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*Research article*

# Finite-time stability of equilibrium solutions for the inertial neural networks via the figure analysis method

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**Abstract:** The existence and finite-time stability (FTS) of equilibrium point (EP) for a kind of inertial neural networks (INNS) with varying-time delays is studied. Firstly, by adopting the degree theory and the maximum-valued method, a sufficient condition in the existence of EP is attained. Then by adopting the maximum-valued approach and the figure analysis approach, without adopting the matrix measure theory, linear matrix inequality (LMI), and FTS theorems, a sufficient condition in the FTS of EP for the discussed INNS is proposed.

**Keywords:** EP; FTS; degree theory; maximum-valued way; INNS; figure analysis way

## 1. Introduction

In 1986, Babcock and Westervelt [1] first introduced an inertial term into neural networks (NNS). Because the addition of inertial terms can generate complicated bifurcation behavior and chaos, the dynamics of the INNS can become more complex when the neuron couplings contain an inertial nature. By now, the dynamic properties of all kinds of INNS have been broadly explored and many results have been achieved in the existence and stability of the EP [2–7], and synchronization [8–10].

So far, the existence and finite-time stability (FTS) of periodic solutions of NNS have been deeply approached, see [11–26]. In [11], the problem of FTS of neutral Hopfield NNs with mixed delays including infinite distributed delays was explored. Some specific results on the FTS of the NNS in the form of linear matrix inequalities (LMLS) were derived by devising different kinds of controllers. In [12], the impulsive effects of the FTS of NNS with time-varying delays were considered. Several novel criteria in the form of LMI in FTS of the networks were achieved by adopting Lyapunov-Krasovskill functional and the average impulsive interval way. In [13], the FTS of fractional-order BAN NNS with mixed time-varying delays was approached. The result in the existence-uniqueness and the FTS of the systems were attained by applying some inequalities. In [14], the FTS of complex-valued BAM NNS with time delay was studied. Using a nonlinear matrix measure way, a condition in the existence-uniqueness of the equilibrium point EP for the system was achieved. Then, a fixed-time stability criterion of the EP was gained in terms of LMIS by the Lyapunov function method. In the article [15], stability analysis for fractional-order NNS was addressed. By adopting the contracting mapping principle, iteration way and inequality skills, the conditions in the existence-uniqueness, and FTS of the EP of the proposed networks were given. In [16], a general class of delayed memristive NNS system described by a functional differential equation with discontinuous right-hand side was investigated. By adopting the FTS theorem and the generalized Lyapunov functional method, several results were given to assure the FTS of the networks. In [17], the FTS and fixed-time stability problem for a class of BAM NNS were concerned. On the basis of FTS theorems, new conditions in the FTS for the networks were derived. In [18], the FTS of Markovian jump NNS with partly unknown transition probabilities was discussed. Via Lyapunov stability theory, two sufficient conditions were derived to assure the finite-time stability of the networks. The existence and FTS conditions of EP for fractional-order BAM NNS were discussed in [19]. Some sufficient conditions in the FTS for the networks were gained by Bellman-Gronwall inequality and contraction mapping. The analysis for the FTS of a kind of fractional-order complex-valued NNS was discussed in [20], adopting the Gronwall inequality, Cauchy-Schiwartz inequality, some criteria in the FTS for the networks were gained. In [21], the authors studied the fix-time stability problem of neutral-type NNS, by using the FTS theorems, some simple conditions were attained to assure the FTS of the networks by using inequality skills. In the [22], the issue of FTS of a kind of fractional-order Cohen-Grossberg BAM NNS was discussed, by applying some inequality skills and contraction mapping principle, and some sufficient conditions were put forward to assure the FTS of the networks. In [23], the authors researched a kind of fuzzy NNS of fractional-order and proportional time delay, some sufficient criteria for the FTS of the networks were attained by means of the FTS theorem, fractional calculus, and differential inequality skills. In [24], FTS for fractional-order complex-valued memristive NNS with proportional delays was discussed, by utilizing Holder inequality, Gronwall inequality and inequality skills, some criteria were attained to assure the FTS of the discussed networks. In [25], a kind of delayed memristor-based fractional-order NNS were studied, by utilizing the way of iteration, contracting mapping principle, and set-valued mapping, a new condition in the existence-uniqueness and the FTS of the EP of the networks was given. In article [22], because of the tangency or non-tangency of the periodic solutions to a certain surface, some sufficient conditions were given to guarantee the global convergence in finite time about the periodic solutions of the networks.

By now, the researchers have studied the existence-uniqueness of the EP of the NNS usually by adopting the homeomorphism theorem [2, 3, 27], the matrix measure approach [14], the contraction mapping principle [15, 19, 22, 23, 25], the LMI approach [2, 3, 11, 12, 14, 27–29] and the degree theory way [30]. At the same time, the researchers have studied the FTS of NNS usually by adopting the FTS theorems [13–19, 21–25], the LMI [11, 12] and the inequality skills [19, 20, 24]. Hence, it is urgent for us to find a new study method to research the FTS of NNS. Since the figure analysis method and maximum-valued approach can greatly reduce the complexity of the proofs of the various finite-time stability theorems, this inspires us to study the finite-time stability of EP for NNS by using the figure analysis method and the maximum-valued approach.

For the NNS without the controllers, it is very difficult for us to achieve the sufficient conditions in the FTS. Hence, we would like to study the FTS of NNS with the controllers by using the figure analysis method and maximum-valued approach.

This constitutes the purpose of the paper. In this paper, we discuss the existence and FTS of the EP for the following INNS:

$$u''_p(t) = -\alpha_p u'_p(t) - \gamma_p u_p(t) + \sum_{q=1}^{n} c_{pq} F_q(u_q(t)) + \sum_{q=1}^{n} d_{pq} F_q(u_q(t-\tau(t))) + I_p, \tag{1.1}$$

in which, $p = 1, 2, \cdots, n$ and $n$ is the number of units in the neural network, $u_p(t) \in R$ corresponds to the state vector of the $p$th unit at time $t$; $\alpha_p \geq 0, \gamma_p \geq 0; c_{pq}, d_{pq} \in R$ are the first and second-order connection weights of the neural network at time $t, \tau(t) \geq 0$ are the transmission delays, $I_p \in R$ is the external input, and $F_q : R \to R$ is the activation function of signal transmission.

The initial values of the system (1.1) are

$$\begin{aligned} u_p(s) &= \phi^*_p(s), \\ u'_p(s) &= \psi^*_p(s), \quad s \in [-\rho, 0], \quad p = 1, 2, \cdots, n, \end{aligned} \tag{1.2}$$

in which $\phi^*_p, \psi^*_p \in C([-\rho, 0], R), \rho = \max_{0<t<\infty}\{\tau(t)\}$.

In this article, firstly, by utilizing degree theory and the maximum-valued way, we study the existence of EP of the system (1.1), then we discuss the FTS of EP of the system (1.1) by devising the novel controller, adopting the maximum-valued way and figure analysis way.

So, the contribution of the paper is reflected as follows: (1) The maximum-valued way is introduced to study the existence of EP in our article; (2) The figure analysis way is cited to study the FTS of EP; (3) Novel sufficient conditions in the existence and FTS of EP for the system (1.1) are gained by adopting the figure analysis way and the maximum-valued way.

## 2. Preliminaries

In this article, we usually suppose that

$(v_1)$

$$\tau'(t) \leq \tau^* < 1$$

.

$(v_2)$ There exists positive constant $L_q$ such that for all $u, v \in R$,

$$|F_q(u) - F_q(v)| \leq L_q|u - v|,$$

where $|.|$ is the norm of R. The following notations will be used in proofs of the main Theorems:

$$k = \max\{-0.5\xi_p, 0.5(\xi_p - \alpha_p)\},$$

$$k_1 = -0.5\xi_p + 0.5L_p \sum_{q=1}^{n} \Big[|c_{pq}| + \frac{|d_{pq}|}{1 - \tau*}\Big],$$

$$k_2 = 0.5(\xi_p - \alpha_p) + 0.5\sum_{q=1}^{n} L_q(|c_{qp}| + |d_{qp}|) + 0.5\sum_{q=1}^{n} |F_q(0)|;$$

$$k_3 = 1 - \xi_p^2 - \gamma_p + \alpha_p, k_4 = -\xi_p + 0.5L_p\sum_{q=1}^{n}\Big[|c_{qp}| + \frac{|d_{qp}|}{1-\tau^*}\Big],$$

$$k_6 = \xi_p - \alpha_p + 0.5\sum_{q=1}^{n} L_q(|c_{pq}| + |d_{pq}|) + 0.5\sum_{q=1}^{n} |F_q(0)|.$$

Let $(u_1, u_2, \cdots, u_n)^T$ be a solution of system (1.1) with (1.2). Making variable transformation: $v_p(t) = u_p'(t) + \xi_p u_p(t), \xi_p > 0$ is a chosen constant, $p = 1, 2, \cdots, n$, then system (1.1) is changed into for $p = 1, 2, \cdots, n$,

$$\begin{cases} u_p'(t) = -\xi_p u_p(t) + v_p(t), \\ v_p'(t) = -[\xi_p^2 + \gamma_p - \alpha_p]u_p(t) + (\xi_p - \alpha_p)v_p(t) + \sum\limits_{q=1}^{n} d_{pq}F_q(u_q(t-\tau(t))) \\ \quad + \sum\limits_{q=1}^{n} c_{pq}F_q(u_q(t)) + I_p, \end{cases} \tag{2.1}$$

with the initial values:

$$u_p(s) = \phi_p^*(s), v_p(s) = \psi_p^*(s) + \xi_p\phi_p^*(s), s \in [-\rho, 0],$$

where $p = 1, 2, \cdots, n$.

Because the existence and the FTS of EP of system (1.1) are equivalent to the existence and FTS of EP of system (2.1), consequently, we only require to show the existence and the FTS of EP of system (2.1).

**Lemma 2.1( [21])**. Let $G(\lambda^*, v) : [0, 1]\times\bar{\Omega}^* \to R^p$ be a continuous homotopic mapping. If $G(\lambda^*, v) = u$ has no solutions in $\partial\Omega^*$ for$\lambda^* \in [0, 1]$ and $u \in R^p/G(\lambda^*, \partial\Omega^*)$, where $\partial\Omega^*$ is the boundary of $\Omega^*$, then topological degree $\deg(G(\lambda^*, v), \Omega^*, u)$ of $G(\lambda^*, v)$ is a constant which is independent of $\lambda^*$. Thus, deg(G(0, v),$\Omega^*, u) = deg(G(1, v), \Omega^*, u)$.

**Lemma 2.2( [21])**. Let $G(v) : \Omega^* \to R^p$ be a continuous mapping. If $deg(G(v), \Omega^*, u)$
$\neq 0$, then there is at least a solution of $G(v) = u$ in $\Omega^*$.

**Lemma 2.3( [21])**. Let $\Omega^* \subset R^p$ be a non-nempty, bounded open set and $g : R^p \to R^p$ be an $\Omega^*$-admissible map, i.e., $g(u) \neq 0$ for all $u \in \partial\Omega^*$, then $deg(g, \Omega^*) = \sum\limits_{u\in g^{-1}(0)\cup\Omega^*} signdetDg(u)$, where, $detDg(u)$ denotes Jacobi-determinant of $g(u)$ at point $u$, $signDg(u)$ is the symbol of Jacobi-determinant of $g(u)$ at point $u$.

**Notation 1**. It is very difficult to show the inequality of exponential type and logarithmic type. In proving the inequality of exponential type and logarithmic type, the proving can be transformed into the comparison of down and up between the two figures of the two functions.

**Definition 2.1** The EP $(u_1^*(t), u_2^*(t), \cdots, u_n^*(t), v_1^*(t), v_2^*(t), \cdots, v_n^*(t))^T$ of system (2.1) is said to be finite-time stable if there exists a time $t_1 > 0$ related to the inertial values of system (4.1) such that for each solution $(u_1(t), u_2(t), \cdots, u_n(t), v_1(t), v_2(t), \cdots, v_n(t))^T$ of system (4.1), we have $\lim_{t\to t_1} |u_p(t) - u_p^*(t)| = 0, \lim_{t\to t_1} |v_p(t) - v_p^*(t)| = 0$.

**Lemma 2.4 (Figure analysis way)** If $m_3 < 0, m_1 > 0, m_2 > 0, m_4 > 0$, then there exists a point $x_0 > 0$ such that when $x < x_0, (m_1 + m_2 e^{m_3 x}) - m_4 x^2 > 0; x > x_0, (m_1 + m_2 e^{m_3 x}) - m_4 x^2 < 0$.
Proof. Setting $F(x) = m_1 + m_2 e^{m_3 x} - m_4 x^2, x > 0$, thus $\lim_{x\to 0^+} F(x) = m_1 + m_2 > 0, \lim_{x\to+\infty} F_1(x) = -\infty < 0$. Because of the continuation of $F(x)$, then there is a point $x_0 \in (0, +\infty)$ such that $F(x_0) = 0$. Namely two figures of two functions $f(x) = m_1 + m_2 e^{m_3 x}$ and $g(x) = m_4 x^2$ have a crossover point at $x_0$. By the figure analysis way, we find that when $x \in (0, x_0)$, the figure of function $f(x)$ is above the image of $g(x)$, while when $x \in (x_0, +\infty)$, the figure of function $f(x)$ is below the image of $g(x)$. Then it follows that when $x < x_0, m_1 + m_2 e^{m_3 x} - m_4 x^2 > 0; x > x_0, m_1 + m_2 e^{m_3 x} - m_4 x^2 < 0$. This finishes the proof of Lemma 2.2. The following Figure 1 is the figures of $f(x)$ and $g(x)$:

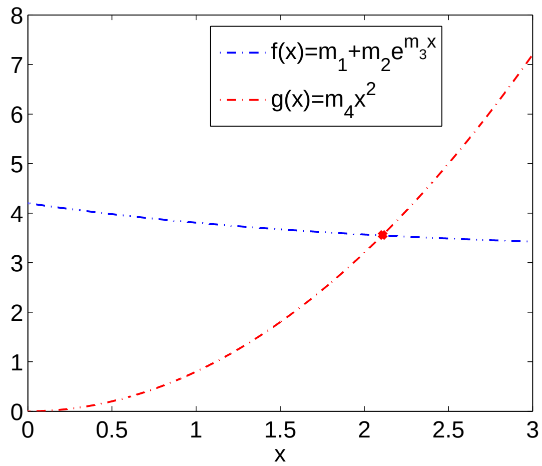


**Figure 1.** Curves of the $f(x)$ and $g(x)$.

## 3. Main results

Let $R^{2n}$ be a $2n$-dimensional Euclidean space, which is endowed with a norm $\|.\|$, where $\|(u, v)^T\| = \sqrt{\sum_{p=1}^{n} (|u_i|^2 + |v_i|^2)}, u = (u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n) \in R^{2n}$, $|.|$ denotes Euclidean norm in $R$.

**Theorem 3.1**. Let $(v_1)$ and $(v_2)$ hold. Furthermore suppose that

$(v_3)$

$$0 < \xi_p < \alpha_p, \quad \xi_p(\xi_p - \alpha_p) + \xi_p^2 + \gamma_p - \alpha_p \neq 0;$$

$(v_4)$

$$k_1 < 0, k_3^2 < 4k_1 k_2.$$

Then system (2.1) has at least an EP.
**Notation 1.** $(v_3)$ implies
$(v_5)$

$$k < 0, k_1 > k_4, k_2 > k_6.$$

$(v_4)$ and $(v_5)$ imply
$(v_6)$

$$k_2 < 0, k_4 < 0, k_6 < 0.$$

The Proof of Theorem 3.1.
**Proof:** Let $(u^*, v^*) \in R^n \times R^n$ be an EP of system (2.1), where $u^* = (u_1^*, u_2^*, \cdots, u_n^*), v^* = (v_1^*, v_2^*, \cdots, v_n^*)$, then the EP $(u^*, v^*)$ satisfies the following equations:

$$\begin{cases} -\xi_p u_p^* + v_p^* = 0, \\ -(\xi_p^2 + \gamma_p - \alpha_p)u_p^* + (\xi_p - \alpha_p)v_p^* + \sum\limits_{q=1}^{n}(c_{pq} + d_{pq})F_q(u_q^*) + I_p = 0. \end{cases} \tag{3.1}$$

Inspired by (3.1), we let

$$\begin{aligned} & G(u_1, u_2, \cdots, u_n; v_1, v_2, \cdots, v_n) \\ = & \begin{Bmatrix} -\xi_1 u_1 + v_1 \\ -\xi_2 u_2 + v_2 \\ \vdots \\ -\xi_n u_n + v_n \\ -(\xi_1^2 + \gamma_1 - \alpha_1)u_1 + (\xi_1 - \alpha_1)v_1 + \sum\limits_{q=1}^{n}(c_{1q} + d_{1q})F_q(u_1) + I_1 \\ -(\xi_2^2 + \gamma_2 - \alpha_2)u_2 + (\xi_2 - \alpha_2)v_2 + \sum\limits_{q=1}^{n}(c_{2q} + d_{2q})F_q(u_2) + I_2 \\ \vdots \\ -(\xi_n^2 + \gamma_n - \alpha_n)u_n + (\xi_n - \alpha_n)v_n + \sum\limits_{q=1}^{n}(c_{nq} + d_{nq})F_q(u_n) + I_n \end{Bmatrix} = 0. \end{aligned} \tag{3.2}$$

Apparently, the solutions to (3.2) are the EP of system (2.1). We devise a mapping as follows:

$$\begin{aligned} & H^*(u_1, u_2, \cdots, u_n; v_1, v_2, \cdots, v_n; \mu) \\ = & \mu G(u_1, u_2, \cdots, u_n; v_1, \cdots, v_2, \cdots, v_n) + (1 - \mu)\Big[ - \xi_1 u_1 + v_1, -\xi_2 u_2 + v_2, \cdots, -\xi_n u_n \\ & +v_n, -(\xi_1^2 + \gamma_1 - \alpha_1)u_1 + (\xi_1 - \alpha_1)v_1 + I_1, \cdots, -(\xi_n^2 + \gamma_n - \alpha_n)u_n + (\xi_n - \alpha_n)v_n + I_n\Big], \end{aligned}$$

where $\mu \in [0, 1]$ ia a parameter. Let $u = (u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n), \Omega^* = \{u \in R^n \times R^n : \|u^T\| < d\}$, where $d > -\frac{1}{k}\Big[0.5n \sum\limits_{q=1}^{n} |F_q(0)| + \sum\limits_{p=1}^{n} \frac{2k_1 I_p^2}{k_3^2 - 4k_1 k_2}\Big]$.
We will show that $H^*(u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n, \mu)^T$ is a homotopic mapping. In order to this end, we prove that $H^*(u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n, \mu) \neq (0, 0, \cdots, 0)^T, u \in \partial\Omega^* \cup R^{2n}$. If $H^*(u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n, \mu) = (0, 0, \cdots, 0)^T, u \in \partial\Omega^* \cup R^{2n}$, then the constant vector $u$ with $\|u\| = d$ satisfies for $p = 1, 2, \cdots, n$,

$$-\xi_p u_p + v_p = 0 \tag{3.3}$$

$$\mu\Big[ - (\xi_p^2 + \gamma_p - \alpha_p)u_p + (\xi_p - \alpha_p)v_p + \sum_{q=1}^{n}(c_{pq} + d_{pq})F_q(u_q) + I_p\Big]$$

$$+(1-\mu)\Big[-(\xi_p^2+\gamma_p-\alpha_p)u_p+(\xi_p-\alpha_p)v_p+I_p\Big]=0. \tag{3.4}$$

Then by (3.3) and (3.4), we get

$$\begin{aligned}
0 &= \sum_{p=1}^{n}\Big\{u_p(-\xi_p u_p+v_p)+v_p\mu\Big[-(\xi_p^2+\gamma_p-\alpha_p)u_p+(\xi_p-\alpha_p)v_p+\sum_{q=1}^{n}(c_{pq}+d_{pq})F_q(u_q)\\
&\quad +I_p\Big]+v_p(1-\mu)\Big[-(\xi_p^2+\gamma_p-\alpha_p)u_p+(\xi_p-\alpha_p)v_p+I_p\Big]\Big\}\\
&= \sum_{p=1}^{n}\Big\{u_p(-\xi_p u_p+v_p)+v_p\Big[-(\xi_p^2+\gamma_p-\alpha_p)u_p+(\xi_p-\alpha_p)v_p+I_p+\mu\Big(\sum_{q=1}^{n}(c_{pq}+d_{pq})\\
&\quad \times F_q(u_q)\Big)\Big]\Big\}\\
&\le \sum_{p=1}^{q}\Big\{-\xi_p u_p^2+(\xi_p-\alpha_p)v_p^2+(1-\xi_p^2-\gamma_p+\alpha_p)u_p v_p+I_p v_p+|v_p|\sum_{q=1}^{n}(|c_{pq}|+|d_{pq}|)\\
&\quad \times|F_q(u_q)|\Big\}.
\end{aligned} \tag{3.5}$$

By $(v_2)$, we get by applying inequality $2ab \le a^2+b^2$,

$$\begin{aligned}
& |v_p||F_q(u_q)|\\
\le\ & |v_p||F_q(u_q)-F_q(0)|+|v_p||F_q(0)|\\
\le\ & L_q|v_p||u_q|+|v_p||F_q(0)|\\
\le\ & 0.5L_q(u_q^2+v_p^2)+0.5|F_q(0)|(v_p^2+1).
\end{aligned} \tag{3.6}$$

Substituting (3.6) into (3.5) gives

$$\begin{aligned}
& 0\\
\le\ & \sum_{p=1}^{n}\Big\{-\xi_p u_p^2+(\xi_p-\alpha_p)v_p^2+(1-\xi_p^2-\gamma_p+\alpha_p)u_p v_p+I_p v_p+\sum_{q=1}^{n}(|c_{pq}|+|d_{pq}|)\times\\
& [0.5L_q(u_q^2+v_p^2)+0.5|F_q(0)|(v_p^2+1)]
\end{aligned}$$

$$\begin{aligned}
&= \sum_{p=1}^{n}\Big\{-0.5\xi_p u_p^2+0.5(\xi_p-\alpha_p)v_p^2-0.5\xi_p u_p^2+0.5(\xi_p-\alpha_p)v_p^2+(1-\xi_p^2-\gamma_p+\alpha_p)u_p v_p\\
&\quad +I_p v_p+\sum_{q=1}^{n}(|c_{pq}|+|d_{pq}|)\Big[0.5L_q(u_q^2+v_p^2)+0.5|F_q(0)|(v_p^2+1)\Big]\\
&\le \sum_{p=1}^{n}\Big\{k(u_p^2+v_p^2)-0.5\xi_p u_p^2+0.5(\xi_p-\alpha_p)v_p^2+(1-\xi_p^2-\gamma_p+\alpha_p)u_p v_p+I_p v_p+\sum_{q=1}^{n}(|c_{pq}|\\
&\quad +|d_{pq}|)\Big[0.5L_q(u_q^2+v_p^2)+0.5|F_q(0)|(v_p^2+1)\Big]
\end{aligned}$$

$$= \sum_{p=1}^{n}\Big\{k(u_p^2+v_p^2)+k_1u_p^2+k_2v_p^2+k_3u_pv_p+I_pv_p+0.5\sum_{q=1}^{n}|F_q(0)|\Big\}. \tag{3.7}$$

Let $H(u_p, v_p) = k_1u_p^2 + k_3u_pv_p + k_2v_p^2 + I_pv_p, p = 1, 2, \cdots, n, u_p, v_p \in (-\infty, \infty)$.
Solving the following equation group of $u_p$ and $v_p$

$$\begin{cases} \frac{\partial H(u_p,v_p)}{\partial u_p} = 2k_1u_p + k_3v_p = 0, \\ \frac{\partial H(u_p,v_p)}{\partial v_p} = k_3u_p + 2k_2v_p + I_p = 0, \end{cases}$$

we get the unique local extreme point of $H(u_p, v_p)$,

$$v_p^0 = \frac{2k_1I_p}{k_3^2-4k_1k_2}, u_p^0 = \frac{k_3I_p}{4k_1k_2-k_3^2}.$$

Because $k_1 < 0, k_3^2 - 4k_1k_2 < 0$, then

$$\begin{aligned} & H(u_p, v_p) \\ = \ & k_1\Big[u_p + \frac{k_3}{2k_1}v_p\Big]^2 + \Big[\frac{4k_1k_2-k_3^2}{4k_1}\Big]v_p^2 + I_pv_p \\ \le \ & I_pv_p. \end{aligned} \tag{3.8}$$

From (3.8), one has

$$H(u_p^0, v_p^0) \le \frac{2k_1I_p^2}{k_3^2-4k_1k_2}. \tag{3.9}$$

Because $k_1 < 0, k_3^2 - 4k_1k_2 < 0$, then

$$\begin{aligned} & \lim_{u_p(t)\to-\infty, v_p(t)\to-\infty} H(u_p, v_p) < 0, \lim_{u_p(t)\to\infty, v_p\to\infty} H(u_p, v_p) < 0, \\ & \lim_{u_p\to-\infty, v_p\to\infty} H(u_p, v_p) < 0, \lim_{u_p(t)\to\infty, v_p\to-\infty} H(u_p, v_p) < 0. \end{aligned} \tag{3.10}$$

By (3.9) and (3.10), we have

$$H(u_p, v_p) \le \max\{H(u_p, v_p\} = H(u_p^0, v_p^0) \le \frac{2k_1I_p^2}{k_3^2-4k_1k_2}. \tag{3.11}$$

Substituting (3.11) into (3.7) yields

$$\begin{aligned} 0 \ \le \ & k\sum_{p=1}^{n}(u_p^2+v_p^2) + 0.5n\sum_{q=1}^{n}|F_q(0)| + \sum_{p=1}^{n}\frac{2k_1I_p^2}{k_3^2-4k_1k_2} \\ = \ & kd + 0.5n\sum_{q=1}^{n}|F_q(0)| + \sum_{p=1}^{n}\frac{2k_1I_p^2}{k_3^2-4k_1k_2} \\ < \ & 0. \end{aligned} \tag{3.12}$$

This leads to a contradiction. So, $H^*(u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n, \mu) \neq 0$, when $u = (u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n)^T \in \partial\Omega^* \cup R^{2n}, \mu \in [0, 1]$. Via Lemma 2.1, it follows that

$$
\begin{aligned}
& d[G(u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n), \Omega^*, (0, 0 \cdots, 0)^T] \\
= \; & d[H^*(u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n, 1), \Omega^*, (0, 0, \cdots, 0)^T] \\
= \; & d[H^*(u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n, 0), \Omega^*, (0, 0, \cdots, 0)^T] \\
= \; & deg\Big( -\xi_1 u_1 + v_1, -\xi_2 u_2 + v_2, -\xi_3 u_3 + v_3, \cdots, -\xi_n u_n + v_n, -(\xi_1^2 + \gamma_1 - \alpha_1)u_1 + \\
& (\xi_1 - \alpha_1)v_1 + I_1, \cdots, -(\xi_n^2 + \gamma_n - \alpha_n)u_n + (\xi_n - \alpha_n)v_n + I_n, \Omega^*, (0, 0, \cdots, 0)^T \Big)
\end{aligned}
$$

$$
\begin{aligned}
= \; & \begin{vmatrix}
-\xi_1 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & -\xi_2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & -\xi_{n-1} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\xi_n & 1 \\
r_1 & m_1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & r_2 & m_2 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & r_{n-1} & m_{n-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & r_n & m_n
\end{vmatrix} \\
= \; & (-1)^{\frac{n(n+1)}{2}} \prod_{p=1}^{n} (\xi_p m_p - r_p) \neq 0,
\end{aligned}
\tag{3.13}
$$

where $r_p = -(\xi_p^2 + \gamma_p - \alpha_p), m_p = \xi_p - \alpha_p, p = 1, 2, \cdots, n$. According to Lemma 2.2, system (3.2), i.e., system (2.1) has at least one EP. The proof of Theorem 3.1 is complete.

## 4.. Finite-time stability of equilibrium solution

The INNS with controllers can be described as

$$
\begin{cases}
u'_p(t) = -\xi_p u_p(t) + v_p(t) + w_p(t) \\
v'_p(t) = -[\xi_p^2 + \gamma_p - \alpha_p]u_p(t) + (\xi_p - \alpha_p)v_p(t) + \sum\limits_{q=1}^{n} d_{pq} F_q(u_q(t - \tau(t))) \\
\qquad + \sum\limits_{q=1}^{n} c_{pq} F_q(u_q(t)) + I_p + l_p(t),
\end{cases}
\tag{4.1}
$$

where, $w_p(t), l_p(t)$ are the controllers and the parameters are the same as those in system (2.1), where, the controllers are devised as

$$
\begin{cases}
w_p(t) = \beta, \\
l_p(t) = \alpha + \frac{l_1 l_2 e^{l_2 t} - 2 l_3 t}{r_p(t)} + F_q(0), r_p(t) \neq 0; \\
l_p(t) = 0, r_p(t) = 0,
\end{cases}
\tag{4.2}
$$

where $r_p(t)$ is defined in Theorem 4.1, $l_1 > 0, l_2 < 0, l_3 > 0$ are constants.

**Theorem 4.1.** Let $(v_1) - (v_3)$ in Theorem 3.1 hold. Furthermore assume that
$(h_1)$

$$k_1 < 0, k_3^2 < 4k_6k_4;$$

$(h_2)$

$$2k_4\alpha + 2k_6\beta > k_3(\alpha + \beta).$$

Then system (2.1) has a unique EP which is finite-time stable under the controllers (4.2).
**Notation 2.** $(h_1)$ and $(v_1) - (v_3)$ imply $(v_4)$ holds. Since the EP of system (2.1) can be finite-time stable, then system (2.1) only has a unique EP.
**Proof:** By Notations 1 and 2, the conditions in Theorem 3.1 are satisfied. According to Theorem 3.1, system (2.1) has at least an EP, say $(u_1^*, u_2^*, \cdots, u_n^*, v_1^*, v_2^*, \cdots, v_n^*)^T$. Let $(u_1(t), u_2(t), \cdots, u_n(t), v_1(t), v_2(t), \cdots, v_n(t))^T$ is a arbitrary solution of system (4.1). Now we will show that the EP of (2.1) can be finite-time stable under the controllers (4.2). From Notation 2, we show the unique EP of system (2.1) is finite-time stable.
Devise a Lyapunov function as follows:

$$M(t) = M_1(t) + M_2(t),$$

where

$$M_1(t) = 0.5\sum_{p=1}^{n}\left(e_p^2(t) + v_p^2(t)\right)$$

and

$$M_2(t) = \frac{0.5}{1-\tau^*}\sum_{p=1}^{n}\sum_{q=1}^{n}|d_{qp}|L_p\int_{t-\tau(t)}^{t}e_p^2(s)ds,$$

where $e_p(t) = u_p(t) - u_p^*, r_p(t) = v_p(t) - v_p^*$.

Then we have via (4.1) and $(v_2)$,

$$\begin{aligned}
&\frac{dM_1(t)}{dt}\\
=\ & \sum_{p=1}^{n}\Big\{e_p(t)[-\xi_p e_p(t) + r_p(t) + w_p(t)] + r_p(t)\Big( - (\xi_p^2 + \gamma_p - \alpha_p)e_p(t) + (\xi_p - \alpha_p)r_p(t) + \\
& \sum_{q=1}^{n} d_{pq}[F_q(u_q(t-\tau(t))) - F_q(u_q^*)] + \sum_{q=1}^{n} c_{pq}[F_q(u_q(t)) - F_q(u_q^*)] + l_p(t)\Big)\Big\}\\
\leq\ & \sum_{p=1}^{n}\Big\{ - \xi_p e_p^2(t) + \beta e_p(t) + (1 - \gamma_p + \alpha_p - \xi_p^2)e_p(t)r_p(t) + (\xi_p - \alpha_p)r_p^2(t) + \sum_{q=1}^{n}|d_{pq}||r_p(t)|\\
& \times|F_q(u_q(t-\tau(t))) - F_q(u^*)| + \sum_{q=1}^{n}|c_{pq}||r_p(t)||F_q(u_q(t)) - F_q(u_q^*)| + \alpha r_p(t) + l_1 l_2 e^{l_2 t}
\end{aligned}$$

$$
\begin{aligned}
&\quad -2l_3t + F_q(0)r_p(t)\Big\} \\
&\le \sum_{p=1}^{n}\Big\{-\xi_p e_p^2(t) + \beta e_p(t) + (1-\gamma_p+\alpha_p-\xi_p^2)e_p(t)v_p(t) + (\xi_p-\alpha_p)r_p^2(t) + \sum_{q=1}^{n}|d_{pq}||r_p(t)| \\
&\quad \times L_q|e_q(t-\tau(t))| + \sum_{q=1}^{n}|c_{pq}||r_p(t)|L_q|e_q(t)| + \alpha r_p(t) + |F_q(0)||r_p(t)| + l_1l_2e^{l_2t} - 2l_3t\Big\}.
\end{aligned}
\tag{4.3}
$$

Applying $2ab \le a^2 + b^2$, we have

$$
\sum_{q=1}^{n}|d_{pq}|L_q|r_p(t)||e_q(t-\tau(t))| \le 0.5\sum_{q=1}^{n}|d_{pq}|L_q[r_p^2(t) + e_q^2(t-\tau(t))],
\tag{4.4}
$$

$$
|F_q(0)||v_p(t)| \le 0.5|F_q(0)|(v_p^2(t)+1)
\tag{4.5}
$$

and

$$
\sum_{q=1}^{n}|c_{pq}|L_q|r_p(t)||e_q(t)| \le 0.5\sum_{q=1}^{n}|c_{pq}|L_q[r_p^2(t) + e_q^2(t)].
\tag{4.6}
$$

Substituting (4.4)–(4.6) into (4.3) gives

$$
\begin{aligned}
&\frac{dM_1(t)}{dt} \\
&\le \sum_{p=1}^{n}\Big\{\Big[-\xi_p + 0.5\sum_{q=1}^{n}L_p|c_{qp}|\Big]e_p^2(t) + \beta e_p(t) + (1-\gamma_p+\alpha_p-\xi_p^2)e_p(t)r_p(t) + \\
&\quad \alpha r_p(t) + \Big[\xi_p - \alpha_p + 0.5\sum_{q=1}^{n}L_q(|c_{pq}|+|d_{pq}|)\Big]r_p^2(t) + 0.5\sum_{q=1}^{n}|d_{qp}|L_pe_p^2(t-\tau(t))\Big\}.
\end{aligned}
\tag{4.7}
$$

At the same time, we have

$$
\begin{aligned}
&\frac{dM_2(t)}{dt} \\
&= \frac{0.5}{1-\tau^*}\sum_{p=1}^{n}\sum_{q=1}^{n}|d_{qp}|L_p\Big[e_p^2(t) - e_p^2(t-\tau(t))[1-\tau'(t)]\Big] \\
&\le \frac{0.5}{1-\tau^*}\sum_{p=1}^{n}\sum_{q=1}^{n}|d_{qp}|L_p\Big[e_p^2(t) - e_p^2(t-\tau(t))[1-\tau^*]\Big] \\
&= \frac{0.5}{1-\tau^*}\sum_{p=1}^{n}\sum_{q=1}^{n}|d_{qp}|L_pe_p^2(t) - 0.5\sum_{p=1}^{n}\sum_{q=1}^{n}|d_{qp}|L_pe_p^2(t-\tau(t)).
\end{aligned}
\tag{4.8}
$$

By (4.7) and (4.8), we have

$$\begin{aligned} &\frac{dM(t)}{dt} \\ \leq\ & \sum_{p=1}^{n}\Big\{k_4 e_p^2(t) + \beta e_p(t) + k_3 e_p(t) r_p(t) + k_6 r_p^2(t) + \alpha r_p(t) + l_1 l_2 e^{l_2 t} - 2l_3 t\Big\}. \end{aligned} \tag{4.9}$$

Let $H(e_p(t), r_p(t)) = k_4 e_p^2(t) + \beta e_p(t) + k_3 e_p(t) r_p(t) + k_6 r_p^2(t) + \alpha r_p(t)$, $p = 1, 2, \cdots, n$, $e_p, r_p \in (-\infty, \infty)$. Solving the following equation group of $e_p(t)$ and $r_p(t)$

$$\begin{cases} \frac{\partial H(e_p(t), r_p(t))}{\partial e_p(t)} = 2k_4 e_p(t) + \beta + k_3 r_p = 0, \\ \frac{\partial H(e_p(t)), r_p(t))}{\partial r_p(t)} = k_3 e_p(t) + 2k_6 r_p(t) + \alpha = 0, \end{cases}$$

we get the unique local extreme point of $H(e_p(t), r_p(t))$ as follows:

$$e_p^0 = \frac{2k_6\beta - k_3\alpha}{k_3^2 - 4k_6k_4}, r_p^0 = \frac{2k_4\alpha - k_3\beta}{k_3^2 - 4k_6k_4}.$$

It is clear that because of $k_4 < 0, k_6 + \frac{k_3^2}{4k_4} < 0$,

$$\begin{aligned} & H(e_p(t), r_p(t)) \\ =\ & k_4\Big[e_p(t) - \frac{k_3 r_p(t)}{2k_4}\Big]^2 + \Big[k_6 + \frac{k_3^2}{4k_4}\Big] r_p^2(t) + \alpha r_p(t) + \beta e_p(t) \\ \leq\ & \alpha r_p(t) + \beta e_p(t). \end{aligned} \tag{4.10}$$

From (4.10), one has due to $(h_2)$

$$H(e_p^0, r_p^0) \leq \frac{-k_3(\alpha+\beta) + 2k_4\alpha + 2k_6\beta}{k_3^2 - 4k_6k_4} < 0. \tag{4.11}$$

Because of $k_4 < 0, k_6 + \frac{k_3^2}{4k_4} < 0$, then

$$\begin{aligned} &\lim_{e_p(t)\to-\infty, r_p(t)\to-\infty} H(e_p(t), r_p(t)) < 0, \lim_{e_p(t)\to\infty, r_p(t)\to\infty} H(e_p(t), r_p(t)) < 0, \\ &\lim_{e_p(t)\to-\infty, r_p(t)\to\infty} H(e_p(t), r_p(t)) < 0, \lim_{e_p(t)\to\infty, r_p(t)\to-\infty} H(e_p(t), r_p(t)) < 0. \end{aligned} \tag{4.12}$$

By (4.11) and (4.12), one has

$$\begin{aligned} & H(e_p(t), r_p(t)) \\ \leq\ & \max\{H(e_p^0, r_p^0), H(\infty, \infty), H(\infty, -\infty), H(-\infty, \infty), H(-\infty, -\infty)\} \\ <\ & 0. \end{aligned} \tag{4.13}$$

Substituting (4.13) into (4.9) gives

$$\frac{dM(t)}{dt} < n(l_1 l_2 e^{l_2 t} - 2l_3 t). \tag{4.14}$$

Integrating (4.14) over $[0, t]$ gives

$$\begin{aligned} 0 \leq M(t) &< M(0) + n(l_1 e^{l_2 t} - l_1 - l_3 t^2) \\ &= M(0) - nl_1 + nl_1 e^{l_2 t} - l_3 t^2 \\ &< M(0) + nl_1 e^{l_2 t} - l_3 t^2. \end{aligned} \tag{4.15}$$

Since $U(0) > 0, l_1 > 0, l_2 < 0, l_3 > 0$, via figure method in Lemma 2.4, it follows that there exists a point $t_1$ such that when $t > t_1$,

$$U(0) + nl_1 e^{l_2 t} - l_3 t^2 < 0. \tag{4.16}$$

Substituting (4.16) into (4.15) yields

$$\lim_{t \to t_1} M^*(t) = 0.$$

As a result,

$$\lim_{t \to t_1} |u_p(t) - u_p^*(t)| = 0, \lim_{t \to t_1} |v_p(t) - v_p^*(t)|, \quad p = 1, 2, \cdots, n.$$

The accomplishes the proof of theorem 4.1.
**Claim 1.** So far, the researchers usually apply the FTS theorems to studying the FTS of EP for NNS. In this article, the maximum valued approach and the figure analysis method are cited to study the FTS of EP for the discussed NNS.
**Claim 2.** The way used in this paper can be used to study the stability of neural networks [7, 31, 32].
**Claim 3.** In this paper, the settling time $T$ is given from the (4.15) by using figure way, which also depends on the initial values. Thus, the setting time $T$ is obtained by the complicated computation.

## 5. Numerical examples

**Example 1.** Consider system (4.1) with the controllers (4.2), where $p, q = 1, 2, F_q(u_q(t)) = 0.04|u_q(t)|, \xi_p = 4, \gamma_p = 1, \alpha_p = 254, c_{pq} = d_{pq} = -5, \tau(t) = 1.5 + 0.5 sint, \tau'(t) \leq \tau^* = 0.5, I_p = 473, \alpha = -1 < 0, \beta = -1 < 0, l_1 = l_3 = 1, l_2 = -1$. Then In Theorem 4.1, the equilibrium point $(u_1^*, u_2^*, v_1^*, v_2^*) = (1, 1, 4, 4), \xi_p(\xi_p - \alpha_p) + \xi_p^2 + \gamma_p - \alpha_p = -1 \neq 0, L_q = 0.25, k_1 = -0.125 < 0, k_3^2 = 121, k_4 k_6 = 150, k_3^2 < 4k_6 k_4$. Hence, the requirements in Theorem 4.1 are satisfied. Via Theorem 4.1, the EP of system 4.1 is finite-time stable under the controllers (4.2). The study approach, the results and the controllers devised on the FTS of the system (4.1) are all different from those in the existing articles [11–27, 30], so our results cannot be tested with those in [11–27, 30]. The curves of variables $u_1(t), u_2(t), v_1(t), v_2(t)$ are shown in the following Figure 2.

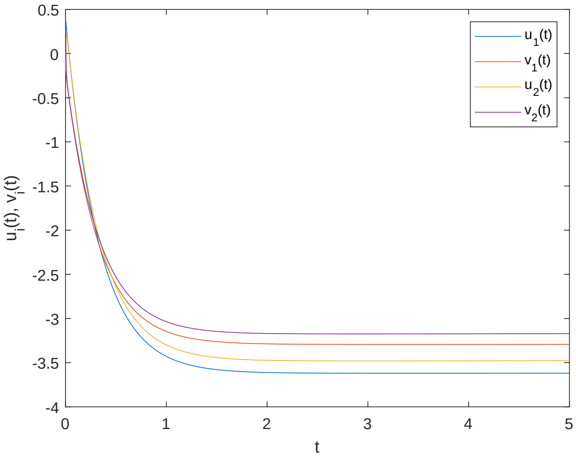


**Figure 2.** Curves of the $u_1(t)$,$u_2(t)$, $v_1(t)$, $v_2(t)$, i.e., the figure of the FTS of the EP of system (4.1) with the controllers (4.2).

**Example 2.** Consider system (4.1) with the controllers (4.2), where $p, q = 1, 2, F_q(u_q(t)) = 2u_q(t) + 1, \xi_p = 5, \gamma_p = 2, \alpha_p = 36, c_{pq} = d_{pq} = -1, \tau(t) = -1 + 0.6cost, \tau'(t) \leq \tau^* = 0.6, I_p = 500, \alpha = -1 < 0, \beta = -9 < 0, l_1 = l_3 = 1, l_2 = -1$. Then In Theorem 4.1, equilibrium point is $(u_1^*, u_2^*, v_1^*, v_2^*) = (0.7960, 0.7998, 3.9798, 3.9989)$, and $\xi_p(\xi_p - \alpha_p) + \xi_p^2 + \gamma_p - \alpha_p = -164 \neq 0, L_q = 2, k_1 = -0.5 < 0, k_3^2 - 4k_6k_4 = -257 < 0$. Hence, the requirements in Theorem 4.1 are satisfied. By Theorem 4.1, the EP of system 4.1 is finite-time stable under the controllers (4.2). The study approach, the results and the controllers devised on the FTS of the system (4.1) are all different with those in the existing articles [11–27, 30], so our results cannot be tested with those in [11–27, 30]. The curves of variables $u_1(t), u_2(t), v_1(t), v_2(t)$ are shown in the following Figure 3.

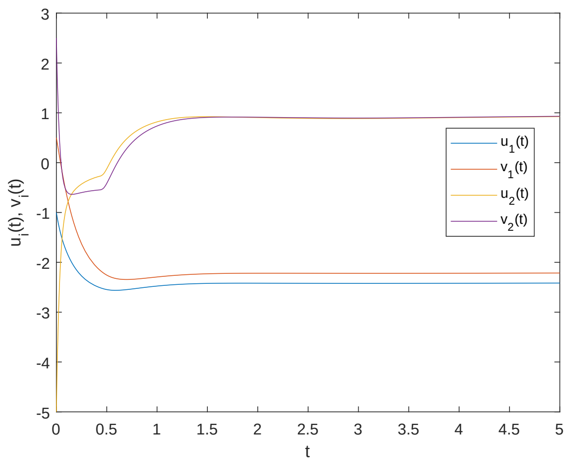


**Figure 3.** Curves of the $u_1(t)$,$u_2(t)$, $v_1(t)$, $v_2(t)$, i.e., the figure of the FTS of the EP of system (4.1) with the controllers (4.2).

## 6. Conclusions

Firstly, the existence of EP for the INNS was discussed by applying the degree theory and maximum-valued way. Then, the condition in the FTS of the networks is proposed by applying the maximum-valued way and the figure analysis way. Our study way and results extend the research fields of FTS for the NNS. In the future, we study the fixed-time stability of EP of neural networks.

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## Conflict of interest

The authors declare that there is no conflict of interest.

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