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Research article

A new structural entropy measurement of networks based on the nonextensive statistical mechanics and hub repulsion

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Abstract: The structure properties of complex networks are an open issue. As the most important parameter to describe the structural properties of the complex network, the structure entropy has attracted much attention. Recently, the researchers note that hub repulsion plays an role in structural entropy. In this paper, the repulsion between nodes in complex networks is simulated when calculating the structure entropy of the complex network. Coulomb's law is used to quantitatively express the repulsive force between two nodes of the complex network, and a new structural entropy based on the Tsallis nonextensive statistical mechanics is proposed. The new structure entropy synthesizes the influence of repulsive force and betweenness. We study several construction networks and some real complex networks, the results show that the proposed structure entropy can describe the structural properties of complex networks more reasonably. In particular, the new structural entropy has better discrimination in describing the complexity of the irregular network. Because in the irregular network, the difference of the new structure entropy is larger than that of degree structure entropy, betweenness structure entropy and Zhang's structure entropy. It shows that the new method has better discrimination for irregular networks, and experiments on Graph, Centrality literature, US Aire lines and Yeast networks confirm this conclusion.

Keywords: complex networks; structure entropy; coulomb's law; tsallis nonextensive statistical mechanics

1. Introduction

In the real world, many systems can be modeled as the complex network [1–5]. It has been proved that it is generally successful to use complex networks to describe their various characteristics [6–8]. Research shows that the complex networks have small world property [9], scale-free property [10] and fractal property [11–13]. Based on these properties, scholars have made a deep exploration and put forward the concepts of node importance [14–16], self similarity of network [16, 17] and community division [18].

With the development of the research, the quantification of network complexity has become an important topic. The structure entropy is developed from information entropy [19, 20], which can be used to quantify the complexity of the network. After analyzing the topology of the network, many researchers have proposed different structure entropy [21–25]. However, these methods have some limitations. For example, the degree structure entropy [22] can only describe the local properties of complex networks, and the global structure properties are not reflected. The betweenness structure entropy [23] is opposite to that, which reflects the global structure property but neglects the local structure property. Even if both properties are taken into account, the existing methods still have insufficient discrimination when describing the complexity of some irregular networks. In order to comprehensively describe the structural properties of complex networks, we need to find a more reasonable structural entropy. The new structural entropy proposed in this paper can fill this gap.

Hubs play an important role in the study of complex networks [26–28]. According to the related research [29], in the dynamic evolution process of complex networks, the hub preferentially connects with nodes with fewer links to generate a more robust network structure. In other words, nodes with high degree do not connect directly, which means hub exclusion plays an important role in network connection. Therefore, it is reasonable to model the repulsion between nodes in complex networks when calculating the structure entropy of networks. In Coulomb's law, if two charges have the same sign, the electrostatic force between them is repulsive. Inspired by the repulsive force model in Coulomb's law, we use this law to quantitatively express the repulsive force between two nodes. In our model, the connecting nodes in the complex network are regarded as charges, and there is interaction between them. Therefore, the importance of each node is no longer measured by the number of nodes connected with it, but by the total force that the linked nodes put on it.

In this paper, a new structure entropy based on the Tsallis nonextensive statistical mechanics [30–32] and Coulomb's laws is proposed. The new entropy synthesizes the repulsive force between nodes and the influence of betweenness. The model is verified on several networks, and the results show that the proposed structural entropy is reasonable.

The rest of this paper is organized as follows. The Section 2 introduces some basic concepts involved in this paper. In Section 3, a new complex network model inspired by Coulomb's law is proposed. In Section 4, the proposed method is verified. Finally, the conclusion is drawn in the last part.

2. Preliminaries

In this section, some basic concepts of complex networks, several centralities and structure entropise, will be briey introduced.

2.1. Degree centrality

In complex network $G = (V, E)$, V and E represent set of nodes and set of links respectively. The degree [33] of node i is defined as $DC(i)$, which represents the number of direct neighbors of node i, The formula is shown as follows:

$$DC(i) = \sum_{j} a_{ij}, \tag{2.1}$$

where $A = \{a_{ij}\}$ is the adjacency matrix of complex network G. The a_{ij} is equal to 1 if node i and node j are directly connected, otherwise it is marked 0. Degree centrality is a local index to describe the importance of nodes. The more links the node has, the more important the node is.

2.2. Betweenness centrality

Betweenness centrality [34] is another index to describe the importance of nodes. It is defined based on the shortest path. Different from degree centrality, it represents the global properties of complex networks, the mathematical expression is as follows:

$$BC(i) = \sum_{i \neq s, i \neq t, s \neq t} \frac{g_i}{g_{st}}, \tag{2.2}$$

where g_{st} is the number of shortest paths between nodes s and t, and g_i is the number of shortest paths which have go to through the vertex i. It is emphasized that $s \neq i \neq t$.

2.3. Coulomb's law

In this paper, Coulomb's law [35] is used to express the repulsive force between two nodes. Coulomb's law is the law of the interaction force of static point charge, which is proportional to the product of their charge quantity and inversely proportional to the square of their distance. The direction of the force is on their line. Suppose that the charge quantity of two point charges is Q_1 and Q_2 respectively, and the distance between them is r, then the force F between them can be calculated by the following formula:

$$F = k\frac{Q_1 Q_2}{r^2}, \tag{2.3}$$

if two charges have the same sign, then the electrostatic force between them is repulsive force. If the charge sign between them is different, then the electrostatic force between them is attractive force.

2.4. Some existing structure entropies

The structure entropy of complex networks is developed from information entropy, which can be used to describe the structure properties of complex networks. Therefore, it is necessary to introduce information entropy briefly. In the signal source, we consider not the uncertainty of a single symbol but the average uncertainty of all possible situations of the signal source. If the signal source symbol has n values: $X = \{x_1, x_2, \cdots, x_n\}$, the corresponding probability is: $P = \{p(x_1), p(x_2), \cdots, p(x_n)\}$ and $\sum_{i=1}^{n} p_i = 1$, where the appearance of each symbol is independent of each other. At this time, the average uncertainty of signal source should be the statistical average (E) of all symbol uncertainties, which can be called information entropy:

$$H = E[-\log p_i] = -\sum_{i=1}^{n} p_i \log p_i, \tag{2.4}$$

where the logarithm is usually 2. However, other logarithm bases can also be taken, which can be converted by the formula of changing base.

2.4.1. Degree structure entropy

At present, there are many scholars, such as Zhang Qi [31] and Wang Bing [32], who have proposed different structure entropy to describe the complexity of network, and most of the structure entropy is based on the degree. When p_i is expressed as the degree distribution of nodes:

$$p_i = \frac{DC(i)}{\sum_{i=1}^{n} DC(i)}, \tag{2.5}$$

we get the degree structure entropy [36]: $H_{\deg} = -\sum_{i=1}^{n} p_i \log p_i$, this is a structure entropy that characterizes the properties of local structure.

2.4.2. Betweenness structure entropy

As mentioned above, degree structure entropy describes the local structure properties of complex networks, which has some limitations. In order to describe the complexity of networks more comprehensively, some scholars proposed the betweenness structure entropy [25]:

$$H_{bet} = -\sum_{i=1}^{n} \beta_i \log \beta_i, \tag{2.6}$$

where $\beta_i = \frac{BC(i)}{\sum_{i=1}^{n} BC(i)}$, $BC(i)$ is the betweenness, which is an index to measure the global properties of complex networks.

2.4.3. Structure entropy based on tsallis nonextensive statistical mechanics

However, the above two kinds of structure entropy have defects. The degree structure entropy only considers the local structure properties of complex networks, but does not take into account the global structure properties, the betweenness structure entropy is just the opposite. For this reason, structural entropy based on Tsallis nonextensive statistical mechanics [31] is proposed:

$$H_{zqs} = -\sum_{i=1}^{n} \frac{p_i^{q_i} - p_i}{1 - q_i}, \tag{2.7}$$

where n is the number of nodes, $p_i = \frac{DC(i)}{\sum_{i=1}^{n} DC(i)}$ is the degree distribution, $q_i = 1 + (BC_{\max} - BC(i)), \{BC_{\max} = \max[BC(i), (i = 1, 2, 3, \cdots, n)]\}$ is given by the denition of the betweenness and the principle of the Tsallis entropy.

3. The proposed method

In this paper, a new structure entropy based on the Tsallis nonextensive statistical mechanics and Coulomb's laws is proposed. The new entropy synthesizes the weighted degree and the influence of betweenness. In contrast, the entropy fuses degree and the influence of betweenness in reference [31],

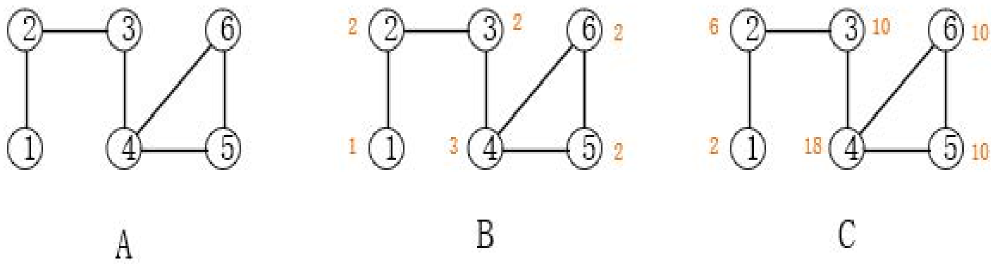


Figure 1. Each network has six nodes. Graph A is the original network, graph B marks the degree value of each node, and graph C calculates the weighted degree of each node by using Coulomb's law on the basis of graph B.

what is more, B wang et al. [32] get a structure entropy based on nonextensive statistical mechanics and similarity of nodes, but the degree or similarity of a node is characterized by the number of nodes connected to it. In real life, the importance of each node is not the same, and the contribution to the neighbor node is not the same too. For example, if a person is the president, there are many people who have intersection with him. But a person is an ordinary people, there are few people who have intersection with him. So they both have different influence on another person. Therefore, the weighted degree is considered for calculating our entropy. Inspired by Coulomb's law, this paper regards the degree of each node as the point charge. The charge of each node is the same symbol, and there is repulsive force between the connected charges. At this time, the importance of a node is equal to the sum of the repulsive scalars between it and its neighbors. $DC(i)$ and $DC(j)$ is the degree of node i and node j respectively. The network studied in this paper is a node weighted network, but the length of the edge e_{ij} between node i and node j is 1, then the repulsive force between them is expressed as follows:

$$f_{ij} = DC(i) \times DC(j), \tag{3.1}$$

where the repulsive force only exists between two directly connected nodes, and its magnitude depends on the degree value of them. That is $e_{ij} = 0$, then there is no repulsive force between node i and node j. Based on the previous description, the weighted degree is defined as follows:

$$\omega_i = \sum\nolimits_j f_{ij}, \tag{3.2}$$

The node j is the neighbor of node i, and the the weighted degree of node i is equal to the sum of the repulsion scalar of all neighbor nodes, it is shown in Figure 1.

In this paper, we propose a new structure entropy of complex network based on Coulomb's law, which we will get through the following steps.

Step 1: Calculate the degree and betweenness of each node with formula which we gave above.

Step 2: The repulsive force between two nodes in complex network is calculated by formula $f_{ij} = DC(i) \times DC(j)$, and get a series of values.

Step 3: Calculate the distribution rate of weighted degree. Here we use this function $\gamma_i = \frac{\omega_i}{\sum_{i=1}^{n} \omega_i}$, the required value is obtained, where n is the number of nodes in a complex network.

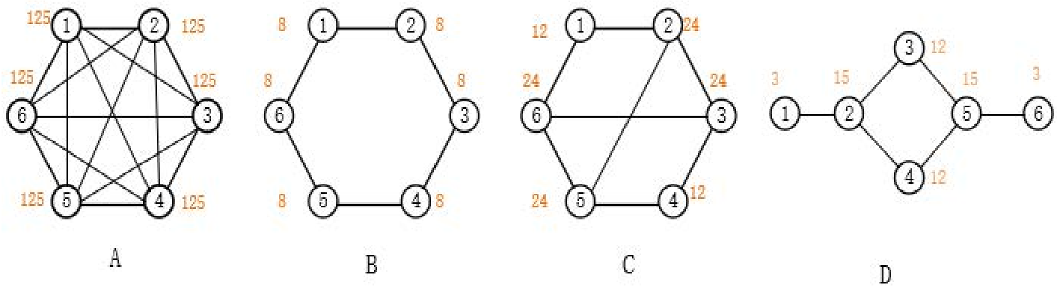


Figure 2. Each network is composed of six nodes, but the connection mode of nodes is different. The weighted degree of nodes in the graph is calculated according to Coulomb's law and marked next to the corresponding node, here we assume that the length of each edge is 1.

Table 1. The structure entropy of the test networks.

Networks	A	B	C	D
$H_{\deg}$	1.7918	1.7918	1.7753	1.7046
H_{bet}	1.7918	1.7918	1.6154	1.3102
H_{zqs}	1.7918	1.7918	0.7861	0.1151
H_{new}	1.7918	1.7918	0.8762	0.0692

Step 4: we define an index q_i, it is a component in the formula of nonexpansive statistical mechanicsand, the formula is defined as follows:$q_i = 1 + (BC_{\max} - BC(i)), \{BC_{\max} = \max[BC(i), (i = 1, 2, 3, \cdots, n)]\}$.

Step 5: The formula combines two new indexes and obtains a different functional equation.

$$H_{new} = -\sum_{i=1}^{n} \frac{\gamma_i^{q_i} - \gamma_i}{1 - q_i}, \tag{3.3}$$

where n is the number of nodes in a complex network.

The new structure entropy is defined above, p_i and q_i are nonexpansive statistical mechanical parameters, where p_i is defined based on degree and q_i is defined based on betweenness. However, considering the different contributions of each node to its neighbors, the importance of nodes can not be simply expressed by degree. We use the weighted degree instead of the traditional degree value to describe the importance of nodes, so in the formula we use γ_i instead of p_i, that is: $p_i = \gamma_i$, in addition, we let q_i keep the same as before, the new structure entropy can better describe the complexity of the network.

Table 2. The degree of the complexity in those test networks.

Networks	A	B	C	D
$H_{\deg}$	Network A ≡	Network B >	Network C >	Network D
H_{bet}	Network A ≡	Network B >	Network C >	Network D
H_{zqs}	Network A ≡	Network B >	Network C >	Network D
H_{new}	Network A ≡	Network B >	Network C >	Network D

Table 3. The structure entropy of the small world networks.

Nodes	Edges	$H_{\deg}$	H_{bet}	H_{zqs}	H_{new}
50	300	3.8841	3.7519	3.2904	3.1475
100	600	4.5812	4.4912	4.2507	4.1730
200	1200	5.2679	5.1636	4.9747	4.8848
400	2400	5.9564	5.8329	5.6085	5.5096
600	3600	6.3644	6.2399	6.0502	5.9521

According to Figure 2, network A is a global coupled network, network B is a nearest neighbor coupled network, network C is a symmetric network, and network D is a spindle network. We use Coulomb's law to calculate the weighted degree of each node, and then calculate the new structure entropy combined with the betweenness of nodes.

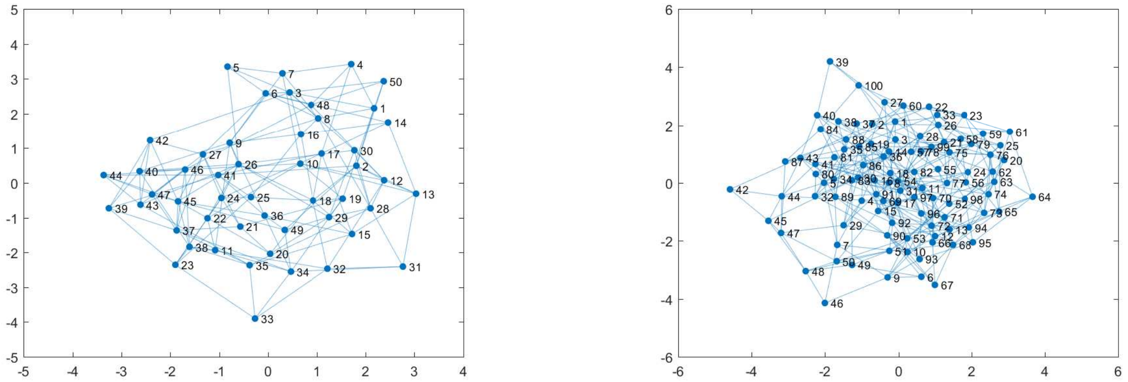
In order to prove the rationality of our proposed structure entropy, we improve the four networks constructed by other one, as shown in Figure 2. Here we calculate the complexity of the four networks by using degree structure entropy, betweenness structure entropy, structure entropy based on Tsallis nonextensive statistical mechanics and the proposed structure entropy respectively. The results are shown in Table 1. The size relationship is shown in Table 2. The degree structure entropy of network A is equal to that of network B, the degree structure entropy of network B is greater than that of network C, and the degree structure entropy of network C is greater than that of network D. The betweenness entropy and Zhang's entropy have the same law as the degree entropy. Finally, let's take a look at the new structural entropy. Its conclusion is consistent with the previous three methods. It shows that our method can not only distinguish some special networks, but also it has the same function as the existing methods in other aspects.

From Table 1, our evaluation results are completely equal to those of the first three methods in global coupled network A and nearest neighbor coupled network B. However, in network C and network D, the difference of the evaluation results of the same method is different. The largest numerical difference is $0.8762 - 0.0692 = 0.8070$, The other three values are 0.0707, 0.0353 and 0.6710. It shows that the new method has better discrimination for irregular networks, so it is better than the previous methods.

4. Application

In order to prove the effectiveness of the proposed method, we construct five small world networks, which are often used for feasibility experiments because of their complex topology. The specific construction steps are as follows: Firstly, input the number of the network nodes. Secondly, given the number of neighbors of each node in the network. Then define the rewiring probability of random connection between nodes. Finally, run the Matlab program to get some required small world networks. We implement the proposed method by ourselves.

Here we calculate the structure entropy by using degree structure entropy, betweenness structure entropy, structure entropy based on Tsallis nonextensive statistical mechanics and the proposed structure entropy respectively. The results are shown in Table 3. From the table, we can find some rules that the value of the same structure entropy increases with the increase of the number of the nodes. That is to say, our method is consistent with other methods. In addition, in the same small world network, the degree entropy is greater than the betweenness entropy, the betweenness entropy is greater than the entropy based on Tsallis nonextensive statistical mechanics, and the entropy based on Tsallis nonex-



(a) (b)

Figure 3. Graph (a) and graph (b) are small world networks constructed by us. It can be clearly seen from the graph that their number of nodes is 50 and 100 respectively. Here we do not mark their weighted degree. The abscissa and ordinate only represent the position of the node in the diagram.

Table 4. The structure entropy of the real networks.

Networks(Nodes)	Edges	$H_{\deg}$	H_{bet}	H_{zqs}	H_{new}
Graph (72)	118	3.9066	3.1337	2.3488	1.7791
Centrality literature (129)	613	4.3731	3.3335	2.7342	2.2395
US Aire lines (332)	2126	5.0250	3.4217	3.0046	2.5550
Yeast (2361)	7182	7.1641	6.1907	6.3025	5.4028

tensive statistical mechanics is greater than our entropy. For example, in a scale-free network with 50 nodes $H_{\deg} > H_{bet} > H_{zqs} > H_{new}$, which is consistent in five networks.

We use the visualization method to process the above data. Figure 3 mainly shows the connection relationship between nodes. The abscissa and ordinate represent the position relationship of nodes in the figure. For example, on the abscissa, 0 represents the middle position of the graph, a negative number represents the left position of the graph, and a positive number represents the right position of the graph. From Figure 3, with the increase of the number of nodes, the fractal properties of the network become more and more obvious, which reflected in the new structural entropy is that the value is getting larger and larger.

In order to further verify the rationality of the proposed structure entropy, the existing structure entropy and the proposed structure entropy are used to calculate the structure entropy of the real network, Here we select Graph and digraph glossary network, Central literature network, US aire lines network and Yeast network. The results are shown in Table 4. These network data come from the website http://vlado.fmf.uni-lj.si/pub/networks/data/, they represent the connection between actual individuals. From the first column of the Table 4, we can see that the number of nodes in the each network increases from top to bottom. From the second column of the table, we can see that the number of edges in eacgh network also increases from top to bottom, they are very representative in real networks, therefore we choose these networks for experimental verification.

From Table 4, the structure entropy of four real networks contains some rules. For every real

Table 5. The size of the complexity in those real networks.

$H_{\deg}$	Yeast >	US Aire lines >	Centrality literature >	Graph
H_{bet}	Yeast >	US Aire lines >	Centrality literature >	Graph
H_{zqs}	Yeast >	US Aire lines >	Centrality literature >	Graph
H_{new}	Yeast >	US Aire lines >	Centrality literature >	Graph

network, the degree entropy is greater than the betweenness entropy, the betweenness entropy is greater than entropy based on Tsallis nonextensive statistical mechanics, and the entropy based on Tsallis nonextensive statistical mechanics is greater than our entropy. This shows that the proposed structural entropy based on Coulomb's law combines the influence of degree entropy and betweenness entropy, what is more, it considers the different influence of neighbor nodes on themselves.

In a word, we get the same conclusion on the real network and the constructed network, which shows that our proposed structural entropy is reasonable, because the new entropy synthesizes the advantages of the existing methods to a certain extent. Experiments show that it can be used to describe the complexity of complex networks. It also provides a new way to describe network complexity.

It can be seen from the Table 5 that the ranking order of the proposed structure entropy is the same as that of the other three structure entropy, which indicates that the new structural entropy can effectively describe the structural properties of complex networks, especially the complexity of complex networks.

5. Conclusions

The quantification of network complexity has always been an important problem. The existing structural entropy mostly describes complex networks from one aspect, so it has some limitations. Inspired by Coulomb's law, this paper puts forward the concept of weighted degree, which combines with betweenness in nonextended statistical mechanics so as to obtain a new structural entropy. The new entropy has better discrimination for irregular networks, which is not the characteristic of the existing methods. Our conclusion is verified by experiments on constructed and actual networks. At the same time, it is also found that the recognition degree of the new entropy and the existing structural entropy on the regular network is consistent, but they all have bad discrimination on these networks, which is also the difficulty and direction of future research.

Conflict of interest

The authors declare that there are no conflicts of interest related to this paper.

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