



Research article

Analysis of medical diagnosis based on variation co-efficient similarity measures under picture hesitant fuzzy sets and their application

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Abstract: One of the most dominant and feasible technique is called the PHF setting is exist in the circumstances of fuzzy set theory for handling intricate and vague data in genuine life scenario. The perception of PHF setting is massive universal is compared to these assumptions, who must cope with two or three sorts of data in the shape of singleton element. Under the consideration of the PHF setting, we utilized some SM in the region of the PHF setting are to diagnose the PHFDSM, PHFWDSM, PHFJSM, PHFWJSM, PHFCSM, PHFWCSM, PHFHVSM, PHFWHVSM and demonstrated their flexible parts. Likewise, a lot of examples are exposed under the invented measures based on PHF data in the environment of medical diagnosis to demonstrate the stability and elasticity of the explored works. Finally, the sensitive analysis of the presented works is also implemented and illuminated their graphical structures.

Keywords: picture hesitant fuzzy sets; variation co-efficient similarity measures; medical diagnosis

Abbreviations

FS: Fuzzy Sets; **IFS:** Intuitionistic Fuzzy Sets; **PFS:** Picture Fuzzy Sets; **HFS:** Hesitant Fuzzy Sets;

IHFS: Intuitionistic Hesitant Fuzzy Sets; **PHFS:** Picture Hesitant Fuzzy Sets; **SM:** Similarity Measures; **TG:** Truth Grade; **AG:** Abstinence Grade; **FG:** Falsity Grade; **PHF:** Picture Hesitant Fuzzy; **PHFDSM:** Picture Hesitant Fuzzy Dice Similarity Measures; **PHFDSM:** Picture Hesitant Fuzzy Weighted Dice Similarity Measures; **PHFJSM:** Picture Hesitant Fuzzy Jaccard Similarity Measures; **PHFWJSM:** Picture Hesitant Fuzzy Weighted Jaccard Similarity Measures; **PHFCSM:** Picture Hesitant Fuzzy Cosine Similarity Measures; **PHFCSM:** Picture Hesitant Fuzzy Weighted Cosine Similarity Measures; **PHFHVSM:** Picture Hesitant Fuzzy Hybrid Vector Similarity Measures; **PHFWHVSM:** Picture Hesitant Fuzzy Weighted Hybrid Vector Similarity Measures

1. Introduction

Decision-making technique is needed in every region of life which covers ambiguity and troubles. A lot of people have exploited the decision-making rule in the region of separated areas under the consideration of a crisp set. A lot of complications have occurred when a person faced fuzzy numbers instead of the crisp number. Zadeh [1] diagnosed the FS, which gives the TG with the value which is taken from the set of attributes limited to the unit interval $[0, 1]$. After their successful utilization, a lot of intellectuals have retained it in the natural setting of divided areas [2,3]. Likewise, Atanassov [4] devised the IFS with a feasible characteristic $0 \leq M_{\mathcal{F}_p}(x) + N_{\mathcal{F}_p}(x) \leq 1$, where $M_{\mathcal{F}_p}(x)$, stated the TG and $N_{\mathcal{F}_p}(x)$, stated the falsity grade (FG). A lot of applications have been implemented in the shape of decision-making [5], medical diagnosis [6], and pattern recognition [7]. The three sorts of data have occurred in individual factual life troubles and the perception of IFS has been unsuccessful. Under such sort of dilemmas, Cuong [8] successfully invented the conception of PFS with a well-known tool $0 \leq M_{\mathcal{F}_p}(x) + A_{\mathcal{F}_p}(x) + N_{\mathcal{F}_p}(x) \leq 1$, where $A_{\mathcal{F}_p}(x)$, stated the AG. Several implementations of the PFS have been exposed in [9–11].

FS, IFS, PFS, and their application have been diagnosed in diverse regions, but a lot of places have occurred which are needed such sort of idea, which are easily managed with these sorts of data which are available in the shape of the group. A well-known researcher, whose name is called Torra [12] invented the perception of HFS, which covers the TG in the form of the finite subset of the $[0, 1]$. HFS has broad consideration from the scientists, and they have exploited in the different regions [13–17]. Moreover, Beg and Rashid [18] invented the perception of IHFS by putting the FG in the shape of the finite subset of the $[0, 1]$. Additionally, Mahmood et al. [19] again settled IHFS with another condition, that is, the amount of the limit of TG and the FG is surpassed from the $[0, 1]$. In any case, this idea is additionally a few intricacies, for taking care of such kind of issues, in this composition we set up the thought of IHFS with another condition and to demonstrate the two ideas have a powerless condition when contrasted with our set-up work. Because of its construction, IHFS is a more powerful method to adapt to abnormal data than existing ideas like HFS and IFS. Even though, HFS and IHFS are effectively used in different fields. Yet, these have different issues, when a chief confronts different sorts of assessment of an individual like indeed, forbearance, no, and refusal as gathering. For adapting such sort of issues, Ullah et al. [20] investigated the idea of PHFSs containing the grades of truth, restraint, deception, and refusal as a subset of the unit span with conditions, the amount of the limit of truth, limit of forbearance and limit of lie grades is having a place on the unit stretch. PHFS is more dependable than IHFS and HFS to adapt to questionable and troublesome data choices. A lot of advantages are demonstrated here:

1. For $A_{i_{\mathcal{F}_P}}(x) = 0$, then the PHFS is changed to IHFS.
2. For $A_{i_{\mathcal{F}_P}}(x) = N_{i_{\mathcal{F}_P}}(x) = 0$, then the PHFS is changed to HFS.
3. By considering the triplet in the shape of a singleton set, then the PHFS is changed to PFS.
4. By considering the duplet in the shape of a singleton set with $A_{i_{\mathcal{F}_P}}(x) = 0$, then the PHFS is changed to IFS.
5. By considering the TG in the shape of a singleton set with $A_{i_{\mathcal{F}_P}}(x) = N_{i_{\mathcal{F}_P}}(x) = 0$, then the PHFS is changed to FS.

Cosine SMs the closeness between two vectors of an inner product space. It is estimated by the cosine of the point between two vectors and decides if two vectors are pointing generally in a similar way. It isn't unexpected used to quantify archive closeness in-text investigation. A measurement characterizes the distance between two items or how far separated two articles are. Assuming we need to gauge closeness as far as likeness, we can utilize another capacity called a comparability measure or similitude coefficient, or now and then a similitude.

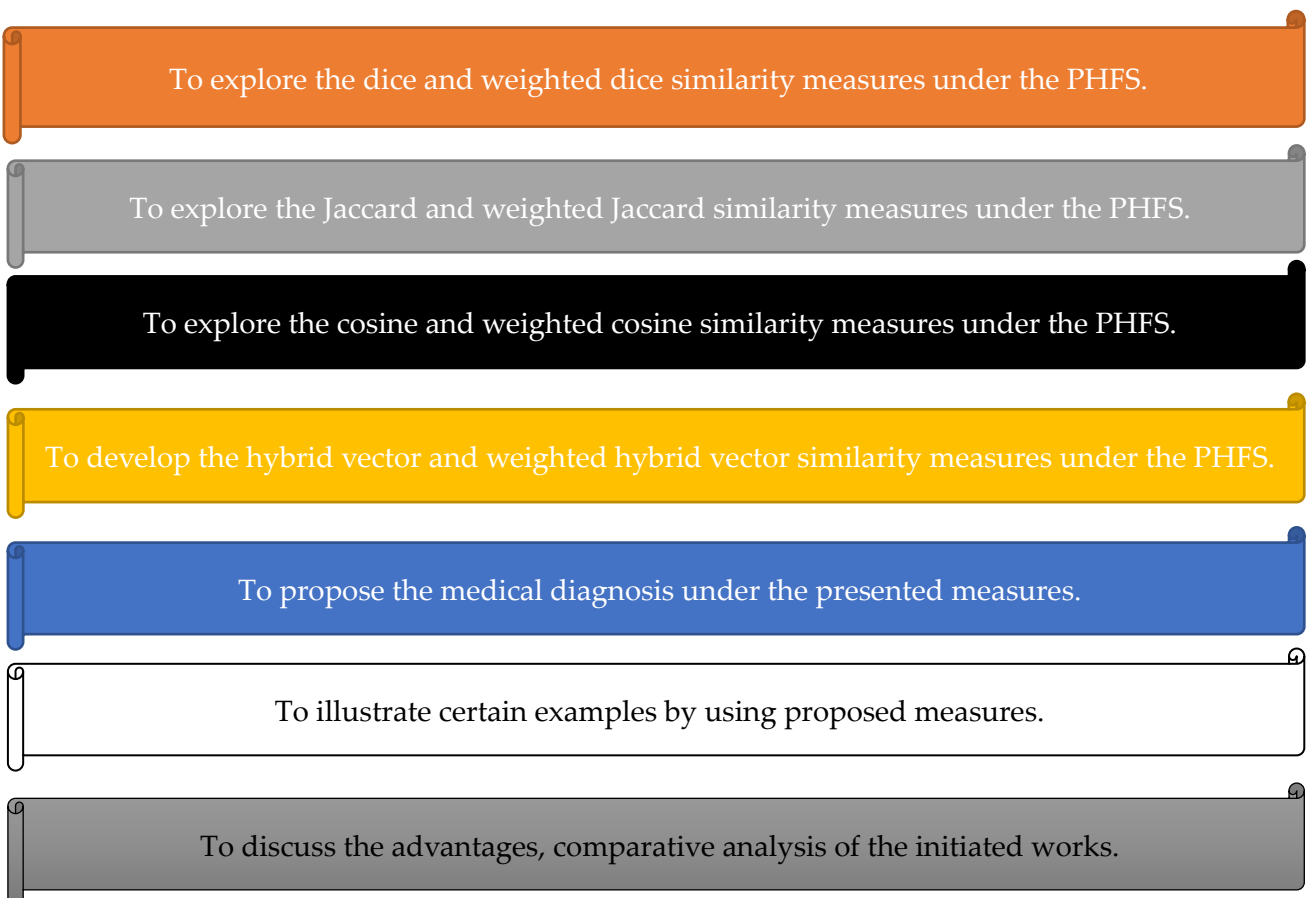


Figure 1. Graphical expressions of the initiated works.

Under the benefits of the PHFS, the summary of invented works is organized here:

1. To demonstrate the perceptions of PHFDSM, PHFWDSM, PHFJSM, PHFWJSM, PHFCSM, PHFWCSM, PHFHVSM, PHFWHVSM, and demonstrated their flexible parts.
2. To elaborate the diseases of the patient under the medical diagnosis to discover the dominancy and feasibility of the invented measures.
3. A lot of examples are exposed under the invented measures based on PHF data in the environment of medical diagnosis to demonstrate the stability and elasticity of the explored works.
4. The sensitive analysis of the presented works is also implemented and illuminated their graphical structures. Figure 1 states the graphical structure of the elaborated works.

Our invented works are implemented in the shape: Section 2 includes some prevailing ideas and section 3 gives the perception of PHFDSM, PHFWDSM, PHFJSM, PHFWJSM, PHFCSM, PHFWCSM, PHFHVSM, and PHFWHVSM, and demonstrated their flexible parts. In Section 4, a lot of examples are exposed under the invented measures based on PHF data in the environment of medical diagnosis to demonstrate the stability and elasticity of the explored works. Finally, the sensitive analysis of the presented works is also implemented and illuminated their graphical structures. The conclusion is implemented at the end of this work.

2. Preliminaries

This section includes several prevailing conceptions in the terms of PFS, HFS, PHFS, and their flexible laws. The meaning of separated mathematical terms is explained in Table 1.

Table 1. Shown of different sorts of symbols and their meanings.

| Symbols | Meanings |
|---------------------|---------------------------|
| $M_{\mathcal{F}_P}$ | Truth grade |
| $N_{\mathcal{F}_P}$ | Falsity grade |
| x | Element of universal sets |
| $M_{H_{HFS}}$ | Finite subset of [0,1] |
| $A_{\mathcal{F}_P}$ | Abstinence grade |
| X | Universal sets |
| $R_{\mathcal{F}_P}$ | Refusal grade |

Definition 1: [8] A PFS \mathcal{F}_P , elaborated by:

$$\mathcal{F}_P = \{(M_{\mathcal{F}_P}(x), A_{\mathcal{F}_P}(x), N_{\mathcal{F}_P}(x)): x \in X\}$$

With $0 \leq M_{\mathcal{F}_P}(x) + A_{\mathcal{F}_P}(x) + N_{\mathcal{F}_P}(x) \leq 1$, where $M_{\mathcal{F}_P}, A_{\mathcal{F}_P}, N_{\mathcal{F}_P}: X \rightarrow [0,1]$. More, the mathematical structure of PFN is designed by: $(M_{\mathcal{F}_P}(x), A_{\mathcal{F}_P}(x), N_{\mathcal{F}_P}(x))$ and $R_{\mathcal{F}_P}(x) = 1 - (M_{\mathcal{F}_P}(x) + A_{\mathcal{F}_P}(x) + N_{\mathcal{F}_P}(x))$, referred to refusal grade.

Definition 2: [12] A HFS H_{HFS} , elaborated by:

$$H_{HFS} = \{ \langle x, M_{H_{HFS}}(x) \rangle : M_{H_{HFS}}(x) \text{ is a finite subset of } [0,1] \forall x \in X \}$$

Moreover, the mathematical structure of hesitant fuzzy number (HFN) is designed by: $M_{H_{HFS}}(x) = \{m_j\}, j = 1, 2, \dots, n$. For instance, $M_{H_{HFS}}(x) = \{0.1, 0.4, 0.2, 0.5\}$.

Definition 3: [12] Let $M_{H_{HFS}} = \{m_j\}, M_{H_{HFS-1}} = \{m_k\}$ and $M_{H_{HFS-2}} = \{m_l\}, j, k, l = 1, 2, \dots, n$ be three HFE with $\delta > 0$. Then

1. $M_{H_{HFS-1}} \oplus M_{H_{HFS-2}} = \coprod_{m_1 \in M_{H_{HFS-1}}, m_2 \in M_{H_{HFS-2}}} \{m_1 + m_2 - m_1 m_2\}$
2. $M_{H_{HFS-1}} \otimes M_{H_{HFS-2}} = \coprod_{m_1 \in M_{H_{HFS-1}}, m_2 \in M_{H_{HFS-2}}} \{m_1 m_2\}$
3. $M_{H_{HFS}}^\delta = \coprod_{m \in M_{H_{HFS}}} \{m^\delta\};$
4. $\delta M_{H_{HFS}} = \coprod_{m \in M_{H_{HFS}}} \{1 - (1 - m)^\delta\}.$

For instance, $M_{H_{HFS-1}} = \{0.5, 0.2, 0.7\}$ and $M_{H_{HFS-2}} = \{0.1, 0.3, 0.4\}$, then $M_{H_{HFS-1}} \oplus M_{H_{HFS-2}} = \{0.95, 0.94, 0.82\}$.

Definition 4: [20] A PHFS \mathcal{F}_P , imagined by:

$$\mathcal{F}_P = \left\{ \left(M_{i_{\mathcal{F}_P}}(x), A_{i_{\mathcal{F}_P}}(x), N_{i_{\mathcal{F}_P}}(x) \right) : x \in X \right\}, i = 1, 2, 3, \dots, z$$

With $0 \leq \max(M_{i_{\mathcal{F}_P}}(x)) + \max(A_{i_{\mathcal{F}_P}}(x)) + \max(N_{i_{\mathcal{F}_P}}(x)) \leq 1$, where $M_{i_{\mathcal{F}_P}}, A_{i_{\mathcal{F}_P}}, N_{i_{\mathcal{F}_P}}$ is a finite subsets of $[0,1]$. More, the mathematical structure of picture HFS (PHFN) is designed by: $(M_{i_{\mathcal{F}_P}}(x), A_{i_{\mathcal{F}_P}}(x), N_{i_{\mathcal{F}_P}}(x))$ and $R_P(s) = 1 - (M_{i_{\mathcal{F}_P}}(x) + A_{i_{\mathcal{F}_P}}(x) + N_{i_{\mathcal{F}_P}}(x))$, stated the refusal grade.

Definition 5: [20] Let $\mathcal{F}_P, \mathcal{F}_{P-1}$ and \mathcal{F}_{P-2} be three PHFE with $\delta > 0$. Then

1. $\mathcal{F}_{P-1} \oplus \mathcal{F}_{P-2} = \coprod_{\substack{m_1 \in \mathcal{F}_{P-1}, m_2 \in \mathcal{F}_{P-2}, \\ a_1 \in \mathcal{F}_{P-1}, a_2 \in \mathcal{F}_{P-2}, \\ n_1 \in \mathcal{F}_{P-1}, n_2 \in \mathcal{F}_{P-2}}} \{m_1 + m_2 - m_1 m_2, a_1 a_2, n_1 n_2\}$
2. $\mathcal{F}_{P-1} \otimes \mathcal{F}_{P-2} = \coprod_{\substack{m_1 \in \mathcal{F}_{P-1}, m_2 \in \mathcal{F}_{P-2}, \\ a_1 \in \mathcal{F}_{P-1}, a_2 \in \mathcal{F}_{P-2}, \\ n_1 \in \mathcal{F}_{P-1}, n_2 \in \mathcal{F}_{P-2}}} \{m_1 m_2, a_1 + a_2 - a_1 a_2, n_1 + n_2 - n_1 n_2\}$

3. $\mathcal{F}_P^\delta = \coprod_{\substack{m_1 \in \mathcal{F}_{P-1}, m_2 \in \mathcal{F}_{P-2}, \\ a_1 \in \mathcal{F}_{P-1}, a_2 \in \mathcal{F}_{P-2}, \\ n_1 \in \mathcal{F}_{P-1}, n_2 \in \mathcal{F}_{P-2}}} \{m^\delta, 1 - (1 - a)^\delta, 1 - (1 - n)^\delta\};$
4. $\delta\mathcal{F}_P = \coprod_{\substack{m_1 \in \mathcal{F}_{P-1}, m_2 \in \mathcal{F}_{P-2}, \\ a_1 \in \mathcal{F}_{P-1}, a_2 \in \mathcal{F}_{P-2}, \\ n_1 \in \mathcal{F}_{P-1}, n_2 \in \mathcal{F}_{P-2}}} \{1 - (1 - m)^\delta, a^\delta, n^\delta\}.$

Definition 6: [21] The VCSM between two vectors $X = (x_1, x_2, \dots, x_n)$ and $Y = (y_1, y_2, \dots, y_n)$ is given by:

$$\begin{aligned} V(X, Y) &= \theta \frac{2XY}{\|X\|^2 + \|Y\|^2} + (1 - \theta) \frac{XY}{\|X\| + \|Y\|} \\ &= \theta \frac{\sum_{i=1}^n 2x_i y_i}{\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2} + (1 - \theta) \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} + \sqrt{\sum_{i=1}^n y_i^2}} \end{aligned}$$

With

1. $0 \leq V(X, Y) \leq 1;$
2. $V(X, Y) = V(Y, X);$
3. $V(X, Y) = 1$ Iff $x_i = y_i, i = 1, 2, \dots, z.$

3. Vector similarity measures of PHFSs

A lot of intellectuals have analyzed a lot of measures based on PHFS, the benefit of these SMs is that they all are the most general form of the prevailing measures under the PHFSs. The invented sort of measure is powerful due to its parameter π . By choosing different values of π , we will get the prevailing measures that are illustrated in existing conceptions. Behind the development of invented measures, cosine SMs (CSMs) are involved. CSMs the closeness between two vectors of an inner product space. It is estimated by the cosine of the point between two vectors and decides if two vectors are pointing generally in a similar way. It isn't unexpected used to quantify archive closeness in-text investigation. A measurement characterizes the distance between two items or how far separated two articles are. Assuming we need to gauge closeness as far as likeness, we can utilize another capacity called a comparability measure or similitude coefficient, or now and then a similitude. This study includes several new conceptions of PHFDMSM, PHFWDSM, PHFJSM, PHFWJSM, PHFCSM, PHFWCSM, PHFHVSM, PHFWHVSM, and their feasible cases. Additionally, the length of TG, AG and FG is denoted by $\#_{MS} = \text{maximum}(\text{order of } M_{i_{\mathcal{F}_{P-1}}}(x), \text{order of } M_{i_{\mathcal{F}_{P-2}}}(x)), \#_{AB} = \text{maximum}(\text{order of } A_{i_{\mathcal{F}_{P-1}}}(x), \text{order of } A_{i_{\mathcal{F}_{P-2}}}(x))$ and $\#_{NMS} = \text{maximum}(\text{order of } N_{i_{\mathcal{F}_{P-1}}}(x), \text{order of } N_{i_{\mathcal{F}_{P-2}}}(x)).$

Definition 7: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i_{\mathcal{F}_{P-1}}}(x), A_{i_{\mathcal{F}_{P-1}}}(x), N_{i_{\mathcal{F}_{P-1}}}(x))$ and $\mathcal{F}_{P-2} =$

$(M_{i_{\mathcal{F}_{P-2}}}(x), A_{i_{\mathcal{F}_{P-2}}}(x), N_{i_{\mathcal{F}_{P-2}}}(x))$, the PHFDSM is of the shape:

$$D_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = \frac{1}{Z} \sum_{j=1}^Z \left(\frac{2 \left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}(x_j) * M_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}(x_j) * A_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}(x_j) * N_{i_{\mathcal{F}_{P-2}}}(x_j) \right)}{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-2}}}^2(x_j) \right)} \right)$$

With:

1. $0 \leq D_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$
2. $D_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = D_P(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$
3. $D_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Definition 8: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i_{\mathcal{F}_{P-1}}}(x), A_{i_{\mathcal{F}_{P-1}}}(x), N_{i_{\mathcal{F}_{P-1}}}(x))$ and $\mathcal{F}_{P-2} = (M_{i_{\mathcal{F}_{P-2}}}(x), A_{i_{\mathcal{F}_{P-2}}}(x), N_{i_{\mathcal{F}_{P-2}}}(x))$, the PHFWDSM is of the shape:

$$D_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = \sum_{j=1}^Z w_j \left(\frac{2 \left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}(x_j) * M_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}(x_j) * A_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}(x_j) * N_{i_{\mathcal{F}_{P-2}}}(x_j) \right)}{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-2}}}^2(x_j) \right)} \right)$$

With:

1. $0 \leq D_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$
2. $D_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = D_{WP}(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$
3. $D_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Where $w_j \in [0,1], \sum_{j=1}^Z w_j = 1$, invented the weight vector.

Definition 9: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i_{\mathcal{F}_{P-1}}}(x), A_{i_{\mathcal{F}_{P-1}}}(x), N_{i_{\mathcal{F}_{P-1}}}(x))$ and $\mathcal{F}_{P-2} = (M_{i_{\mathcal{F}_{P-2}}}(x), A_{i_{\mathcal{F}_{P-2}}}(x), N_{i_{\mathcal{F}_{P-2}}}(x))$, the PHFJSM is of the shape:

$$J_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = \frac{1}{Z} \sum_{j=1}^Z \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}(x_j) * M_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}(x_j) * A_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}(x_j) * N_{i\mathcal{F}_{P-2}}(x_j) \right)}{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-2}}^2(x_j) - \left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}(x_j) * M_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}(x_j) * A_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}(x_j) * N_{i\mathcal{F}_{P-2}}(x_j) \right)} \right) \right)$$

With:

1. $0 \leq J_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$
2. $J_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = J_P(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$
3. $J_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Definition 10: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i\mathcal{F}_{P-1}}(x), A_{i\mathcal{F}_{P-1}}(x), N_{i\mathcal{F}_{P-1}}(x))$ and $\mathcal{F}_{P-2} = (M_{i\mathcal{F}_{P-2}}(x), A_{i\mathcal{F}_{P-2}}(x), N_{i\mathcal{F}_{P-2}}(x))$, the PHFWJSM is of the shape:

$$J_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = \sum_{j=1}^Z w_j \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}(x_j) * M_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}(x_j) * A_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}(x_j) * N_{i\mathcal{F}_{P-2}}(x_j) \right)}{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-2}}^2(x_j) - \left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}(x_j) * M_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}(x_j) * A_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}(x_j) * N_{i\mathcal{F}_{P-2}}(x_j) \right)} \right) \right)$$

With:

1. $0 \leq J_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$

2. $J_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = J_{WP}(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$
3. $J_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Where $w_j \in [0,1], \sum_{j=1}^z w_j = 1$, invented the weight vector.

Definition 11: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i_{\mathcal{F}_{P-1}}}(x), A_{i_{\mathcal{F}_{P-1}}}(x), N_{i_{\mathcal{F}_{P-1}}}(x))$ and $\mathcal{F}_{P-2} = (M_{i_{\mathcal{F}_{P-2}}}(x), A_{i_{\mathcal{F}_{P-2}}}(x), N_{i_{\mathcal{F}_{P-2}}}(x))$, the PHFCSM is of the shape:

$$C_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = \frac{1}{z} \sum_{j=1}^z \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}(x_j) * M_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}(x_j) * A_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}(x_j) * N_{i_{\mathcal{F}_{P-2}}}(x_j) \right)}{\left(\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}^2(x_j) \right)^{\frac{1}{2}} \left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-2}}}^2(x_j) \right)^{\frac{1}{2}} \right)} \right)$$

With:

1. $0 \leq C_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$
2. $C_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = C_P(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$
3. $C_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Definition 12: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i_{\mathcal{F}_{P-1}}}(x), A_{i_{\mathcal{F}_{P-1}}}(x), N_{i_{\mathcal{F}_{P-1}}}(x))$ and $\mathcal{F}_{P-2} = (M_{i_{\mathcal{F}_{P-2}}}(x), A_{i_{\mathcal{F}_{P-2}}}(x), N_{i_{\mathcal{F}_{P-2}}}(x))$, the PHFWCSM is of the shape:

$$C_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = \sum_{j=1}^z w_j \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}(x_j) * M_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}(x_j) * A_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}(x_j) * N_{i_{\mathcal{F}_{P-2}}}(x_j) \right)}{\left(\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}^2(x_j) \right)^{\frac{1}{2}} \left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-2}}}^2(x_j) \right)^{\frac{1}{2}} \right)} \right)$$

With:

1. $0 \leq C_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$
2. $C_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = C_{WP}(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$

3. $C_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Where $w_j \in [0,1], \sum_{j=1}^z w_j = 1$, invented the weight vector.

Definition 13: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i_{\mathcal{F}_{P-1}}}(x), A_{i_{\mathcal{F}_{P-1}}}(x), N_{i_{\mathcal{F}_{P-1}}}(x))$ and $\mathcal{F}_{P-2} = (M_{i_{\mathcal{F}_{P-2}}}(x), A_{i_{\mathcal{F}_{P-2}}}(x), N_{i_{\mathcal{F}_{P-2}}}(x))$, the PHFHVSM is of the shape:

$$H_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = \frac{1}{z} \left(\pi \sum_{j=1}^z \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}(x_j) * M_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}(x_j) * A_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}(x_j) * N_{i_{\mathcal{F}_{P-2}}}(x_j) \right)}{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-2}}}^2(x_j) \right)} \right) + (1 - \pi) \sum_{j=1}^z \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}(x_j) * M_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}(x_j) * A_{i_{\mathcal{F}_{P-2}}}(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}(x_j) * N_{i_{\mathcal{F}_{P-2}}}(x_j) \right)}{\left(\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-1}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-1}}}^2(x_j) \right)^{\frac{1}{2}} \left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i_{\mathcal{F}_{P-2}}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i_{\mathcal{F}_{P-2}}}^2(x_j) \right)^{\frac{1}{2}} \right)} \right) \right)$$

With:

1. $0 \leq H_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$
2. $H_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = H_P(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$
3. $H_P(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Definition 14: For any two PHFSs $\mathcal{F}_{P-1} = (M_{i_{\mathcal{F}_{P-1}}}(x), A_{i_{\mathcal{F}_{P-1}}}(x), N_{i_{\mathcal{F}_{P-1}}}(x))$ and $\mathcal{F}_{P-2} = (M_{i_{\mathcal{F}_{P-2}}}(x), A_{i_{\mathcal{F}_{P-2}}}(x), N_{i_{\mathcal{F}_{P-2}}}(x))$, the PHFWHVSM is of the shape:

$$\begin{aligned}
& H_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \\
& \left(\pi \sum_{j=1}^z w_j \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}(x_j) * M_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}(x_j) * A_{i\mathcal{F}_{P-2}}(x_j) + \right)}{\frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}(x_j) * N_{i\mathcal{F}_{P-2}}(x_j)} \right) + \right. \\
& \left. \frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}^2(x_j) + \right)}{\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-2}}^2(x_j)} \right) \right) + \\
& = \left((1 - \pi) \sum_{j=1}^z w_j \left(\frac{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}(x_j) * M_{i\mathcal{F}_{P-2}}(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}(x_j) * A_{i\mathcal{F}_{P-2}}(x_j) + \right)}{\frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}(x_j) * N_{i\mathcal{F}_{P-2}}(x_j)} \right) \right. \\
& \left. \frac{\left(\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-1}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-1}}^2(x_j) \right)^{\frac{1}{2}} \right)}{\left(\frac{1}{\#_{MS}} \sum_{i=1}^{\#_{MS}} M_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{AB}} \sum_{i=1}^{\#_{AB}} A_{i\mathcal{F}_{P-2}}^2(x_j) + \frac{1}{\#_{NMS}} \sum_{i=1}^{\#_{NMS}} N_{i\mathcal{F}_{P-2}}^2(x_j) \right)^{\frac{1}{2}}} \right) \right) \right)
\end{aligned}$$

With:

1. $0 \leq H_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) \leq 1$
2. $H_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = H_{WP}(\mathcal{F}_{P-2}, \mathcal{F}_{P-1})$
3. $H_{WP}(\mathcal{F}_{P-1}, \mathcal{F}_{P-2}) = 1$ iff $\mathcal{F}_{P-1} = \mathcal{F}_{P-2}$.

Where $w_j \in [0,1], \sum_{j=1}^z w_j = 1$, invented the weight vector.

4. Medical diagnosis

The medical finding is the most common way of figuring out which infection or condition clarifies an individual's manifestations and signs. It is regularly alluded to as a conclusion with the clinical setting being implied. The data needed for the conclusion is ordinarily gathered from a set of experiences and actual assessments of the individual looking for clinical consideration. Regularly, at least one indicative method, like clinical trials, is additionally done during the cycle. At times after death determination is viewed as a sort of clinical conclusion.

A lot of people have employed the concept of IHFS and PHFS in the region of medical diagnosis by using several operators and measures. The major aim of this scenario is to choose some prevailing examples from [22] and diagnose their solution based on invented works.

Medical diagnosis analysis is dependent on data from sources like discoveries from an actual assessment, meeting with the patient or family or both, clinical history of the patient and family, and clinical discoveries as announced by lab tests and radiologic considers. nursing finding sees nursing determination. The characterizing side effects for each dysfunctional behavior are definite in the Diagnostic and Statistical Manual of Mental Disorders (DSM-5), distributed by the American Psychiatric Association. This manual is utilized by psychological well-being experts to analyze psychological circumstances and by insurance, agencies to repay for treatment. Classes of

psychological instability. The major analysis of this contribution is to analyze the diseases of the patient under the consideration of different sorts of symptoms.

Example 1: A doctor wants to examine the diseases of the patients whose detail is of the form $P = \{James, Jak, Harry, Oliver, Robert\}$ and their possible diseases names are of the form $D = \{Coronavirus, Typhoid, Malaria, Stomach Problem, Chest Problem\}$ based on the following symptoms $V = \{Fever, Cough, Shortness of breath, Chest pain, Stomach pain, Headache\}$, for solving such kinds of problems we choose a piece of information and evaluate by using established measures. Table 2 includes PHFNs.

Table 2. Stated several PHFNs.

| Representations | Fever | Cough | Shortness of breath | Chest pain | Headache |
|-----------------|--|--|--|--|--|
| Coronavirus | $\left(\begin{matrix} \{0.4, 0.3, 0.1\}, \\ \{0.12, 0.23\}, \\ \{0.11, 0.01, 0.02\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.4, 0.3, 0.1\}, \\ \{0.12, 0.23\}, \\ \{0.11, 0.01, 0.02\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.14, 0.13, 0.11\}, \\ \{0.112, 0.123\}, \\ \{0.121, 0.012, 0.202\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.23, 0.13, 0.01\}, \\ \{0.2, 0.3\}, \\ \{0.1, 0.1, 0.2\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.4, 0.3, 0.1\}, \\ \{0.12, 0.23\}, \\ \{0.11, 0.01, 0.02\} \end{matrix} \right)$ |
| Typhoid | $\left(\begin{matrix} \{0.23, 0.13, 0.01\}, \\ \{0.2, 0.3\}, \\ \{0.1, 0.1, 0.2\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.4, 0.3, 0.1\}, \\ \{0.12, 0.23\}, \\ \{0.11, 0.01, 0.02\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.23, 0.13, 0.01\}, \\ \{0.2, 0.3\}, \\ \{0.1, 0.1, 0.2\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.14, 0.13, 0.11\}, \\ \{0.112, 0.123\}, \\ \{0.121, 0.012, 0.202\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.4, 0.3, 0.1\}, \\ \{0.12, 0.23\}, \\ \{0.11, 0.01, 0.02\} \end{matrix} \right)$ |
| Malaria | $\left(\begin{matrix} \{0.14, 0.13, 0.11\}, \\ \{0.112, 0.123\}, \\ \{0.121, 0.012, 0.202\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.23, 0.13, 0.01\}, \\ \{0.2, 0.3\}, \\ \{0.1, 0.1, 0.2\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.14, 0.13, 0.11\}, \\ \{0.112, 0.123\}, \\ \{0.121, 0.012, 0.202\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.04, 0.03, 0.01\}, \\ \{0.012, 0.023\}, \\ \{0.311, 0.31, 0.32\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.23, 0.13, 0.01\}, \\ \{0.2, 0.3\}, \\ \{0.1, 0.1, 0.2\} \end{matrix} \right)$ |
| Stomch Problem | $\left(\begin{matrix} \{0.04, 0.03, 0.01\}, \\ \{0.012, 0.023\}, \\ \{0.311, 0.31, 0.32\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.14, 0.13, 0.11\}, \\ \{0.112, 0.123\}, \\ \{0.121, 0.012, 0.202\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.04, 0.03, 0.01\}, \\ \{0.012, 0.023\}, \\ \{0.311, 0.31, 0.32\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.23, 0.13, 0.01\}, \\ \{0.2, 0.3\}, \\ \{0.1, 0.1, 0.2\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.14, 0.13, 0.11\}, \\ \{0.112, 0.123\}, \\ \{0.121, 0.012, 0.202\} \end{matrix} \right)$ |
| Chest Problem | $\left(\begin{matrix} \{0.44, 0.43, 0.41\}, \\ \{0.212, 0.123\}, \\ \{0.011, 0.101, 0.102\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.04, 0.03, 0.01\}, \\ \{0.012, 0.023\}, \\ \{0.311, 0.31, 0.32\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.4, 0.3, 0.1\}, \\ \{0.12, 0.23\}, \\ \{0.11, 0.01, 0.02\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.14, 0.13, 0.11\}, \\ \{0.112, 0.123\}, \\ \{0.121, 0.012, 0.202\} \end{matrix} \right)$ | $\left(\begin{matrix} \{0.04, 0.03, 0.01\}, \\ \{0.012, 0.023\}, \\ \{0.311, 0.31, 0.32\} \end{matrix} \right)$ |

We will examine the values of the data which are available in Tables 2 and Table 3 by using def. (14) to explore the diseases of the patient whose weighted vector is of the form 0.1,0.2,0.22,0.18,0.3, such that

Table 3. Stated known values in the shape of PHFNs.

| | | | | | |
|------------|--|--|--|--|--|
| Cronavirus | $\left(\begin{matrix} \{1,1,1,1\}, \\ \{0,0,0,0\}, \\ \{0,0,0,0\} \end{matrix} \right)$ | $\left(\begin{matrix} \{1,1,1,1\}, \\ \{0,0,0,0\}, \\ \{0,0,0,0\} \end{matrix} \right)$ | $\left(\begin{matrix} \{1,1,1,1\}, \\ \{0,0,0,0\}, \\ \{0,0,0,0\} \end{matrix} \right)$ | $\left(\begin{matrix} \{1,1,1,1\}, \\ \{0,0,0,0\}, \\ \{0,0,0,0\} \end{matrix} \right)$ | $\left(\begin{matrix} \{1,1,1,1\}, \\ \{0,0,0,0\}, \\ \{0,0,0,0\} \end{matrix} \right)$ |
|------------|--|--|--|--|--|

Table 4. Calculated values with the help of Definition. 14.

| Patient Name | James | Jak | Harry | Oliver | Robert |
|-------------------------|-------------|---------|---------|-----------------|---------------|
| Diseases of the Patient | Coronavirus | Typhoid | Malaria | Stomach Problem | Chest Problem |
| Ranking values | 0.73212 | 0.2341 | 0.6612 | 0.5432 | 0.24411 |

From Table 4, we expose that the patient James has the disease coronavirus because its ranking result is greater than other all diseases. The ranking results are stated by:

$$\text{Coronavirus} \geq \text{Malaria} \geq \text{Stomch Problem} \geq \text{Chest Problem} \geq \text{Typhoid}$$

Thus, the disease of the patient James has coronavirus.

4.1. Comparative analysis

Several peoples have demonstrated the sensitive analysis of the invented works by using different sorts of operators, measures, and methods. The major collaboration of this study is to evaluate the comparative analysis of the invented works with several prevailing works like the work of Jan et al. [23], the work of Ahmad et al. [24] utilized under the PHFS, and with some other existing methods. Table 5 invented the comparative works by considering the data in Example 1.

Table 5. Stated the comparative analysis.

| Methods | Using the formula of the Score vales | Ranking Results of the different methods | Best alternatives |
|-------------------|--|--|-------------------|
| Jan et al. [23] | Coronavirus = 0.82, Typhoid = 0.34, Malaria = 0.77, Stomch Probelm = 0.54, Chest Problem = 0.39 | $Cr \geq M \geq S \geq Ch \geq T$ | COVID-19 |
| Ahmad et al. [24] | Coronavirus = 0.76, Typhoid = 0.25, Malaria = 0.710, Stomch Problem = 0.612, Chest Problem = 0.31 | $Cr \geq M \geq S \geq Ch \geq T$ | COVID-19 |
| Proposed Measures | Coronavirus = 0.73212, Typhoid = 0.2341, Malaria = 0.6612, Stomch Probelm = 0.5432, Chest Problem = 0.24411 | $Cr \geq M \geq S \geq Ch \geq T$ | COVID-19 |

Table 4 stated that the COVID-19 is the best alternative identified by all prevailing methods and the intended methods, although the ranking results are the same, which is provided all ideas. The graphical interpretation of the compared data in Table 5 has shown in Figure 2.

4.2. Advantages

The explored work in this article is more reliable and more feasible than the existing drawbacks. Because the constraint of the PHFS is that the sum of a maximum of truth, abstinence, and falsity grades are not exceeded in the unit interval. PHFS is a useful technique to describe the awkward and complicated information in fuzzy set theory. The advantage of the established work is stated by:

First, we discuss the advantage of the PHFS is that, if we choose the abstinence grade will be zero, then the PHFS is converted into IHFS [18]. If we choose the truth, abstinence and falsity grade will be a singleton set, then the PHFS is converted into PFS [8]. If we choose the truth and falsity grade will be a singleton set and the abstinence grade will be zero, then the PHFS is converted into IFS [4]. Similarly, if we choose the abstinence and falsity grade will be zero, then the PHFS is converted into HFS [12].

4.3. Graphical interpretations

The graphical interpretation between established measures and existing measures for Example 1, which is discussed in Table 5, is illustrated below.

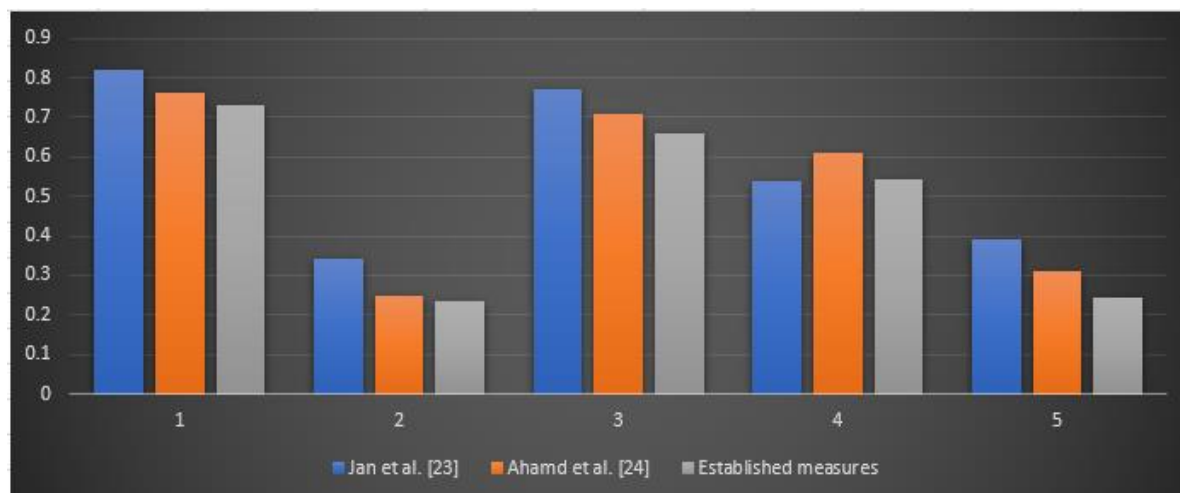


Figure 2. Geometrical interpretation of the established measures and existing measures.

For our convenience, we diagnosed Figure 2 for the data in Table 5, which covers five alternatives, and three sorts of operators have expressed the form of distinct colors. Each color is shown in the shape of different forms some are crossed 0.8 and some have below from 0.4. From the above figure, without studying the data in Table 5, we easily demonstrated the best optimal from the family of alternatives. Which is represented by the very dangerous coronavirus. Thus, the patient first (James) has coronavirus.

In several aspects of the invented works, we have chosen a lot of prevailing works and demonstrated that the elaborated works are massively practicable, and influential is compared to prevailing works [15,27,28].

Li et al. [15] diagnosed SMs under HFSs, by combing the SMs with HFSs and investigating the beneficial optimal from the family of alternatives, but the perception in [15] is the subpart of the PHFSs. For instance, by consideration of initiated sort of works are very complicated for the Li et al. [15] to survive with it, our invented works are massive powerful then prevailing works [15]. Additionally, intuitionistic fuzzy SMs, invented by Hwang et al. [27], combing the SMs with IFSs and investigating the beneficial optimal from the family of alternatives, but the perception in [27] is the subpart of the PHFSs. For instance, by consideration of initiated sort of works are very complicated for the Hwang et al. [27] to survive with it, our invented works are massive powerful then prevailing works [27]. Khan et al. [28] diagnosed SMs under PFSs, by combing the SMs with PFSs and investigating the beneficial optimal from the family of alternatives, but the perception in [28] is the subpart of the PHFSs. For instance, by consideration of initiated sort of works are very complicated for the Khan et al. [28] to survive with it, our invented works are massive powerful than prevailing works [28].

By discussing a lot of invented and prevailing data, we get that the invented works are adaptable and practicable is compared to prevailing operators.

5. Conclusion

SMs are plays a massively important role to demonstrate the beneficial optimal from the group of alternatives in genuine life dilemmas. The major contribution of this work is suggested below:

1. We diagnosed the PHFDSM, PHFWDSM, PHFJSM, PHFWJSM, PHFCSM, PHFWCSM, PHFHVSM, PHFWHVSM and demonstrated their flexible parts.
2. A lot of examples are exposed under the invented measures based on PHF data in the environment of medical diagnosis to demonstrate the stability and elasticity of the explored works.
3. The sensitive analysis of the presented works is also implemented and illuminated their graphical structures.

PHFS has a lot of benefits, but they have also a lot of limitations are invented here:

1. If someone suggested data in the shape of spherical HFS (SHFS), then the perception of PHFS has been unsuccessful.
2. If someone suggested data in the shape of T-spherical HFS (T-SHFS), then the perception of PHFS has been unsuccessful.
3. If someone suggested data in the shape of Complex SHFS (CSHFS), then the perception of PHFS has been unsuccessful.
4. If someone suggested data in the shape of Complex T-SHFS (CT-SHFS), then the perception of PHFS has been unsuccessful.

In the impending times, a lot of theories will be proposed for instance the work of cubic bipolar fuzzy sets [25], complex q-rung orthopair FS [26], q-rung orthopair fuzzy sets [27], Complex q-rung orthopair fuzzy 2-tuple linguistic sets [28], decision-making techniques [29], SMs under the IFSs [30], Medical diagnosis [31], Decision-making technique [32], Sampling method [33], CSFS [34], and Certain Properties of Single-Valued Neutrosophic Graph [35], and q-rung orthopair fuzzy sets [36-38] in the situation of PHFS to advance the excellence and extent of the PHFS.

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Data availability

The data used in the manuscript are hypothetical and can be used by anyone by just citing this article.

Conflicts of interest

The authors declare no conflicts of interest about the publication of the research article.

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