Mathematical Biosciences and Engineering

MBE, 17(4): 3721–3735.
DOI: 10.3934/mbe.2020209
Received: 18 March 2020
Accepted: 18 May 2020
Published: 22 May 2020

http://www.aimspress.com/journal/MBE

Research article

Image edge detection based on singular value feature vector and gradient operator

Jiali Tang[1], Yan Wang[2], Chenrong Huang[3,*], Huangxiaolie Liu[4] and Najla Al-Nabhan[5]

[1] College of Computer Engineering, Jiangsu University of Technology, Changzhou 213001, China
[2] School of Foreign Languages, Jiangsu University of Technology, Changzhou 213001, China
[3] School of Computer Engineering, Nanjing Institute of Technology, Nanjing 211167, China
[4] Nanjing Vivo Software Technology Co., Ltd., Nanjing 211106, China
[5] Department of Computer Science, King Saud University, Riyadh 11362, Saudi Arabia

* **Correspondence:** Email: huangcr@njit.edu.cn.

Abstract: This paper presents an edge detection algorithm based on singular value eigenvector and gradient operator. In the proposed algorithm, the singular values of image blocks are first calculated, and the Sobel gradient template is extended to eight other directions. Then the gradient values of image pixels are determined according to the stability of the singular values of image blocks. The determination of gradient threshold is considered from both global and local aspects. After calculating the global and local gradient thresholds of the original image, the gradient threshold of the whole image is determined by weighting function. Then the edge pixels of the image are filtered according to the gradient threshold, and the edge information image of the original image is obtained. The experimental data show that the proposed algorithm can resist a certain degree of noise interference, and the accuracy and efficiency of edge extraction are better than other similar algorithms.

Keywords: singular value feature vector; gradient operator; edge detection

1. Introduction

Image segmentation technology plays a very important role in the field of machine vision [1–5], and it is the basis of subsequent image processing work. Image edge detection technology is the most important part of image segmentation technology [6]. Edge is an area in which the local gray level changes significantly [7,8]. It mainly exists between different objects, objects and backgrounds, and different regions. How to improve the accuracy of edge detection is an active topic in the field of machine vision [9,10]. For image signals, drastically changing parts, such as edges and contours, carry important feature information. Edge detection can find the boundary contours of objects to achieve the goal of target recognition [11,12]. Therefore, it has a wide range of applications in image processing, computer vision, fault diagnosis, hydrodynamics, seismic signal analysis, data compression, etc. [13]. Gradient-based edge detection operators are actually high-pass filters used to enhance edge points in images, such as Roberts [14], Sobel [15] and Prewitt operators [16]. Classical edge detection operators are very effective for high SNR image processing, but because they use a small template to convolute with the image, the effect of image processing with noise is not good [17]. The standard wavelet transform studied by Mallat and Zhong is also a representative gradient method [18]. However, when the detected image contains noise, the detection effect of the wavelet edge detection operator is greatly reduced. Many scholars have proposed various improved methods for wavelet edge detection, such as applying New Wavelet to traditional wavelet edge detection [19], and combining wavelet with other methods [20]. However, most of the above methods aim at non-noisy image edge detection, which can achieve good detection results, but the effect for noisy image is not good [21]. In this paper, singular value decomposition is introduced into edge detection. Local gradient singular value decomposition is applied in gradient field. The calculation methods of gradient modulus and gradient direction are improved. The gradient modulus defined by singular value can enhance image edge features and suppress noise. This method can not only extract weak edges and detect clear and complete single edges in noiseless images, but also can extract ideal edges in the presence of noise interference.

The remainder of the paper is organized as follows. In Section 2, basic principles of image gradient and Obel operator are introduced. Then we extend the Sobel gradient template to other directions. Section 3 designs an image edge detection algorithm based on singular value feature vector and gradient operator. The experimental results are given in Section 4. Finally we summarize the full paper in Section 5.

2. Image edge detection algorithm based on gradient operator

2.1. Gradient of image pixels

Image gradient is a highly discriminatory feature representation method, which has a certain anti-noise ability [22]. In gray image, gradient is usually used to reflect the gray change at the edge. The gradient of image has been proved to be a highly discriminatory feature representation method, and has a certain degree of noise resistance [23,24].

Let f be a continuous function, so the gradient of (x, y) is

$$
\begin{aligned}
\nabla f(x,y) &= \left[P_x(i,j), P_y(i,j)\right]^T \\
&= \left[\frac{\partial P(i,j)}{\partial x}, \frac{\partial P(i,j)}{\partial y}\right]^T \\
M(i,j) = \left|\nabla f(x,y)\right| &= \left(\left|\frac{\partial P(i,j)}{\partial x}\right|^2 + \left|\frac{\partial P(i,j)}{\partial y}\right|^2\right)^{1/2}
\end{aligned} \tag{1}
$$

where $\nabla f(x,y)$ is the first order differential of $f(x,y)$ in different direction. $M(i,j)$ is the gradient of (x,y). For discrete pixels of gray image, the discrete first-order difference is used to replace the continuous first-order differential operation for gradient calculation.

$$
\left|\nabla f(x,y)\right| = \left\{\begin{matrix}\left[f(x,y) - f(x+1,y)\right]^2 \\ +\left[f(x,y) - f(x,y+1)\right]^2\end{matrix}\right\}^{1/2} \tag{2}
$$

where $f(x,y)$ is the pixel value located at (x,y) in the image f.

2.2. Principles of Sobel operators

At present, Roberts operator, Prewitt operator, Kirsch operator and Sobel operator are commonly used to detect image edges by finding the maximum. Laplacian algorithm and its derivative algorithm are used to detect image edges by finding zero-crossing points of second derivative. Sobel algorithm is a very important algorithm in the field of image detection. It plays an irreplaceable role in both machine vision and image processing. It is a discrete first-order difference operator, which is mainly used to calculate the approximation of image gray function under the first-order gradient. Using this algorithm for every point of digital image, different gradient vectors or other vectors can be obtained. Sobel algorithm uses these vectors to get image edge points [25,26].

Generally, Sobel operator contains two sets of matrices, namely, two directional templates, the size of which is 3 × 3. Generally, the principle of edge detection by Sobel operator is convolution, that is, image and directional template are convoluted, and the result can be obtained as

$$
\begin{aligned}
f_x(x,y) &= \left\{f(x+1,y-1) + 2f(x+1,y) + f(x+1,y+1)\right\} - \\
&\quad \left\{f(x-1y-1) + 2f(x-1,y) + f(x-1,y+1)\right\} \\
f_y(x,y) &= \left\{f(x-1,y+1) + 2f(x,y+1) + f(x+1,y+1)\right\} - \\
&\quad \left\{f(x-1,y-1) + 2f(x,y-1) + f(x+1,y-1)\right\}
\end{aligned} \tag{3}
$$

So the direction template is also called convolution core. Two commonly used horizontal and vertical templates are as Figure 1.

In edge extraction, each pixel in the image is convoluted with the template separately, and the larger value of the convoluted results is used to replace the pixel value of the corresponding point in the center of the template. If the pixel value of the processed pixel is greater than or equal to the smear value, it can be preliminarily judged that the pixel is an edge point. Since the adjacent pixels of a pixel point in an image are not only in its horizontal and vertical directions, in order to better reflect the gradient of the pixels in each direction, the Sobel operator is extended from horizontal and vertical directions to other directions as Figure 2.

−1	0	1
−2	0	2
−1	0	1

1	2	1
0	0	0
−1	−2	1

Figure 1. Convolution template of image block in horizontal and vertical directions.

1	2	1
0	0	0
−1	-2	-1

(a)

2	1	0
1	0	−1
0	-1	-2

(b)

1	0	-1
2	0	−2
1	0	-1

(c)

0	-1	-2
1	0	−1
2	1	0

(d)

−1	0	1
−2	0	2
−1	0	1

(e)

−2	-1	0
−1	0	1
0	1	2

(f)

1	2	1
0	0	0
−1	−2	1

(g)

0	1	2
−1	0	1
−2	-1	0

(h)

Figure 2. Convolution template in eight directions of image block, the direction angles of (a–h) are 0, 45, 90, 135, 180, 225, 270, 315.

The gradient of the pixels can be obtained by convolution template in eight directions as

$$s(x,y)=\sqrt{f_{\theta=0}^{2}(x,y)+\ldots+f_{\theta=315}^{2}(x,y)} \tag{4}$$

where θ is the gradient direction of Sobel template. (x,y) represents the coordinate position of the image pixels. For the boundary pixels of gray image, there may be some incomplete gradient calculation model in some direction. Because of the particularity of the boundary pixels, we can regard the area outside the gray image as the same area as the boundary pixel value, so that the gradient value of the boundary pixels of gray image is only determinated by the pixel within the image.

3. Algorithm for image edge detection

This section proposed a detailed algorithm for image edge detection. For simplicity, we analyze the singular values of image blocks firstly, then determine the threshold of image pixel gradient based on global and local thresholds, and finally extract edge pixels. The solving process is divided into three stages.

3.1. Gradient of image pixels

Singular Value Decomposition (SVD) is an important matrix decomposition method in linear algebra. Singular values of images have good characteristics in describing the distribution characteristics of image pixel matrix, and can capture the important basic structure of image data, thus reflecting the essence of image pixel values [27,28]. Singular value decomposition is defined as follows:

Let $Z\left(Z \in R\right)$ be the image pixel matrix, where R is the real number field. The singular value decomposition of matrix Z is defined as

$$Z = U\lambda V^T \tag{5}$$

where $U \in R^{m\times m}, V \in R^{n\times n}$ are the unitary matrixs, they satisfy the condition $UU^T = E, VV^T = E$. T represents the transpose of a matrix. $\lambda \in R_{m\times n}$ is a diagonal matrix whose elements on the non-diagonal line are non-negative. The element λ_i on the diagonal line is called the singular value of matrix Z. The common practice is to arrange singular values from large to small as $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_i = \lambda_{i+1} = ... = 0$, then λ_i can be uniquely determined by A.

3.2. Stability analysis of singular values

In order to prove the stability of singular values, the concept of norm is introduced in this section. The norm of image matrix has the following relationship with singular values:

Let $A = USV^T$ be the singular value decomposition of matrix $A \in R^{m\times n}$, where $S = diag\{\sigma_1, \cdots, \sigma_p\}$. Then its 2-norm is as

$$\|A\|_2 = \sqrt{\lambda_{\max}(A^T A)} = \sigma_1 \tag{6}$$

Let $A, B \in C^{m\times n}$ be two sets of image blocks, which singular values are $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_n \geq 0$, $\tau_1 \geq \tau_2 \geq ... \geq \tau_n \geq 0$. Then for each unitary invariant norm $\| \ \|$,

$\left\|diag\left(\tau_1-\sigma_1,\tau_2-\sigma_2,...,\tau_n-\sigma_n\right)\right\|\leq\left\|B-A\right\|$. From known conditions we can get $\sigma_1\geq\sigma_2\geq...\geq\sigma_p\geq0$, for each unitary invariant norm on $C^{m\times n}$, we can get

$$\left\|diag\left(\tau_1-\sigma_1,\tau_2-\sigma_2,...,\tau_n-\sigma_n\right)\right\|\leq\left\|A'-A\right\| \tag{7}$$

Obviously $\left|\sigma_1-\delta_1\right|\leq\left\|diag\left(\tau_1-\sigma_1,\tau_2-\sigma_2,...,\tau_n-\sigma_n\right)\right\|$ and $\left\|A'-A\right\|=\left\|\Delta A\right\|$, so we can get

$$\left|\sigma_1-\delta_1\right|\leq\left\|A'-A\right\|=\left\|\Delta A\right\| \tag{8}$$

where $\left\|\Delta A\right\|$ represents the difference between two matrices. For the gray image in this paper, A can be regarded as the difference between the gradient operator matrices of two adjacent pixels. From the adjacency of the pixels, it can be seen that the difference between the gradient operator matrices of adjacent pixels is small. From the above process, we can see that the maximum singular value of the image pixel matrix has good stability, that is, the maximum singular value of the image pixel matrix changes little after slight noise interference. Moreover, the maximum singular value of a matrix is often much larger than other singular values, and the stability of the matrix is guaranteed without affecting the order of the size of the singular values.

3.3. Adaptive gradient operator threshold

Global threshold refers to the use of only one threshold in all image processing, which is called global threshold. We usually determine the global threshold based on the image histogram. Certainly, it is also necessary to use the distribution of gray space to determine the global threshold. The principle of binarization based on global thresholds is simple. It only needs to compare the gray values of all the pixels in the image with the global thresholds. If the gray values of the pixels are larger than the global thresholds, they are taken as white; otherwise, they are taken as black. It can be seen that the global threshold algorithm is relatively simple to run. Local threshold is a threshold set for each locality, which is called local threshold. That is to say, the local threshold is determined according to the distribution of the pixels in the neighborhood for the pixels in the location. The advantage of local thresholds is that they are flexible. Different thresholds can be determined according to different situations without generalization and specific analysis. However, the design of local threshold has strong pertinence, and there is no general algorithm to realize automatic threshold processing.

Therefore, this paper combines global threshold with local threshold to seek adaptive threshold setting. The adaptive global threshold is based on the method of maximum inter-class variance, and the local threshold is based on Bernsen algorithm.

Suppose the number of gray levels contained by image is L, the number of pixels with gray value i is N_i, and the total number of pixels in the image is $N=N_0+N_1+...+N_{L-1}$. Then the probability of the point whose gray value is i is

$$P_i=N_i/N \tag{9}$$

Set a threshold t, the whole image can be divided into dark areas $c1(0,1,...,t)$ and bright areas $c2(t+1,...,L-1)$. The global threshold TM can be obtained as

$$
\begin{aligned}
& TM = \arg\max\left\{a1(t)\times a2(t)\Delta u^2\right\} \\
& s.t. \quad TM = u1 - u2 \\
& \qquad \sigma = a1\times a2(u1-u2)^2 \\
& \qquad a1 = sum(P_i)(0\to t), a2 = 1-a1 \\
& \qquad u1 = sum(i\times P_i)/a1(0\to t), \\
& \qquad u2 = sum(i\times P_i)/a2(t+1\to L-1)
\end{aligned} \tag{10}
$$

Let $f(i,j)$ be the gray value of the image, the window size centered on the pixels (i,j) is $(2\omega+1)\times(2\omega+1)$, then the local threshold TL of each pixel is

$$TL(i,j) = 1/2\times\left(\max f(i+m, j+m) + \min f(i+m, j+m)\right) \tag{11}$$

The final threshold of this algorithm is determined as the weighting of global and local thresholds. The formula is

$$T = TM\times\alpha + TL(i,j)\times(1-\alpha) \tag{12}$$

where α represents the weight factor, TM and TL represent the global and local gradient threshold. When α is assigned to an initial value, the final threshold T of the image pixel gradient can be calculated automatically by Eq (12).

3.4. Proposed edge detection algorithm

Based on the above theoretical basis, this section gives the specific steps of the edge detection algorithm in this paper as follows:

Stage 1 (Image Pixel Gradient Calculation Based on Singular Value). Let f be original image. Firstly, for each pixel, we can find a 3×3 image block centered on it. For example, for an image pixel $f(i,j)$, the set of image blocks is $\sum_{m=i-1,n=j-1}^{i+1,j+1} f(m,n)$. For each image block, we decompose it into singular values, and then replace the original pixels with the maximum of the singular values of the image block. The image is divided into 3×3 image block sets $\sum_{l=1}^{N}\sum_{m=i-1,n=j-1}^{i+1,j+1} f_l(m,n)$, where N is the number of the image blocks. Using the eight templates in Figure 2 to convolute each image blocks, we can get the gradient set of each pixel by Eq (4).

Stage 2 (Determination of Adaptive Pixel Gradient Threshold). For the gradient set E in stage 1, we first calculate the global gradient threshold and local gradient threshold of the original image gradient according to Eqs (10) and (11), then assign an initial value to the weight factor α, and calculate the final threshold T of the image pixel gradient according to Eq (12). The value of weight factor α can be adjusted according to the final edge information. For example, if the edge information image obtained by this algorithm is too complex for an original gray image, that is, there are more edge pixels, then the value of weight factor α can be reduced and the gradient threshold of the image can be recalculated. Conversely, if the edge information image obtained by this algorithm is not clear for an original gray image, that is, fewer edge pixels can be calculated, the weight factor α can be increased, the gradient threshold of the image is recalculated after the new value of α. Repeat this process until the final satisfactory edge information image is obtained.

Stage 3 (Extraction of Image Edge Pixels). On the basis of the first two stages, we determine whether the pixel of an image is an edge point according to the threshold T and the gradient value of the pixel. If the gradient of the pixel of an image is greater than the threshold gradient, we determine that the pixel is an edge pixel of the image. If the gradient value of the pixel of an image is less than the threshold gradient, we determine that the pixel is a non-edge pixel of the image. For edge pixels, we use black dots instead of original pixels (let the gray value of the pixel be 0); for non-edge pixels, we use white dots instead of original pixels (let the gray value of the pixel be 255). The formula is

$$f'(x,y)=\begin{cases}0, & s(x,y)\geq T\\ 255, & s(x,y)<T\end{cases} \tag{13}$$

where $f'(x,y)$ represents the pixel value of the edge information image. We use this method to process each pixel and finally get the edge information image of the original image.

4. Experimental results and analysis

In this section, we tested the performance of our algorithm with images from the UCID test database. All the experiments are performed using MATLAB v7.0 on a Windows XP platform with a CPU at 1.9 GHz and 4 GB memory. The algorithms to be compared include the proposed algorithm, WT algorithm and Curvelet algorithm.

4.1. Parameters setting

In this paper, we adjust the value of weighting factor α to make the extracted edge information most obvious. The quality of edge information image can be evaluated by imperceptibility. Peak signal-to-noise ratio (PSNR) is used to evaluate the algorithm by comparing the original image with the edge information image and expressing the peak mean square error of the image in logarithmic form. The calculation formula of PSNR is

$$PSNR=-10\log_{10}\frac{\sum_{k=1}^{d}\sum_{m=0}^{M}\sum_{n=0}^{N}\left[I_k(m,n)-I_k'(m,n)\right]^2}{d\times M\times N\times\max_{m,n}\left\{I_k^2(m,n)\right\}} \tag{14}$$

where $I_k(m,n)$ is the pixel value of the original image, $I_k'(m,n)$ is the pixel value of an edge image, $d=3$ represents the image is a color image, $d=1$ represents image is gray image. $M \times N$ is the size of image, $\max_{m,n}\left\{I_k^2(m,n)\right\}$ represents the square of the maximum pixel, and for gray image, $\max_{m,n}\left\{I_k^2(m,n)\right\}=255^2$.

4.2. Results and analysis

Figure 3 shows the edge detection results of the algorithm under different noise conditions, where (a) is the original image; (b) is the image after adding salt and pepper noise; the noise intensity $\delta = 0.01$; (c) is the original image after adding Gauss noise, the noise intensity $\delta = 20$; (d) is the edge detection result of the original image; (e) is the edge detection result of the image (b); and (f) is the edge detection result of the original image (a). The edge detection result of image (c) shows that the edge information of image can still be detected by this algorithm after adding salt and pepper or Gauss noise. Experiments show that the proposed algorithm has a certain anti-noise performance.

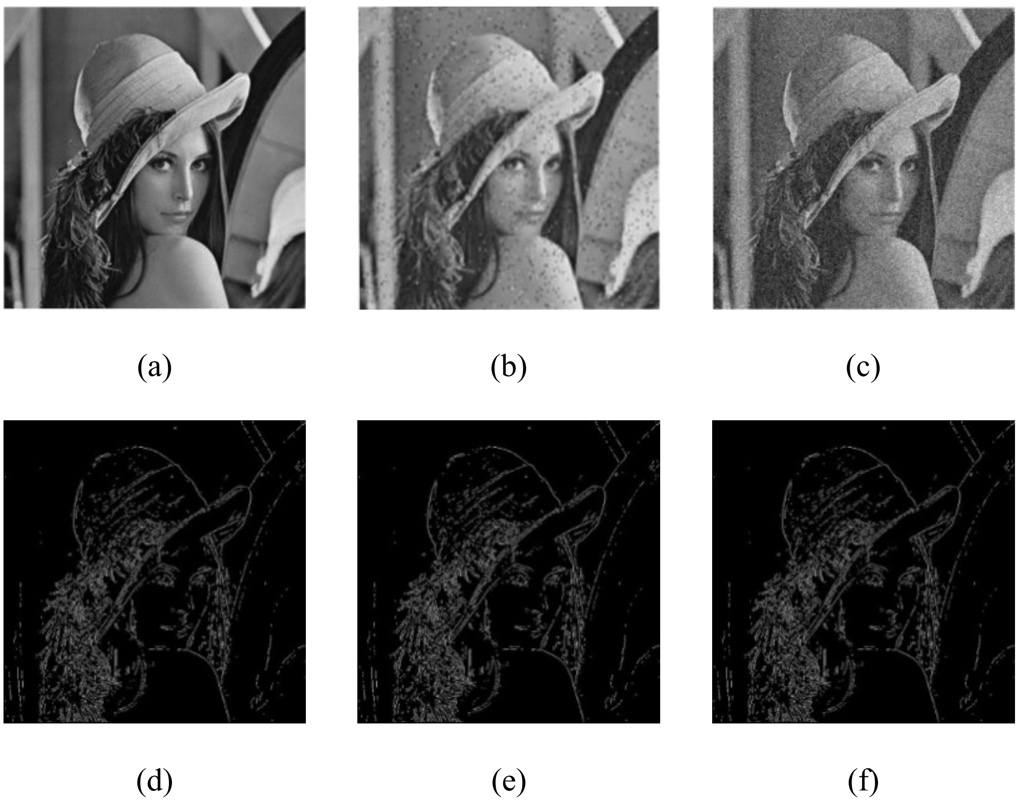

Figure 3. Edge detection results of proposed algorithm under different noise interference conditions: Salt and pepper noise and Gaussian noise.

Figure 4 shows the experimental results of three different edge detection algorithms. Three representative gray-level images Lena, House and Peppers are selected in this paper. The first line are original image, the second are WT detection algorithm, the third are Curvelet algorithm and the fourth behavior our proposed algorithm. From the data in the graph, it can be seen that the edge information extracted by the algorithm in this paper is clearer than that extracted by the other two algorithms. That is to say, the detection effect of the proposed algorithm is better than that of the other two algorithms.

Figure 4. Comparisons of edge extraction results of three algorithms: WT algorithm, Curvelet algorithm and our proposed algorithm.

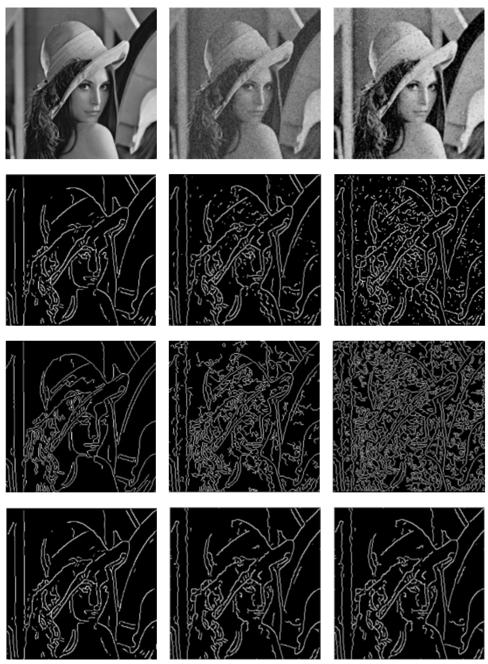

Figure 5. Comparison of edge detection results between three algorithms under different noise conditions.

Figure 5 shows the detection results of three detection algorithms for Lena image under different noise conditions. The first column is the detection results of three algorithms for the original image, the second the detection results of three algorithms after adding Gauss noise, and the third the detection results of three algorithms after adding salt and pepper noise. From the information in the graph, it can be seen that the edge information extracted by this algorithm is clearer than that of the other two algorithms after adding Gauss noise, and the edge information extracted by this algorithm is obviously better than that of the other two algorithms after adding salt and pepper noise. The experimental results show that the proposed algorithm is superior to other similar algorithms in anti-jamming.

The experimental test images are 8 images in Figure 6.

Figure 6. Experimental images for testing running time and PSNR.

Table 1 presents the comparison of PSNR results of three algorithms for image detection time and edge information extracted. From the experimental results in Table 1, we can see that the algorithm in this paper runs faster than the other two algorithms, and the edge information image extracted by this algorithm has higher PSNR value.

Table 1. Comparisons of runtime and PSNR between three methods.

Image	Our method		WT method		Curvelet method	
	PSNR	Time (s)	PSNR	Time (s)	PSNR	Time (s)
Barbara	11.918	8.2124	8.928	16.0680	7.917	25.0420
Baboon	11.930	6.6189	9.491	18.7680	8.836	21.3780
Boat	11.712	8.8901	9.908	16.9420	8.959	25.4220
Cartoon	10.991	10.3894	8.663	16.3340	7.900	24.6960
Lake	12.065	15.1090	9.597	15.8810	8.926	33.4920
House	11.890	9.2783	10.910	21.2630	9.884	33.0670
Lena	10.910	8.3820	9.652	21.2910	8.950	23.7630
Peppers	11.790	11.2891	8.886	17.4220	8.836	36.8600

In order to test the edge detection accuracy of our proposed algorithm and other algorithms, images with known number of edge pixels are selected as the test images, as shown in Figure 7.

Three algorithms are used to detect the edge information, and then according to the ratio of the number of extracted edge pixels to the number of known edge pixels, the edge detection rate of the three algorithms can be obtained.

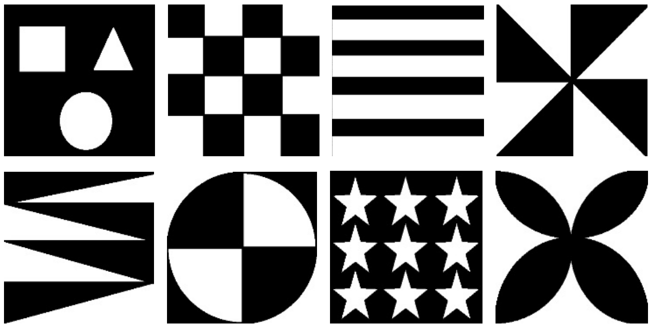

Figure 7. Experimental images with known number of edge pixels.

Three detection algorithms are used to detect the edge of the eight gray-scale images and extract the edge information image. Then the number of black points in the edge information image (i.e. the number of pixels whose pixel value is 0) is calculated. The detection rate of the algorithm is the ratio of the number of black points in the edge information image extracted by the algorithm to the number of known edge pixels.

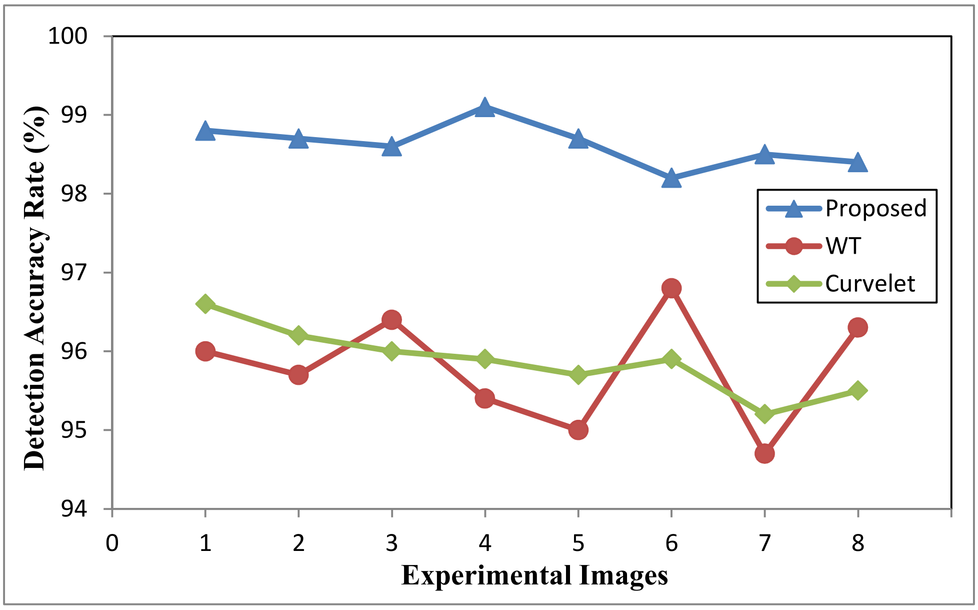

Figure 8. Experimental results of three different algorithms.

Figure 8 is a comparison of the detection rates of three algorithms for detecting experimental images. The abscissa is eight different test images, and the ordinate is the percentage of edge pixels detected by the algorithm. From the data in the graph, it can be seen that the detection rate of this algorithm is higher than that of the other two algorithms for different experimental images.

5. Conclusions

An image edge detection algorithm based on singular value vector and gradient operator is proposed in this paper. The effectiveness and advantages of this method are verified by experiments. In this paper, the gradient operator is extended to eight other directions to make the calculated gradient value more accurate. By analyzing the stability of the image block singular value, the maximum singular value of the image is used to improve the anti-interference ability of the algorithm. The threshold of image gradient is determined adaptively according to global and local image information. Experiments are given to compare the proposed algorithm with other similar algorithms in terms of visual perception, anti jamming and running time of the algorithm. The results show that the algorithm presented in this paper has good performance.

Acknowledgments

The authors extend their appreciation to the Deanship of Scientific Research at King Saud University for funding this work through research group No. RG-1441-331. This work was also supported by the Opening Project of Jiangsu Key Laboratory of Advanced Numerical Control Technology (SYKJ201804), by the Project funded by Jiangsu Postdoctoral Science Foundation (2019K041), by Nanjing Subsidy Project of IUR Cooperation (201722043) and by Changzhou Sci&Tech Program (CE20195030).

Conflict of Interests

The authors declare no conflict of interest.

References

1. H. Zhang, Z. Duan, Z. Zhu, Y. Wang, Fast moving object segmentation based on active contours, *J. Comput.*, **7** (2012), 863–869.
2. Z. Pan, C. N. Yang, V. S. Sheng, N. Xiong, W. Meng, Machine learning for wireless multimedia data security, *Secur. Commun. Networks*, **2019** (2019).
3. T. Ma, H. Rong, Y. Hao, J. Cao, Y. Tian, M. A. Al-Rodhaan, A Novel Sentiment Polarity Detection Framework for Chinese, *IEEE Trans. Affective Comput.*, **2019** (2019).
4. Y. Tian, M. M. Kaleemullah, M. A. Rodhaan, B. Song, A. Al-Dhelaan, T. Ma, A privacy preserving location service for cloud-of-things system, *J. Parallel Distrib. Comput.*, **123** (2019), 215–222.
5. B. Song, M. M. Hassan, A. Alamri, A. Alelaiwi, Y. Tian, M. Pathan, et al., A two-stage approach for task and resource management in multimedia cloud environment, *Computing*, **98** (2016), 119–145.
6. S. Mallat, W. L. Hwang, Singularity detection and processing with wavelets, *IEEE Trans. Inf. Theory*, **38** (1992), 617–643.
7. W. B. Wei, X. T. Rui, Study on edge detection method, *Comput. Eng. Appl.*, **42** (2006), 88–91.

8. Y. Tian, B. Song, M. Al-Rodhaan, C. R. Huang, M. A. Al-Dhelaan, A. Al-Dhelaan, et al., A stochastic location privacy protection scheme for edge computing, *Math. Biosci. Eng.*, **17** (2020), 2636–2649.
9. P. Selvakumar, S. Hariganesh, *The performance analysis of edge detection algorithms for image processing*, IEEE International Conference on Computing Technologies and Intelligent Data Engineering, 2016, 1–5. Available from: https://ieeexplore.ieee.org/abstract/document/7725371.
10. Y. Li, X. Chang, J. Chang, Image edge detection based on Gaussian mixture model in non-subsampled contourlet domain, *J. Electr. Comput. Eng.*, **2016** (2016), 4125909.
11. L. Zhang, P. Bao, Edge Detection by Scale Multiplication in Wavelet Domain, *Pattern Recognit. Lett.*, **23** (2002), 1771–1784.
12. F. Y. Wang, M. Chen, Q. S. Fei, The improved Method for Image edge detection based on wavelet Transform with Modulus Maxima, *Adv. Mat. Res.*, **850** (2014), 897–900.
13. S. Rani, A novel mathematical morphology based edge detection method for medical images, *CSI Trans. ICT*, **4** (2016), 217–225.
14. M. Raman, A. Himanshu, Study and comparison of various image edge detection techniques, *Int. J. Image Process.*, **3** (2009), 1–11.
15. E. Nadernejad, S. Sharifzadeh, H. Hassanpour, Edge detection techniques: Evaluation and comparisons, *Appl. Math. Sci.*, **2** (2008), 1507–1520.
16. M. Samuel, M. Mohamad, S. M. Saad, M. Hussein, Development of edge-based lane detection algorithm using image processing, *JOIV Int. J. Inf. Visualization*, **2** (2018), 19–22.
17. O. P. Verma, N. Agrawal, S. Sharma, *An optimal edge detection using modified artificial bee colony algorithm*, Proceedings of the National Academy of Sciences India, 2016, 1–12. Available from: https://link.springer.com/article/10.1007/s40010-015-0256-7.
18. S. Mallat, S. Zhong, Characterization of signals from multiscale edges, *IEEE Trans. Pattern Anal. Mach. Intell.*, **14** (1992), 710–732.
19. W. F. Ma, C. X. Deng, *An improved wavelet multi-scale edge detection algorithm*, IEEE International Conference on Wavelet Analysis and Pattern Recognition, 2010, 302–306. Available from: https://ieeexplore.ieee.org/abstract/document/6294797.
20. R. Silva, R. Minetto, W. R. Schwartz, H. Pedrini, Adaptive edge-preserving image denoising using wavelet transforms, *Pattern Anal. Appl.*, **16** (2013), 567–580.
21. C. I. Gonzalez, P. Melin, J. R. Castro, O. Mendoza, O. Castillo, An improved sobel edge detection method based on generalized type-2 fuzzy logic, *Soft Comput.*, **20** (2016), 773–784.
22. D. G. Lowe, Distinctive image features from scale-invariant keypoints, *Int. J. Comput. Vision*, **60** (2004), 91–110.
23. N. Nausheen, A. Seal, P. Khanna, S. Halder, A FPGA based implementation of Sobel edge detection, *Microprocess. Microsyst.*, **56** (2018), 84–91.
24. A. Fabijańska, A survey of subpixel edge detection methods for images of heat-emitting metal specimens, *Int. J. Appl. Math. Comput. Sci.*, **22** (2012), 695–710.
25. T. Chen, X. Sun, H. Han, X. You, Image edge detection based on ACO-PSO algorithm, *Image*, **6** (2015), 47–54.
26. G. T. Shrivakshan, C. Chandrasekar, A Comparison of Various Edge Detection Techniques Used in Image Processing, *Int. J. Comput. Sci. Issues*, **9** (2012), 269–276.
27. R. Liu, T. Tan, Digital image watermarking method based on SVD, *Acta Electron. Sin.*, **29** (2001), 168–171.

28. F. Guo, Y. Yang, B. Chen, L. Guo, A novel multi-scale edge detection technique based on wavelet analysis with application in multiphase flows, *Powder Technol.*, **202** (2010), 171–177.