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Research article

Prespecified-time bipartite synchronization of coupled reaction-diffusion memristive neural networks with competitive interactions

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Abstract: In this paper, we investigate the prespecified-time bipartite synchronization (PTBS) of coupled reaction-diffusion memristive neural networks (CRDMNNs) with both competitive and cooperative interactions. Two types of bipartite synchronization are considered: leaderless PTBS and leader-following PTBS. With the help of a structural balance condition, the criteria for PTBS for CRDMNNs are derived by designing suitable Lyapunov functionals and novel control protocols. Different from the traditional finite-time or fixed-time synchronization, the settling time obtained in this paper is independent of control gains and initial values, which can be pre-set according to the task requirements. Lastly, numerical simulations are given to verify the obtained results.

Keywords: memristor; reaction-diffusion term; neural networks; competitive interactions; bipartite synchronization; prespecified-time synchronization

1. Introduction

The memristor is the fourth basic circuit element, and it was firstly proposed by Chua in 1971 [1]. The first memristor device was realized in 2008 [2], and since then it has received much attention from research area, such as neural networks, signal processing and artificial intelligence. As a particular type of neural networks, memristive neural networks (MNNs) have become an interesting topic for researchers [3–6]. MNNs can be regarded as a switch system since the memristive connection weights switch according to the system states. Compared with the traditional network model, MNNs have more equilibrium points and can better modify the dynamics of neurons in human brains. Lately, the dynamical analysis of MNNs has attracted great attention [7–9].

As we know, reaction-diffusion influences are important and common in chemistry, biology and neuroscience. Since the electrons in circuits move in a nonuniform electric field, the state of the electrons dose not only depend on time but also on the spatial information. Due to this reason, MNNs with reaction-diffusion terms can better modify the real-world systems than the traditional MNNs. Recently, the research on dynamics of reaction-diffusion MNNs has made great progress, such as stability [10], passivity [11, 12], and asymptotical synchronization [13, 14]. However, most of these results only focus on the cooperative interactions between neurons, while the competitive relationships have not been considered yet.

In the real world, competition and cooperation relationships coexist in many dynamic systems. For instance, in biological systems, interactions between cells may be cooperative or competitive; in social networks, relationships between people may be hostile or friendly. Hence, it is meaningful and necessary to consider the model of coopetition networks, in which the competition and cooperation relationships coexist [15–21]. Due to the existence of competition relationships, the traditional synchronization cannot be realized, and instead we will consider bipartite synchronization. In Reference [15], the idea of bipartite consensus in coopetition networks was first proposed, it requires that all nodes' states will reach the consensus in modulus but may not in sign. Recently, bipartite synchronization of coopetition networks has attracted the attention of many scholars [18–21]. In Reference [19], by designing an adaptive control strategy, the bipartite synchronization of coupled coopetition MNNs is investigated. In Reference [20], the model of inertial MNNs with antagonistic interactions is considered, and the criteria for bipartite synchronization are derived in both the leaderless and leader-following case. However, the above works only focus on the asymptotic or exponential synchronization, and how to improve the convergence speed in bipartite synchronization still remains an open topic.

Synchronization is an important phenomenon in engineering and biological systems. As we know, the convergence speed is an important issue in synchronization. To overcome the constraints of long convergence time, the issue of finite-time synchronization (FTS) is proposed [24]. Different from asymptotic synchronization [22, 23], FTS guarantees the system will reach synchronization in a finite time. However, the convergence time of FTS is related to the initial conditions [24–27], which may be unmeasurable in practice. To solve this problem, the idea of fixed-time synchronization (FxT) is introduced [28]. Although the FxTS control removes the limitation of initial dependence, its settling time is related to the control parameters [28–31]. Hence, the settling time of traditional FTS and FxTS control still depend on the initial conditions and system parameters, which is a great limitation for some practical applications. To solve this problem, prespecified-time synchronization (PTS) control is proposed [32–35], in which the settling time is independent of both initial value and control parameters. As we know, the investigation of prespecified-time control is still very limited, especially for the coupled networks with competitive interactions.

This paper is intended to explore the prespecified-time bipartite synchronization (PTBS) of CRDMNNs with both cooperative and competitive relationships. The main contributions of this work are as follows:

1) Different from most previous results that only focus on cooperative interactions, this paper considers the model of CRDMNNs with both cooperative and competitive relationships between neurons.

2) By designing a novel control protocol, criteria for leader-following PTBS and leaderless PTBS of CRDMNNs are derived. It is shown that under the proposed protocol, all nodes' states will reach synchronization in modulus (but might not in sign) within a prescribed time.

3) Different from the traditional FTS and FxTS methods, the prespecified-time control method in this work can lead to a predetermined settling time, which is independent of both initial values and system parameters.

Notations. In this paper, $sgn(\cdot)$ denotes the signum function. The symbol T denotes vector transposition. C^1 denotes the set of continuous functions. $\lambda_{\max}(A)$ is the maximal eigenvalue of matrix A. $\otimes$ denotes the Kronecker product. $\Omega = \{x = (x_1, \cdots, x_\vartheta)^T | h_k^m \leq |x_k| \leq h_k^M, k = 1, \cdots, \vartheta\}$ is a bounded compact set with smooth boundary $\partial\Omega$.

2. Preliminaries and model formulation

A signed graph is represented by $\mathcal{G} = \{\mathbb{P}, \mathbb{J}, \mathbb{G}\}$, where $\mathbb{P} = \{p_1, p_2, \cdots, p_N\}$ is the node set, $\mathbb{J} \subset \mathbb{P} \times \mathbb{P} = \{(p_i, p_j) | p_i, p_j \in \mathbb{P}\}$ denotes the edge set, $\mathbb{G} = [G_{ij}] \in \mathbb{R}^{N \times N}$ is the adjacent matrix satisfying that $G_{ij} \neq 0$ if $(p_i, p_j) \in \mathbb{J}$, and $G_{ij} = 0$ otherwise. It is assumed that $G_{ii} = 0$. If the relationship between nodes p_i and p_j is competitive (cooperative), then $G_{ij} < 0$ ($G_{ij} > 0$). Due to the existence of negative edges, the elements in the Laplacian matrix L of a signed graph are given as

$$l_{ik} = \begin{cases} \sum_{j \in \mathcal{N}_i} |G_{ij}|, & k = i, \\ G_{ik}, & k \neq i. \end{cases}$$

Definition 1. *For a signed graph, it is called structurally balanced if the node set $\mathbb{P}$ can be divided into two subgroups $\mathbb{P}_1$ and $\mathbb{P}_2$, where $\mathbb{P}_1 \cup \mathbb{P}_2 = \mathbb{P}$ and $\mathbb{P}_1 \cap \mathbb{P}_2 = \emptyset$, such that $G_{ij} \geq 0$ for $\forall p_i, p_j \in \mathbb{P}_l (l \in \{1, 2\})$ and $G_{ij} \leq 0$ for $\forall p_i \in \mathbb{P}_l, p_j \in \mathbb{P}_k (k \neq l, k, l \in \{1, 2\})$ [15].*

Lemma 1. *Suppose L is the Laplacian matrix of a signed graph $\mathcal{G}$. If $\mathcal{G}$ is structurally balanced, then there is a gauge transformation matrix $S = \text{diag}(s_1, \cdots, s_N)$ with $s_i \in \{-1, 1\}$, such that SLS has all nonnegative entries [15].*

The model of CRDMNNs with competitive interactions is:

$$\begin{aligned} \frac{\partial z_p(t,x)}{\partial t} =& D\Delta z_p(t,x) - Cz_p(t,x) + A(z_p(t,x)) f(z_p(t,x)) \\ &+ \sigma \sum_{q \in \mathcal{N}_p} |G_{pq}| \Gamma\Big(sgn(G_{pq}) z_q(t,x) - z_p(t,x)\Big) + u_p(t,x), \ p = 1, \cdots, n \end{aligned} \tag{2.1}$$

where $z_p(t,x) = (z_{p1}(t,x), \cdots, z_{pn}(t,x))^T \in \mathbb{R}^n$ denotes the state of the pth neural network at time t and space $x \in \Omega \subseteq \mathbb{R}^z$. $f(z_p(t,x)) = (f_1(z_{p1}(t,x)), \cdots, f_n(z_{pn}(t,x)))^T \in \mathbb{R}^n$ denotes the activation function. $\Delta = \sum_{k=1}^{\vartheta} (\frac{\partial^2}{\partial x_k^2})$ is the Laplace diffusion operator. $D = \text{diag}(d_1, \cdots, d_n) > 0$. $C = \text{diag}(c_1, c_2, \cdots, c_n) > 0$. $A(z_p(t,x)) = [a_{kj}(z_{pj}(t,x))]_{n \times n}$ stands for the memristive connection weight matrix. σ is a positive number denoting the coupling strength, and $\mathcal{N}_p$ is the neighbor set of node p. $\Gamma = \text{diag}(\gamma_1, \gamma_2, \cdots, \gamma_n) \in \mathbb{R}^{n \times n}$ is the inner coupling matrix. $u_p(t,x)$ is the control input to be designed. $G = (G_{pq})_{N \times N}$ is the symmetric weighted adjacency matrix of the signed graph, representing the competitive-cooperation relationship between neurons. The memristive connection weights $a_{kj}(z_{pj}(t,x))$ can be described as

$$a_{kj}(z_{pj}(t,x)) = \begin{cases} \acute{a}_{kj}, |z_{pj}(t,x)| \leq T_j, \\ \grave{a}_{kj}, |z_{pj}(t,x)| > T_j, \end{cases} \tag{2.2}$$

where $T_j > 0$ is the switching jump, and $\acute{a}_{kj}, \grave{a}_{kj}$ are real numbers. Define

$$\underline{a}_{kj} = \min\{\acute{a}_{kj}, \grave{a}_{kj}\},\ \bar{a}_{kj} = \max\{\acute{a}_{kj}, \grave{a}_{kj}\},\ a^{+}_{kj} = \max\{|\acute{a}_{kj}|, |\grave{a}_{kj}|\}.$$

The initial condition and Dirichlet boundary condition of $z_p(t,x)$ are

$$\begin{cases} z_p(t_0, x) = \psi_p(x), x \in \Omega, \\ z_p(t, x) = 0, (t, x) \in [t_0, +\infty) \times \partial\Omega. \end{cases} \tag{2.3}$$

Remark 1. *In this article, the reaction-diffusion term is contained in the MNNs model, and it is closer to the real networks because spatial location is considered. So, our results are more general than previous results in [19, 20].*

Next, we make the following assumptions.

Assumption 1. *The cooperative-competitive network $\mathcal{G}$ is structurally balanced and connected.*

Assumption 2. *The activation function $f_p(\cdot)$ is bounded, and there exist constants σ_p and Υ such that*

$$|f_p(x) - s_q f_p(y)| \leq \sigma_p |x - s_q y|, \forall x, y \in \mathbb{R},$$
$$|f_p(x)| \leq \Upsilon, \forall x \in \mathbb{R}.$$

In addition, define $F = \mathrm{diag}(\sigma_1, \cdots, \sigma_n) \in \mathbb{R}^{n\times n}$.

Lemma 2. *If $\eta(x) = \eta(x_1, \cdots, x_\vartheta) : \Omega \to \mathbb{R}$ satisfies that $\eta(x) \in C^1(\Omega)$ and $\eta(x)|_{\partial\Omega} = 0$, then [12]*

$$\int_\Omega \sum_{k=1}^{\vartheta} \Big(\eta(x)\frac{\partial^2 \eta(x)}{\partial x_k^2}\Big)dx \leq -\sum_{k=1}^{\vartheta} \frac{\pi^2}{(h_k^M - h_k^m)^2} \int_\Omega \eta^2(x)dx.$$

Based on the feature of Laplacian matrix L of the signed graph, we have:

$$\begin{aligned} &\sum_{q\in\mathcal{N}_p} |G_{pq}|\Gamma\Big(sgn(G_{pq})z_q(t,x) - z_p(t,x)\Big) \\ =&\sum_{q\in\mathcal{N}_p} G_{pq}\Gamma z_q(t,x) - \sum_{q\in\mathcal{N}_p} |G_{pq}|\Gamma z_q(t,x) = \sum_{q=1}^{N} l_{pq}\Gamma z_q(t,x). \end{aligned} \tag{2.4}$$

Hence, CRDMNN (2.1) can be written as

$$\begin{aligned} \frac{\partial z_p(t,x)}{\partial t} =& D\Delta z_p(t,x) - Cz_p(t,x) + A(z_p(t,x))f(z_p(t,x)) \\ &+ \sigma \sum_{q=1}^{N} l_{pq}\Gamma z_q(t,x) + u_p(t,x). \end{aligned} \tag{2.5}$$

Definition 2. *The CRDMNNs System (2.5) is said to reach leaderless PTBS, if there exists a time T, which is a prespecified constant for any initial values, such that*

$$\lim_{t\to T^-} \|s_p z_p(t,x) - s_q z_q(t,x)\| = 0,\ p, q = 1, \cdots, N,$$
$$\|s_p z_p(t,x) - s_q z_q(t,x)\| \equiv 0,\ \forall t > T.$$

Remark 2. *In this work, we use $z_q(t,x) - z_p(t,x)$ to represent the cooperation relationship between nodes p and q, and we use $z_q(t,x) + z_p(t,x)$ to depict the competitive relationship between nodes p and q.*

The System (2.5) can be rewritten as the following matrix form:

$$\begin{aligned}\frac{\partial z(t,x)}{\partial t} =& \check{D}\Delta z(t,x) - \check{C}z(t,x) + \check{A}(z(t,x))f(z(t,x)) \\ &+ \sigma(L\otimes\Gamma)z(t,x) + u(t,x).\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}&z(t,x) = (z_1^T(t,x),\cdots,z_N^T(t,x))^T, \check{D} = I_N\otimes D,\\ &u(t,x) = (u_1^T(t,x),\cdots,u_N^T(t,x))^T,\ \check{C} = I_N\otimes C,\\ &\check{A}(z(t,x)) = \mathrm{diag}(A(z_1(t,x)),\cdots,A(z_N(t,x))),\\ &f(z(t,x)) = (f^T(z_1(t,x)),\cdots,f^T(z_N(t,x)))^T\end{aligned}$$

To achieve the main results, two classes of matrices are given. Define

$$M = \begin{bmatrix} s_1 & -s_2 & 0 & 0 & \cdots & 0 & 0\\ 0 & s_2 & -s_3 & 0 & \cdots & 0 & 0\\ 0 & 0 & s_3 & -s_4 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \ldots & s_{N-1} & -s_N\end{bmatrix}_{(N-1)\times N}$$

$$J = \begin{bmatrix} s_1 & s_1 & s_1 & \cdots & s_1 & s_1\\ 0 & s_2 & s_2 & \cdots & s_2 & s_2\\ 0 & 0 & s_3 & s_3 & \cdots & s_3\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \ldots & 0 & s_{N-1}\\ 0 & 0 & 0 & \ldots & 0 & 0\end{bmatrix}_{N\times(N-1)}$$

Definition 3. *For the CRDMNN System (2.6), define:*

$$d(z) = \|\mathbf{M}z(t,x)\|^2 = \int_\Omega z^T(t,x)\mathbf{M}^T\mathbf{M}z(t,x)dx,$$

where $\mathbf{M} = M\otimes I_n$. Then, the networks' states can reach leaderless PTBS, if there exists a time T, which is a prescribed constant for any initial values, such that

$$\lim_{t\to T^-} d(z) = 0,\ d(z)\equiv 0, \forall t > T.$$

Lemma 3. *For the Laplacian matrix $L\in\mathbb{R}^{N\times N}$, there is a matrix $H\in\mathbb{R}^{(N-1)\times(N-1)}$ given by $H = MLJ$, so that the following condition is satisfied:*

$$HM = ML,\ H_{ij} = \sum_{k=1}^{j} s_i s_k l_{ik} - s_{i+1}s_k l_{i+1,k}$$

For convenience, define

$$\bar{\mathbf{A}} = I_N \otimes \bar{A},\ \bar{\mathbf{A}}_1 = I_{N-1} \otimes \bar{A},\ \mathbf{H} = H \otimes \Gamma,$$
$$\hat{C}_1 = I_{N-1} \otimes C, \hat{D}_1 = I_{N-1} \otimes D,\ \mathbf{F} = I_{N-1} \otimes F,$$
$$\mathbf{D}^* = \sum_{k=1}^{\vartheta} \frac{\pi^2}{(h_k^M - h_k^m)^2} \hat{D}_1$$

3. Main results

To achieve leaderless PTBS, the following controller for System (2.5) is introduced:

$$\begin{aligned} u_p(t,x) =& - K s_p \sum_{j=p}^{N-1} sgn(s_j z_j(t,x) - s_{j+1} z_{j+1}(t,x)) \\ & - \xi_p z_p(t,x) - \psi \frac{r}{T} \mu(t) z_p(t,x) \end{aligned} \tag{3.1}$$

where $K = \mathrm{diag}(k_1, k_2, \cdots, k_n) > 0$, $\xi_p > 0$ and $\psi > 0$ are control gains to be designed. r is a positive constant, and $T > 0$ is a prespecified time chosen by users. Let $t_b = t_0 + T$, and $\mu(t)$ is a function defined as:

$$\mu(t) = \begin{cases} \frac{T}{t_b - t}, & t \in [t_0, t_b), \\ 1, & t \in [t_b, +\infty). \end{cases} \tag{3.2}$$

The derivative of $\mu(t)$ can be calculated:

$$\dot{\mu}(t) = \begin{cases} \frac{1}{T}\mu^2(t), & t \in [t_0, t_b), \\ 0, & t \in [t_b, +\infty). \end{cases} \tag{3.3}$$

With the control protocol (3.1), the theorem can be achieved.

Theorem 1. *Under Assumptions 1 and 2, if the control gains constants k_j, ξ_p satisfy the following conditions*

$$k_j \geq 2 \sum_{s=1}^{n} (\bar{a}_{js} - \underline{a}_{js}) \Upsilon, \quad j = 1, 2, \cdots, n,$$
$$- \tilde{\xi} + \lambda_{\max}(-\mathbf{D}^* - \hat{C}_1 + \bar{\mathbf{A}}_1 \bar{\mathbf{A}}_1^T + \mathbf{F}^2 + \sigma \mathbf{H}) \leq -\frac{\phi}{2}$$

where $\tilde{\xi} = \min_p\{\xi_p\}$, and $\phi > 0$ is a positive constant, then the leaderless bipartite prespecified-time synchronization for CRDMNN (2.6) can be achieved with controller (3.1).

Proof. Before proceeding, let $y(t,x) = \mathbf{M}z(t,x)$, and then we get $y_i(t,x) = s_i z_i(t,x) - s_{i+1} z_{i+1}(t,x), i = 1, \cdots, N-1$. Consider the Lyapunov functional

$$V(t) = \frac{1}{2} \int_{\Omega} z^T(t,x) \mathbf{M}^T \mathbf{M} z(t,x) dx. \tag{3.4}$$

Computing the derivative along (2.6), it follows that

$$\begin{aligned}\frac{dV(t)}{dt} &= \int_{\Omega} z^T(t,x)\mathbf{M}^T\mathbf{M}\dot{z}(t,x)dx \\ &= \int_{\Omega} z^T(t,x)\mathbf{M}^T\mathbf{M}\Big(\check{D}\Delta z(t,x) - \check{C}z(t,x) \\ &\quad + \check{A}(z(t,x))f(z(t,x)) + \sigma(L\otimes\Gamma)z(t,x) + u(t,x)\Big)dx. \end{aligned} \tag{3.5}$$

From Lemma 2, we get

$$\begin{aligned}&\int_{\Omega} z^T(t,x)\mathbf{M}^T\mathbf{M}\check{D}\Delta z(t,x)dx \\ =&\int_{\Omega} z^T(t,x)\mathbf{M}^T\hat{D}_1\mathbf{M}\Delta z(t,x)dx \\ =&\int_{\Omega}\sum_{i=1}^{N-1} y_i^T(t,x)D\Delta y_i(t,x)dx \\ \le& -\sum_{k=1}^{\vartheta}\frac{\pi^2}{(h_k^M - h_k^m)^2}\int_{\Omega}\sum_{i=1}^{N-1} y_i^T(t,x)Dy_i(t,x)dx \\ =& -\sum_{k=1}^{\vartheta}\frac{\pi^2}{(h_k^M - h_k^m)^2}\int_{\Omega} z^T(t,x)\mathbf{M}^T\hat{D}_1\mathbf{M}z(t,x)dx \\ =&\int_{\Omega} z^T(t,x)\mathbf{M}^T\mathbf{D}^*\mathbf{M}z(t,x)dx. \end{aligned} \tag{3.6}$$

Note that

$$\begin{aligned}&\mathbf{M}\hat{A}(z(t,x))f(z(t,x)) \\ =&\mathbf{M}\bar{\mathbf{A}}f(z(t,x)) + \mathbf{M}(\check{A}(z(t,x)) - \bar{\mathbf{A}})f(z(t,x)). \end{aligned} \tag{3.7}$$

Based on Assumption 2, we have

$$\begin{aligned}&z^T(t,x)\mathbf{M}^T\mathbf{M}\Big(\check{A}(z(t,x)) - \bar{\mathbf{A}}\Big)f(z(t,x)) \\ =&\sum_{i=1}^{N-1} y_i^T(t,x)\Big(s_i(A(z_i(t,x)) - \bar{A})f(z_i(t,x)) \\ &- s_{i+1}(A(z_{i+1}(t,x)) - \bar{A})f(z_{i+1}(t,x))\Big) \\ =&\sum_{i=1}^{N-1}\sum_{j=1}^{n} y_{ij}(t,x)\sum_{s=1}^{n}[(a_{js}(z_{is}) - \bar{a}_{js})s_i f_s(z_{is}(t,x)) \\ &- (a_{js}(z_{i+1,s}) - \bar{a}_{js})s_{i+1}f_s(z_{i+1,s}(t,x))] \\ \le&\sum_{i=1}^{N-1}\sum_{j=1}^{n} 2\sum_{s=1}^{n}(\bar{a}_{js} - \underline{a}_{js})\Upsilon|y_{ij}(t,x)|. \end{aligned} \tag{3.8}$$

Similarly, we get

$$
\begin{aligned}
& z^T(t,x)\mathbf{M}^T\mathbf{M}\bar{\mathbf{A}}f(z(t,x)) \\
=& z^T(t,x)\mathbf{M}^T\bar{\mathbf{A}}_1\mathbf{M}f(z(t,x)) \\
=& \sum_{i=1}^{N-1}(s_i z_i(t,x) - s_{i+1}z_{i+1}(t,x))^T\bar{A}(s_i f(z_i(t,x)) - s_{i+1}f(z_{i+1}(t,x))) \\
\leq& \sum_{i=1}^{N-1} y_i^T(t,x)\bar{A}\bar{A}^T y_i(t,x) + \sum_{i=1}^{N-1} y_i^T(t,x)F^2 y_i(t,x) \\
=& z^T(t,x)\mathbf{M}^T\bar{\mathbf{A}}_1\bar{\mathbf{A}}_1^T\mathbf{M}z(t,x) + z^T(t,x)\mathbf{M}^T\mathbf{F}^2\mathbf{M}z(t,x).
\end{aligned} \tag{3.9}
$$

By Lemma 3, we get

$$
\begin{aligned}
& z^T(t,x)\mathbf{M}^T\mathbf{M}(L\otimes\Gamma)z(t,x) \\
=& z^T(t,x)\mathbf{M}^T(M\otimes I_n)(L\otimes\Gamma)z(t,x) \\
=& z^T(t,x)\mathbf{M}^T(ML\otimes\Gamma)z(t,x) \\
=& z^T(t,x)\mathbf{M}^T(HM\otimes\Gamma)z(t,x) \\
=& z^T(t,x)\mathbf{M}^T(H\otimes\Gamma)(M\otimes I_n)z(t,x) \\
=& z^T(t,x)\mathbf{M}^T\mathbf{H}\mathbf{M}z(t,x),
\end{aligned} \tag{3.10}
$$

and

$$
\begin{aligned}
& z^T(t,x)\mathbf{M}^T\mathbf{M}u(t,x) \\
=& -\sum_{i=1}^{N-1}\sum_{j=1}^{n} k_j y_{ij}(t,x)sgn(y_{ij}(t,x)) - \tilde{\xi}z^T(t,x)\mathbf{M}^T\mathbf{M}z(t,x) \\
& -\psi\frac{r}{T}\mu(t)z^T(t,x)\mathbf{M}^T\mathbf{M}z(t,x) \\
=& -\sum_{i=1}^{N-1}\sum_{j=1}^{n} k_j|y_{ij}(t,x)| - (\tilde{\xi} + \psi\frac{r}{T}\mu(t))z^T(t,x)\mathbf{M}^T\mathbf{M}z(t,x).
\end{aligned} \tag{3.11}
$$

From the condition in Theorem 1, we derive that

$$
\begin{aligned}
\dot{V}(t) \leq & \int_\Omega z^T(t,x)\mathbf{M}^T(-\tilde{\xi} - \mathbf{D}^* - \hat{C}_1 + \bar{\mathbf{A}}_1\bar{\mathbf{A}}_1^T + \mathbf{F}^2 \\
& + \sigma\mathbf{H})\mathbf{M}z(t,x)dx - 2\psi\frac{r}{T}\mu(t)V(t) \\
\leq & -\phi V(t) - 2\psi\frac{r}{T}\mu(t)V(t).
\end{aligned} \tag{3.12}
$$

According to the definition of $\mu(t)$, defining $\Omega(t) = \mu^r(t)$ yields

$$
\frac{r}{T}\mu(t) = \frac{\dot{\Omega}(t)}{\Omega(t)},\ t\in[t_0,t_b). \tag{3.13}
$$

Multiplying $\Omega^{2\psi}(t)$ on both sides of (3.12) yields

$$\frac{d[\Omega^{2\psi}(t)V(t)]}{dt} \leq -\phi[\Omega^{2\psi}(t)V(t)]. \tag{3.14}$$

Solving the above inequality yields

$$\Omega^{2\psi}(t)V(t) \leq e^{-\phi(t-t_0)}\Omega^{2\psi}(t_0)V(t_0). \tag{3.15}$$

Then, we have

$$V(t) \leq e^{-\phi(t-t_0)}\mu(t)^{-2r\psi}\mu(t_0)^{2r\psi}V(t_0). \tag{3.16}$$

Note that

$$\mu(t_0)^{2r\psi} = 1, \lim_{t\to t_b^-}\mu(t)^{-2r\psi} = 0. \tag{3.17}$$

which yields

$$\lim_{t\to t_b^-} V(t) = 0. \tag{3.18}$$

Hence, $\lim_{t\to t_b^-} \|y(t,x)\| = 0$, and the systems can reach synchronization in prespecified time T.
Next, we prove that the synchronization can be maintained on $[t_b, +\infty)$.

$$\dot{V}(t) \leq -(\phi + 2\psi\frac{r}{T})V(t) = -\hat{\theta}V(t),\ t \in [t_b, +\infty). \tag{3.19}$$

where $\hat{\theta} = \phi + 2\psi\frac{r}{T} > 0$.
Then, we can get

$$0 \leq V(t) \leq V(t_b) = 0,\ t \in [t_b, +\infty). \tag{3.20}$$

So, we obtain that $V(t) \equiv 0$ over $[t_b, +\infty)$. Hence, $\|y(t,x)\| \equiv 0$ on $[t_b, +\infty)$. Based on Definition 3, leaderless PTBS for Systems (2.6) can be realized with controller (3.1). The proof is complete.

Remark 3. *Different from [33–35], the competitive relationship between network nodes is considered in this paper, which is described by negative edges in the signed graph. It can be seen that the prescribed-time synchronization in the above papers is a special case of the bipartite prescribed-time synchronization in this work. On the other hand, compared with the asymptotic bipartite synchronization criteria in [19,20], our approach ensures that the CRDMNNs can reach bipartite synchronization within prespecified time t_b, which is independent of initial values and control parameters.*

Remark 4. *Since there are switching parameters in the memristor system, the first term of controller (3.1) is designed to remove the influence of the memristor parameters. Then, the terms $-\xi_p z_p(t,x)$ and $-\psi\frac{r}{T}\mu(t)z_p(t,x)$ are devoted to realizing the convergence in a prespecified time. The function $\mu(t)$ plays an important role to ensure the prespecified-time synchronization. It can be seen from (3.17) that $\mu(t_0)^{2r\psi} = 1, \lim_{t\to t_b^-}\mu(t)^{-2r\psi} = 0$. In practical applications, there may exist a leader in the community. In this case, leader-following synchronization is a meaningful issue to be considered.*

Next, we aim to investigate leader-following PTBS of CRDMNNs. Considering the following system with coupling delays:

$$\begin{aligned}\frac{\partial z_p(t,x)}{\partial t} =&D\Delta z_p(t,x) - Cz_p(t,x) + A(z_p(t,x))f(z_p(t,x)) \\ &+ \sigma\sum_{q\in\mathcal{N}_p}|G_{pq}|\Gamma\Big(sgn(G_{pq})z_q(t-\tau,x) - z_p(t-\tau,x)\Big) \\ &+ u_p(t,x). \end{aligned} \tag{3.21}$$

The reference target system is:

$$\frac{\partial z_0(t,x)}{\partial t} =D\Delta z_0(t,x) - Cz_0(t,x) + A(z_0(t,x))f(z_0(t,x)). \tag{3.22}$$

where $z_0(t,x) = (z_{01}(t,x), z_{02}(t,x), \cdots, z_{0n}(t,x))^T \in \mathbb{R}^n$ is the leader node.

Definition 4. *For the CRDMNN System (3.21), it is said that the network can reach leader-following PTBS, if there exists a prespecified constant T, which is independent of any initial values or control parameters, such that*

$$\begin{aligned}&\lim_{t\to T^-}\|z_p(t,x) - s_p z_0(t,x)\| = 0, \\ &\|z_p(t,x) - s_p z_0(t,x)\| \equiv 0, \forall t > T,\end{aligned}$$

where $s_p = 1$ if $p \in \mathbb{P}_1$, and $s_p = -1$ if $p \in \mathbb{P}_2$.

Define synchronization error $\delta_p(t,x) = z_p(t,x) - s_p z_0(t,x)$, and since $\sum_{q=1}^{N} l_{pq}s_q = 0$, we have

$$\begin{aligned}&\sigma\sum_{q\in\mathcal{N}_p}|G_{pq}|\Gamma\Big(sgn(G_{pq})z_q(t-\tau,x) - z_p(t-\tau,x)\Big) \\ =&\sigma\sum_{q=1}^{N} l_{pq}\Gamma\Big(z_q(t-\tau,x) - s_q z_0(t-\tau,x) + s_q z_0(t-\tau,x)\Big) \\ =&\sigma\sum_{q\in\mathcal{N}_p}|G_{pq}|\Gamma\Big(sgn(G_{pq})\delta_q(t-\tau,x) - \delta_p(t-\tau,x)\Big) \end{aligned} \tag{3.23}$$

Then, the error system can be obtained:

$$\begin{aligned}\frac{\partial \delta_p(t,x)}{\partial t} =&D\Delta\delta_p(t,x) - C\delta_p(t,x) + A(z_p(t,x))f(z_p(t,x)) \\ &- s_p A(z_0(t,x))f(z_0(t,x)) \\ &+ \sigma\sum_{q=1}^{N} l_{pq}\Gamma\delta_q(t-\tau,x) + u_p(t,x). \end{aligned} \tag{3.24}$$

Define the norm

$$\|\delta_p(t,x)\|_2 = \Big(\int_\Omega \delta_p^T(t,x)\delta_p(t,x)dx\Big)^{\frac{1}{2}}$$

The prespecified-time controller $u_p(t,x)$ is designed as

$$u_p = \begin{cases} -Ksgn(\delta_p(t,x)) - \xi_p\delta_p(t,x) - 2\psi\dfrac{r}{T}\mu(t)\delta_p(t,x) \\ -\phi\dfrac{\delta_p(t,x)}{\|\delta_p(t,x)\|_2^2}\displaystyle\int_{t-\tau}^{t}\delta_p^T(\theta,x)\delta_p(\theta,x)d\theta \\ -2\psi\dfrac{r}{T}\mu(t)\dfrac{\delta_p(t,x)}{\|\delta_p(t,x)\|_2^2}\displaystyle\int_{t-\tau}^{t}\delta_p^T(\theta,x)\delta_p(\theta,x)d\theta, \\ \text{if } \delta_p(t,x)\neq 0, \\ 0, \quad \text{otherwise}, \end{cases} \tag{3.25}$$

where $K = \text{diag}(k_1, k_2, \cdots, k_n)$ with $k_i > 0$, $\xi_p, \phi, \psi > 0$ are the control gains. $sgn(\delta_p(t,x)) = (sgn(\delta_{p1}(t,x)), \cdots, sgn(\delta_{pn}(t,x)))^T \in \mathbb{R}^n$. T, r and $\mu(t)$ are defined the same as in controller (3.1).

Theorem 2. *Under Assumptions 1 and 2, suppose the following conditions hold:*

$$k_i > 2\sum_{s=1}^{n} a_{is}^{+}\Upsilon, \quad i = 1, 2, \cdots, n,$$

$$-\tilde{\xi} - c_{\min} - \sum_{k=1}^{\vartheta}\frac{\pi^2 d_{\min}}{(h_k^M - h_k^m)^2} + \sigma^2\lambda_{\max}[(L\otimes\Gamma)^2] \leq -\frac{\phi}{2}$$

where $\phi > 0$ *is given in controller (3.25),* $\tilde{\xi} = \min_p\{\xi_p\}$, $c_{\min} = \min\{c_1, c_2, \cdots, c_n\}$, *and* $d_{\min} = \min\{d_1, d_2, \cdots, d_n\}$. *Then, leader-following PTBS for Systems (3.21) and (3.22) can be achieved under controller (3.25).*

Proof. Design the Lyapunov functional

$$V(t) = \frac{1}{2}\sum_{p=1}^{N}\int_{\Omega}\delta_p^T(t,x)\delta_p(t,x)dx + \int_{\Omega}\int_{t-\tau}^{t}\delta^T(\theta,x)\delta(\theta,x)d\theta dx.$$

Computing the derivative along (3.24), we get

$$\begin{aligned}\frac{dV(t)}{dt} =& \sum_{p=1}^{N}\int_{\Omega}\delta_p^T(t,x)\dot{\delta}_p(t,x)dx + \int_{\Omega}\delta^T(t,x)\delta(t,x)dx \\ &- \int_{\Omega}\delta^T(t-\tau,x)\delta(t-\tau,x)dx \\ =& \sum_{p=1}^{N}\int_{\Omega}\delta_p^T(t,x)\Big(-C\delta_p(t,x) + D\Delta\delta_p(t,x) \\ &+ A(z_p(t,x))f(z_p(t,x)) - s_pA(z_0(t,x))f(z_0(t,x))\end{aligned}$$

$$
\begin{aligned}
&+\sigma\sum_{q=1}^{N} l_{pq}\Gamma\delta_q(t-\tau,x)+u_p(t,x)\Big)dx\\
&+\int_{\Omega}\delta^T(t,x)\delta(t,x)dx-\int_{\Omega}\delta^T(t-\tau,x)\delta(t-\tau,x)dx. \qquad (3.26)
\end{aligned}
$$

Based on Lemma 2, we get

$$
\begin{aligned}
&\int_{\Omega}\delta_p^T(t,x)D\Delta\delta_p(t,x)dx\\
\leq&-\sum_{k=1}^{\vartheta}\frac{\pi^2}{(h_k^M-h_k^m)^2}\int_{\Omega}\delta_p^T(t,x)D\delta_p(t,x)dx\\
\leq&-\sum_{k=1}^{\vartheta}\frac{\pi^2 d_{\min}}{(h_k^M-h_k^m)^2}\int_{\Omega}\delta_p^T(t,x)\delta_p(t,x)dx. \qquad (3.27)
\end{aligned}
$$

According to Assumption 2, we have

$$
\begin{aligned}
&\sum_{p=1}^{N}\delta_p^T(t,x)\Big(A(z_p)f(z_p(t,x))-s_pA(z_0)f(z_0(t,x))\Big)\\
=&\sum_{p=1}^{N}\sum_{i=1}^{n}\delta_{pi}(t,x)\sum_{s=1}^{n}\Big(a_{is}(z_{ps})f_s(z_{ps}(t,x))-s_pa_{is}(z_{0s})f_s(z_{0s}(t,x))\Big)\\
\leq&\sum_{p=1}^{N}\sum_{i=1}^{n}2\sum_{s=1}^{n}a_{is}^{+}\Upsilon|\delta_{pi}(t,x)|. \qquad (3.28)
\end{aligned}
$$

For the coupling terms, it can be yielded that

$$
\begin{aligned}
&\sigma\sum_{p=1}^{N}\delta_p^T(t,x)\sum_{q=1}^{N}l_{pq}\Gamma\delta_q(t-\tau,x)\\
=&\sigma\delta^T(t,x)(L\otimes\Gamma)\delta(t-\tau,x)\\
\leq&\delta^T(t-\tau,x)\delta(t-\tau,x)+\sigma^2\delta^T(t,x)(L\otimes\Gamma)^2\delta(t,x)\\
\leq&\delta^T(t-\tau,x)\delta(t-\tau,x)+\sigma^2\lambda_{\max}[(L\otimes\Gamma)^2]\delta^T(t,x)\delta(t,x) \qquad (3.29)
\end{aligned}
$$

For the control term (3.25), we get

$$
\begin{aligned}
&\sum_{p=1}^{N}\delta_p^T(t,x)u_p(t,x)\\
=&\sum_{p=1}^{N}\delta_p^T(t,x)\Big(-Ksgn(\delta_p(t,x))-\xi_p\delta_p(t,x)\\
&-\psi\frac{r}{T}\mu(t)\delta_p(t,x)-\frac{\phi\delta_p(t,x)}{\|\delta_p(t,x)\|_2^2}\Big|\int_{t-\tau}^{t}\delta_p^T(\theta,x)\delta_p(\theta,x)d\theta\Big|
\end{aligned}
$$

$$
\begin{aligned}
&-2\psi\frac{r}{T}\mu(t)\frac{\delta_p(t,x)}{\|\delta_p(t,x)\|_2^2}\Big|\int_{t-\tau}^{t}\delta_p^T(\theta,x))\delta_p(\theta,x)d\theta\Big|\Big)\\
=&-\sum_{p=1}^{N}\sum_{i=1}^{n}k_i|\delta_{pi}(t,x)|-\sum_{p=1}^{N}\xi_p\delta_p^T(t,x)\delta_p(t,x)\\
&-\sum_{p=1}^{N}\phi\int_{t-\tau}^{t}\delta_p^T(\theta,x)\delta_p(\theta,x)d\theta-2\psi\frac{r}{T}\mu(t)\sum_{p=1}^{N}\frac{1}{2}\delta_p^T(t,x)\delta_p(t,x)\\
&-2\psi\frac{r}{T}\mu(t)\sum_{p=1}^{N}\int_{t-\tau}^{t}\delta_p^T(\theta,x)\delta_p(\theta,x)d\theta. \qquad (3.30)
\end{aligned}
$$

By combining the above inequalities, one has

$$
\begin{aligned}
\frac{dV(t)}{dt}=&\int_{\Omega}\delta^T(t,x)\Big(-\tilde{\xi}-c_{\min}-\sum_{k=1}^{\vartheta}\frac{\pi^2d_{\min}}{(h_k^M-h_k^m)^2}\\
&+\lambda_{\max}[(L\otimes\Gamma)^2]\Big)\delta(t,x)dx\\
&-\phi\sum_{p=1}^{N}\int_{\Omega}\int_{t-\tau}^{t}\delta_p^T(\theta,x))\delta_p(\theta,x)d\theta dx\\
&-2\psi\frac{r}{T}\mu(t)\cdot\frac{1}{2}\sum_{p=1}^{N}\int_{\Omega}\delta_p^T(t,x)\delta_p(t,x)dx\\
&-2\psi\frac{r}{T}\mu(t)\sum_{p=1}^{N}\int_{\Omega}\int_{t-\tau}^{t}\delta_p^T(\theta,x)\delta_p(\theta,x)d\theta dx\\
\le&-\phi V(t)-2\psi\frac{r}{T}\mu(t)V(t). \qquad (3.31)
\end{aligned}
$$

Following the discussion in Theorem 1, we derive $\lim_{t\to t_b^-}\|z_p(t,x)-s_pz_0(t,x)\|_2=0$ and $\|z_p(t,x)-s_pz_0(t,x)\|_2\equiv 0$ on $[t_b,+\infty)$. Thus, leader-following PTBS for Systems (3.21) and (3.22) can be realized with controller (3.25).

Remark 5. *Different from [12–14], the competitive relationship and coupling delay are considered in this paper, which can better modify the real network model. Due to the existence of antagonistic interactions and coupling delays, the traditional synchronization methods in [12–14] cannot be directly used to realize PTBS. Moreover, the convergence speed is required to be prescribed instead of asymptotic in this work, and thus the Theorem in our work greatly generalizes the previous results on bipartite synchronization.*

Remark 6. *In Reference [27], by applying nonsmooth analysis and some novel inequality techniques in the complex field, several nonseparation method-based fixed-time synchronization criteria are derived. In Reference [30], the FXTS of dynamic systems are reconsidered in this article based on special functions from the view of improving the estimate accuracy for settling time and reducing the chattering caused by the sign function. In this paper, the convergence time is prescribed in advance according to actual requirements, and thus our criteria are less conservative and more flexible.*

4. Numerical examples

To verify our theorem, two simulation examples are proposed in this section. We consider a network with 3 nodes, and since the structural balance condition is satisfied, the node set V can be separated into $V_1 = \{1, 2\}$ and $V_2 = \{3\}$. Then, one has $s_1 = s_2 = 1$ and $s_3 = -1$. We first consider Theorem 1.

Example 1. *Choose the CRDMNNs with 2 neurons as below:*

$$\begin{aligned}\frac{\partial z_p(t,x)}{\partial t} =& D\Delta z_p(t,x) - Cz_p(t,x) + A(z_p(t,x))f(z_p(t,x))\\ &+ \sigma\sum_{q\in\mathcal{N}_p}|G_{pq}|\Gamma\Big(sgn(G_{pq})z_q(t,x) - z_p(t,x)\Big) + u_p(t,x)\end{aligned} \tag{4.1}$$

where $z_p(t,x) = (z_{p1}(t,x), z_{p2}(t,x))^T$, $D = \mathrm{diag}(1,1), C = \mathrm{diag}(2,2)$, *and* $\Omega = \{x| -1 \leq x \leq 1\}$. *The memristive matrix* $A(z_p(t,x))$ *switches between* $\check{A}$ *and* $\hat{A}$, *where*

$$\hat{A} = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad \check{A} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & -0.1 \end{bmatrix} \tag{4.2}$$

The activation functions are chosen as

$$f_p(x) = 0.5(|x+1| - |x-1|). \tag{4.3}$$

It can be checked that $F = 0.5I_2$ *and* $\Upsilon = 0.5$. *Hence, the conditions in Assumption 2 are satisfied. Choose* $\Gamma = I_2$ *and* $\sigma = 1$. *The Laplacian matrix is*

$$L = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$

The controller is

$$\begin{aligned}u_p(t,x) =& -Ks_p\sum_{j=p}^{N-1} sgn(s_jz_j(t,x) - s_{j+1}z_{j+1}(t,x))\\ &- \xi_pz_p(t,x) - \psi\frac{r}{T}\mu(t)z_p(t,x).\end{aligned} \tag{4.4}$$

Choose feedback matrix $K = 0.6I_2$, $\xi_p = 0.5, \psi = 1.5, r = 2, \phi = 1$. *Let* $t_0 = 0$, *and set the prespecified time* $t_b = 2$. *It can be checked that*

$$\begin{aligned}&k_j \geq 2\sum_{s=1}^{2}(\bar{a}_{js} - \underline{a}_{js})\Upsilon, \quad j = 1,2\\ &-\tilde{\xi} + \lambda_{\max}(-\mathbf{D}^* - \hat{C}_1 + \bar{\mathbf{A}}_1\bar{\mathbf{A}}_1^T + \mathbf{F}^2 + \sigma\mathbf{H}) \leq -\frac{\phi}{2}\end{aligned}$$

Thus, the conditions in Theorem 1 are satisfied. Choosing initial values in $[-3,3]$, *Figures 1 and 2 depict the evolutions of states* $z_{p1}(t,x)$ *and* $z_{p2}(t,x)$. *It can be seen that system (4.1) can reach leaderless PTBS in prespecified time* t_b *under control law (4.4).*

Next, we consider the leader-following PTBS in Theorem 2.

Example 2. *Consider the following 2-node CRDMNN with coupling delays:*

$$\begin{aligned}\frac{\partial z_p(t,x)}{\partial t} =& D\Delta z_p(t,x) - Cz_p(t,x) + A(z_p(t,x))f(z_p(t,x)) \\ &+ \sigma \sum_{q\in\mathcal{N}_p} |G_{pq}|\Gamma\Big(sgn(G_{pq})z_q(t-\tau,x) \\ &- z_p(t-\tau,x)\Big) + u_p(t,x) \end{aligned} \tag{4.5}$$

where $\tau = 0.5$*. Other parameters are the same as in Example 1. The desired reference target is*

$$\frac{\partial z_0(t,x)}{\partial t} = D\Delta z_0(t,x) - Cz_0(t,x) + A(z_0(t,x))f(z_0(t,x)). \tag{4.6}$$

Let $\varepsilon_p(t,x) = z_p(t,x) - s_p z_0(t,x)$*, and the error system is*

$$\begin{aligned}\frac{\partial \varepsilon_p(t,x)}{\partial t} =& D\Delta \varepsilon_p(t,x) - C\varepsilon_p(t,x) + A(z_p(t,x))f(z_p(t,x)) \\ &- s_pA(z_0(t,x))f(z_0(t,x)) \\ &+ \sigma\sum_{q=1}^{N} l_{pq}\Gamma\varepsilon_q(t-\tau,x) + u_p(t,x). \end{aligned} \tag{4.7}$$

The prespecified-time controller $u_p(t,x)$ *is chosen as (3.25). Choose* $k_i = 2, \xi_p = 2$*. Other parameters are the same as in Example 1. It can be checked that*

$$k_i > 2\sum_{s=1}^{2} a_{is}^{+}\Upsilon,$$

$$-\tilde{\xi} - c_{\min} - \sum_{k=1}^{\vartheta} \frac{\pi^2 d_{\min}}{(h_k^M - h_k^m)^2} + \sigma^2\lambda_{\max}[(L\otimes\Gamma)^2] \le -\frac{\phi}{2}$$

Thus, the condition in Theorem 2 holds, and the leader-following PTBS is achieved. Take initial random conditions in the interval $[-3, 3]$*, and Figure 3 describes the synchronization error* $\varepsilon_p(t,x)$ *between system (4.5) and (4.6) with controller (3.25). From the simulations, leader-following synchronization can be achieved in prescribed time* $t_b = 2$*.*

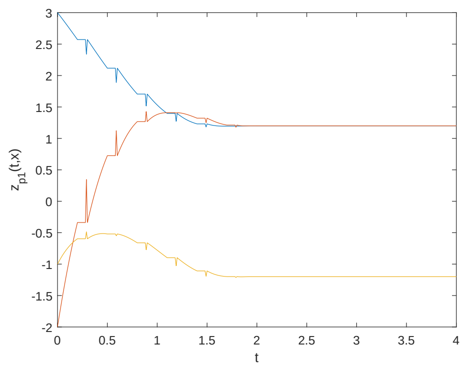

Figure 1. The trajectories $z_{p1}(t,x)$ of system (4.1) under controller (4.4).

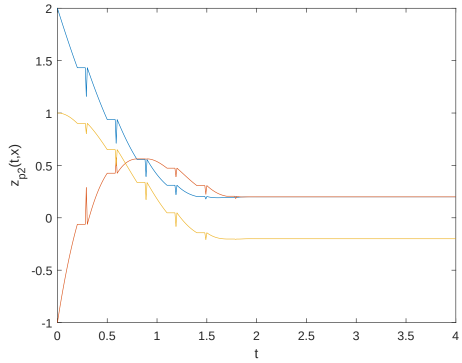

Figure 2. The trajectories $z_{p2}(t, x)$ of system (4.1) under controller (4.4).

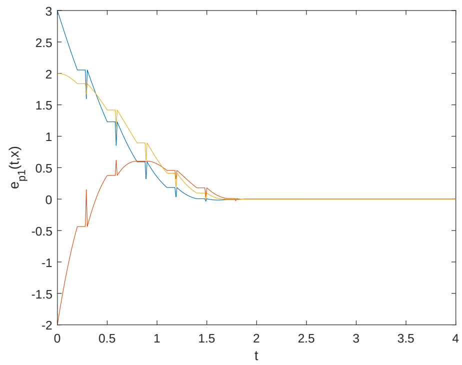

Figure 3. The trajectories $e_{p1}(t, x)$ of system (4.5) under controller (3.25).

5. Conclusions

This paper investigates the prespecified-time bipartite synchronization (PTBS) of CRDMNNs with both cooperative and competitive interactions. Two types of PTBS are considered in our work: leaderless PTBS and leader-following PTBS. In addition, the coupling delays are considered in the leader-following case. By designing suitable Lyapunov functionals and novel control protocols, two criteria are derived for leaderless PTBS and leader-following PTBS based on a structural balance condition. Compared with FTS and FxTS, the settling time in our theorem can be predetermined according to the task, which is independent of the initial values and control parameters.

Our future study will focus on 1) PTBS control of neural networks under time-scale and 2) PTBS control of quaternion-valued neural networks.

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Conflict of interest

The authors have no conflict of interest.

References

1. L. O. Chua, Memristor-the missing circuit element, *IEEE Trans. Circuit Theory.*, **18** (1971), 507–519. https://doi.org/10.1109/TCT.1971.1083337
2. D. Strukov, G. Snider, D. Stewart, R. S. Williams, The missing memristor found, *Nature*, **453** (2008), 80–83. https://doi.org/10.1038/nature06932
3. K. Miller, K. S. Nalwa, A. Bergerud, N. M. Neihart, S. Chaudhary, Memristive behavior in thin anodic titania, *IEEE Electron Device Lett.*, **37** (2010), 737–739. https://doi.org/10.1109/LED.2010.2049092
4. J. Sun, Y. Shen, Q. Yin, C. Xu, Compound synchronization of four memristor chaotic oscillator systems and secure communication, *Chaos*, **23** (2012), 1–10. https://doi.org/10.1063/1.4794794
5. F. Corinto, A. Ascoli, M. Gilli, Nonlinear dynamics of memristor oscillators, *IEEE Trans. Circuits Syst. I*, **58** (2011), 1323–1336. https://doi.org/10.1109/TCSI.2010.2097731
6. Y. V. Pershin, M. Di Ventra, Experimental demonstration of associative memory with memristive neural networks, *Neural Netw.*, **23** (2010), 881–886. https://doi.org/10.1016/j.neunet.2010.05.001
7. L. Wang, H. He, Z. Zeng, Global synchronization of fuzzy memristive neural networks with discrete and distributed delays, *IEEE Trans. Fuzz. Syst.*, **28** (2020), 2022–2034. https://doi.org/10.1109/TFUZZ.2019.2930032
8. R. Wei, J. Cao, W. Qian, C. Xue, X. Ding, Finite-time and fixed-time stabilization of inertial memristive Cohen-Grossberg neural networks via non-reduced order method, *AIMS Math.*, **6** (2021), 6915–6932. https://doi.org/10.3934/math.2021405
9. L. Wang, Z. Zeng, M. Ge, A disturbance rejection framework for finite-time and fixed-time stabilization of delayed memristive neural networks, *IEEE Trans. Syst. Man Cybern. Syst.*, **51** (2021), 905–915. https://doi.org/10.1109/TSMC.2018.2888867
10. Y. Sheng, H. Zhang, Z. Zeng, Stability and robust stability of stochastic reaction-diffusion neural networks with infinite discrete and distributed delays, *IEEE Trans. Syst. Man. Cybern. Syst.*, **50** (2020), 1721–1732. https://doi.org/10.1109/TSMC.2017.2783905
11. Y. Sheng, Z. Zeng, Passivity and robust passivity of stochastic reaction-diffusion neural networks with time-varying delays, *J. Franklin Inst.*, **354** (2017), 3995–4012. https://doi.org/10.1016/j.jfranklin.2017.03.014
12. Z. Wang, J. Cao, G. Lu, M. Abdel-Aty, Fixed-time passification analysis of interconnected memristive reaction-diffusion neural networks, *IEEE Trans. Netwowk Sci. Eng.*, **7** (2020), 1814–1824. https://doi.org/10.1109/TNSE.2019.2954463
13. Z. Guo , S. Wang, J. Wang, Global exponential synchronization of coupled delayed memristive neural networks with reaction-diffusion terms via distributed pinning controls, *IEEE Trans. Neural Networks Learn. Syst*., **32** (2021), 105–116. https://doi.org/10.1109/TNNLS.2020.2977099

14. X. Yang, J. Cao, Z. Yang, Synchronization of coupled reaction-diffusion neural networks with time-varying delays via pining-impulsive controller, *Siam J. Control Optim.*, **51** (2013), 3486–3510. https://doi.org/10.1137/120897341

15. C. Altafini, Consensus problems on networks with antagonistic interactions, *IEEE Trans. Autom. Control*, **58** (2013), 935–946. https://doi.org/10.1109/TAC.2012.2224251

16. J. Hu, Y. Wu, T. Li, B. K. Ghosh, Consensus control of general linear multiagent systems with antagonistic interactions and communication noises, *IEEE Trans. Autom. Control*, **64** (2019), 2122–2127. https://doi.org/10.1109/TAC.2018.2872197

17. Y. Wu, L. Liu, J. Hu, G. Feng, Adaptive antisynchronization of multilayer reaction-diffusion neural networks, *IEEE Trans. Neural Netw. Learn. Syst.*, **29** (2018), 807–818. https://doi.org/10.1109/TNNLS.2017.2647811

18. F. Liu, Q. Song, G. Wen, J. Cao, X. Yang, Bipartite synchronization in coupled delayed neural networks under pinning control, *Neural Networks*, **108** (2018), 146–154. https://doi.org/10.1016/j.neunet.2018.08.009

19. N. Li, W. Zheng, Bipartite synchronization of multiple memristor-based neural networks with antagonistic interactions, *IEEE Trans. Neural Networks Learn. Syst.*, **32** (2021), 1642–1653. https://doi.org/10.1109/TNNLS.2020.2985860

20. N. Li, W. Zheng, Bipartite synchronization for inertia memristor-based neural networks on coopetition networks, *Neural Networks*, **124** (2020), 39–49. https://doi.org/10.1016/j.neunet.2019.11.010

21. K. Mao, X. Liu, J. Cao, Y. Hu, Finite-time bipartite synchronization of coupled neural networks with uncertain parameters, *Phys. A*, **585** (2022), 126431. https://doi.org/10.1016/j.physa.2021.126431

22. X. Li, D. Peng, J. Cao, Lyapunov stability for impulsive systems via event-triggered impulsive control, *IEEE Trans. Autom. Control*, **65** (2020), 4908–4913. https://doi.org/10.1109/TAC.2020.2964558

23. X. Li, X. Yang, J. Cao, Event-triggered impulsive control for nonlinear delay systems, *Automatica*, **117** (2020), 108981. https://doi.org/10.1016/j.automatica.2020.108981

24. S. P. Bhat, D. S. Bernstein, Finite-time stability of continuous autonomous systems, *Siam J. Control Optim.*, **38** (2000), 751–766. https://doi.org/10.1137/S0363012997321358

25. X. Li, D. W. C. Ho, J. Cao, Finite-time stability and settling-time estimation of nonlinear impulsive systems, *Automatica*, **99** (2019), 361–368. https://doi.org/10.1016/j.automatica.2018.10.024

26. Z. Zhang, X. Liu, D. Zhou, C. Lin, J. Chen, H. Wang, Finite-time stabilizability and instabilizability for complex-valued memristive neural networks with time delays, *IEEE Trans. Syst. Man Cybern. Syst.*, **48** (2018), 2371–2382. https://doi.org/10.1109/TSMC.2017.2754508

27. L. Feng, J. Yu, C. Hu, C. Yang, H. Jiang, Nonseparation method-based finite/fixed-time synchronization of fully complex-valued discontinuous neural networks, *IEEE Trans. Cybern.*, **51** (2021), 3212–3223. https://doi.org/10.1109/TCYB.2020.2980684

28. A. Polyakov, Nonlinear feedback design for fixed-time stabilization of linear control systems, *IEEE Trans. Automat. Control*, **57** (2012), 2106–2110. https://doi.org/10.1109/TAC.2011.2179869
29. R. Wei, J. Cao, Fixed-time synchronization of second-order MNNs in quaternion field, *IEEE Trans. Syst. Man Cybern. Syst.*, **51** (2021), 3587–3598. https://doi.org/10.1109/TSMC.2019.2931091
30. C. Hu, H. Jiang, Special functions-based fixed-time estimation and stabilization for dynamic systems, *IEEE Trans. Syst. Man Cybern. Syst.*, **52** (2022), 3251–3262. https://doi.org/10.1109/TSMC.2021.3062206
31. X. Ding, J. Cao, A. Alsaedi, T. Hayat, Robust fixed-time synchronization for uncertain complex-valued neural networks with discontinuous activation functions, *Neural Networks*, **90** (2017), 42–55. https://doi.org/10.1016/j.neunet.2017.03.006
32. C. Hu, H. He, H. Jiang, Fixed/Preassigned-time synchronization of quaternion-valued neural networks via pure power-law control, *Neural Networks*, **146** (2022), 341–349. https://doi.org/10.1016/j.neunet.2021.11.023
33. C. Hu, H. He, H. Jiang, Fixed/preassigned-time synchronization of complex networks via improving fixed-Time stability, *IEEE Trans. Cybern.*, **51** (2021), 2882–2892. https://doi.org/10.1109/TCYB.2020.2977934
34. S. Shao, X. Liu, J. Cao, Prespecified-time synchronization of switched coupled neural networks via smooth controllers, *Neural Networks*, **133** (2021), 32–39. https://doi.org/10.1016/j.neunet.2020.10.007
35. X. Liu, D. W. C. Ho, and C. Xie, Prespecified-time cluster synchronization of complex networks via a smooth control approach, *IEEE Trans. Cybern.*, **50** (2020), 1771–1775. https://doi.org/10.1109/TCYB.2018.2882519