



Research article

Dynamic analysis of two fishery capture models with a variable search rate and fuzzy biological parameters

Hua Guo¹, Yuan Tian^{1,*}, Kaibiao Sun^{2,*} and Xinyu Song³

¹ School of Science, Dalian Maritime University, Dalian 116026, China

² School of Control Science and Engineering, Dalian University of Technology, Dalian 116024, China

³ College of Mathematics and Statistics, Xinyang Normal University, Xinyang 464000, China

* **Correspondence:** Email: tianyuan1981@163.com, sunkb@dlut.edu.cn.

Abstract: The fishery resource is a kind of important renewable resource and it is closely connected with people's production and life. However, fishery resources are not inexhaustible, so it has become an important research topic to develop fishery resources reasonably and ensure their sustainability. In the current study, considering the environment changes in the system, a fishery model with a variable predator search rate and fuzzy biological parameters was established first and then two modes of capture strategies were introduced to achieve fishery resource exploitation. For the fishery model in a continuous capture mode, the dynamic properties were analyzed and the results show that predator search rate, imprecision indexes and capture efforts have a certain impact on the existence and stability of the coexistence equilibrium. The bionomic equilibrium and optimal capture strategy were also discussed. For the fishery model in a state-dependent feedback capture mode, the complex dynamics including the existence and stability of the periodic solutions were investigated. Besides the theoretical results, numerical simulations were implemented step by step and the effects of predator search rate, fuzzy biological parameters and capture efforts on the system were demonstrated. This study not only enriched the related content of fishery dynamics, but also provided certain reference for the development and utilization of fishery resources under the environment with uncertain parameters.

Keywords: fuzzy biological parameters; periodic solution; stability; switch capture strategy; variable search rate

1. Introduction

In the natural ecosystem, forests, grasslands, wildlife and fishery resources are different important components, providing rich natural resources for human production and life, and playing an important role in promoting human development. Fishery resource, as a valuable renewable resource, is closely

related to our daily life. The Food and Agriculture Organization of the United Nations (FAO) reported in 2018 that the total world fisheries production recorded new highs, but nearly one third of fisheries production was wasted. Meanwhile, overfishing is widespread, which poses a huge challenge to the sustainability of the world's fish supply [1]. The ocean is a large ecosystem, and the marine species are complex and form both competition and predation relationships with each other. The research on predator-prey interactions and their dynamic behavior among populations has long been a topic of common interest to mathematicians and biologists. Especially in fishery, studying the rationality behind the dynamic behavior of fish species can help humans develop and utilize fishery resources more scientifically and rationally.

The essence of biomathematics is to transform a complex biological problem into a mathematical problem by means of establishing mathematical models. The pioneer work investigating the predation relationship was carried out by Lotka [2] and Volterra [3], which is widely referred to the Lotka-Volterra model. Since then, many scholars have improved the classic Lotka-Volterra model to describe various biological phenomena. For example, Gause et al. [4] introduced a Gause-type model with a general form of uptake function. Smith [5] introduced a Smith model in which a species growth is low in the presence of limited food and it is assumed that after the population reaches saturation, the population will not increase any more and the food is only used to maintain the survival of the species. Compared with the logistic growth function, the Smith growth function in some cases is more accurate in describing the growth law. Based on this consideration, Sivakumar et al. [6] introduced a diffusive model with Smith growth and analyzed the stability and hopf bifurcation. Han et al. [7] introduced a spatiotemporal discrete model with Smith growth rate function and analyzed the bifurcation and turing instability. Feng et al. [8] introduced a modified Leslie-Gower incorporated with the Smith growth rate and Beddington-DeAngelis uptake function. Another aspect of the work to improve the model is reflected in the functional reaction functions [9–12], among others the Holling-II type uptake function is widely applicable to many species, and the model with the Holling-II type uptake function has been widely concerned by scholars. In Holling-II uptake function, the search rate is often assumed to be constant. However, in the actual system, the density of prey and the search environment of predators have obvious effects on the search rate of predators. Based on the above biological background, Hassell and Comins [13] introduced a saturated search rate, and subsequently Yu et al. [14] analyzed the predator-prey model with the variable predator search rate and fear effect. In this study, we introduced a Smith growth predator-prey model with the Holling-II functional response and a variable search rate.

With the change of the external environment, such as extreme environment and temperature, the biological parameters of some species including the birth and death rates will show certain fluctuations, which will further influence the dynamics of species populations. Some findings suggest that fish species are more affected by climate change [15–17]. Kwon et al. [15] showed that climate changes can make fish populations decrease. Although environmental factors are known to influence population change, it is not clear how these factors affect population change due to certain fluctuations and uncertainties. To describe this uncertainty, scholars introduced some specific representations in predator-prey models, such as random disturbances [18–21], fuzzy numbers and fuzzy sets [22–32]. Pal et al. [22] introduced a predator-prey model with fuzzy interval numbers represented by interval-valued function. Pal et al. [23] investigated a fishery model with fuzzy parameters and human capture activities. Pal et al. [24] studied a delayed predator-prey harvesting model with interval-valued imprecise parameters and presented a stability and bifurcation analysis method. Yu et al. [25] analyzed a predator-prey capture model with interval-

valued imprecise parameters considering the mutual interference between predators in the system as well as the prey refuge effect. Das et al. [26] considered the phenomenon of pest disease transmission in the system and studied a class of predator-prey models with interval-valued parameters. In most of the above papers and related studies, the same index is used to describe different interval-valued parameters. In order to describe different parameters, Xiao et al. [27] used interval-valued numbers with different indexes to describe imprecise parameters and studied a two-species competitive model with interval-valued fuzzy parameters. Tian et al. [28–30] used interval-valued fuzzy numbers with different indicators to describe the parameters of different populations, and studied two fishery capture models with parameters represented by species-dependent interval-valued functions. Yu et al. [31] investigated a fuzzy predator-prey model with parameters represented by a triangular fuzzy number. Xu et al. [32] developed a triangular fuzzy water hyacinth-fish model with a Kuznets curve effect.

Fishery resource is closely connected with people's production and life, and fishery capture activity is an indispensable link for humans to obtain natural resources, which can be divided into continuous ways [33–38] or intermittent ways [39–45]. The continuous harvesting activity is easy to implement in practice, and easy to simulate in the modeling process. However, this kind of harvesting method is relatively ideal and does not take into account the fishery environment and the current situation of fish resources. In order to realize the rational development and utilization of fish resources and promote the healthy and sustainable development of fish resources, it is necessary to adopt more reasonable and effective ways. Compared with continuous harvesting activity, intermittent harvesting activity, which occurs at discrete moments, is more realistic in practice. Among the many types of intermittent harvesting activities, state-dependent harvesting activity is a typical one, which takes full account of the current situation of prey or predators and promotes the sustainable development of fish species as far as possible. For such kinds of human interventions, impulsive differential equations presents an effective description [46–59]. In the above studies, the models involved can be roughly divided into four types: Prey-dependent [41, 45, 51–57], predator-dependent [39, 40], ratio-dependent [58] and hybrid-dependent [42–44]. From the perspective of keeping ecological balance, setting certain threshold values for both prey populations and predator populations is necessary. In this study, we develop a switch capture strategy, where fishing activities can be carried out only if prey population reaches or exceeds a preset threshold. At the same time, if predator populations are below a low value, we will not only carry out fishing activities but also release some predator pups.

Inspired by the above work, in the current study we propose two fishing models with variable predator search speed and fuzzy parameters and discuss the effects of different predator search rates and parameter imprecision indicators, as well as the dynamic behavior guided by different fishing methods. We organize the article in the following way: In the subsequent section, we develop two fishery capture models with variable search rate and triangular fuzzy biological parameters, followed by a presentation of basic concepts of impulsive semi-continuous dynamic system. In section three, we mainly investigate the system dynamics under continuous capture strategy and complex dynamic behaviour induced by the switched capture strategy. In the next section, we conduct numerical simulations to illustrate the conclusions of the previous section. At the end of the article, we make a brief summary of the research.

2. Mathematical model and basic knowledge

In this work, we consider a Gause-type fishery model with Smith growth and Holling-II type uptake function, i.e.,

$$\begin{cases} \frac{dx}{dt} = rx \left(\frac{K-x}{K+mx} \right) - \frac{sx}{sHx+1}y, \\ \frac{dy}{dt} = -dy + \frac{csx}{sHx+1}y, \end{cases} \quad (2.1)$$

in which x and y indicate the density of prey and predator species, respectively; parameters r and K are intrinsic growth rate and maximum environmental capacity, respectively; the term $rx(K-x)/(K+mx)$ is the Smith growth function with maintenance constant m ; H denotes the handling time of a prey by one predator; d is the predator's death rate; c denotes the conversion coefficient. The parameter s describes the search rate of the predator and is generally taken as a constant. However, in a real system, the predator's search speed will depend on the density of the prey and the search environment. Considering this phenomenon, Hassell and Comins [13] introduced the following type of search rate

$$s(x) = \frac{ax}{x+g},$$

where a is the maximum search rate and g is a constant, then the extended form of Model (2.1) is described as follows:

$$\begin{cases} \frac{dx}{dt} = rx \left(\frac{K-x}{K+mx} \right) - \frac{ax^2}{aHx^2+x+g}y, \\ \frac{dy}{dt} = -dy + \frac{cax^2}{aHx^2+x+g}y. \end{cases} \quad (2.2)$$

In natural systems, biological species are inevitably affected by environmental changes, so it is meaningful and necessary to consider biological models with imprecise parameters (2.2). In order to describe the uncertainty of such parameters, we use triangular fuzzy number (TFN) [31]. For a triangular fuzzy number $\tilde{U} \equiv (u_1, u_2, u_3)$, the ω -cut set of \tilde{U} is $[U_{l(\omega)}, U_{r(\omega)}]$, where $U_{l(\omega)} = \{x : \mu_{\tilde{U}}(x) \geq \omega\} = u_1 + \omega(u_2 - u_1)$ and $U_{r(\omega)} = \{x : \mu_{\tilde{U}}(x) \geq \omega\} = u_3 + \omega(u_3 - u_2)$ for $\omega \in [0, 1]$. Considering that the mortality and conversion rates of predators and prey are most susceptible to environmental changes, imprecisions of these three parameters are assumed in this paper, and represented by triangular fuzzy number $\tilde{r} = (r_1, r_2, r_3)$, $\tilde{d} = (d_1, d_2, d_3)$ and $\tilde{c} = (c_1, c_2, c_3)$. Using theory of ω -cut fuzzy number, Model (2.2) can be expressed as

$$\begin{cases} \left(\frac{dx}{dt} \right)_{l(\omega)} = r_{l(\omega)}x \left(\frac{K-x}{K+mx} \right) - \frac{ax^2}{aHx^2+x+g}y, \\ \left(\frac{dx}{dt} \right)_{r(\omega)} = r_{r(\omega)}x \left(\frac{K-x}{K+mx} \right) - \frac{ax^2}{aHx^2+x+g}y, \\ \left(\frac{dy}{dt} \right)_{l(\omega)} = -d_{r(\omega)}y + \frac{c_{l(\omega)}ax^2}{aHx^2+x+g}y, \\ \left(\frac{dy}{dt} \right)_{r(\omega)} = -d_{l(\omega)}y + \frac{c_{r(\omega)}ax^2}{aHx^2+x+g}y. \end{cases} \quad (2.3)$$

Using the utility function method (UFM) [23, 32], one can get

$$\begin{cases} \frac{dx}{dt} = w_1 \left(\frac{dx}{dt} \right)_{l(\omega)} + (1 - w_1) \left(\frac{dx}{dt} \right)_{r(\omega)}, \\ \frac{dy}{dt} = w_2 \left(\frac{dy}{dt} \right)_{l(\omega)} + (1 - w_2) \left(\frac{dy}{dt} \right)_{r(\omega)}, \end{cases} \quad (2.4)$$

where $0 \leq w_1 \leq 1$, $0 \leq w_2 \leq 1$.

For convenience, let's write: $\hat{r} = w_1 r_{l(\omega)} + (1 - w_1) r_{r(\omega)}$, $\hat{d} = w_2 d_{r(\omega)} + (1 - w_2) d_{l(\omega)}$, $\hat{c} = w_1 c_{l(\omega)} + (1 - w_2) c_{r(\omega)}$. By combining formulas (2.3) and (2.4), we get

$$\begin{cases} \frac{dx}{dt} = \hat{r}x \left(\frac{K - x}{K + mx} \right) - \frac{ax^2}{aHx^2 + x + g}y, \\ \frac{dy}{dt} = \hat{d}y + \frac{\hat{c}ax^2}{aHx^2 + x + g}y. \end{cases} \quad (2.5)$$

In order to meet people's daily life, it is necessary to capture two kinds of fish. Let q_1 and E_1 be the capture rate and capture effort of the prey species, q_2 and E_2 be that of the predators. To analyze the impact of different fishing strategies on the system, we consider two different forms of fishing strategies. The first capture strategy is a continuous mode, and the fishery model based on the continuous capture mode is formulated as follows:

$$\begin{cases} \frac{dx}{dt} = \hat{r}x \left(\frac{K - x}{K + mx} \right) - \frac{ax^2}{aHx^2 + x + g}y - q_1 E_1 x, \\ \frac{dy}{dt} = -\hat{d}y + \frac{\hat{c}ax^2}{aHx^2 + x + g}y - q_2 E_2 y. \end{cases} \quad (2.6)$$

The second is a state-dependent feedback capture strategy. Let x_T be the prey's reference threshold. When the prey populations reach the threshold x_T , a capture activity is implemented. In addition, to maintain the balance of the ecosystem, it is also necessary to consider whether to release predator fish. Let y_T be the predator's reference threshold. When the predator population is below the threshold y_T , in addition to capture activity, a certain quantity of predators needs to be put into the system, which is denoted by τ . Then, the fishery model based on the switch capture strategy is formulated as follows:

$$\begin{cases} \left. \begin{aligned} \frac{dx}{dt} &= \hat{r}x \left(\frac{K - x}{K + mx} \right) - \frac{ax^2}{aHx^2 + x + g}y \\ \frac{dy}{dt} &= -\hat{d}y + \frac{\hat{c}ax^2}{aHx^2 + x + g}y \end{aligned} \right\} x \neq x_T, \\ \left. \begin{aligned} \Delta x &= -q_1 E_1 x \\ \Delta y &= -q_2 E_2 y \end{aligned} \right\} x = x_T, y > y_T, \\ \left. \begin{aligned} \Delta x &= -q_1 E_1 x \\ \Delta y &= -q_2 E_2 y + \tau \end{aligned} \right\} x = x_T, y \leq y_T. \end{cases} \quad (2.7)$$

The objective is to investigate the effects of variable search rate and imprecise biological parameters on Model (2.5), while exploring the complex dynamic behavior of Models (2.6) and (2.7) under different capture modes.

2.1. Basic knowledge

2.1.1. Fuzzy set and ω -cut set

Definition 2.1 (Interval number [22]). A closed interval $\tilde{U} = [u_l, u_r] = \{u | u_l \leq u \leq u_r, u \in \mathbb{R}\}$ is called an interval number \tilde{U} , in which u_r and u_l are upper and lower boundaries of \tilde{U} , respectively.

Definition 2.2 (Fuzzy set [23, 32]). A fuzzy set \tilde{U} is defined as $\tilde{U} = \{(u, \mu_{\tilde{U}}(u)) : u \in \mathbb{R}\}$, where $\mu_{\tilde{U}} : \mathbb{R} \rightarrow [0, 1]$ is called the membership function of \tilde{U} and $\mu_{\tilde{U}}(u)$ is the membership value.

Definition 2.3 (TFN [23, 32]). A TFN $\tilde{U} \equiv (u_1, u_2, u_3)$ is a fuzzy set with $\mu_{\tilde{U}} : \mathbb{R} \rightarrow [0, 1]$ defined as:

$$\mu_{\tilde{U}}(u) = \begin{cases} \frac{u - u_1}{u_2 - u_1}, & \text{if } u_1 \leq u \leq u_2, \\ \frac{u_3 - u}{u_3 - u_2}, & \text{if } u_2 \leq u \leq u_3, \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.4 (ω -cut set [23, 32]). For a fuzzy number \tilde{A} , the ω -cut set is defined as $\tilde{A}_\omega = \{x : \mu_{\tilde{A}}(x) \geq \omega, \omega \in (0, 1]\}$. For $\omega = 0$, \tilde{A}_0 is defined as $\tilde{A}_0 = \{x : \mu_{\tilde{A}}(x) \geq 0\}$.

Undoubtedly, ω -cut set of TFN $\tilde{U} \equiv (a_1, a_2, a_3)$ is a closed interval $[U_{l(\omega)}, U_{r(\omega)}]$, where $U_{l(\omega)} = \inf \{u : \mu_{\tilde{U}}(u) \geq \omega\} = u_1 + \omega(u_2 - u_1)$, $U_{r(\omega)} = \sup \{u : \mu_{\tilde{U}}(u) \geq \omega\} = u_3 + \omega(u_3 - u_2)$.

Definition 2.5 (Utility function [23, 32]). For given items U_i , $i = 1, 2, \dots, n$, let w_i be the weight assigned to item U_i , then a utility function is defined based on the relative importance of item U_i , which can be expressed by

$$U = \sum_{i=1}^n w_i U_i,$$

where $w_i \geq 0$ and $\sum_{i=1}^n w_i = 1$.

2.1.2. Concept of impulsive semi-continuous dynamic systems

For a planar model

$$\left\{ \begin{array}{l} \frac{dx}{dt} = F(x, y) \\ \frac{dy}{dt} = G(x, y) \end{array} \right\} \quad \phi(x, y) \neq 0, \quad (2.8)$$

$$\left\{ \begin{array}{l} \Delta N = I_1(x, y) \\ \Delta P = I_2(x, y) \end{array} \right\} \quad \phi(x, y) = 0,$$

where F, G, ϕ, I_1 and I_2 are continuously differentiable functions of x and y , $(x, y) \in \Omega \subset \mathbb{R}_+^2$. Let $\pi = (\pi_1, \pi_2)$ be the solution map. Given $P \in \Omega$ and $t_1, t_2 \in \mathbb{R}$, $\pi(P, 0) = P$, $\pi(\pi(P, t_1), t_2) = \pi(P, t_1 + t_2)$, then $(\Omega, \pi; \mathbf{I}, \mathcal{M})$ is called an impulsive semi-continuous dynamic system, where $\mathcal{M} \triangleq \{(x, y) | \phi(x, y) = 0, (x, y) \in \Omega\}$ and $\mathbf{I} \triangleq \mathbf{1} + (I_1, I_2)$.

The solution of $(\Omega, \pi; \mathbf{I}, \mathcal{M})$ is represented by $\mathbf{z}(t) = (x(t), y(t))'$, with orbit represented by $\gamma_{S_0}(\mathbf{z}) \triangleq \{\mathbf{z}(t), t \geq 0, \mathbf{z}(0) = S_0\}$. Denote $\Sigma \triangleq \{t_i | i = 1, 2, \dots\}$ as the time set of the impulses. Assume that the first partial derivative of $\phi(x, y)$ with respect to x (or y) is constant, as illustrated in Figure 1.

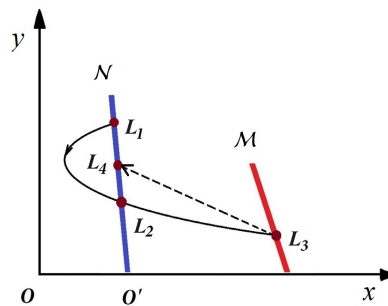


Figure 1. Schematic representation of the successor points.

Let $L_1 \in \mathcal{N} \triangleq \mathbf{I}(\mathcal{M})$. Denote $\mathcal{N} \cap x\text{-axis} = \{O'\}$, $\gamma_{L_1}(\mathbf{z}) \cap \mathcal{M} = \{L_3\}$, $\gamma_{L_1}(\mathbf{z}) \cap \mathcal{N} = \{L_2\}$ and $L_4 = \mathbf{I}(L_3)$. In Figure 1, we can define the Poincaré map on \mathcal{N} by $\mathcal{P}_{\mathcal{M}} \triangleq \mathbf{I} \cdot \pi$, i.e., $L_4 = \mathcal{P}_{\mathcal{M}}(L_2) = \mathcal{P}_{\mathcal{M}}(L_1)$. Then two types of successor functions are defined: $f_{\mathcal{M}}^I(L_2) \triangleq y_{\mathcal{P}(L_2)} - y_{L_2}$ and $f_{\mathcal{M}}^{II}(L_1) \triangleq y_{\mathcal{P}(L_1)} - y_{L_1}$.

Definition 2.6 (Order- k periodic solution [28, 42, 43]). For a given $\hat{\mathbf{z}}(t) = (\hat{x}(t), \hat{y}(t))'$ with $\hat{\mathbf{z}}_0 \in \mathcal{N}$, if $\exists n \geq 1$ such that $\hat{\mathbf{z}}_n = \hat{\mathbf{z}}_0$, then $\hat{\mathbf{z}}(t)$ is said to be periodic. Let $k \triangleq \min\{l \mid 1 \leq l \leq n, \hat{\mathbf{z}}_l = \hat{\mathbf{z}}_0\}$, then we call $\hat{\mathbf{z}}(t) = (\hat{x}(t), \hat{y}(t))'$ ($(i-1)t_k \leq t \leq it_k$) an order- k periodic solution. Moreover, if $\forall \epsilon > 0$, $\exists U_\delta$ of $\hat{\mathbf{z}}$ such that $\forall \mathbf{z} \in U_\delta$, $\exists \hat{t}(t)$, there is $|\mathbf{z}(t) - \hat{\mathbf{z}}(\hat{t}(t))| < \epsilon$, $\forall t \geq t_0$, then the orbit $\gamma(\hat{\mathbf{z}})$ is orbitally asymptotically stable.

Lemma 2.1. [28, 42, 43] Assume that $\tilde{\mathbf{z}}(t) = (\xi(t), \eta(t))'$ ($(i-1)T \leq t \leq iT$) is an order- k periodic solution. If $|\mu_k| < 1$, then the orbit $\gamma(\tilde{\mathbf{z}})$ is orbitally asymptotically stable, where

$$\mu_k = \prod_{j=1}^k \Delta_j \exp \int_0^T \left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} \right) \Big|_{(\xi(t), \eta(t))} dt,$$

$$\Delta_j = \frac{F_+ [(1 + I_{2y})\phi_x - I_{2x}\phi_y] + G_+ [(1 + I_{1x})\phi_y - I_{1y}\phi_x]}{F_0\phi_x + G_0\phi_y},$$

$F_+ = F(\xi(\tau_j^+), \eta(\tau_j^+))$, $G_+ = G(\xi(t_j^+), \eta(t_j^+))$ with $(\xi(t_j^+), \eta(t_j^+)) \in \mathcal{N}$; $F_0 = F(\xi(t_j), \eta(t_j))$, $G_0 = G(\xi(t_j), \eta(t_j))$ with $(\xi(t_j), \eta(t_j)) \in \mathcal{M}$.

3. Main results

For convenience of description, let us denote $\mathbf{E} = (E_1, E_2)$ and define

$$K_{\mathbf{E}} \triangleq \frac{(\hat{r} - q_1 E_1)K}{\hat{r} + mq_1 E_1},$$

$$x_{\mathbf{E}}^* \triangleq \frac{-(\hat{d} + q_2 E_2) - \sqrt{(\hat{d} + q_2 E_2)^2 - 4(\hat{d}aH + q_2 E_2 aH - ca)(\hat{d} + q_2 E_2)g}}{2(\hat{d}aH + q_2 E_2 aH - ca)},$$

$$y_{\mathbf{E}}^* \triangleq \frac{\left[\hat{r} \left(\frac{K - x_{\mathbf{E}}^*}{K + mx_{\mathbf{E}}^*} \right) - q_1 E_1 \right] (aHx_{\mathbf{E}}^* + x_{\mathbf{E}}^* + g)}{ax_{\mathbf{E}}^*}.$$

3.1. Dynamic characteristics of Model (2.6)

3.1.1. Existence and stability of equilibrium

Theorem 3.1. For Model (2.6), an extinction equilibrium $Q_0(0, 0)$ always exists; $Q_1(K_E, 0)$ exists as long as $0 \leq E_1 < \bar{E}_1 \triangleq \hat{r}/q_1$; a coexistent equilibrium $Q_2(x_E^*, y_E^*)$ exists when $0 \leq E_1 < \bar{E}_1$, $0 \leq E_2 < \bar{E}_2 \triangleq (\hat{c} - H\hat{d})/Hq_2$ and $x_E^* < K_E$.

Proof. The existence of trivial and predator extinction equilibria is evident. The coexistent equilibrium should satisfy the following equations:

$$\begin{cases} \hat{r} \left(\frac{K-x}{K+mx} \right) - \frac{ax}{aHx^2+x+g} y - q_1 E_1 = 0, \\ -\hat{d} + \frac{\hat{c}ax^2}{aHx^2+x+g} - q_2 E_2 = 0. \end{cases} \quad (3.1)$$

From Eq (3.1), a quadratic equation can be obtained:

$$Ax^2 + Bx + C = 0, \quad (3.2)$$

where

$$A = \hat{d}aH + q_2 E_2 aH - ca, B = \hat{d} + q_2 E_2, C = (\hat{d} + q_2 E_2)g.$$

It can be concluded that Eq (3.3) has a positive root x_E^* if $\hat{d}aH + q_2 E_2 aH - ca < 0$, i.e., $0 \leq E_2 < \bar{E}_2$. To ensure that y_E^* is positive, x_E^* should be in the range $[0, K_E)$ and E_1 should be in the range of $[0, \bar{E}_1)$.

To sum up, if $0 \leq E_1 < \bar{E}_1$, $0 \leq E_2 < \bar{E}_2$ and $x_E^* < K_E$, $Q_2(x_E^*, y_E^*)$ is a coexistence equilibrium of system (2.6). \square

Define

$$F(x, y; w_1, w_2, \omega, E_1) \triangleq \hat{r}x \left(\frac{K-x}{K+mx} \right) - \frac{ax^2}{aHx^2+x+g} y - q_1 E_1 x,$$

$$G(x, y; w_1, w_2, \omega, E_2) \triangleq \hat{d}y + \frac{\hat{c}ax^2}{aHx^2+x+g} y - q_2 E_2 y.$$

Theorem 3.2. When $E_1 > \bar{E}_1$, $Q_0(0, 0)$ is globally asymptotically stable; when $0 \leq E_1 < \bar{E}_1$, $0 \leq E_2 < \bar{E}_2$, $x_E^* > K_E$ or $0 \leq E_1 < \bar{E}_1$, $E_2 > \bar{E}_2$ hold, $Q_1(K_E, 0)$ is locally asymptotically stable; when $\Gamma_E^1 < 0$, $\Gamma_E^2 < 0$ hold simultaneously, $Q_2(x_E^*, y_E^*)$ is locally asymptotically stable, where

$$\Gamma_E^1 \triangleq \hat{r} \left(\frac{K-x_E^*}{K+mx_E^*} \right) - rx_E^* \left(\frac{(1+m)K}{(K+mx_E^*)^2} \right) - \frac{2ax(aHx_E^{*2} + x_E^* + g) - ax_E^{*2}(2aHx_E^* + 1)}{(aHx_E^* + x_E^* + g)^2} y_E^* - q_1 E_1,$$

$$\Gamma_E^2 \triangleq 2\hat{c}ax(aHx_E^{*2} + x_E^* + g) - \hat{c}ax_E^{*2}(2aHx_E^* + 1).$$

Proof. At $\bar{Q}(\bar{x}, \bar{y})$, the Jacobian matrix $J(\bar{Q})$ is

$$J(\bar{Q}) = \begin{pmatrix} F_x(\bar{x}, \bar{y}) & F_y(\bar{x}, \bar{y}) \\ G_x(\bar{x}, \bar{y}) & G_y(\bar{x}, \bar{y}) \end{pmatrix}.$$

For Q_0 , there is

$$J(Q_0) = \begin{pmatrix} \hat{r}K - q_1 E_1 & 0 \\ 0 & -\hat{d} - q_2 E_2 \end{pmatrix}.$$

The eigenvalues are $\lambda_1 = \hat{r}K - q_1E_1$ and $\lambda_2 = -\hat{d} - q_2E_2 < 0$. When $E_1 > \bar{E}_1$, there is $\lambda_1 = \hat{r} - q_1E_1 < 0$, hence $Q_0(0, 0)$ is locally asymptotically stable. Moreover, if $E_1 > \bar{E}_1$, then $dx/dt < 0$, i.e., prey populations decrease to zero, which causes the number of predators to decrease to zero as well. Therefore, $Q_0(0, 0)$ is globally asymptotically stable.

For Q_1 , there is

$$J(Q_1) = \begin{pmatrix} -\frac{\hat{r}K_E(1+mK)}{(1+mK_E)^2} & \frac{aK_E^2}{aHK_E^2 + K_E + g} \\ 0 & -\hat{d} - q_2E_2 + \frac{\hat{c}aK_E^2}{aHK_E^2 + K_E + g} \end{pmatrix}.$$

The eigenvalues are

$$\lambda_1 = -\frac{\hat{r}K_E((1+m)K)}{(K+mK_E)^2}, \lambda_2 = -\hat{d} - q_2E_2 + \frac{\hat{c}aK_E^2}{aHK_E^2 + K_E + g}.$$

Define

$$f(x) \triangleq -\hat{d} - q_2E_2 + \frac{\hat{c}ax^2}{aHx^2 + x + g},$$

then $f'(x) > 0$. Case i) x_E^* exists, i.e., $f(x_E^*) = 0$, which means $0 \leq E_2 < \bar{E}_2$, if $K_E < x_E^*$, $f(K_E)$ will be less than zero; Case ii) $E_2 > \bar{E}_2$, i.e., $\hat{d} + q_2E_2 > \hat{c}/H$. Since $\lim_{x \rightarrow \infty} f(x) = -\hat{d} - q_2E_2 + \hat{c}/H$, then $f(K_E) < 0$. Combine cases i) and ii) with the existence condition of K_E , i.e., $0 \leq E_1 < \bar{E}_1$, and it can be derived that $\lambda_1 < 0$ and $\lambda_2 < 0$, thus, $Q_1(K_{(E_1, E_2)}, 0)$ is locally asymptotically stable.

For Q_2 , there is

$$J(Q_2) = \begin{pmatrix} C_{21} & C_{22} \\ C_{23} & C_{24} \end{pmatrix},$$

where

$$\begin{aligned} C_{21} &= \hat{r} \left(\frac{K - x_E^*}{K + mx_E^*} \right) - rx_E^* \left(\frac{(1+m)k}{(K + mx_E^*)^2} \right) - \frac{2ax_E^* (aHx_E^{*2} + x_E^* + g) - ax_E^{*2} (2aHx_E^* + 1)}{(aHx_E^* + x_E^* + g)^2} y_E^* - q_1E_1, \\ C_{22} &= \frac{\hat{d} + q_2E_2}{\hat{c}}, \\ C_{23} &= \frac{2\hat{c}ax (aHx_E^{*2} + x_E^* + g) - \hat{c}ax_E^{*2} (2aHx_E^* + 1)}{(aHx_E^* + x_E^* + g)^2} y_E^*, \\ C_{24} &= 0. \end{aligned}$$

Since $\text{Tr}(J(Q_2)) = C_{21}$ and $\text{Det}(J(Q_2)) = -C_{22}C_{23}$, when $\Gamma_E^1 < 0$, $\Gamma_E^2 < 0$ hold simultaneously, $Q_2(x_E^*, y_E^*)$ is locally asymptotically stable. \square

3.1.2. Bionomic equilibrium

Economic benefit is important for human activities; thus, it is meaningful to combine biological balance with economic benefit balance. Let s_1 and s_2 be the selling price of prey and predator, c_1 and c_2 be the costs per unit of capture effort accordingly, then the net profit of the capture process can be characterized as

$$\text{Prof}_{net} = (q_1s_1x - c_1)E_1 + (q_2s_2y - c_2)E_2.$$

The bionomic equilibrium $(x_r, y_r, E_{1r}, E_{2r})$ satisfies that

$$\hat{r} \left(\frac{K-x}{K+mx} \right) - \frac{ax}{aHx^2+x+g} y - q_1 E_1 = 0, \quad (3.3)$$

$$-\hat{d} + \frac{\hat{c}ax^2}{aHx^2+x+g} - q_2 E_2 = 0, \quad (3.4)$$

$$\text{Prof}_{net} = (s_1 q_1 x - c_1) E_1 + (s_2 q_2 y - c_2) E_2 = 0. \quad (3.5)$$

To determine a bionomic equilibrium $(x_r, y_r, E_{1r}, E_{2r})$, we will discuss it in four cases:

Case 1: $c_2 > s_2 q_2 y$. In this case, the predator's fishing costs outweigh its benefits, which means that $E_{2r} = 0$. Thus, only the prey stocks are fished and ($c_1 < s_1 q_1 x$), then $x_r = c_1 / s_1 q_1$ and (y_r, E_{1r}) satisfy

$$\hat{r} \frac{q_1 s_1 K - c_1}{q_1 s_1 K + mc_1} - \frac{ac_1 q_1 s_1}{aHc_1^2 + c_1 q_1 s_1 + g q_1^2 s_1^2} y - q_1 E_1 = 0. \quad (3.6)$$

Case 2: $c_1 > s_1 q_1 x$. In this case, the prey's fishing costs outweigh its benefits, which means that $E_{1r} = 0$. Thus, only the predator stocks are fished and ($c_2 < s_2 q_2 y$), then we have $y_r = c_2 / s_2 q_2$. Substituting y_r, E_{1r} for y, E_1 respectively in Eq (3.3) yields that

$$\hat{r} \left(\frac{K-x}{K+mx} \right) - \frac{ax}{aHx^2+x+g} \frac{c_2}{s_2 q_2} = 0. \quad (3.7)$$

If there exists positive solution of Eq (3.7), denoted as x_r , then we have

$$E_{2r} = \frac{1}{q_2} \left(-\hat{d} + \frac{\hat{c}ax_r^2}{aHx_r^2+x_r+g} \right).$$

Thus, $E_{2r} > 0$ in the case of $\hat{d} < \hat{c}ax_r^2 / (aHx_r^2 + x_r + g)$.

Case 3: If $c_1 > s_1 q_1 x$ and $c_2 > s_2 q_2 y$, fishery activities for both prey and predator will be quit.

Case 4: If $c_1 < s_1 q_1 x$ and $c_2 < s_2 q_2 y$, there are $x_r = c_1 / s_1 q_1$ and $y_r = c_2 / s_2 q_2$. Substituting into Eqs (3.3) and (3.4), one can get

$$E_{1r} = \frac{1}{q_1} \left[\hat{r} \frac{q_1 s_1 K - c_1}{q_1 s_1 K + mc_1} - \frac{ac_1 q_1 s_1}{aHc_1^2 + c_1 q_1 s_1 + g q_1^2 s_1^2} \frac{c_2}{s_2 q_2} \right],$$

$$E_{2r} = \frac{1}{q_2} \left[-\hat{d} + \frac{\hat{c}ac_1^2}{aHc_1^2 + c_1 q_1 s_1 + g q_1^2 s_1^2} \right].$$

Therefore, $E_{1r} > 0$ if

$$\hat{r} \frac{q_1 s_1 K - c_1}{q_1 s_1 K + mc_1} > \frac{ac_1 q_1 s_1}{aHc_1^2 + c_1 q_1 s_1 + g q_1^2 s_1^2} \frac{c_2}{s_2 q_2}, \quad (3.8)$$

and $E_{2r} > 0$ if

$$\frac{\hat{c}ac_1^2}{aHc_1^2 + c_1 q_1 s_1 + g q_1^2 s_1^2} > \hat{d}. \quad (3.9)$$

Therefore, it can be concluded that the nontrivial bionomic equilibrium point $(x_r, y_r, E_{1r}, E_{2r})$ exists if inequations (3.8) and (3.9) hold.

3.1.3. Optimal harvesting strategy

Let δ be the discount rate, then the net profit $J(E)$ is defined as

$$J(E_1, E_2) = \int_0^{\infty} e^{-\delta t} [(q_1 s_1 x - c_1) E_1(t) + (q_2 s_2 y - c_2) E_2(t)] dt, \quad (3.10)$$

where $E_{1\min} \leq E_1(t) \leq E_{1\max}$ and $E_{2\min} \leq E_2(t) \leq E_{2\max}$. According to Pontryagin's maximal principle [60], denote

$$H = e^{-\delta t} [(q_1 s_1 x - c_1) E_1 + (q_2 s_2 y - c_2) E_2] + \lambda_1 F(x, y; E_1) + \lambda_2 G(x, y; E_2),$$

in which λ_1 and λ_2 are to be determined.

clearly, H is linear dependent on E_1 and E_2 . We suppose that the control are not bangbang ones, which means the optimal harvesting efforts would not be $E_{i\min}$ or $E_{i\max}$, and the singular control is obtained by

$$\begin{aligned} \frac{\partial H}{\partial E_1} &= e^{-\delta t} (q_1 s_1 x - c_1) - \lambda_1 q_1 x = 0 \Rightarrow \lambda_1 = e^{-\delta t} \left(s_1 - \frac{c_1}{q_1 x} \right), \\ \frac{\partial H}{\partial E_2} &= e^{-\delta t} (q_2 s_2 y - c_2) - \lambda_2 q_2 y = 0 \Rightarrow \lambda_2 = e^{-\delta t} \left(s_2 - \frac{c_2}{q_2 y} \right). \end{aligned}$$

The adjoint equations are

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x} = -\left\{ e^{-\delta t} q_1 s_1 E_1 + \lambda_1 F_x(x, y; w_1, w_2, \omega, E_1) + \lambda_2 G_y(x, y; w_1, w_2, \omega, E_2) \right\}, \quad (3.11)$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y} = -\left\{ e^{-\delta t} q_2 s_2 E_2 + \lambda_1 F_x(x, y; w_1, w_2, \omega, E_1) + \lambda_2 G_y(x, y; w_1, w_2, \omega, E_2) \right\}. \quad (3.12)$$

Substituting λ_1 and λ_2 into Eq (3.11) and simplifying, there is

$$\begin{aligned} \delta \left(s_1 - \frac{c_1}{q_1 x} \right) &= q_1 s_1 E_1 + \left(s_1 - \frac{c_1}{q_1 x} \right) \left\{ \hat{r} \left(\frac{K-x}{K+mx} \right) - \hat{r} x \left(\frac{(1+m)K}{(K+mx)^2} \right) - \frac{2ax(aHx^2+x+g)-ax^2(2aHx+1)}{(aHx^2+x+g)^2} y - q_1 E_1 \right\} \\ &+ \left(s_2 - \frac{c_2}{q_2 y} \right) \left\{ \frac{2\hat{c}ax(aHx^2+x+g)-\hat{c}ax^2(2aHx+1)}{(aHx^2+x+g)^2} y \right\}. \end{aligned} \quad (3.13)$$

Substituting λ_1 and λ_2 into Eq (3.12) and simplifying, there is

$$\delta \left(s_2 - \frac{c_2}{q_2 y} \right) = q_2 s_2 E_2 - \left(s_1 - \frac{c_1}{q_1 x} \right) \frac{ax^2}{aHx^2 + x + g} + \left(s_2 - \frac{c_2}{q_2 y} \right) \left\{ -\hat{d} - q_2 E_2 + \frac{\hat{c}ax^2}{aHx^2 + x + g} \right\}. \quad (3.14)$$

Solving steady state Eqs (3.3) and (3.4) together with Eqs (3.13) and (3.14), we will find the optimal state (x_δ, y_δ) and the optimal capture efforts $E_{i\delta}$, $i = 1, 2$.

3.2. Complex dynamics of Model (2.7)

Define

$$\mathcal{M}_1 = \{(x, y) \in \mathbb{R}_2^+ \mid x = x_T, y_T < y \leq y_L(x_T)\}, \mathcal{M}_2 = \{(x, y) \in \mathbb{R}_2^+ \mid x = x_T, 0 \leq y \leq y_T\},$$

$$\mathcal{N}_1 = \mathbf{I}(\mathcal{M}_1) = \{(x, y) \mid x = (1 - q_1 E_1)x_T, (1 - q_2 E_2)y_T < y \leq (1 - q_2 E_2)y_L(x_T)\},$$

$$\mathcal{N}_2 = \mathbf{I}(\mathcal{M}_2) = \{(x, y) \mid x = (1 - q_1 E_1)x_T, \tau \leq y \leq (1 - q_2 E_2)y_T + \tau\}.$$

3.2.1. Predator-extinction periodic solution

For any given $w_1, w_2, \omega \in [0, 1]$, if $y(0) = 0$, then $y(t) \equiv 0 \forall t \geq 0$ in the case of $\tau = 0$. Therefore, Model (2.7) reduces to

$$\left\{ \begin{array}{l} \frac{dx}{dt} = \hat{r}x \left(\frac{K-x}{K+mx} \right) \\ \frac{dy}{dt} = 0 \\ \Delta x = -q_1 E_1 x \\ \Delta y = 0 \end{array} \right\} \begin{array}{l} x < x_T, \\ x = x_T. \end{array} \quad (3.15)$$

Define

$$T_0 \triangleq \frac{1}{\hat{r}} \ln \left(\frac{1}{(1-q_1 E_1)} \right) - \frac{1+m}{\hat{r}} \ln \left(\frac{K-x_T}{K-(1-q_1 E_1)x_T} \right).$$

Let $x = \psi(t; w_1, w_2, \omega)$ be the solution determined by the following equation

$$\frac{x}{(K-x)^{1+m}} = \frac{(1-q_1 E_1)x_T}{(K-(1-q_1 E_1)x_T)^{1+m}} \exp(\hat{r}t).$$

$$\Phi(u) \triangleq \frac{(K-ux_T)(K+mx_T)}{(K+umx_T)(K-x_T)} \cdot \exp \left(\int_0^{T_0} \left[-\frac{\hat{r}(1+m)Kx}{K+mx} + \frac{\hat{c}ax^2}{aHx^2+x+g} - \hat{d} \right]_{\psi(t;w_1,w_2,\omega)} dt \right)$$

Theorem 3.3. For any given $w_1, w_2, \omega \in [0, 1]$, if $0 < x_T < K_E$, then a predator-extinction periodic solution $\bar{\mathbf{z}}(t) = (\psi(t - (j-1)T_0; w_1, w_2, \omega), 0)$ ($(j-1)T_0 \leq t \leq jT_0$) exists in Model (2.7), which is orbitally asymptotically stable when $E_2 > \bar{E}_2(E_1) \triangleq \max\{0, 1 - \Phi^{-1}(1 - q_1 E_1)\}/q_2$.

Proof. Let $x_0 = x(0) \triangleq (1 - q_1 E_1)x_T$, then for $(j-1)T_0 \leq t \leq jT_0$, there is

$$\frac{x(t)}{(K-x(t))^{1+m}} = \frac{(1-q_1 E_1)x_T \exp(\hat{r}K(t - (j-1)T_0))}{(K-(1-q_1 E_1)x_T)^{1+m}}.$$

clearly, $\bar{\mathbf{z}}(t) = (\psi(t - (j-1)T_0; w_1, w_2, \omega), 0)$ ($(j-1)T_0 \leq t \leq jT_0$) is a periodic solution.

Since

$$F(x, y; w_1, w_2, \omega) = \hat{r}x \left(\frac{K-x}{K+mx} \right) - \frac{ax^2}{aHx^2+x+g}y, \quad G(x, y; w_1, w_2, \omega) = -\hat{d}y + \frac{\hat{c}ax^2}{aHx^2+x+g}y,$$

$$\phi(x, y) = x - x_T, \quad I_2(x, y) = -q_2 E_2 y, \quad I_1(x, y) = -q_1 E_1 x,$$

then

$$\begin{aligned} \frac{\partial F}{\partial x} &= \hat{r} \left(\frac{K-x}{K+mx} \right) - rx \left(\frac{(1+m)K}{(K+mx)^2} \right) - \frac{2ax(aHx^2+x+g) - ax^2(2aHx+1)}{(aHx^2+x+g)^2}y, \\ \frac{\partial G}{\partial y} &= -\hat{d} + \frac{\hat{c}ax^2}{aHx^2+x+g}, \\ \frac{\partial I_1}{\partial x} &= -q_1 E_1, \quad \frac{\partial I_2}{\partial x} = 0, \quad \frac{\partial \phi}{\partial x} = 1, \quad \frac{\partial I_1}{\partial y} = 0, \quad \frac{\partial I_2}{\partial y} = -q_2 E_2, \quad \frac{\partial \phi}{\partial y} = 0. \end{aligned}$$

For $\bar{\mathbf{z}}(t) = (\bar{\xi}(t; w_1, w_2, \omega), 0)$, there is

$$\Delta_1 = (1 - q_2 E_2) \frac{F((1 - q_1 E_1)x_T, 0)}{F(x_T, 0)} = (1 - q_2 E_2)(1 - q_1 E_1) \frac{[K - (1 - q_1 E_1)x_T](K + mx_T)}{[K + m(1 - q_1 E_1)x_T](K - x_T)}$$

and

$$\begin{aligned} & \exp\left(\int_0^{T_0} \left[\left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y}\right) \Big|_{(\psi(t; w_1, w_2, \omega), 0)}\right] dt\right) \\ &= \frac{1}{1 - q_1 E_1} \exp\left(\int_0^{T_0} \left[-\frac{\hat{r}(1+m)Kx}{K+mx} + \frac{\hat{c}ax^2}{aHx^2+x+g} - \hat{d}\right] \Big|_{\psi(t; w_1, w_2, \omega)} dt\right), \end{aligned}$$

then by Lemma 2.1, there is $\mu_1 = (1 - q_2 E_2)\Phi(1 - q_1 E_1)$. Therefore, when $E_2 > \widetilde{E}_2(E_1)$, the predator extinction periodic solution $\bar{\mathbf{z}}(t) = (\psi(t - (j-1)T_0; w_1, w_2, \omega), 0)$ ($(j-1)T_0 \leq t \leq jT_0$) is orbitally asymptotically stable. \square

3.2.2. Coexisting order-1 periodic solution

Define

$$y_L(x) \triangleq \hat{r} \left(\frac{K-x}{K+mx} \right) \frac{aHx^2+x+g}{ax}, 0 \leq x \leq K.$$

Denote $A((1 - q_1 E_1)x_T, y_L((1 - q_1 E_1)x_T))$, $B(x_T, y_L(x_T))$, $D(x_T, y_T)$ and $G((1 - q_1 E_1)x_T, \tau)$. Define

$$x_T^{\max} \triangleq \max\{x_T \mid \gamma_A(\mathbf{z}) \cap \mathcal{M}_2 \neq \emptyset\}.$$

For $0 < x_T < x_T^{\max}$, denote $C((1 - q_1 E_1)x_T, y_C) \in \mathcal{N}$ with $\gamma_C(\mathbf{z}) \cap \mathcal{M}_1 = \{D\}$ and $\gamma_A(\mathbf{z}) \cap \mathcal{M}_1 = \{A^-\}$.

Theorem 3.4. For $0 < x_T \leq x_T^{\max}$, if i) $y_{A^-}(x_T) \leq y_T$ or ii) $y_{A^-}(x_T) > y_T$, $\tau \leq \bar{\tau} \triangleq y_C - (1 - q_2 E_2)y_T$ holds, system (2.7) has a coexistence order-1 periodic solution; if iii) $y_{A^-}(x_T) > y_T$, $E_2 > \max\{0, (1 - y_C/y_{A^-})/q_2\}$ and $y_C \leq \tau \leq y_L(x_T) - (1 - q_2 E_2)y_T$, Model (2.7) has a coexistence order-2 periodic solution.

Proof. To show the existence of the order-1 periodic solution, it is only necessary to find a point $S \in \mathcal{N}_2$ satisfying $f_{\mathcal{M}_2}^I(S) = 0$ or $f_{\mathcal{M}_2}^{II}(S) = 0$.

i) $y_{A^-}(x_T) \leq y_T$, as shown in Figure 2(a). Clearly, $f_{\mathcal{M}_2}^I(G) = y_{G^+} - y_G = (1 - q_2 E_2)G^- > 0$. If $\tau \leq \bar{\tau} \triangleq y_L((1 - q_1 E_1)x_T) - (1 - q_2 E_2)y_{A^-}$, then $f_{\mathcal{M}_2}^I(A) = y_{A^+} - y_A \leq 0$. Thus, $\exists S \in \overline{AG}$, satisfying $f_{\mathcal{M}_2}^I(S) = 0$. For $\tau > \bar{\tau}$, there is $f_{\mathcal{M}_2}^I(A) > 0$, then $f_{\mathcal{M}_2}^{II}(A^+) = y_{A^{++}} - y_{A^+} = (1 - q_2 E_2)(y_{A^{++}} - y_{A^+}) < 0$. Define $\epsilon = (y_{A^+} - y_A)/3$. For $0 < \delta < \epsilon$ and $\hat{A} \in U_o(A, \delta) \cap \mathcal{N}$, there is $0 < y_{A^+} - y_{\hat{A}^+} < \epsilon$. Thus, $f_{\mathcal{M}_2}^{II}(\hat{A}) = y_{\hat{A}^+} - y_{\hat{A}} > 0$, and $\exists S \in \overline{AG}$ satisfies $f_{\mathcal{M}_2}^{II}(S) = 0$.

ii) $y_{A^-}(x_T) > y_T$, $\tau \leq \bar{\tau} \triangleq y_C - (1 - q_2 E_2)y_T$, there is $f_{\mathcal{M}_2}^I(A) = y_{A^+} - y_A \leq 0$. Together with $f_{\mathcal{M}_2}^I(G) = y_{G^+} - y_G = (1 - q_2 E_2)G^- > 0$, it deduces that $\exists S \in \overline{GG_2} \subset \mathcal{N}_2$ with $f_{\mathcal{M}_2}^{II}(S) = 0$.

iii) $y_{A^-}(x_T) > y_T$, $E_2 > \max\{0, (1 - y_C/y_{A^-})/q_2\}$ and $y_C \leq \tau \leq y_L(x_T) - (1 - q_2 E_2)y_T$, as shown in Figure 2(b). Since $f_{\mathcal{M}_2}^I(C) = (1 - q_2 E_2)y_T + \tau - y_C > 0$ and $f_{\mathcal{M}_1}^I(C^+) = (1 - q_2 E_2)y_{C^{++}} - y_{C^+} < 0$, then $\mathcal{P}_{\mathcal{M}_1} \cdot \mathcal{P}_{\mathcal{M}_2}(C) < 0$. Similarly, $\mathcal{P}_{\mathcal{M}_1} \cdot \mathcal{P}_{\mathcal{M}_2}(G) < 0$, then $\exists S \in \overline{CG} \subset \mathcal{N}_2$ such that $\mathcal{P}_{\mathcal{M}_1} \cdot \mathcal{P}_{\mathcal{M}_2}(S) = S$.

To sum up, for case i) or ii), $\exists S \in \mathcal{N}_2$ with $f_{\mathcal{M}_2}^I(S) = 0$ or $f_{\mathcal{M}_2}^{II}(S) = 0$, which means the existence of coexistence order-1 periodic solutions; for case iii), $\exists S \in \mathcal{N}_2$ with $\mathcal{P}_{\mathcal{M}_1} \cdot \mathcal{P}_{\mathcal{M}_2}(S) = S$, which means the existence of coexistence order-2 periodic solution. \square

Let $\tilde{\mathbf{z}}(t) = (\tilde{\xi}(t; w_1, w_2, \omega), \tilde{\eta}(t; w_1, w_2, \omega))$ ($(j-1)T \leq t \leq jT$) be the coexistence order-1 periodic solution. For the sake of simplification, let's denote

$$\tilde{\xi}_0 \triangleq \tilde{\xi}(0; w_1, w_2, \omega), \tilde{\xi}_1 \triangleq \tilde{\xi}(T; w_1, w_2, \omega), \tilde{\eta}_0 \triangleq \tilde{\eta}(0; w_1, w_2, \omega), \tilde{\eta}_1 \triangleq \tilde{\eta}(T; w_1, w_2, \omega)$$

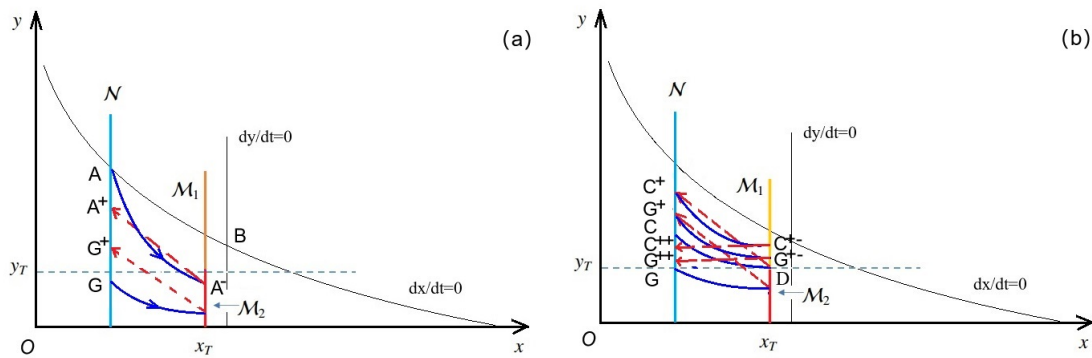


Figure 2. System trajectory trend diagram: a) $y_{A^-}(x_T) \leq y_T$; b) $y_{A^-}(x_T) > y_T$, $E_2 > \max\{0, (1 - y_C/y_{A^-})/q_2\}$ and $y_C \leq \tau \leq y_L(x_T) - (1 - q_2 E_2)y_T$.

and

$$F_0 = F(\tilde{\xi}_0, \tilde{\eta}_0; w_1, w_2, \omega), F_1 = F(\tilde{\xi}_1, \tilde{\eta}_1; w_1, w_2, \omega),$$

$$G_0 = G(\tilde{\xi}_0, \tilde{\eta}_0; w_1, w_2, \omega), G_1 = G(\tilde{\xi}_1, \tilde{\eta}_1; w_1, w_2, \omega).$$

The coexistence order-2 periodic solution is denoted by

$$\widehat{\mathbf{z}}_2(t) = \begin{cases} (\hat{\xi}_1(t; w_1, w_2, \omega), \hat{\eta}_1(t; w_1, w_2, \omega)), & (j-1)T_1 \leq t \leq jT_1, \\ (\hat{\xi}_2(t; w_1, w_2, \omega), \hat{\eta}_2(t; w_1, w_2, \omega)), & jT_1 \leq t \leq j(T_1 + T_2). \end{cases}$$

Similarly, denote

$$\hat{\xi}_{10} \triangleq \hat{\xi}_1(0; w_1, w_2, \omega), \hat{\xi}_{11} \triangleq \hat{\xi}_1(T_1; w_1, w_2, \omega), \hat{\eta}_{10} \triangleq \hat{\eta}_1(0; w_1, w_2, \omega), \hat{\eta}_{11} \triangleq \hat{\eta}_1(T_1; w_1, w_2, \omega),$$

$$\hat{\xi}_{20} \triangleq \hat{\xi}_2(T_1; w_1, w_2, \omega), \hat{\xi}_{21} \triangleq \hat{\xi}_2(T_1 + T_2; w_1, w_2, \omega), \hat{\eta}_{20} \triangleq \hat{\eta}_2(T_1; w_1, w_2, \omega), \hat{\eta}_{21} \triangleq \hat{\eta}_2(T_1 + T_2; w_1, w_2, \omega)$$

and

$$F_{10} = F(\hat{\xi}_{10}, \hat{\eta}_{10}; w_1, w_2, \omega), F_{11} = F(\hat{\xi}_{11}, \hat{\eta}_{11}; w_1, w_2, \omega),$$

$$G_{10} = G(\hat{\xi}_{10}, \hat{\eta}_{10}; w_1, w_2, \omega), G_{11} = G(\hat{\xi}_{11}, \hat{\eta}_{11}; w_1, w_2, \omega),$$

$$F_{20} = F(\hat{\xi}_{20}, \hat{\eta}_{20}; w_1, w_2, \omega), F_{21} = F(\hat{\xi}_{21}, \hat{\eta}_{21}; w_1, w_2, \omega),$$

$$G_{20} = G(\hat{\xi}_{20}, \hat{\eta}_{20}; w_1, w_2, \omega), G_{21} = G(\hat{\xi}_{21}, \hat{\eta}_{21}; w_1, w_2, \omega).$$

Define

$$\Theta_1 \triangleq \exp\left(\int_0^T \left[\hat{r} \left(\frac{K-x}{K+mx} \right) - rx \left(\frac{(1+m)K}{(K+mx)^2} \right) - \hat{d} + \frac{\hat{c}ax^2}{aHx^2 + x + g} \right] \Big|_{(\tilde{\xi}(t; w_1, w_2, \omega), \tilde{\eta}(t; w_1, w_2, \omega))} dt\right).$$

Denote

$$\Theta_2 \triangleq \exp\left(\int_0^{T_1} \left[\hat{r} \left(\frac{K-x}{K+mx} \right) - rx \left(\frac{(1+m)K}{(K+mx)^2} \right) - \hat{d} + \frac{\hat{c}ax^2}{aHx^2 + x + g} \right] \Big|_{(\hat{\xi}_1(t; w_1, w_2, \omega), \hat{\eta}_1(t; w_1, w_2, \omega))} dt\right)$$

$$\cdot \exp\left(\int_{T_1}^{T_2} \left[\hat{r} \left(\frac{K-x}{K+mx} \right) - rx \left(\frac{(1+m)K}{(K+mx)^2} \right) - \hat{d} + \frac{\hat{c}ax^2}{aHx^2 + x + g} \right] \Big|_{(\hat{\xi}_2(t; w_1, w_2, \omega), \hat{\eta}_2(t; w_1, w_2, \omega))} dt\right).$$

Theorem 3.5. *The coexistence order-1 periodic solution $\tilde{\mathbf{z}}(t) = (\tilde{\xi}(t; w_1, w_2, \omega), \tilde{\eta}(t; w_1, w_2, \omega))((k-1)T \leq t \leq kT)$ is orbitally asymptotically stable if $(1 - q_2 E_2)\Theta_1 F_0/F_1 < 1$; the coexistence order-2 periodic solution $\tilde{\mathbf{z}}(t)$ is orbitally asymptotically stable if $(1 - q_2 E_2)^2 \Theta_2 F_{10} F_{20}/F_{11} F_{22} < 1$.*

Proof. Similar to the proof of Theorem , there is

$$\Delta_1 = (1 - q_2 E_2) \frac{F_{20}}{F_{11}}, \quad \Delta_2 = (1 - q_2 E_2) \frac{F_{10}}{F_{21}}$$

and

$$\begin{aligned} & \exp\left(\int_0^T \left[\left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y}\right)\Big|_{(\xi(t; w_1, w_2, \omega), \eta(t; w_1, w_2, \omega))} dt\right]\right) \\ &= \exp\left(\int_0^T \left[\hat{r} \left(\frac{K-x}{K+mx}\right) - rx \left(\frac{(1+m)K}{(K+mx)^2}\right) - \hat{d} + \frac{\hat{c}ax^2}{aHx^2 + x + g}\right]\Big|_{(\xi(t; w_1, w_2, \omega), \eta(t; w_1, w_2, \omega))} dt\right), \\ & \exp\left(\int_0^{T_1} \left[\left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y}\right)\Big|_{(\hat{\xi}(t; w_1, w_2, \omega), \hat{\eta}(t; w_1, w_2, \omega))} dt\right] + \int_{T_1}^{T_1+T_2} \left[\left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y}\right)\Big|_{(\tilde{\xi}(t; w_1, w_2, \omega), \tilde{\eta}(t; w_1, w_2, \omega))} dt\right]\right) \\ &= \exp\left(\int_0^{T_1} \left[\hat{r} \left(\frac{K-x}{K+mx}\right) - rx \left(\frac{(1+m)K}{(K+mx)^2}\right) - \hat{d} + \frac{\hat{c}ax^2}{aHx^2 + x + g}\right]\Big|_{(\hat{\xi}(t; w_1, w_2, \omega), \hat{\eta}(t; w_1, w_2, \omega))} dt\right) \\ &\cdot \exp\left(\int_{T_1}^{T_1+T_2} \left[\hat{r} \left(\frac{K-x}{K+mx}\right) - rx \left(\frac{(1+m)K}{(K+mx)^2}\right) - \hat{d} + \frac{\hat{c}ax^2}{aHx^2 + x + g}\right]\Big|_{(\tilde{\xi}(t; w_1, w_2, \omega), \tilde{\eta}(t; w_1, w_2, \omega))} dt\right). \end{aligned}$$

Therefore, it can be concluded that the coexistence order-1 periodic solution $\tilde{\mathbf{z}}(t) = (\tilde{\xi}(t; w_1, w_2, \omega), \tilde{\eta}(t; w_1, w_2, \omega))((k-1)T \leq t \leq kT)$ is orbitally asymptotically stable if $(1 - q_2 E_2)\Theta_1 F_0/F_1 < 1$; the coexistence order-2 periodic solution $\tilde{\mathbf{z}}(t)$ is orbitally asymptotically stable if $(1 - q_2 E_2)^2 \Theta_2 F_{10} F_{20}/F_{11} F_{22} < 1$. \square

4. Numerical simulations and discussion

To illustrate the theoretical results, the model parameters are assumed to be $K = 50$, $m = 1$, $a = 0.005$, $H = 2$. The capture rates are set to be $q_1 = 0.1$, $q_2 = 0.02$. The parameters r , d and c are set to be the TFNs $\tilde{r} = (0.18, 0.20, 0.22)$, $\tilde{d} = (0.04, 0.05, 0.06)$, $\tilde{c} = (0.30, 0.33, 0.36)$.

4.1. Simulations for Model (2.6)

We will illustrate the effects of the imprecision indicators w_1, w_2, ω , search rate constant g and capture efforts $\mathbf{E} = (E_1, E_2)$ on the system, respectively.

First, we illustrate the impact of the imprecision indicators w_1, w_2, ω on Model (2.6). For $g = 0.05$, $\mathbf{E} = \mathbf{0}$ and $(x_0, y_0) = (10, 4)$, the time series and stable steady states for different w_1, w_2, ω are presented in Figures 3–5 and Table 1.

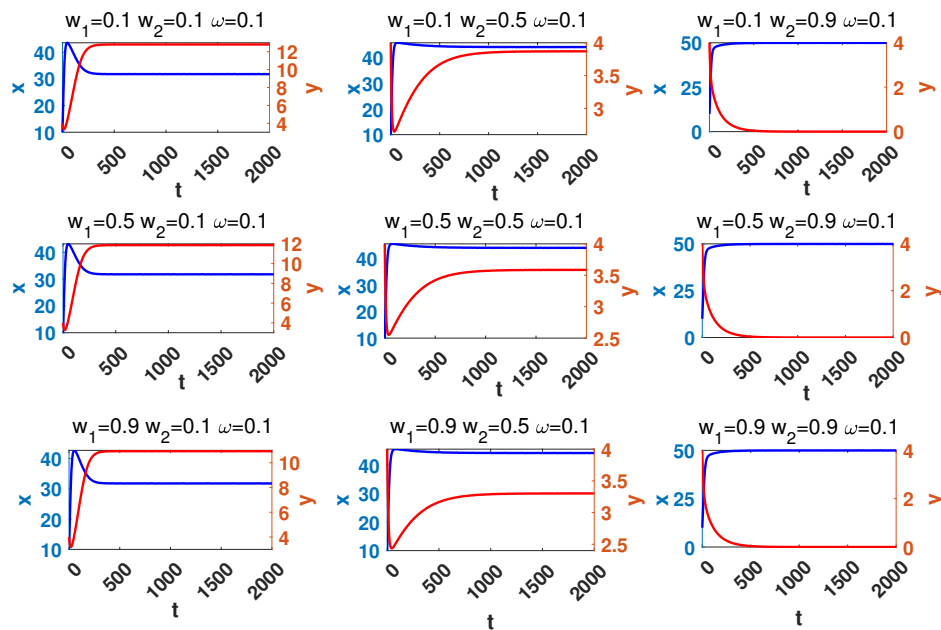


Figure 3. Impact of imprecision indicators on Model (2.6): Time series for different (w_1, w_2) when $\omega = 0.1$, where the density of prey is marked blue and the density of predators is marked red.

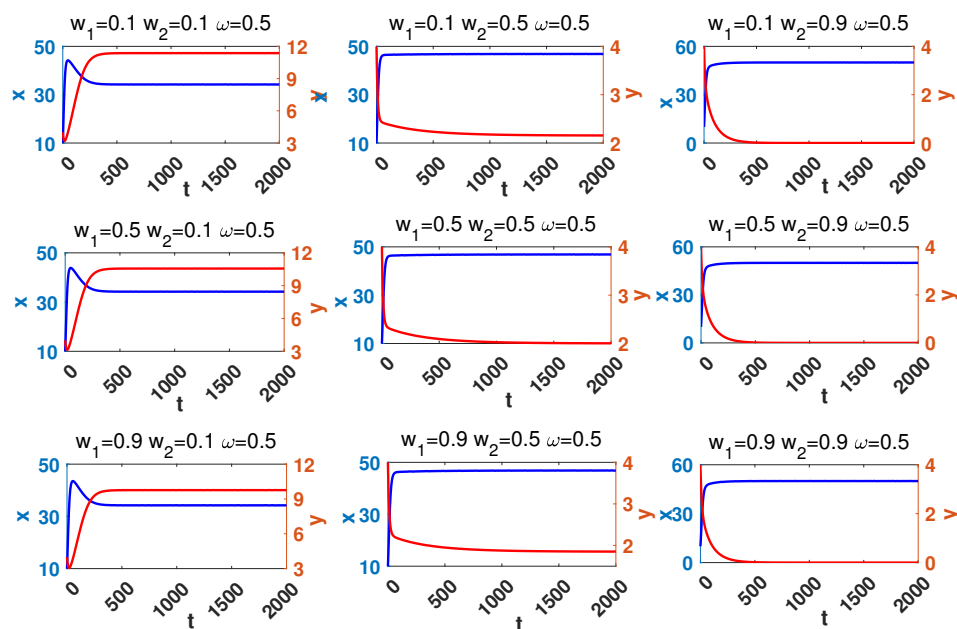


Figure 4. Impact of imprecision indicators on Model (2.6): Time series for different (w_1, w_2) when $\omega = 0.5$, where the density of prey is marked blue and the density of predators is marked red.

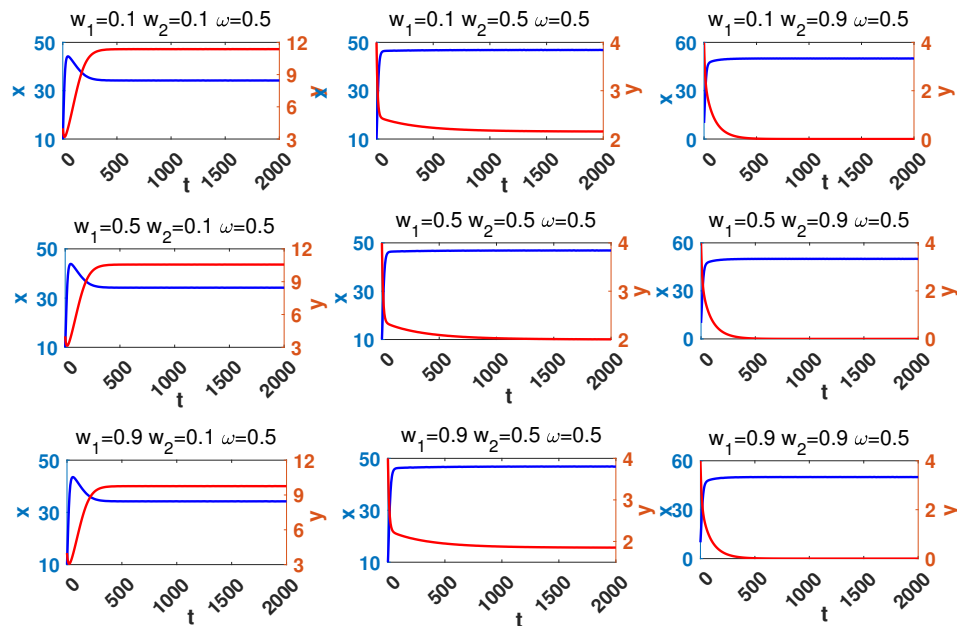


Figure 5. Impact of imprecision indicators on Model (2.6): Time series for different (w_1, w_2) when $\omega = 0.9$, where the density of prey is marked blue and the density of predators is marked red.

Table 1. The impact of w_1, w_2, ω on coexistence equilibrium Q_2 .

w_1	w_2	$Q_2(x_E^*, y_E^*)$		
		$\omega = 0.1$	$\omega = 0.5$	$\omega = 0.9$
0.1	0.1	(31.78, 12.81)	(34.23, 11.37)	(36.61, 9.89)
	0.5	(44.21, 3.87)	(46.86, 2.15)	(49.42, 0.41)
	0.9	Nonexistence	Nonexistence	Nonexistence
0.5	0.1	(31.78, 11.87)	(34.23, 10.57)	(36.61, 9.22)
	0.5	(44.21, 3.59)	(46.86, 2.00)	(49.42, 0.38)
	0.9	Nonexistence	Nonexistence	Nonexistence
0.9	0.1	(31.78, 10.93)	(34.23, 9.76)	(36.61, 8.54)
	0.5	(44.21, 3.30)	(46.86, 1.85)	(49.42, 0.35)
	0.9	Nonexistence	Nonexistence	Nonexistence

From Figures 3–5 and Table 1 it can be concluded that: 1) When w_1 is increasing, x_E^* doesn't change, while y_E^* decreases; 2) when w_2 is increasing, x_E^* is increasing, while y_E^* is decreasing; 3) when ω is increasing, x_E^* is also increasing, while y_E^* is decreasing; 4) as w_2 decreases to a certain level, the coexistence equilibrium Q_2 disappears and the predator species goes extinct.

Second, we illustrate the impact of predator search rate on Model (2.6). Here, the parameter g is selected as the relative size of predator search rate. For $w_1 = 0.2, w_2 = 0.2, \omega = 0.5, (E_1, E_2) = (0.015, 0.001)$ and $(x_0, y_0) = (10, 4)$, the impact of g on Model (2.6) is presented in Figure 6 and Table 2. It can be observed that the predator search rate has a certain impact on the stability of the coexistence

equilibrium Q_2 . As g increases, i.e., predator search speed decreases, x_E^* increases and y_E^* decreases. Especially, when g increases to a certain level, the predator search rate becomes extremely small and leads to the extinction of predator species.

Third, we illustrate the impact of (E_1, E_2) on Model (2.6). For $g = 0.05$, $w_1 = 0.2$, $w_2 = 0.2$, $\omega = 0.5$, the impact of (E_1, E_2) on the system with the initial value $(x_0 = 10, y_0 = 4)$ is presented in Figure 7. It can be observed that the fishing activities contribute to the stability of the system. Moreover, as long as the fishing intensity is appropriate, the system can achieve a coexistence steady state. However, if the fishing effort is too large, especially if the capture effort of the prey population exceeds the given threshold, the system will become extinct. These observations and the conclusion are consistent with Theorem 1.

Next, we illustrate the impact of the imprecision indicators w_1, w_2, ω on bionomic equilibrium. For $s_1 = 0.5$, $s_2 = 20$, $c_1 = 1.8$, $c_1 = 2$, the impact of w_1, w_2, ω on bionomic equilibrium (if existing) is presented in Table 3. It can be observed that bionomic equilibrium does not exist unconditionally and needs to satisfy inequalities (3.8) and (3.9).

Lastly, we illustrate the optimal harvesting policy. Let $q_1 = 0.1$, $q_2 = 0.02$, $s_1 = 1$, $s_2 = 20$, $c_1 = 1.8$, $c_1 = 2$, $\delta = 0.008$ and imprecision indexes $w_1 = 0.1$, $w_2 = 0.1$, $\alpha = 0.5$, then we obtain the optimal equilibrium $(42.00, 8.51)$ and the optimal harvesting efforts $E_1 = 0.12$, $E_2 = 0.38$.

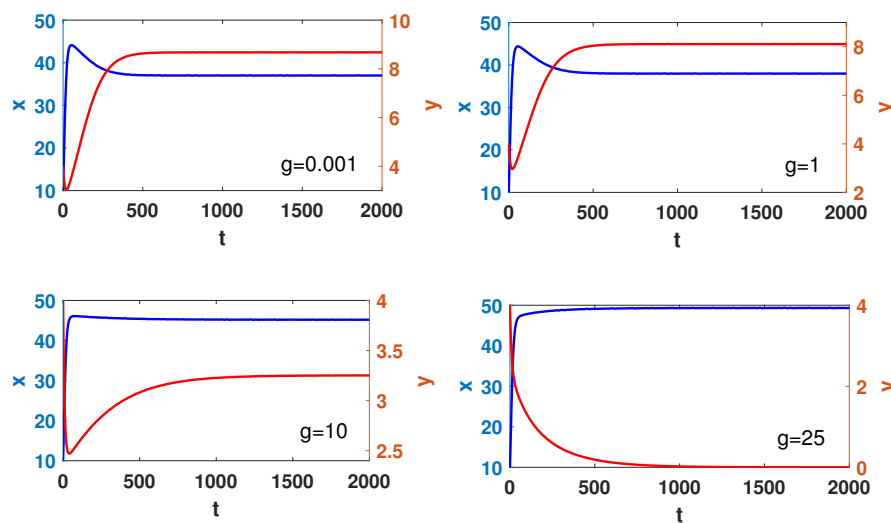


Figure 6. Impact of predator search speed g on the Model (2.6): the density of prey is marked blue and the density of predators is marked red.

Table 2. The impact of predator search speed on coexistence equilibrium Q_2 .

g	Coexistence equilibrium	Nature
0.001	(37.00, 8.68)	Globally asymptotically stable
1	(37.98, 8.11)	Globally asymptotically stable
10	(45.19, 3.25)	Globally asymptotically stable
25	Nonexistence	-

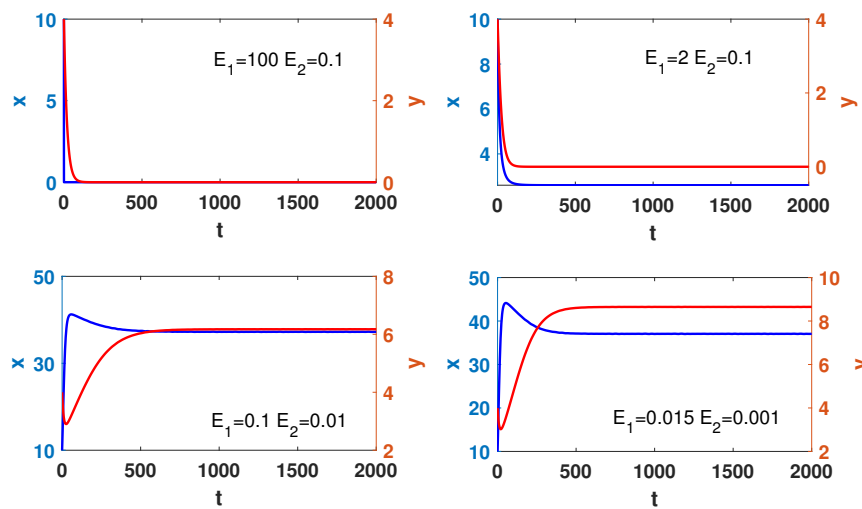


Figure 7. Impact of capture effects (E_1, E_2) on Model (2.6): the density of prey is marked blue and the density of predators is marked red.

Table 3. The impact of w_1, w_2, ω on bionomic equilibria.

w_1	w_2	Bionomic equilibrium $(x_r, y_r, E_{1r}, E_{2r})$		
		$\omega = 0.1$	$\omega = 0.5$	$\omega = 0.9$
0.1	0.1	(36,5,0.1712,0.2101)	(36,5,0.1843,0.0894)	Nonexistence
	0.5	Nonexistence	Nonexistence	Nonexistence
	0.9	Nonexistence	Nonexistence	Nonexistence
0.5	0.1	(36,5,0.1452,0.2101)	(36,5,0.1582,0.0894)	Nonexistence
	0.5	Nonexistence	Nonexistence	Nonexistence
	0.9	Nonexistence	Nonexistence	Nonexistence
0.9	0.1	(36,5,0.1192,0.2101)	(36,5,0.1322,0.0894)	Nonexistence
	0.5	Nonexistence	Nonexistence	Nonexistence
	0.9	Nonexistence	Nonexistence	Nonexistence

4.2. Simulations for Model (2.7)

Here, the dynamic behavior of Model (2.7) induced by the switch capture strategy is presented. For $w_1 = 0.2, w_2 = 0.2, \omega = 0.5, x_T = 28, y_T = 2.3, E_1 = 5, E_2 = 40$ and other parameters presented at the beginning of section four, it can be observed from Figure 8 that Model (2.7) has a predator-extinction periodic solution.

For $w_1 = 0.2, w_2 = 0.2, \omega = 0.5, x_T = 28, y_T = 2.3, E_1 = 5, E_2 = 10$, Model (2.7) has a different coexistence periodic solution, as shown in Figures 9–12. It can be observed that the periodic solution depends on the value of τ .

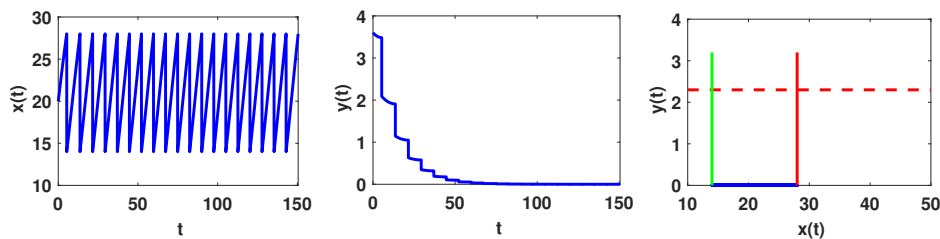


Figure 8. Time series and phase diagram of the predator-extinction periodic solution in case of $\tau = 0$.

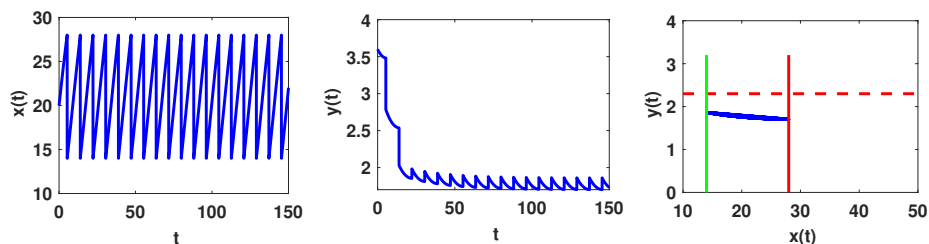


Figure 9. Time series and phase diagram of the coexistence order-1 periodic solution in case of $\tau = 0.5$.

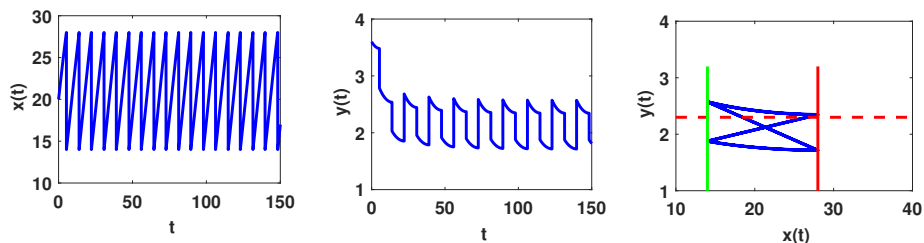


Figure 10. Time series and phase diagram of the coexistence order-2 periodic solution in case of $\tau = 1.2$.

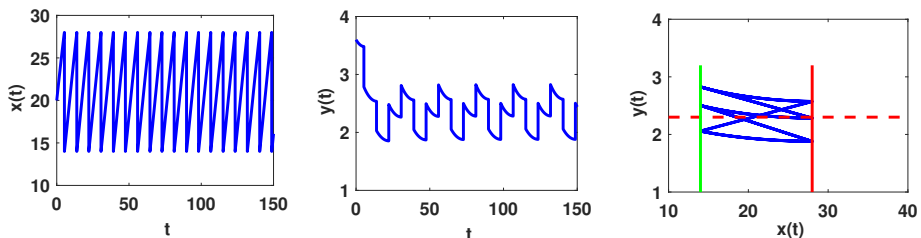


Figure 11. Time series and phase diagram of the coexistence order-3 periodic solution in case of $\tau = 1$.

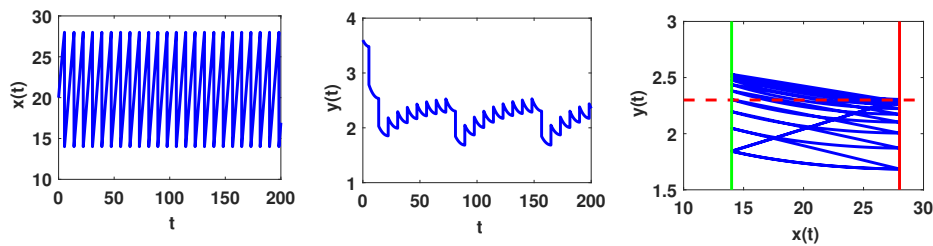


Figure 12. Time series and phase diagram of the coexistence order-9 periodic solution in case of $\tau = 0.7$.

5. Conclusions

Considering that some biological parameters of different species in an ecosystem fluctuate to a certain extent due to changes in the external environment, it is of practical significance to study biological models with imprecise parameters. In addition, the search rate of predators varies rather than is fixed in response to changes in the environment and the distribution of prey. In view of the above phenomena, we proposed a Gause-type fishery model incorporated with the Smith growth function, variable search rate and triangular fuzzy biological parameters. Moreover, from the perspective of rational exploitation of fishery resources, we introduced two fishing strategies into the system and analyzed the effects of different fishing strategies on fish resources.

For the continuous type capture system, we figured out the effects of variable predator search speed and imprecision indicators on the system's dynamics (Theorems 1–2, Tables 1,2 and Figures 3–6). The results show that when the parameters change obviously, the imprecise indicators have a certain influence on the dynamic characteristics of the system. This is reasonable since the gradual change of the environment will not change the related characteristics of biological species, but dramatic changes will change its living habits to adapt to this change. In addition, for the sake of maximizing economic benefit of the capture process, we discussed the bionomic equilibrium of the system (Table 3) and the optimal capture strategy.

For the switch capture system, we provided the existence and stability conditions of predator-extinction periodic solution and coexistence order-1 or 2 periodic solution. To prevent predators from going extinct, predator populations should not be captured too aggressively. Through computer simulations, we found different order coexistence periodic solutions. This further indicates that the ecological balance of species can be achieved with the switch capture strategy.

The research indicated that the triangular imprecise biological parameters, variable predator search rate and capture activities further enriched the dynamic characteristics of the biological system, and also provided a theoretical reference basis for scientific and effective exploitation and utilization of fishery resources under uncertain parameter environments.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. FAO, The State of World Fisheries and Aquaculture 2018 - Meeting the sustainable development goals, Rome, 2018 Licence: CC BY-NC-SA 3.0 IGO.
2. A. J. Lotka, Elements of physical biology, in *Science Progress in the Twentieth Century (1919–1933)*, Sage Publications, (1926), 341–343.
3. V. Volterra, Fluctuations in the abundance of a species considered mathematically, *Nature*, **119** (1926). <https://doi.org/10.1038/119012a0>
4. G. F. Gause, N. P. Smaragdova, A. A. Witt, Further studies of interaction between predator and prey, *J. Anim. Ecol.*, **5** (1936), 1–18.
5. J. M. Smith, *Models in Ecology*, Cambridge University Press, Cambridge, 1974.
6. M. Sivakumar, M. Sambath, K. Balachandran, Stability and hopf bifurcation analysis of a diffusive predator–prey model with Smith growth, *Int. J. Biomath.*, **8** (2015), 1550013. <https://doi.org/10.1142/S1793524515500138>
7. X. L. Han, C. Y. Lei, Bifurcation and turing instability analysis for a space- and time-discrete predator-prey system with Smith growth function, *Chaos Solitons Fractals*, **166** (2022), 112910. <https://doi.org/10.1016/j.chaos.2022.112910>
8. X. Feng, X. Liu, C. Sun, Stability and Hopf bifurcation of a modified Leslie-Gower predator-prey model with Smith growth rate and B-D functional response, *Chaos Solitons Fractals*, **174** (2023), 113794. <https://doi.org/10.1016/j.chaos.2023.113794>
9. V. S. Ivlev, *Experimental Ecology of the Feeding of Fishes*, Yale University Press, New Haven, 1961.
10. C. S. Holling, The functional response of predators to prey density and its role in mimicry and population regulation, *Mem. Entomol. Soc. Can. Suppl.*, **97** (2012), 5–60. <https://doi.org/10.4039/entm9745fv>
11. T. Kuang, E. Beretta, Global qualitative analysis of a ratio-dependent predator-prey system, *J. Math. Biol.*, **36** (1998), 389–406. <https://doi.org/10.1007/s002850050105>
12. R. S. Cantrell, C. Cosner, On the dynamics of predator–prey models with the Beddington-DeAngelis functional response, *J. Math. Anal. Appl.*, **257** (2001), 206–222. <https://doi.org/10.1006/jmaa.2000.7343>
13. M. P. Hassell, H. N. Comins, Sigmoid functional responses and population stability, *Theor. Popul. Biol.*, **46** (1978), 62–67. [https://doi.org/10.1016/0040-5809\(78\)90004-7](https://doi.org/10.1016/0040-5809(78)90004-7)

14. F. Yu, Y. S. Wang, Hopf bifurcation and Bautin bifurcation in a prey-predator model with prey's fear cost and variable predator search speed, *Math. Comput. Simulat.*, **196** (2022), 192–209. <https://doi.org/10.1016/j.matcom.2022.01.026>
15. Y. S. Kwon, M. J. Bae, S. J. Hwang, S. H. Kim, Y. S. Park, Predicting potential impacts of climate change on freshwater fish in Korea, *Ecol. Inf.*, **29** (2015), 156–165. <https://doi.org/10.1016/j.ecoinf.2014.10.002>
16. N. W. Pankhurst, P. L. Munday, Effects of climate change on fish reproduction and early life history stages, *Mar. Freshwater Res.*, **9** (2011), 1015–1026. <https://doi.org/10.1071/MF10269>
17. T. M. Van Zuiden, M. M. Chen, S. Stefanoff, L. Lopez, S. Sharma, Projected impacts of climate change on three freshwater fishes and potential novel competitive interactions, *Divers. Distrib.*, **22** (2016), 603–614. <https://doi.org/10.1111/ddi.12422>
18. M. Liu, K. Wang, Global stability of a nonlinear stochastic predator-prey system with Beddington-DeAngelis functional response, *Commun. Nonlinear Sci.*, **16** (2011), 1114–1121. <https://doi.org/10.1016/j.cnsns.2010.06.015>
19. J. Lv, K. Wang, Asymptotic properties of a stochastic predator-prey system with Holling II functional response, *Commun. Nonlinear Sci.*, **16** (2011), 4037–4048. <https://doi.org/10.1016/j.cnsns.2011.01.015>
20. S. Zhang, S. Yuan, T. Zhang, Dynamic analysis of a stochastic eco-epidemiological model with disease in predators, *Stud. Appl. Math.*, **149** (2022), 5–42. <https://doi.org/10.1111/sapm.12489>
21. J. Xu, Z. Yu, T. Zhang, S. Yuan, Near-optimal control of a stochastic model for mountain pine beetles with pesticide application, *Stud. Appl. Math.*, **149** (2022), 678–704. <https://doi.org/10.1111/sapm.12517>
22. D. Pal, G. S. Mahapatra, G. P. Samanta, Optimal harvesting of prey-predator system with interval biological parameters: A bioeconomic model, *Math. Biosci.*, **241** (2013), 181–187. <https://doi.org/10.1016/j.mbs.2012.11.007>
23. D. Pal, G. S. Mahapatra, G. P. Samanta, Stability and bionomic analysis of fuzzy parameter based prey-predator harvesting model using UFM, *Nonlinear Dyn.*, **79** (2015), 1939–1955. <https://doi.org/10.1007/s11071-014-1784-4>
24. D. Pal, G. S. Mahapatra, G. P. Samanta, New approach for stability and bifurcation analysis on predator-prey harvesting model for interval biological parameters with time delays, *Comp. Appl. Math.*, **37** (2018), 3145–3171. <https://doi.org/10.1007/s40314-017-0504-3>
25. X. W. Yu, S. L. Yuan, T. H. Zhang, About the optimal harvesting of a fuzzy predator-prey system: a bioeconomic model incorporating prey refuge and predator mutual interference, *Nonlinear Dyn.*, **94** (2018), 2143–2160. <https://doi.org/10.1007/s11071-018-4480-y>
26. S. Das, P. Mahato, S. K. Mahato, A Prey Predator Model in Case of Disease Transmission via Pest in Uncertain Environment, *Differ. Equation Dyn. Syst.*, **31** (2023), 457–483. <https://doi.org/10.1007/s12591-020-00551-7>

27. Q. Z. Xiao, B. X. Dai, L. Wang, Analysis of a competition fishery model with interval-valued parameters: extinction, coexistence, bionomic equilibria and optimal harvesting policy, *Nonlinear Dyn.*, **80** (2015), 1631–1642. <https://doi.org/10.1007/s11071-015-1967-7>
28. Y. Tian, C. X. Li, J. Liu, Complex dynamics and optimal harvesting strategy of competitive harvesting models with interval-valued imprecise parameters, *Chaos Solitons Fractals*, **167** (2023), 113084. <https://doi.org/10.1016/j.chaos.2022.113084>
29. Y. Tian, H. Guo, K. Sun, Complex dynamics of two prey-predator harvesting models with prey refuge and interval-valued imprecise parameters, *Math. Meth. Appl. Sci.*, **46** (2023), 14278–14298. <https://doi.org/10.1002/mma.9319>
30. H. Guo, Y. Tian, K. B. Sun, X. Y. Song, Study on dynamic behavior of two fishery harvesting models: effects of variable prey refuge and imprecise biological parameters, *J. Appl. Math. Comput.* <https://doi.org/10.1007/s12190-023-01925-0>
31. X. Yu, S. Yuan, T. Zhang, About the optimal harvesting of a fuzzy predator-prey system: A bioeconomic model incorporating prey refuge and predator mutual interference, *Nonlinear Dyn.*, **94** (2018), 2143–2160. <https://doi.org/10.1007/s11071-018-4480-y>
32. J. Xu, S. Yuan, T. Zhang, Optimal harvesting of a fuzzy water hyacinth-fish model with Kuznets curve effect, *Int. J. Biomath.*, **16** (2023), 2250082. <https://doi.org/10.1142/S1793524522500826>
33. R. P. Gupta, M. Banerjee, P. Chandra, Bifurcation analysis and control of Leslie-Gower predator-prey model with Michaelis-Menten type prey-harvesting, *Differ. Equation Dyn. Syst.*, **20** (2012), 339–366. <https://doi.org/10.1007/s12591-012-0142-6>
34. Y. F. Lv, R. Yuan, Y. Z. Pei, A prey-predator model with harvesting for fishery resource with reserve area, *Appl. Math. Model.*, **37** (2013), 3048–3062. <https://doi.org/10.1016/j.apm.2012.07.030>
35. D. P. Hu, H. J. Cao, Stability and bifurcation analysis in a predator-prey system with Michaelis-Menten type predator harvesting, *Nonlinear Anal.-Real.*, **33** (2017), 58–82. <https://doi.org/10.1016/j.nonrwa.2016.05.010>
36. T. K. Ang, H. M. Safuan, Dynamical behaviors and optimal harvesting of an intraguild prey-predator fishery model with Michaelis-Menten type predator harvesting, *Biosystems*, **202** (2021), 104357. <https://doi.org/10.1016/j.biosystems.2021.104357>
37. X. Y. Meng, J. Li, Dynamical behavior of a delayed prey-predator-scavenger system with fear effect and linear harvesting, *Int. J. Biomath.*, **14** (2021), 2150024. <https://doi.org/10.1142/S1793524521500248>
38. S. Debnath, P. Majumdar, S. Sarkar, U. Ghosh, Global dynamics of a prey-predator model with holling type III functional response in the presence of harvesting, *J. Biol. Syst.*, **30** (2022), 225–260. <https://doi.org/10.1142/S0218339022500073>
39. L. F. Nie, Z. D. Teng, H. Lin, J. G. Peng, The dynamics of a Lotka-Volterra predator-prey model with state dependent impulsive harvest for predator, *Biosystems*, **98** (2009), 67–72. <https://doi.org/10.1016/j.biosystems.2009.06.001>

40. H. J. Guo, L. S. Chen, X. Y. Song, Qualitative analysis of impulsive state feedback control to an algae-fish system with bistable property, *Appl. Math. Comput.*, **271** (2015), 905–922. <https://doi.org/10.1016/j.amc.2015.09.046>
41. Y. Tian, H. M. Li, The study of a predator-prey model with fear effect based on state-dependent harvesting strategy, *Complexity*, **2022** (2022), 9496599. <http://dx.doi.org/10.1155/2022/9496599>
42. Y. Tian, Y. Gao, K. B. Sun, Global dynamics analysis of instantaneous harvest fishery model guided by weighted escapement strategy, *Chaos Soliton Fractals*, **164** (2022), 112597. <https://doi.org/10.1016/j.chaos.2022.112597>
43. Y. Tian, Y. Gao, K. B. Sun, A fishery predator-prey model with anti-predator behavior and complex dynamics induced by weighted fishing strategies, *Math. Biosci. Eng.*, **20** (2022), 1558–1579. <http://dx.doi.org/10.3934/mbe.2023071>
44. Y. Tian, Y. Gao, K. B. Sun, Qualitative analysis of exponential power rate fishery model and complex dynamics guided by a discontinuous weighted fishing strategy, *Commun. Nonlinear Sci.*, **118** (2023), 107011. <https://doi.org/10.1016/j.cnsns.2022.107011>
45. H. Li, Y. Tian, Dynamic behavior analysis of a feedback control predator-prey model with exponential fear effect and Hassell-Varley functional response, *J. Franklin I.*, **360** (2023), 3479–3498. <https://doi.org/10.1016/j.jfranklin.2022.11.030>
46. X. N. Liu, L. S. Chen, Complex dynamics of Holling type II Lotka-Volterra predator-prey system with impulsive perturbations on the predator, *Chaos Soliton Fractals*, **16** (2004), 311–320. [https://doi.org/10.1016/S0960-0779\(02\)00408-3](https://doi.org/10.1016/S0960-0779(02)00408-3)
47. B. Liu, Y. J. Zhang, L. S. Chen, Dynamic complexities of a Holling I predator-prey model concerning periodic biological and chemical control, *Chaos Soliton Fractals*, **22** (2004), 123–134. <https://doi.org/10.1016/j.chaos.2003.12.060>
48. X. Y. Song, Y. F. Li, Dynamic complexities of a Holling II two-prey one-predator system with impulsive effect, *Chaos Soliton Fractals*, **33** (2007), 463–478. <https://doi.org/10.1016/j.chaos.2006.01.019>
49. G. R. Jiang, Q. S. Lu, L. N. Qian, Complex dynamics of a Holling type II prey-predator system with state feedback control, *Chaos Soliton Fractals*, **31** (2007), 448–461. <https://doi.org/10.1016/j.chaos.2005.09.077>
50. Y. Tian, K. B. Sun, L. S. Chen, Geometric approach to the stability analysis of the periodic solution in a semi-continuous dynamic system, *Int. J. Biomath.*, **7** (2014), 1450018. <https://doi.org/10.1142/S1793524514500181>
51. S. Y. Tang, W. H. Pang, R. A. Cheke, J. Wu, Global dynamics of a state-dependent feedback control system, *Adv. Differ. Equation*, **2015** (2015), 322. <https://doi.org/10.1186/s13662-015-0661-x>
52. S. Y. Tang, B. Tang, A. L. Wang, Y. N. Xiao, Holling II predator-prey impulsive semi-dynamic model with complex Poincaré map, *Nonlinear Dyn.*, **81** (2015), 1575–1596. <https://doi.org/10.1007/s11071-015-2092-3>

53. T. Q. Zhang, W. B. Ma, X. Z. Meng, T. H. Zhang, Periodic solution of a prey-predator model with nonlinear state feedback control, *Appl. Math. Comput.*, **266** (2015), 95–107. <https://doi.org/10.1016/j.amc.2015.05.016>
54. J. Yang, S. Y. Tang, Holling type II predator-prey model with nonlinear pulse as state-dependent feedback control, *J. Comput. Appl. Math.*, **291** (2016), 225–241. <https://doi.org/10.1016/j.cam.2015.01.017>
55. S. Tang, C. Li, B. Tang, X. Wang, Global dynamics of a nonlinear state-dependent feedback control ecological model with a multiple-hump discrete map, *Commun. Nonlinear Sci.*, **79** (2019), 104900. <https://doi.org/10.1016/j.cnsns.2019.104900>
56. Q. Zhang, B. Tang, T. Cheng, S. Tang, Bifurcation analysis of a generalized impulsive Kolmogorov model with applications to pest and disease control, *SIAM J. Appl. Math.*, **80** (2020), 1796–1819. <https://doi.org/10.1137/19M1279320>
57. Q. Zhang, S. Tang, X. Zou, Rich dynamics of a predator-prey system with state-dependent impulsive controls switching between two means, *J. Differ. Equations*, **364** (2023), 336–377. <https://doi.org/10.1016/j.jde.2023.03.030>
58. W. Li, J. Ji, L. Huang, Global dynamic behavior of a predator-prey model under ratio-dependent state impulsive control, *Appl. Math. Model.*, **77** (2020), 1842–1859. <https://doi.org/10.1016/j.apm.2019.09.033>
59. Q. Zhang, S. Tang, Bifurcation analysis of an ecological model with nonlinear state-dependent feedback control by Poincaré map defined in phase set, *Commun. Nonlinear Sci.*, **108** (2022), 106212. <https://doi.org/10.1016/j.cnsns.2021.106212>
60. L. S. Pontryagin, *The Mathematical Theory of Optimal Processes*, CRC press, London, 1987. <https://doi.org/10.1201/9780203749319>



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