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Research article

Extinction and stationary distribution of stochastic predator-prey model with group defense behavior

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Abstract: Considering that many prey populations in nature have group defense behavior, and the relationship between predator and prey is usually affected by environmental noise, a stochastic predator-prey model with group defense behavior is established in this paper. Some dynamical properties of the model, including the existence and uniqueness of global positive solution, sufficient conditions for extinction and unique ergodic stationary distribution, are investigated by using qualitative theory of stochastic differential equations, Lyapunov function analysis method, *Itô* formula, etc. Furthermore, the effects of group defense behavior and environmental noise on population stability are also discussed. Finally, numerical simulations are carried out to illustrate that the effects of environmental noise on both populations are negative, the appropriate group defense level of prey can maintain the stability of the relationship between two populations, and the survival threshold is strongly influenced by the intrinsic growth rate of prey population and the intensity of environmental noise.

Keywords: stochastic predator-prey model; group defense; extinction; stationary distribution; *Itô* formula

1. Introduction

In ecosystems, predation is not the only relationship between predator and prey. Many preys can perceive threats from predators, and thus make a variety of different response behaviors [1], which has an impact on the population dynamics of predators and prey. Studies have shown that the impact of this indirect behavior can be as great as the direct capture and killing their prey by predators [2–6].

For example, sparrowhawks are more likely to attack individual redshanks or small groups of redshanks during hunting, while their predation success rate is significantly lower for larger groups of redshanks [7]. Swarming locusts release large amounts of the volatile compound phenylacetonitrile, while dispersing locusts hardly synthesize phenylacetonitrile. When locust is threatened by natural enemies, phenylacetonitrile, as an olfactory warning compound, can further synthesize highly toxic substance hydrocyanic acid to achieve the purpose of defense against natural enemies [8]. As a result, locusts can reproduce recklessly, which causes locust plagues and results in serious economic losses [9, 10]. This behavior of prey is called group defense, which reduces the defense cost for individuals through 'number security' and thus increases survival rates. Freedman and Wolkowicz first investigated predator-prey interactions under the effects of prey group defense behavior and concluded that group defense behavior of sufficiently abundant prey would lead to extinction of predator populations [11]. From then on, more and more scholars have studied the more complex interspecific relationships of predator-prey systems under the effects of group defense [12–17]. Xiao and Ruan analyzed the dynamical complexities of the following predator-prey model with group defense behavior in [18]:

$$\begin{cases} \frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - \alpha xye^{-\beta x}, \\ \frac{dy}{dt} = y\left(\mu\alpha xe^{-\beta x} - D\right), \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ represent the population densities of prey and predator at time t, respectively. All parameter values of model (1.1) are non-negative. r denotes the intrinsic growth rate of prey population; K is the environmental capacity; α is the predation intensity; μ is the conversion efficiency of biomass; D is the natural death rate of predator. The response function $f(x) = \alpha xe^{-\beta x}$ represents the capture rate of predators on preys with group defense behavior, which satisfies

$$f(0) = 0,\ f(x) > 0\ for\ x > 0,$$

and

$$\begin{cases} f'(x) > 0,\ 0 \leq x < \frac{1}{\beta}, \\ f'(x) < 0,\ x > \frac{1}{\beta}, \end{cases}$$

where $\beta \in [0, 1)$ reflects the group defense level of prey population. The predation rate of predator reaches the maximum value at $x = \frac{1}{\beta}$, and decreases if the group defense level of prey increases when $x > \frac{1}{\beta}$. The larger β indicates that prey population can produce group defense behavior under low density. From [18], we know model (1.1) has complicated dynamics, when $\mu\alpha > e\beta D$, if $x_1 < K < 1/\beta$, model (1.1) has three equilibria: two hyperbolic saddles $(0, 0)$ and $(K, 0)$, and a globally asymptotically stable equilibrium (x_1, y_1) in the interior of the first quadrant; if $K > 1/\beta + x_2$, model (1.1) has four equilibria: two hyperbolic saddles $(0, 0)$ and (x_2, y_2), a hyperbolic stable node $(K, 0)$ and an unstable equilibrium (x_1, y_1), and in this case, model (1.1) has no closed orbits, where x_1 and x_2 are roots of $\mu\alpha xe^{-\beta x} - D = 0$. Moreover, model (1.1) may have a unique limit cycle or a homoclinic loop for different parameter values.

However, considering that in nature, various systems are inevitably affected by some uncertain environmental noises (weather change, human activities, etc.), which cannot be ignored, and it is more realistic and appropriate to use stochastic dynamical systems to describe these phenomena. Recently, many scholars considered introducing the stochastic pertubations into dynamical models [19–23] and have achieved meaningful results, such as the prediction and control of infectious disease [24–27], pest management [28] and various predator-prey systems [29–32]. Based on [18] and above biological background, considering that small environmental fluctuations in nature mainly affect the parameters of the model, which can be characterized as white noise [33], the following stochastic perturbation predator-prey model with group defense is established:

$$\begin{cases} dx = \left[rx\left(1-\frac{x}{K}\right) - \alpha xye^{-\beta x}\right]dt + \sigma_1 x dB_1(t), \\ dy = \left(\mu\alpha xye^{-\beta x} - Dy\right)dt + \sigma_2 y dB_2(t). \end{cases} \tag{1.2}$$

Let $B_1(t)$ and $B_2(t)$ be independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$, $\sigma_i > 0\,(i = 1, 2)$ is the intensity of environmental noise.

By constructing suitable Lyapunov functions and applying *Itô* formula, scholars have studied the dynamics of the stochastic predator-prey models with hunting cooperation [3], prey refuge and fear effect [4, 34], Beddington-DeAngelis functional response [35], Holling type II-III functional response [36], Allee effect [37], habitat complexity and prey aggregation [38], etc. As mentioned above, the dynamic properties of the deterministic predator-prey model (1.1) with group defense behavior are very complicated, so far, there is no literature on the stochastic predator-prey model (1.2) with group defense behavior. Therefore, one purpose of this paper is to study the effects of stochastic noise on the dynamics of the deterministic model with group defense behavior. We give how does the globally asymptotically stable equilibrium point or limit cycle of a deterministic model change under the effects of white noise. The other purpose of this paper is to analyze the impact of group defense behavior and other key factors on the extinction or persistence of the population.

The rest of this paper is arranged as follows. In Section 2, Lyapunov function analysis method is used for proving the existence and uniqueness of positive solution of model (1.2). In Section 3, the extinction and persistence of the population in model (1.2) is discussed, and proves that there is a unique ergodic stationary distribution. Numerical simulations are given in Section 4 and Section 5 to support our conclusions.

2. Existence and uniqueness of the positive solution

In this paper, we define the state space as $\mathbb{R}^2_+ = \{(x_1, x_2) \in \mathbb{R}^2 | x_1 > 0, x_2 > 0\}$.

Theorem 2.1. *For any initial value $(x(0), y(0)) \in \mathbb{R}^2_+$, there is a unique solution $(x(t), y(t))$ of model (1.2) on $t \geq 0$ and the solution will remain in $\mathbb{R}^2_+$ with probability one, that is to say, $(x(0), y(0)) \in \mathbb{R}^2_+$ for all $t \geq 0$ almost surely (a.s.).*

Proof. Since the coefficients of model (1.2) satisfy the local Lipschitz condition, for any given initial value $(x(0), y(0)) \in \mathbb{R}^2_+$, there exists a unique local positive solution $(x(t), y(t))$ on $t \in [0, \tau_e)$, where τ_e denotes the explosion time. In order to prove that solution $(x(t), y(t))$ is global, it is sufficient to prove $\tau_e = \infty$.

Take a sufficiently large non-negative number n_0, such that $x(0) \in \left[\frac{1}{n_0}, n_0\right]$, $y(0) \in \left[\frac{1}{n_0}, n_0\right]$. For each integer $n \geq n_0$, define the stopping time as

$$\tau_n = \inf\left\{t \in [0, \tau_e) \middle| x(t) \notin \left(\frac{1}{n}, n\right) \text{ or } y(t) \notin \left(\frac{1}{n}, n\right)\right\}.$$

Obviously, τ_n is increasing as $n \to \infty$. Set $\tau_\infty = \lim_{n\to\infty} \tau_n$, hence $\tau_\infty \leq \tau_e$ a.s. If we can prove $\tau_\infty = \infty$ a.s., then $\tau_e = \infty$ a.s.

If the statement is false, then there are a pair of constants $T > 0$ and $\varepsilon \in (0, 1)$ such that

$$P\{\tau_\infty \leq T\} > \varepsilon.$$

Thus, there is an integer $n_1 \geq n_0$ such that

$$P\{\tau_n \leq T\} \geq \varepsilon, \forall n \geq n_1. \tag{2.1}$$

Define a C^2-function $V : \mathbb{R}_+^2 \to \mathbb{R}_+$ by

$$V(x, y) = \left(x - \frac{D}{\mu\alpha} - \frac{D}{\mu\alpha} \ln \frac{\mu\alpha}{D} x\right) + \frac{1}{\mu}(y - 1 - \ln y) \geq 0.$$

Making use of *Itô* formula, we obtain

$$dV(x, y) = LV(x, y)\,dt + \sigma_1 \left(x - \frac{D}{\mu\alpha}\right) dB_1(t) + \frac{\sigma_2}{\mu}(y - 1)\,dB_2(t),$$

where

$$\begin{aligned}
LV(x,y) =& \left(1 - \frac{D}{\mu\alpha x}\right)\left(rx\left(1 - \frac{x}{K}\right) - \alpha xye^{-\beta x}\right) + \frac{1}{\mu}(1 - \frac{1}{y})\left(\mu\alpha xye^{-\beta x} - Dy\right) \\
&+ \frac{1}{2}\frac{D}{\mu\alpha x^2}(\sigma_1 x)^2 + \frac{1}{2}\frac{1}{\mu y^2}(\sigma_2 y)^2 \\
\leq& rx - \frac{r}{K}x^2 + \frac{D}{\mu\alpha}\frac{r}{K}x + \frac{D}{\mu} + \frac{1}{2}\frac{D}{\mu\alpha x^2}\sigma_1^2 + \frac{1}{2\mu}\sigma_2^2 \\
\leq& \frac{K}{4r}\left(r + \frac{Dr}{K\mu\alpha}\right)^2 + \frac{2}{\mu}\left(2D + \frac{D}{\alpha}\sigma_1^2 + \sigma_2^2\right) \\
\triangleq& \tilde{P},
\end{aligned}$$

$\tilde{P}$ is a positive constant. So we have

$$dV(x, y) \leq \tilde{P}dt + \sigma_1 \left(x - \frac{D}{\mu\alpha}\right) dB_1(t) + \frac{\sigma_2}{\mu}(y - 1)\,dB_2(t). \tag{2.2}$$

Integrating both sides of (2.2) from 0 to $\tau_n \wedge T = \min\{\tau_n, T\}$ and then taking the expectation, we obtain

$$EV(x(\tau_n \wedge T), y(\tau_n \wedge T)) \leq V(x(0), y(0)) + \tilde{P}E(\tau_n \wedge T),$$

therefore

$$EV(x(\tau_n \wedge T), y(\tau_n \wedge T)) \leq V(x(0), y(0)) + \tilde{P}T. \tag{2.3}$$

Let $\Omega_n = \{\tau_n \leq T\}$ for $n > n_1$. From (2.1), we obtain $P(\Omega_n) > \varepsilon$. For each $\omega \in \Omega_n$, $x(\tau_n, \omega)$ or $y(\tau_n, \omega)$ equals either n or $\frac{1}{n}$. So $V(x(\tau_n, \omega), y(\tau_n, \omega))$ is no less than either

$$n - 1 - \ln n \quad \text{or} \quad \frac{1}{n} - 1 - \ln \frac{1}{n} = \frac{1}{n} - 1 + \ln n.$$

Thus

$$V\left(x\left(\tau_n,\omega\right),y\left(\tau_n,\omega\right)\right) \geq \left(n-1-\ln n\right) \wedge \left(\frac{1}{n}-1+\ln n\right)$$

holds. According to (2.3), we have

$$V\left(x\left(0\right),y\left(0\right)\right)+\tilde{P}T \geq E\left[I_{\Omega_n(\omega)}V\left(x\left(\tau_n,\omega\right),y\left(\tau_n,\omega\right)\right)\right] \geq \varepsilon\left(n-1-\ln n\right) \wedge \left(\frac{1}{n}-1+\ln n\right),$$

where I_{Ω_n} denotes the indicator function of Ω_n. Letting $n \to \infty$, then we obtain

$$\infty > V\left(x\left(0\right),y\left(0\right)\right)+\tilde{P}T = \infty,$$

which leads to a contradiction, so we must have $\tau_\infty = \infty$ a.s. This completes the proof.

3. Extinction and stationary distribution

3.1. Extinction

In this section, we first discuss the conditions for the extinction of population. The following definition is given:

Definition 3.1. *If* $\lim\limits_{t\to+\infty} x(t) = 0$ *a.s. holds, the prey population will be extinct with probability one; If* $\lim\limits_{t\to+\infty} y(t) = 0$ *a.s. holds, the predator population will be extinct with probability one.*

Theorem 3.1. *For model (1.2), if* $\sigma_1 > \sqrt{2r}$ *holds, then the prey population* $x(t)$ *will be extinct with probability one.*

Proof. Applying *Itô's* formula to the first equation of model (1.2), we have

$$d\ln x = \tfrac{1}{x}dx - \tfrac{1}{2x^2}(dx)^2 = \left(r - \tfrac{r}{K}x - \alpha y e^{-\beta x} - \tfrac{1}{2}\sigma_1^2\right)dt + \sigma_1 dB_1(t).$$

Integrating both sides from 0 to t and dividing by t, we obtain

$$\begin{aligned}\frac{\ln x(t)-\ln x(0)}{t} &= \frac{1}{t}\int_0^t\left(r-\frac{r}{K}x-\alpha y e^{-\beta x}-\frac{1}{2}\sigma_1^2\right)dt+\frac{\sigma_1 B_1(t)}{t}\\ &\leq r-\frac{1}{2}\sigma_1^2+\frac{\sigma_1 B_1(t)}{t}.\end{aligned}$$

Making use of the strong law of large numbers for local martingales [39] leads to $\lim\limits_{t\to\infty}\frac{\sigma_1 B_1(t)}{t} = 0$ a.s. Therefore

$$\limsup_{t\to\infty}\frac{\ln x(t)}{t} \leq r-\frac{1}{2}\sigma_1^2 < 0 \ a.s.$$

That is to say $\lim\limits_{t\to+\infty} x(t) = 0$ a.s., according to definition 3.1, the prey population will be extinct with probability one. This completes the proof.

Remark 3.1. *Theorem 3.1 gives a sufficient condition for the extinction of prey population. When* $\sigma_1 > \sqrt{2r}$*, the prey population will be extinct, and it is not difficult to draw from model (1.2) that if the prey population is extinct, it will lead to the extinction of predator population. When* $\sigma_1 \neq 0$*, from the above proof process, it can be seen that even small effects of environmental noise will reduce the density of prey population in mean. Once the environmental noise reaches a certain level, the prey population will be extinct.*

Theorem 3.2. *For model (1.2), if* $\sigma_2 > \sqrt{2\left(\frac{\mu\alpha}{\beta} - D\right)}$ *holds, then the predator population* $y(t)$ *will be extinct with probability one.*

Proof. Applying *Itô's* formula to the second equation of model (1.2), we have

$$\begin{aligned} d\ln y(t) =& \frac{1}{y}dy - \frac{1}{2y^2}(dy)^2 \\ =& \left(\mu\alpha x e^{-\beta x} - D - \frac{1}{2}\sigma_2^2\right)dt + \sigma_2 dB_2(t) \\ \leq& \left(\frac{\mu\alpha}{\beta} - D - \frac{1}{2}\sigma_2^2\right)dt + \sigma_2 dB_2(t). \end{aligned}$$

Integrating both sides from 0 to t and dividing by t, we obtain

$$\begin{aligned} \frac{\ln y(t) - \ln y(0)}{t} \leq& \frac{1}{t}\int_0^t \left(\frac{\mu\alpha}{\beta} - D - \frac{1}{2}\sigma_2^2\right)dt + \frac{\sigma_2 B_2(t)}{t} \\ =& \left(\frac{\mu\alpha}{\beta} - D - \frac{1}{2}\sigma_2^2\right) + \frac{\sigma_2 B_2(t)}{t}. \end{aligned}$$

So

$$\limsup_{t\to\infty} \frac{\ln y(t)}{t} \leq \frac{\mu\alpha}{\beta} - D - \frac{1}{2}\sigma_2^2 < 0 \ a.s.$$

Then we have $\lim_{t\to\infty} y(t) = 0$. This completes the proof.

Remark 3.2. *Note that when* $\sigma_2 \neq 0$*, that is, in the presence of environmental noise, once the stochastic noise* σ_2 *is larger than the critical threshold (* $\sqrt{2\left(\frac{\mu\alpha}{\beta} - D\right)}$*), then the predator population will tend to be extinct. From the expression of this threshold, it is not difficult to see that the lower group defense level* β *of the prey is conductive to the survival of the predator population.*

3.2. Stationary distribution

Whether the predator and prey populations will continue to survive is also an important issue in the study of predator-prey model. Therefore, in this section, we will prove that model (1.2) has a unique ergodic stationary distribution through the equivalent conditions of Hasminskii theorem, indicating that both predators and prey populations will survive [40].

Theorem 3.3. *If* $R = \left(D + \frac{\sigma_2^2}{2}\right)^{-1}\beta\left(r - \frac{\sigma_1^2}{2} - \frac{D\left(\frac{r}{K} + \frac{\mu\alpha}{D}r\right)^2}{4\mu\alpha}\right) > 1$*, then model (1.2) has a unique stationary distribution* $\pi(\cdot)$ *with ergodicity.*

Proof. Model (1.2) can be written as the following form

$$d\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} rx - \frac{r}{K}x^2 - \alpha xye^{-\beta x} \\ \mu\alpha xye^{-\beta x} - Dy \end{pmatrix}dt + \begin{pmatrix} \sigma_1 x \\ 0 \end{pmatrix}dB_1(t) + \begin{pmatrix} 0 \\ \sigma_2 y \end{pmatrix}dB_2(t),$$

then we get its diffusion matrix

$$A(x,y) = \begin{pmatrix} \sigma_1^2 x^2 & 0 \\ 0 & \sigma_2^2 y^2 \end{pmatrix},$$

and there exists a positive number $G = \min_{(x,y)\in\bar{U}} \left\{\sigma_1^2 x^2, \sigma_2^2 y^2\right\}$, such that $\sum_{i,j=1}^{2} a_{ij}\xi_i\xi_j = \sigma_1^2 x^2 {\xi_1}^2 + \sigma_2^2 y^2 {\xi_2}^2 \geq G|\xi|^2$, $(x, y) \in \bar{U}$, $\xi = (\xi_1, \xi_2) \in \mathbb{R}^2$.

Construct a C^2-function $V : \mathbb{R}_+^2 \to \mathbb{R}_+$ by

$$\bar{V}(x,y) = -P\left[\alpha \ln x + \ln y - \frac{\beta\alpha}{D}(\mu x + y)\right] + \frac{1}{2}(\mu x + y)^2,$$

where

$$P = \frac{2}{R-1} \max\left\{2, \sup_{(x,y)\in\mathbb{R}_+^2} \left\{\left(r + \frac{\sigma_1^2}{2}\right)\mu^2 x^2 - \frac{1}{2}\frac{r}{K}\mu^2 x^3 - \frac{1}{2}\left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right)y^2\right\}\right\}.$$

It is easy to prove that $\bar{V}(x, y)$ has a unique minimum point (x_0, y_0). Define a C^2-function $V : \mathbb{R}_+^2 \to \mathbb{R}_+$ by

$$V(x,y) = \bar{V}(x,y) - \bar{V}(x_0,y_0) \triangleq V_1 + V_2, \tag{3.1}$$

where $V_1 = -P\left[\alpha \ln x + \ln y - \frac{\beta\alpha}{D}(\mu x + y)\right]$, $V_2 = \frac{1}{2}(\mu x + y)^2 - \bar{V}(x_0, y_0)$. Applying *Itô's* formula to (3.1), we have

$$\begin{aligned}
LV_1 =& P\left(-\beta r + \beta\frac{r}{K}x + \alpha\beta y e^{-\beta x} - \mu\alpha x e^{-\beta x} + D + \frac{\mu\alpha\beta}{D} r x\right. \\
&\left. -\frac{\mu\alpha\beta}{D}\frac{r}{K}x^2 - \frac{\mu\alpha\beta}{D} x y e^{-\beta x} + \frac{\mu\alpha\beta}{D} x y e^{-\beta x} - \alpha\beta y + \frac{\beta}{2}\sigma_1^2 + \frac{1}{2}\sigma_2^2\right) \\
\leq& -P\left[\beta\left(r - \frac{\sigma_1^2}{2} - \frac{D\left(\frac{r}{K} + \frac{\mu\alpha}{D}r\right)^2}{4\mu\alpha}\right) - \left(D + \frac{\sigma_2^2}{2}\right)\right] \\
=& -\frac{P}{D + \frac{\sigma_2^2}{2}}(R-1) \\
\triangleq& -P^*(R-1).
\end{aligned}$$

$$\begin{aligned}
LV_2 =& r\mu^2 x^2 - \frac{r}{K}\mu^2 x^3 - \mu^2\alpha y e^{-\beta x} x^2 + r\mu x y - \frac{r}{K}\mu x^2 y - \mu\alpha x y^2 e^{-\beta x} \\
&+ \mu^2\alpha y e^{-\beta x} x^2 - \mu x y + \mu\alpha x y^2 e^{-\beta x} - D y^2 + \frac{1}{2}\mu^2\sigma_1^2 x^2 + \frac{1}{2}\sigma_2^2 y^2 \\
\leq& \left(r - \frac{\sigma_1^2}{2}\right)\mu^2 x^2 - \frac{r}{K}\mu^2 x^3 + r\mu x y - \left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right)y^2.
\end{aligned}$$

Therefore

$$\begin{aligned}
LV =& LV_1 + LV_2 \\
\leq& -P^*(R-1) + \left(r + \frac{\sigma_1^2}{2}\right)\mu^2 x^2 - \frac{r}{K}\mu^2 x^3 + r\mu x y - \left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right)y^2.
\end{aligned}$$

Choosing sufficiently small ε_1 and ε_2 such that

$$0 < \varepsilon_1 < (r\mu)^{-1} \min\left\{\frac{1}{2}\left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right), \frac{P^*(R-1)}{4}\right\}, \tag{3.2}$$

$$0<\varepsilon_2<(r\mu)^{-1}\min\left\{\tfrac{r\mu^2}{2K},\tfrac{P^*(R-1)}{4}\right\},\tag{3.3}$$

$$\min\left\{\tfrac{r\mu^2}{2K\varepsilon_1},\tfrac{D-\beta^{-1}\mu\alpha-\frac{1}{2}\sigma_2^2}{2\varepsilon_2}\right\}\geq M-P^*(R-1)+1.\tag{3.4}$$

Then we consider the bounded open set

$$S=\left\{(x,y)\in\mathbb{R}_+^2\mid \varepsilon_1\leq x\leq\frac{1}{\varepsilon_1},\quad \varepsilon_2\leq y\leq\frac{1}{\varepsilon_2}\right\}.$$

Clearly, $S^c=\mathbb{R}_+^2\backslash S=S_1\cup S_2\cup S_3\cup S_4$, where

$$S_1=\left\{(x,y)\in\mathbb{R}_+^2\mid 0<x<\varepsilon_1\right\},\quad S_2=\left\{(x,y)\in\mathbb{R}_+^2\mid 0<y<\varepsilon_2\right\},$$

$$S_3=\left\{(x,y)\in\mathbb{R}_+^2\mid x>\tfrac{1}{\varepsilon_1}\right\},\quad S_4=\left\{(x,y)\in\mathbb{R}_+^2\mid y>\tfrac{1}{\varepsilon_2}\right\}.$$

Case 1. When $(x,y)\in S_1$, and $xy<\varepsilon_1 y<\varepsilon_1\left(1+y^2\right)$, we have

$$\begin{aligned}LV\leq&-P^*(R-1)+r\mu\varepsilon_1+\left(r+\frac{\sigma_1^2}{2}\right)\mu^2x^2-\frac{r}{K}\mu^2x^3-\left(D-\frac{\mu\alpha}{\beta}-\frac{\sigma_2^2}{2}-r\mu\varepsilon_1\right)y^2\\ \leq&-\frac{P^*(R-1)}{4}+\left(-\frac{P^*(R-1)}{4}+r\mu\varepsilon_1\right)+\left(-\frac{1}{2}\left(D-\frac{\mu\alpha}{\beta}-\frac{\sigma_2^2}{2}\right)+r\mu\varepsilon_1\right)y^2\\ &+\left(-\frac{P^*(R-1)}{2}+\left(r+\frac{\sigma_1^2}{2}\right)\mu^2x^2-\frac{1}{2}\frac{r}{K}\mu^2x^3-\frac{1}{2}\left(D-\frac{\mu\alpha}{\beta}-\frac{\sigma_2^2}{2}\right)y^2\right).\end{aligned}$$

By (3.2) and the definition of P, we can obtain

$$LV\leq-\frac{P^*(R-1)}{4}\leq-1,\ \text{for}\,(x,y)\in S_1.$$

Case 2. When $(x,y)\in S_2$, and $xy<\varepsilon_2 x<\varepsilon_2\left(1+x^3\right)$, we have

$$\begin{aligned}LV\leq&-P^*(R-1)+r\mu\varepsilon_2-\left(\frac{r}{K}\mu^2-r\mu\varepsilon_2\right)x^3+\left(r+\frac{\sigma_1^2}{2}\right)\mu^2x^2-\left(m-\frac{k\alpha}{\beta}-\frac{\sigma_2^2}{2}\right)y^2\\ \leq&-\frac{P^*(R-1)}{4}+\left(-\frac{P^*(R-1)}{4}+r\mu\varepsilon_2\right)-\frac{1}{2}\left(D-\frac{\mu\alpha}{\beta}-\frac{\sigma_2^2}{2}\right)y^2+\left(-\frac{r\mu^2}{2K}+r\mu\varepsilon_2\right)x^3\\ &+\left(-\frac{P^*(R-1)}{2}+\left(r+\frac{\sigma_1^2}{2}\right)\mu^2x^2-\frac{r\mu^2}{2K}x^3-\frac{1}{2}\left(D-\frac{\mu\alpha}{\beta}-\frac{\sigma_2^2}{2}\right)y^2\right).\end{aligned}$$

By (3.3) and the definition of P again, we can obtain $LV\leq-\frac{P^*(R-1)}{4}\leq-1$, for $(x,y)\in S_2$.
In addition, by Young inequality, we have $xy\leq\frac{2}{5}x^{\frac{5}{2}}+\frac{3}{5}y^{\frac{5}{3}}$, for $\forall x,y>0$. Hence,

$$LV\leq-P^*(R-1)+\left(r+\frac{\sigma_1^2}{2}\right)\mu^2x^2-\frac{r}{K}\mu^2x^3+r\mu\left(\frac{2}{5}x^{\frac{5}{2}}+\frac{3}{5}y^{\frac{5}{3}}\right)-\left(D-\frac{\mu\alpha}{\beta}-\frac{\sigma_2^2}{2}\right)y^2.\tag{3.5}$$

Case 3. When $(x, y) \in S_3$, we have by (3.2) and (3.5)

$$\begin{aligned}
LV \le & -P^*(R-1) + \left[r\mu\left(\frac{2}{5}x^{\frac{5}{2}} + \frac{3}{5}y^{\frac{5}{3}}\right) - \frac{1}{2}\frac{r}{K}\mu^2x^3 + \left(r + \frac{\sigma_1^2}{2}\right)\mu^2x^2 - \frac{1}{2}\left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right)y^2 \right] \\
& - \frac{1}{2}\frac{r}{K}\mu^2x^3 - \frac{1}{2}\left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right)y^2 \\
\le & -P^*(R-1) - \frac{r\mu^2}{2K\varepsilon_1^3} + M,
\end{aligned}$$

where

$$M = \max_{(x,y)\in D_3} \left\{ r\mu\left(\frac{2}{5}x^{\frac{5}{2}} + \frac{3}{5}y^{\frac{5}{3}}\right) - \frac{1}{2}\frac{r}{K}\mu^2x^3 + \left(r + \frac{\sigma_1^2}{2}\right)\mu^2x^2 - \frac{1}{2}\left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right)y^2 \right\}. \tag{3.6}$$

By (3.4) and (3.6), we have $LV \le -1$, for $\forall (x, y) \in S_3$.

Case 4. When $(x, y) \in S_4$, similar to the case 3, by (3.4) we have ,

$$\begin{aligned}
LV & \le -P^*(R-1) - \frac{1}{2}\left(D - \frac{\mu\alpha}{\beta} - \frac{\sigma_2^2}{2}\right)y^2 - \frac{1}{2}\frac{r}{K}\mu^2x^3 + M \\
& \le -P^*(R-1) - \frac{D - \beta^{-1}\mu\alpha - \frac{1}{2}\sigma_2^2}{2\varepsilon_2^2} + M \le -1.
\end{aligned}$$

In conclusion, $LV \le -1$, $\forall (x, y) \in S^c$. According to the equivalent conditions of Hasminskii theorem in [40], we know that the model (1.2) is ergodic and has a unique stationary distribution.

4. The effects of environmental noise on model (1.2)

In this section, we will discuss the effects of environmental noise on extinction and persistence of populations, and further investigate how the environmental noise affects the dynamics of deterministic system through numerical simulations.

Take parameters as follows: $r = 0.5, \mu = 0.4, \alpha = 0.4, K = 4, D = 0.15, \beta = 0.38$. According to Theorem 2.1 in [18], since $e\beta D < \mu\alpha$ and $x_1 < K < 1/\beta$, deterministic model (1.1) has a globally asymptotically stable equilibrium (x_1, y_1), which means that both populations are persistent (see the dotted line in Figure 1(a) and the red curve in Figure 1(b)). Now consider the effects of small environmental noise on stochastic model (1.2). Taking $\sigma_1 = \sigma_2 = 0.01$, then $R = \left(D + \frac{\sigma_2^2}{2}\right)^{-1}\beta\left(r - \frac{\sigma_1^2}{2} - \frac{D\left(\frac{r}{K} + \frac{\mu\alpha}{D}r\right)^2}{4\mu\alpha}\right) \approx 1.0089 > 1$, it is clear from Theorem 3.3 that there is a unique ergodic stationary distribution for model (1.2), which implies that both populations are still persistent(see the solid line in Figure 1(a) and the blue curve in Figure 1(b)), and the solution of stochastic model (1.2) fluctuates near (x_1, y_1). In addition, we can also get the density function distribution of $x(t)$, $y(t)$, see Figure 1(c)(d). Moreover, we simulate the effects of environmental noises σ_1 and σ_2 on the threshold R. It is not difficult to find that $R > 1$ in Theorem 3.3 is a strong sufficient condition, which can only be satisfied when σ_1 and σ_2 are small, see Figure 2.

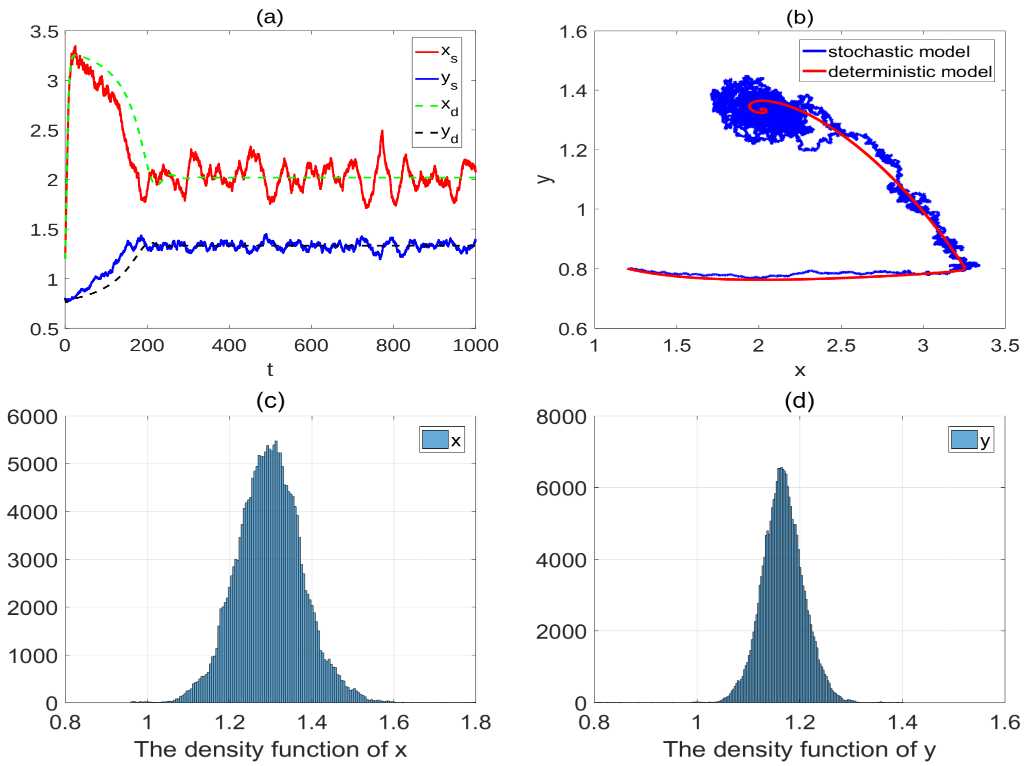

Figure 1. The effects of small environmental noise on stochastic model (1.2), where x_s and y_s are stochastic predator-prey model, x_d and y_d are deterministic predator-prey model. (a) Time sequence diagrams of model (1.2); (b) Phase portraits of model (1.2); (c)(d) Density function distribution of $x(t)$, $y(t)$.

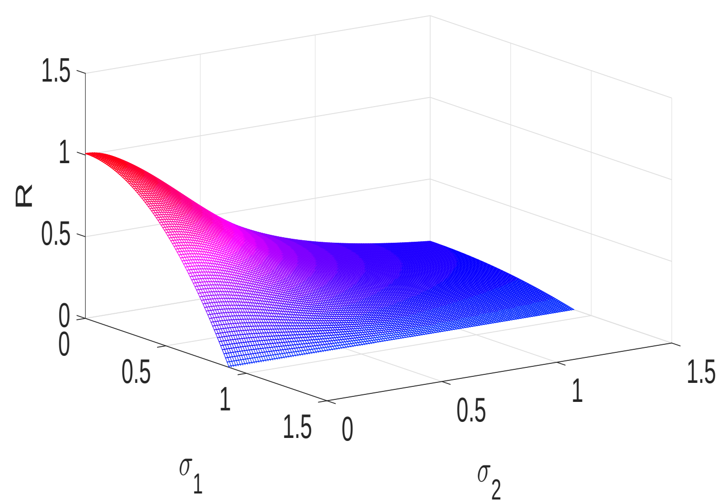

Figure 2. The influence of environmental noises σ_1 and σ_2 on the threshold R.

However, when the environmental noise σ_1 is small and σ_2 is large ($\sigma_1 = 0.01, \sigma_2 = 1$), $\beta = 0.25$, by calculations we have $\sigma_2 > \sqrt{2\left(\frac{\mu\alpha}{\beta} - D\right)} \approx 0.9899$. From Theorem 3.2, we conclude that the predator population $y(t)$ goes extinct (see Figure 3(a)). We also find that the solution of stochastic model (1.2) fluctuates around the hyperbolic saddle $(K, 0)$ of deterministic model (1.1), which means that environmental noise causes the solution of a deterministic model (1.1) to transform from tending to a globally asymptotically stable equilibrium point (x_1, y_1) to fluctuating near another equilibrium point $(K, 0)$. In this case, predator population is extinct, and the prey fluctuates around the environmental capacity K.

Let environmental noise σ_1 continue to increase and take $\sigma_1 = 1.3$, $\sigma_2 = 1$. By calculations, we get $\sigma_1 > \sqrt{2r} = 1$. Theorem 3.1 and Remark 3.1 show that both populations are extinct, see Figure 3(b). That is to say the environmental noise σ_1 is unfavorable for the persistence of prey and predator population.

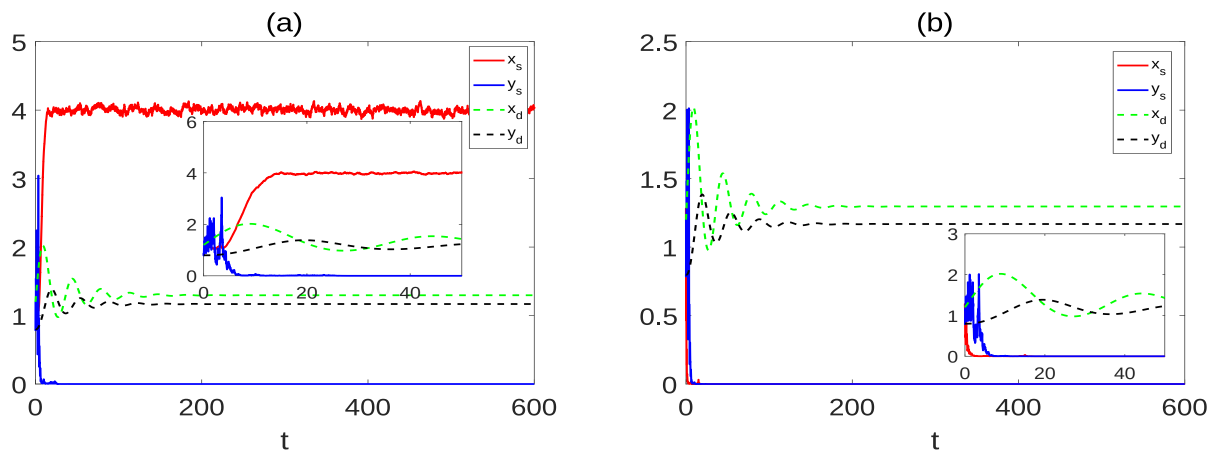

Figure 3. Time series of the population in model (1.2) under different environmental noise, $\beta = 0.25$. (a) $\sigma_1 = 0.01$, $\sigma_2 = 1$; (b) $\sigma_1 = 1.3$, $\sigma_2 = 1$.

In fact, the conditions in Theorem 3.1, Theorem 3.2 and 3.3 are only sufficient conditions. When the condition is not satisfied, the extinction or persistence of the population cannot be determined. Take Theorem 3.1 as an example, when $\sigma_1 = 1$, $\sigma_2 = 0.01$, although the condition $\sigma_1 > \sqrt{2r}$ in Theorem 3.1 cannot hold, the prey population is still extinct, see Figure 4.

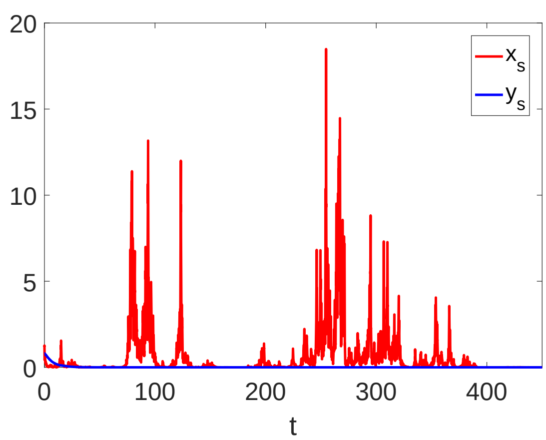

Figure 4. Time series of the population in model (1.2) under $\sigma_1 = 1$, $\sigma_2 = 0.01$, $\beta = 0.25$.

Take $r = 0.5$, $\mu = 0.4$, $\alpha = 0.4$, $K = 6$, $D = 0.15$, $\beta = 0.25$. Since $e\beta D < \mu\alpha$ and $x_1 + 1/\beta < K < x_2$, by the results in [18] and numerical simulation, we know deterministic model (1.1) has a stable limit cycle in the interior of the first quadrant. Now we investigate how σ_1 or σ_2 affects the dynamics of deterministic model (1.1) in this case, see Figure 5(a)–(d). When σ_1 and σ_2 are small ($\sigma_1 = \sigma_2 = 0.01$ and $\sigma_1 = \sigma_2 = 0.05$), the solutions of model (1.2) still fluctuate around the stable limit cycle of model (1.1), see Figure 5(a),(b); As σ_1 and σ_2 increase gradually, environmental noises make the dynamics of deterministic model(1.1) very complex, see Figure 5(c) (($\sigma_1 = \sigma_2 = 0.1$)); If σ_1 and σ_2 continue to increase, according to Theorem 3.1 and Theorem 3.2, the population may become extinct. For example, take $\sigma_1 = 0.05$, $\sigma_2 = 0.65$, the solutions of model (1.2) tends to the boundary equilibrium point $(K, 0)$ of model (1.1) and predator population will be extinct, see Figure 5(d).

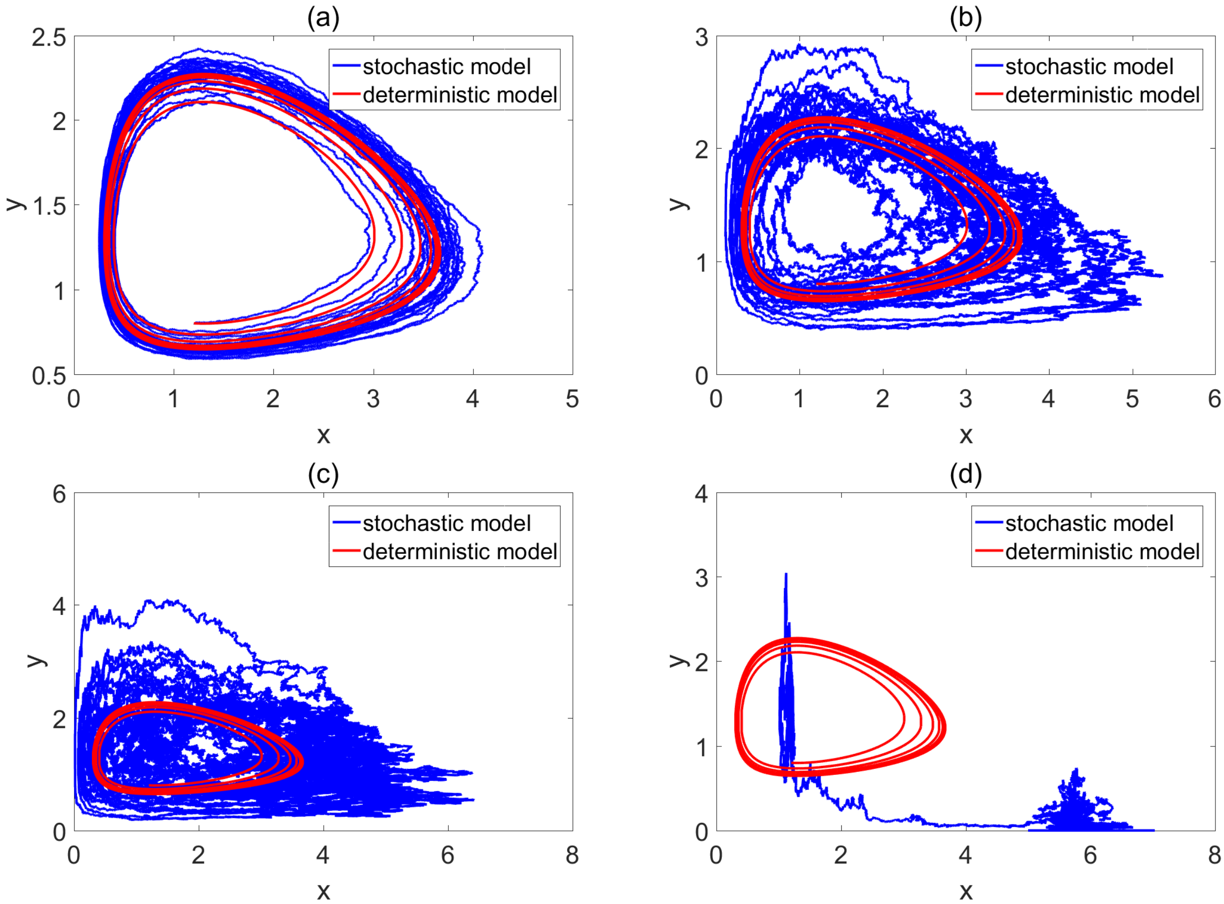

Figure 5. Phase portraits of model (1.2). (a) $\sigma_1 = \sigma_2 = 0.01$; (b) $\sigma_1 = \sigma_2 = 0.05$; (c) $\sigma_1 = \sigma_2 = 0.1$; (d) $\sigma_1 = 0.05$, $\sigma_2 = 0.65$.

In summary, the environmental noise makes the dynamics of the deterministic system more complex, and the greater the intensity of the environmental noise, the less conducive to the survival of the population.

5. Sensitivity analysis on threshold R

In order to identify the key factors affecting populations stability, a sensitivity analysis is performed on threshold R.

Sensitivity analysis is a method of quantitative uncertainty analysis for any complex model. Here we analyze the sensitivity of the parameter to R by calculating the Partial Rank Correlation Coefficient (PRCC) value of the parameter, and determine the key parameters that affect the threshold. When the $PRCC$ value of the parameter is positive, the parameter is positively correlated with R. Conversely, the parameter is negatively correlated with R. We make the following provisions: if $|PRCC| > 0.4$, then there is a strong correlation between the input parameter and the output variable, i.e., the parameter has a strong influence on R; if $0.2 < |PRCC| < 0.4$, this parameter has a moderate influence on R. If $|PRCC| < 0.2$, then this parameter has a weak influence on R. For more details, see [41].

Take $\beta = 0.25$, $\sigma_1 = \sigma_2 = 0.01$, the sample size is 1500 and all parameters vary simultaneously. From Figure 6, the $PRCC$ value of r, β, K, D are positive, it shows that parameter r, β, K, D are positively correlated with the threshold R. The increase of these parameters increases R, which is conductive to the persistence of populations. And the $PRCC$ value of μ, α , σ_1, σ_2 are negative, so μ, α, σ_1, σ_2 are negatively correlated with the threshold R. The increase of these parameters decreases R, which is harmful to the populations. Among them, $|PRCC|_{r,\sigma_1} > 0.4$, so r, σ_1 have a strong correlation with R; $0.2 < |PRCC|_{\mu,\alpha,D} < 0.4$, so μ, α and D have a moderate influence on R; $|PRCC|_{\beta,K,\sigma_2} < 0.2$, so β, K and σ_2 have a weak influence on R.

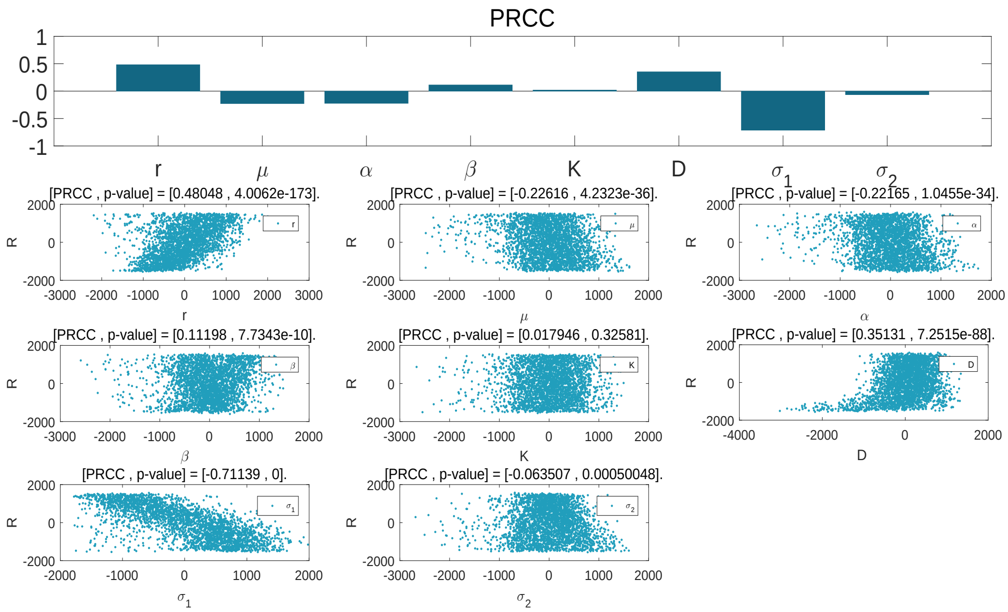

Figure 6. $PRCC$ values of main parameters on threshold R and scatter plot of $PRCC$ on parameters r, μ, α, K, β, D, σ_1 and σ_2.

6. Conclusions

In this paper, a stochastic predator-prey model with group defense behavior is established. The dynamics of stochastic model (1.2) are studied through theoretical analysis and numerical simulations.

According to Theorem 3.1 and 3.2, when the environmental noise has a great impact on prey population, both populations will be extinct; when the environmental noise has a great impact on predator population, the predator population will be extinct, which will destroy the stability between two populations. By numerical simulations, we show that the environmental noise makes the dynamics of the deterministic system more complex.

Under the influence of environmental noise, the appropriate group defense level of prey can help the survival of two populations, and maintain the stability of the relationship between two populations(see Figure 7(a), $\beta = 0.25$). From Theorem 2.2 in [18], if $e\beta D < \mu\alpha$ and $K > \frac{1}{\beta} + x_2$, then model (1.1) has a hyperbolic stable node $(K, 0)$. So the larger group defense level is not conducive to the persistence of predator population, resulting in inundation of prey population (see Figure 7(b), $\beta = 0.39$). The threshold R, which plays a crucial role in maintaining stability of the relationship between two populations, is strongly influenced by the intrinsic growth rate of prey population r and the intensity of environmental noise σ_1.

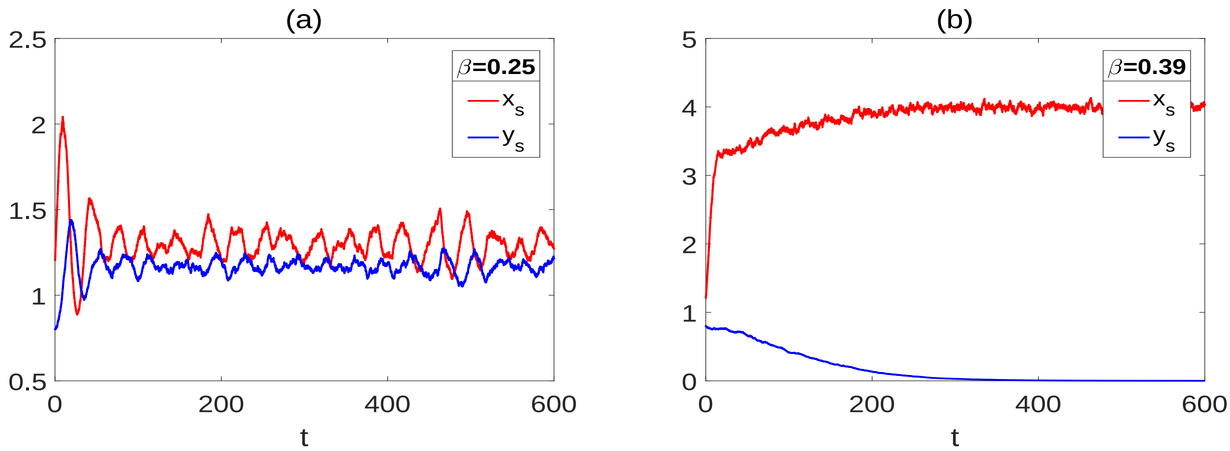

Figure 7. Time series of model (1.2) under different prey population group defense level. (a) $\beta = 0.25$; (b) $\beta = 0.39$.

In this paper, we only consider the influence of group defense behavior and environmental noise on the dynamics of stochastic predator-prey model. However, there are other more complex influences in nature. For example, in the study of pest management with group defense behavior, the effects of releasing natural enemies, impulsive spraying pesticide and environmental noise on agricultural production should be considered simultaneously. We will enrich it in future work.

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Conflict of interest

No potential conflict of interest was reported by the authors.

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