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Research article

A new structure entropy of complex networks based on nonextensive statistical mechanics and similarity of nodes

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Abstract: Entropy has been widely measured the complexity of complex networks. At present, many measures about entropies were defined based on the directed connection of nodes. The similarity of nodes can better represent the relationship among all nodes in complex networks. In the definition of similarity of nodes, the importance of a node in the network depends not only on the degree of the node itself, but also on the extent of dependence of neighboring nodes on the node. In this paper, we proposed a new structure entropy based on nonextensive statistical mechanics and similarity of nodes. In the proposed method, the similarity of nodes and the betweenness of nodes are both quantified. The proposed method considers the extent of dependence between neighbouring nodes. For some complex networks, the proposed structure entropy can distinguish complexity of that while other entropies can not be. Meanwhile, we construct five ER random networks and small-world networks and some real-world complex networks such as the US Air Lines networks, the GD'01-GD Proceedings Self-Citing networks, the Science Theory networks, the Centrality Literature networks and the Yeast networks are measured by the proposed method. The results illustrated our method for quantifying the complexity of complex networks is effective.

Keywords: complex network; structure entropy; similarity of nodes; nonextensive statistical mechanics; Tsallis entropy

1. Introduction

For a large number of real-world complex systems, they can be modeled into complex networks for analysis [1]. For examples, analyzing the statistical properties of urban transportation networks [2, 3], describing the interactions between proteins in biology [4–6], studying the spatio-temporal complexity and evolutionary mechanism of power systems [7–9]. Meanwhile, these concepts such as topology and dynamics [10, 11], self-similarity and fractal [12–14], the centralities of nodes [15, 16], the relationship between statistical mechanics [17, 18], the vulnerability of communities in complex networks [19] are gradually discovered. Recently, the quantification of the complexity of complex networks has always been an ongoing research topic. The complexity of networks is not only measured by numbers of nodes and edges but also affected by a structure of networks. As a result, there are many methods to calculate the structure entropy of complex networks. Some structure entropies based on degree distribution or the betweenness of nodes to quantify the complexity of networks [20–24]. However, these methods also have some limitations. On the one hand, the degree distribution may lose some global information [25, 26]. On the other hand, the degree and betweenness centrality distribution methods both are defined based on the number of directly connect neighbouring nodes. However, for measuring complexity of complex networks, the structure entropy should include information about all nodes in complex networks.

At present, the structure entropy of some complex networks is defined based on the degree of nodes [21, 27, 28]. In ref. [29], the neighborhood similarity of nodes can more accurately evaluate the importance of nodes compared with the degree of nodes. This suggests that the similarity of nodes contains more information than traditional methods [30, 31]. For nodes of complex networks, the definition of similarity of nodes, the importance of a node in the network depends not only on the degree of the node itself, but also on the extent of dependence of neighboring nodes on the node [29,32]. That is, the higher the neighborhood similarity of nodes, the easier the nodes are to be replaced, and the lower the structure importance of nodes [33, 34]. Inspired by this, this paper define the structure entropy of the complex networks based on the similarity of nodes. In fact, the complex network is composed of many nodes and the links between nodes. The importance of a node depends not only on the importance of the node itself in the complex network, but also on the influence of the local area network where the node is located in the whole network. Therefore, it is feasible to measure the complexity of complex networks based on the importance of nodes in the network.

The proposed method combines the similarity of nodes and the betweenness of nodes to quantify the complexity of the complex networks. The similarity of nodes considers the extent of dependence between nodes and neighbouring nodes. The betweenness of nodes reflect the importance of the nodes in the entire network and are independent of the topology of the network. Meanwhile, the betweenness index can find the importance of the nodes in the network thoroughly, and truly reflect the actual situation of the network. Some existing structure entropies are based on the betweenness of nodes. In the proposed method, the degree entropy and the betweenness structure entropy are a special case. This method is applied for five ER random networks [35] and five small-world networks [7] and some real-world complex networks [36], which including the US Air Lines networks, the GD'01-GD Proceedings Self-citing networks, the Science Theory networks, the Centrality Literature networks and the Yeast networks. The results show that the proposed method for quantifying the complexity of networks is reasonable.

The rest of this paper is organized as follows. In Section 2, the preliminaries briefly introduce some works about the similarity of nodes, the betweenness centrality, the degree distribution and some existing structure entropies. In Section 3, the new structure entropy of the complex networks is presented. In Section 4, some applications are given to illustrate with new structure entropy. In Section 5, the conclusions are given.

2. Preliminaries

In this section, some preliminaries briefly are introduced, including the similarity of nodes, the betweenness centrality, the degree distribution, and some existing structure entropies.

2.1. The similarity of nodes

For a given node in the network, its edges can be known easily, that is to say its neighbors are known [37]. If nodes "*a*" and "*b*" have more common neighbors, then nodes "*a*" and "*b*" are likely to be connected. In other words, for example, if two people have more mutual friends, the more likely they are to know each other [29]. The amount of intersection between the two nodes in networks, which noted as $S(a, b)$, is given as follows:

$$S(a,b) = \mid n(a) \bigcap n(b) \mid, \tag{2.1}$$

where, $n(a)$ is the neighbor of node "*a*" and $n(b)$ is the neighbor of node "*b*". And then, the similarity between nodes "*a*" and "*b*" which noted as $sim(a, b)$, is defined as follows:

$$sim(a,b) \begin{cases} \frac{|n(a) \bigcap n(b)|}{|n(a) \bigcup n(b)|}, & \text{if no edges between neighbouring nodes "}a\text{" and "}b\text{"} \\ 1, & \text{if there are edges between neighbouring nodes "}a\text{" and "}b\text{"}, \end{cases} \tag{2.2}$$

where $|n(a) \bigcap n(b)|$ represents the common neighbouring node of two nodes. The $|n(a) \bigcup n(b)|$ represents the sum of two node neighbors, that is to say all neighbors of two nodes [38]. To illustrate similarity of nodes, an example is given as Figure 1.

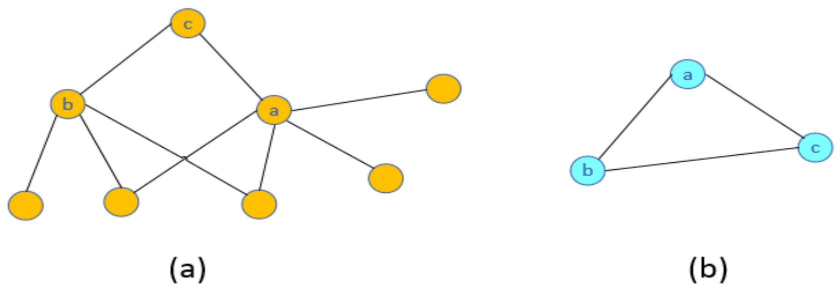

Figure 1. Overlap between the topologies of the neighbors of node "*c*". (a): Node "*a*" has five neighbouring nodes; node "*b*" has four neighbouring nodes; nodes "*a*" and "*b*" have three common neighbouring nodes, then $sim(a, b) = |n(a) \bigcap n(b)|/|n(a) \bigcup n(b)| = 3/6 = 1/2$. (b): There is an edge between nodes "*a*" and "*b*", then $sim(a, b) = 1$.

In Figure 1 (a), there is no connecting edge between nodes "*a*" and "*b*". Therefore, according to Eq. (2.2), $|n(a) \bigcap n(b)| = 3$ and $|n(a) \bigcup n(b)| = 6$, then $sim(a, b) = 3/6 = 1/2$. If there are edges between neighbouring nodes "*a*" and "*b*", as shown in Figure 1 (b), then $sim(a, b) = 1$ [29]. Meanwhile, the value of "*sim*" is between 0 and 1. The greater the similarity of node pairs is, the higher the coincidence extent of node structure is. The more neighbors a node has, the more important the node is. Based on the similarity of nodes, Chen et al. [29] given an index $LLS(i)$, which is defined as follows:

$$LLS(i) = \sum_{a,b\in n(i)} (1 - sim(a,b)), \tag{2.3}$$

where $LLS(i)$ represented the importance evaluation index of neighborhood similarity of node i. The larger the value of $LLS(i)$ is, the higher degree of topology agreement between neighbouring nodes. Therefore, the less important the node is.

2.2. *Degree distribution*

For networks, k_i represents the degree of the ith node. The degree distribution of the complex networks, denoted as p_i, is defined as follows [21,28]:

$$p_i = \frac{k_i}{\sum_{i=1}^{n} k_i}, \tag{2.4}$$

where p_i is often used to calculate the structure entropy of complex networks.

2.3. *Betweenness centrality*

The betweenness centrality is defined based on the number of shortest paths through a node to another node in the network. The betweenness of the ith node, which denoted as $v(i)$, is defined as follows [39]:

$$v(i) = \sum_{s\neq i\neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}, \tag{2.5}$$

where $\sigma_{st}(i)$ represents the number of the shortest paths that pass through the ith node. The σ_{st} represents the total number of the shortest paths from node s to node t.

2.4. *Tsallis nonextensive statistical mechanics*

A generalized form is postulated for entropy was given using a quantity normally scaled in multifractals by Tsallis and shown as follows [40,41]:

$$S_q = k\frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \qquad (q \in R), \tag{2.6}$$

where k is the Boltzmann universal constant for thermodynamic systems. The value of k will be taken to be unity in information theory. p_i is the associated probabilities and $\sum_{i=1}^{W} p_i = 1$. q is used to describe the nonextension additivity of the system. When the nonextensive parameter $q = 1$, the nonextensive additivity of the system degrades to classical additivity (extension additivity) [40]. The Eq. (2.6) is immediately verified that:

$$S_1 = \lim_{q\to 1} S_q = k\lim_{q\to 1} \frac{1 - \sum_{i=1}^{W} p_i exp[(q-1)lnp_i]}{q-1} = -k\sum_{i=1}^{W} p_i lnp_i. \tag{2.7}$$

Based on the replica-trick type of expansion, S_q can be written as follows:

$$S_q = \frac{k}{q-1} \sum_{i=1}^{W} p_i (1 - p_i^{q-1}) . \tag{2.8}$$

This nonextensive theory has been widely applied to various phenomena and methods such as nonextensive networks, generalized simulated annealing algorithm and so on [42–44].

2.5. *Some existing structure entropies*

Structure entropy can quantify the complexity of complex networks. In this subsection, some existing structure entropies, such as the Shannon entropy [27], the degree structure entropy [21, 28], the betweenness structure entropy [22] and a structure entropy based on nonextensive statistical mechanics are introduced [45].

Shannon put forward the concept of "information entropy" and solved the problem of quantitative measurement of information [46]. The information entropy is also known as the Shannon entropy, which is defined as follows [27]:

$$E_{Shannon} = \sum_{i=1}^{n} p_i log(p_i) . \tag{2.9}$$

The degree distribution is widely used in most of the existing structure entropies of complex networks, such as the degree structure entropy. It is defined as follows [21, 28]:

$$E_{deg} = \sum_{i=1}^{n} p_i log_2 \frac{1}{p_i} = -k \sum_{i=1}^{n} p_i log(p_i) , \tag{2.10}$$

where k is the Boltzmann constant and p_i is defined as in Eq. (2.4).

The betweenness structure entropy of complex network proposed in ref. [22], is defined as follows:

$$E_{bet} = -\sum_{i=1}^{n} p_i^{'} log(p_i^{'}) , \tag{2.11}$$

where $p_i^{'}$ is defined as follows:

$$p_i^{'} = \frac{v(i)}{\sum_{i=1}^{n} v(i)} , \tag{2.12}$$

where $v(i)$ is defined as in Eq. (2.5).

Recently, a structure entropy was given by Zhang et al. The structure entropy based on nonextensive statistical mechanics, combining degree entropy and betweenness entropy. It is defined as follows [45]:

$$S_q^{'} = -k \sum_{i=1}^{N} \frac{p_i^{q_i} - p_i}{1 - q_i} , \tag{2.13}$$

where p_i is defined as in Eq. (2.4). The q_i represents the ith entropic index which is defined based on the betweenness of the ith node. According to the definition of the betweenness and the principle of the Tsallis entropy, the entropic index set q is defined as follows [45]:

$$q = \{q_1, q_2, q_3, \cdots q_i \cdots q_n\}\,. \tag{2.14}$$

The entropic index q_i in the entropic index set is defined as follows:

$$q_i = 1 + (v(max) - v(i)), \{v(max) = max[v(i), (i = 1, 2, 3, \cdots, n)]\}\,. \tag{2.15}$$

This is a special structure entropy, when the entropic index set $q = \{q_1, q_2, q_3, \cdots q_i \cdots q_n\}$ is equal to $q = \{1, 1, 1, \cdots 1 \cdots 1\}$, the structure entropy is degenerated to the degree structure entropy.

All of these structure entropies are used to measure the complexity of complex networks. Both degree structure entropy and betweenness structure entropy are defined based on the Shannon entropy, and only a single network index is considered. However, in most complex networks, there are more than one statistical indexes of the network. Besides, the degree structure entropy ignores the global property of the complex networks; only the number of neighbouring nodes directly connected are considered. The betweenness structure entropy considers the global information of the complex networks, but when the network has big scales, the calculation process is very difficult. The structure entropy is given by Zhang et al. [21, 22, 45], which combines degree structure entropy and betweenness structure entropy. But the degree structure entropy only depends on the ratio of the number of edges connecting a single node to the sum of the number of edges of all nodes, so this method will not be able to get the relationship between unrelated nodes.

3. The proposed method

3.1. New structure entropy

In this section, a new structure entropy is proposed based on the nonextensive statistical mechanics and similarity of nodes. The new structure entropy is denoted by $S^{'}_{Q}$ and defined as follows:

$$S^{'}_{Q} = -k \sum_{i=1}^{N} \frac{P_i^{q_i} - P_i}{1 - q_i}\,, \tag{3.1}$$

where q_i represents the ith entropic index, which is defined based on the betweenness of the ith node. When the entropic index set $q = \{q_1, q_2, q_3, \cdots q_i \cdots q_n\}$ is equal to $q = \{1, 1, 1, \cdots 1 \cdots 1\}$, the structure entropy is degenerated to the degree structure entropy. And the value of q_i is bigger than 1. The P_i represents the new importance evaluation index of neighborhood similarity of the ith node common which is defined as follows:

$$P_i = \frac{LLS(i)}{\sum_{i=1}^{n} LLS(i)}\,. \tag{3.2}$$

In this method, the value of P_i is between 0 and 1. When $LLS(i) = \{LLS_1, LLS_2, LLS_3, \cdots LLS_i \cdots LLS_n\}$ is equal to $LLS(i) = \{0, 0, 0, \cdots 0 \cdots 0\}$, the structure entropy is degenerated to the betweenness structure entropy. In addition, the similarity between neighbouring node pairs can indicate their importance in networks. It is convenient to see the degree of association among nodes. The higher the similarity degree of the node is, the easier it is to be replaced by other nodes. In other words, the lower the similarity, the stronger the importance of the node.

3.2. Numerical examples

In order to illustrate our method, some examples are used to present. Four different structures of weighted networks are given and shown in Figure 2.

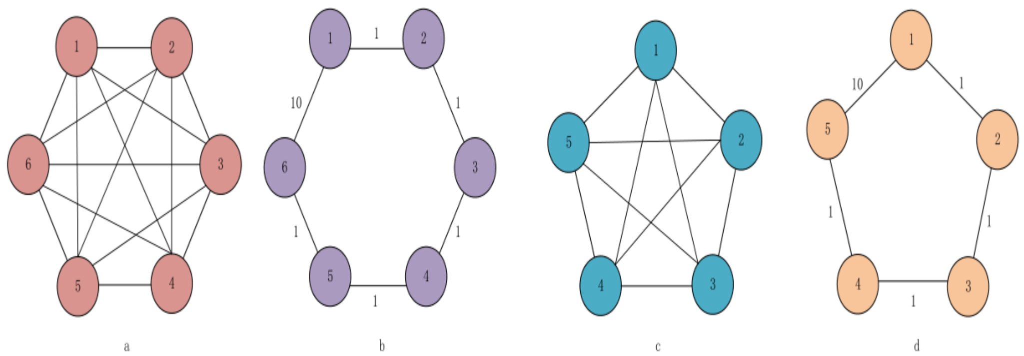

Figure 2. The Network *a* and *b* with 6 nodes, the Network *c* and *d* with 5 nodes. All the nodes of the Network *a* and *c* is connected, the Network *b* and *d* is the ring network.

From Figure 2, each node in the Network *a* and *c* has the same value of degree and betweenness. the Network *b* and *d*, the degree of each node is equal. Therefore, four networks can be divided into two parts, the Network *a* and *b* have the same degree distribution, the Network *c* and *d* also have the same degree distribution. But each of these four networks has its own structure.

The structure entropy of four networks is calculated by the degree entropy, the betweenness entropy, the structure entropy was given by Zhang et al. and the proposed structure entropy, respectively. The values of structure entropy are shown in Table 1. A more intuitive sorting of four networks complexities are shown in Table 2.

Table 1. The structure entropy results of the test networks.

test networks	*Network* a	*Network* b	*Network* c	*Network* d
E_{deg}	1.7918	1.7918	1.6049	1.6049
E_{bet}	1.7918	1.3660	1.6094	1.0887
$S_q^{'}$	1.7918	1.6308	1.6094	1.2996
$S_Q^{'}$	1.7918	0.2990	1.6094	0.1399

Table 2. The sort of the complexity in those test networks.

E_{deg}	Network a =	Network b	Network c =	Network d
E_{bet}	Network a >	Network b	Network c >	Network d
$S_q^{'}$	Network a >	Network b	Network c >	Network d
$S_Q^{'}$	Network a >	Network b	Network c >	Network d

From the second line of Table 1, when using the method E_{deg} to measure the complexity of four networks of Figure 2, the values of structure entropy are same ($E_{deg}(a)$=$E_{deg}(b)$=1.7918,

$E_{deg}(c)$=$E_{deg}(d)$=1.6049). On the contrary, when using the proposed method S'_Q to measure the complexity of four networks of Figure 2, the values of structure entropy are different ($S'_Q(a)$=1.7918, $S'_Q(b)$=0.2990, $S'_Q(c)$=1.6094, $S'_Q(d)$=0.1399). Besides, from the second and fourth columns of Table 1 that the values of structure entropy by the methods S'_q and S'_Q are same, the reason is that both the Network *a* and the Network *C* are global coupling networks. To sum up, these results shown that the proposed method can measure the complex networks with similar topological properties.

From Table 2, can more intuitively see the values of the complexity of the four networks is measured by the E_{deg}, E_{bet}, S'_q and the proposed method S'_Q. The values of degree structure entropy of the Network *a* and the Network *b* is equal to each other, and the values of degree structure entropy of the Network *c* is equal to the Network *d*. It means that the extent of complexity for the Network *a* is equal to the Network *b* in the method of degree structure entropy. The extent of complexity for the Network *c* is also equal to the Network *d*. However, from Figure 2, these four networks have different structures. This shown that degree structure entropy cannot accurately measure the complexity of the networks. From the third line to the fifth line of Table 1 that four different data can be obtained by using the betweenness entropy, the structure entropy was given by Zhang et al. and the proposed structure entropy to calculate four networks. This shows that the betweenness entropy, the structure entropy was given by Zhang et al. and the proposed method based on nonextensive statistical mechanics and similarity of nodes can effectively measure the complexity of different networks.

But in some cases, the betweenness structure entropy also has some shortcomings in the process of quantifying networks. For example, from Figure 3, the Network *a* and the Network *b* have the same betweenness structure entropy, the Network *c* and the Network *d* also have the same betweenness structure entropy. The specific calculation results are shown in Table 3.

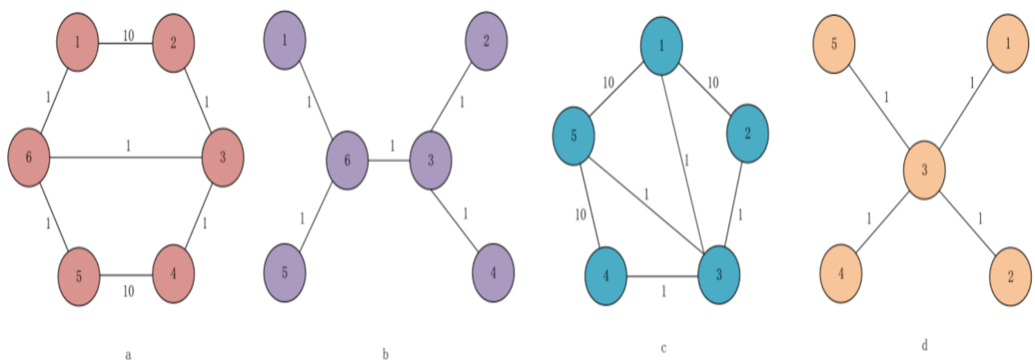

Figure 3. The Network *a* and *b* with 6 nodes, the Network *c* and *d* with 5 nodes. The four networks can be divided into two parts, the Network *a* and the Network *b* have the same betweenness, and the Network *c* and the Network *d* also have the same betweenness.

From the third line of Table 3, when using the method E_{bet} to measure the complexity of four networks of Figure 3, the values of structure entropy are same ($E_{bet}(a)$=$E_{bet}(b)$=0.6927, $E_{bet}(c)$=$E_{bet}(d)$=10.7510). On the contrary, when using the proposed method S'_Q to measure the complexity of four networks of Figure 3, the values of structure entropy are different ($S'_Q(a)$=0.6690, $S'_Q(b)$=0.6931, $S'_Q(c)$=0.1717, $S'_Q(d)$=10.7510). This shows that the proposed method can measure complex networks with similar topological properties. From the fifth column of Table 3, for the same network the values of S'_q and S'_Q are same, the reason is that these methods are degenerated to the

Table 3. The structure entropy results of the test networks.

test networks	*Network* a	*Network* b	*Network* c	*Network* d
E_{deg}	1.7721	1.6434	1.5741	1.3863
E_{bet}	0.6927	0.6927	10.7510	10.7510
S'_q	1.5475	1.4564	0.9926	10.7510
S'_Q	0.6690	0.6931	0.1717	10.7510

Table 4. The sort of the complexity in those test networks.

E_{deg}	Network a >	Network b	Network c >	Network d
E_{bet}	Network a =	Network b	Network c =	Network d
S'_q	Network a >	Network b	Network c <	Network d
S'_Q	Network a <	Network b	Network c <	Network d

betweenness structure entropy. Besides, from Table 1 and Table 3, for the same network the values of the proposed method S'_Q are smaller than other methods and the specific reasons need futher study.

Sort the complexity of four networks in Figure 3, as shown in Table 4. From Table 4, can more intuitively see the values of the complexity of four networks is measured by the E_{deg}, E_{bet}, S'_q and the proposed method S'_Q. The values of the betweenness structure entropy of the Network a and b have the same complexity, the Network c and d have the same complexity. However, from Figure 3, these four networks have different complexity and different structures. This indicates that betweenness structure entropy cannot accurately measure some weighted networks with special structure. Again, from the fifth line of Table 3 that have four different data. The results shown that our method can quantify the complexity difference between four networks.

In conclusion, neither degree structure entropy nor betweenness structure entropy can quantify some weighted networks with special structures. However, the proposed method based on nonextensive statistical mechanics and similarity of nodes can distinguish the complexity of these networks with special structures.

4. Measuring the complexity of ER random networks, small-world networks and some real-world complex networks

In this section, the constructed ER random networks, small-world networks and five real-world complex networks are used to illustrate the feasibility of the proposed method. Due to the ER random networks with complex topology and unknown organizational scale, the ER random networks are often used in complex networks research. Five constructed ER random networks are applied to test the feasibility of the proposed method. One of the ER random network is shown in Figure 4 (a). From Figure 4 (a), the ER random network has 200 nodes, and each pair of nodes is connected with a probability of p. The construction steps of the ER random networks are as follows [35],

Step 1. Given N nodes and the probability p of node to be connected by other nodes;

Step 2. Select two different nodes with no edges connected and generate a random number r ($0 < r < 1$). If $r < p$, then add an edge between these two nodes, otherwise there is no edge;

Step 3. Repeat the Step 2 until all node pairs are selected.

For these constructed ER random networks, the complexity are measured by the proposed method, Eq. (2.10), Eq. (2.11) and Eq. (2.13), the results are shown in Table 5. From Table 5, the values of the structural entropy obtained by these methods increases with the increase of the number of nodes. Meanwhile, for the same network most of the values of E_{deg}, E_{bet} and S'_q are bigger than the proposed method S'_Q. This is a question that needs to be studied.

Table 5. The structure entropy of the ER random networks.

Nodes	Edges	E_{deg}	E_{bet}	S'_q	S'_Q
50	31	3.4435	2.2227	2.6619	1.5887
200	396	5.1506	4.8519	4.1936	4.4422
400	1603	5.9319	5.7816	5.5078	5.6880
600	3484	6.3556	6.2517	6.1866	6.0594
800	6375	6.6551	6.5736	6.5501	6.4573

Although the ER random networks has short average distance, it does not have the characteristics of large clustering coefficients. Therefore, in order to more convincingly verify the effectiveness of the proposed method, five small-world networks are constructed. The five small-world networks with 50, 200, 400, 600, 800 nodes, respectively. A small-world network with 200 nodes is shown in Figure 4 (b). The construction steps of the small-world networks are as follows [7],

Step 1. Start with a regular network. Construct a nearest-neighbor coupled network with N nodes, which are enclosed as a ring. At the same time, each node is connected to its neighbouring $K/2$ nodes, where K is an even;

Step 2. Randomized reconnection. Add an edge between a randomly selected pair of nodes with probability p. Among them, there can be at most one edge between any two different nodes, and each node cannot have an edge connected to itself.

(a) (b)

Figure 4. (a): The ER random network with 200 nodes. (b): The small-world network with 200 nodes.

From the second line of Table 6, the small-world network of 50 nodes is a circular structure, $q = \{1, 1, 1, \cdots 1 \cdots 1\}$ is obtained by calculation, so it is reduced to degree structure entropy. From the first line of Table 6 that the calculation results of the four methods are the same. Besides, although the five networks are all the small-world networks, the values of S'_Q are different after the complexity is measured by the proposed method. Meanwhile, as the number of nodes in the small-world networks gradually increases, the values of structure entropy also gradually increases. When comparing with methods E_{deg}, E_{bet} and $S^{'}_{q}$ it can be seen that the proposed method $S^{'}_{Q}$ is also can measure the complexity of complex networks and measure the interval of complex values is smaller. Thus, the Table 6 shows that when a network is given, the complexity can be measured by the proposed method.

Table 6. The structure entropy of the small-world networks.

Nodes	Edges	E_{deg}	E_{bet}	$S^{'}_{q}$	$S^{'}_{Q}$
50	50	3.9120	3.9120	3.9120	3.9120
200	1189	5.2699	5.1827	4.9910	4.8654
400	4299	5.9701	5.9067	5.8471	5.7687
600	9529	6.3816	6.3356	6.3029	6.2488
800	16518	6.6738	6.6405	6.6046	6.5646

In order to prove the proposed method can measure the complexity of the real-world complex networks. Some real-world complex networks, which including the US Air Lines networks, the GD'01-GD Proceedings Self-citing networks, the Science Theory networks, the Centrality Literature networks, and the Yeast networks are given [36]. Some statistics of these real-world complex networks are shown in the second to fifth lines of Table 7. And the complexity of these real-world complex networks are calculated by the existing structure entropy and the proposed structure entropy, respectively. The values of complexity are shown in last four lines of Table 7.

Table 7. The structure entropy of the real networks.

Network	Yeast	GD'01-GD	Science theory	US Air lines	Centrality literature
Nodes	2361	311	1589	332	129
Edges	7182	645	2742	2126	613
Diameter	7	8	11	6	4
Average degree	5.8568	4.1286	3.4512	12.8072	9.5039
E_{deg}	7.1641	5.2709	6.9677	5.0250	4.3731
E_{bet}	6.1907	4.7234	3.7416	3.4217	3.3335
$S^{'}_{q}$	6.3025	4.2295	6.3604	3.0046	2.7342
$S^{'}_{Q}$	5.1442	3.5994	2.9494	1.9874	1.8060

From Table 7, in one hand, for the same network the results shown the values of the structure entropy calculated by different methods are different. The reason is that four methods from different perspectives to measure the complexity of the complex networks. Meanwhile, the results shown that both the existing structure entropy and the proposed structure entropy can measure these complexity of the real-world complex networks. In the other hand, from the third and fifth columns of Table 7 that there is no proportional relationship between the values of structure entropy and the number of nodes and edges. Besides, the calculation results of the proposed method is slightly smaller, and the specific reasons need further study.

5. Conclusions

For complex networks, measuring the complexity is an important issue. Entropy is an effective way to measure the complexity of complex networks. In order to overcome the shortcomings of existing entropy methods, this paper proposed a new entropy measure based on the nonextensive statistical mechanics and each nodes similarity. In our method, the relationship of all node in the complex network is considered. Meanwhile, complexity of the five ER random networks, the five small-world networks and the five real-world complex networks are considered by using our method. The structure of these networks is distinguished. The results shown the proposed entropy can be used to quantify the complexity of networks, including some networks with special structures. Meanwhile, both the degree structure entropy and the betweenness structure entropy are special cases of our method.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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