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Research article

Single-species population models with stage structure and partial tolerance in polluted environments

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Abstract: We propose stage-structured single-species population models with psychological effects and partial tolerance in polluted environments in this paper. First, the conditions of extinction and the time for extinction are investigated respectively. Especially, the time for extinction takes longer as the value of the psychological effects increases. Then the weak persistence in the mean around the pollution-free equilibrium and the stochastic permanence have been derived under some moderate conditions. Further, the existence of a periodic solution for the periodic single-species population has been determined. The corresponding numerical simulations verify the efficiency of the main theoretical results.

Keywords: single-species population model; stage-structured; psychological effects; periodic solution; survival level

1. Model formulation

Pollution in ecological environments has always been a widespread concern of the public. With the rapid development of the modern industrial society, the environmental pollution caused by human activities has become an issue and it has series impacts on the survival of single-species populations in the environment. Under the assumption that the growth rate of a local population was linearly dependent on the absorbed toxicant concentration, Hallam et al. [1] studied the ecological impact of toxicant concentration on single-species populations for the first time. Later, single-species population models gradually attracted the attentions of researchers around the world. For example, Ma et al. [2] considered the survival threshold of small populations when the capacity of the environment was limited, and they observed the changes in the environment by uptake and egestion of organisms. Wang et al. [3] later generalized the results in [2] into periodic toxicant inputs and discussed the permanence of single-species populations. After two decades, motivated by Hallam's model in [1], Liu et al. [4]

investigated a single-species model in a polluted closed environment with pulsed toxicant inputs; they found that impulsive toxicant inputs led to periodic behaviors and oscillations for their solution. Then He and Wang [5] followed Hallam's model and assumed that newborns carried some amount of toxicant within each living organism that is diffused into the environment after death, they further analyzed the sufficient conditions for uniform persistence, weak persistence in the mean and the extinction of a single-species population. By simplifying Hallam's model, Liu and Wang [6] investigated the thresholds for local extinction and weak persistence in the mean for single species by using a deterministic model and stochastic model respectively. In [7], they continued to study the survival threshold for a stochastic generalized logistic model, stochastic Leslie model and stochastic Gallopin model; they obtained that the intensity of white noise made sense. Further, survival thresholds were extensively discussed in [8–10] for a single-species model with Markov switching and pulsed toxicant inputs in a polluted environment with two types of noises, for an n-species stochastic Lotka-Volterra cooperative model and for a stochastic cooperative species model in a polluted environment, respectively. Moreover, single-species population models with partial pollution tolerance and psychological effects in a polluted environment were respectively paid more attention in [11–15]. Especially, Wei and Chen [12] proposed a single-species population model with psychological effects in a polluted environment:

$$\begin{aligned} \mathrm{d}x(t) &= \left[x(b-d) - cx^2 - \alpha x C_o - \frac{\lambda x C_e}{1+\beta C_e^2}\right]\mathrm{d}t - \frac{\sigma x C_e}{1+\beta C_e^2}\mathrm{d}B(t), \\ \mathrm{d}C_o(t) &= [kC_e - (g+m+b)C_o]\mathrm{d}t, \\ \mathrm{d}C_e(t) &= [u_e - hC_e]\mathrm{d}t, \end{aligned} \tag{1.1}$$

where $x(t)$ is the density of a single-species population in a polluted environment at time t; $C_o(t)$ and $C_e(t)$ are the concentration of toxicants in the organism, and the concentration of toxicants in the environment at time t respectively. They derived the sufficient conditions for local extinction, weak persistence in the mean and stochastic permanence; they also found that psychological effects had an impact on the density of populations.

In this paper we consider stage-structured single-species population models because there exist differences between juveniles and adults at the distinct stages in their lifetimes . Usually, in a population, adults produce juveniles, hunt for juveniles and protect juveniles against attacks from their predators; juveniles cannot produce newborns and have less experience for hunting untill they turn into adults. Motivated by the previous contributions in [16–18] and the assumptions proposed in [12, 16], we establish a stage-structured population model with pollution and psychological effects as follows:

$$\begin{aligned} \mathrm{d}J(t) &= [aA - e_J J - r_J C_o J - \alpha J - c_J J^2]\mathrm{d}t, \\ \mathrm{d}A(t) &= [\alpha J - e_A A - r_A C_o A - c_A A^2 - \lambda g(A, C_e)]\mathrm{d}t - \sigma g(A, C_e)\mathrm{d}B(t), \\ \mathrm{d}C_o(t) &= [kC_e - gC_o - mC_o - bC_o]\mathrm{d}t, \\ \mathrm{d}C_e(t) &= [u_e - hC_e]\mathrm{d}t, \end{aligned} \tag{1.2}$$

where $J(t)$ and $A(t)$ respectively indicate the densities of juveniles and adults in a population at time t; $g(A, C_e)$ describes the assumptions that adults are directly affected by a heavily polluted environment in a nonlinear form, and by a lightly polluted environment in a linear form as follows:

$$g(A, C_e) = \begin{cases} \dfrac{AC_e}{1+\beta C_e^2}, & C_e > c \quad \text{(heavy pollution)}, \\ AC_e, & C_e < c \quad \text{(light pollution)}. \end{cases} \tag{1.3}$$

In other words, the nonlinear form in Eq (1.3) means that adults produce the psychological effects with a rate β to avoid being harmed by pollution because adults with good vertebrate organs and highly differentiated nervous systems can transmit the information on their surroundings to their nervous systems well; the single-species population is in a heavily polluted environment when $C_e > c$ is valid, and the population is in a lightly polluted environment when $C_e < c$ holds; here, c is a threshold value for a polluted environment. We further assume that juveniles are indirectly affected by the environmental pollution, and that juveniles are produced by adults at a constant rate a; e_J and e_A are the natural mortality rates of juveniles and adults respectively; r_J and r_A refer to the loss rates for the juveniles and adults respectively; α indicates the conversion rate from juveniles to adults; c_J and c_A are the intra-specific competition rates for juveniles and adults; λ denotes the contact rate between adults and toxicants in the environment. Because the contact rate is always affected by weather conditions, temperature and other types of noise, the constant contact rate λ is replaced by a random variable $\tilde{\lambda} = \lambda + \sigma\xi(t)$ with white noise $\xi(t)$ satisfying $\xi(t)\mathrm{d}t = \mathrm{d}B(t)$, where $B(t)$ is a scalar standard Brownian motion process. β represents the inhibition rate or psychological effects of adults when they are surrounded by the polluted environment; it also describes the sensitivity of adults to the polluted environment; k is the uptake rate from the polluted environment; g, m and b express the loss rates due to egestion, metabolic process and reproduction, respectively; $u_e(t)$ is regarded as the external toxicant input into the environment; h represents the natural purification rate of the environment itself.

2. Fitness

Theorem 2.1. *Model* (1.2) *with heavy pollution has a unique solution* $(J(t), A(t), C_o(t), C_e(t))$ *for any* $(J(0), A(0), C_o(0), C_e(0))$ *and* $t \geqslant 0$*; the solution will remain in* $\mathbb{R}^4_+$ *with a probability of* 1.

Proof. Since the coefficients of System (1.2) satisfy the local Lipschitz condition [19], there is a unique local solution $(J(t), A(t), C_o(t), C_e(t)) \in \mathbb{R}^4_+, t \in [0, \tau_e)$ (where τ_e is the explosion time) for any initial value $(J(0), A(0), C_o(0), C_e(0)) \in \mathbb{R}^4_+$. In order to prove that this solution is global, we only need to prove that $\tau_e = \infty$. We assume that l_0 is a sufficiently large integer such that $(J(0), A(0), C_o(0), C_e(0))$ is in the interval $[\frac{1}{l_0}, l_0]$; for each integer $l \geqslant l_0$, we define a stopping time [19]:

$$\tau_l = \inf\left\{t \in [0, \tau_e) : \min\{J(t), A(t), C_o(t), C_e(t)\} \leqslant \frac{1}{l} \text{ or } \max\{J(t), A(t), C_o(t), C_e(t)\} \geqslant l\right\}. \tag{2.1}$$

Throughout this paper, we set $\inf \emptyset = \infty$. It is apparent that τ_l increases with respect to l. We set $\tau_\infty = \lim_{l\to\infty} \tau_l$; therefore, $\tau_\infty \leqslant \tau_e$ holds almost surely. We claim that $\tau_\infty = \infty$ is valid almost surely. The proof goes by contradiction from now on. If the statement is false, then there exists a pair of constants $T > 0$ and $\varepsilon \in (0, 1)$ such that $\mathbb{P}\{\tau_\infty \leqslant T\} > \varepsilon$. Hence there exists an integer $l_1 \geqslant l_0$ such that

$$\mathbb{P}\{\tau_l \leqslant T\} \geqslant \varepsilon, \text{ for all } l \geqslant l_1. \tag{2.2}$$

We define a Lyapunov-function $V_1 : \mathbb{R}^4_+ \to \mathbb{R}$ as follows:

$$V_1 = J - 1 - \ln J + A - 1 - \ln A + C_o - 1 - \ln C_o + C_e - 1 - \ln C_e. \tag{2.3}$$

The generalized Itô's formula gives

$$\mathrm{d}V_1 = \mathcal{L}V_1 \mathrm{d}t - (A - 1)\frac{\sigma C_e}{1 + \beta C_e^2}\mathrm{d}B(t); \tag{2.4}$$

together with $0 < C_o < 1$ and $0 < C_e < 1$, we have

$$
\begin{aligned}
\mathcal{L}V_1 < & -(e_J + c_J)J - c_J J^2 - (e_A + c_A - a)A - c_A A^2 + (r_J + r_A)C_o \\
& + (\lambda + k)C_e + e_J + e_A + \alpha + g + m + b + u_e + h + \frac{\sigma^2 C_e^2}{2(1 + \beta C_e^2)^2} \\
\leqslant & \, C_1 + r_J + r_A + e_J + e_A + \alpha + u_e + h + \lambda + g + m + b + k + \frac{\sigma^2}{2} := M,
\end{aligned}
\tag{2.5}
$$

where C_1 is the maximum for the quadratic forms of A and J with negative signs for their highest-order terms; we therefore have

$$
\mathrm{d}V_1 \leqslant M\mathrm{d}t - (A - 1)\frac{\sigma C_e}{1 + \beta C_e^2}\mathrm{d}B(t). \tag{2.6}
$$

Integrating both sides of Inequality (2.6) from 0 to $\tau_l \wedge T$ implies that

$$
\int_0^{\tau_l \wedge T} \mathrm{d}V_1 \leqslant \int_0^{\tau_l \wedge T} M\mathrm{d}t - \int_0^{\tau_l \wedge T} (A(t) - 1)\frac{\sigma C_e(t)}{1 + \beta C_e^2(t)}\mathrm{d}B(t), \tag{2.7}
$$

and then taking expectations yields

$$
\mathbb{E}V_1(\tau_l \wedge T) \leqslant V_1(0) + MT. \tag{2.8}
$$

We set $\Omega_l = \{\tau_l \leqslant T\}$ for $l \geqslant l_1$; then, Expression (2.2) turns into $\mathbb{P}\{\Omega_l\} \geqslant \varepsilon$. Note that for every $\omega \in \Omega_l$, each component $J(\tau_l \wedge T), A(\tau_l \wedge T), C_o(\tau_l \wedge T)$ or $C_e(\tau_l \wedge T)$ equals either l or $\frac{1}{l}$; hence, Inequality (2.8) could be rewritten as

$$
V_1(0) + MT \geqslant \mathbb{E}V_1(\tau_l \wedge T) \geqslant \varepsilon \min\left\{l - 1 - \ln l, \frac{1}{l} - 1 + \ln l\right\}. \tag{2.9}
$$

This, as $l \to \infty$, leads to a contradiction

$$
\infty > V_1(0) + MT \geqslant \infty. \tag{2.10}
$$

Therefore the assertion $\tau_\infty = \infty$ is valid almost surely. The proof is complete.

Corollary 2.2. *Model* (1.2) *with light pollution has a unique solution* $(J(t), A(t), C_o(t), C_e(t))$ *for any* $(J(0), A(0), C_o(0), C_e(0))$ *and* $t \geqslant 0$*; the solution will remain in* $\mathbb{R}_+^4$ *with a probability of* 1.

3. Survival analysis

3.1. Extinction

Lemma 3.1. [12] *The upper boundaries of toxicant concentrations in organisms and in a polluted environment are given as*

$$
C_e^* \leqslant \frac{u_e^*}{h}, \quad C_o^* \leqslant \frac{k u_e^*}{h(g + m + b)}. \tag{3.1}
$$

Lemma 3.2. *For any given constant T, the lower boundaries of toxicant concentrations in organisms and in a polluted environment are obtained as*

$$
\begin{aligned}
(C_e)_* &\geqslant \frac{(u_e)_*}{h} + e^{-hT}\Big(1 - \frac{(u_e)_*}{h}\Big), \\
(C_o)_* &\geqslant \frac{(\frac{(u_e)_*}{h} + e^{-hT})_*}{g + m + b} + e^{-(g+m+b)T}\Big(1 - \frac{(C_e)_*}{g + m + b}\Big).
\end{aligned}
\tag{3.2}
$$

Proof. From the fourth equation of Model (1.2), we have

$$\dot{C}_e(t) \geqslant (u_e)_* - hC_e.$$

By a method of constant variation, we get

$$C_e(t) \geqslant \frac{(u_e)_*}{h} + e^{-hT}\Big(1 - \frac{(u_e)_*}{h}\Big),$$

therefore the first assertion is valid. Similarly, by the third equation of Model (1.2), we obtain

$$\dot{C}_o(t) \geqslant k(C_e)_* - (g + m + b)C_o.$$

The method of constant variation gives that

$$C_o(t) \geqslant \frac{(C_e)_*}{g + m + b} + e^{-(g+m+b)T}\Big(1 - \frac{(C_e)_*}{g + m + b}\Big),$$

which further implies the second assertion.

Theorem 3.1. *If the coefficients of Model* (1.2) *with heavy pollution satisfy*

$$a < e_A + c_A + r_A(C_o)_* + \frac{\lambda(C_e)_*}{1 + \beta(C_e^*)^2}, \tag{3.3}$$

then the densities of juveniles and adults in a local population will go to extinction.

Proof. Let us define $V_2 = \ln(J + A)$ according to Itô's formula

$$\mathrm{d}V_2 = \mathcal{L}V_2\mathrm{d}t - \frac{\sigma AC_e}{(J + A)(1 + \beta C_e^2)}\mathrm{d}B(t). \tag{3.4}$$

We claim that

$$\begin{aligned} \mathcal{L}V_2 = & \frac{1}{J + A}\Big(aA - e_J J - r_J C_o J - c_J J^2 - e_A A - r_A C_o A - c_A A^2 - \frac{\lambda AC_e}{1 + \beta C_e^2}\Big) \\ & - \frac{A^2}{2(J + A)^2} \cdot \frac{\sigma^2 C_e^2}{(1 + \beta C_e^2)^2} \end{aligned} \tag{3.5}$$

given $J > 1$ and $A > 1$; it is thus estimated by

$$\mathcal{L}V_2 < \frac{1}{J + A}\Big[(-e_J - c_J - r_J(C_o)_*)J + \Big(a - e_A - c_A - r_A(C_o)_* - \frac{\lambda(C_e)_*}{1 + \beta(C_e^*)^2}\Big)A\Big] \leqslant q_1, \tag{3.6}$$

and together with Lemma 3.1

$$q_1 = \max\Big\{ - e_J - c_J - r_J(C_o)_*, a - e_A - c_A - r_A(C_o)_* - \frac{\lambda(C_e)_*}{1 + \beta(C_e^*)^2}\Big\}.$$

Integrating both sides of Eq (3.4) and dividing by t gives

$$\frac{1}{t}\ln(J(t) + A(t)) < \frac{1}{t}\ln(J(0) + A(0)) + q_1 - \frac{1}{t}\int_0^t \frac{A(s)\sigma C_e(s)}{(J(s) + A(s))(1 + \beta C_e^2(s))}\mathrm{d}B(s). \tag{3.7}$$

According to the strong law of large numbers of the local martingale [19]

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t \frac{\sigma A(s)C_e(s)}{(J(s)+A(s))(1+\beta C_e^2(s))}\mathrm{d}B(s) = 0 \ \text{ almost surely} \tag{3.8}$$

Taking the upper limit on both sides of Inequality (3.7), we get

$$\limsup_{t\to\infty}\frac{1}{t}\ln(J(t)+A(t)) < q_1 < 0. \tag{3.9}$$

Hence

$$\lim_{t\to\infty} J(t) = 0 \quad \textit{and} \quad \lim_{t\to\infty} A(t) = 0. \tag{3.10}$$

Corollary 3.1. *If the coefficients of Model* (1.2) *with light pollution satisfy*

$$a < e_A + c_A + r_A(C_o)_* + \lambda(C_e)_*, \tag{3.11}$$

then the densities of juveniles and adults in a local population will go to extinction.

3.2. Stochastic permanence

Considering the following n-dimensional stochastic differential equation:

$$\mathrm{d}x(t) = f(x(t))\mathrm{d}t + g(x(t))\mathrm{d}B(t), \quad x(t) \in \mathbb{R}_+^n, \tag{3.12}$$

we introduce the following useful definitions and lemmas.

Definition 3.1. [20] The solution $x(t)$ of Eq (3.12) is called stochastically ultimately bounded if, for any $\varepsilon \in (0,1)$, there is a positive constant $\chi(=\chi(\varepsilon))$ such that for any initial value $x(0) \in \mathbb{R}_+^n$, the solution has the property that

$$\limsup_{t\to\infty}\mathbb{P}\{|x(t)| > \chi\} < \varepsilon. \tag{3.13}$$

Definition 3.2. [21] The solution $x(t)$ of Eq (3.12) is said to be stochastically permanent if, for any $\varepsilon > 0$, there are constants $\delta_1 > 0$ and $\delta_2 > 0$ such that

$$\liminf_{t\to\infty}\mathbb{P}\{|x(t)| \geqslant \delta_1\} \geqslant 1-\varepsilon, \qquad \limsup_{t\to\infty}\mathbb{P}\{|x(t)| \leqslant \delta_2\} \geqslant 1-\varepsilon. \tag{3.14}$$

Lemma 3.3. *The densities of juveniles and adults in Model* (1.2) *with heavy pollution are stochastically ultimately bounded.*

Proof. We define $V_3 = e^t(J+A)$; applying Itô's formula gives

$$\mathcal{L}V_3 = e^t(J + A + \mathcal{L}J + \mathcal{L}A) = e^t F(J,A) < C_2 e^t, \tag{3.15}$$

where $F(J,A)$ yields a positive boundary C_2 as follows:

$$F(J,A) = (1 - e_J - r_J C_o)J - c_J J^2 + (1 + a - e_A - r_A C_o)A - c_A A^2 - \frac{\lambda A C_e}{1+\beta C_e^2} < C_2.$$

We further derive

$$J(t) + A(t) < e^{-t}[J(0)+A(0)] + C_2(1-e^{-t}) - e^{-t}\int_0^t \frac{\sigma A(s)C_e(s)}{1+\beta C_e^2(s)}\mathrm{d}B(s). \tag{3.16}$$

Let $|X| = \sqrt{J^2 + A^2}$; given that

$$\mathbb{E}\, e^{-t} \int_0^t \frac{\sigma A(s) C_e(s)}{1 + \beta C_e^2(s)} \mathrm{d}B(s) = 0, \qquad \sqrt{J^2 + A^2} \leqslant J + A,$$

and taking expectations on both sides of Inequlity (3.16), we obtain that

$$\limsup_{t\to\infty} \mathbb{E}|X(t)| \leqslant C_2. \tag{3.17}$$

The Chebyshev's inequality and Definition 3.1 gives

$$\limsup_{t\to\infty} \mathbb{P}\{|X(t)| > \chi\} < \varepsilon. \tag{3.18}$$

Lemma 3.4. *Model* (1.2) *with heavy pollution has the following property:*

$$\limsup_{t\to\infty} \mathbb{E} \frac{1}{|X(t)|^{2+\theta}} \leqslant H, \tag{3.19}$$

provided that θ is a positive number and satisfies the following condition:

$$(a \wedge \alpha) - (e_J + r_J) \vee (e_A + r_A + \lambda) - 0.5(3 + \theta)\sigma^2 > 0. \tag{3.20}$$

Proof. Motivated by the approaches in Theorem 4.5 of [6] and Lemma 3.5 of [20], we define $V_4 = (J + A)^{-1}$; the generalized Itô's formula gives that

$$\mathrm{d}V_4 = \mathcal{L}V_4 \mathrm{d}t + V_4^2 \frac{\sigma A C_e}{1 + \beta C_e^2} \mathrm{d}B(t), \tag{3.21}$$

where

$$\begin{aligned}
\mathcal{L}V_4 &= -V_4^2 \Big[aA - (e_J + r_J C_o) J + \alpha J - \alpha J - \Big(e_A + r_A C_o + \frac{\lambda C_e}{1 + \beta C_e^2} \Big) A - c_J J^2 - c_A A^2 \Big] \\
&\quad + V_4^3 \frac{\sigma^2 C_e^2 A^2}{(1 + \beta C_e^2)^2} \\
&\leqslant -V_4^2 [(a \wedge \alpha) - (e_J + r_J) \vee (e_A + r_A + \lambda)](J + A) \\
&\quad + V_4^2 (\alpha J + c_J J^2 + c_A A^2) + V_4^3 \sigma^2 A^2 \\
&\leqslant -[(a \wedge \alpha) - (e_J + r_J) \vee (e_A + r_A + \lambda) - \sigma^2] V_4 + (\alpha + c_J) \vee c_A.
\end{aligned} \tag{3.22}$$

We define $V_5 = V_4^{2+\theta}$; the generalized Itô's formula again yields that

$$\mathrm{d}V_5 = \mathcal{L}V_5 \mathrm{d}t + (2 + \theta) V_4^{3+\theta} \frac{\sigma A C_e}{1 + \beta C_e^2} \mathrm{d}B(t), \tag{3.23}$$

where

$$\begin{aligned}
\mathcal{L}V_5 &= (2 + \theta) V_4^{1+\theta} \mathcal{L}V_4 + 0.5(2 + \theta)(1 + \theta) V_4^{4+\theta} \frac{\sigma^2 A^2 C_e^2}{(1 + \beta C_e^2)^2} \\
&\leqslant (2 + \theta) V_4^{1+\theta} \Big\{ -[(a \wedge \alpha) - (e_J + r_J) \vee (e_A + r_A + \lambda) - \sigma^2] V_4 + (\alpha + c_J) \vee c_A \Big\} \\
&\quad + 0.5(2 + \theta)(1 + \theta) \sigma^2 V_4^{2+\theta} \\
&\leqslant C_3 V_4^{1+\theta} - C_4 V_4^{2+\theta},
\end{aligned} \tag{3.24}$$

with

$$\begin{aligned} C_3 &= (2+\theta)[(\alpha + c_J) \vee c_A], \\ C_4 &= (2+\theta)[(a \wedge \alpha) - (e_J + r_J) \vee (e_A + r_A + \lambda) - 0.5(3+\theta)\sigma^2]. \end{aligned} \tag{3.25}$$

We differentiate Eq (3.23) with factor e^{Kt}, so

$$\mathrm{d}(e^{Kt}V_5) = \mathcal{L}(e^{Kt}V_5)\mathrm{d}t - (Ke^{Kt}V_5 + e^{Kt}V_5\mathcal{L}V_5)\frac{\sigma AC_e}{1+\beta C_e^2}\mathrm{d}B(t). \tag{3.26}$$

We choose a moderate constant K such that $C_4 - K > 0$ and obtain

$$\mathcal{L}(e^{Kt}V_5) = e^{Kt}[KV_5 + \mathcal{L}V_5] \leqslant C_5 e^{Kt}. \tag{3.27}$$

Therefore, Eq (3.26) gives

$$\mathbb{E}[e^{Kt}V_4^{2+\theta}(t)] \leqslant V_4^{2+\theta}(0) + \frac{C_5}{K}e^{Kt},$$

which leads to

$$\limsup_{t\to\infty} \mathbb{E}V_4^{2+\theta}(t) \leqslant \frac{C_5}{K} := H.$$

The Cauchy inequality implies

$$(J+A)^{2+\theta} \leqslant \sqrt{2}^{2+\theta}|X|^{2+\theta};$$

thus

$$\limsup_{t\to\infty} \mathbb{E}\frac{1}{|X(t)|^{2+\theta}} \leqslant \limsup_{t\to\infty} \mathbb{E}[\sqrt{2}^{2+\theta}V_4^{2+\theta}(X(t))] \leqslant \sqrt{2}^{2+\theta}\cdot\frac{C_5}{K} := H. \tag{3.28}$$

Theorem 3.2. *If Condition* (3.20) *is valid, then the densities of juveniles and adults in Model* (1.2) *with heavy pollution are stochastically permanent.*

Proof. Given Inequality (3.28) in Lemma 3.4, we have

$$\liminf_{t\to\infty} \mathbb{E}\frac{1}{|X(t)|^{2+\theta}} \leqslant H. \tag{3.29}$$

For any given $\varepsilon > 0$, we set

$$\delta_1^{2+\theta} = \frac{\varepsilon}{H}. \tag{3.30}$$

The Chebyshev's inequality and Expression (3.30) gives that

$$\mathbb{P}\{|X(t)| < \delta_1\} = \mathbb{P}\Big\{\frac{1}{|X(t)|} > \frac{1}{\delta_1}\Big\} \leqslant \delta_1^{2+\theta}\mathbb{E}\frac{1}{|X(t)|^{2+\theta}} \leqslant \varepsilon; \tag{3.31}$$

therefore,

$$\mathbb{P}\{|X(t)| \geqslant \delta_1\} \geqslant 1 - \varepsilon. \tag{3.32}$$

According to Definition 3.2, the left side of the definition is valid. For the above $\varepsilon > 0$, we define

$$\delta_2 = \frac{C_2}{\varepsilon}.$$

The Chebyshev's inequality and Inequality (3.17) yields that

$$\mathbb{P}\{|X(t)| > \delta_2\} \leqslant \frac{\mathbb{E}|X(t)|}{\delta_2} \leqslant \varepsilon; \tag{3.33}$$

therefore,

$$\mathbb{P}\{|X(t)| \leqslant \delta_2\} \geqslant 1 - \varepsilon. \tag{3.34}$$

That is to say, by Definition 3.2, the densities of juveniles and adults in a local population are stochastically permanent.

Corollary 3.2. *If Condition* (3.20) *is valid, then the densities of juveniles and adults in Model* (1.2) *with light pollution are stochastically permanent.*

3.3. Weak persistence in the mean

Theorem 3.3. *If the coefficients of Model* (1.2) *with heavy pollution satisfy*

$$a < c_J + 2c_J\hat{J} + 0.5\alpha + e_J,\ 2\sigma^2 < 2c_A + 4c_A\hat{A} + 2e_A - a - 2\alpha,\ k < \min\{2(g+m+b), 2h-1\}, \tag{3.35}$$

then the densities of juveniles and adults are weakly persistent in the mean under the expectation around the pollution-free equilibrium $(\hat{J}, \hat{A}, 0, 0)$ *in the long run; that is,*

$$\limsup_{t\to\infty} \frac{1}{t}\mathbb{E}\int_0^t \Big(\zeta_1(J(s)-\hat{J})^2 + \zeta_2(A(s)-\hat{A})^2 + \zeta_3 C_o^2(s) + \zeta_4 C_e^2(s)\Big)\mathrm{d}s \leqslant \zeta_5, \tag{3.36}$$

where $\hat{J}$ *and* $\hat{A}$ *simultaneously satisfy* $\alpha\hat{J} = e_A\hat{A} + c_A\hat{A}^2$ *and* $a\hat{A} = e_J\hat{J} + \alpha\hat{J} + c_J\hat{J}^2$, *as well as*

$$\begin{aligned} &\zeta_1 = 2c_J + 4c_J\hat{J} + \alpha + 2e_J - 2a,\ \zeta_2 = 2c_A + 4c_A\hat{A} + 2e_A - a - 2\alpha - 2\sigma^2, \\ &\zeta_3 = 2(g+m+b) - k,\ \zeta_4 = 2h - k - 1,\ \zeta_5 = u_e^2 + (a + 2\sigma^2)\hat{A}^2 + a\hat{J}^2. \end{aligned} \tag{3.37}$$

Proof. New variables are governed as follows:

$$w_1 = J - \hat{J}, \quad w_2 = A - \hat{A}, \quad w_3 = C_o, \quad w_4 = C_e. \tag{3.38}$$

Hence, Model (1.2) is rewritten as given by the following equations:

$$\begin{aligned} \mathrm{d}w_1 &= [a(w_2+\hat{A}) - (e_J + r_J w_3 + \alpha)(w_1+\hat{J}) - c_J(w_1+\hat{J})^2]\mathrm{d}t, \\ \mathrm{d}w_2 &= \Big[\alpha(w_1+\hat{J}) - \Big(r_A w_3 + e_A + \frac{\lambda w_4}{1+\beta w_4^2}\Big)(w_2+\hat{A}) - c_A(w_2+\hat{A})^2\Big]\mathrm{d}t \\ &\quad - \frac{\sigma w_4}{1+\beta w_4^2}(w_2+\hat{A})\mathrm{d}B(t), \\ \mathrm{d}w_3 &= [kw_4 - (g+m+b)w_3]\mathrm{d}t, \\ \mathrm{d}w_4 &= [u_e - hw_4]\mathrm{d}t. \end{aligned} \tag{3.39}$$

We define a C^2-function

$$V_6 = w_1^2 + w_2^2 + w_3^2 + w_4^2.$$

Applying Itô's formula implies that

$$\mathrm{d}V_6 = \mathcal{L}V_6\mathrm{d}t - \frac{2\sigma w_2 w_4}{1+\beta w_4^2}(w_2+\hat{A})\mathrm{d}B(t), \tag{3.40}$$

where

$$
\begin{aligned}
\mathcal{L}V_6 &= 2w_1\Big[a(w_2+\hat{A}) - (e_J + r_J w_3 + \alpha)(w_1+\hat{J}) - c_J(w_1+\hat{J})^2\Big] \\
&\quad + 2w_2\Big[\alpha(w_1+\hat{J}) - \Big(r_A w_3 + e_A + \frac{\lambda w_4}{1+\beta w_4^2}\Big)(w_2+\hat{A}) - c_A(w_2+\hat{A})^2\Big] \\
&\quad + 2w_3[kw_4 - (g+m+b)w_3] + 2w_4[u_e - hw_4] + \frac{\sigma^2 w_4^2}{(1+\beta w_4^2)^2}(w_2+\hat{A})^2 \\
&= 2aw_1w_2 + 2a\hat{A}w_1 - 2(e_J+\alpha)w_1^2 - 2r_Jw_3w_1^2 - 2\hat{J}w_1(e_J + r_Jw_3 + \alpha) \\
&\quad - 2c_Jw_1(w_1^2 + 2\hat{J}w_1 + (\hat{J})^2) + 2\alpha w_1w_2 + 2\alpha\hat{J}w_2 - 2e_Aw_2^2 - 2w_2^2(r_Aw_3 + \frac{\lambda w_4}{1+\beta w_4^2}) \\
&\quad - 2w_2\hat{A}(r_Aw_3 + e_A + \frac{\lambda w_4}{1+\beta w_4^2}) - 2c_Aw_2(w_2^2 + 2\hat{A}w_2 + (\hat{A})^2) + 2kw_3w_4 \\
&\quad - 2(g+m+b)w_3^2 + 2u_ew_4 - 2hw_4^2 + \frac{\sigma^2w_2^2w_4^2}{(1+\beta w_4^2)^2} + \frac{2\sigma^2\hat{A}w_2w_4^2}{(1+\beta w_4^2)^2} + \frac{\sigma^2(\hat{A})^2w_4^2}{(1+\beta w_4^2)^2},
\end{aligned}
$$

Given $w_1 > 1$ and $w_2 > 1$, it is thus estimated by

$$
\begin{aligned}
\mathcal{L}V_6 &< aw_1^2 + aw_2^2 + aw_1^2 + a(\hat{A})^2 - 2(e_J+\alpha)w_1^2 - 2c_Jw_1^2 - 4c_J\hat{J}w_1^2 \\
&\quad + \alpha w_1^2 + \alpha w_2^2 + \alpha w_2^2 + \alpha(\hat{J})^2 - 2e_Aw_2^2 - 2c_Aw_2^2 - 4c_A\hat{A}w_2^2 \\
&\quad + kw_3^2 + kw_4^2 - 2(g+m+b)w_3^2 + u_e^2 + w_4^2 - 2hw_4^2 + 2\sigma^2w_2^2 + 2\sigma^2(\hat{A})^2 \\
&= (2a - 2e_J - \alpha - 2c_J - 4c_J\hat{J})w_1^2 + (a + 2\alpha - 2e_A - 2c_A - 4c_A\hat{A} + 2\sigma^2)w_2^2 \\
&\quad + (k - 2(g+m+b))w_3^2 + (k+1-2h)w_4^2 + u_e^2 + (a+2\sigma^2)\hat{A}^2 + a\hat{J}^2.
\end{aligned}
$$

Therefore, by Expression (3.37), one gets that

$$
\mathcal{L}V_6 < -\zeta_1w_1^2 - \zeta_2w_2^2 - \zeta_3w_3^2 - \zeta_4w_4^2 + \zeta_5. \tag{3.41}
$$

Integrating from 0 to t on both sides of Eq (3.40) and taking the expectation, we derive the required Expression (3.36) as time t tends to infinity. The proof is complete.

Corollary 3.3. *If coefficients of Model* (1.2) *with light pollution satisfy Condition* (3.35)*, then the densities of juveniles and adults are weakly persistent in the mean under the expectation around the pollution-free equilibrium* $(\hat{J}, \hat{A}, 0, 0)$ *in the long run.*

4. Existence of periodic solution

We assume that $a(t), r_J(t), c_J(t), r_A(t), c_A(t), \alpha(t), \lambda(t)$ and $\sigma(t)$ are both positive and continuous functions with a period T. We further investigate the periodic model given by Model (4.1) due to the existence of periodic phenomena in the real world.

$$
\begin{aligned}
&\mathrm{d}J(t) = [a(t)A - e_J(t)J - r_J(t)C_oJ - \alpha(t)J - c_J(t)J^2]\mathrm{d}t, \\
&\mathrm{d}A(t) = \Big[\alpha(t)J - e_A(t)A - r_A(t)C_oA - c_A(t)A^2 - \lambda(t)g(A, C_e)\Big]\mathrm{d}t - \sigma(t)g(A, C_e)\mathrm{d}B(t), \\
&\mathrm{d}C_o(t) = [k(t)C_e - g(t)C_o - m(t)C_o - b(t)C_o]\mathrm{d}t, \\
&\mathrm{d}C_e(t) = [u_e(t) - h(t)C_e]\mathrm{d}t.
\end{aligned} \tag{4.1}
$$

Lemma 4.1. [22, 23] *Consider the following periodic stochastic differential equation:*

$$dx(t) = f(t, x(t))dt + g(t, x(t))dB(t), \quad x \in \mathbb{R}^n, \tag{4.2}$$

where both $f(t, x(t))$ and $g(t, x(t))$ are T-periodic functions with respect to t. If System (4.2) *has a global positive solution, suppose that there exists a T-periodic Lyapunov function $V(t, x)$ with respect to t; the following conditions are satisfied outside of a certain compact set:*

$$(i) \quad \inf_{|x|>r} V(t, x) \to \infty, \quad r \to \infty, \tag{4.3}$$

$$(ii) \quad \mathcal{L}V(t, x) \leqslant -1. \tag{4.4}$$

Then, System (4.2) *has a T-periodic solution.*
Theorem 4.1. *If the coefficients of Model* (4.1) *satisfy*

$$m_1 - Q^* - M_1 > 2, \tag{4.5}$$

then Model (4.1) *has a positive periodic solution, where*

$$m_1 = \frac{1}{T}\int_0^T \Big(2\sqrt{a(t)\alpha(t)} + (e_J(t) \wedge e_A(t))\Lambda_1 - \frac{1}{2}\sigma^2(t)\Big)dt,$$

$$M_1 = \max\Big\{\frac{c_J^*}{4}, \frac{(a^* + c_A^*)^2}{4(c_A)_*^2}\Big\}, \quad Q^*(t) = e_J^*(t) + e_A^*(t) + (r_J^*(t) + r_A^*(t))C_o^* + \alpha^*(t) + \frac{\lambda^*(t)C_e^*}{1 + \beta C_e^2 *},$$

and Λ_1 represents the minimum of the densities of juveniles and adults in a local population.
Proof. The existence and uniqueness of the global positive solution of Model (4.1) is derived by a similar way as shown in Theorem 2.1, so we will omit the details. Next, we define the C^2-function $V_7(t, J, A)$ as follows:

$$V_7 = J - 1 - \ln J + A - 1 - \ln A + \omega(t), \tag{4.6}$$

where $\omega(t)$ is a function defined in $[0, \infty)$ with $\omega(0) = 0$ that satisfies the following equation:

$$\dot{\omega}(t) = 2\sqrt{a(t)\alpha(t)} + (e_J(t) \wedge e_A(t))\Lambda_1 - \frac{1}{2}\sigma^2(t) - m_1. \tag{4.7}$$

Integrating $\dot{\omega}(t)$ from t to $t + T$ yields that

$$\omega(t + T) - \omega(t) = \int_t^{t+T} \dot{\omega}(s)\mathrm{d}s = 0, \tag{4.8}$$

so $\omega(t)$ is a periodic function defined in $[0, \infty)$ with a period T. It is easy to check that

$$\liminf_{(J,A)\in\mathbb{R}_+^2\setminus D} V_7(t, J, A) \to \infty \tag{4.9}$$

in the set

$$D = \Big\{(J, A)\Big|(J, A) \in \Big[\varepsilon, \frac{1}{\varepsilon}\Big] \times \Big[\varepsilon, \frac{1}{\varepsilon}\Big]\Big\},$$

where ε is a sufficiently small positive number that satisfies

$$2c_J^*\varepsilon < -\frac{(a^*+c_A^*)^2}{4c_A^*} - Q^* + m_1, \tag{4.10}$$

$$2(a^*+c_A^*)\varepsilon < -\frac{c^*}{4} - Q^* + m_1, \tag{4.11}$$

$$-\frac{(c_J)_*}{2\varepsilon^2} + G_3 < -1, \tag{4.12}$$

$$-\frac{(c_A)_*}{2\varepsilon^2} + G_4 < -1. \tag{4.13}$$

The generalized Itô's formula yields that

$$\begin{aligned}
\mathcal{L}V_7 =\ & a(t)A - e_J(t)J - r_J(t)C_oJ - \alpha(t)J - c_J(t)J^2 - a(t)\frac{A}{J} + Q(t) \\
& + c_J(t)J + \alpha(t)J - e_A(t)A - r_A(t)C_oA - c_A(t)A^2 - \frac{\lambda(t)AC_e}{1+\beta(t)C_e^2} \\
& - \alpha(t)\frac{J}{A} + c_A(t)A + \frac{\sigma^2(t)C_e^2}{2(1+\beta(t)C_e^2)^2} + \dot{\omega}(t) \\
<\ & -c_J(t)J^2 + c_J(t)J - c_A(t)A^2 + (a(t)+c_A(t))A - 2\sqrt{a(t)\alpha(t)} \\
& - (e_J(t)\wedge e_A(t))\Lambda_1 + \frac{1}{2}\sigma^2(t) + \dot{\omega}(t) + Q^* \\
=\ & -c_J(t)J^2 + c_J(t)J - c_A(t)A^2 + (a(t)+c_A(t))A - m_1 + Q^* \\
<\ & C_7 - m_1 + Q^*,
\end{aligned} \tag{4.14}$$

with

$$Q(t) = e_J(t) + e_A(t) + (r_J(t)+r_A(t))C_o + \alpha(t) + \frac{\lambda(t)C_e}{1+\beta C_e^2},$$

where $-(c_J)_*J^2 + c_J^*J - (c_A)_*A^2 + (a^*+c_A^*)A$ yields a maximum C_7. To show $\mathcal{L}V_7(t,J,A) < -1$ in $\mathbb{R}_+^2 \setminus D$, we separate $\mathbb{R}_+^2 \setminus D$ into four parts:

$$\begin{aligned}
D_1 &= \{(J,A)\in\mathbb{R}_+^2 | 0 < J < \varepsilon\}, \quad D_2 = \{(J,A)\in\mathbb{R}_+^2 | 0 < A < \varepsilon\}, \\
D_3 &= \Big\{(J,A)\in\mathbb{R}_+^2 \Big| J > \frac{1}{\varepsilon}\Big\}, \qquad D_4 = \Big\{(J,A)\in\mathbb{R}_+^2 \Big| A > \frac{1}{\varepsilon}\Big\}.
\end{aligned}$$

Case 1. When $(J,A)\in D_1$, we get

$$\begin{aligned}
\mathcal{L}V_7 &< c_J(t)J - c_J(t)J^2 - c_A(t)A^2 + (a(t)+c_A(t))A - m_1 + Q^* \\
&< c_J^*J - (c_J)_*J^2 - (c_A)_*\Big(A - \frac{a^*+c_A^*}{2(c_A)_*}\Big)^2 + \frac{(a^*+c_A^*)^2}{(4c_A)_*} - m_1 + Q^* \\
&< c_J^*\varepsilon + \frac{(a^*+c_A^*)^2}{4(c_A)_*} - m_1 + Q^*.
\end{aligned} \tag{4.15}$$

Inequality (4.10) gives

$$\mathcal{L}V_7 < \frac{1}{2}\Big(\frac{(a^*+c_A^*)^2}{4(c_A)_*} - m_1 + Q^*\Big) := G_1 < -1.$$

Case 2. When $(J, A) \in D_2$, we obtain

$$\begin{aligned}\mathcal{L}V_7 &< (a(t) + c_A(t))A - c_J(t)\Big(J - \frac{1}{2}\Big)^2 + \frac{c_J(t)}{4} - m_1 + Q^* \\ &< (a^* + c_A^*)\varepsilon + \frac{c_J^*}{4} - m_1 + Q^*.\end{aligned} \tag{4.16}$$

Inequality (4.11) yields

$$\mathcal{L}V_7 < \frac{1}{2}\Big(\frac{c_J^*}{4} - m_1 + Q^*\Big) := G_2 < -1.$$

Case 3. When $(J, A) \in D_3$, we have

$$\begin{aligned}\mathcal{L}V_7 &< -\frac{1}{2}c_J(t)J^2 - \frac{1}{2}c_J(t)J^2 + c_J(t)J - c_A(t)A^2 + (a(t) + c_A(t))A - m_1 + Q^* \\ &< -\frac{1}{2}(c_J)_*J^2 - \frac{(c_J)_*}{2}(J-1)^2 + \frac{1}{2}c_J^* - (c_A)_*A^2 + (a^* + c_A^*)A - m_1 + Q^*.\end{aligned} \tag{4.17}$$

Inequality (4.12) shows that

$$\mathcal{L}V_7 < -\frac{(c_J)_*}{2\varepsilon^2} + G_3 < -1,$$

with

$$G_3 = C_7 + \frac{1}{2}c_J^* - m_1 + Q^*.$$

Case 4. When $(J, A) \in D_4$, we derive

$$\begin{aligned}\mathcal{L}V_7 &< -\frac{1}{2}c_A(t)A^2 - \frac{1}{2}c_A(t)A^2 + (a(t) + c_A(t))A - c_J(t)J^2 + c_J(t)J - m_1 + Q^* \\ &< -\frac{1}{2}(c_A)_*A^2 - \Big(\sqrt{\frac{(c_A)_*}{2}}A - \frac{a^* + c_A^*}{\sqrt{2(c_A)_*}}\Big)^2 + \frac{(a^* + c_A^*)^2}{2(c_A)_*} - (c_J)_*J^2 + c_J^*J - m_1 + Q^*.\end{aligned} \tag{4.18}$$

Inequality (4.13) implies that

$$\mathcal{L}V_7 < -\frac{(c_A)_*}{2\varepsilon^2} + G_4 < -1,$$

where

$$G_4 = C_7 + \frac{(a^* + c_A^*)^2}{2(c_A)_*} - m_1 + Q^*.$$

So, $\mathcal{L}V_7(t, J, A) < -1$ is checked when $(J, A) \in \mathbb{R}_+^2 \setminus D$.

5. Examples and simulations

In this section, we present the numerical simulations to verify the aforementioned main results. Due to the linear dependence of the diffusion coefficients in [24, 25], we govern the data from [12] and the Milstein's method in [26]; we let the threshold value for the polluted environment in Expression (1.3) be $c = 0.30$ and take $k = 0.20, g = 0.08, m = 0.04, h = 0.40, b = 0.20, \theta = 0.10$ and the initial values $(J(0), A(0), C_o(0), C_e(0)) = (0.40, 0.20, 0.05, 0.10)$. Then, the the discretization equations corresponding to Model (1.2) are written as

$$\begin{aligned} J_n &= J_{n-1} + [aA_{n-1} - J_{n-1}(e_J + r_J C_{o,n-1} + \alpha + c_J J_{n-1})]\Delta t, \\ A_n &= A_{n-1} + \alpha J_{n-1} - [A_{n-1}(e_A + r_A C_{o,n-1} + c_A A_{n-1}) + \lambda g_{n-1}]\Delta t \\ &\quad -\sigma g_{n-1}\xi\sqrt{\Delta t} - 0.5\sigma^2 g_{n-1}^2(\xi^2\Delta t - \Delta t), \\ C_{o,n} &= C_{o,n-1} + [kC_{e,n-1} - (g + m + b)C_{o,n-1}]\Delta t, \\ C_{e,n} &= C_{e,n-1} + [u_e - hC_{e,n-1}]\Delta t, \end{aligned} \tag{5.1}$$

where

$$g_{n-1}(A_{n-1}, C_{e,n-1}) = \begin{cases} \dfrac{A_{n-1}C_{e,n-1}}{1 + \beta C_{e,n-1}^2}, & C_{e,n-1} > c, \\ A_{n-1}C_{e,n-1}, & C_{e,n-1} < c, \end{cases} \tag{5.2}$$

and ξ is a Gaussian random variable with a standard normal distribution $\mathcal{N}(0, 1)$.

Example 5.1. The parameters of extinction in Model (1.2) are presented in Table 1. We choose (I)–(III) as the parameters of Model (1.2) in a heavily polluted environment; and choose (IV) as the parameters of Model (1.2) in a lightly polluted environment. It is easy to verify that Condition (3.3) of Theorem 3.1 and Condition (3.11) of Corollary 3.1 are satisfied.

$$a - e_A - c_A - r_A(C_o)_* - \frac{\lambda(C_e)_*}{1 + \beta(C_e^*)^2} = -0.0359 < 0, \tag{5.3}$$

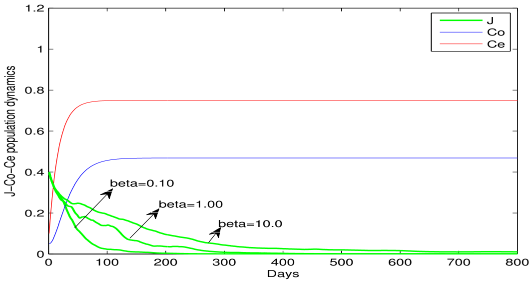
$$a - e_A - c_A - r_A(C_o)_* - \lambda(C_e)_* = -0.0406 < 0, \tag{5.4}$$

where $(C_o)_* = 0.05, C_e^* = 0.10, (C_e)_* = 0.75$.

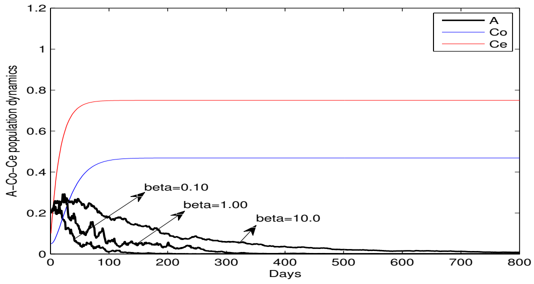
The results of numerical simulations indicate that the densities of juveniles and adults tend to extinction as shown in Figures 1 and 2. We conclude that the densities of juveniles and adults are slightly different between Figures 1 and 2, and that the time for extinction was extended as a result of increasing the psychological effects β from 0.10 to 10.0. By the same arguments, the simulations for extinction of juveniles and adults with external toxicant periodic inputs to Model (1.2) are demonstrated in Figure 2, and the time for extinction behaves in the similar way as shown in Figure 1.

Table 1. Parameters for extinction for Model (1.2).

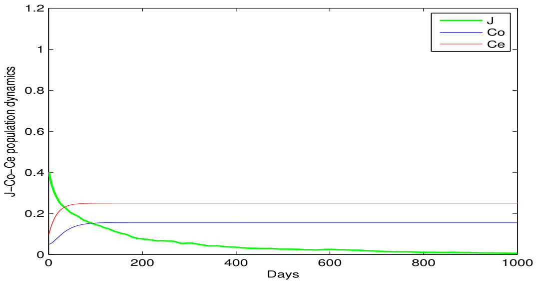
No.	a	e_J	e_A	r_J	r_A	α	c_J	c_A	λ	u_e	σ	β
(I)	0.35	0.10	0.15	0.10	0.15	0.25	0.05	0.20	0.30	0.30	0.30	0.10
(II)	0.35	0.10	0.15	0.10	0.15	0.25	0.05	0.20	0.30	0.30	0.30	1.00
(III)	0.35	0.10	0.15	0.10	0.15	0.25	0.05	0.20	0.05	0.30	0.30	10.0
(IV)	0.32	0.10	0.15	0.10	0.15	0.25	0.05	0.20	0.05	0.10	0.30	null



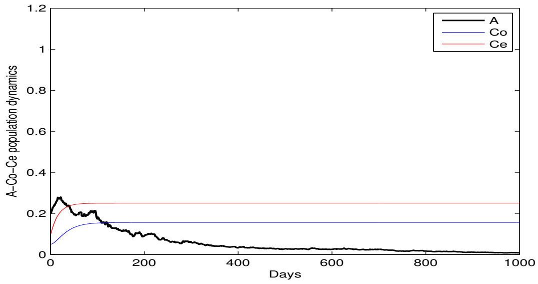
(a) heavy pollution



(b) heavy pollution

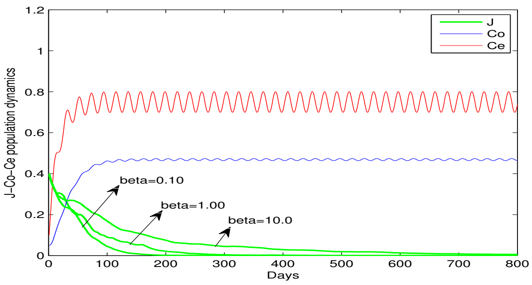


(c) light pollution

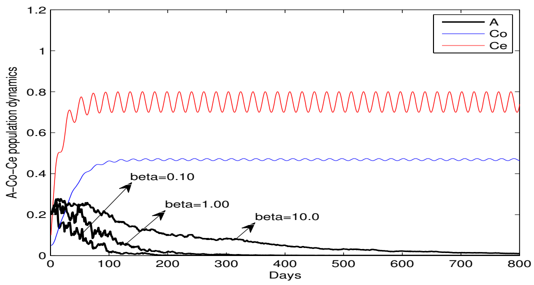


(d) light pollution

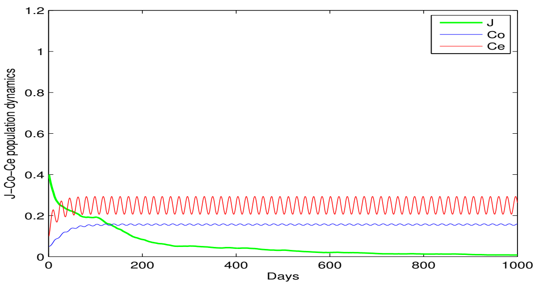
Figure 1. Extinction derived from Model (1.2), with constant input for heavy pollution, $u_e(t) = 0.30$, and for light pollution, $u_e(t) = 0.10$, in Table 1.



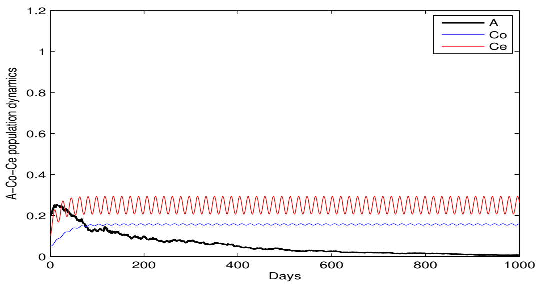
(a) heavy pollution



(b) heavy pollution



(c) light pollution



(d) light pollution

Figure 2. Extinction derived from Model (1.2), with periodic input for heavy pollution, $u_e(t) = 0.30 + 0.10\sin(0.35t)$, and for light pollution, $u_e(t) = 0.10 + 0.10\sin(0.35t)$.

Example 5.2. Main parameters for the stochastic permanence of Model (1.2) are given in Table 2. We next investigate the impacts of the psychological effects β and external toxicant input u_e for juveniles and adults in a local population. We take (V)–(VII) as the parameters of Model (1.2) in a heavily polluted environment, and take (VIII) as the parameters of Model (1.2) in a lightly polluted environment. It is easy to verify that Condition (3.20) of Theorem 3.2 is satisfied, and that

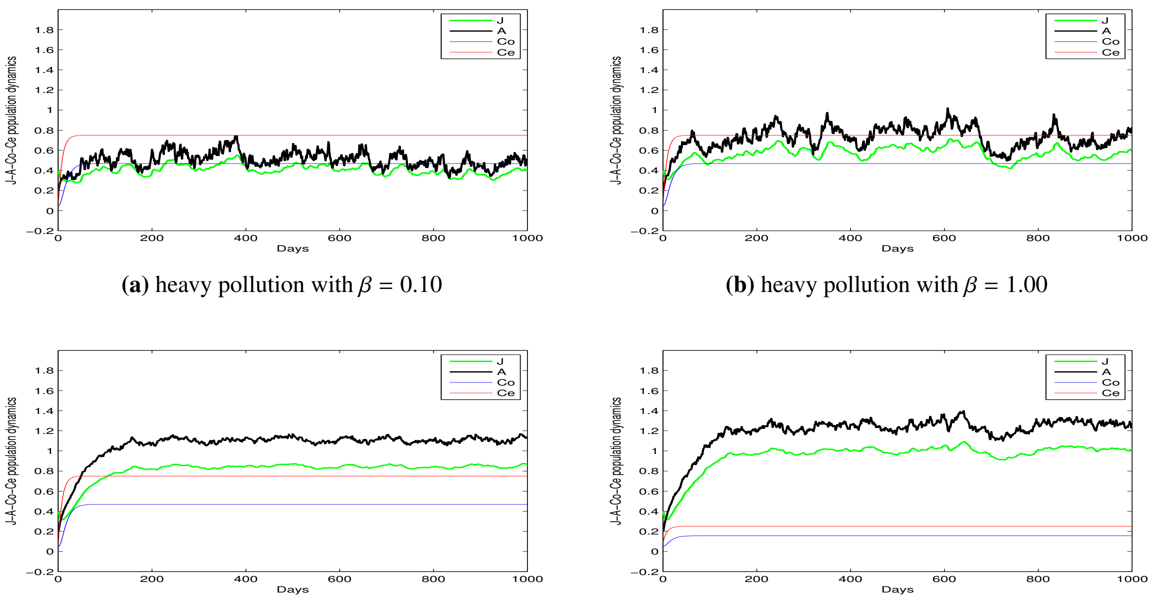
Table 2. Parameters for stochastic permanence in Model (1.2).

No.	a	e_J	e_A	r_J	r_A	α	c_J	c_A	λ	u_e	σ	β
(V)	0.35	0.05	0.10	0.10	0.05	0.32	0.05	0.10	0.10	0.30	0.10	0.10
(VI)	0.35	0.05	0.10	0.10	0.05	0.32	0.05	0.10	0.10	0.30	0.10	1.00
(VII)	0.35	0.05	0.10	0.10	0.05	0.32	0.05	0.10	0.10	0.30	0.10	10.0
(VIII)	0.35	0.05	0.10	0.10	0.05	0.32	0.05	0.10	0.10	0.10	0.10	null

$$(a \wedge \alpha) - (e_J + r_J) \vee (e_A + r_A + \lambda) - 0.5(3 + \theta)\sigma^2 = 0.0345 > 0.$$

Thus, juveniles and adults in a lightly polluted environment and in a heavily polluted environment are stochastically permanent as shown in Figures 3 and 4.

The simulations shown in (a)–(c) of Figures 3 and 4 reveal that the densities for juveniles and adults in a heavily polluted environment grow as the value of the psychological effects β increases from 0.10 to 10.0. Further, the densities for juveniles and adults in a lightly polluted environment, as shown in (d), present higher levels than that in the heavily polluted environment shown in (c) of Figure 3. We take the parameters in Table 2 and external toxicant periodic inputs $u_e(t) = 0.30 + 0.10\sin(0.35t)$ for heavy pollution and $u_e(t) = 0.10 + 0.10\sin(0.35t)$ for light pollution; then, the stochastic permanence of the juveniles and adults in Model (1.2) in Figure 4 are similar to those in Figure 3.



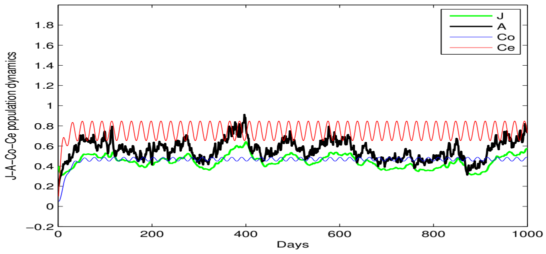
(a) heavy pollution with $\beta = 0.10$

(b) heavy pollution with $\beta = 1.00$

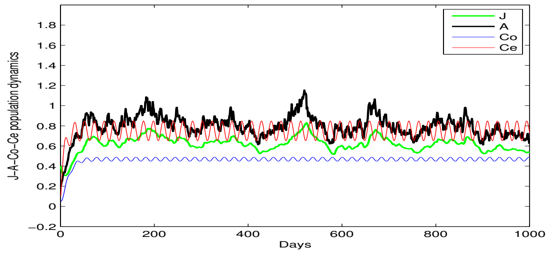
(c) heavy pollution with $\beta = 10.0$

(d) light pollution

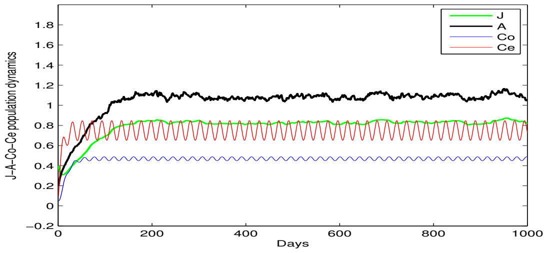
Figure 3. Stochastic permanence of Model (1.2), with constant input for heavy pollution, $u_e(t) = 0.30$, and for light pollution, $u_e(t) = 0.10$, in Table 2.



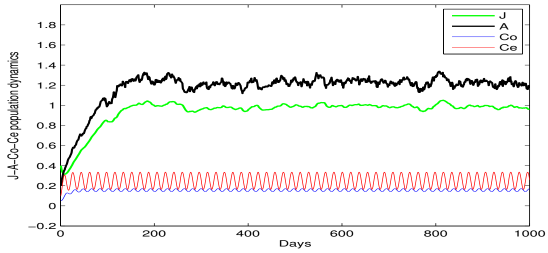
(a) heavy pollution with $\beta = 0.10$



(b) heavy pollution with $\beta = 1.00$



(c) heavy pollution with $\beta = 10.0$



(d) light pollution

Figure 4. Stochastic permanence of Model (1.2), with periodic input for heavy pollution, $u_e(t) = 0.30 + 0.10\sin(0.35t)$, and for light pollution $u_e(t) = 0.10 + 0.10\sin(0.35t)$.

Example 5.3. We take the parameters in Table 3, and verify that Conditions (3.35) of Theorem 3.3 are satisfied. That is

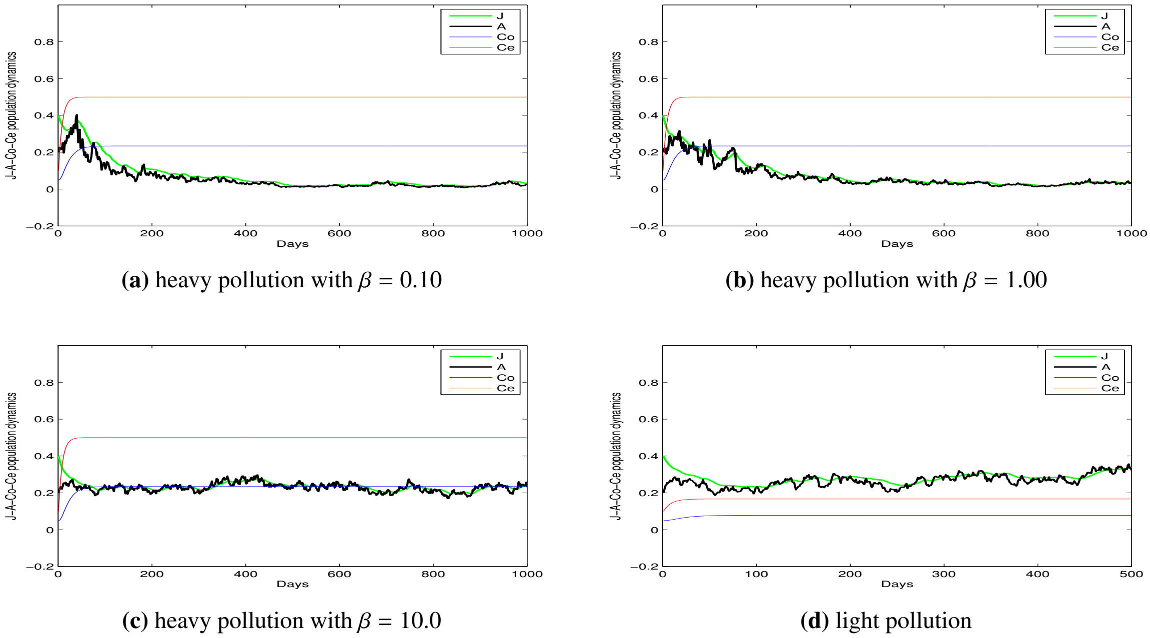
$$\begin{aligned} &c_J + 2c_J\hat{J} + 0.5\alpha + e_J - a = 0.05 > 0, \quad 2c_A + 4c_A\hat{A} + 2e_A - a - 2\alpha - 2\sigma^2 = 0.05 > 0, \\ &2(g + m + b) - k = 0.49 > 0, \quad 2h - 1 - k = 0.05 > 0. \end{aligned} \tag{5.5}$$

The pollution-free equilibrium of Model (1.2) is $(\hat{J}, \hat{A}, 0, 0) = (0.50, 0.50, 0, 0)$; then, the weak persistence in the mean of Model (1.2) is demonstrated in Figures 5 and 6. Taking (IX)–(XI) as the parameters of Model (1.2) for a heavily polluted environment, we find that the densities for weak persistence in the means for juveniles and adults in a heavily polluted environment arise when the value of the psychological effects β is increasing from 0.10 to 10.0 as shown in (a)–(c) of Figure 5. Again, taking (XII) as the parameters of Model (1.2) for a lightly polluted environment, the corresponding numerical simulations reveal that the survival levels of juveniles and adults, as shown in (d), are higher those shown in (c) of Figure 5.

The external toxicant periodic inputs $u_e(t) = 0.30 + 0.10\sin(0.35t)$ and $u_e(t) = 0.10 + 0.10\sin(0.35t)$ were taken into account for heavy pollution and light pollution respectively; then, the stochastic permanence of juveniles and adults, as according to Model (1.2) in Figure 6, the same as those in Figure 5.

Table 3. Parameters for weak persistence in the means according to Model (1.2).

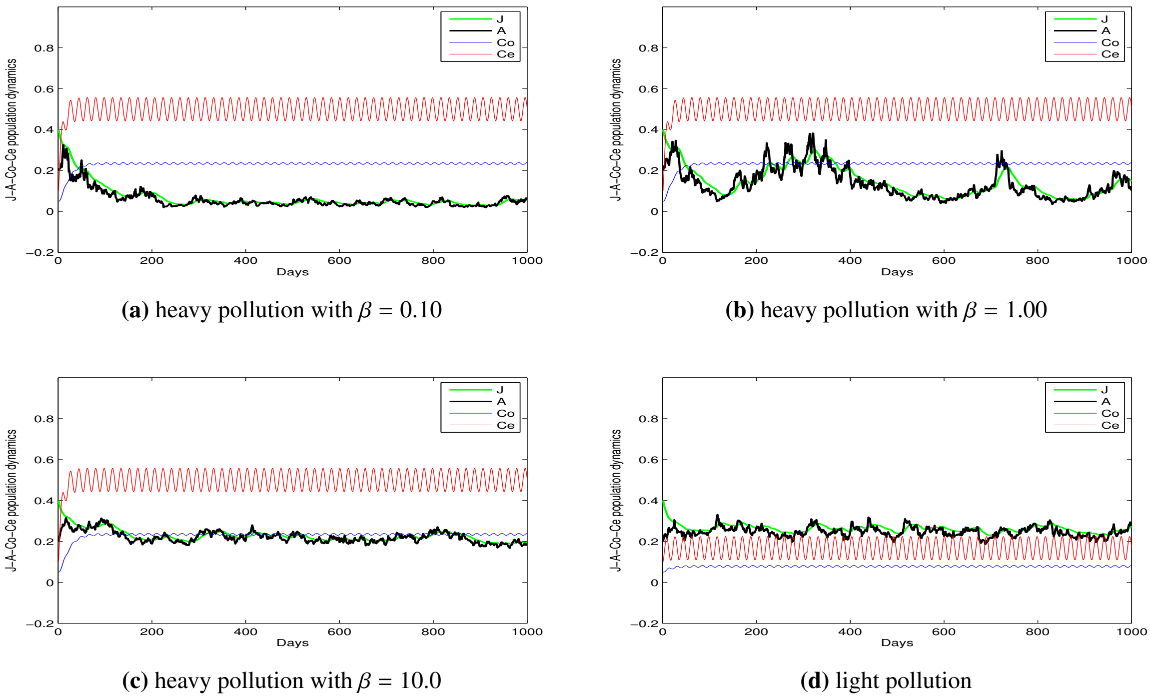
No.	a	e_J	e_A	r_J	r_A	α	c_J	c_A	λ	k	u_e	h	σ	β
(IX)	0.35	0.05	0.10	0.10	0.05	0.20	0.10	0.20	0.30	0.15	0.30	0.60	0.20	0.10
(X)	0.35	0.05	0.10	0.10	0.05	0.20	0.10	0.20	0.30	0.15	0.30	0.60	0.20	1.00
(XI)	0.35	0.05	0.10	0.10	0.05	0.20	0.10	0.20	0.30	0.15	0.30	0.60	0.20	10.0
(XII)	0.35	0.05	0.10	0.10	0.05	0.20	0.10	0.20	0.35	0.15	0.10	0.60	0.20	null



(**a**) heavy pollution with β = 0.10 (**b**) heavy pollution with β = 1.00

(**c**) heavy pollution with β = 10.0 (**d**) light pollution

Figure 5. Weak persistence in the means of Model (1.2), with constant input for heavy pollution, $u_e(t) = 0.30$, and for light pollution $u_e(t) = 0.10$, in Table 3.



(**a**) heavy pollution with β = 0.10 (**b**) heavy pollution with β = 1.00

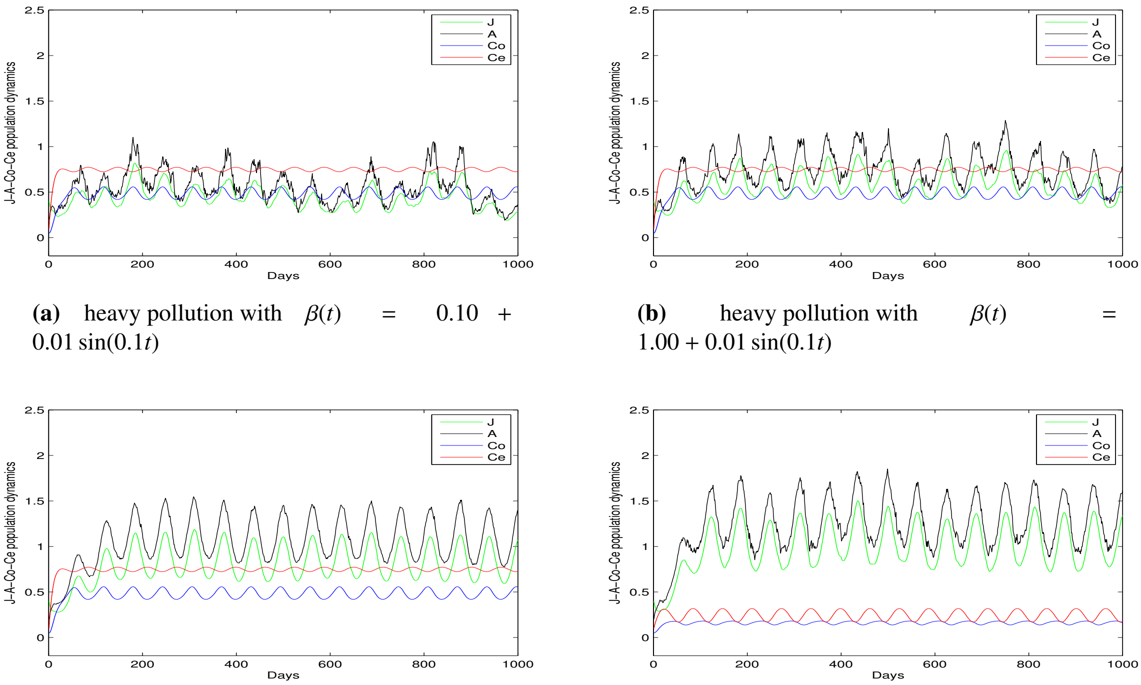
(**c**) heavy pollution with β = 10.0 (**d**) light pollution

Figure 6. Weak persistence in the means of Model (1.2), with periodic input for heavy pollution, $u_e(t) = 0.30 + 0.10\sin(0.35t)$, and for light pollution, $u_e(t) = 0.10 + 0.10\sin(0.35t)$.

Example 5.4. Let all parameters be periodic functions of t as follows:

$$\begin{array}{lll} a(t) = 0.35 + 0.01\sin(0.1t), & e_J(t) = 0.05 + 0.01\sin(0.1t), & e_A(t) = 0.10 + 0.01\sin(0.1t), \\ r_J(t) = 0.10 + 0.01\sin(0.1t), & r_A(t) = 0.05 + 0.01\sin(0.1t), & \alpha(t) = 0.32 + 0.01\sin(0.1t), \\ c_J(t) = 0.05 + 0.01\sin(0.1t), & c_A(t) = 0.10 + 0.01\sin(0.1t), & \lambda(t) = 0.10 + 0.01\sin(0.1t), \\ k(t) = 0.20 + 0.01\sin(0.1t), & g(t) = 0.08 + 0.01\sin(0.1t), & m(t) = 0.04 + 0.01\sin(0.1t), \\ h(t) = 0.40 + 0.01\sin(0.1t), & b(t) = 0.20 + 0.01\sin(0.1t), & \sigma(t) = 0.15 + 0.01\sin(0.1t). \end{array} \tag{5.6}$$

It is obvious that Condition (4.5) of Theorem 4.1 is satisfied. When the psychological effect were varied from 0.10 to 10.0, as shown in (a)–(c) of Figure 7, the corresponding simulations showed that the periodicity of Model (1.2) becomes apparent in a heavily polluted environment, and also that the survival levels for juveniles and adults in a lightly polluted environment , as shown in (d), are higher than those in a heavily polluted environment , as shown in (c) of Figure 7.



(a) heavy pollution with $\beta(t) = 0.10 + 0.01\sin(0.1t)$

(b) heavy pollution with $\beta(t) = 1.00 + 0.01\sin(0.1t)$

(c) heavy pollution with $\beta(t) = 10.0 + 0.01\sin(0.1t)$

(d) light pollution

Figure 7. Periodic solution of Model (1.2) with heavy pollution, $u_e(t) = 0.30 + 0.01\sin(0.1t)$, and light pollution, $u_e(t) = 0.10 + 0.01\sin(0.1t)$.

6. Conclusions

We investigate the survival levels and periodicities of single-species population models with stage structure and psychological effects within a polluted environment in this study. We always assumed that within a local population, adults produced juveniles at a constant rate, juveniles matured and turned into adults at a constant rate and that juveniles were not involved in the hunting. Adults frequently hunt in polluted environments for survival and reproduction, inevitably taking losses because of the polluted environment. Both juveniles and adults experienced losses due to toxicants within organisms.

The main results demonstrate that the extinction of juveniles and adults in heavily polluted environments depends on the psychological effects, and the time to extinction of the juveniles and adults becomes shorter in Model (1.2) as the value of the psychological effects varies from 0.10 to 10.0; also, the time to extinction in a lightly polluted environment is around 1000 days longer than that in a heavily polluted environment as presented in Figures 1 and 2.

Further, the research results also show that under the conditions of constant toxicant inputs and a periodic toxicant input, the survival levels including the stochastic permanence in Theorem 3.2 and weak persistence in the mean in Theorem 3.3 in a heavily polluted environment, decrease when the value of the psychological effects increases as presented in (a)–(c) of Figures 3–6. Meanwhile, the densities for juveniles and adults , as according to Model (1.2) are higher in a lightly polluted environment than that in a heavily polluted environment; the corresponding numerical simulations are shown in (d) of Figures 3–6. Finally, the existence of the periodic solution to Model (1.2) has been derived in Theorem 4.1 for both heavily and lightly polluted environments. The corresponding numerical simulations were carried out by employing Milstein's method as presented in Figure 7.

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Conflict of interest

All authors declare that they have no conflict of interests regarding this study.

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