Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/mbe

MBE, 19(10): 9709–9729.
DOI: 10.3934/mbe.2022451
Received: 09 March 2022
Revised: 21 May 2022
Accepted: 06 June 2022
Published: 04 July 2022

Research article

T-S fuzzy observer-based adaptive tracking control for biological system with stage structure

Yi Zhang[1], Yue Song[1,*] and Song Yang[2]

[1] School of Science, Shenyang University of Technology, Shenyang 110870, China
[2] Station for Drug and Instrument Control of Shenyang Joint Logistics Support Center, Shenyang 110026, China

* **Correspondence:** Email: songyue@smail.sut.edu.cn.

Abstract: In this paper, the T-S fuzzy observer-based adaptive tracking control of the biological system with stage structure is studied. First, a biological model with stage structure is established, and its stability at the equilibrium points is analyzed. Considering the impact of reducing human activities on the biological population, an adaptive controller is applied to the system. Since it is difficult to measure density directly, a fuzzy state observer is designed, which is used to estimate the density of biological population. At the same time, the density of predators can track the desired density through the adjustment of adaptive controller. The stability of the biological system is guaranteed, and the observer error and tracking error are shown to converge to zero. Finally, the effectiveness of the proposed adaptive control method is verified by numerical simulation.

Keywords: stage structure; adaptive control; tracking control; T-S fuzzy; fuzzy state observer; biological system

1. Introduction

For a biological system, the biological individual development has a trend from infancy to maturity in the natural state. The corresponding structural and functional characteristics will also change significantly. When studying biological dynamic systems, researchers often assumed that there was little difference between infancy and maturity [1]. However, because different stages of the biological population have different viabilities, this characteristic will affect the population density to a certain extent. The biological model with stage structure is closer to reality. The prey-predator model with stage structure was studied in [2–4]. In addition, the equilibrium attraction and biological control were studied for this model in [4]. The dynamics of biological systems with stage structure were studied in [5].

From the above research, it can be seen that the exploration of biological systems with stage structure is deepening. Many studies have been done to analyze the stability of biological systems, which is also an important property in biological systems. However, considering the impact of external factors on the density of a biological population, it is necessary to explore the prey-predator model from multiple perspectives. For example, due to the threat posed by the spread of COVID-19, we need to implement a lockdown policy immediately. The sudden outbreak led many countries to implement lockdown policies [6–9]. To a certain extent, it limits human behavior and has a certain impact on the density of the biological population. Therefore, it is natural to study the effects of reducing human behavior on biological populations. In order to ensure the sustainable development of the biological environment under human intervention, intelligent control is needed to manage the prey-predator population.

In recent years, more and more people have begun to study adaptive control methods, and the research results are constantly emerging. By continuously monitoring the controlled object and adjusting the control parameters according to its changes, the adaptive control method can make the system run in the optimal or suboptimal state. Applying adaptive control to biological systems has important implications for managing population density. The problem of epidemic outbreak with adaptive gain was studied in [10], and a nonlinear robust adaptive integral sliding mode controller was proposed. On the basis of reference [10, 11] not only studied the adaptive control of the influenza model but also studied the sliding mode control of the influenza model with uncertainty. For an active suspension system, an adaptive control method was proposed in [12], and a fault-tolerant controller was also designed for actuator faults. Aiming at the problems of unknown faults and actuator sticking in [13], a solution was proposed by combining the adaptive fault-tolerant control and boundary vibration method. Different from [12, 13], an observer-based adaptive centralized control algorithm was proposed using adaptive backstepping control design technology. The adaptive control problem of a multiple-input multiple-output uncertain nonlinear system was studied in [14]. Same as in [14], a new adaptive fuzzy controller was proposed by using backstepping technology to study the finite time tracking problem of a nonlinear pure feedback system in [15]. In [16], an event-triggered adaptive control strategy was proposed in order to solve the tracking control problem of a strict feedback system with uncertainty in a fixed time. For uncertain nonlinear systems with time-varying disturbance, a sliding mode control strategy based on an adaptive reaching law was proposed in [17]. For a nonlinear system with uncertainty and external disturbance, a new adaptive fuzzy sliding mode controller was proposed in [18]. In the autopilot system, in order to improve the control performance, an adaptive controller was designed in [19]. Taking the same approach, an adaptive fuzzy control scheme using fuzzy logic system was designed in [20]. It can be seen that adaptive control can be widely used in aerospace and mechanical systems. Applying adaptive control to biological systems, the research on adaptive control of biological systems is relatively rare, which is an important reason for our research.

Many physical controlled objects can be portrayed using nonlinear systems in real life. For a stochastic nonlinear system, the classical quadratic function was used to construct an adaptive tracking controller in [21]. The problem of adaptive tracking control for a class of uncertain switched nonlinear systems was studied in [22]. Based on backstepping and Lyapunov functions, an adaptive tracking controller was proposed. An adaptive tracking control was developed based on fuzzy approximation technology and event-triggered for a nonlinear system with uncertainty in [23]. A novel event-triggered tracking controller was designed by introducing a fuzzy logic system to approximate the unknown nonlinear term in [24]. A new speed curve optimization tracking control method was proposed in [25], and a sliding mode controller was designed for the speed curve tracking problem under bounded disturbance. The tracking control problem of a nonlinear system was studied in [26], which introduced an adaptive control strategy and constructed an adaptive tracking controller using a new Lyapunov Krasovskii functional. An adaptive sliding mode control scheme based on backstepping was proposed in [27] to make the tracking error converge to zero in the presence of uncertain nonlinearity and disturbance. Based on the backstepping design, an adaptive control strategy was proposed to offset the error in [28]. Applying the adaptive tracking control to the biological system not only ensures the stability of the system, but also ensures the tracking performance.

The purpose of this paper is to control the density of predators to track a desired density through an adaptive tracking controller, which is undoubtedly a very meaningful and challenging work. Adaptive controllers for observer-based nonlinear systems were designed in [29, 30]. Due to the difficulty of measuring density involved in this paper, the T-S fuzzy method is used to construct a fuzzy observer, which is used to estimate the population density in biological system. An adaptive tracking controller is designed to make sure the density of predators can track the desired density. The main contributions of this paper are as follows: 1) A fuzzy state observer is designed, which can effectively observe the state of the system. In biological systems, it is difficult to accurately determine the density of a biological population, and an observer is required to estimate the biological population density. 2) An adaptive controller is designed, which can guarantee the effective tracking of predator population density. This is convenient for people to accurately grasp the density of predators, and it is conducive to the stability of ecological balance.

This paper is organized as follows. In Section 2, a biological system with stage structure is established, and its stability is analyzed. In Section 3, a fuzzy state observer is designed to estimate the density of biological population. Then, an adaptive controller is proposed. In Section 4, the validity of the observer, and the stability and tracking convergence of the biological system under the regulation of the controller are verified. Finally, a numerical simulation is carried out to support viewpoints in this paper.

Notations: The superscript T of a matrix represents its matrix transpose. $det(\cdot)$ stands for the determinant of a square matrix. $Re(c)$ means the real part of a complex number c. $\|.\|$ represents Euclidean norm.

2. Model establishment

Based on [31], the following system is established:

$$\begin{cases} \dot{x}_1(t) = ax_2(t) - b_1x_1(t) - \beta x_1(t) - s_1x_1^2(t) - \sigma x_1(t)y(t) \\ \dot{x}_2(t) = \beta x_1(t) - b_2x_2(t) \\ \dot{y}(t) = \sigma x_1(t)y(t) - b_3y(t) - s_2y^2(t), \end{cases} \tag{2.1}$$

where $x_1(t)$ and $x_2(t)$ represent the juveniles, density and adults, density of the prey population at time t, respectively; $y(t)$ represents the density of the predator population at time t; a represents the birth rate of the juvenile population, and β represents the transformation rate from the juvenile population to the adult population; b_1, b_2, b_3 represent the mortality rates of the juvenile prey population, adult prey population and predator population, respectively; s_1, s_2 represent the intraspecific competition intensities of the prey adult population and predator population in order to compete for food, respectively; σ represents the predation rate of the predator population to the prey population.

In system (2.1), all constants involved are positive. Considering the limitations of the actual ecological environment, the juveniles and adults density of the prey population and the density of the predator cannot exceed the maximum carrying capacity of the environment. The state variables and parameters meet the following conditions:

$$0 < x_1 < x_{1\max}, 0 < x_2 < x_{2\max}, 0 < y < y_{\max},$$

where $x_{1\max}, x_{2\max}, y_{\max}$ represent the maximum environmental carrying capacities of the juvenile prey population, adult prey population and predator population, respectively.

It can be seen that system (2.1) has four equilibrium points $P_1(0,0,0)$, $P_2(x_{12}, x_{22}, y_2)$, $P_3(x_{13}, x_{23}, 0)$, $P_4\left(0, 0, -\frac{b_3}{s_2}\right)$, where

$$\begin{cases} x_{12} = \dfrac{a\beta s_2 - b_1 b_2 s_2 - b_2\beta s_2 + b_2 b_3\sigma}{b_2\sigma^2 + b_2 s_1 s_2} \\ x_{22} = \dfrac{\beta\left(a\beta s_2 - b_1 b_2 s_2 - b_2\beta s_2 + b_2 b_3\sigma\right)}{b_2\left(b_2\sigma^2 + b_2 s_1 s_2\right)} \\ y_2 = \dfrac{a\beta\sigma - b_2 b_3 s_1 - b_1 b_2\sigma - b_2\beta\sigma}{b_2\sigma^2 + b_2 s_1 s_2} \\ x_{13} = \dfrac{a\beta - b_1 b_2 - b_2\beta}{b_2 s_1} \\ x_{23} = \dfrac{a\beta^2 - b_1 b_2\beta - b_2\beta^2}{b_2^2 s_1} \end{cases}$$

Because $-\frac{b_3}{s_2} < 0$, and the biological population density involved in this paper is positive, the equilibrium points P_1, P_2, P_3 are considered. At the equilibrium points P_2, P_3, in order to ensure the existence of biological populations, the following conditions should be satisfied:

$$x_{12} > 0, x_{22} > 0, y_2 > 0, x_{13} > 0, x_{23} > 0$$

that is, $a\beta\sigma - b_2 b_3 s_1 - b_1 b_2\sigma - b_2\beta\sigma > 0, a\beta - b_1 b_2 - b_2\beta > 0$.

Theorem 2.1. *1) System* (2.1) *is unstable at the equilibrium points* $P_1(0,0,0)$.
2) System (2.1) *is stable at the equilibrium points* $P_2(x_{12}, x_{22}, y_2)$ *when the following inequality holds.*

$$\begin{cases} -\sigma x_{12} + b_3 + 2s_2 y_2 > 0 \\ \sigma^2 x_{12} y_2 - a\beta > 0 \\ \sigma^2 x_{12} y_2 - a\beta\left(-\sigma x_{12} + b_3 + 2s_2 y_2\right) > 0 \end{cases}$$

3) System (2.1) *is stable at the equilibrium point* $P_3(x_{13}, x_{23}, 0)$ *when the following inequality holds.*

$$\begin{cases} -\sigma x_{13} + b_3 > 0 \\ b_2\left(b_1 + \beta + 2s_1 x_{13}\right) - a\beta > 0 \end{cases}$$

Proof. The Jacobian matrix of system (2.1) is

$$J = \begin{bmatrix} -b_1 - \beta - 2s_1x_1 - \sigma y & a & -\sigma x_1 \\ \beta & -b_2 & 0 \\ \sigma y & 0 & \sigma x_1 - b_3 - 2s_2 y \end{bmatrix}$$

It can be obtained that the characteristic polynomial of system (2.1) at the equilibrium point $P_1(0,0,0)$ is

$$\begin{aligned} det(\lambda E - J)\big|_{P_1} &= \begin{vmatrix} \lambda + b_1 + \beta & -a & 0 \\ -\beta & \lambda + b_2 & 0 \\ 0 & 0 & \lambda + b_3 \end{vmatrix} \\ &= (\lambda + b_3)\left[(\lambda + b_1 + \beta)(\lambda + b_2) - a\beta\right] \\ &= (\lambda + b_3)\left[\lambda^2 + (b_1 + b_2 + \beta)\lambda + b_2(b_1 + \beta) - a\beta\right] \end{aligned}$$

Since $b_1 + b_2 + \beta > 0, b_2(b_1 + \beta) - a\beta < 0$, it can be obtained that the system is unstable at the equilibrium point $P_1(0,0,0)$.

It can be obtained that the characteristic polynomial of the system at the equilibrium point $P_2(x_{12}, x_{22}, y_2)$ is

$$\begin{aligned} det(\lambda E - J)\big|_{P_2} &= \begin{vmatrix} \lambda + \Phi_1 & -a & \sigma x_{12} \\ -\beta & \lambda + b_2 & 0 \\ -\sigma y_2 & 0 & \lambda + \Phi_2 \end{vmatrix} \\ &= (\lambda + \Phi_2)\left[(\lambda + \Phi_1)(\lambda + b_2) - a\beta\right] + \sigma^2 x_{12} y_2 (\lambda + b_2), \end{aligned}$$

where $\Phi_1 = b_1 + \beta + 2s_1x_{12} + \sigma y_2, \Phi_2 = -\sigma x_{12} + b_3 + 2s_2y_2$. Integrating the above formula, we have

$$\lambda^3 + \Upsilon_1\lambda^2 + \Upsilon_2\lambda + \Upsilon_3 = 0,$$

where

$$\begin{aligned} \Upsilon_1 &= b_2 + \Phi_1 + \Phi_2 \\ \Upsilon_2 &= b_2(\Phi_1 + \Phi_2) + \Phi_1\Phi_2 + \sigma^2 x_{12} y_2 - a\beta \\ \Upsilon_3 &= b_2\Phi_1\Phi_2 + \sigma^2 x_{12} y_2 - a\beta\Phi_2 \end{aligned}$$

Since $-\sigma x_{12} + b_3 + 2s_2y_2 > 0, \sigma^2 x_{12}y_2 - a\beta > 0, \sigma^2 x_{12}y_2 - a\beta(-\sigma x_{12} + b_3 + 2s_2y_2) > 0$, we have $Re\lambda_1 < 0, Re\lambda_2 < 0, Re\lambda_3 < 0$, and $\Upsilon_1 > 0, \Upsilon_2 > 0, \Upsilon_3 > 0, \Upsilon_1\Upsilon_2 - \Upsilon_3 > 0$. According to the Routh-Hurwitz theorem [32], system (2.1) is stable at the equilibrium point $P_2(x_{12}, x_{22}, y_2)$.

Similarly, it can be obtained that the characteristic polynomial of the system at equilibrium point $P_3(x_{13}, x_{23}, 0)$ is

$$\begin{aligned} det(\lambda E - J)\big|_{P_2} &= \begin{vmatrix} \lambda + \Omega_1 & -a & \sigma x_{13} \\ -\beta & \lambda + b_2 & 0 \\ 0 & 0 & \lambda + \Omega_2 \end{vmatrix} \\ &= (\lambda + \Omega_2)\left[(\lambda + \Omega_1)(\lambda + b_2) - a\beta\right] \end{aligned}$$

$$= (\lambda + \Omega_2)\left[\lambda^2 + (b_2 + \Omega_1)\lambda + b_2\Omega_1 - a\beta\right]$$

where $\Omega_1 = b_1 + \beta + 2s_1x_{13}, \Omega_2 = -\sigma x_{13} + b_3$.

Since $-\sigma x_{13} + b_3 > 0, b_2(b_1 + \beta + 2s_1x_{13}) - a\beta > 0$, according to the Routh-Hurwitz theorem [32], system (2.1) is stable at the equilibrium point $P_3(x_{13}, x_{23}, 0)$. □

From a developmental perspective, we should adopt some appropriate strategies to control the density of the biological population and avoid the loss of biological population diversity. The following assumptions are proposed:

Assumption 2.1. The predator population mainly preys on the juvenile population $x_1(t)$, and the capture of the adult population $x_2(t)$ by predators can be ignored.

Assumption 2.2. Considering that reducing human activities will have a certain impact on the population of predators, $u(t)$ indicates that human activities are constrained at time t.

Under the above assumptions, the following biological system is established.

$$\begin{cases} \dot{x}_1(t) = ax_2(t) - b_1x_1(t) - \beta x_1(t) - s_1x_1^2(t) - \sigma x_1(t)y(t) \\ \dot{x}_2(t) = \beta x_1(t) - b_2x_2(t) \\ \dot{y}(t) = \sigma x_1(t)y(t) - b_3y(t) - s_2y^2(t) + u(t) \end{cases} \tag{2.2}$$

System (2.2) is obtained by applying control in the third equation of (2.1). When $u(t) = 0$, it implies that the biological system has not been interfered with by human behavior.

3. Main results

3.1. Observer design

In real life, the direct measurement of density is difficult, so the density of biological population must be estimated by introducing a fuzzy state observer. The T-S fuzzy method is used to construct an observer to estimate the state of the system.

For the design of the observer, we recall the following lemma.

Lemma 3.1. *In [33]: Given constant matrices Q and S, the symmetric constant matrices Z, J with $J > 0$, and a time-varying matrix $L(t)$ with appropriate dimension, the following matrix inequality holds*

$$QL(t)S + (QL(t)S)^T \leq Z$$

if $L(t)$ satisfies $L^T(t)L(t) \leq J$.

System (2.2) can be rewritten as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + BU(t) \tag{3.1}$$

where

$$A = \begin{bmatrix} -b_1 - \beta - s_1x_1 & a & -\sigma x_1 \\ \beta & -b_2 & 0 \\ \sigma y & 0 & -s_2y - b_3 \end{bmatrix}, \hat{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) & y(t) \end{bmatrix}^T$$

$$B=\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix},U(t)=\begin{bmatrix}1&1&u(t)\end{bmatrix}^T$$

Let

$$\Gamma_1(t)=-b_1-\beta-s_1x_1(t),\Gamma_2(t)=-\sigma x_1(t),\Gamma_3(t)=\sigma y(t),\Gamma_4(t)=-s_2y(t)-b_3;$$

we have

$$\begin{aligned}&\max\Gamma_1(t)=-b_1-\beta-s_1x_1^{\min}(t),\min\Gamma_1(t)=-b_1-\beta-s_1x_1^{\max}(t),\\&\max\Gamma_2(t)=-\sigma x_1^{\min}(t),\min\Gamma_2(t)=-\sigma x_1^{\max}(t),\\&\max\Gamma_3(t)=\sigma y^{\max}(t),\min\Gamma_3(t)=\sigma y^{\min}(t),\\&\max\Gamma_4(t)=-s_2y^{\min}(t)-b_3,\min\Gamma_4(t)=-s_2y^{\max}(t)-b_3.\end{aligned}$$

Using the principle of maximum and minimum, $\Gamma_1(t),\Gamma_2(t),\Gamma_3(t),\Gamma_4(t)$ can be expressed as

$$\begin{aligned}\Gamma_1(t)&=\mu_{11}(\Gamma_1(t))\max\Gamma_1(t)+\mu_{12}(\Gamma_1(t))\min\Gamma_1(t),\\\Gamma_2(t)&=\mu_{21}(\Gamma_2(t))\max\Gamma_2(t)+\mu_{22}(\Gamma_2(t))\min\Gamma_2(t),\\\Gamma_3(t)&=\mu_{31}(\Gamma_3(t))\max\Gamma_3(t)+\mu_{32}(\Gamma_3(t))\min\Gamma_3(t),\\\Gamma_4(t)&=\mu_{41}(\Gamma_4(t))\max\Gamma_4(t)+\mu_{42}(\Gamma_4(t))\min\Gamma_4(t),\end{aligned}$$

where $\mu_{i1}+\mu_{i2}=1,i=1,2,3,4$ are used to denote the membership function. The fuzzy rules can be found in the Appendix.

Let $\Gamma(t)=[\Gamma_1(t)\quad\Gamma_2(t)\quad\Gamma_3(t)\quad\Gamma_4(t)]^T$, $\Gamma_1(t)\in[\min\Gamma_1(t),\max\Gamma_1(t)],\Gamma_2(t)\in[\min\Gamma_2(t),\max\Gamma_2(t)]$, $\Gamma_3(t)\in[\min\Gamma_3(t),\max\Gamma_3(t)]$, $\Gamma_4(t)\in[\min\Gamma_4(t),\max\Gamma_4(t)]$. $k_{ij}(\Gamma_j(t))$ represents the grade of membership of $\Gamma_j(t)$ in the fuzzy set k_{ij}. Using the standard fuzzy blending method, we have

$$R_i(\Gamma(t))=\frac{\prod_{j=1}^{4}k_{ij}(\Gamma_j(t))}{\sum_{i=1}^{16}\prod_{j=1}^{4}k_{ij}(\Gamma_j(t))}\geq0,\sum_{i=1}^{16}R_i(\Gamma(t))=1.$$

Then, system (3.1) can be rewritten as

$$\dot{\hat{x}}(t)=\sum_{i=1}^{16}R_i(\Gamma(t))(A_i\hat{x}(t)+B_iU(t)).\tag{3.2}$$

Let $\sum_{i=1}^{16}R_i(\Gamma(t))A_i$, $\sum_{i=1}^{16}R_i(\Gamma(t))B_i$ be $\bar{A},\bar{B}$, respectively. Then, system (3.2) can be rewritten as

$$\dot{\hat{x}}(t)=\bar{A}\hat{x}(t)+\bar{B}U(t).\tag{3.3}$$

In order to facilitate the design of the observer, adding the output vector Y to system (3.3), we have

$$\begin{cases} \dot{\hat{x}}(t) = \bar{A}\hat{x}(t) + \bar{B}U(t) \\ Y(t) = C\hat{x}(t) \end{cases}, \tag{3.4}$$

where C is a matrix with appropriate dimension. For system (3.4), an observer of the following form is designed

$$\begin{cases} \dot{\tilde{x}}(t) = \bar{A}\tilde{x}(t) + \bar{B}U(t) + G\left(Y(t) - \tilde{Y}(t)\right) \\ \tilde{Y}(t) = C\tilde{x}(t) \end{cases}, \tag{3.5}$$

where $\tilde{x}(t) = \begin{bmatrix} \tilde{x}_1(t) & \tilde{x}_2(t) & \tilde{y}(t) \end{bmatrix}^T$ represents the state estimate of $\hat{x}(t)$, $\tilde{Y}(t)$ represents the output vector of the observer, $\left(\bar{A} - GC\right)^T\left(\bar{A} - GC\right) \leq J$, and $J > 0$ is a symmetric constant matrix.

By Lemma 3.1, we can have

$$\left(\bar{A} - GC\right) + \left(\bar{A} - GC\right)^T \leq Z,$$

where Z is a symmetric constant matrix. By choosing the matrices G, C such that all characteristic values of Z have negative real parts less than $-\varpi\,(\varpi > 0)$.

The observer error $\varsigma(t)$ can be obtained as $\varsigma(t) = \hat{x}(t) - \tilde{x}(t)$. From (3.4) and (3.5), the observer error system can be obtained:

$$\begin{aligned} \dot{\varsigma}(t) &= \dot{\hat{x}}(t) - \dot{\tilde{x}}(t) \\ &= \bar{A}\hat{x}(t) + \bar{B}U(t) - \bar{A}\tilde{x}(t) - \bar{B}U(t) - G\left(Y(t) - \tilde{Y}(t)\right) \\ &= \left(\bar{A} - GC\right)\varsigma(t). \end{aligned} \tag{3.6}$$

3.2. Controller design

Set $y_d(t)$ as the desired density of the predator population. The goal of our proposed adaptive control strategy is to steer the state variable $y(t)$ to track the desired density.

Assumption 3.1. The desired density $y_d(t)$ is bounded and satisfies

$$0 < y_d(t) < y_{\max}.$$

Remark 1. Assumption 3.1 is reasonable, because $y_d(t)$ represents the actual biological meaning.

It can be obtained from system (2.2):

$$u(t) = \dot{y}(t) - \sigma x_1(t)y(t) + b_3 y(t) + s_2 y^2(t). \tag{3.7}$$

Let $e(t) = y(t) - y_d(t)$ represent the error between the predator population and the desired density. By introducing a variable $\varphi(t) = \dot{y}_d(t) - \theta e(t)$ without considering $\dot{y}(t)$, where $\theta > 0$ is a positive constant, we have

$$\varphi(t) - \sigma x_1(t)y(t) + b_3 y(t) + s_2 y^2(t) = \varphi(t) + X(x_1, y)\delta, \tag{3.8}$$

where X is a vector about x_1, y. δ is a vector containing parameters, which can be obtained from (3.8).

$$X = \begin{bmatrix} -x_1 y & y & y^2 \end{bmatrix}, \delta = \begin{bmatrix} \sigma & b_3 & s_2 \end{bmatrix}^T$$

The adaptive controller can be designed as

$$u(t) = \dot{y}_d(t) - \theta e(t) - \sigma \tilde{x}_1(t) \tilde{y}(t) + b_3 \tilde{y}(t) + s_2 \tilde{y}^2(t) + \hat{\lambda}(t) \operatorname{sgn}(e(t)), \tag{3.9}$$

where $\tilde{x}_1(t)$ and $\tilde{y}(t)$ represent estimates of biological population density $x_1(t)$ and $y(t)$, respectively. In addition, $\hat{\lambda}(t)$ denotes the adaptive law to ensure the stability of the system when the model is uncertain.

Design an update law $\hat{\lambda}(t)$

$$\dot{\hat{\lambda}}(t) = -\varepsilon e(t), \tag{3.10}$$

where ε is a positive constant, representing the update rate of tracking error $e(t)$.

Therefore, (3.9) can be rewritten as

$$u(t) = \dot{y}_d(t) - \theta e(t) + \tilde{X}(\tilde{x}_1, \tilde{y}) \delta + \hat{\lambda}(t) \operatorname{sgn}(e(t)), \tag{3.11}$$

where $\tilde{X} = \begin{bmatrix} -\tilde{x}_1 \tilde{y} & \tilde{y} & \tilde{y}^2 \end{bmatrix}$.

Remark 2. Furthermore, in order to avoid the undesired chatter in the controller (3.11), the sign function can be replaced by an approximately continuous alternative in the implementation. We consider the tangent hyperbolic function $\tanh(\rho e(t))$ or the smoothing function $sat\left(\frac{e}{\chi}\right)$ instead of $\operatorname{sgn}(e(t))$, where

$$sat\left(\frac{e}{\chi}\right) = \begin{cases} \operatorname{sgn}(e(t)), |e| \geq \chi \\ \frac{e}{\chi}, |e| < \chi \end{cases}$$

ρ, χ are two constants. Besides, we can also avoid chattering by adding a low-pass filter at the output of the controller to eliminate high-frequency signals at the control input.

4. Stability analysis

In this section, a Lyapunov function is used to prove the effectiveness of the proposed observer and controller.

Firstly, using the proposed adaptive controller, (3.7) is substituted into (3.9):

$$\begin{aligned} \dot{y} - \dot{y}_d(t) + \theta e(t) = &-\sigma(\tilde{x}_1(t) \tilde{y}(t) - x_1(t) y(t)) + b_3(\tilde{y}(t) - y(t)) + s_2\left(\tilde{y}^2(t) - y^2(t)\right) \\ &+ \hat{\lambda}(t) \operatorname{sgn}(e(t)) \end{aligned}. \tag{4.1}$$

Equation (4.1) can be rewritten as

$$\dot{y} - \dot{y}_d(t) + \theta e(t) = \hat{X}(\hat{x}_1, \hat{y}) \delta + \hat{\lambda}(t) \operatorname{sgn}(e(t)), \tag{4.2}$$

where $\hat{X}(\hat{x}_1, \hat{y}) = \tilde{X}(\tilde{x}_1, \tilde{y}) - X(x_1, y) = \begin{bmatrix} \tilde{x}_1(t) \tilde{y}(t) - x_1(t) y(t) & \tilde{y}(t) - y(t) & \tilde{y}^2(t) - y^2(t) \end{bmatrix}^T$ represents the error vector, which is bounded.

Since $\dot{e}(t) = \dot{y}(t) - \dot{y}_d(t)$, it can be obtained that

$$\dot{e}(t) = -\theta e(t) + \hat{X}(\hat{x}_1, \hat{y})\delta + \hat{\lambda}(t)\,\mathrm{sgn}(e(t)). \tag{4.3}$$

Since the parameter vector δ is bounded, and the estimation error $\hat{X}(\hat{x}_1, \hat{y})$ of the state variable is bounded,it can be assumed that there is a positive constant γ such that $\hat{X}(\hat{x}_1, \hat{y})\delta$ is bounded, that is

$$\left|\hat{X}(\hat{x}_1, \hat{y})\delta\right| \le \gamma. \tag{4.4}$$

Next, the Lyapunov function is established as

$$V(t) = \frac{1}{2}\left[e^2(t) + \frac{1}{\varepsilon}\left(\hat{\lambda}(t) + \gamma\right)^2 + \varsigma^T\varsigma\right] \tag{4.5}$$

It can be seen from (4.5) that the Lyapunov function is a positive definite function. Substitute (3.6) into (4.5), and the time derivative of (4.5) can be obtained:

$$\dot{V}(t) = e(t)\dot{e}(t) + \frac{1}{\varepsilon}\dot{\hat{\lambda}}(t)\left(\hat{\lambda}(t) + \gamma\right) + \frac{1}{2}\varsigma^T\left[\left(\bar{A} - GC\right)^T + \left(\bar{A} - GC\right)\right]\varsigma. \tag{4.6}$$

Substitute (4.3) into the above equation to obtain

$$\begin{aligned}\dot{V}(t) = &-\theta e^2(t) + e(t)\hat{X}(\hat{x}_1, \hat{y})\delta + e(t)\hat{\lambda}(t)\,\mathrm{sgn}(e(t)) + \frac{1}{\varepsilon}\dot{\hat{\lambda}}(t)\left(\hat{\lambda}(t) + \gamma\right)\\ &+ \frac{1}{2}\varsigma^T\left[\left(\bar{A} - GC\right)^T + \left(\bar{A} - GC\right)\right]\varsigma\end{aligned}. \tag{4.7}$$

Since $e(t)\,\mathrm{sgn}(e(t)) = |e(t)|$, substitute (3.10) into (4.7), and we have

$$\begin{aligned}\dot{V}(t) = &-\theta e^2(t) + e(t)\hat{X}(\hat{x}_1, \hat{y})\delta + \hat{\lambda}(t)|e(t)| - e(t)\left(\hat{\lambda}(t) + \gamma\right)\\ &+ \frac{1}{2}\varsigma^T\left[\left(\bar{A} - GC\right)^T + \left(\bar{A} - GC\right)\right]\varsigma\end{aligned}. \tag{4.8}$$

Substitute (4.4) into (4.8), and we can get

$$\dot{V}(t) \le -\theta e^2(t) + |e(t)|\gamma + \hat{\lambda}(t)|e(t)| - |e(t)|\left(\hat{\lambda}(t) + \gamma\right) - \varpi\|\varsigma(t)\|^2 \tag{4.9}$$

This can be obtained by integrating the above formula.

$$\dot{V}(t) \le -\theta e^2(t) - \varpi\|\varsigma(t)\|^2 \tag{4.10}$$

Since θ, ϖ are two positive constants, $\dot{V}(t) \le 0$.

Theorem 4.1. *The proposed observer can effectively estimate the state of the system. The designed adaptive controller ensures the tracking convergence. When* $t \to \infty$, $\varsigma(t) \to 0$, $e(t) \to 0$. *That is the predator population will converge to the desired density,* $y(t) \to y_d(t)$.

Proof. According to Barbalat's Lemma [34], when $t \geq 0$, if κ is a uniformly continuous function, $\lim_{t\to\infty} \int_0^t \kappa(\tau)\, d\tau$ exists and is limited, and we have $\lim_{t\to\infty} \kappa(t) = 0$.

Suppose $\psi(t) = \theta e^2(t) + \varpi \|\varsigma(t)\|^2 \geq 0$, and (4.10) can be rewritten as

$$\dot{V}(t) \leq -\theta e^2(t) - \varpi \|\varsigma(t)\|^2 = -\psi(t). \tag{4.11}$$

Integrate (4.11) from 0 to $t \to \infty$ to obtain

$$V(\infty) - V(0) \leq -\lim_{t\to\infty} \int_0^t \psi(\tau)\, d\tau$$

that is,

$$\lim_{t\to\infty} \int_0^t \psi(\tau)\, d\tau \leq V(0) - V(\infty), \tag{4.12}$$

$\dot{V}(t) \leq 0$ is known from (4.10), and $V(\infty) - V(0) \leq 0$ can be obtained after integrating it, so $V(0) - V(\infty) \geq 0$. Combined with (4.12), $\lim_{t\to\infty} \int_0^t \psi(\tau)\, d\tau$ exists and is limited, and $\lim_{t\to\infty} \int_0^t \psi(\tau)\, d\tau$ is positive due to $\psi(t) \geq 0$. From Barbalat's Lemma

$$\lim_{t\to\infty} \psi(t) = 0,$$

that is,

$$\lim_{t\to\infty} \left(\theta e^2(t) + \varpi \|\varsigma(t)\|^2\right) = 0. \tag{4.13}$$

Since θ, ϖ are two positive constants, $e^2(t) \geq 0, \|\varsigma(t)\|^2 \geq 0$, and it can be seen from (4.13) that when $t \to \infty$, $e(t) \to 0, \varsigma(t) \to 0$. Therefore, the proposed adaptive controller can track the desired density of the predator population, that is, $y(t) \to y_d(t)$. □

5. Simulation

In recent years, human activities have seriously damaged the ecological environment. People have gradually realized the importance of protecting the ecological environment. In some of the latest research, it can be seen that human activities have led to the destruction of the ecological environment of a large number of biological species. Among them, *Lampetra japonica* is affected by soil erosion, and the living environments of spawning grounds and young fish are damaged. In addition, water pollution affects the living environment. So, its resources are quite small and in a vulnerable state. Therefore, it is urgent to reduce the impact on biological population density by controlling human behavior. In this paper, considering the survival of *Lampetra japonica*, the following parameters are selected according to its actual situation:

$$a = 0.25, \beta = 0.19, \sigma = 0.6, s_1 = 0.05,$$
$$s_2 = 0.06, b_1 = 0.09, b_2 = 0.07, b_3 = 0.07.$$

Then, the biological system can be obtained as

$$\begin{cases} \dot{x}_1(t) = 0.25x_2(t) - 0.09x_1(t) - 0.19x_1(t) - 0.05x_1^2(t) - 0.6x_1(t)y(t) \\ \dot{x}_2(t) = 0.19x_1(t) - 0.07x_2(t) \\ \dot{y}(t) = 0.6x_1(t)y(t) - 0.07y(t) - 0.06y^2(t) \end{cases}. \tag{5.1}$$

Considering the actual situation of the biological system, it follows that

$$x_1(t) \in [0,3], x_2(t) \in [0,6], y(t) \in [0,10].$$

At this time,

$$a\beta\sigma - b_2b_3s_1 - b_1b_2\sigma - b_2\beta\sigma = 0.016495 > 0, a\beta - b_1b_2 - b_2\beta = 0.0279 > 0.$$

When $u(t) = 0$, we have four equilibrium points of system (5.1) and take the positive equilibrium point as $P_2 = (0.1816, 0.4929, 0.6492)$.

After adding the controller, the system is

$$\begin{cases} \dot{x}_1(t) = 0.25x_2(t) - 0.09x_1(t) - 0.19x_1(t) - 0.05x_1^2(t) - 0.6x_1(t)y(t) \\ \dot{x}_2(t) = 0.19x_1(t) - 0.07x_2(t) \\ \dot{y}(t) = 0.6x_1(t)y(t) - 0.07y(t) - 0.06y^2(t) + u(t) \end{cases}.$$

Then, we have

$$\begin{aligned} &\max\Gamma_1(t) = -0.28, \min\Gamma_1(t) = -0.43, \\ &\max\Gamma_2(t) = 0, \min\Gamma_2(t) = -1.8, \\ &\max\Gamma_3(t) = 6, \min\Gamma_3(t) = 0, \\ &\max\Gamma_4(t) = -0.07, \min\Gamma_4(t) = -0.67. \end{aligned}$$

So, the fuzzy model is

$$\dot{\hat{x}}(t) = \sum_{i=1}^{16} R_i(\Gamma(t))(A_i\hat{x}(t) + B_iU(t)),$$

where

$$A_1 = \begin{bmatrix} -0.28 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.07 \end{bmatrix}, A_2 = \begin{bmatrix} -0.28 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.67 \end{bmatrix}, A_3 = \begin{bmatrix} -0.28 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.07 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -0.28 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.67 \end{bmatrix}, A_5 = \begin{bmatrix} -0.28 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.07 \end{bmatrix}, A_6 = \begin{bmatrix} -0.28 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.67 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} -0.28 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.07 \end{bmatrix}, A_8 = \begin{bmatrix} -0.28 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.67 \end{bmatrix}, A_9 = \begin{bmatrix} -0.43 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.07 \end{bmatrix},$$

$$A_{10} = \begin{bmatrix} -0.43 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.67 \end{bmatrix}, A_{11} = \begin{bmatrix} -0.43 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.07 \end{bmatrix}, A_{12} = \begin{bmatrix} -0.43 & 0.25 & 0 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.67 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} -0.43 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.07 \end{bmatrix}, A_{14} = \begin{bmatrix} -0.43 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 6 & 0 & -0.67 \end{bmatrix}, A_{15} = \begin{bmatrix} -0.43 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.07 \end{bmatrix},$$

$$A_{16} = \begin{bmatrix} -0.43 & 0.25 & -1.8 \\ 0.19 & -0.07 & 0 \\ 0 & 0 & -0.67 \end{bmatrix}, B_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, i = 1, 2, \ldots, 16.$$

The membership function can be obtained:

$$\mu_{11}=\frac{100\Gamma_1(t)+43}{15},\mu_{12}=\frac{-28-100\Gamma_1(t)}{15},\mu_{21}=\frac{5\Gamma_2(t)+9}{9},\mu_{22}=\frac{-5\Gamma_2(t)}{9},$$
$$\mu_{31}=\frac{\Gamma_3(t)}{6},\mu_{32}=\frac{6-\Gamma_3(t)}{6},\mu_{41}=\frac{100\Gamma_4(t)+67}{60},\mu_{42}=\frac{-7-100\Gamma_4(t)}{60}.$$

Let

$$Q=\begin{bmatrix}0.2&0.1&1\\0.1&0.2&10\\1&10&0.2\end{bmatrix},G=\begin{bmatrix}-2.0797&0.5029&-1.2395\\0.2227&0.3810&0.5019\\1.7537&-1.4476&4.5462\end{bmatrix},C=\begin{bmatrix}-1&-3&10\\-3&-2&0.5\\10&0.5&-5\end{bmatrix}$$

The observer error can be obtained as

$$\dot{\varsigma}(t)=\begin{bmatrix}-38.32&-19&-64.4\\-78.04&-27.13&-7.5\\-63.5&-36&-144.33\end{bmatrix}\varsigma(t).$$

When $t<100$, set the desired density of the predator population as $y_d(t)=0.3$. The tracking error is $e(t)=y(t)-0.3$. When $t\geq 100$, set $y_d(t)=0.75$. Then, the tracking error is $e(t)=y(t)-0.75$. Let $\theta=0.15$, and then the adaptive controller is

$$\begin{cases}u(t)=-0.15[y(t)-0.3]-0.6\tilde{x}_1(t)\tilde{y}(t)+0.07\tilde{y}(t)+0.06\tilde{y}^2(t)+\hat{\lambda}(t)\operatorname{sgn}[y(t)-0.3],t<100\\u(t)=-0.15[y(t)-0.75]-0.6\tilde{x}_1(t)\tilde{y}(t)+0.07\tilde{y}(t)+0.06\tilde{y}^2(t)+\hat{\lambda}(t)\operatorname{sgn}[y(t)-0.75],t\geq 100\end{cases}.$$

If the parameter of the designed adaptive update law is $\varepsilon=0.09$, the adaptive update law is

$$\begin{cases}\dot{\hat{\lambda}}(t)=-0.09[y(t)-0.3],t<100\\\dot{\hat{\lambda}}(t)=-0.09[y(t)-0.75],t\geq 100.\end{cases}$$

The initial values are as follows: $x_1(0)=1.9, x_2(0)=2.05, y(0)=5.03$.

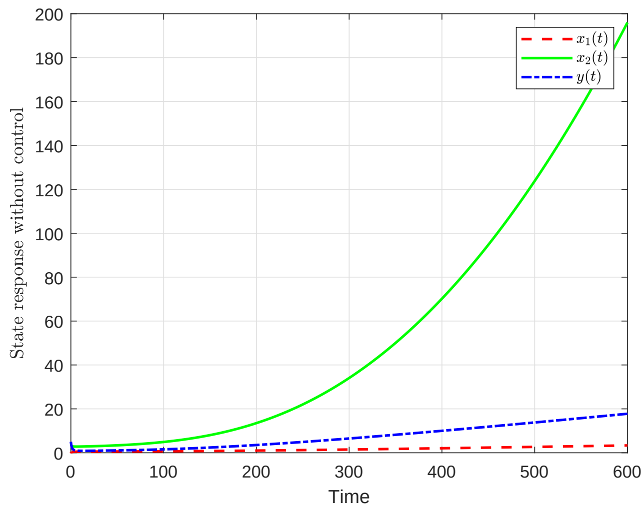

Figure 1. The state response without control.

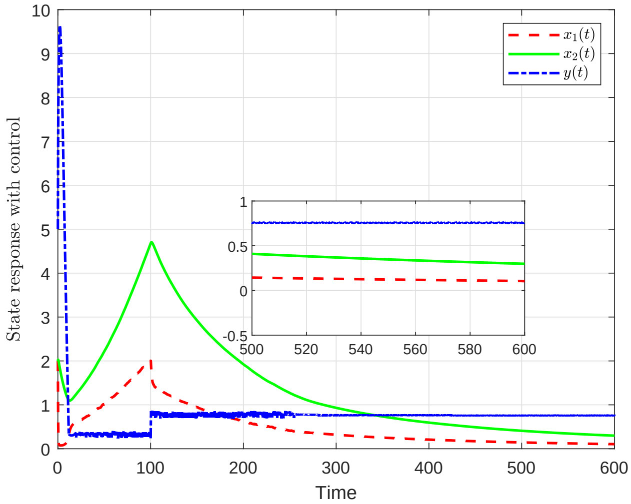

Figure 2. The state response with control.

Figure 1 shows the state response without control. It can be seen that the density of the biological species is divergent and unstable. Figure 2 shows the state response of the biological population after applying the controller. It can be seen that the biological model converges and tends to be stable with control. It can be verified that the designed adaptive controller is effective.

Figure 3 shows the tracking error between the predator population density and the desired density. It can be seen that the error almost converges to 0, which can be confirmed by the enlarged subgraph. Figure 4 shows the trajectory of y tracking the desired density y_d with control. It can be seen that y can track well the desired density y_d under the regulation of the adaptive controller.

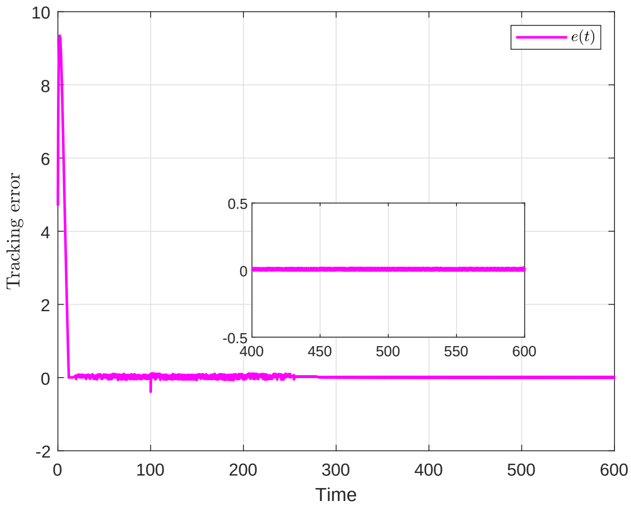

Figure 3. Tracking error after applying controller.

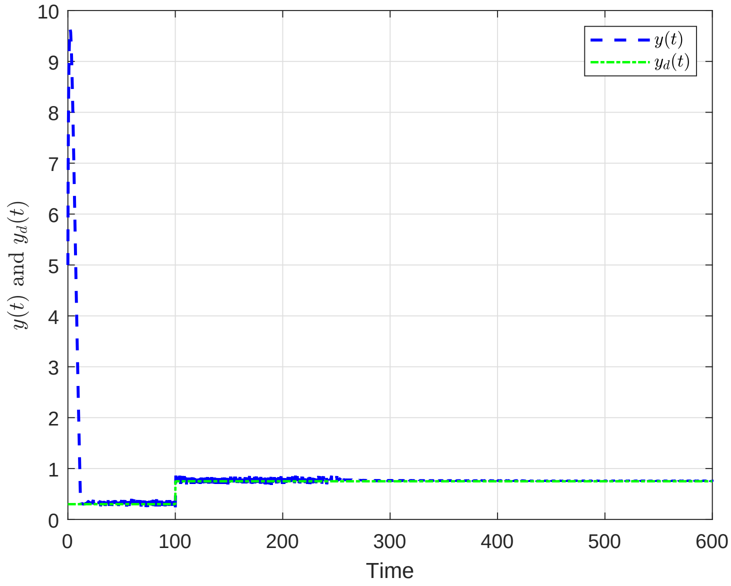

Figure 4. Trajectory of $y(t)$ and $y_d(t)$.

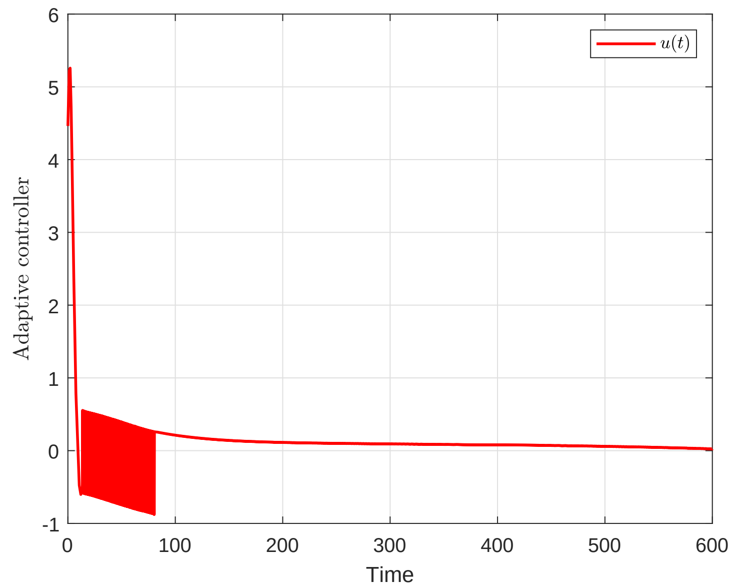

Figure 5. Adaptive controller.

The trajectory of the adaptive controller is shown in Figure 5. Figure 6 shows the observer error. From the above figures, it can be verified that under the action of the proposed controller and update law, the biological system can remain stable, and the density of predators can well follow the desired density. In other words, under the regulation of the adaptive controller designed in this paper, the ecosystem of *Lampetra japonica* will maintain natural balance and benign conditions. The adaptive controller designed in this paper can stabilize the ecosystem of *Lampetra japonica*. *Lampetra japonica* is small in size, inhabits the river all its life and has no migratory habit. Due to the influence of human activities, its population is small and in a vulnerable situation. This may lead to the destruction of the ecological balance. According to the data, *lampetra japonica* has been listed in the second level of China's national key protected wildlife list. In this case, we need to reduce human activities. Therefore, a controller is applied to the third differential equation in (2.1). Through the adaptive controller, we study the effect of limiting human activities on the density of *Lampetra japonica*, and we make the density of *Lampetra japonica* able to track the given density. The adaptive controller also ensures the stability of the whole ecosystem.

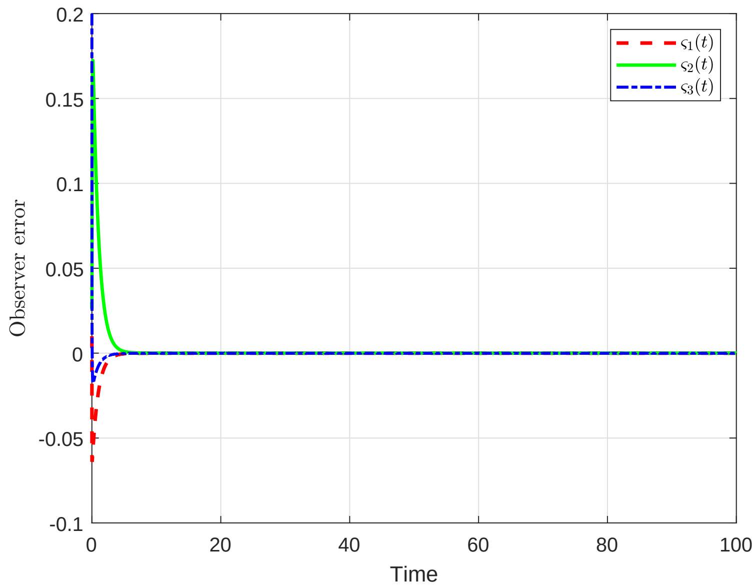

Figure 6. Observer error.

6. Conclusions

In this paper, an adaptive tracking control scheme has been investigated for a biological model with stage structure. A state observer is constructed by the T-S fuzzy method, and an adaptive tracking controller is designed on this basis. The proposed control scheme not only guarantees the stability of the biological model but also ensures that the predator density can track a desired density. Finally, a simulation example is given to illustrate the effectiveness of the results. From the biological point of view, the proposed adaptive tracking control method can appropriately adjust the biological population density to stabilize the whole biological system. At the same time, it can realize effective tracking and finally achieve the sustainable development of biological resources. Therefore, our results have a far-reaching impact on the study of biological population density.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant 62103289.

Conflict of interest

The authors declare there is no conflict of interest.

References

1. C. Liu, L. P. Wang, Q. L. Zhang, Y. Yan, Dynamical analysis in a bioeconomic phytoplankton zooplankton system with double time delays and environmental stochasticity, *Phys. A Stat. Mech. Appl.*, **482** (2017), 682–698. https://doi.org/10.1016/j.physa.2017.04.104
2. Q. Liu, D. Q. Jiang, T. Hayat, A. Alsaedi, B. Ahmad, Dynamical behavior of a stochastic predator-prey model with stage structure for prey, *Stochastic Anal. Appl.*, **38** (2020), 647–667. https://doi.org/10.1080/07362994.2019.1710188
3. X. Y. Meng, H. F. Huo, X. B. Zhang, Stability and global Hopf bifurcation in a Leslie-Gower predator-prey model with stage structure for prey, *J. Appl. Math. Comput.*, **60** (2019), 1–25. https://doi.org/10.1007/s12190-018-1201-0
4. A. Ruiz-Herrera, Attraction to equilibria in stage-structured predator prey models and bio-control problems, *J. Dyn. Diff. Equations*, **31** (2019), 435–450. https://doi.org/10.1007/s10884-018-9673-z
5. K. S. Mathur, A. Srivastava, J. Dhar, Dynamics of a stage-structured SI model for food adulteration with media-induced response function, *J. Eng. Math.*, **127** (2021), 1. https://doi.org/10.1007/s10665-021-10089-4
6. G. Q. Sun, S. F. Wang, M. T. Li, L. Li, J. Zhang, W. Zhang, et al., Transmission dynamics of COVID-19 in Wuhan, China: effects of lockdown and medical resources, *Nonlinear Dyn.*, **101** (2020), 1981–1993. https://doi.org/10.1007/s11071-020-05770-9
7. M. C. Wang, F. Liu, M. N. Zheng, Air quality improvement from COVID-19 lockdown: evidence from China, *Air Qual. Atmos. Health*, **14** (2021), 591–604. https://doi.org/10.1007/s11869-020-00963-y
8. H. Lau, V. Khosrawipour, P. Kocbach, A. Mikolajczyk, J. Schubert, J. Bania, et al., The positive impact of lockdown in Wuhan on containing the COVID-19 outbreak in China, *Int. Soc. Travel Med.*, **27** (2020), taaa037. https://doi.org/10.1093/jtm/taaa037
9. Z. B. Sun, H. Zhang, Y. F. Yang, H. Wan, Y. X. Wang, Impacts of geographic factors and population density on the COVID-19 spreading under the lockdown policies of China, *Sci. Total Environ.*, **746** (2020), 141347. https://doi.org/10.1016/j.scitotenv.2020.141347
10. A. Rajaei, A. Vahidi-Moghaddam, A. Chizfahm, M. Sharifi, Control of malaria outbreak using a non-linear robust strategy with adaptive gains, *IET Control Theory Appl.*, **13** (2019), 2308–2317. https://doi.org/10.1049/iet-cta.2018.5292
11. M. Sharifi, H. Moradi, Nonlinear robust adaptive sliding mode control of influenza epidemic in the presence of uncertainty, *J. Process Control*, **56** (2017), 48–57. https://doi.org/10.1016/j.jprocont.2017.05.010

12. X. Zhang, L. Liu, Y. J. Liu, Adaptive fuzzy fault-tolerant control of seat active suspension systems with actuator fault, *IET Control Theory Appl.*, **15** (2021), 1104–1114. https://doi.org/10.1049/cth2.12107

13. S. Q. Gao, J. K. Liu, Adaptive fault-tolerant vibration control of a wind turbine blade with actuator stuck, *Int. J. Control*, **93** (2020), 713–724. https://doi.org/10.1080/00207179.2018.1484572

14. J. P. Zhao, S. C. Tong, Y. Li, Observer-based fuzzy adaptive control for MIMO nonlinear systems with non-constant control gain and input delay, *IET Control Theory Appl.*, **15** (2021), 1488–1505. https://doi.org/10.1049/cth2.12138

15. L. L. Zhang, B. Chen, C. Lin, Y. Shang, Fuzzy adaptive finite-time consensus tracking control for nonlinear multi-agent systems, *Int. J. Syst. Sci.*, **52** (2021), 1346–1358. https://doi.org/10.1080/00207721.2020.1856450

16. Y. K. Yang, Y. G. Niu, Fixed-time adaptive fuzzy control for uncertain non-linear systems under event-triggered strategy, *IET Control Theory Appl.*, **14** (2020), 1845–1854. https://doi.org/10.1049/iet-cta.2019.1274

17. C. C. Sun, G. F. Gong, H. Y. Yang, Sliding mode control with adaptive fuzzy immune feedback reaching law, *Int. J. Control*, **18** (2020), 363–373. https://doi.org/10.1007/s12555-019-0285-0

18. C. V. Kien, N. N. Son, H. P. H. Anh, Adaptive fuzzy sliding mode control for nonlinear uncertain SISO system optimized by differential evolution algorithm, *Int. J. Fuzzy Syst.*, **21** (2019), 755–768. https://doi.org/10.1007/s40815-018-0558-4

19. L. Y. Zhu, T. S. Li, R. H. Yu, Y. Wu, J. Ning, Observer-based adaptive fuzzy control for intelligent ship autopilot with input saturation, *Int. J. Fuzzy Syst.*, **22** (2020), 1416–1429. https://doi.org/10.1007/s40815-020-00880-3

20. J. Na, Y. B. Huang, X. Wu, S. F. Su, G. Li, Adaptive finite-time fuzzy control of nonlinear active suspension systems with input delay, *IEEE Trans. Cybern.*, **50** (2020), 2639–2650. https://doi.org/10.1109/TCYB.2019.2894724

21. H. Y. Yue, J. M. Li, J. R. Shi, W. Yang, Adaptive fuzzy tracking control for stochastic nonlinear systems with time-varying input delays using the quadratic functions, *Int. J. Uncertainty Fuzziness Knowl. Based Syst.*, **26** (2018), 109–142. https://doi.org/10.1142/S0218488518500071

22. L. Ma, X. Huo, X. D. Zhao, G. D. Zong, Adaptive fuzzy tracking control for a class of uncertain switched nonlinear systems with multiple constraints: a small-gain approach, *Int. J. Fuzzy Syst.*, **21** (2019), 2609–2624. https://doi.org/10.1007/s40815-019-00708-9

23. Y. H. Hu, L. Zhao, L. B. Wu, N. N. Zhao, Y. J. Zhang, Adaptive event-triggered fuzzy tracking control of nonlinear systems with dead-zones and unmeasurable states, *Int. J. Syst. Sci.*, **51** (2020), 3251–3268. https://doi.org/10.1080/00207721.2020.1814445

24. B. M. Li, J. W. Xia, H. S. Zhang, H. Shen, Z. Wang, Event-triggered adaptive fuzzy tracking control for stochastic nonlinear systems, *J. Franklin Inst.*, **357** (2020), 9505–9522. https://doi.org/10.1016/j.jfranklin.2020.07.023

25. Y. Cao, Z. C. Wang, F. Liu, P. Li, G. Xie, Bio-inspired speed curve optimization and sliding mode tracking control for subway trains, *IEEE Trans. Veh. Technol.*, **68** (2019), 6331–6342. https://doi.org/10.1109/TVT.2019.2914936

26. Z. G. Liu, B. H. Li, T. Jiao, L. R. Xue, Adaptive tracking control of uncertain nonlinear system with time delays, external disturbances and dead-zone input, *Int. J. Syst. Sci.*, **51** (2020), 1987–2004. https://doi.org/10.1080/00207721.2020.1783021

27. C. J. Xi, J. X. Dong, Adaptive exact sliding tracking control of high-order strict-feedback systems with mismatched nonlinearities and external disturbances, *Int. J. Robust Nonlinear Control*, **30** (2020), 8228–8243. https://doi.org/10.1002/rnc.5231

28. K. W. Xia, Y. H. Eun, T. Y. Lee, S. Y. Park, Integrated adaptive control for spacecraft attitude and orbit tracking using disturbance observer, *Int. J. Aeronaut. Space Sci.*, **22** (2021), 936–947. https://doi.org/10.1007/s42405-021-00359-x

29. M. Li, Y. Chen, Y. Z. Liu, Adaptive disturbance observer-based event-triggered fuzzy control for nonlinear system, *Inf. Sci.*, **575** (2021), 485–498. https://doi.org/10.1016/j.ins.2021.06.055

30. W. H. Zhang, W. Wei, Disturbance-observer-based finite-time adaptive fuzzy control for non-triangular switched nonlinear systems with input saturation, *Inf. Sci.*, **561** (2021), 152–167. https://doi.org/10.1016/j.ins.2021.01.026

31. Q. L. Zhang, C. Liu, X. Zhang, *Complexity, Analysis and Control of Singular Biological Systems*, Springer-Verlag, London, 2012. https://doi.org/10.1007/978-1-4471-2303-3

32. Y. Zhang, Z. H. Shi, Sliding mode control for uncertain T-S fuzzy singular biological economic system, *IEEE Access*, **7** (2019), 14387–14395. https://doi.org/10.1109/ACCESS.2019.2891821

33. S. Hwang, H. S. Kim, Extended disturbance observer-based integral sliding mode control for nonlinear system via T-S fuzzy model, *IEEE Access*, **8** (2020), 116090–116105. https://doi.org/10.1109/ACCESS.2020.3004241

34. Z. J. Wu, Y. Q. Xia, X. J. Xie, Stochastic barbalat's lemma and its applications, *IEEE Trans. Autom. Control*, **57** (2012), 1537–1543. https://doi.org/10.1109/TAC.2011.2175071

Appendix

The fuzzy rules are given as follows:

Rule 1: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_1\hat{x}(t) + B_1U(t)$.

Rule 2: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_2\hat{x}(t) + B_2U(t)$.

Rule 3: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_3\hat{x}(t) + B_3U(t)$.

Rule 4: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_4\hat{x}(t) + B_4U(t)$.

Rule 5: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_5\hat{x}(t) + B_5U(t)$.

Rule 6: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_6\hat{x}(t) + B_6U(t)$.

Rule 7: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_7\hat{x}(t) + B_7U(t)$.

Rule 8: If $\Gamma_1(t)$ is $\mu_{11}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_8\hat{x}(t) + B_8U(t)$.

Rule 9: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_9\hat{x}(t) + B_9U(t)$.

Rule 10: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_{10}\hat{x}(t) + B_{10}U(t)$.

Rule 11: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_{11}\hat{x}(t) + B_{11}U(t)$.

Rule 12: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{21}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_{12}\hat{x}(t) + B_{12}U(t)$.

Rule 13: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_{13}\hat{x}(t) + B_{13}U(t)$.

Rule 14: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{31}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_{14}\hat{x}(t) + B_{14}U(t)$.

Rule 15: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{41}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_{15}\hat{x}(t) + B_{15}U(t)$.

Rule 16: If $\Gamma_1(t)$ is $\mu_{12}(\Gamma_1(t))$, $\Gamma_2(t)$ is $\mu_{22}(\Gamma_2(t))$, $\Gamma_3(t)$ is $\mu_{32}(\Gamma_3(t))$, $\Gamma_4(t)$ is $\mu_{42}(\Gamma_3(t))$, then $\dot{\hat{x}}(t) = A_{16}\hat{x}(t) + B_{16}U(t)$.

In the above,

$$A_1 = \begin{bmatrix} \max\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_2 = \begin{bmatrix} \max\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix},$$

$$A_3 = \begin{bmatrix} \max\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_4 = \begin{bmatrix} \max\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix},$$

$$A_5 = \begin{bmatrix} \max\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_6 = \begin{bmatrix} \max\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix},$$

$$A_7 = \begin{bmatrix} \max\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_8 = \begin{bmatrix} \max\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix},$$

$$A_9 = \begin{bmatrix} \min\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_{10} = \begin{bmatrix} \min\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} \min\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_{12} = \begin{bmatrix} \min\Gamma_1(t) & a & \max\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} \min\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_{14} = \begin{bmatrix} \min\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \max\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix},$$

$$A_{15} = \begin{bmatrix} \min\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \max\Gamma_4(t) \end{bmatrix}, A_{16} = \begin{bmatrix} \min\Gamma_1(t) & a & \min\Gamma_2(t) \\ \beta & -b_2 & 0 \\ \min\Gamma_3(t) & 0 & \min\Gamma_4(t) \end{bmatrix}.$$

$B_i = B, i = 1, 2, \ldots, 16.$