Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/mbe

MBE, 19(12): 12334–12352.
DOI: 10.3934/mbe.2022575
Received: 20 May 2022
Revised: 28 July 2022
Accepted: 05 August 2022
Published: 24 August 2022

Research article

Adaptive fuzzy output-feedback event-triggered control for fractional-order nonlinear system

Chaoyue Wang, Zhiyao Ma and Shaocheng Tong*

College of Science, Liaoning University of Technology, Jinzhou 121001, China

* **Correspondence:** Email: jztongsc@163.com.

Abstract: This paper studies the issue of adaptive fuzzy output-feedback event-triggered control (ETC) for a fractional-order nonlinear system (FONS). The considered fractional-order system is subject to unmeasurable states. Fuzzy-logic systems (FLSs) are used to approximate unknown nonlinear functions, and a fuzzy state observer is founded to estimate the unmeasurable states. By constructing appropriate Lyapunov functions and utilizing the backstepping dynamic surface control (DSC) design technique, an adaptive fuzzy output-feedback ETC scheme is developed to reduce the usage of communication resources. It is proved that the controlled fractional-order system is stable, the tracking and observer errors are able to converge to a neighborhood of zero, and the Zeno phenomenon is excluded. A simulation example is given to verify the availability of the proposed ETC algorithm.

Keywords: fractional-order nonlinear system; output-feedback control; backstepping control; dynamic surface control; event-triggered control

1. Introduction

Fractional-order nonlinear systems (FONSs) are a class of complex systems modeled by fractional calculus. In recent years, the application of fractional calculus has attracted wide-ranging attention. In practice, examples such as the blood ethanol concentration system by Qureshi et al. [1], the dynamics of the TB virus by Ullah et al. [2], the fractional Brusselator reaction-diffusion system by Jena et al. [3], etc. can be modeled as fractional order systems. This kind of system can well describe the genetic effect and long memory effect of a real physical system. Nevertheless, it is noteworthy that conventional control scheme is no longer applicable to fractional-order systems due to some general rules for integer-order calculation, such as Chain rules and Leibniz rules, not being well built with regard to fractional derivatives. Hence, how to solve the stability analysis or controller of fractional order systems has attracted extensive attention of scholars. For example, in [4], a fractional order controller designed by Liu et al. to ensure that the synchronization errors of the fractional order chaotic system reached the specified performance whether there is external interference or not. Wei et al. [5] designed an adaptive tracking controller and extended the result to the case of $1 < \alpha < 2$ by the semigroup property of the fractional derivative and fractional tracking differentiator. Afterwards, Li et al. [6] studied the adaptive control issue for a type of commensurate FONS with parametric uncertainty and external interference, and unlike the discontinuous function, the auxiliary function is used to get smooth control input and achieve perfect property tracking in case of bounded interferences. However, the above literature demands that nonlinear functions in the plant are known. So, these techniques are not appropriate for resolving the control issue of FONSs with unknown nonlinear functions.

In practice, there are always completely unknown nonlinear functions in the modeled control systems, which cannot be ignored. Therefore, [7–11] used FLSs or neural networks (NNs) with approximation capability to solve this problem. The authors in [7–9] put forward adaptive intelligent (fuzzy and NN) control schemes for FONSs, in which Wang and Liang [8] and Li et al. [9] are robust to input saturation and fault, respectively. Because the above intelligent adaptive control methods adopt a conventional backstepping control technology, there is a problem of computational complexity. In order to avoid this problem, Ma and Ma [10] and Sui et al. [11] proposed several adaptive intelligent state feedback control methods by introducing fractional-order filters. However, the existing methods in [7–11] are merely applicable to those FONSs whose states are only measured. Therefore, the above scheme is not suitable for the control design of FONSs with unmeasurable state variables. Then, [12–14] presented some adaptive intelligent output-feedback DSC methods for FONSs with unmeasurable states. It should be noted that in [12], the authors established a new fractional-order reduced-order observer, which not only obtained the information of unmeasurable states, but also reduced errors caused by the full-order state observer in the estimation of some measurable states. As far as we know, there are few results on output-feedback and DSC control simultaneously for fractional-order nonlinear systems, which prompted us to study this problem.

Notably, that the above results adopt conventional periodic control methods, and the control signal needs to be transferred to the actuator in actual time, which will lead to unnecessary sampling and communication. Recently, the ETC theorem has been exploited rapidly, which can reduce the communication load of the controlled system [15–17]. An event-triggered pulse control method for impulsive systems is proposed in [15], which successfully solves the stabilization matter of nonlinear impulsive systems and eliminates Zeno behavior. Later, in [16,17], Sui et al. and Wang et al. designed intelligent adaptive ETC schemes for stochastic systems and multi-agent systems to achieve system stability while reducing the waste of communication resources. Note that the systems considered by above intelligent controllers are only applicable to integer-order nonlinear systems, not FONSs. Meanwhile, it is not easy to extend the direct Lyapunov algorithm and its related control schemes from integer order to FONSs. Although [18–20] proposed several intelligent adaptive control algorithms for the FONSs, they are designed for situation in which the state is completely measurable and there is a problem of computational complexity. To our knowledge, there is a short age of studies on the observer-based ETC control methods for FONSs, which inspires us to study this problem.

According to the above-mentioned discussions, we study the adaptive output-feedback event-triggered control for FONS with unmeasurable state variables. The proposed ETC scheme can significantly decrease the consumption of communication resources. The main innovations of this paper are as follows.

1) This article first designs an output-feedback event-triggered controller of the FONS. The

proposed control algorithm erases the restrictive condition in [18–20], with which the state of the system must be completely measured.

2) Due to the DSC technique being used to control design, the put forward control method settles the computational complexity issue in current works [18–20].

3) In this paper, the adaptive control law and the event-triggered mechanism are designed together. The stability of the controlled system can be guaranteed by using the fractional-order Lyapunov criterion. Unlike [9–12], the control signal needs to be sampled and updated regularly. The system drive will be generated only when the preset conditions are met in this paper, which greatly reduces the consumption of network resources.

2. Preliminaries

2.1. System statements

Consider the following FONS:

$$\begin{aligned} {}_0^C D_t^\alpha x_i &= x_{i+1} + f_i(\bar{x}_i) \\ {}_0^C D_t^\alpha x_n &= u + f_n(\bar{x}_n) \\ y &= x_1 \end{aligned} \tag{2.1}$$

where $\bar{x}_n = [x_1, x_2, ..., x_n]^T \in R^n$ are the system state vectors, and $y \in R$ and $u \in R$ denote the output variable and control input of the system. $f_i(\cdot) \in R, i = 1, \ldots, n$, denotes an unknown smooth nonlinear function. This paper assumes that only the output variable y is measurable.

Assumption 1 [9–14]: The given reference signals $y_d, {}_0^C D_t^\alpha y_d$ and ${}_0^C D_t^\alpha({}_0^C D_t^\alpha y_d)$ are smooth and bounded. Furthermore, assumed that there exists known constant $Z_0 > 0$ satisfying that $y_d^2 + ({}_0^C D_t^\alpha y_d)^2 + ({}_0^C D_t^\alpha({}_0^C D_t^\alpha y_d))^2 \leq Z_0$.

Control Objectives: In this article, a fuzzy adaptive event-triggered controller is designed for System (1) such that all signals in the considered system are bounded, and the tracking error converges to the compact set of the origin.

2.2. Preliminaries

Some useful definitions and lemmas are given first.

Definition 1 [21]: The αth Caputo derivative is defined as:

$${}_0^C D_t^\alpha F(t) = \frac{1}{\Gamma(\omega - \alpha)} \int_0^t \frac{F^{(\omega)}(\tau)}{(t-\tau)^{\alpha+1-\omega}} d\tau \tag{2.2}$$

where $\omega - 1 < \alpha \leq \omega$, ω is a positive integer. $\Gamma(\cdot) = \int_0^{+\infty} \tau^{-1} e^{-\tau} d\tau$ denotes Euler's gamma function with $\Gamma(1) = 1$.

Definition 2 [21]: The Mittag-Leffler function is formulated as

$$E_{\alpha,\phi}(\gamma) = \sum_{j=0}^{\infty} \frac{\gamma^j}{\Gamma(j\alpha + \phi)} \tag{2.3}$$

where $\alpha > 0, \phi > 0$, and γ is a complex number.

Lemma 1 [21]: Let $\alpha \in (0,2)$, $\eta \in R$ and $\phi \in (\pi\alpha/2, \min\{\pi, \pi\alpha\})$, and then one has

$$E_{\alpha,\eta}(\zeta) \leq \frac{r}{1+|\zeta|} \tag{2.4}$$

In (2.4), $r > 0$, $|\zeta| \geq 0$, and $\phi \leq |\arg(\zeta)| \leq \pi$.

Lemma 2: Suppose that$f(x)$ is a continuous function on a compact set Ω. There exists an FLS such as

$$\sup_{x\in\Omega}\left|f(x) - \xi^{*T}\psi(x)\right| \leq \varepsilon \tag{2.5}$$

where $\varepsilon > 0$ is any positive constant.

3. Fuzzy state observer

Write the FONS (2.1) as follows:

$$\begin{aligned} {}_0^C D_t^\alpha x &= Ax + Ky + \sum_{i=1}^{n} B_i f_i(\bar{x}_i) + Bu \\ y &= Cx \end{aligned} \tag{3.1}$$

where $A = \begin{bmatrix} -k_1 & 1 & \cdots & 0 \\ -k_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -k_n & 0 & \cdots & 0 \end{bmatrix}_{n\times n}$, $K = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}_{n\times 1}$, $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{n\times 1}$, $B_i = \begin{bmatrix} \underbrace{0 \quad \cdots \quad 1}_{i} & \cdots & 0 \end{bmatrix}_{1\times n}$, $C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$, K is chosen such that $(A + CK)$ is a Hurwitz matrix. $Q = Q^T > 0$ is a positive definite matrix, and exist a positive definite matrix $P = P^T > 0$ such that

$$A^T P + PA = -2Q, \tag{3.2}$$

It is worth noting that $f_i(\bar{x}_i)$ in (3.1) is an unknown continuous function, so it is necessary to approximate $f_i(\bar{x}_i)$ with the help of an FLS $\hat{f}_i(\bar{x}_i|\xi_i) = \xi_i^T \phi_i(\bar{x}_i)$,. In the bounded sets Ω, the definition of ideal parameter vectors ξ_i^* are described as:

$$\xi_i^* = \arg\min_{\xi\in\Omega}\left[\sup_{\bar{x}_i\in U}\left|\hat{f}_i(\bar{x}_i|\,\xi_i) - f_i(\bar{x}_i)\right|\right] \tag{3.3}$$

The definition of the optimal approximation errorε_i is described as

$$\varepsilon_i = f_i(\bar{x}_i) - \hat{f}_i(\bar{x}_i|\,\xi_i^*) \tag{3.4}$$

with $\varepsilon_i(\bar{x}_i)$ is bounded by constant $\varepsilon_i^* > 0$.

Design the fuzzy state observer for (3.1) as

$$\begin{aligned} {}_0^C D_t^\alpha \hat{x} &= A\hat{x} + Ky + \sum_{i=1}^{n} B_i \hat{f}_i(\hat{\bar{x}}_i|\xi_i) + Bu \\ \hat{y} &= C\hat{x} \end{aligned} \tag{3.5}$$

where $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T$ are the estimations of x.

Define the observer error as $e = x - \hat{x}$.

From (3.1), (3.4) and (3.5), one has

$$ {}_0^C D_t^\alpha e = \sum_{i=1}^{n} B_i \xi_i^{*T} (\phi_i(\bar{x}_i) - \phi_i(\hat{\bar{x}}_i)) + Ae + \sum_{i=1}^{n} B_i \tilde{\xi}_i^T \phi_i(\hat{\bar{x}}_i) + \varepsilon \tag{3.6} $$

where $\varepsilon = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n]^T, \xi_i$ are the estimations of ideal parameters ξ_i^*, and $\tilde{\xi} = \xi^* - \xi$.

Construct the Lyapunov function candidate as $V_0 = \frac{1}{2} e^T P e$, and then the following Theorem can be obtained.

Theorem 1: For controlled System (2.1), the fuzzy state observer (3.3) has the following property:

$$ {}_0^C D_t^\alpha V_0 \le -\lambda_0 \|e\|^2 + \frac{1}{2} \|P\|^2 \sum_{i=1}^{n} \|\tilde{\xi}_i\|^2 + \delta_0 \tag{3.7} $$

where $\lambda_0 = (\lambda_{\min}(Q) - 2) > 0$, and $\delta_0 = \|P\|^2 \sum_{i=1}^{n} \|\xi_i^*\|^2 + \frac{1}{2} \|P\|^2 \sum_{i=1}^{n} \varepsilon_i^{*2}$.

Proof: We choose the Lyapunov function candidate as $V_0 = \frac{1}{2} e^T P e$. From (3.5) and (3.6), by the inequality ${}_0^C D_t^\alpha (x^T(t) x(t)) \big/ 2 \le x^T(t) {}_0^C D_t^\alpha x(t)$, ${}_0^C D_t^\alpha V_0$ can be calculated as

$$ \begin{array}{c} {}_0^C D_t^\alpha V_0 \le \frac{1}{2} e^T (PA + A^T P) e + e^T P (\sum\limits_{i=1}^{n} B_i \xi_i^{*T} (\psi_i(\bar{x}_i) \\ -\psi_i \ (\hat{\bar{x}}_i)) + \sum\limits_{i=1}^{n} B_i \tilde{\xi}_i^T \psi_i(\hat{\bar{x}}_i) + \varepsilon), \end{array} \tag{3.8} $$

By employing Young's inequality and $\psi_i^T(\hat{x}_i) \psi_i(\hat{x}_i) \le 1$, we can gain

$$ \begin{array}{l} e^T P (\sum\limits_{i=1}^{n} B_i \xi_i^{*T} (\psi_i(\bar{x}_i) - \psi_i(\hat{\bar{x}}_i)) \le \|e\|^2 + \|P\|^2 \sum_{i=1}^{n} \left\|\xi_i^*\right\|^2 \\ e^T P \sum\limits_{i=1}^{n} B_i \tilde{\xi}_i^T \psi_i(\hat{\bar{x}}_i) \le \frac{1}{2} \|e\|^2 + \frac{1}{2} \|P\|^2 \sum_{i=1}^{n} \tilde{\xi}_i^T \tilde{\xi}_i \\ e^T P \varepsilon \le \frac{1}{2} \|e\|^2 + \frac{1}{2} \|P\|^2 \sum_{i=1}^{n} \varepsilon_i^{*2} \end{array} \tag{3.9} $$

Substituting (3.9) into (3.8), one can have (3.7). This completes the proof of Theorem 1.

Remark 1: Theorem 1 shows that if $\tilde{\xi}_i^T \tilde{\xi}_i$ is bounded, smaller observation errors e_i can be obtained by selecting a large enough λ_0. It is further concluded that the constructed fuzzy state observer (3.5) can better estimate the unknown states.

4. ETC controller design and stability analysis

This part will use the adaptive fuzzy backstepping control algorithm to provide the observer-based adaptive ETC control design program and give its stability analysis.

4.1. Design procedure

Make the coordinate transforms as

$$ \begin{array}{r} S_1 = x_1 - y_d \\ S_i = \hat{x}_i - \upsilon_{i-1} \quad i = 2, \ldots, n \\ \eta_{i-1} = \upsilon_{i-1} - \tau_{i-1} \end{array} \tag{4.1} $$

where S_1 is tracking error, S_i are dynamic surface errors, υ_i are filter variables, and η_i are filter output errors, and τ_{i-1} are the virtual control functions.

Step1: Via (3.3), (4.1), and $x_2 = e_2 + \hat{x}_2$, one has

$$\begin{aligned} {}_0^C D_t^\alpha S_1 &= x_2 + f_1(x_1) - {}_0^C D_t^\alpha y_d \\ &= e_2 \ + S_2 + \eta_1 + \tau_1 + \xi_1^{*T}(\phi_1(x_1) - \phi_1(\hat{x}_1)) \\ &\quad + \xi_1^T \phi_1(\hat{x}_1) + \tilde{\xi}_1^T \phi_1(\hat{x}_1) - {}_0^C D_t^\alpha y_d + \varepsilon_1 \end{aligned} \tag{4.2}$$

Choose the Lyapunov function as

$$V_1 = V_0 + \frac{1}{2} S_1^2 + \frac{1}{2\gamma_1} \tilde{\xi}_1^T \tilde{\xi}_1 \tag{4.3}$$

where $\gamma_1 > 0$ is a known constant.

The virtual controller τ_1 and the adaptive law ${}_0^C D_t^\alpha \xi_1$ are designed as

$$\tau_1 = -c_1 S_1 - \frac{5}{2} S_1 - \xi_1^T \phi_1(\hat{x}_1) + {}_0^C D_t^\alpha y_d \tag{4.4}$$

$${}_0^C D_t^\alpha \xi_1 = \gamma_1 S_1 \phi_1(\hat{x}_1) - \kappa_1 \xi_1 \tag{4.5}$$

where $c_1 > 0$ and $\kappa_1 > 0$ are known constants.

Introduce dynamic surface filter in [15] as

$$\sigma_1 {}_0^C D_t^\alpha \upsilon_1 + \upsilon_1 = \tau_1, \upsilon_1(0) = \tau_1(0) \tag{4.6}$$

where σ_1 is a constant.

By using (4.2) and (4.6), one has

$${}_0^C D_t^\alpha \eta_1 = {}_0^C D_t^\alpha \upsilon_1 - {}_0^C D_t^\alpha \tau_1 = -\frac{\eta_1}{\sigma_1} + W_1(\cdot) \tag{4.7}$$

where $W_1(\cdot)$ is a continuous function.

Remark 2: In the backstepping ETC design of FONSs, it is difficult to obtain the mathematical analytical expression of the fractional derivative of the virtual controllers. To solve this problem, some authors used the packaged approximation technology in [18–20] to repeatedly approximate the virtual controllers. Because this method takes all the signals of the closed-loop system as the input variables of the NN or FLS, it will increase the dimensions of the adjusted parameter vector, resulting in the problem of computational complexity. Therefore, this paper adopts the DSC technology to effectively avoid this problem.

Step i: From (3.5) and (4.1), one has

$$\begin{aligned} {}_0^C D_t^\alpha S_i &= {}_0^C D_t^\alpha \hat{x}_i - {}_0^C D_t^\alpha \upsilon_{i-1} \\ &= S_{i+1} + \eta_i + \tau_i + \xi_i^T \phi_i(\hat{\bar{x}}_i) + \tilde{\xi}_i^T \phi_i(\hat{\bar{x}}_i) \\ &\quad - \tilde{\xi}_i^T \phi_i(\hat{\bar{x}}_i) - {}_0^C D_t^\alpha \upsilon_{i-1} + k_i e_1 \end{aligned} \tag{4.8}$$

The Lyapunov function candidate is chosen as

$$V_i = V_{i-1} + \frac{1}{2}S_i^2 + \frac{1}{2\gamma_i}\tilde{\xi}_i^T\tilde{\xi}_i + \frac{1}{2}\eta_{i-1}^2 \tag{4.9}$$

where $\gamma_i > 0$ is a known constant.

The virtual controller τ_i and the adaptive law ${}_0^C D_t^\alpha \xi_i$ are designed as

$$\tau_i = -c_i S_i - S_i - S_{i-1} - \xi_i^T \phi_i(\hat{\bar{x}}_i) + {}_0^C D_t^\alpha \upsilon_{i-1} - k_i e_1 \tag{4.10}$$

$${}_0^C D_t^\alpha \xi_i = \gamma_i S_i \phi_i(\hat{\bar{x}}_i) - \kappa_i \xi_i \tag{4.11}$$

where $c_i > 0$ and $\kappa_i > 0$ are known constants.

Introduce dynamic surface filter as

$$\sigma_i {}_0^C D_t^\alpha \upsilon_i + \upsilon_i = \tau_i, \upsilon_i(0) = \tau_i(0) \tag{4.12}$$

where σ_i is a constant.

By using (4.8) and (4.12), one can obtain

$${}_0^C D_t^\alpha \eta_i = {}_0^C D_t^\alpha \upsilon_i - {}_0^C D_t^\alpha \tau_i = -\frac{\eta_i}{\sigma_i} + W_i(\cdot) \tag{4.13}$$

where $W_i(\cdot)$ is a continuous function.

Step n: We first devise an event-triggered controller as

$$\tau_n = -c_n S_n - k_n e_1 - \xi_n^T \phi_n(\hat{\bar{x}}_n) - S_{n-1} + {}_0^C D_t^\alpha \upsilon_n - \frac{1}{2}S_n \tag{4.14}$$

$$\omega(t) = -(1+\bar{\delta})(\tau_n \tanh(\frac{S_n \tau_n}{\psi}) + \bar{m}\tanh(\frac{S_n \bar{m}}{\psi})) \tag{4.15}$$

$$u(t) = \omega(t_k), \forall t \in [t_k, t_{k+1}) \tag{4.16}$$

where $c_n > 0$ is a known constant, and $t_k (k \in z^+)$ defines input updating time.

Thus, in order to get a lower communication rate, the event-triggered condition can be designed as

$$t_{k+1} = \inf\{t \in R \,|\, |\nu(t)| \geq \bar{\delta}\,|u(t)| + m\} \tag{4.17}$$

where $\bar{\delta} \in (0,1), m > 0$ and $\bar{m} > [m \big/ (1-\bar{\delta})]$ are given as the known parameters, and $\nu(t) = \omega(t) - u(t)$ is called as the measurement error. When (4.17) is triggered, the time will be marked as t_{k+1}, and the controller $u(t_{k+1})$ will be utilized to the system. At the time $t \in [t_k, t_{k+1})$ the control signal is always unchanging.

In order to discuss the event-triggered rule, we consider that the actuator normally operates, i.e., $|\omega(t) - u(t)| \leq \bar{\delta}\,|u(t)| + m$.

If $\nu(t_k) > 0$, the measured error can be rewritten as

$$\begin{aligned} -\bar{\delta}u(t_k) - m \leq \omega(t) - u(t) \leq \bar{\delta}u(t_k) + m \\ u(t) - \omega(t) = \lambda_1(\bar{\delta}u(t_k) + m) \end{aligned} \tag{4.18}$$

where $\lambda_1(t) \in [-1, 1]$.

If $v(t_k) < 0$, we can transform the event-triggered condition (4.17) as

$$\begin{aligned} &\bar{\delta}u(t_k) - m \leq \omega(t) - u(t) \leq -\bar{\delta}u(t_k) + m \\ &u(t) - \omega(t) = \lambda_2(\bar{\delta}u(t_k) - m) \end{aligned} \tag{4.19}$$

where $\lambda_2(t) \in [-1, 1]$.

From (4.18) and (4.19), one obtains

$$\omega(t) = u(t) + \bar{\delta}\lambda_1(t)u(t) + \lambda_2(t)m \tag{4.20}$$

where $|\lambda_i| \leq 1, i = 1, 2$ are time-varying variables. Then, one has

$$u(t) = \frac{\omega(t)}{1 + \lambda_1(t)\bar{\delta}} - \frac{\lambda_2(t)m}{1 + \lambda_1(t)\bar{\delta}} \tag{4.21}$$

Remark 3: The event-triggered parameters $\bar{\delta}$ and m in (4.17) are determined according to the required communication rate. Therefore, in practical applications, while ensuring satisfactory tracking performance, we should try to reduce the communication burden.

According to (3.5), (4.1) and (4.21), one has

$$\begin{aligned} {}_0^C D_t^\alpha S_n &= {}_0^C D_t^\alpha \hat{x}_n - {}_0^C D_t^\alpha \upsilon_{n-1} \\ &= k_n e_1 + \xi_n^T \phi_n(\hat{\bar{x}}_n) + \frac{\omega(t)}{1+\lambda_1(t)\bar{\delta}} - \frac{\lambda_2(t)m}{1+\lambda_1(t)\bar{\delta}} \\ &\quad + \tilde{\xi}_n^T \phi_n(\hat{\bar{x}}_n) - {}_0^C D_t^\alpha \upsilon_{n-1} - \tilde{\xi}_n^T \phi_n(\hat{\bar{x}}_n), \end{aligned} \tag{4.22}$$

Choose the Lyapunov function as

$$V = V_{n-1} + \frac{1}{2}S_n^2 + \frac{1}{2}\eta_{n-1}^2 + \frac{1}{2\gamma_n}\tilde{\xi}_n^T \tilde{\xi}_n, \tag{4.23}$$

where $\gamma_n > 0$ is a known constant.

Assumption 2 [10, 12]: For any initial conditions, there exists a constant $q > 0$, such that $V(0) \leq q$.

The adaptive law ${}_0^C D_t^\alpha \xi_n$ is designed as:

$${}_0^C D_t^\alpha \xi_n = \gamma_n S_n \phi_n(\hat{\bar{x}}_n) - \kappa_n \xi_n \tag{4.24}$$

where $\kappa_n > 0$ is a known constant.

Remark 4: A backstepping control algorithm is indicated for FONSs in [17]. It applies the stability analysis of integer-order Lyapunov methods to known fractional-order systems. However, in this article, the system model may be completely unknown. In addition, the stability of the control algorithm is analyzed by the fractional order adaptive stability criterion.

4.2. Closed-loop systems stability analysis

Theorem 2: Consider System (2.1), under Assumptions 1–2, and then the put forward adaptive fuzzy output feedback event-triggered controller (4.21) with the event-triggered mechanism (4.17) can keep that controlled fractional-order system is stable, and the tracking error is able to regulate to a small residual set of the origin. Meanwhile, Zeno behavior is removed effectively.

Proof: Construct the whole Lyapunov function as

$$V = V_0 + \sum_{i=1}^{n} V_i = V_0 + \sum_{i=1}^{n} (\frac{1}{2}S_i^2 + \frac{1}{2\gamma_i}\tilde{\xi}_i^T\tilde{\xi}_i) + \sum_{i=1}^{n-1}\frac{1}{2}\eta_i^2 \tag{4.25}$$

By utilizing the inequality ${}_0^C D_t^\alpha (x^T(t)x(t))\Big/2 \le x^T(t){}_0^C D_t^\alpha x(t)$, one can obtain

$${}_0^C D_t^\alpha V \le {}_0^C D_t^\alpha V_0 + \sum_{i=1}^{n} (S_i {}_0^C D_t^\alpha S_i + \frac{1}{\gamma_i}\tilde{\xi}_i^T {}_0^C D_t^\alpha \tilde{\xi}_i) + \sum_{i=1}^{n-1}\eta_i {}_0^C D_t^\alpha \eta_i \tag{4.26}$$

According to (4.4), (4.10), (4.14), (4.22), (4.23), (4.26) and adding and subtracting $S_n\tau_n$, $|S_n\bar{m}|$ in the right of (4.26), the following equality can be obtained:

$$\begin{aligned} S_1 {}_0^C D_t^\alpha S_1 = {} & S_1(e_2 + S_2 + \eta_1 + \tau_1 + \xi_1^{*T}(\phi_1(x_1) - \phi_1(\hat{x}_1)) \\ & + \xi_1^T\phi_1(\hat{x}_1) + \tilde{\xi}_1^T\phi_1(\hat{x}_1) - {}_0^C D_t^\alpha y_d + \varepsilon_1), \end{aligned} \tag{4.27}$$

$$\begin{aligned} S_i {}_0^C D_t^\alpha S_i = {} & S_i(S_{i+1} + \eta_i + \tau_i + \xi_i^T\phi_i(\hat{\bar{x}}_i) + \tilde{\xi}_i^T\phi_i(\hat{\bar{x}}_i) \\ & - \tilde{\xi}_i^T\phi_i(\hat{\bar{x}}_i) - {}_0^C D_t^\alpha \upsilon_{i-1} + k_i e_1), \end{aligned} \tag{4.28}$$

$$\begin{aligned} S_n {}_0^C D_t^\alpha S_n = {} & S_n(k_n e_1 + \xi_n^T\phi_n(\hat{\bar{x}}_n) + \tfrac{\omega(t)}{1+\lambda_1(t)\bar{\delta}} - {}_0^C D_t^\alpha \upsilon_{n-1} \\ & - \tfrac{\lambda_2(t)m}{1+\lambda_1(t)\bar{\delta}} + \tilde{\xi}_n^T\phi_n(\hat{\bar{x}}_n) - \tilde{\xi}_n^T\phi_n(\hat{\bar{x}}_n) + \tau_n - \tau_n) \\ & + |S_n\bar{m}| - |S_n\bar{m}|. \end{aligned} \tag{4.29}$$

In view of $|\lambda_i(t)| \le 1$ and $S_n\omega(t) \le 0$, we can obtain

$$\frac{S_n\omega(t)}{1 + \lambda_1(t)\bar{\delta}} \le \frac{S_n\omega(t)}{1 + \bar{\delta}} \tag{4.30}$$

$$\frac{\lambda_2 m}{1 + \lambda_1(t)\bar{\delta}} \le \left|\frac{m}{1 - \bar{\delta}}\right| \tag{4.31}$$

According to (4.26),(4.30) and (4.31), (4.29) can be rewritten as

$$\begin{aligned} S_n {}_0^C D_t^\alpha S_n \le {} & S_n(k_n e_1 + \xi_n^T\phi_n(\hat{\bar{x}}_n) - {}_0^C D_t^\alpha \upsilon_{n-1} - \tau_n \\ & + \tilde{\xi}_n^T\phi_n(\hat{\bar{x}}_n) - \tilde{\xi}_n^T\phi_n(\hat{\bar{x}}_n)) + |S_n\tau_n| + |S_n\bar{m}| \\ & - S_n\tau_n \tanh(\tfrac{S_n\tau_n}{\psi}) - S_n\bar{m}\tanh(\tfrac{S_n\bar{m}}{\psi}). \end{aligned} \tag{4.32}$$

Then, utilizing the inequality $|\tau| - \tau\tanh(\tau/\psi \le \chi\psi)$, $\chi = 0.2785$, one can get

$$\begin{aligned} S_n {}_0^C D_t^\alpha S_n \le {} & S_n(k_n e_1 + \xi_n^T\phi_n(\hat{\bar{x}}_n) - {}_0^C D_t^\alpha \upsilon_{n-1} - \tau_n \\ & + \tilde{\xi}_n^T\phi_n(\hat{\bar{x}}_n) - \tilde{\xi}_n^T\phi_n(\hat{\bar{x}}_n)) + 0.557\varepsilon^*. \end{aligned} \tag{4.33}$$

For (4.14)–(4.17), (4.33) applying Young's inequality, one gets

$$S_1(e_2 + \varepsilon_1 + \xi_1^{*T}(\phi_1(x_1) - \phi_1(\hat{x}_1))) \le 2S_1^2 + \frac{\|e\|^2}{2} + \frac{\varepsilon_1^{*2}}{2} + \|\xi_1^*\|^2 \tag{4.34}$$

$$S_i\eta_i \le \frac{S_i^2}{2} + \frac{\eta_i^2}{2} \tag{4.35}$$

$$-S_i\tilde{\xi}_i^T\phi_i(\hat{\bar{x}}_i)) \le \frac{S_i^2}{2} + \frac{\tilde{\xi}_i^T\tilde{\xi}_i}{2} \tag{4.36}$$

Substituting (4.9)–(4.11), (4.14)–(4.17), (4.10)–(4.31), (4.34)–(4.36) into (4.33) yields

$$\begin{aligned} {}_0^C D_t^\alpha V \le & -\bar{\lambda}\|e\|^2 + \tfrac{1}{2}\|P\|^2 \sum_{i=1}^{n} \tilde{\xi}_i^T\tilde{\xi}_i + \sum_{h=1}^{n}(-c_h S_h^2 + \tfrac{\kappa_h}{\gamma_h}\tilde{\xi}_h^T\xi_h) \\ & + \sum_{h=1}^{n-1}(\eta_h(-\tfrac{\eta_h}{\sigma_h} + \tfrac{\eta_h}{2} + W_h)) + \sum_{h=2}^{n}\tfrac{\tilde{\xi}_i^T\tilde{\xi}_i}{2} + \delta_1 + 0.557\varepsilon^* \end{aligned} \tag{4.37}$$

Note that $\Xi_0 = \{y_d^2 + ({}_0^C D_t^\alpha y_d)^2 + ({}_0^C D_t^\alpha({}_0^C D_t^\alpha y_d))^2 \le Z_0\}$ and $\Xi = \{V(t) \le q\}$ are compact sets, and thus $\Xi_0 \times \Xi$ is still a compact set. Since $W_i(\cdot)$ are continuous functions on $\Xi_0 \times \Xi$, there exist positive constants K_i such that $|W_i(\cdot)| \le K_i$.

Applying Young's inequality again, yields:

$$\tilde{\xi}_h^T\xi_h \le -\frac{1}{2}\tilde{\xi}_h^T\tilde{\xi}_h + \frac{1}{2}\|\xi_h^*\|^2 \tag{4.38}$$

$$\eta_h W_h \le \frac{\eta_h^2}{2} + \frac{K_h^2}{2} \tag{4.39}$$

Substituting (4.38) and (4.39) into (4.37), one has

$$\begin{aligned} {}_0^C D_t^\alpha V \le & -\bar{\lambda}\|e\|^2 + \tfrac{1}{2}\|P\|^2 \sum_{i=1}^{n}\tilde{\xi}_i^T\tilde{\xi}_i + \sum_{h=1}^{n-1}((1-\tfrac{1}{\sigma_h})\eta_h^2 + \tfrac{K_h^2}{2}) + \delta_1 \\ & + \sum_{h=1}^{n}(-\tfrac{\kappa_h}{2\gamma_h}\tilde{\xi}_i^T\tilde{\xi}_i - c_h S_h^2 + \tfrac{\kappa_h}{2\gamma_h}\|\xi_h^*\|^2 + \tfrac{\tilde{\xi}_i^T\tilde{\xi}_i}{2}) + 0.557\varepsilon^* \end{aligned} \tag{4.40}$$

Then, we have

$${}_0^C D_t^\alpha V \le -\mu V + D \tag{4.41}$$

where $\mu = \min\{2(\bar{\lambda}\big/\lambda_{\max}(P)), 2c_h, \kappa_h - \gamma_h(\|P\|^2 + 1), 2/\sigma_h - 2\}$, and $D = \delta_1 + \sum_{h=1}^{n-1}(K_h^2\big/2) + \sum_{h=1}^{n}(\kappa_h/2\gamma_h)\|\xi_h^*\|^2 + 0.557\varepsilon^*$.

Now, from (4.41) and according to Liu et al. [7] and Gong and Lan [22], one obtains

$${}_0^C D_t^\alpha V + \beta(t) = -\mu V + D \tag{4.42}$$

where $\beta(t) > 0$.

Applying the Lyapunov transform on (4.42) yields

$$\begin{aligned} V(s) &= \tfrac{s^{\alpha-1}V(0)}{s^\alpha+\mu} + \tfrac{D}{s(s^\alpha+\mu)} - \tfrac{\beta(s)}{s^\alpha+\mu} \\ &= \tfrac{s^{\alpha-1}V(0)}{s^\alpha+\mu} + \tfrac{s^{\alpha-(\alpha+1)}D}{s^\alpha+\mu} - \tfrac{\beta(s)}{s^\alpha+\mu} \end{aligned} \tag{4.43}$$

Using the inverse Laplace transform on (4.43), yields

$$\begin{aligned} V(t) &= E_{\alpha,1}(-\mu t^{\alpha})V(0) + t^{\alpha}E_{\alpha,\alpha+1}(-\mu t^{\alpha})D \\ &\quad -\beta(t) * t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^{\alpha}) \end{aligned} \tag{4.44}$$

where $*$ denotes the convolution operator.

Notably, $\beta(t) \geq 0$ and $t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^{\alpha}) \geq 0$ in (4.44), so $\beta(t) * t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^{\alpha}) \geq 0$. Thus, one has from (4.44)

$$V(t) \leq E_{\alpha,1}(-\mu t^{\alpha})V(0) + t^{\alpha}E_{\alpha,\alpha+1}(-\mu t^{\alpha})D \tag{4.45}$$

From Lemma 1, it follows that

$$\left|t^{\alpha}E_{\alpha,\alpha+1}(-\mu t^{\alpha})D\right| \leq \frac{Dt^{\alpha}d}{1+|\mu t^{\alpha}|} \leq \frac{Dd}{\mu} \tag{4.46}$$

Therefore, for $t \geq 0$, it follows that

$$V(t) \leq V(0)E_{\alpha,1}(-\mu t^{\alpha}) + \frac{Dd}{\mu} \tag{4.47}$$

Due to Lemma 1, one obtains

$$\left|E_{\alpha,1}(-\mu t^{\alpha})\right| \leq \frac{\lambda}{1+\mu t^{\alpha}} \tag{4.48}$$

Then, one writes (4.48) as follows:

$$V(t) \leq V(0)\frac{\lambda}{1+\mu t^{\alpha}} + \frac{Dd}{\mu} \tag{4.49}$$

From (4.49), it follows:

$$\frac{1}{2}S_1^2 \leq V(t) \leq V(0)\frac{\lambda}{1+\mu t^{\alpha}} + \frac{Dd}{\mu} \tag{4.50}$$

$$\frac{1}{2}e^T Pe \leq V(t) \leq V(0)\frac{\lambda}{1+\mu t^{\alpha}} + \frac{Dd}{\mu} \tag{4.51}$$

Therefore, one has

$$|S_1| \leq \sqrt{\frac{2\lambda V(0)}{1+\mu t^{\alpha}} + \frac{2Dd}{\mu}} \tag{4.52}$$

$$\|e\| \leq \sqrt{\frac{2\lambda V(0)}{1+\mu t^{\alpha}(\lambda_{\min}(P))} + \frac{2Dd}{\mu(\lambda_{\min}(P))}} \tag{4.53}$$

Based on (4.49), (4.52) and (4.53), we proved that the controlled system can be stable, and $\lim_{t\to\infty}|S_1| = \sqrt{2Dd/\mu}$ and $\lim_{t\to\infty}\|e\| = \sqrt{2Dd/(\mu(\lambda_{\min}(P)))}$, which means that $S_i (i = 1, ..., n)$, e and $\tilde{\xi}$ are bounded. Furthermore, as ξ_i^* is bounded, ξ_i as also bounded. Finally, by choosing design parameters, the tracking error and observer error are reduced.

Remark 5: From (4.46), it can be concluded that smaller tracking error can be obtained by increasing μ or decreasing D. Consequently, for smaller D, we can appropriately decrease κ_h or increase γ_h according on the definition of $D = \delta_1 + \sum_{h=1}^{n-1} (K_h^2 \big/ 2) + \sum_{h=1}^{n} (\kappa_h/2\gamma_h)\|\xi_h^*\|^2 + 0.557\varepsilon^*$. So as to get larger μ, we can appropriately increase c_h and κ_h or decrease γ_h and σ_h by the definition of $\mu = \min\{2(\bar{\lambda} \big/ \lambda_{\max}(P)), 2c_h, \kappa_h - \gamma_h(\|P\|^2 + 1), 2/\sigma_h - 2\}$.

Ultimately, Zeno behavior is removed through the following proof. By recalling the measurement error $\nu(t) = \omega(t) - u(t)$, we have

$$ {}_0^C D_t^\alpha |\nu| = {}_0^C D_t^\alpha \sqrt{\nu \cdot \nu} = sign(\nu) {}_0^C D_t^\alpha \nu \leq \left| {}_0^C D_t^\alpha \omega \right| \tag{4.54} $$

From (4.20) that $\omega(t)$ is a differentiable signal of order α, and ${}_0^C D_t^\alpha \omega$ is a bounded function.

Thus, the existence of $\rho > 0$ makes $\left| {}_0^C D_t^\alpha \omega \right| \leq \rho$ hold. According to $\nu(t_k) = 0$ and $\lim_{t \to t_{k+1}} \nu(t) = m$, one can have $t_{k+1} - t_k \geq m/\rho$. So, Zeno behavior does not occur.

Remark 6: It is worth noting that the control schemes designed by the authors in [7–14] are based on time triggered control. Because the control signal is sampled and updated periodically, it leads to a waste of communication resources. In order to solve this issue, an event-triggered mechanism is introduced in the backstepping technology. This mechanism enables the control signal to be sampled and updated only when the given conditions cannot be met, thus decreasing the communication load.

Remark 7: The repeated differentiation of virtual control function will lead to complexity explosion, so filter is introduced to solve this problem. By using event-triggered mechanism and dynamic surface filter at the same time, this paper greatly saves the waste of computing resources.

Remark 8: In [15, 16], it is important to study the adaptive ETC for integer-order nonlinear systems. However, we have designed an event-triggered rule for FONSs with unmeasurable states. Within the framework of event-triggered DSC scheme, we solved the computing explosion caused by duplicate derivation of virtual controllers. Unlike event-triggered rule in Wang et al. [17], threshold is a function of system state or tracking error. This paper determines the improved event-triggered mechanism based on the size of the control signal itself.

5. Simulation example

This part gives a simulation example to verify the availability of theoretical results.

Example: The fractional-order strict-feedback system is described as follows:

$$ \begin{cases} {}_0^C D_t^\alpha x_1 = x_2 + (x_1 - x_1^3) \\ {}_0^C D_t^\alpha x_2 = u + 20x_1 - x_2 \\ \qquad y = x_1 \end{cases} \tag{5.1} $$

when $f_1(x_1) = x_1 - x_1^3, f_2(x_1, x_2) = 20x_1 - x_2$.

The membership functions can be chosen as

$$ \mu_{A_1^l}(x_1) = e^{-\frac{(x_1 - 2 + l)^2}{16}}, \mu_{A_2^l}(x_2) = e^{-\frac{(x_2 - 2 + l)^2}{16}}, l = 1, ..., 5 \tag{5.2} $$

The given reference signal is $y_d = \cos(2t)$. The observer gain is selected as $K_1 = [k_1, k_2]^T = [10, 220]^T$. Then, the observer (3.3) can be written as

$$ \begin{aligned} {}_0^C D_t^\alpha \hat{x}_i &= A\hat{x}_i + Ly + \sum_{i=1}^{n} B_i \hat{f}_i(\hat{\bar{x}}_i | \xi_i) + Bu \\ \hat{y} &= C\hat{x}_1 \end{aligned} \tag{5.3} $$

Further, given $Q = 3I$, a positive definite matrix $P = \begin{bmatrix} 0.3014 & 0.0136 \\ 0.0136 & 66.4364 \end{bmatrix}$ can be obtained by solving the Lyapunov Eq (3.2).

The control law can be given as

$$\begin{aligned} \tau_1 &= -c_1 S_1 - \tfrac{5}{2} S_1 - \xi_1^T \phi_1(\hat{x}_1) + {}_0^C D_t^\alpha y_d \\ \tau_2 &= -c_2 S_2 - k_2 e_1 - \xi_2^T \phi_2(\hat{\bar{x}}_i) - S_1 + {}_0^C D_t^\alpha \upsilon_1 - \tfrac{1}{2} S_2 \end{aligned} \tag{5.4}$$

The event-triggered controller is devised as

$$\omega(t) = -(1+\bar{\delta})(\tau_2 \tanh(\frac{S_2 \tau_2}{\psi}) + \bar{m} \tanh(\frac{S_2 \bar{m}}{\psi})). \tag{5.5}$$

$$u(t) = \omega(t_k), \forall t \in [t_k, t_{k+1}) \tag{5.6}$$

The event-triggered condition can be designed as

$$t_{k+1} = \inf\{t \in R \,|\, |v(t)| \geq \bar{\delta}\, |u(t)| + m\}. \tag{5.7}$$

where $\bar{\delta} \in (0,1)$, $m > 0$ and $\bar{m} > [m \big/ (1-\bar{\delta})]$. With the following adaptive laws:

$$\begin{aligned} {}_0^C D_t^\alpha \xi_1 &= \gamma_1 S_1 \phi_1(\hat{x}_1) - \kappa_1 \xi_1 \\ {}_0^C D_t^\alpha \xi_2 &= \gamma_2 S_2 \phi_2(\hat{\bar{x}}_i) - \kappa_2 \xi_2 \end{aligned} \tag{5.8}$$

In the simulation, choose the parameters as follows: $\alpha = 0.98$, $c_1 = 5$, $c_2 = 15$, $\kappa_1 = 0.01$, $\kappa_2 = 0.02$, $\bar{\delta} = 0.45$, $\bar{m} = 0.15$ and $\psi = 0.4$. The system states initial conditions are chosen as $x_1(0) = 0.4$, $x_2(0) = 0.5$, $\hat{x}_1(0) = 0.5$, $\hat{x}_2(0) = 0.5$, $\xi_1(0) = [-0.1, -0.3, -0.5, -0.7, -0.9]^T$ and $\xi_2(0) = [-0.1, -0.3, -0.5, -0.7, -0.9]^T$.

The simulation results are shown in Figures 1–6. Figure 1 shows the curves of reference signal y_d and the system output y. Figure 2 shows the curves of reference signal y_d and the system output y without ETC. From Figures 1 and 2, we can see that ETC can ensure satisfactory system performance. The trajectories of the tracking error are shown in Figure 3. Figures 4 and 5 response of x_i and $\hat{x}_i$, $i = 1, 2$. Figure 6 responses of u. Figure 7 shows the trigger time intervals with the event-triggered control. However, by calculating, we know that the controller executes 2000 times without event-triggered control; under the event-triggered control method, the number of samples is only 1007, which greatly reduces the waste of communication resources. From Figures 1–7, it is concluded that the proposed event-triggered controller can achieve the stability of the controlled system and effectively decrease the communication load.

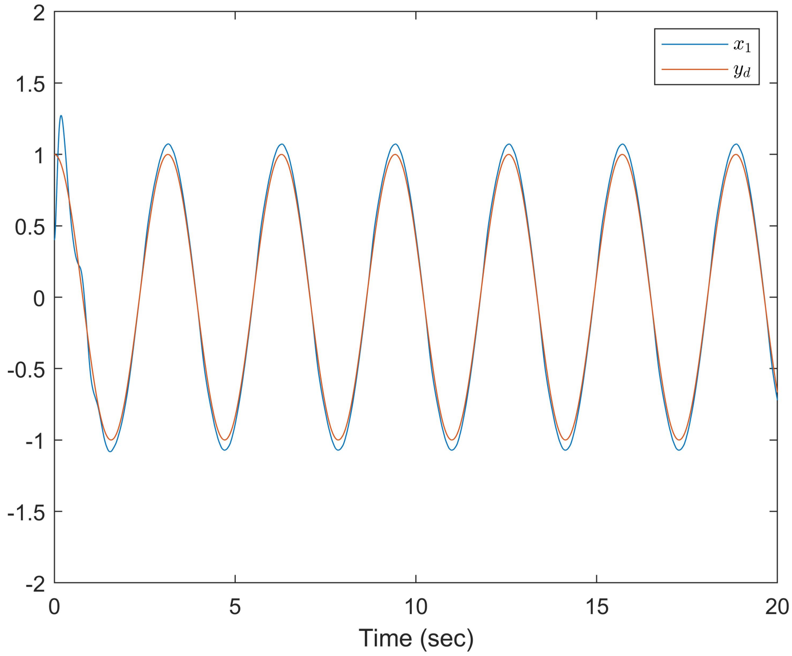

Figure 1. Curves of $y(t)$ and $y_d(t)$.

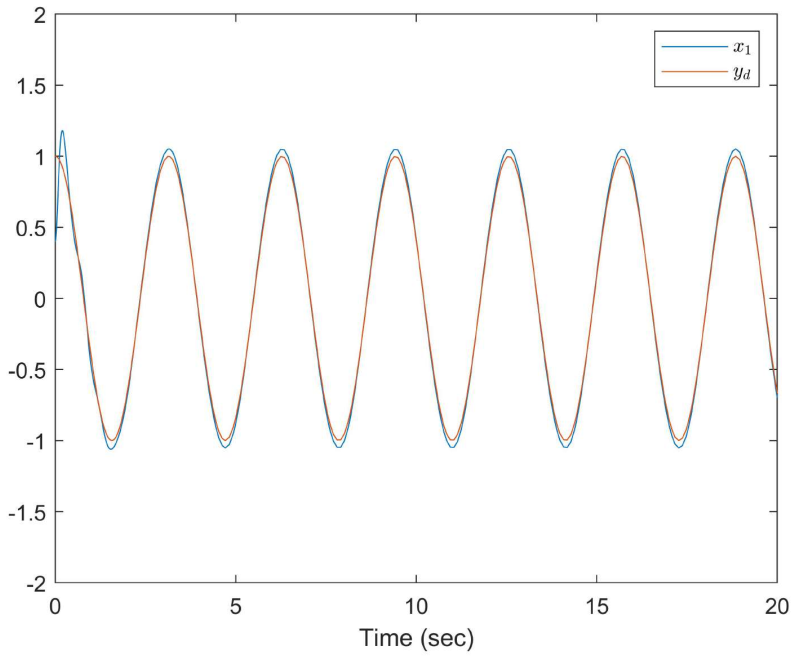

Figure 2. Curves of $y(t)$ and $y_d(t)$ (without event-triggered control).

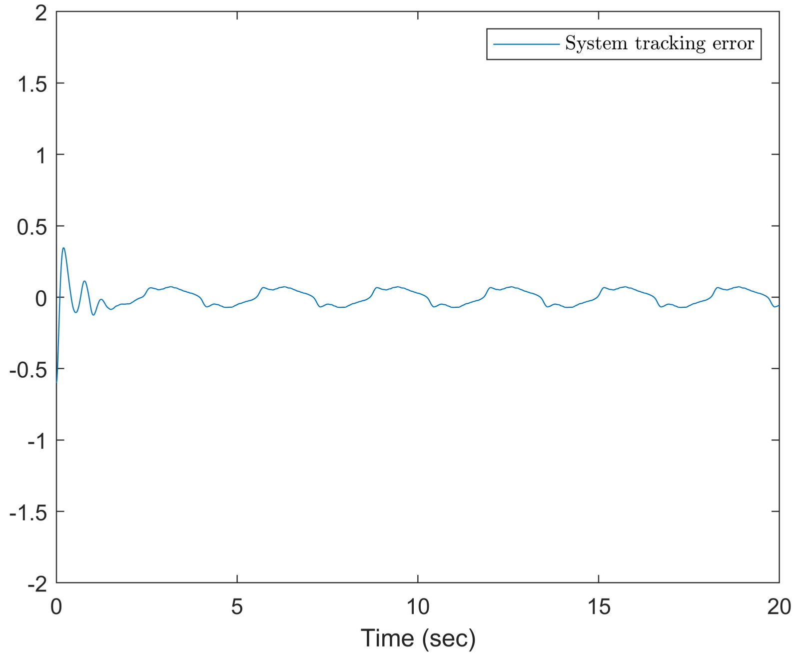

Figure 3. Curve of S_1.

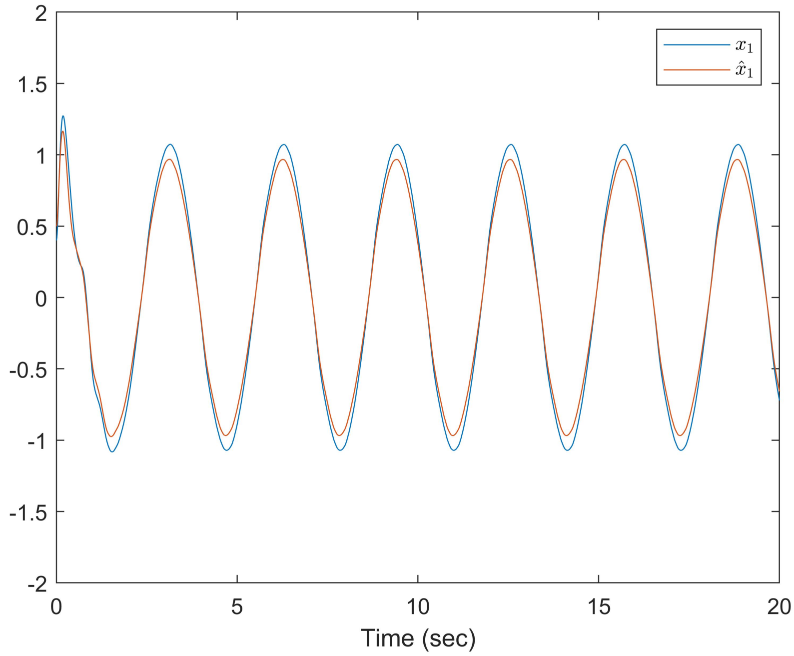

Figure 4. Curves of $x_1(t)$ and $\hat{x}_1(t)$.

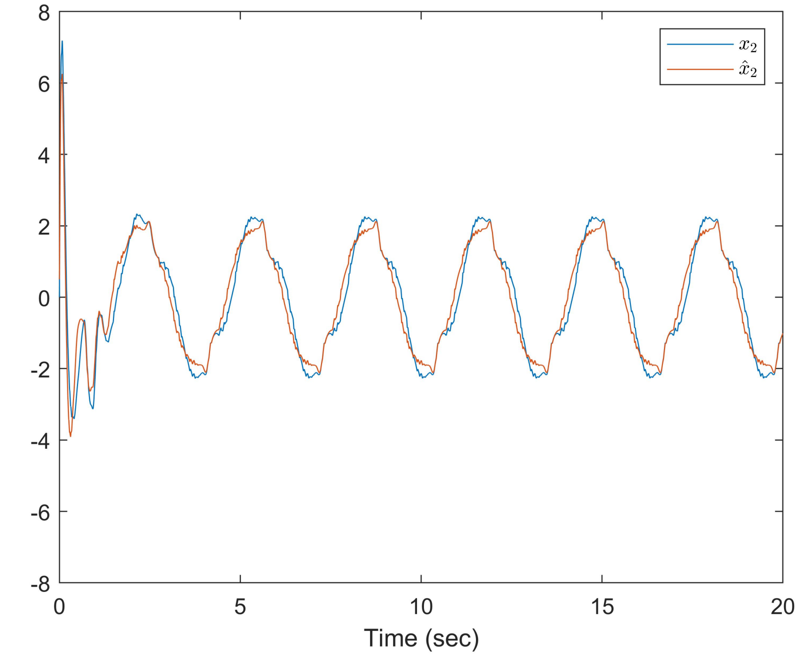

Figure 5. Curves of $x_2(t)$ and $\hat{x}_2(t)$.

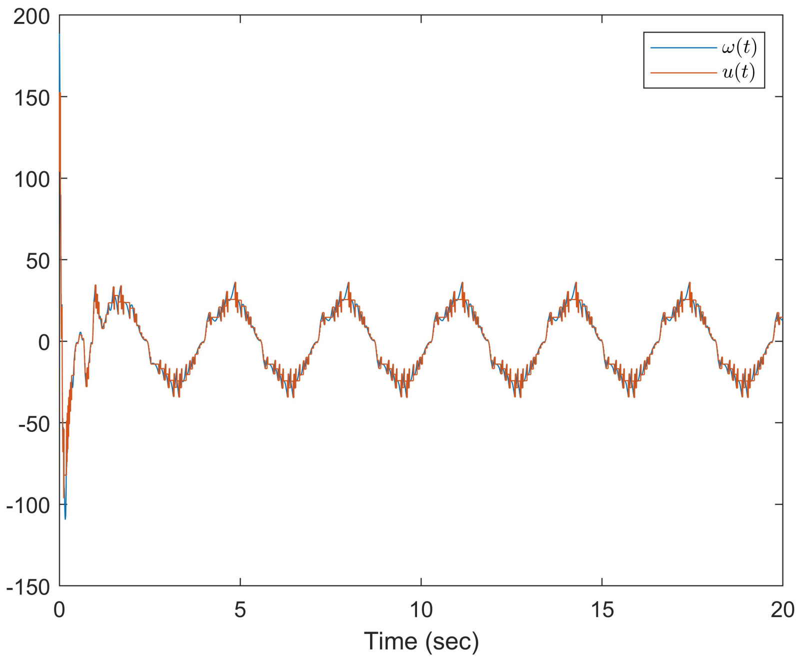

Figure 6. Control input.

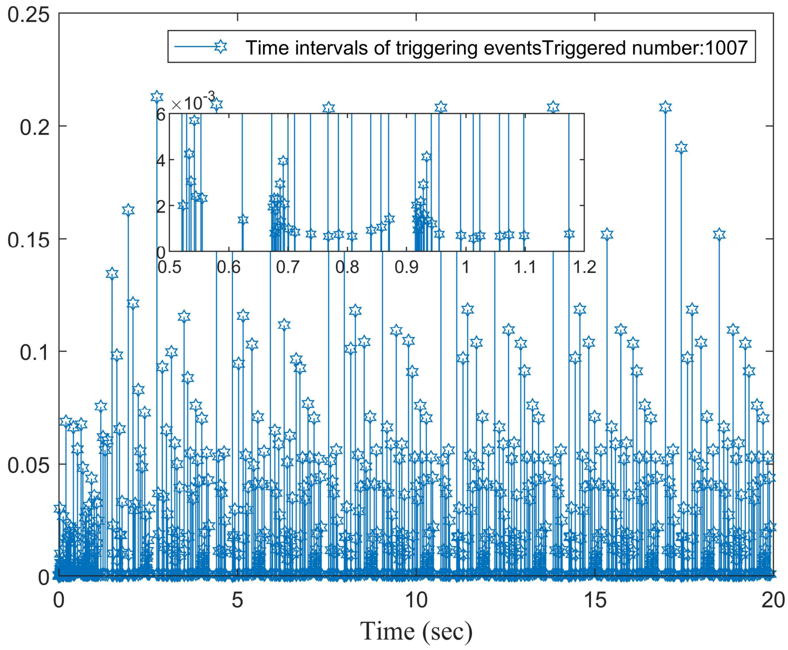

Figure 7. Nonzero interevent times.

6. Conclusions

The observer-based adaptive ETC algorithm for FONS was investigated in this study. By employing FLS to model the unknown dynamics, a fuzzy state observer is constructed for the unmeasurable state vectors. Using an adaptive backstepping control algorithm, an observer-based adaptive fuzzy DSC method is proposed. The put forward control algorithm has avoided computational complexity problem resulted in the repeated iteration of virtual controllers in the inherent backstepping method. Additionally, it has reduced the burden of communication and removed the Zeno behavior. The simulation results testify the validity of the controller. The further research will focus on the intelligent adaptive ETC problem of fractional-order nonlinear impulsive systems based on this study and literature [23,24].

Acknowledgments

This work is supported in part by the National Natural Science Foundation of China (under Grant No. 62173172), and in part by the Doctoral Startup Fund of Liaoning University of Technology (under Grant No. XB2021015).

Conflict of interest

The authors declare that there is no conflict of interest.

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