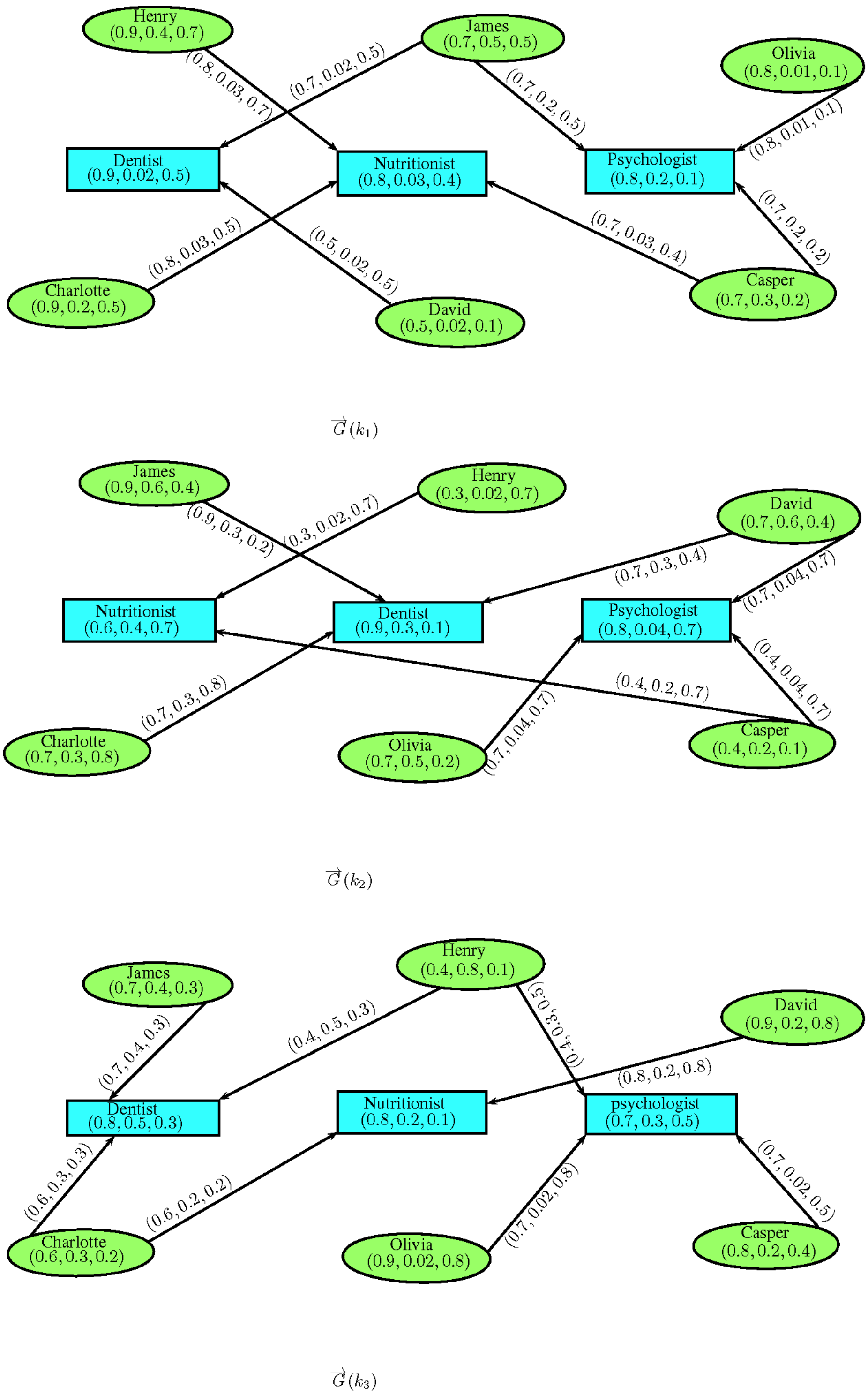

Figure 21. SVNS digraph $\vec{\mathscr{G}} = \{\vec{\mathscr{G}}(k_1), \vec{\mathscr{G}}(k_2), \vec{\mathscr{G}}(k_3)\}$.

The truth membership grade of each candidate corresponding to parameters k_1, k_2, and k_3 indicates the degree of honesty, adaptable, and confidence to get a job. The indeterminacy membership grade expresses the indeterminate state of honesty, adaptable, and confidence. The false membership grade shows the degree of dishonesty, unadaptable, and uncertain towards their careers. For instance, the membership grades of Henry corresponding to a parameter k_1 are $(0.9, 0.4, 0.7)$ which represents that Henry is 90% honest, 40% is indeterminate under the specific conditions, and 70% dishonest toward their career.

The membership grades of each job represent the honesty, adaptable, and confidence level of candidates after getting that particular job. For instance, the membership values of psychologist corresponding to a parameter k_1 are $(0.8, 0.2, 0.1)$ which represent that candidates which are competing for this job 80% honest, 20% indeterminant, and 10% dishonest if they succeed to get this job.

The truth membership grade of each directed edge between a candidate and a job corresponding to parameters k_1, k_2, and k_3 indicates the eligibility for that job, the indeterminate value depicts the indeterminate state of that job, and the false membership grade express the non-eligibility for that particular career. For example, the directed edge between James and psychologist corresponding to a parameter k_1 has a membership grades $(0.7, 0.2, 0.5)$ which manifest that James is 70% eligible for psychologist, 20% is indeterminate either James is eligible for this job or not, and 50% is ineligible on the basis of their honesty for this job.

The SVNS out neighborhood of each candidate is given in Table 14.

Table 14. SVNS out neighborhood of each candidate.

Candidates	$\mathscr{N}^+$(Candidates)
Henry	$\{\frac{Nutritionist(0.8,0.03,0.7)}{k_1}, \frac{Nutritionist(0.3,0.02,0.7)}{k_2}, \frac{Dentist(0.4,0.5,0.3), Psychologist(0.4,0.3,0.5)}{k_3}\}$
James	$\{\frac{Dentist(0.7,0.02,0.5), Psychologist(0.7,0.2,0.5)}{k_1}, \frac{Dentist(0.9,0.3,0.2)}{k_2}, \frac{Dentist(0.7,0.4,0.3)}{k_3}\}$
Olivia	$\{\frac{Psychologist(0.8,0.01,0.1)}{k_1}, \frac{Psychologist(0.7,0.04,0.7)}{k_2}, \frac{Psychologist(0.7,0.02,0.8)}{k_3}\}$
Charlotte	$\{\frac{Nutritionist(0.8,0.03,0.5)}{k_1}, \frac{Dentist(0.7,0.3,0.8)}{k_2}, \frac{Dentist(0.6,0.3,0.3), Nutritionist(0.6,0.2,0.2)}{k_3}\}$
David	$\{\frac{Dentist(0.5,0.02,0.5)}{k_1}, \frac{Dentist(0.7,0.3,0.4), Psychologist(0.7,0.04,0.7)}{k_2}, \frac{Nutritionist(0.8,0.2,0.8)}{k_3}\}$
Casper	$\{\frac{Nutritionist(0.7,0.03,0.4), Psychologist(0.7,0.2,0.2)}{k_1}, \frac{Nutritionist(0.4,0.2,0.7), Psychologist(0.4,0.04,0.7)}{k_2}, \frac{Psychologist(0.7,0.02,0.5)}{k_3}\}$

The SVNS competition graph of a SVNS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k_1), \overrightarrow{\mathscr{G}}(k_2), \overrightarrow{\mathscr{G}}(k_3)\}$ is given in Figure 22.

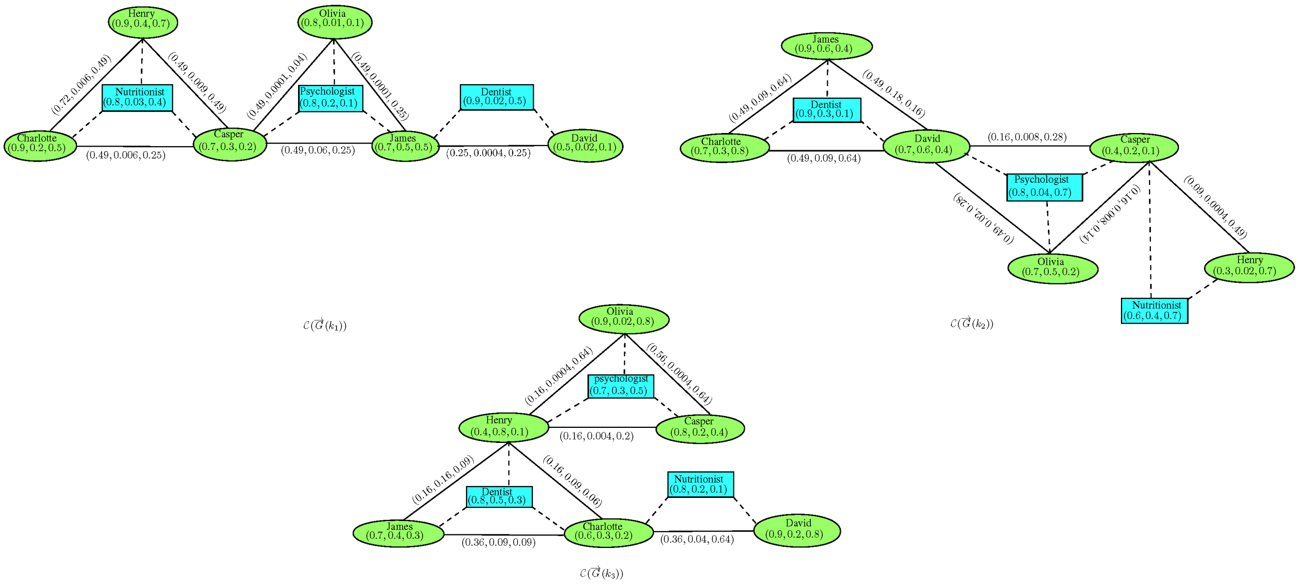

Figure 22. SVNS competition graph $C(\overrightarrow{\mathscr{G}}) = (C(\overrightarrow{\mathscr{G}}(k_1)), C(\overrightarrow{\mathscr{G}}(k_2)), C(\overrightarrow{\mathscr{G}}(k_3)))$.

The solid lines represent the strength of competition among each candidate and dashed lines express the candidates that are competing for the particular job. For instance, Casper, Olivia, and James are competing for the job psychologist and the truth membership value of each edge indicates the degree of eligibility of both candidates for the job psychologist. After performing some operations (AND or OR); we obtain the resultant SVN graph $\mathscr{G}(k) = (\mathscr{U}(k), \mathscr{V}(k))$ as given in Figure 23, where $k = k_1 \wedge k_2 \wedge k_3$.

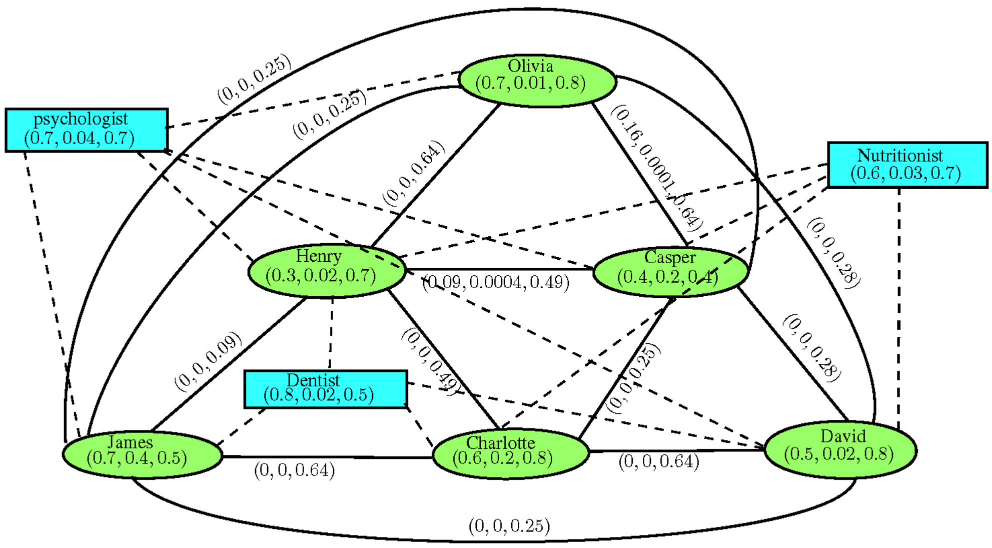

Figure 23. SVN graph $\mathscr{G}(k) = (\mathscr{U}(k), \mathscr{V}(k))$.

We evaluate the strength of competition of each candidate by using Algorithm 3.1. The strength of job competition among candidates with respect to a particular job is evaluated in Table 15. In Table 15, $\mathscr{W}(x, a)$ express the competition of candidate x for job a.

Table 15. Strength of job competition among candidates.

(Candidates, Job)	In competition	$\mathscr{W}(x,a)$	$\mathscr{S}(x,a)$
(James, Dentist)	(Henry, David, Charlotte)	$(0, 0, 0.326)$	0.674
(Henry, Dentist)	(James, David, Charlotte)	$(0, 0, 0.193)$	0.807
(David, Dentist)	(James, Henry, Charlotte)	$(0, 0, 0.296)$	0.704
(Charlotte, Dentist)	(James, Henry, David)	$(0, 0, 0.59)$	0.41
(Henry, Nutritionist)	(Casper, Charlotte, David)	$(0.03, 0.0001, 0.326)$	0.703
(Casper, Nutritionist)	(Henry, Charlotte, David)	$(0.03, 0.0001, 0.34)$	0.689
(David, Nutritionist)	(Henry, Charlotte, Casper)	$(0, 0, 0.306)$	0.694
(Charlotte, Nutritionist)	(Henry, Casper, David)	$(0, 0, 0.46)$	0.54
(James, Psychologist)	(Henry, David, Casper, Olivia)	$(0, 0, 0.21)$	0.79
(Henry, Psychologist)	(James, David, Casper, Olivia)	$(0.022, 0.0001, 0.305)$	0.716
(David, Psychologist)	(James, Henry, Casper, Olivia)	$(0, 0, 0.202)$	0.798
(Casper, Psychologist)	(James, Henry, David, Olivia)	$(0.062, 0.0001, 0.415)$	0.646
(Olivia, Psychologist)	(James, Henry, David, Casper)	$(0.04, 0.00002, 0.452)$	0.587

From Table 15, we concluded that Henry has highest number of chances to be selected for the job of Dentist, Henry has higest number of chances to be selected for the job of Nutritionist, and David has highest number of chances to be selected for the job of Psychologist. The graphical view of these consequences is given in Figure 24.

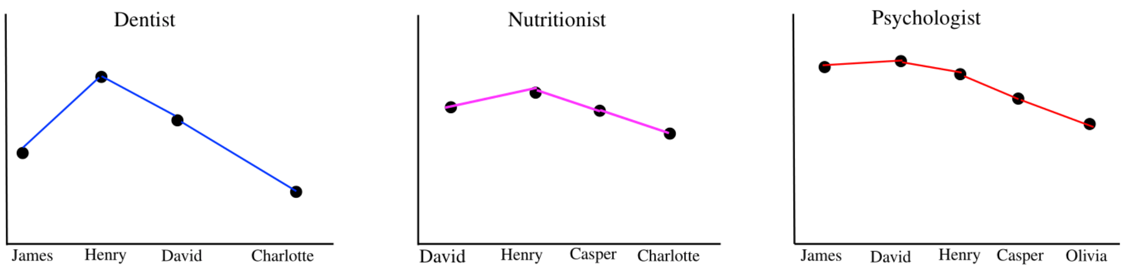

Figure 24. Graphical representation of strength of job competition among candidates.

4. Discussions and comparison analysis

The proposed method SVNS competition graph is more efficient and appropriate to manifest the competition in decision support network as compared to existing approaches. In this section, we verify the effectiveness of our proposed technique with present frameworks.

4.1. Comparison with fuzzy soft competition graphs

The energetic technique of FS competition graphs was defined and illustrated by Nawaz and Akram [31] that tackles competitive relationships among the objects in the presence of parametrization. To puzzle out the above decision-making problem with this model, we treat the above SVNS digraph as FS digraph in which the uncertainty was discussed only in one direction. The FS digraph which depicts the correlation between candidates and jobs is specified in Figure 25.

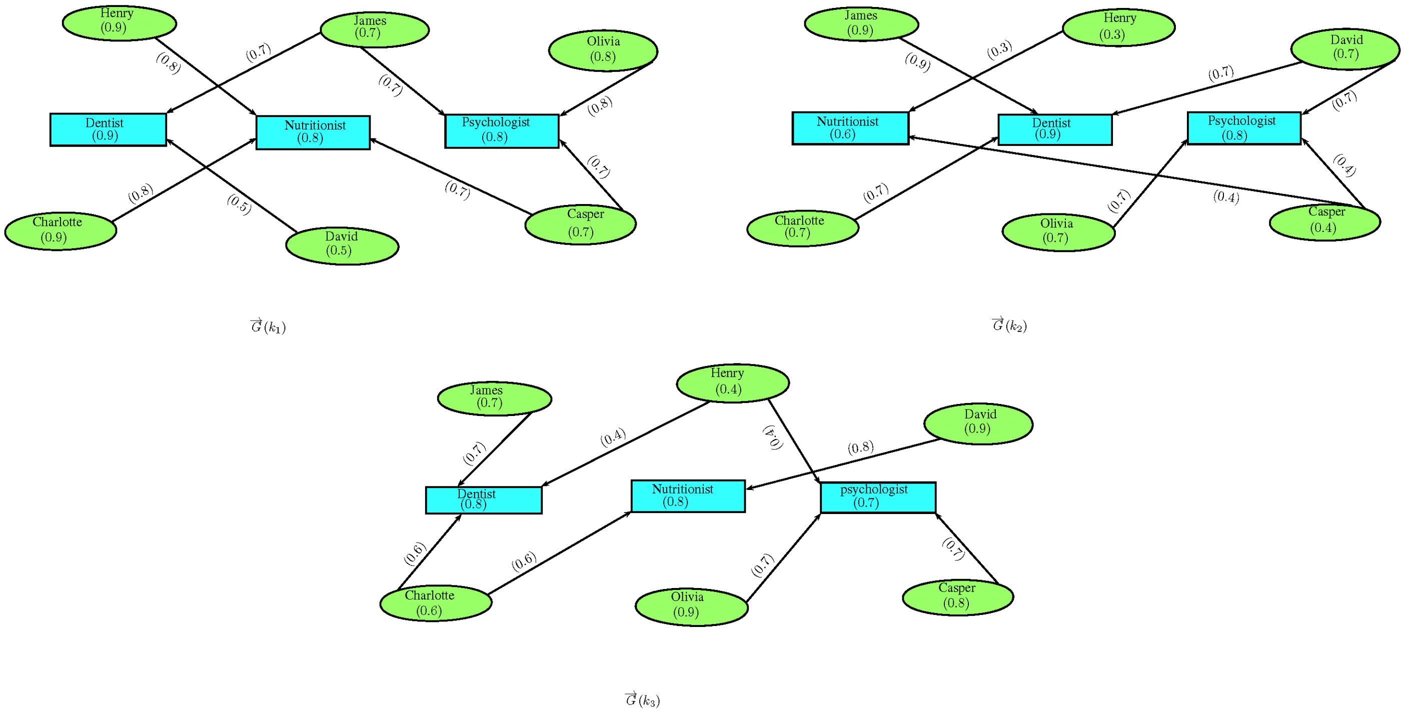

Figure 25. FS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k_1), \overrightarrow{\mathscr{G}}(k_2), \overrightarrow{\mathscr{G}}(k_3)\}$.

The membership grades corresponding to parameters k_1, k_2, and k_3 indicate the degree of honesty to each candidate. The membership grades of each job represent the honesty of candidates after getting that particular job. The membership grades of each directed edge between a candidate and a job corresponding to parameters k_1, k_2, and k_3 indicate the eligibility for that particular career.

The FS out neighborhood of each candidate is given in Table 16.

Table 16. FS out neighborhood of each candidate.

Candidates	$\mathscr{N}^+$(Candidates)
Henry	$\{\frac{(Nutritionist,0.8)}{k_1}, \frac{(Nutritionist,0.3)}{k_2}, \frac{(Dentist,0.4),(Psychologist,0.4)}{k_3}\}$
James	$\{\frac{(Dentist,0.7),(Psychologist,0.7)}{k_1}, \frac{(Dentist,0.9)}{k_2}, \frac{(Dentist,0.7)}{k_3}\}$
Olivia	$\{\frac{(Psychologist,0.8)}{k_1}, \frac{(Psychologist,0.7)}{k_2}, \frac{(Psychologist,0.7)}{k_3}\}$
Charlotte	$\{\frac{(Nutritionist,0.8)}{k_1}, \frac{(Dentist,0.7)}{k_2}, \frac{(Dentist,0.6),(Nutritionist,0.6)}{k_3}\}$
David	$\{\frac{(Dentist,0.5)}{k_1}, \frac{(Dentist,0.7),(Psychologist,0.7)}{k_2}, \frac{(Nutritionist,0.8)}{k_3}\}$
Casper	$\{\frac{(Nutritionist,0.7),(Psychologist,0.7)}{k_1}, \frac{(Nutritionist,0.4),(Psychologist,0.4)}{k_2}, \frac{(Psychologist,0.7)}{k_3}\}$

The FS competition graph of FS digraph $\overrightarrow{\mathscr{G}} = \{\overrightarrow{\mathscr{G}}(k_1), \overrightarrow{\mathscr{G}}(k_2), \overrightarrow{\mathscr{G}}(k_3)\}$ is given in Figure 26.

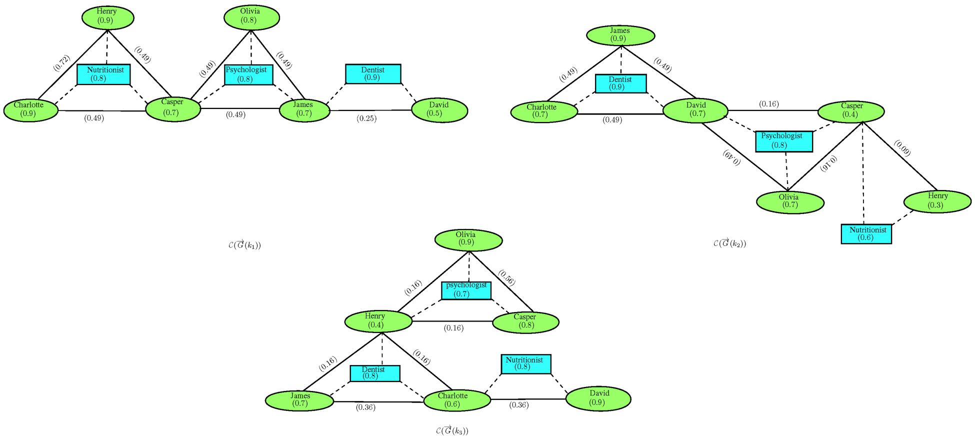

Figure 26. FS competition graph $C(\overrightarrow{\mathscr{G}}) = (C(\overrightarrow{\mathscr{G}}(k_1)), C(\overrightarrow{\mathscr{G}}(k_2)), C(\overrightarrow{\mathscr{G}}(k_3)))$.

The resultant FS graph $\mathscr{G}(k) = (\mathscr{U}(k), \mathscr{V}(k))$ is given in Figure 27, where $k = k_1 \wedge k_2 \wedge k_3$.

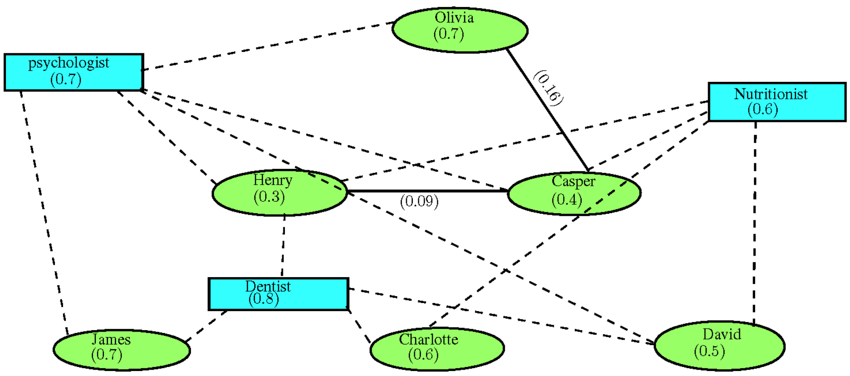

Figure 27. FS graph $\mathscr{G}(k)$.

If $x, c_1, c_2, c_3, \cdots, c_n$ are the candidates that are competing for job a, then the strength of job competition can be evaluated by the following formula:

$$\mathscr{Z}(x, a) = \frac{\mu_{\mathscr{V}(k)}(xc_1) + \mu_{\mathscr{V}(k)}(xc_2) + \cdots + \mu_{\mathscr{V}(k)}(xc_n)}{n}.$$

The strength of job competition among candidates with respect to a particular job is evaluated in Table 17.

From Table 17, we concluded that Henry and Casper have equal number of chances to be selected for the job of Nutritionist, and Casper has highest number of chances to be selected for the job of Psychologist. The graphical view of these consequences is given in Figure 28.

Table 17. Strength of job competition among candidates.

(Candidates, Job)	In competition	$\mathscr{X}(x,a)$
(James, Dentist)	(Henry, David, Charlotte)	0
(Henry, Dentist)	(James, David, Charlotte)	0
(David, Dentist)	(James, Henry, Charlotte)	0
(Charlotte, Dentist)	(James, Henry, David)	0
(Henry, Nutritionist)	(Casper, Charlotte)	0.03
(Casper, Nutritionist)	(Henry, Charlotte)	0.03
(David, Nutritionist)	(Henry, Charlotte, Casper)	0
(Charlotte, Nutritionist)	(Henry, Casper)	0
(James, Psychologist)	(Henry, David, Casper, Olivia)	0
(Henry, Psychologist)	(James, David, Casper, Olivia)	0.022
(David, Psychologist)	(James, Henry, Casper, Olivia)	0
(Casper, Psychologist)	(James, Henry, David, Olivia)	0.062
(Olivia, Psychologist)	(James, Henry, David, Casper)	0.04

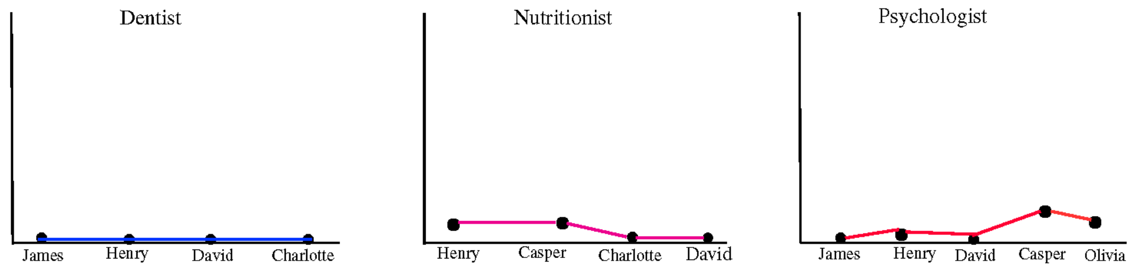

Figure 28. Graphical representation of strength of job competition among candidates.

The theoretical differences of these two approaches are given in Table 18.

Table 18. Theoretical comparison of two different framework.

Proposed technique	Existing technique
1- Neutrosophic occurs when a piece of information contains ambiguities in three different directions.	Fuzziness arises when a piece of information contains uncertainty in one direction only.
2- Give information of an object about truth, falsity, and indeterminant state at a time.	Due to limitation, it tells the vagueness of an object either truth, falsity, or indeterminant state.
3- Give more information in competing networks due to to the presence of three membership values.	Due to the absence of membership values, this model lacks a lot of important facts.
4- This model generalizes the existing techniques including crisp, fuzziness, and bipolarity based theories.	Only generalize the crisp theory fails in other cases.
5- We discuss the correspondence, association, or relation among objects in the presence of truth, falsity, or indeterminant state.	Fail to tell the relations, conflicts, and influences among objects in the presence of truth, falsity, or indeterminant state.

The strength of job competition of SVNS competition graph and the evaluation of strength which is obtained from existing technique are given in Table 19. Graphical views of these numerical consequences are given in Figure 29.

Table 19. Comparison of numerical values with two different techniques.

(Candidates, Job)	In competition	$\mathscr{S}(x,a)$	$\mathscr{Z}(x,a)$
(James, Dentist)	(Henry, David, Charlotte)	0.674	0
(Henry, Dentist)	(James, David, Charlotte)	0.807	0
(David, Dentist)	(James, Henry, Charlotte)	0.704	0
(Charlotte, Dentist)	(James, Henry, David)	0.41	0
(Henry, Nutritionist)	(Casper, Charlotte)	0.703	0.03
(Casper, Nutritionist)	(Henry, Charlotte)	0.689	0.03
(David, Nutritionist)	(Henry, Charlotte, Casper)	0.694	0
(Charlotte, Nutritionist)	(Henry, Casper)	0.54	0
(James, Psychologist)	(Henry, David, Casper, Olivia)	0.79	0
(Henry, Psychologist)	(James, David, Casper, Olivia)	0.716	0.022
(David, Psychologist)	(James, Henry, Casper, Olivia)	0.798	0
(Casper, Psychologist)	(James, Henry, David, Olivia)	0.646	0.062
(Olivia, Psychologist)	(James, Henry, David, Casper)	0.587	0.04

It is observed from Table 19 that Henry has highest number of chances for the Dentist job selection. While on the other way, the results that we estimate from existing model indicate that Henry has no chance for the selection of this job. Due to the absence of indeterminacy and falsity membership grades in present model, we obtained huge fluctuations and get not more accurate and satisfied consequences from existing technique. To tackle truth, indeterminacy as well as falsity membership grade the proposed framework give a better illustration of competition in the presence of parameterizations. This proposed technique investigates the most common yet significant problem of competition among the objects and provides more explicit and error free consequences. Furthermore, this model also facilitates to analyze the data when the researchers and decision-makers assign the SVN membership grades to elements. Thus, any SVNS theory permits for a wider range of real-life applications than the corresponding FS-based theory. SVNS texture possess better traits than FS structure. Graphical view of proposed and existing techniques is given in Figure 29.

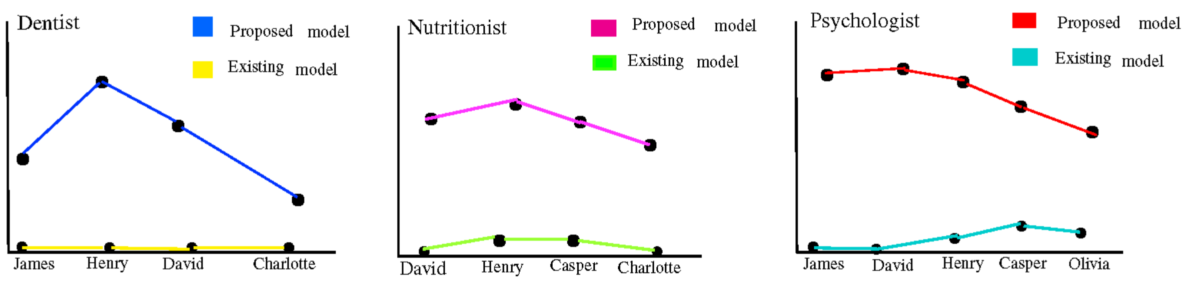

Figure 29. Graphical representation of proposed and existing technique.

4.2. Comparison with single-valued neutrosophic competition graphs

The SVN competition graph theory studied and introduced by Akram and Siddique [22] that is very generous in order to cope with competing communications of actual world riddles. In this framework, the uncertainties and haziness are discussed in three paths which yields very beneficial facts and information. Due to absence of parametric tools, we consider the uncertain relations between candidates and jobs in only one way either we discuss the competitive relations of the above solved example with respect to honest, or adaptable, or confidence level of the candidates. In case of SVN competition graph, we cannot discuss the correspondence, association, or relation among objects in the presence of parameters. Due to the absence of parametrization tools, this model lacks a lot of important facts. In proposed method, we sort out reality-based problems in the presence of parameters as per requirement. So, our proposed model generalize this existing model.

5. Conclusions and future directions

Neutrosophic soft sets theory is a rational approach to deal with vagueness and approximate thinking in the presence of parameterizations and it helps to solve many mathematical problems to minimize uncertainty and also used in decision support system. Neutrosophic soft models provide more accuracy, elasticity, and consistency to the system as compared to the fuzzy and bipolar models. We have introduced the notion of SVNS competition graphs to study the competition among objects with respect to different parameters. We have discussed the generalization of SVNS competition graph as SVNS $\boldsymbol{k}$-competition graph and $\boldsymbol{p}$-competition SVNS graph. Various results related to strong edges of all these graphs have also been presented. We have discussed the importance of SVNS competition graph with an application as job competition among candidates. We are planning to prolong our research work to (i) the exploration of the HyperSoft Set, (ii) IndetermSoft Set, (iii) SuperHyperGraph, embedded with the neutrosophic set.

Acknowledgments

The research of Santos-García was funded by the Spanish project ProCode-UCM (PID2019-108528RB-C22) from the Ministerio de Ciencia e Innovación.

Conflict of interest

The authors declare no conflict of interest.

References

1. J. E. Cohen, Interval graphs and food webs: a finding and a problem, *Document 17696-PR, RAND Corporation, Santa Monica*, 1968.
2. J. R. Lundgren, Food webs, competition graphs, competition-common enemy graphs, and niche graphs, in *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, (1989), 221–243. https://doi.org/10.1007/978-1-4684-6381-1_9

3. S. R. Kim, T. A. McKee, F. R. McMorris, F. S. Roberts, p-Competition graphs, *Linear Algebra Appl.*, **217** (1995), 167–178. https://doi.org/10.1016/0024-3795(94)00060-Q
4. R. C. Brigham, F. R. McMorris, R. P. Vitray, Tolerance competition graphs, *Linear Algebra Appl.*, **217** (1995), 41–52. https://doi.org/10.1016/0024-3795(94)00059-M
5. H. H. Cho, S. R. Kim, Y. Nam, The m-step competition graph of a digraph, *Discrete Appl. Math.*, **105** (2000), 115–127. https://doi.org/10.1016/S0166-218X(00)00214-6
6. L. A. Zadeh, Fuzzy sets, *Inf. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
7. A. Kaufmann, Introduction à la théorie des sous-ensembles flous: À l'usage des ingénieurs (Fuzzy Sets Theory), Tome III, Paris, French, 1975.
8. L. A. Zadeh, Similarity relations and fuzzy orderings, *Inf. Sci.*, **3** (1971), 177–200. https://doi.org/10.1016/S0020-0255(71)80005-1
9. A. Rosenfeld, Fuzzy graphs, in *Fuzzy Sets and their Applications to Cognitive and Decision Processes*, Academic Press, (1975), 77–95. https://doi.org/10.1016/B978-0-12-775260-0.50008-6
10. P. Bhattacharya, Some remarks on fuzzy graphs, *Pattern Recognit. Lett.*, **6** (1987), 297–302. https://doi.org/10.1016/0167-8655(87)90012-2
11. J. N. Mordeson, P. S. Nair, *Fuzzy Graphs and Fuzzy Hhypergraphs*, Physica, Springer Verlag, Heidelberg, 2012.
12. S. Samanta, M. Pal, Fuzzy k-competition graphs and p-competition fuzzy graphs, *Fuzzy Inf. Eng.*, **5** (2013), 191–204. https://doi.org/10.1007/s12543-013-0140-6
13. M. Sarwar, M. Akram, Novel concepts of bipolar fuzzy competition graphs, *J. Appl. Math. Comput.*, **54** (2017), 511–547. https://doi.org/10.1007/s12190-016-1021-z
14. R. Sahu, S. R. Dash, S. Das, Career selection of students using hybridized distance measure based on picture fuzzy set and rough set theory, *Decis. Mak. Appl. Manag. Eng.*, **4** (2021), 104–126. https://doi.org/10.31181/dmame2104104s
15. A. Ashraf, K. Ullah, A. Hussain, M. Bari, Interval-valued picture fuzzy Maclaurin symmetric mean operator with application in multiple attribute decision-making, *Rep. Mech. Eng.*, **3** (2022), 301–317. https://doi.org/10.31181/rme20020042022a
16. S. Shahzadi, A. Rasool, M. Sarwar, M. Akram, A framework of decision making based on bipolar fuzzy competition hypergraphs, *J. Intell. Fuzzy Syst.*, **41** (2021), 1319–1339. https://doi.org/10.3233/JIFS-210216
17. K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets Syst.*, **20** (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3
18. S. Sahoo, M. Pal, Intuitionistic fuzzy competition graphs, *J. Appl. Math. Comput.*, **52** (2016), 37–57. https://doi.org/10.1007/s12190-015-0928-0
19. F. Smarandache, *Neutrosophy, Neutrosophic Probability, Set and Logic*, American Research Press, Rehoboth, USA, **105** (1998).
20. M. Akram, G. Shahzadi, Operations on single-valued neutrosophic graphs, *J. Uncertain Syst.*, **11** (2017), 1–26.

21. M. Akram, A. Luqman, Certain network models using single-valued neutrosophic directed hypergraphs, *J. Intell. Fuzzy Syst.*, **33** (2017), 575–588. https://doi.org/10.3233/JIFS-162347
22. M. Akram, S. Siddique, Neutrosophic competition graphs with applications, *J. Intell. Fuzzy Syst.*, **33** (2017), 921–935. https://doi.org/10.3233/JIFS-162207
23. C. Karamasa, E. Demir, S. Memis, S. Korucuk, Weighting the factors affecting logistics outsourcing, *Decis. Mak. Appl. Manag. Eng.*, **4** (2020), 19–32.
24. D. Molodtsov, Soft set theory-first results, *Comput. Math. Appl.*, **37** (1999), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5
25. P. K. Maji, Neutrosophic soft set, *Ann. Fuzzy Math. Inf.*, **5** (2013), 157–168.
26. B. Ahmad, A. Kharal, On fuzzy soft sets, *Adv. Fuzzy Syst.*, **2009** (2009), 1–6. https://doi.org/10.1155/2009/586507
27. F. Feng, C. Li, B. Davvaz, M. I. Ali, Soft sets combined with fuzzy sets and rough sets: A tentative approach, *Soft Comput.*, **14** (2010), 899–911. https://doi.org/10.1007/s00500-009-0465-6
28. I. Deli, S. Broumi, Neutrosophic soft matrices and NSM-decision making, *J. Intell. Fuzzy Syst.*, **28** (2015), 2233–2241. https://doi.org/10.3233/IFS-141505
29. I. Deli, S. Broumi, Neutrosophic soft relations and some properties, *Ann. Fuzzy Math. Inf.*, **9** (2015), 169–182.
30. M. Akram, S. Shahzadi, A. Rasool, M. Sarwar, Decision-making methods based on fuzzy soft competition hypergraphs, *Complex Intell. Syst.*, **8** (2022), 2325–2348. https://doi.org/10.1007/s40747-022-00646-4
31. H. S. Nawaz, M. Akram, Oligopolistic competition among the wireless internet service providers of Malaysia using fuzzy soft graphs, *J. Appl. Math. Comput.*, **67** (2021), 855–890. https://doi.org/10.1007/s12190-021-01514-z
32. H. S. Nawaz, M. Akram, J. C. R. Alcantud, An algorithm to compute the strength of competing interactions in the Bering Sea based on Pythagorean fuzzy hypergraphs, *Neural Comput. Appl.*, **34** (2022), 1099–1121. https://doi.org/10.1007/s00521-021-06414-8
33. M. Akram, S. Shahzadi, Neutrosophic soft graphs with application, *J. Intell. Fuzzy Syst.*, **32** (2017), 841–858. https://doi.org/ 10.3233/JIFS-16090
34. M. Akram, H. S. Nawaz, Implementation of single-valued neutrosophic soft hypergraphs on human nervous system. *Artif. Intell. Rev.*, **2022** (2022). https://doi.org/10.1007/s10462-022-10200-w
35. M. Akram, H. S. Nawaz, Algorithms for the computation of regular single-valued neutrosophic soft hypergraphs applied to supranational asian bodies, *J. Appl. Math. Comput.*, **2022** (2022). https://doi.org/10.1007/s12190-022-01714-1
36. H. Wang, F. Smarandache, Y. Zhang, R. Sunderraman, *Single Valued Neutrosophic Sets*, Infinite Study, Coimbatore, India, 2010.

AIMS Press