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Research article

Robust finite-time stability of nonlinear systems involving hybrid impulses with application to sliding-mode control

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Abstract: This paper is concerned with the robust finite-time stability and stabilization of impulsive systems subject to hybrid disturbances that consists of external disturbances and hybrid impulses with time-varying jump maps. First, the global finite-time stability and local finite-time stability of a scalar impulsive system are ensured by the analysis of cumulative effect of hybrid impulses. Then, asymptotic stabilization and finite-time stabilization of second-order system subject to hybrid disturbances are achieved by linear sliding-mode control and non-singular terminal sliding-mode control. It shows that the stable systems under control are robust to external disturbances and hybrid impulses with non-destabilizing cumulative effect. If the hybrid impulses have destabilizing cumulative effect, the systems are also capable of absorbing the hybrid impulsive disturbances by the designed sliding-mode control strategies. Finally, the effectiveness of theoretical results is verified by numerical simulation and the tracking control of linear motor.

Keywords: robust finite-time stability; hybrid impulse; sliding-mode control; second-order dynamics

1. Introduction

Since developed by V. Utkin in the pioneering work [1], sliding-mode control (SMC) has been intensively investigated by many researchers [2–5]. Normally, there are two stages to design SMC for nonlinear systems: the reaching mode and the sliding mode. In reaching mode, the trajectories of nonlinear systems are enforced to reach a predefined surface defined by a constraint on the sliding variable within finite time. In sliding mode, the trajectories of nonlinear systems stay on and slide on the predefined surface called sliding surface towards the equilibrium. SMC advances in aspect of fast response, transient performance, robustness against disturbances, and consequently enables the solution of numerous control problems in engineering such as the control of robotic manipulators, the tracking control of motors, the attitude control of unmanned aerial vehicles and so forth (see [6–11] and references therein).

In the SMC design of nonlinear systems, the linear sliding variable usually results in asymptotic convergence to the desired equilibrium, which is not sufficient for control task in engineering such as missile guidance [12, 13]. Then, SMC is refined, for instance, with terminal SMC (TSMC) to achieve the finite-time stability (FTS) of nonlinear systems. For instance, the TSMC strategies involving non-singular terminal sliding manifold were developed for the FTS of rigid manipulators in [2, 14], which were extended to servo motor systems in [15]. In [16], a non-singular TSMC law for systems with mismatched disturbances was given by designing observer-based sliding mode surface and advanced in nominal performance recovery and chattering alleviation. Recently, several non-singular TSMC strategies were also proposed for fixed-time stability and derived fruitful results in engineering [17–19].

It has been shown in above-mentioned work that the nonlinear systems under SMC possess disturbance rejection and robustness properties where the disturbances have historically been considered as bounded function in continuous dynamics. This is natural as the disturbances model, for instance, the lumped uncertainty that consists of unmodeled system dynamics and external disturbances in linear motor [11]. However, the abrupt changes and instantaneous disturbances such as leg disturbances in bipedal robot demand that the control technique is able to absorb an impulsive disturbance [20]. Then, the impulsive disturbances are considered in the SMC design of nonlinear systems [21–24]. For instance, a SMC strategy based on linear sliding variable was developed for the asymptotic stability of linear systems with destabilizing impulsive effect in [23], which is further extended to TSMC design for FTS of second-order system in [22]. In [21], an integral SMC law for exponential stability of linear uncertain systems was constructed where the impulsive disturbances exhibit different effect in system components. Note that the impulse actions in existing results concerning SMC and TSMC of impulsive systems are modeled to have time-invariant jump maps. As reported in [20, 25–27], the impulsive disturbances in practical systems vary with time and have time-varying jump maps. But how to design SMC and TSMC for impulsive systems with time-dependent hybrid impulses remains unknown due to the difficulty in three aspects: (i) The impulsive disturbances together with matched disturbances have hybrid effect to the dynamics of nonlinear systems; (ii) The states may not reach the sliding surface or slide along the sliding surface because of time-dependent impulsive disturbances; (iii) When suffering hybrid impulsive disturbances with destabilizing effect, the reachability of the sliding surface and the convergence to the origin depend on a region of initial condition related to the impulses, which implies that the global stability may vanish. To study the influence of the time-varying jumps and retain the FTS of system subject to hybrid disturbances via SMC is the primary concern of this paper.

Thus motivated, this paper aims to design the SMC and TSMC strategies to stabilize the impulsive systems subject to matched disturbances and hybrid impulsive disturbances. The main contributions reside in the following:

- The global and local FTS of a scalar impulsive system with external disturbances are established to show the influence of time-dependent hybrid impulses.
- It shows that the FTS of continuous system can be retained if the hybrid impulses have non-destabilizing cumulative effect. If the hybrid impulses have destabilizing cumulative effect, the FTS of impulsive systems can also be ascertained under fixed dwell-time condition.
- Based on the robust FTS results, a linear SMC strategy and a non-singular TSMC strategy are designed for second-order system subject to hybrid disturbances that consists of matched

disturbances and hybrid impulsive disturbances. The designed SMC strategies are capable of absorbing the hybrid impulsive disturbances with both non-destabilizing and destabilizing cumulative effect.

The rest of this paper is organized as follows. In Section 2, the robust FTS of scalar impulsive system is given. Then, the asymptotic stability and FTS of second-order system with hybrid disturbances are achieved by SMC and TSMC strategies in Section 3. Finally, Section 4 presents the numerical examples and Section 5 collects the main findings and future works.

2. Robust finite-time stability

Throughout this paper, unless otherwise specified, the following notations are used. $\bar{n} = \{1, 2, \cdots, n\}$. $\mathbb{R}$, $\mathbb{N}$ and $\mathbb{N}_+$ denote the sets of real numbers, nonnegative integers and positive integers. $\mathbb{R}^n$ denote the n-dimensional real space equipped with the Euclidean norm $|\cdot|$.

First, we consider a scalar impulsive system as follows

$$\begin{cases} \dot{s}(t) = -p\,\mathrm{sign}(s(t)) + b(t)s(t) + d(t), \ t \in [t_k, t_{k+1}), \ k \in \mathbb{N}, \\ s(t) = \gamma_k s(t^-), \ t = t_k, \ k \in \mathbb{N}_+, \\ s(t_0) = s_0, \end{cases} \tag{2.1}$$

where $t_0 \geq 0$, $s(t) \in \mathbb{R}$, $b(t) \leq -p'$, $p' \geq 0$, $d(t)$ is the measurable external disturbance with bound d^*, $p > d^* \geq 0$, $\gamma_k > 0$, $k \in \mathbb{N}_+$. The impulse time sequence $\{t_k\}_{k=1}^{\infty}$ ($\{t_k\}$ for short) satisfies $t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ and $t_k \to +\infty$ as $k \to +\infty$. We denote the set of impulse time sequences with the above property by $\mathcal{F}_0$. We also define several sets of impulse time sequences for later use. For $\tau > 0$, $\mathcal{F}_\tau^1$ denotes the set of impulse time sequences $\{t_k\}$ satisfying $t_{k+1} - t_k \geq \tau$, $k \in \mathbb{N}$. For integer $N > 0$, $\mathcal{F}_N^2$ denotes the set of impulse time sequences with N impulse times, that is $\mathcal{F}_N^2 = \{t_k\}_{k=1}^N$. In addition, the impulses are assumed to have time-varying periodic jump maps, that is, there exists a positive integer N such that $\gamma_k = \gamma_{k+N}$ for $k \in \mathbb{N}_+$. Denote $\bar{\gamma} = 1 \vee \max_{\Lambda \subset \bar{N}}\{\prod_{i \in \Lambda} \gamma_i\}$ and $\underline{\gamma} = 1 \wedge \min_{\Lambda \subset \bar{N}}\{\prod_{i \in \Lambda} \gamma_i\}$. According to Definition 7 and Theorem 3 of [28], the discontinuous system (2.1) has a unique solution in the sense of Filippov. For a locally Lipschitz continuous function $V : \mathbb{R} \to \mathbb{R}_+$, the upper right-hand Dini derivative along the system (2.1) is defined by $D^+V(s) = \limsup_{h \to 0^+} \frac{V(s+\alpha f) - V(s)}{\alpha}$ where $f = -p\,\mathrm{sign}(s) + bs + d$ [29].

Definition 1 ([30]). *The system* (2.1) *is said to be finite-time stable if there exists an open neighborhood $U \subset \mathbb{R}$ of the origin and a function $T : U \setminus \{0\} \to (0, \infty)$ such that system* (2.1) *is Lyapunov stable and every solution $s(t, s_0)$ of system* (2.1) *starting from the initial state $s_0 \in U$ is defined on* $[0, T)$ *and $s(t, s_0) \equiv 0$ for all $t \in [T, \infty)$. T is called the settling-time function. If $U = \mathbb{R}$, system* (2.1) *is said to be globally finite-time stable.*

For a set of admissible impulse time sequences $\mathcal{F}$, the system (2.1) is said to be (globally) finite-time stable over the class $\mathcal{F}$ if it is (globally) finite-time stable for any impulse time sequence $\{t_k\}$ belonging to $\mathcal{F}$ where $\mathcal{F}$ is $\mathcal{F}_0$, $\mathcal{F}_\tau^1$, or $\mathcal{F}_N^2$.

Theorem 1. *If $\prod_{i=1}^N \gamma_i \leq 1$, the system* (2.1) *is globally finite-time stable over the class $\mathcal{F}_0$ with settling time $\frac{\bar{\gamma}|s_0|}{\underline{\gamma}(p-d^*)}$.*

Proof. Consider the Lyapunov function $V(t) = \frac{1}{2}s^2(t)$. Then, it follows from (2.1) that for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$,

$$\begin{aligned} D^+V(t) &= s(t)[-p\text{sign}(s(t)) + b(t)s(t) + d(t)] \\ &= -p|s(t)| + b(t)s^2(t) + s(t)d(t) \\ &\leq -\sqrt{2}(p-d^*)V^{\frac{1}{2}}(t), \end{aligned} \tag{2.2}$$

which implies that

$$\begin{aligned} V^{\frac{1}{2}}(t) &\leq V^{\frac{1}{2}}(t_k) - \frac{p-d^*}{\sqrt{2}}(t-t_k) \\ &= \gamma_k V^{\frac{1}{2}}(t_k^-) - \frac{p-d^*}{\sqrt{2}}(t-t_k) \\ &\leq \gamma_k V^{\frac{1}{2}}(t_{k-1}) - \frac{p-d^*}{\sqrt{2}}[\gamma_k(t_k - t_{k-1}) - (t-t_k)] \\ &= \gamma_k\gamma_{k-1} V^{\frac{1}{2}}(t_{k-1}^-) - \frac{p-d^*}{\sqrt{2}}[\gamma_k(t_k - t_{k-1}) - (t-t_k)] \\ &\leq \gamma_k\gamma_{k-1} V^{\frac{1}{2}}(t_{k-2}) - \frac{p-d^*}{\sqrt{2}}[\gamma_k\gamma_{k-1}(t_{k-1} - t_{k-2}) - \gamma_k(t_k - t_{k-1}) - (t-t_k)] \\ &\leq \cdots \\ &\leq \prod_{i=1}^{k}\gamma_i V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}}[\sum_{j=1}^{k}\prod_{i=j}^{k}\gamma_i(t_j - t_{j-1}) - (t-t_k)] \\ &\leq \bar{\gamma}V^{\frac{1}{2}}(t_0) - \frac{\underline{\gamma}(p-d^*)}{\sqrt{2}}(t-t_0). \end{aligned} \tag{2.3}$$

Consequently, it obtains that $V(t) \equiv 0$ for all $t \geq t_0 + \frac{\bar{\gamma}|s_0|}{\underline{\gamma}(p-d^*)}$. Thus, the system (2.1) is globally finite-time stable over the class $\mathcal{F}_0$. □

Theorem 2. *If* $\prod_{i=1}^{N}\gamma_i > 1$, *the system* (2.1) *with initial condition* $s_0 \in U_\sigma^1$ *is finite-time stable over the class* $\mathcal{F}_\tau^1$ *with settling time* $\frac{\tau(\ln\sigma_0 - \ln(\sigma_0-\sigma))}{\ln q}$, *where* $U_\sigma^1 = \{s \in \mathbb{R} \| s| \leq \sigma\}$, $\sigma \in (0, \sigma_0)$, $\sigma_0 = \frac{\tau(p-d^*)}{\bar{\gamma}\ln q}$ *and* $q = \sqrt[N]{\prod_{i=1}^{N}\gamma_i}$.

Proof. The proof is similar to Theorem 1, so we just sketch the outline and focus on the different parts. It follows from (2.3) that

$$\begin{aligned} V^{\frac{1}{2}}(t) &\leq \prod_{i=1}^{k}\gamma_i V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}}[\sum_{j=1}^{k}\prod_{i=j}^{k}\gamma_i(t_j - t_{j-1}) + (t-t_k)] \\ &= \prod_{i=1}^{k}\gamma_i V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}}[\sum_{j=1}^{k}\int_{t_{j-1}}^{t_j}\prod_{i=j}^{k}\gamma_i ds + \int_{t_k}^{t} ds] \\ &= \prod_{i=1}^{k}\gamma_i\Big\{V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}}[\sum_{j=1}^{k}\int_{t_{j-1}}^{t_j}\prod_{i=1}^{j-1}\gamma_i^{-1} ds + \int_{t_k}^{t}\prod_{i=1}^{k}\gamma_i^{-1} ds]\Big\} \end{aligned}$$

$$\leq \prod_{i=1}^{k} \gamma_i \Big\{ V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}\bar{\gamma}} \Big[\int_{t_k}^{t} \exp(-\frac{\ln q}{\tau}(s-t_0))ds + \sum_{j=1}^{k} \int_{t_{j-1}}^{t_j} \exp(-\frac{\ln q}{\tau}(s-t_0))ds \Big] \Big\}$$

$$= \prod_{i=1}^{k} \gamma_i \Big\{ V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}\bar{\gamma}} \int_{t_0}^{t} \exp(-\frac{\ln q}{\tau}(s-t_0))ds \Big\}$$

$$= \prod_{i=1}^{k} \gamma_i \Big\{ V^{\frac{1}{2}}(t_0) - \frac{\sigma_0}{\sqrt{2}} \Big[1 - \exp(-\frac{\ln q}{\tau}(t-t_0))\Big] \Big\}. \tag{2.4}$$

Therefore, $V(t) \equiv 0$ for all $t \geq t_0 + \frac{\tau(\ln \sigma_0 - \ln(\sigma_0 - \sigma))}{\ln q}$. The proof is completed. □

Theorem 3. *If $\prod_{i=1}^{N} \gamma_i > 1$ and $-p' + \frac{\ln q}{\tau} < 0$, the system (2.1) is global finite-time stable over the class $\mathcal{F}_\tau^1$ with settling time $\frac{1}{\lambda} \ln(1 + \frac{\lambda \bar{\gamma} |s_0|}{p-d^*})$ where $\lambda = p' - \frac{\ln q}{\tau}$ and $q = \sqrt[N]{\prod_{i=1}^{N} \gamma_i}$.*

Proof. According to (2.2), it obtains that

$$D^+ V(t) \leq -\sqrt{2}(p-d^*)V^{\frac{1}{2}}(t) - 2p'V(t), \tag{2.5}$$

for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. For $t \in [t_0, t_1)$, it yields

$$V^{\frac{1}{2}}(t) \leq \exp(-p'(t-t_0))V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}} \int_{t_0}^{t} \exp(-p'(t-s))ds. \tag{2.6}$$

Then, it will be derived that

$$V^{\frac{1}{2}}(t) \leq \prod_{i=1}^{k} \gamma_i \exp(-p'(t-t_0))V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}} \Big\{ \int_{t_k}^{t} \exp(-p'(t-s))ds + \sum_{j=1}^{k} \prod_{i=j}^{k} \gamma_i \int_{t_{j-1}}^{t_j} \exp(-p'(t-s))ds \Big\}, \tag{2.7}$$

for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$ where $\sum_{i=1}^{0} = 1$. From (2.6), this assertion is true for $k = 0$. Suppose that it is true for $k = 0, 1, \cdots, m-1$, $m \in \mathbb{N}$, it follows from (2.5) and (2.7) that

$$V^{\frac{1}{2}}(t) \leq \exp(-p'(t-t_m))V^{\frac{1}{2}}(t_m) - \frac{p-d^*}{\sqrt{2}} \int_{t_m}^{t} \exp(-p'(t-s))ds$$

$$= \gamma_m \exp(-p'(t-t_m))V^{\frac{1}{2}}(t_m^-) - \frac{p-d^*}{\sqrt{2}} \int_{t_m}^{t} \exp(-p'(t-s))ds$$

$$\leq \gamma_m \exp(-p'(t-t_m)) \Big\{ \prod_{i=1}^{m-1} \gamma_i \exp(-p'(t_m-t_0))V^{\frac{1}{2}}(t_0) - \frac{p-d^*}{\sqrt{2}} \Big[\int_{t_{m-1}}^{t_m} \exp(-p'(t_m-s))ds + \sum_{j=1}^{m-1} \prod_{i=j}^{m-1} \gamma_i \int_{t_{j-1}}^{t_j} \exp(-p'(t_m-s))ds \Big] \Big\} - \frac{p-d^*}{\sqrt{2}} \int_{t_m}^{t} \exp(-p'(t-s))ds$$

$$= \prod_{i=1}^{m} \gamma_i \exp(-p'(t-t_0))V^{\frac{1}{2}}(t_0)$$

$$-\frac{p-d^*}{\sqrt{2}}\Big[\int_{t_m}^{t}\exp(-p'(t-s))ds+\sum_{j=1}^{m}\prod_{i=j}^{m}\gamma_i\int_{t_{j-1}}^{t_j}\exp(-p'(t-s))ds\Big], \tag{2.8}$$

for $t \in [t_m, t_{m+1})$. Therefore, (2.7) holds for $k = m$. Based on mathematical induction, the assertion (2.7) is true for all $k \in \mathbb{N}$. Subsequently, it follows from (2.7) that

$$\begin{aligned}
V^{\frac{1}{2}}(t) \le& \prod_{i=1}^{m}\gamma_i\exp(-p'(t-t_0))\Big\{V^{\frac{1}{2}}(t_0)\\
&-\frac{p-d^*}{\sqrt{2}}\Big[\int_{t_m}^{t}\prod_{i=1}^{m}\gamma_i^{-1}\exp(p'(s-t_0))ds+\sum_{j=1}^{m}\int_{t_{j-1}}^{t_j}\prod_{i=1}^{j-1}\gamma_i^{-1}\exp(p'(s-t_0))ds\Big]\Big\}\\
\le& \prod_{i=1}^{m}\gamma_i\exp(-p'(t-t_0))\Big\{V^{\frac{1}{2}}(t_0)\\
&-\frac{p-d^*}{\sqrt{2}\bar{\gamma}}\Big[\int_{t_m}^{t}\exp((-\frac{\ln q}{\tau}+p')(s-t_0))ds+\sum_{j=1}^{m}\int_{t_{j-1}}^{t_j}\exp((-\frac{\ln q}{\tau}+p')(s-t_0))ds\Big]\Big\}\\
=& \prod_{i=1}^{m}\gamma_i\exp(-p'(t-t_0))\Big\{V^{\frac{1}{2}}(t_0)-\frac{p-d^*}{\sqrt{2}\bar{\gamma}}\int_{t_0}^{t}\exp(\lambda(s-t_0))ds\Big\}\\
=& \prod_{i=1}^{m}\gamma_i\exp(-p'(t-t_0))\Big\{V^{\frac{1}{2}}(t_0)-\frac{p-d^*}{\sqrt{2}\lambda\bar{\gamma}}\big[\exp(\lambda(t-t_0))-1\big]\Big\},
\end{aligned} \tag{2.9}$$

which indicates that $V(t) \equiv 0$ for all $t \ge t_0 + \frac{1}{\lambda}\ln(1+\frac{\lambda\bar{\gamma}|s_0|}{p-d^*})$. Thus, the system (2.1) is globally finite-time stable over the class $\mathcal{F}_\tau^1$. □

Remark 1. *Recently, great efforts have been devoted to FTS of impulsive systems [29, 31–33]. For instance, in [31], the FTS of impulsive systems was established under requirement that the impulses have time-invariant non-destabilizing effect. Then, several Lyapunov-based theorems with settling-time estimation were given for FTS of impulsive systems with destabilizing impulses in [29]. Note that the impulses considered in existing results are either non-destabilizing or destabilizing and have time-invariant jump maps. However, the impulse action in practical systems varies with time and has time-varying jump maps. To establish the FTS of impulsive systems with time-dependent impulses is more practical and meaningful. Even though the FTS results given in [34] can deal with the hybrid impulses with time-varying jump maps, the comparison of two constructed functions related to impulse times is not easy to be implemented. In comparison, Theorems 1–3 present the explicit conditions for FTS of impulsive systems with time-dependent hybrid impulses. The first condition in Theorem 1 indicates that the cumulative effect of time-dependent hybrid impulses is non-destabilizing. Consequently, Theorem 1 shows that the FTS of continuous system is retained, even though the system is subject to time-dependent hybrid impulses with non-destabilizing cumulative effect. The first conditions in Theorems 2 and 3 imply that the cumulative effect of time-dependent hybrid impulses is destabilizing. Thus, Theorems 2 and 3 show that the FTS of impulsive systems can be ascertained under fixed dwell-time condition, even though the system is subject to time-dependent hybrid impulses with destabilizing cumulative effect. If there are no jumps in system* (2.1) *($\gamma_k = 1$ for all $k \in \mathbb{N}_+$), the FTS can also be ascertained from Theorem 1. Thus, the established results also show the robustness of stable system against hybrid impulses.*

3. SMC of second-order system

In this section, the linear SMC and non-singular TSMC strategies are designed for the asymptotic stabilization and finite-time stabilization of second-order system subject to hybrid disturbances that consist of matched disturbances and hybrid impulsive disturbances. Consider the following second-order system

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x) + g(x)u + d, \end{cases} \tag{3.1}$$

subject to impulsive disturbances

$$\begin{cases} x_1(t_k) = c_{1k}x_1(t_k^-), \\ x_2(t_k) = c_{2k}x_2(t_k^-), \end{cases} \tag{3.2}$$

where $x = [x_1, x_2]^T \in \mathbb{R}^2$ is the system state, f and g are continuous functions, u is the control input, d is the measurable bounded matched disturbance such that $|d(t)| \leq d^*$, c_{1k} and c_{2k} are the time-varying impulsive strength of the impulsive disturbances. Assume that there exists a positive integer N such that $c_{1k} = c_{1(k+N)}$ and $c_{2k} = c_{2(k+N)}$ for $k \in \mathbb{N}_+$. Denote $\bar{c} = 1 \vee \max_{\Lambda \subset \bar{N}}\{\prod_{i \in \Lambda} c_{1i}\}$ and $\underline{c} = 1 \wedge \min_{\Lambda \subset \bar{N}}\{\prod_{i \in \Lambda} c_{1i}\}$. In addition, f and g are assumed to satisfy appropriate conditions to ensure the existence and uniqueness of the solution to systems (3.1) and (3.2).

Definition 2. *The second-order systems* (3.1) *and* (3.2) *is said to be asymptotically stable if there exists an open neighborhood $U \subset \mathbb{R}^2$ of the origin such that, for all $x_0 = [x_{10}, x_{20}]^T \in U$, the system is stable and $x(t, x_0) \to 0$ as $t \to +\infty$. If $U = \mathbb{R}$, systems* (3.1) *and* (3.2) *is said to be globally asymptotically stable.*

For a set of admissible impulse time sequences $\mathcal{F}$, the second-order systems (3.1) and (3.2) is said to be (globally) asymptotically stable over the class $\mathcal{F}$ if it is (globally) asymptotically stable for any impulse time sequence $\{t_k\}$ belonging to $\mathcal{F}$ where $\mathcal{F}$ is $\mathcal{F}_0$, $\mathcal{F}_\tau^1$, or $\mathcal{F}_N^2$. The defintions of FTS and globally FTS of the second-order systems (3.1) and (3.2) are similar as Definition 1.

Assumption 1. $c_{1k} = c_{2k}$, $k \in \mathbb{N}_+$.

Under Assumption 1, the linear SMC strategy is designed for asymptotic stabilization of second-order systems (3.1) and (3.2). Consider the sliding variable

$$s(x) = x_2 + \beta x_1, \tag{3.3}$$

and controller

$$u(x) = -g^{-1}(x)[f(x) + \beta x_2 + p\,\mathrm{sign}(s) + p's], \tag{3.4}$$

where $\beta > 0$, $p > d^*$, $p' \geq 0$. Then, the second-order system is asymptotically stabilized as follows.

Theorem 4. *If $\prod_{i=1}^{N} c_{1i} \leq 1$, the second-order systems* (3.1) *and* (3.2) *is globally asymptotically stable over the class $\mathcal{F}_0$ by the control laws* (3.3), (3.4).

Proof. From the definition of silding variable, we get that

$$\dot{s} = \dot{x}_2 + \beta\dot{x}_1 = f(x) + g(x)u + d + \beta x_2 = -p\,\mathrm{sign}(s) - p's + d, \tag{3.5}$$

for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. When $t = t_k$, $k \in \mathbb{N}_+$, $s(t_k) = x_2(t_k) + \beta x_1(t_k) = c_{2k}x_2(t_k^-) + \beta c_{1k}x_1(t_k^-) = c_{1k}s(t_k^-)$. Based on Theorem 1, $s(t) \equiv 0$ for all $t \geq T$ where $T = t_0 + \frac{\bar{c}|s_0|}{\underline{c}(p-d^*)}$. Thus, the closed-loop systems (3.1) and (3.2) reaches the sliding surface $s(t) = 0$ within finite time. After reaching the sliding surface, we claim that the solution of closed-loop systems (3.1), (3.2) will stay on the surface. For $t \in [t_k, t_{k+1}) \cap [T, \infty)$, one has the following estimation of $s^2(t)$

$$D^+(s^2(t)) = 2s(t)[-p\text{sign}(s(t)) - p's(t) + d(t)] \leq -2(p - d^*)|s(t)|. \tag{3.6}$$

Denote the first impulse time in the interval (T, ∞) by t_K. From (3.6), one gets that $s(t) = 0$ for $[T, t_K)$. When $t = t_K$, $s(t_K) = c_{2K}x_2(t_K^-) + \beta c_{1K}x_1(t_K^-) = c_{1K}s(t_K^-) = 0$. Repeating the same analysis of $s(t)$ in each interval $[t_k, t_{k+1})$, $k \geq K$, we obtain that the solution of closed-loop systems (3.1) and (3.2) will stay on the surface $s(t) = 0$. On the sliding surface, it holds that

$$\begin{cases} \dot{x}_1(t) = -\beta x_1(t), \ t \in [t_k, t_{k+1}), \\ x_1(t_k) = c_{1k}x_1(t_k^-), k \in \mathbb{N}_+, \ k \geq K, \end{cases} \tag{3.7}$$

whose solution is expressed by

$$x_1(t) = \prod_{i=K}^{k} c_{1i}x_1(t_K)\exp(-\beta(t - t_K)), \tag{3.8}$$

for $t \in [t_k, t_{k+1})$, $k \geq K$, $k \in \mathbb{N}_+$. Since $\prod_{i=1}^{N} c_{1i} \leq 1$, $\prod_{i=K}^{k} c_{1i} \leq \bar{c}$ for all $k \geq K$ and consequently $x_1(t) \leq \bar{c}x_1(t_K)\exp(-\beta(t - t_K))$ for $t \geq t_K$. Thus, the second-order systems (3.1) and (3.2) is globally asymptotically stable over the class $\mathcal{F}_0$. □

Theorem 5. *If $\prod_{i=1}^{N} c_{1i} > 1$, the second-order systems* (3.1) *and* (3.2) *with initial condition $x_0 \in U_\sigma^2$ is asymptotically stable over the class $\mathcal{F}_\tau^1$ by control laws* (3.3), (3.4) *with β satisfying $-\beta + \frac{\ln q}{\tau} < 0$, where $U_\sigma^2 = \{x = [x_1, x_2]^T \in \mathbb{R}^2 \| s| \leq \sigma, s = x_2 + \beta x_1\}$, $\sigma \in (0, \sigma_0)$, $\sigma_0 = \frac{\tau(p-d^*)}{\bar{c}\ln q}$, and $q = \sqrt[N]{\prod_{i=1}^{N} c_{1i}}$.*

Proof. The proof is similar to Theorem 4, so we just sketch the outline and focus on the different part. Denote $\sigma_0 = \frac{\tau(p-d^*)}{\bar{c}\ln q}$. For each $x_0 \in U_\sigma^2$, $s_0 \in U_\sigma^1$. From Theorem 2, it obtains that $s(t) \equiv 0$ for all $t \geq T$ where $T = t_0 + \frac{\tau(\ln\sigma_0 - \ln(\sigma_0 - \sigma))}{\ln q}$. Thus, the closed-loop systems (3.1) and (3.2) reaches the sliding surface $s(t) = 0$ within finite time and stay on the surface afterwards. Since $\prod_{i=1}^{N} c_{1i} = q^N$, $\prod_{i=K}^{k} c_{1i} \leq \bar{c}\exp(\frac{\ln q}{\tau}(t - t_K))$ for $t \in [t_k, t_{k+1})$, $k \geq K$. On the sliding surface, the solution of system (3.7) is estimated by

$$x_1(t) \leq \bar{c}x_1(t_K)\exp((-\beta + \frac{\ln q}{\tau})(t - t_K)),$$

for $t \in [t_k, t_{k+1})$, $k \geq K$. Since $-\beta + \frac{\ln q}{\tau} < 0$, the closed-loop systems (3.1) and (3.2) is asymptotically stable. The proof is completed. □

Theorem 6. *If $\prod_{i=1}^{N} c_{1i} > 1$, the second-order systems* (3.1) *and* (3.2) *is globally asymptotically stable over the class $\mathcal{F}_\tau^1$ by the control laws* (3.3), (3.4) *with β, p' satisfying $-p' + \frac{\ln q}{\tau} < 0$ and $-\beta + \frac{\ln q}{\tau} < 0$, where $q = \sqrt[N]{\prod_{i=1}^{N} c_{1i}}$.*

Proof. The theorem is the direct consequence of Theorem 3 by the same line as the proof of Theorem 5. The details are omitted. □

Remark 2. *Based on the technique proposed in [29], a linear SMC strategy was designed for linear impulsive systems with destabilizing impulses in [23] where the reachability of the sliding surface depends on the initial condition, which agrees with Theorem 5. Therefore, Theorem 5 extends Theorem 2 of [23] to the case of hybrid impulses involving destabilizing cumulative effect. Since the considered impulsive disturbances have time-varying jump maps, the techniques developed in [21, 29] cannot be used for SMC design. From Theorems 4 and 6, the proposed SMC strategy is capable of not only dealing with hybrid impulses involving time-varying jump maps but also stabilizing the second-order system in global sense instead of local one dependent on the initial condition.*

Remark 3. *It should be pointed out that Assumption 1 ensures the continuity of the sliding surface. After the sliding surface is reached, the continuity of the sliding variable at impulse time remains, because* $s(t_k) = c_{2k}x_2(t_k^-) + \beta c_{1k}x_1(t_k^-) = c_{1k}s(t_k^-) = 0$ *for* $k \geq K$ *where* K *is defined in the proof of Theorem 4. Thus, the sliding variable is continuous along the solution of systems* (3.1) *and* (3.2) *after the sliding surface is reached.*

In what follows, a non-singular TSMC strategy is designed for finite-time stabilization of second-order systems (3.1) and (3.2). To this end, the following assumption is given to capture the characteristics of hybrid impulses.

Assumption 2. $c_{1k}^{\alpha_1/\alpha_2} = c_{2k}$, $k \in \mathbb{N}$, *where* $0 < \alpha_1 < \alpha_2$ *are odd integers.*

Consider the sliding variable

$$s(x) = x_2 + \beta x_1^{\alpha_1/\alpha_2}, \tag{3.9}$$

and controller

$$u(x) = \begin{cases} -g^{-1}(x)[f(x) + p\text{sign}(x_2) + p'x_2], & x \in \mathbb{S}_1, \\ -g^{-1}(x)[f(x) + p\text{sign}(s) + p's - \frac{\alpha_1 x_2^2}{\alpha_2 x_1}], & x \in \mathbb{S}_2, \end{cases} \tag{3.10}$$

where $\beta > 0$, $p > d^*$, $p' \geq 0$, $\mathbb{S}_1 = \{x \in \mathbb{R}^2 | x_1x_2 > 0 \text{ or } x_1 = 0, x_2 \neq 0\}$ and $\mathbb{S}_2 = \{x \in \mathbb{R}^2 | x_1x_2 < 0 \text{ or } x_2 = 0\}$. Then, the second-order system is stabilized within finite time as follows.

Theorem 7. *If* $\prod_{i=1}^{N} c_{1i} \leq 1$, *the second-order systems* (3.1) *and* (3.2) *is globally finite-time stable over the class* $\mathcal{F}_0$ *by the control laws* (3.9), (3.10).

Proof. The proof is partitioned into two parts: (I) The systems (3.1) and (3.2) reaches the sliding surface $s(t) = 0$ within finite time; (II) The systems (3.1) and (3.2) converges to the origin within finite time on the sliding surface.

Part I: According to the initial condition, there two cases: $(x_{10}, x_{20}) \in \mathbb{S}_2$ and $(x_{10}, x_{20}) \in \mathbb{S}_1$.

Case i: When $(x_{10}, x_{20}) \in \mathbb{S}_2$, skip to Step 2 of Case ii.

Case ii: When $(x_{10}, x_{20}) \in \mathbb{S}_1$, there are two steps to complete the proof.

Step 1: It will be shown that the trajectories of systems (3.1) and (3.2) move into $\mathbb{S}_2$ within finite time. Under control law (3.10), the dynamics of x_2 reads as

$$\begin{cases} \dot{x}_2(t) = -p\text{sign}(x_2(t)) - p'x_2(t) + d(t), \ t \in [t_{k-1}, t_k), \\ x_2(t_k) = c_{2k}x_2(t_k^-), \ k \in \mathbb{N}_+. \end{cases} \tag{3.11}$$

Based on Theorem 1, $x_2(t)$ converges to zero with settling time $T_1 = \frac{\bar{c}^{\alpha_1/\alpha_2}|x_{20}|}{\underline{c}^{\alpha_1/\alpha_2}(p-d^*)}$, that is, the trajectories of systems (3.1) and (3.2) move into $\mathbb{S}_2$ within finite time. In addition, the moment of x_1 is estimated as follows

$$\begin{aligned} x_1(T_1) &= \prod_{i=1}^{k_1} c_{1k} x_{10} + \sum_{i=1}^{k_1} \prod_{j=i}^{k_1} c_{1j} \int_{t_{i-1}}^{t_i} x_2(s)ds + \int_{t_{k_1}}^{T_1} x_2(s)ds \\ &\le \prod_{i=1}^{k_1} c_{1k} x_{10} + \sum_{i=1}^{k_1} \prod_{j=i}^{k_1} c_{1j} \int_{t_{i-1}}^{t_i} |x_2(s)|ds + \int_{t_{k_1}}^{T_1} |x_2(s)|ds \\ &\le \bar{c}|x_{10}| + \bar{c}^{1+\alpha_1/\alpha_2}|x_{20}|T_1. \end{aligned} \tag{3.12}$$

Step 2: It will be shown that systems (3.1) and (3.2) reaches the sliding surface $s(t) = 0$ within finite time in $\mathbb{S}_2$. First, we claim that the system will stay in $\mathbb{S}_2$ after reaching $\mathbb{S}_2$. If this assertion is false, the trajectory has to intercept the boundary of $\mathbb{S}_2$ at non-impulse time. If it intercepts the boundary $\{x \in \mathbb{R}^2 | x_2 = 0, x_1 > 0\}$, there exists $t^* \ge T_1$ such that

$$x_1(t^*) > 0,\ x_2(t^*) = 0,\ \dot{x}_2(t^*) > 0. \tag{3.13}$$

However, it follows from (3.1) that

$$\begin{aligned} \dot{x}_2(t^*) &= -p\text{sign}(s(t^*)) - p' s(t^*) + d(t^*) + \frac{\alpha_1 x_2^2(t^*)}{\alpha_2 x_1(t^*)} \\ &= -p\text{sign}(\beta x_1^{\alpha_1/\alpha_2}(t^*)) - p'\beta x_1^{\alpha_1/\alpha_2}(t^*) + d(t^*) \\ &\le -p + d^* - p'\beta x_1^{\alpha_1/\alpha_2}(t^*) < 0, \end{aligned} \tag{3.14}$$

which is contradiction. If it intercepts the boundary $\{x \in \mathbb{R}^2 | x_2 = 0, x_1 < 0\}$, there exists t^* such that

$$x_1(t^*) < 0,\ x_2(t^*) = 0,\ \dot{x}_2(t^*) < 0. \tag{3.15}$$

However, it follows from (3.1) that

$$\dot{x}_2(t^*) \ge p - d^* - p'\beta x_1^{\alpha_1/\alpha_2}(t^*) > 0, \tag{3.16}$$

which is also contradiction. If the system intercepts the boundary $\{x \in \mathbb{R}^2 | x_1 = 0, x_2 > 0\}$, there exists $t^* > T_1$ such that

$$x_1(t^*) = 0,\ x_2(t^*) > 0. \tag{3.17}$$

However, it follows from (3.1) that

$$\begin{aligned} \dot{x}_2(t) &= -p\text{sign}(s(t)) - p' s(t) + d(t) + \frac{\alpha_1 x_2^2(t)}{\alpha_2 x_1(t)} \\ &\le -p + d^* - p'[x_2(t) + \beta x_1^{\alpha_1/\alpha_2}(t)] + \frac{\alpha_1 x_2^2(t)}{\alpha_2 x_1(t)} \\ &< -p'[x_2(t) + \beta x_1^{\alpha_1/\alpha_2}(t)] + \frac{\alpha_1 x_2^2(t)}{\alpha_2 x_1(t)}, \end{aligned} \tag{3.18}$$

for $[t^* - \delta, t^*)$ with sufficiently small $\delta > 0$. Consequently, $\lim_{t\to t^{*-}} \dot{x}_2(t) = -\infty$, which contradicts with $x_2(t^*) > 0$ from the continuity of x_2 at t^*. If the system intercepts the boundary $\{x \in \mathbb{R}^2 | x_1 = 0, x_2 < 0\}$, there exists $t^* > T_1$ such that

$$x_1(t^*) = 0,\ x_2(t^*) < 0. \tag{3.19}$$

However, it follows from (3.1) that

$$\begin{aligned}\dot{x}_2(t) &= -p\text{sign}(s(t)) - p' s(t) + d(t) + \frac{\alpha_1 x_2^2(t)}{\alpha_2 x_1(t)}\\ &\geq -p + d^* - p'[x_2(t) + \beta x_1^{\alpha_1/\alpha_2}(t)] + \frac{\alpha_1 x_2^2(t)}{\alpha_2 x_1(t)}\\ &> -p'[x_2(t) + \beta x_1^{\alpha_1/\alpha_2}(t)] + \frac{\alpha_1 x_2^2(t)}{\alpha_2 x_1(t)},\end{aligned} \tag{3.20}$$

for $[t^* - \delta, t^*)$ with sufficiently small $\delta > 0$. Consequently, $\lim_{t\to t^{*-}} \dot{x}_2(t) = +\infty$, which contradicts with $x_2(t^*) < 0$ from the continuity of x_2 at t^*. In each case, the contradiction follows that the trajectory intercepts the boundary of $\mathbb{S}_2$. Thus, the system will stay in $\mathbb{S}_2$ after T_1. In $\mathbb{S}_2$, one has

$$\begin{aligned}\dot{s}(t) &= -p\text{sign}(s(t)) - p' s(t) + d(t) + \frac{\alpha_1 x_2^2(t)}{\alpha_2 x_1(t)} + \beta\frac{\alpha_1}{\alpha_2}(x_1(t))^{\alpha_1/\alpha_2 - 1} x_2(t)\\ &= -p\text{sign}(s(t)) - p' s(t) + d(t) + \frac{\alpha_1 x_2(t)}{\alpha_2 x_1(t)} s(t),\end{aligned} \tag{3.21}$$

for $t \in [t_k, t_{k+1}) \cap [T_1, \infty)$. When $t = t_k$ and $t \geq T_1$, $s(t) = c_{2k} s(t^-)$. Since $\frac{x_2}{x_1} \leq 0$ in $\mathbb{S}_2$, it follows from Theorem 1 that $s(t)$ converges to zero within finite time and the settling time is bounded by $T_2 = \frac{\bar{c}^{\alpha_1/\alpha_2}|s_0|}{\underline{c}^{\alpha_1/\alpha_2}(p-d^*)}$ where $s_0 = \beta(\bar{c}|x_{10}| + \bar{c}^{1+\alpha_1/\alpha_2}|x_{20}|T_1)^{\alpha_1/\alpha_2}$.

Theorefore, the systems (3.1) and (3.2) with either type of initial condition reaches the sliding surface $s(t) = 0$ within finite time.

Part II: After reaching the sliding surface $s(t) = 0$, we claim that the solution of closed-loop systems (3.1) and (3.2) will stay on the surface and converge to the origin within finite time. Denote the first impulse time in interval $(t_0 + T_1 + T_2, \infty)$ by t_K . For $t \in [t_k, t_{k+1}) \cap [t_0 + T_1 + T_2, \infty)$, $k \in \mathbb{N}$, one has the following estimation of $s^2(t)$

$$D^+(s^2(t)) = 2s(t)[-p\text{sign}(s(t)) - p' s(t) + d(t) + \frac{\alpha_1 x_2(t)}{\alpha_2 x_1(t)} s(t)] \leq -2(p - d^*)|s(t)|,$$

which implies that that $s(t) = 0$ for $[T, t_K)$. For $k \geq K$, $s(t_K) = c_{2K} x_2(t_K^-) + \beta c_{1K}(x_1(t_K^-))^{\alpha_1/\alpha_2} = c_{2K} s(t_K^-) = 0$. Repeating the same analysis of $s(t)$ in each interval $[t_k, t_{k+1})$, $k \geq K$, we obtain that the solution of closed-loop systems (3.1) and (3.2) will stay on the surface $s(t) = 0$. On the sliding surface, it holds that

$$\begin{cases} \dot{x}_1(t) = -\beta(x_1(t))^{\alpha_1/\alpha_2},\ t \in [t_k, t_{k+1}),\\ x_1(t_k) = c_{1k} x_1(t_k^-), k \in \mathbb{N}_+,\ k \geq K. \end{cases} \tag{3.22}$$

Choose the Lyapunov function $V(t) = \frac{1}{2}x_1^2(t)$. Then, it follows from (3.22) that

$$V^{\alpha_3}(t) = V^{\alpha_3}(t_k) - \alpha_3 k_0 \beta(t - t_k)$$

$$
\begin{aligned}
&= c_{1k}^{2\alpha_3} V^{\alpha_3}(t_k^-) - \alpha_3 k_0 \beta (t - t_k) \\
&= c_{1k}^{2\alpha_3} V^{\alpha_3}(t_{k-1}) - \alpha_3 k_0 \beta [c_{1k}^{2\alpha_3}(t_k - t_{k-1}) + (t - t_k)] \\
&= (c_{1k} c_{1(k-1)})^{2\alpha_3} V^{\alpha_3}(t_{k-1}^-) - \alpha_3 k_0 \beta [c_{1k}^{2\alpha_3}(t_k - t_{k-1}) + (t - t_k)] \\
&= (c_{1k} c_{1(k-1)})^{2\alpha_3} V^{\alpha_3}(t_{k-2}) - \alpha_3 k_0 \beta [(c_{1k} c_{1(k-1)})^{2\alpha_3}(t_{k-1} - t_{k-1}) + c_{1k}^{2\alpha_3}(t_k - t_{k-1}) + (t - t_k)] \\
&= \cdots \\
&= \prod_{i=K}^{k} c_{1i}^{2\alpha_3} V^{\alpha_3}(t_K) - \alpha_3 k_0 \beta [\sum_{i=K}^{k} \prod_{j=i}^{k} c_{1j}^{2\alpha_3}(t_i - t_{i-1}) + (t - t_k)],
\end{aligned}
$$

for $t \in [t_k, t_{k+1})$, $k \geq K$, where $\alpha_3 = \frac{1}{2}(1 - \frac{\alpha_1}{\alpha_2})$ and $k_0 = 2^{1-\alpha_3}$. Since $\prod_{i=1}^{N} c_{1i} \leq 1$, $\underline{c} \leq \prod_{i=m}^{k} c_{1i} \leq \bar{c}$ for all $k \geq m \geq K$ and consequently $V^{\alpha_3}(t) \leq \bar{c}^{2\alpha_3} V^{\alpha_3}(t_K) - \underline{c}^{2\alpha_3}(t - t_K)$ for $t \geq t_K$. Thus, $x_1(t)$ converges to the origin within finite time.

In conclusion, the second-order systems (3.1) and (3.2) is globally finite-time stable over the class $\mathcal{F}_0$. The proof is completed. □

Obviously, Theorem 7 presents the global FTS result for system subject to hybrid impulsive disturbances involving non-destabilizing cumulative effect. Next, the global FTS theorem for the case with destabilizing cumulative effect is given under the condition about the sliding variable as follows.

Condition 1. $\beta > \frac{2\bar{c}^2 \beta_0 \ln q}{k_0 \tau}$ *where* $\beta_0 = (\bar{c}|x_{10}| + \bar{c}^{1+\alpha_1/\alpha_2} q^{\frac{T_1}{\tau}} |x_{20}| T_1)^{2\alpha_3}$, $\alpha_3 = \frac{1}{2}(1 - \frac{\alpha_1}{\alpha_2})$ *and* $k_0 = 2^{1-\alpha_3}$.

Theorem 8. *If* $\prod_{i=1}^{N} c_{1i} > 1$, *the second-order systems* (3.1) *and* (3.2) *is globally finite-time stable over the class* $\mathcal{F}_\tau^1$ *by the control laws* (3.9) *and* (3.10) *with* β, p' *satisfying* $-p' + \frac{\ln q}{\tau} < 0$, $-\beta + \frac{\ln q}{\tau} < 0$, *and Condition 1, where* $q = \sqrt[N]{\prod_{i=1}^{N} c_{1i}}$.

Proof. The proof is similar to Theorem 7, so we just sketch the outline and focus on the different part. Denote $\lambda_1 = p' - \frac{\ln q}{\tau}$, $\lambda_2 = p' - \frac{\ln q'}{\tau}$ and $\lambda_3 = \beta_1 - \frac{\ln q}{\tau}$ where $q' = \sqrt[N]{\prod_{i=1}^{N} c_{2i}}$ Obviously, $q' = q^{\alpha_1/\alpha_2}$, $\lambda_2 > \lambda_1 > 0$ and $\lambda_3 > 0$.

In Step 1, it follows from (3.11) and Theorem 3 that $x_2(t)$ converges to zero with settling time $T_1 = \frac{1}{\lambda_2} \ln(1 + \frac{\lambda_2 \bar{c}^{\alpha_1/\alpha_2}|x_{20}|}{p - d^*})$, that is, the trajectories of system move into $\mathbb{S}_2$ within finite time.

In Step 2, it follows from (3.21) and Theorem 3 that the sliding surface $s(t) = 0$ is reached within finite time and the settling time is bounded by $T_2 = \frac{1}{\lambda_2} \ln(1 + \frac{\lambda_2 \bar{c}^{\alpha_1/\alpha_2}|s_0|}{p - d^*})$.

In Part II, on the sliding surface, it follows from (3.22) that

$$
\begin{aligned}
V^{\alpha_3}(t) &= \prod_{i=K}^{k} c_{1i}^{2\alpha_3} V^{\alpha_3}(t_K) - \alpha_3 k_0 \beta [\sum_{i=K}^{k} \prod_{j=i}^{k} c_{1j}^{2\alpha_3}(t_i - t_{i-1}) + (t - t_k)] \\
&= \prod_{i=K}^{k} c_{1i}^{2\alpha_3} \Big\{ V^{\alpha_3}(t_K) - \alpha_3 k_0 \beta [\sum_{i=K}^{k} \int_{t_{i-1}}^{t_i} \prod_{j=K}^{i-1} c_{1j}^{-2\alpha_3} ds + \int_{t_k}^{t} \prod_{j=K}^{k} c_{1j}^{-2\alpha_3} ds] \Big\} \\
&\leq \prod_{i=K}^{k} c_{1i}^{2\alpha_3} \Big\{ V^{\alpha_3}(t_K) \\
&\quad - \frac{\alpha_3 k_0 \beta}{\bar{c}^2} [\sum_{i=K}^{k} \int_{t_{i-1}}^{t_i} \exp(-\frac{2\alpha_3 \ln q}{\tau}(s - t_K)) ds + \int_{t_k}^{t} \exp(-\frac{2\alpha_3 \ln q}{\tau}(s - t_K)) ds] \Big\}
\end{aligned}
$$

$$= \prod_{i=K}^{k} c_{1i}^{2\alpha_3} \Big\{ V^{\alpha_3}(t_K) - \frac{\alpha_3 k_0 \beta}{\bar{c}^2} \int_{t_K}^{t} \exp(-\frac{2\alpha_3 \ln q}{\tau}(s - t_K)) ds \Big\}$$

$$= \prod_{i=K}^{k} c_{1i}^{2\alpha_3} \Big\{ V^{\alpha_3}(t_K) - \frac{k_0 \beta \tau}{2\bar{c}^2 \ln q} [1 - \exp(-\frac{2\alpha_3 \ln q}{\tau}(t - t_K))] \Big\},$$

for $t \in [t_k, t_{k+1})$, $k \geq K$. Since $x_1 x_2 \leq 0$ in $\mathbb{S}_2$, $\dot{V}(t) = x_1 \dot{x}_1 = x_1 x_2 \leq 0$. Consequently, $V(t_K) \leq V(T_1) \leq (\bar{c}|x_{10}| + \bar{c}^{1+\alpha_1/\alpha_2} q^{\frac{T_1}{\tau}} |x_{20}| T_1)^2$. Under Condition 1, one has that $\lim_{t\to\infty} V^{\alpha_3}(t) < 0$. Therefore, $x_1(t)$ converges to zero within finite time. In conclusion, the second-order systems (3.1) and (3.2) is globally finite-time stable over the class $\mathcal{F}_\tau^1$. □

Remark 4. *In [22], a non-singular TSMC was designed for the finite-time stabilization of second-order system subject to impulsive disturbances involving time-invariant jump maps. However, the established FTS results are local sense, since the convergence of the state on the sliding surface depends on the reached point. In contrast, Theorem 8 presents the global FTS theorem for hybrid impulsive disturbances involving destabilizing cumulative effect. Even though the hybrid impulsive disturbances have destabilizing cumulative effect, the system is capable of absorbing the impulsive disturbances by the designed sliding-mode control strategies.*

Remark 5. *It should be pointed out that Assumption 2 ensures the continuity of the sliding variable at impulse time after the sliding surface is reached, because* $s(t_K) = c_{2K} x_2(t_K^-) + \beta c_{1K}(x_1(t_K^-))^{\alpha_1/\alpha_2} = c_{2K} s(t_K^-) = 0$ *for* $k \geq K$ *where* K *is defined in the proof of Theorem 7. Thus, the sliding variable is continuous along the solution of systems* (3.1) *and* (3.2) *after the sliding surface is reached. Note that Assumptions 1 and 2 are indispensable to achieve asymptotic stabilization and finite-time stabilization of the second-order system, since they avoid the escapement of states on the sliding surface. However, these assumptions can be relaxed, if the impulsive disturbance has finite jump times (See the following corollaries and Example 2 for details).*

Corollary 1. *For any positive integer* N *such that* $\prod_{i=1}^{N} c_{2i} \leq 1$*, the second-order systems* (3.1) *and* (3.2) *is globally finite-time stable over the class* $\mathcal{F}_N^2$ *by the control laws* (3.9) *and* (3.10).

Proof. The proof is similar to Theorem 7, so we just sketch the outline and focus on the different part. Denote $t_{N+1} = \infty$, $\bar{c}_2 = 1 \vee \max_{\Lambda\subset\bar{N}}\{\prod_{i\in\Lambda} c_{2i}\}$ and $\underline{c}_2 = 1 \wedge \min_{\Lambda\subset\bar{N}}\{\prod_{i\in\Lambda} c_{2i}\}$.

In Step 1, it follows from (3.11) and Theorem 1 that $x_2(t)$ converges to zero with settling time $T_1 = \frac{\bar{c}_2 |x_{20}|}{\underline{c}_2 (p-d^*)}$, that is, the trajectories of the system move into $\mathbb{S}_2$ within finite time.

In Step 2, (3.21) holds for $t \geq T_N$ in $\mathbb{S}_2$. Denote $b(t) = \frac{\alpha_1 x_2(t)}{\alpha_2 x_1(t)}$ for $t \geq T_N$. Then, the comparison system is expressed by

$$\begin{cases} \dot{r}(t) = -p\text{sign}(r(t)) - p' r(t) + d(t) + d' + b(t) r(t), \ t \geq T_N, \\ r(T_N) = r_0, \end{cases} \tag{3.23}$$

where d' is a constant which satisfies $0 < d' < p - d^*$. Based on Theorem 1 with $\gamma_i = 1$, $r(t)$ is finite-time stable and consequently the systems (3.1) and (3.2) reaches the sliding surface $s(t) = 0$ within finite time in $\mathbb{S}_2$.

In Part II, it holds that

$$\dot{x}_1(t) = -\beta(x_1(t))^{\alpha_1/\alpha_2}, \tag{3.24}$$

on the sliding surface. Based on Example 2.1 in [30], the system converges to the origin within finite time. □

Corollary 2. *For any positive integer N such that* $\prod_{i=1}^{N} c_{2i} > 1$, *the second-order system* (3.1)-(3.2) *is globally finite-time stable over the class* $\mathcal{F}_\tau^1 \cap \mathcal{F}_N^2$ *by the control law* (3.9)-(3.10) *with* p' *satisfying* $-p' + \frac{\ln q}{\tau} < 0$, *where* $q = \sqrt[N]{\prod_{i=1}^{N} c_{2i}}$.

Proof. The corollary is the direct consequence of Theorem 3 by the same line as the proof of Corollary 1. The details are omitted. □

Remark 6. *Note that the restriction of time-varying jump maps is relaxed in Corollaries 1 and 2. Then, the trajectories of the system may escape from the sliding surface due to the hybrid impulsive disturbances. In spite of this, the designed non-singular TSMC strategy is able to stabilize the second-order system within finite time.*

4. Numerical examples

Example 1. *Consider the second-order systems* (3.1) *and* (3.2) *with* $f(x) = x_2^2$, $g(x) = 1$, $d(t) = \sin(\frac{\pi t}{2})$. *The impulse strength is set by* $c_{1k} = c_{2k}$ *for* $k \in \mathbb{N}_+$, $c_{1k} = \frac{13}{10}$ *for* $k = 3m + 1$, $c_{1k} = \frac{3}{5}$ *for* $k = 3m + 2$, $c_{1k} = \frac{6}{5}$ *for* $k = 3m + 3$, $m \in \mathbb{N}$, *to satisfy Assumption 1. Consider the sliding variable* $s = x_2 + \frac{1}{2}x_1$ *and controller* $u(x) = -x_2^2 - \frac{1}{2}x_2 - \frac{11}{10}\mathrm{sign}(s)$. *According to Theorem 4, the system is globally asymptotically stable over the class* $\mathcal{F}_0$. *Figure 1 presents the state trajectories of the system subject to hybrid impulsive disturbances with different impulse times: (a)* $t_k = 0.1k$, $k \in \mathbb{N}_+$; *(b)* $t_{3(k-1)+1} = 0.06(k-1) + 0.03$, $t_{3(k-1)+2} = 0.06(k-1) + 0.05$, $t_{3k} = 0.06k$, $k \in \mathbb{N}_+$. *As shown in Figure 1, the state trajectories converge to the origin, which verifies the theoretical conclusion.*

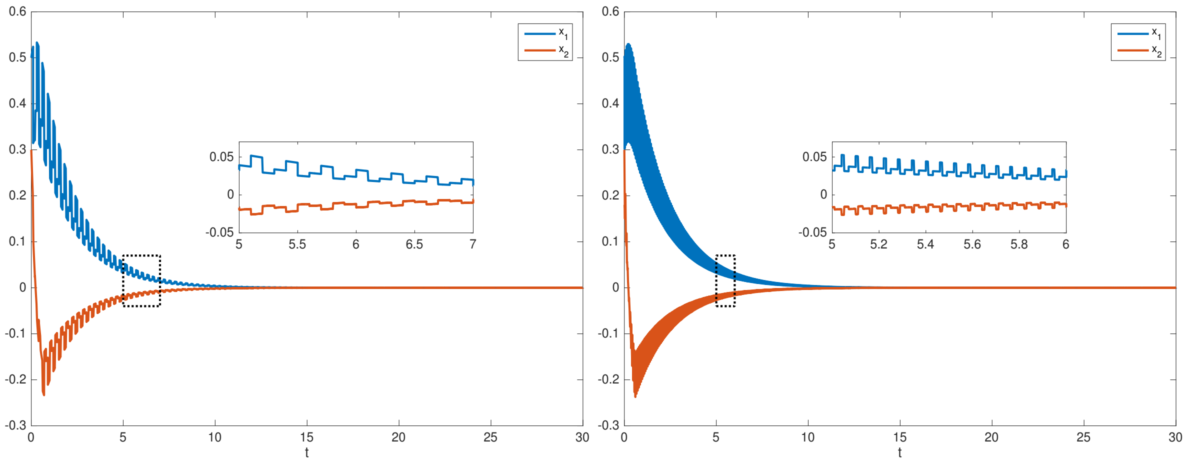

Figure 1. Trajectories of second-order systems (3.1) and (3.2) in Example 1.

Example 2. *Consider the second-order systems* (3.1) *and* (3.2) *with* $f(x) = -\frac{1}{2}x_2$, $g(x) = 1$, $d(t) = \frac{1}{2}\sin(\frac{\pi t}{2})$. *Here, we consider two types of impulsive disturbances: (i) The hybrid impulses with stabilizing cumulative effect; (ii) The hybrid impulses with destabilizing cumulative effect.*

Type i: $c_{1k} = c_{2k}^3$ *for* $k \in \mathbb{N}_+$, $c_{2k} = \frac{6}{5}$ *for* $k = 3m + 1$, $c_{2k} = \frac{7}{10}$ *for* $k = 3m + 2$, $c_{2k} = \frac{11}{10}$ *for* $k = 3m + 3$, $m \in \mathbb{N}$. *Figure 2 depicts the state trajectories of the second-order system with initial* *conditions* $[x_{10}, x_{20}]^T = [0.5, 0.7]^T$ *and* $[x_{10}, x_{20}]^T = [-0.5, -0.7]^T$ *which are obviously unstable. To ensure the FTS of the second-order system, the sliding variable* (3.9) *and controller* (3.10) *are designed by* $\beta = \frac{1}{2}$, $p = \frac{6}{5}$ *and* $p' = 0$. *Then, Assumption 2 is satisfied with* $\alpha_1 = 1$ *and* $\alpha_2 = 3$. *According to Theorem 7, the system is globally finite-time stable over the class* $\mathcal{F}_0$. *The trajectories of the system are presented in Figure 3, from which we can see the convergence of trajectories to the origin.*

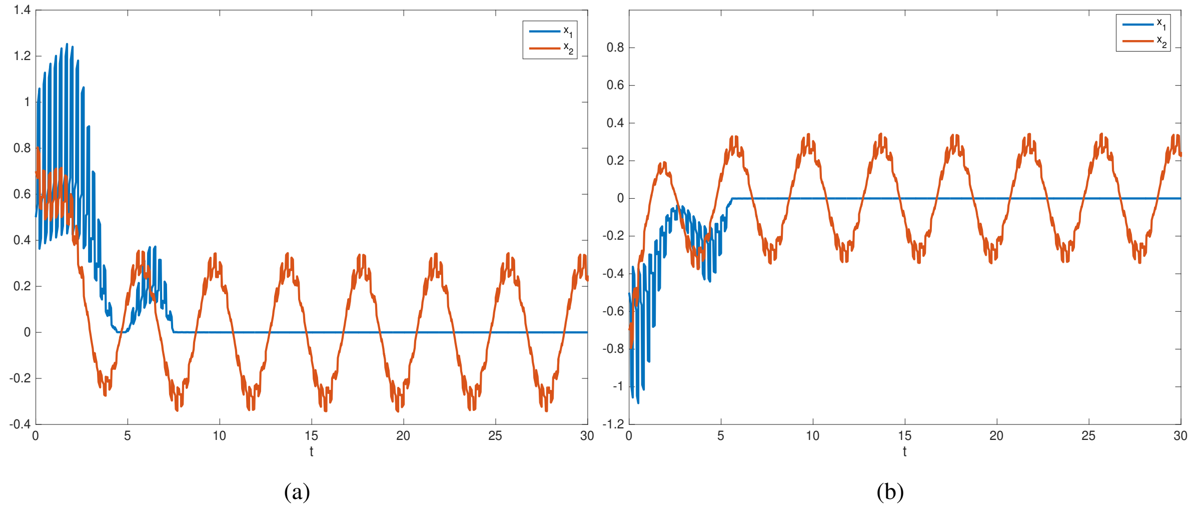

Figure 2. Trajectories of second-order system subject to hybrid impulsive disturbances of type i without control in Example 2. (a) $[x_{10}, x_{20}]^T = [0.5, 0.7]^T$. (b) $[x_{10}, x_{20}]^T = [-0.5, -0.7]^T$.

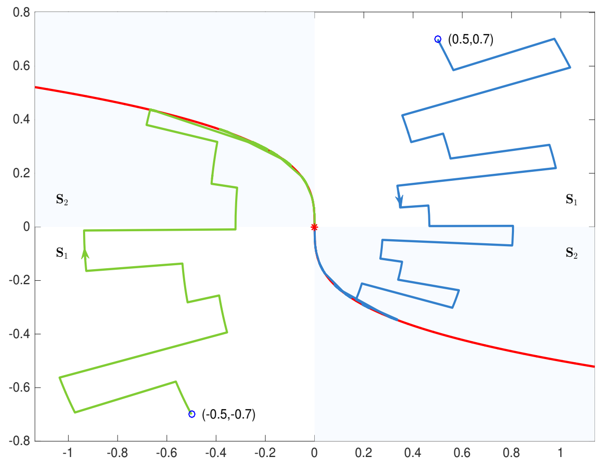

Figure 3. Trajectories of second-order system subject to hybrid impulsive disturbances of type i under control in Example 2.

Type ii: $c_{1k} = 1$ *for* $k \in \mathbb{N}_+$, $c_{2k} = \frac{7}{5}$ *for* $k = 3m + 1$, $c_{2k} = \frac{9}{10}$ *for* $k = 3m + 2$, $c_{2k} = \frac{3}{2}$ *for* $k = 3m + 3$, $m \in \mathbb{N}$. *Here, we consider the impulses occur finite times:* $t_k = 0.1k$, $k = \overline{50}$. *Figure 4 depicts the trajectories of the second-order system subject to this kind of hybrid impulsive disturbance. From the figure, we see that the system is unstable even though impulses occur finite times. For FTS, the sliding variable* (3.9) *and controller* (3.10) *are designed by* $\beta = \frac{1}{2}$, $p = \frac{6}{5}$, *and* $p' = 2.13$. *According* *to Corollary 2, the system is globally finite-time stable over the class* $\mathcal{F}_N \cap \mathcal{F}_\tau$ *where* $N = 50$ *and* $\tau = 0.1$. *The state trajectories of the system are presented in Figure 5, from which we can see that the trajectories move towards the origin even though they may escape from the sliding surface.*

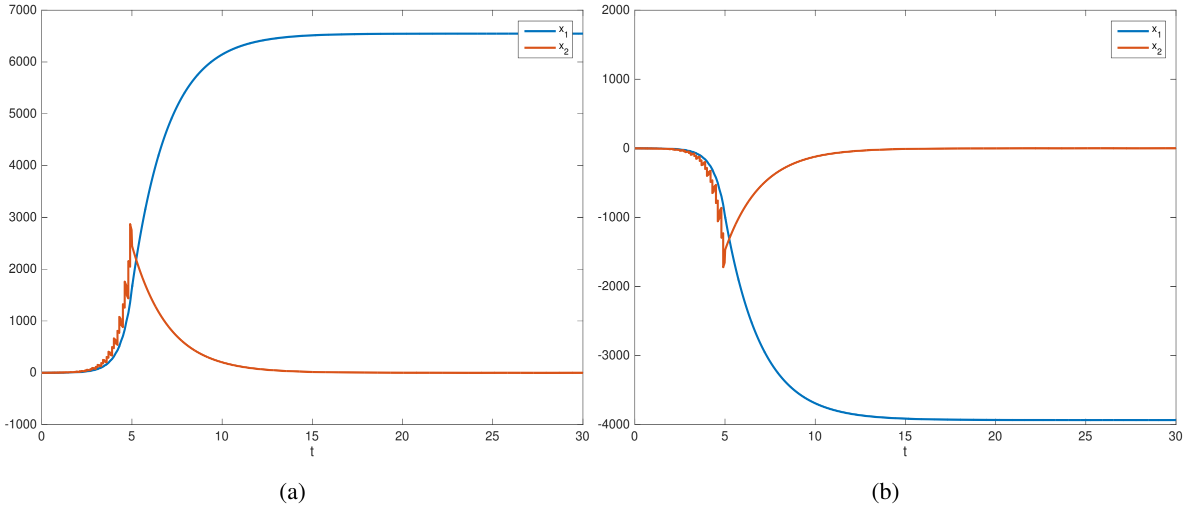

Figure 4. Trajectories of second-order system subject to hybrid impulsive disturbances of type ii without control in Example 2. (a) $[x_{10}, x_{20}]^T = [0.5, 0.7]^T$. (b) $[x_{10}, x_{20}]^T = [-0.5, -0.7]^T$.

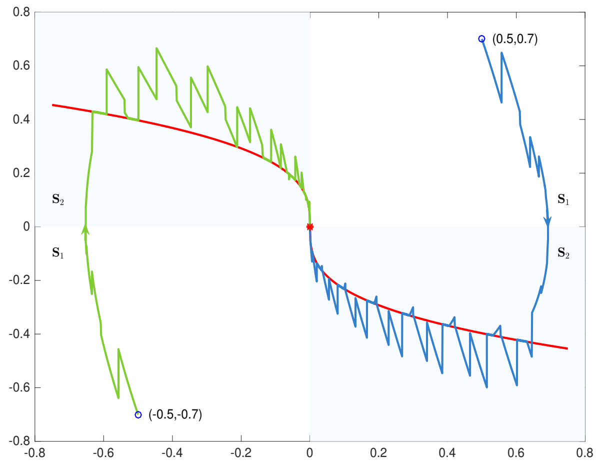

Figure 5. Trajectories of second-order system subject to hybrid impulsive disturbances of type ii under control in Example 2.

Example 3. *As an application example, we consider the tracking control of linear motor [11] subject to hybrid disturbances that consists of lumped uncertainty and hybrid impulses. The simplified model can be expressed by*

$$\begin{cases} \dot{w}(t) = v(t),\ t \geq 0,\ t \neq t_k, \\ m\dot{v}(t) = (1 - k_m)(k_f u(t) - h(v(t)) + d(t)),\ t \geq 0,\ t \neq t_k, \\ v(t_k) = I_k(v(t_k^-)),\ t_k = 0.1k,\ k \in \overline{300}, \end{cases} \tag{4.1}$$

where w is the position, v is the velocity, m stands for the mass of the moving stage, $0 \le k_m < 1$ *refers to the gain variation due to the load mass,* k_f *represents the voltage-to-force amplifier gain, u is the control input,* $h(v) = k_c sign(v) + k_v v$ *is the simplified friction that consists of Coulomb friction and viscous friction where* k_c *is the Coulomb friction coefficient and* k_v *is the viscous friction coefficient, d is the lumped uncertainty, and* I_k *is the impulsive disturbance. Denote the desired position and velocity by* w_d *and* v_d *and define the tracking error and velocity error by* $w_e = w - w_d$ *and* $v_e = v - v_d$*. Then, the error system is expressed by*

$$\begin{cases} \dot{w}_e(t) = v_e(t), \ t \ge 0, \ t \ne t_k, \\ m\dot{v}_e(t) = k_{mf}u(t) + f(t) + d_e(t), \ t \ge 0, \ t \ne t_k, \\ v_e(t_k) = IE_k, \ t_k = 0.1k, \ k \in \overline{300}, \end{cases} \tag{4.2}$$

where $k_{mf} = k_f(1 - k_m)$, $f(t) = -(1 - k_m)h - m\dot{v}_d$, $d_e = (1 - k_m)d$, $IE_k = I_k(v(t_k^-)) - v_d(t_k^-)$*. Suppose that* $IE_k = c_k v_e(t_k^-)$ *where* c_k *is the time-varying strength of impulsive disturbances. The simulated position of the tracking system* (4.1) *is presented in Figure 6(a) where* $m = 11$ *kg,* $k_m = 0.03$, $k_f = 10$ *N/V,* $k_c = 2.91$ *N,* $k_v = 0.08$ *Ns/mm,* $c_{3m+1} = 1.5$, $c_{3m+2} = 0.3$, $c_{3m+3} = 2$, $m \in \mathbb{N}$*. From the numerical simulation, we can see that the tracking position fails to track the desired one. To accomplish the tracking task, the sliding variable* (3.9) *and controller* (3.10) *are designed by* $\beta = \frac{1}{2}$, $p = 2$*, and* $p' = 0$*. According to Corollary 1, the error system is finite-time stable by the control laws* (3.9) *and* (3.10) *over the class* $\mathcal{F}_N^2$ *where* $N = 300$*. Figure 6(b) depicts the trajectories of system* (4.1) *under control. Obviously, the position of the tracking system is capable of tracking the desired position within finite time, which corresponds to the global FTS of the error system.*

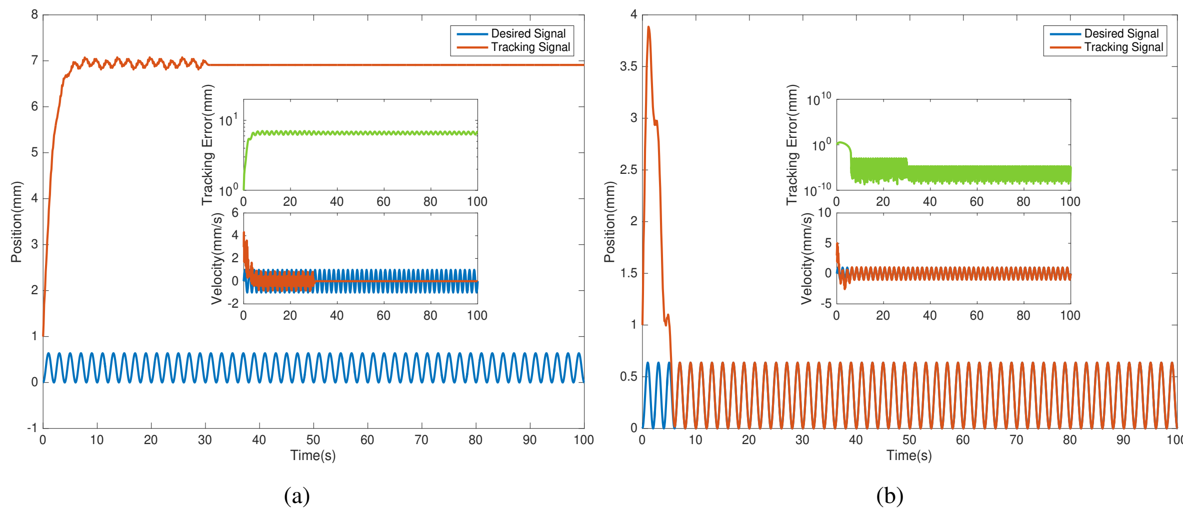

Figure 6. Simulation of linear motor in Example 3. (a) System without control. (b) System under SMC.

5. Conclusions

In this paper, several criteria for robust finite-time stability of a scalar impulsive system are given where the hybrid effect of time-dependent impulses with time-varying jump maps is fully considered. Based on the robust finite-time stability results, a linear SMC strategy and a non-singular TSMC strategy are designed for asymptotic stabilization and finite-time stabilization of second-order system subject to hybrid disturbances that consists of matched disturbances and hybrid impulsive disturbances. It shows that the designed SMC strategies are robust against the hybrid disturbances, especially the hybrid impulsive disturbances with both non-destabilizing and destabilizing cumulative effect. Note that the FTS results in this paper are established for impulsive systems with stable flows. In [35, 36], novel dwell-time conditions are given to guarantee the input-to-state stability of impulsive systems with unstable continuous and discrete dynamics. Thus, the FTS of such systems will be investigated under the novel dwell-time condition in the future. In addition, the SMC strategies are designed by the system state which is not easily fetched in practical systems such as aircraft engine and robot arm [9, 37]. Therefore, another future work will be the observer-based SMC design for nonlinear systems subject to hybrid disturbances.

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Conflict of interest

The authors declare there is no conflict of interest.

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