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Research article

Periodic solution of a stage-structured predator-prey model incorporating prey refuge

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Abstract: In this paper, we consider a delayed stage-structured predator-prey model incorporating prey refuge with Holling type II functional response. It is assumed that prey can live in two different regions. One is the prey refuge and the other is the predatory region. Moreover, in real world application, we should consider the stage-structured model. It is assumed that the prey in the predatory region can divided by two stages: Mature predators and immature predators, and the immature predators have no ability to attack prey. Based on Mawhin's coincidence degree and novel estimation techniques for a priori bounds of unknown solutions to $Lu = \lambda Nu$, some sufficient conditions for the existence of periodic solution is obtained. Finally, an example demonstrate the validity of our main results.

Keywords: periodic solution; stage-structured; prey refuge

1. Introduction

From the initial work by Lotka [1] and Volterra [2], predator-prey model has become and will continue to be one of the main themes in mathematical biology. In the interaction between predator and prey, the phenomenon of prey refuge always exists. It is assumed that prey species can live in two different regions. One is the prey refuge and the other is the predatory region. From biological view, prey refuge can exists and there are no predators in the prey refuge, so it can help increase the population density of the prey. Further, refuge is an effective strategy for reducing predator-prey model with a refuge. Moreover, Magalhães et al. [5] studied the dynamics of thrips prey and their mite predators in a refuge, and they predicted the small effect of the refuge on the density of prey under the equilibrium state. Ghosh et al. [6] investigated the impact of additional food for predator on the dynamics of the predator-prey model with a prey refuge. When in a high-prey refuge ecological

system, it was observed that the predator extinction possibility may be removed by supplying additional food [7, 8] to predator population. Ufuktepe [9] investigated the stability of a prey refuge predator-prey model with Allee effects. Fractional-order factor was introduced into prey-predator model with prey refuge in Xie [10]. On the other hand, Holling [11] argued that the functional response is an important factor to affect the predator-prey model. The predator may reduce its feeding rate when it is fully saturated, and the feeding rate no longer varies with the increase in prey density. Thus he proposed three types of Holling functional response [12–25]. For the above reasons, Jana [26] considered the following predator-prey system incorporating a prey refuge:

$$\begin{cases} \frac{dx}{dt} = r_1 x (1 - \frac{x}{k_1}) - \sigma_1 x + \sigma_2 y, \\ \frac{dy}{dt} = r_2 y (1 - \frac{y}{k_2}) + \sigma_1 x - \sigma_2 y - \frac{\alpha y z}{a + y}, \\ \frac{dz}{dt} = \frac{\beta y (t - \tau) z (t - \tau)}{a + y (t - \tau)} - dz - \gamma z^2, \end{cases}$$
(1.1)

where x and y denote the density of the prey in the refuge and in the predatory region at any time t, r_1 and r_2 denote intrinsic growth rate respectively for the prey population x and y at any time t. Further, the environment carrying capacity are denoted by k_1 and k_2 respectively for the prey x and y. Then, at any time t, σ_1 denotes the per unit migration of the prey in the refuge to the predatory region and σ_2 denotes from the predatory region to the refuge. Next, z denotes density of the predator in the predatory region at any time t. In addition, the predator consumes the prey at Holling type II functional response $\frac{\alpha y}{a+y}$, where α is the maximal predator per capita consumption rate and a is the half capturing saturation constant. Furthermore, d is the natural death rate of predator consumes prey (assume that $0 < \beta \leq \alpha$). Because the reproduction of predators after predating the prey is not instantaneous, we assumed that the time interval between the prey are killed and the corresponding increase in the number of predators are thought to be time delayed τ of the system (1.1).

In the real world application, some authors argued that predators living in the predatory region are classified by two fixed ages [27–31], one is immature predator and the other is mature predator, the immature predator have no ability to attack prey.

Motivated by the above mentioned works, in the present paper, we investigate the periodic solution of the following delayed model:

$$\begin{pmatrix}
\frac{dx}{dt} = r_1(t)x(t)(1 - \frac{x(t)}{k_1(t)}) - \sigma_1(t)x(t) + \sigma_2(t)y(t), \\
\frac{dy}{dt} = r_2(t)y(t)(1 - \frac{y(t)}{k_2(t)}) + \sigma_1(t)x(t) - \sigma_2(t)y(t) - \frac{\alpha(t)y(t)z_2(t)}{a + y(t)}, \\
\frac{dz_1}{dt} = \frac{\beta(t)y(t)z_2(t)}{a + y(t)} - \frac{\beta(t - \tau)y(t - \tau)z_2(t - \tau)}{a + y(t - \tau)} - d_1(t)z_1(t), \\
\frac{dz_2}{dt} = \frac{\beta(t - \tau)y(t - \tau)z_2(t - \tau)}{a + y(t - \tau)} - d_2(t)z_2(t),
\end{cases}$$
(1.2)

where $z_1(t)$ and $z_2(t)$ denote the density of immature predator and mature predator respectively at any time t. Next, $r_1(t)$, $r_2(t)$, $k_1(t)$, $k_2(t)$, $\sigma_1(t)$, $\sigma_2(t)$, $\alpha(t)$, $\beta(t)$, $d_1(t)$ and $d_2(t)$ are continuously positive

periodic functions with period ω . Moreover, $d_1(t)$ and $d_2(t)$ are the death rate of predator at any time *t*. The nomenclature $\frac{y(t-\tau)z_2(t-\tau)}{a+y(t-\tau)}$ stands for the number of immature predator that were born at time $(t-\tau)$ which still survive at time *t* and become mature predator. The initial conditions for the system (1.2) are

$$(x(t), y(t), z_1(t), z_2(t)) \in C_+ = C([-\tau, 0], \mathbb{R}^4_+), x(0) > 0, y(0) > 0, z_1(0) > 0, z_2(0) > 0.$$

The aim of this paper is to obtain some sufficient conditions for the existence of positive periodic solution of system (1.2). However, we encounter with some difficulties when we use Mawhin's coincidence degree theory to obtain the periodic solutions. Firstly, the forth equation of system (1.2) has a term $y(t - \tau)z_2(t - \tau)$, rather than $y(t - \tau)z_2(t)$. If we follow the skill in [32], it will lead us to $u_3(\xi_3) \leq \bar{\alpha}^{-1}(a + l_+)[(\bar{r}_2 - \bar{\sigma}_2) + l_-^{-1}\bar{\sigma}_1 e^{u_1(\eta_1)}]$, which contains $u_1(\eta_1)$. Consequently, we can not get the bound of u_3 or u_1 . We will overcome this difficulty in the following paper.

The rest of our paper is organized as follows: section 2 is to prove the existence of the positive periodic solution of system (1.2). In section 3, an example is to demonstrate the obtained result.

2. The existence of the positive periodic solution

It is not difficult to see that we can separate the third equation of (1.2) from the whole system and obtain the following subsystem of (1.2):

$$\begin{cases} \frac{dx}{dt} = r_1(t)x(t)(1 - \frac{x(t)}{k_1(t)}) - \sigma_1(t)x(t) + \sigma_2(t)y(t), \\ \frac{dy}{dt} = r_2(t)y(t)(1 - \frac{y(t)}{k_2(t)}) + \sigma_1(t)x(t) - \sigma_2(t)y(t) - \frac{\alpha(t)y(t)z(t)}{a + y(t)}, \\ \frac{dz_2}{dt} = \frac{\beta(t - \tau)y(t - \tau)z_2(t - \tau)}{a + y(t - \tau)} - d_2(t)z_2(t). \end{cases}$$
(2.1)

The initial conditions for the system (2.1) are

$$(x(t), y(t), z_2(t)) \in C_+ = C([-\tau, 0], \mathbb{R}^3_+), x(0) > 0, y(0) > 0, z_2(0) > 0.$$

In order to obtain the positive periodic solution of (2.1), we need some known and preliminary results.

Lemma 2.1. Let $\Omega \in U$ be an open bounded set. Let L be a Fredholm operator of index zero and let N be L-compact on $\overline{\Omega}$. Suppose that the following conditions are satisfied: (a) for each $\lambda \in (0, 1), u \in \partial\Omega \cap DomL, Lu \neq \lambda Nu$; (b) for each $u \in \partial\Omega \cap \ker L, QNu \neq 0$; (c) deg[$JQN, \Omega \cap \ker L, 0$] $\neq 0$. Then Lu = Nu has at least one solution in $\overline{\Omega} \cap DomL$.

For the notations, concepts and further details of *Lemma* 2.1, one can refer to [33–36].

Lemma 2.2. If f(t) is a continuously periodic function with period ω , then

$$\int_{\omega}^{t+\omega} f(s)ds = \int_{0}^{t} f(s)ds, \text{ for any } t.$$

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Proof. Let $j = s - \omega$, then

$$\int_{\omega}^{t+\omega} f(s)ds = \int_{0}^{t} f(j+\omega)dj = \int_{0}^{t} f(j)dj = \int_{0}^{t} f(s)ds.$$

Lemma 2.3. [37] If u(t) is a continuously differentiable periodic function with period ω , then there is $a \tilde{t} \in [0, \omega]$ such that

$$|u(t)| \le |u(\tilde{t})| + \int_0^\omega |\dot{u}(s)| ds \text{ or } |u(t)| \ge |u(\tilde{t})| - \int_0^\omega |\dot{u}(s)| ds.$$

For convenience, we adopt the following notations in our paper: $\bar{c} = \int_{-\infty}^{\infty} \int_{-$

$$\begin{split} f &= \frac{1}{\omega} \int_{0}^{-r} f(t) dt, \qquad f^{L} = \min_{t \in [0, \omega]} f(t), \qquad f^{M} = \max_{t \in [0, \omega]} f(t), \\ l_{-} &= \frac{a d_{2}^{L}}{\beta^{M} e^{2\bar{\sigma}_{2}\omega} - d_{2}^{L}}, \qquad l_{+} = \frac{a d_{2}^{M}}{\beta^{L} e^{-2\bar{\sigma}_{2}\omega} - d_{2}^{M}}, \qquad u_{0} = \frac{a d_{2}^{L}}{\bar{\beta} - a}, \\ b_{1} &= \frac{1}{2} (\frac{\bar{k}_{1}}{r_{1}}) \{ (\bar{r}_{1} - \bar{\sigma}_{1}) + [(\bar{r}_{1} - \bar{\sigma}_{1})^{2} + 4\bar{\sigma}_{2} l_{+} (\frac{\bar{r}_{1}}{k_{1}})]^{\frac{1}{2}} \}, \qquad b_{2} = (\frac{\bar{k}_{1}}{r_{1}}) (\bar{r}_{1} - \bar{\sigma}_{1}), \\ b_{3} &= \bar{\alpha}^{-1} a [(\bar{r}_{2} - \bar{\sigma}_{2}) - (\frac{\bar{r}_{2}}{k_{2}}) l_{+}], \\ b_{4} &= \bar{\alpha}^{-1} (a + l_{+}) [(\bar{r}_{2} - \bar{\sigma}_{2}) + \bar{\sigma}_{2} l_{-} e^{B_{1}}], \\ b_{5} &= \frac{1}{2} (\frac{\bar{k}_{1}}{r_{1}}) \{ (\bar{r}_{1} - \bar{\sigma}_{1}) + [(\bar{r}_{1} - \bar{\sigma}_{1})^{2} + 4\bar{\sigma}_{2} u_{0} (\frac{\bar{r}_{1}}{k_{1}})]^{\frac{1}{2}} \}, \\ b_{6} &= \bar{\alpha}^{-1} (a + u_{0}) [(\bar{r}_{2} - \bar{\sigma}_{2}) - (\frac{\bar{r}_{1}}{k_{1}}) u_{0} + \bar{\sigma}_{2} \frac{D_{0}}{u_{0}}], \\ B_{1} &= \max\{|B_{11}|, |B_{12}|\}, \qquad B_{2} &= \max\{|B_{21}|, |B_{22}|\}, \qquad B_{3} &= \max\{|B_{31}|, |B_{32}|\}. \\ \text{And we assume that:} \\ (H_{1}) : d_{2}^{L} &\leq \beta^{M} e^{2\bar{\sigma}_{2}\omega}; \\ (H_{2}) : \beta^{L} &\leq (d_{2}^{L})^{-1} d_{2}^{M} \beta^{M}; \\ (H_{3}) : \bar{r}_{2} &< \bar{\alpha} a^{-1} e^{B_{3}} + (\frac{\bar{r}_{2}}{k_{2}}) l_{+} + \bar{\sigma}_{2}. \\ \text{Now, we are in a position to state our main results.} \\ \end{split}$$

Theorem 2.1. If system (1.2) satisfies: (H_1) , (H_2) and (H_3) , then it has at least one positive periodic solution.

Proof. We prove this theorem for two steps. Step 1: We prove subsystem (2.1) has at least one periodic solution. Letting

 $u_1(t) = \ln x(t),$ $u_2(t) = \ln y(t),$ $u_3(t) = \ln z_2(t),$

then we have

$$\begin{cases} \dot{u}_{1}(t) = r_{1}(t)(1 - \frac{e^{u_{1}(t)}}{k_{1}(t)}) + \sigma_{2}(t)e^{u_{2}(t) - u_{1}(t)} - \sigma_{1}(t), \\ \dot{u}_{2}(t) = r_{2}(t)(1 - \frac{e^{u_{2}(t)}}{k_{2}(t)}) + \sigma_{1}(t)e^{u_{1}(t) - u_{2}(t)} - \frac{\alpha(t)e^{u_{3}(t)}}{a + e^{u_{2}}(t)} - \sigma_{2}(t), \\ \dot{u}_{3}(t) = \frac{\beta(t - \tau)e^{u_{2}(t - \tau) + u_{3}(t - \tau) - u_{3}(t)}}{a + e^{u_{2}(t - \tau)}} - d_{2}(t). \end{cases}$$

$$(2.2)$$

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Take

$$U = V = \{ u = (u_1, u_2, u_3) \in C(\mathbb{R}, \mathbb{R}^3) | u(t + \omega) = u(t) \}.$$

It is easy to see that U, V are both Banach Spaces with the norm $\|\cdot\|$,

 $||u|| = \max_{t \in [0,\omega]} |u_1| + \max_{t \in [0,\omega]} |u_2| + \max_{t \in [0,\omega]} |u_3|, u = (u_1, u_2, u_3) \in U \text{ or } V.$ For any $u = (u_1, u_2, u_3) \in U$, by the periodicity of the coefficients of system (2.2). We can check that: $r_1(t)(1-\frac{e^{u_1(t)}}{k_1(t)})+\sigma_2(t)e^{u_2(t)-u_1(t)}-\sigma_1(t):=\Theta_1(u,t),$

$$r_2(t)(1 - \frac{e^{u_2(t)}}{k_2(t)}) + \sigma_1(t)e^{u_1(t) - u_2(t)} - \frac{\alpha(t)e^{u_3(t)}}{a + e^{u_2}(t)} - \sigma_2(t) := \Theta_2(u, t)$$

and

 $\frac{\beta(t-\tau)e^{u_2(t-\tau)+u_3(t-\tau)-u_3(t)}}{a+e^{u_2(t-\tau)}} - d_2(t) := \Theta_3(u,t)$ are all ω -periodic functions.

In fact,

$$\begin{split} \Theta_1(u(t+\omega), t+\omega) &= r_1(t+\omega)(1 - \frac{e^{u_1(t+\omega)}}{k_1(t+\omega)}) + \sigma_2(t+\omega)e^{u_2(t+\omega)-u_1(t+\omega)} - \sigma_1(t+\omega) \\ &= r_1(t)(1 - \frac{e^{u_1(t)}}{k_1(t)}) + \sigma_2(t)e^{u_2(t)-u_1(t)} - \sigma_1(t) \\ &= \Theta_1(u, t). \end{split}$$

In a similar way, one can obtain

 $\Theta_2(u(t+\omega), t+\omega) = \Theta_2(u, t), \\ \Theta_3(u(t+\omega), t+\omega) = \Theta_3(u, t).$ Set

 $L: DomL \cap U \qquad L(u_1(t), u_2(t), u_3(t)) = (\frac{du_1(t)}{dt}, \frac{du_2(t)}{dt}, \frac{du_3(t)}{dt}),$ where $DomL = \{(u_1, u_2, u_3) \in C(\mathbb{R}, \mathbb{R}^3)\}$ and $N : U \to U$ is defined by

$$N\begin{pmatrix} u_1\\ u_2\\ u_3 \end{pmatrix} = \begin{pmatrix} \Theta_1(u,t)\\ \Theta_2(u,t)\\ \Theta_3(u,t) \end{pmatrix},$$

Define

$$P\begin{pmatrix} u_1\\ u_2\\ u_3 \end{pmatrix} = Q\begin{pmatrix} u_1\\ u_2\\ u_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\omega} \int_0^{\omega} u_1(t)dt\\ \frac{1}{\omega} \int_0^{\omega} u_2(t)dt\\ \frac{1}{\omega} \int_0^{\omega} u_3(t)dt \end{pmatrix}, \begin{pmatrix} u_1\\ u_2\\ u_3 \end{pmatrix} \in U = V.$$

It is not difficult to know that

ker $L = \{u \in U | u = C_0, C_0 \in \mathbb{R}^3\}$ and $ImL = \{v \in V | \int_0^{\omega} v(t)dt = 0\}.$ Consequently, dim ker $L = codim ImL = 3 < +\infty$, and P and Q are continuous projectors such that

 $ImP = \ker L$, $\ker Q = ImL = Im(I - Q)$.

It follows that L is a Fredholm mapping of index zero. Moreover, the generalized inverse (of L) K_p : $ImL \rightarrow DomL \cap \ker P$ exists and is given by

 $K_p(v) = \int_0^t v(s)ds - \frac{1}{\omega} \int_0^\omega \int_0^t v(s)dsdt,$ then,

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$$QNu = \begin{pmatrix} \frac{1}{\omega} \int_0^{\omega} \Theta_1(u, t) dt \\ \frac{1}{\omega} \int_0^{\omega} \Theta_2(u, t) dt \\ \frac{1}{\omega} \int_0^{\omega} \Theta_3(u, t) dt \end{pmatrix}$$

and

 $K_p(I-Q)Nu = \int_0^t Nu(s)ds - \frac{1}{\omega} \int_0^{\omega} \int_0^t Nu(s)dsdt - (\frac{t}{\omega} - \frac{1}{2}) \int_0^{\omega} Nu(s)ds.$ Obviously, QN and $K_p(I-Q)N$ are continuous. By using the Arzela-Ascoli theorem, it's not difficult to see that N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset U$.

Next, our aim is to search for an appropriate open bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Lu = \lambda Nu, \lambda \in (0, 1)$, we have

$$\dot{u}_1(t) = \lambda [r_1(t)(1 - \frac{e^{u_1(t)}}{k_1(t)}) + \sigma_2(t)e^{u_2(t) - u_1(t)} - \sigma_1(t)],$$
(2.3)

$$\dot{u}_2(t) = \lambda [r_2(t)(1 - \frac{e^{u_2(t)}}{k_2(t)}) + \sigma_1(t)e^{u_1(t) - u_2(t)} - \frac{\alpha(t)e^{u_3(t)}}{a + e^{u_2}(t)} - \sigma_2(t)],$$
(2.4)

$$\dot{u}_{3}(t) = \lambda \left[\frac{\beta(t-\tau)e^{u_{2}(t-\tau)+u_{3}(t-\tau)-u_{3}(t)}}{a+e^{u_{2}(t-\tau)}} - d_{2}(t)\right].$$
(2.5)

Suppose $u = (u_1(t), u_2(t), u_3(t))^T \in U$ is a solution of (2.3), (2.4) and (2.5), for a certain $\lambda \in (0, 1)$. Integrating (2.3), (2.4) and (2.5) over the interval $[0, \omega]$, we obtain

$$\bar{\sigma}_1 \omega = \int_0^\omega [\sigma_2(t)e^{u_2(t) - u_1(t)} + r_1(t) - \frac{r_1(t)}{k_1(t)}e^{u_1(t)}]dt,$$
(2.6)

$$\bar{\sigma}_2 \omega = \int_0^\omega [\sigma_1(t)e^{u_1(t) - u_2(t)} - \frac{\alpha(t)e^{u_3(t)}}{a + e^{u_2(t)}} + r_2(t) - \frac{r_2(t)}{k_2(t)}e^{u_1(t)}]dt,$$
(2.7)

$$\bar{d}_2\omega = \int_0^\omega \frac{\beta(t-\tau)e^{u_2(t-\tau)+u_3(t-\tau)-u_3(t)}}{a+e^{u_2(t-\tau)}}dt.$$
(2.8)

From the Eqs (2.3) and (2.6), we have

$$\begin{split} \int_{0}^{\omega} |\dot{u}_{1}(t)| dt &= \lambda \int_{0}^{\omega} \left| r_{1}(t) (1 - \frac{e^{u_{1}(t)}}{k_{1}(t)}) + \sigma_{2}(t) e^{u_{2}(t) - u_{1}(t)} - \sigma_{1}(t) \right| dt \\ &< \int_{0}^{\omega} \left| r_{1}(t) - \frac{r_{1}(t)}{k_{1}(t)} e^{u_{1}(t)} + \sigma_{2}(t) e^{u_{2}(t) - u_{1}(t)} \right| dt + \int_{0}^{\omega} |\sigma_{1}(t)| dt \\ &< \bar{\sigma}_{1} \omega + \bar{\sigma}_{1} \omega \\ &= 2\bar{\sigma}_{1} \omega, \end{split}$$

that is

$$\int_0^\omega |\dot{u}_1(t)| dt < 2\bar{\sigma}_1 \omega.$$
(2.9)

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Similarly, it follows from (2.4) and (2.7), (2.5) and (2.8) that

$$\int_0^\omega |\dot{u}_2(t)| dt < 2\bar{\sigma}_2 \omega,\tag{2.10}$$

$$\int_0^\omega |\dot{u}_3(t)| dt < 2\bar{d}_2\omega,\tag{2.11}$$

Since $(u_1(t), u_2(t), u_3(t)) \in U$, there exists $\xi_i, \eta_i \in [0, \omega]$, such that

$$u_i(\xi_i) = \min_{t \in [0,\omega]} u_i(t), u_i(\eta_i) = \max_{t \in [0,\omega]} u_i(t), i = 1, 2, 3.$$

Multiplying (2.5) by $e^{u_3(t)}$ and integrating over $[0, \omega]$, we have

$$\int_0^{\omega} d_2(t) e^{u_3(t)} dt = \int_0^{\omega} \frac{\beta(t-\tau) e^{u_2(t-\tau)+u_3(t-\tau)}}{a+e^{u_2(t-\tau)}} dt.$$

Now we make the change of a variable $j = t - \tau$ and *Lemma* 2, we obtain

$$\int_{0}^{\omega} \frac{\beta(t-\tau)e^{u_{2}(t-\tau)+u_{3}(t-\tau)}}{a+e^{u_{2}(t-\tau)}}dt = \int_{-\tau}^{\omega-\tau} \frac{\beta(j)e^{u_{2}(j)+u_{3}(j)}}{a+e^{u_{2}(j)}}dj = \int_{0}^{\omega} \frac{\beta(t)e^{u_{2}(t)+u_{3}(t)}}{a+e^{u_{2}(t)}}dt,$$

that is

$$\int_{0}^{\omega} d_{2}(t)e^{u_{3}(t)}dt = \int_{0}^{\omega} \frac{\beta(t)e^{u_{2}(t)+u_{3}(t)}}{a+e^{u_{2}(t)}}dt.$$
(2.12)

From the Eq (2.12), we obtain

$$d_2^L \int_0^\omega e^{u_3(t)} dt \le \int_0^\omega d_2(t) e^{u_3(t)} dt = \int_0^\omega \frac{\beta(t) e^{u_2(t) + u_3(t)}}{a + e^{u_2(t)}} dt \le \beta^M \frac{e^{u_2(\eta_2)}}{a + e^{u_2(\xi_2)}} \int_0^\omega e^{u_3(t)} dt,$$

which is

$$d_2^L \le \beta^M \frac{e^{u_2(\eta_2)}}{a + e^{u_2(\xi_2)}},$$

thus

$$u_2(\eta_2) \ge \ln \frac{d_2^L(a + e^{u_2(\xi_2)})}{\beta^M}.$$
(2.13)

It follows from (2.10), (2.13) and Lemma 3 that we have

$$u_{2}(t) \ge u_{2}(\eta_{2}) - \int_{0}^{\omega} |\dot{u}_{2}(t)| dt > \ln \frac{d_{2}^{L}(a + e^{u_{2}(\xi_{2})})}{\beta^{M}} - 2\bar{\sigma}_{2}\omega := B_{21}.$$
(2.14)

In particular, we obtain

$$u_2(\xi_2) > \ln \frac{d_2^L(a + e^{u_2(\xi_2)})}{\beta^M} - 2\bar{\sigma}_2\omega,$$

or

$$(\beta^{M}e^{2\bar{\sigma}_{2}\omega}-d_{2}^{L})e^{u_{2}(\xi_{2})}-ad_{2}^{L}>0.$$

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And in view of (H_1) , we have

$$u_2(\xi_2) > \ln \frac{ad_2^L}{\beta^M e^{2\bar{\sigma}_2 \omega} - d_2^L} = \ln l_-.$$

In a similar way, from the Eq (2.12) we obtain

$$d_2^M \ge \beta^L \frac{e^{u_2(\xi_2)}}{a + e^{u_2(\eta_2)}},$$

therefore

$$u_2(\xi_2) \le \ln \frac{d_2^M(a + e^{u_2(\eta_2)})}{\beta^L}.$$
(2.15)

It follows from (2.10), (2.15) and Lemma 3 that we have

$$u_2(t) \le u_2(\xi_2) + \int_0^\omega |\dot{u}_2(t)| dt < \ln \frac{d_2^M(a + e^{u_2(\eta_2)})}{\beta^L} + 2\bar{\sigma}_2\omega := B_{22}.$$
(2.16)

In particular, we have

$$u_2(\eta_2) < \ln \frac{d_2^M(a + e^{u_2(\eta_2)})}{\beta^L} + 2\bar{\sigma}_2\omega,$$

or

$$(\beta^L e^{-2\bar{\sigma}_2\omega} - d_2^M)e^{u_2(\eta_2)} - ad_2^M < 0.$$

In view of (H_1) , we have

$$u_2(\eta_2) < \ln \frac{ad_2^M}{\beta^L e^{-2\bar{\sigma}_2\omega} - d_2^M} = \ln l_+.$$

In view of (H_2) , we have

$$\frac{ad_2^M}{\beta^L e^{-2\bar{\sigma}_2\omega} - d_2^M} \geq \frac{ad_2^L}{\beta^M e^{-2\bar{\sigma}_2\omega} - d_2^L}.$$

It follows from (2.14) and (2.16) that

 $\max_{t \in [0,\omega]} |u_2(t)| = \max_{t \in [0,\omega]} \{|B_{21}|, |B_{22}|\} := B_2.$

It follows from (2.6) that

$$\bar{\sigma}_1 \omega \leq \frac{\bar{\sigma}_2 \omega e^{\ln l_+}}{e^{u_1(\xi_1)}} + \bar{r}_1 \omega - (\frac{\bar{r}_1}{k_1}) \omega e^{u_1(\xi_1)},$$

thus

$$u_1(\xi_1) \le \ln\left\{\frac{1}{2}(\frac{\bar{k_1}}{r_1})\left[(\bar{r}_1 - \bar{\sigma}_1) + \sqrt{(\bar{r}_1 - \bar{\sigma}_1)^2 + 4\bar{\sigma}_2 l_+(\frac{\bar{r}_1}{k_1})}\right]\right\} = \ln b_1.$$

This combined with (2.9), give

$$u_1(t) \le u_1(\xi_1) + \int_0^\omega |\dot{u}_1(t)| dt < \ln b_1 + 2\bar{\sigma}_1 \omega := B_{11}.$$
(2.17)

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Similarly, we have

$$\bar{\sigma}_1 \omega \geq \bar{r}_1 \omega - (\frac{\bar{r}_1}{k_1}) \omega e^{u_1(\eta_1)},$$

therefore

$$u_1(\eta_1) \ge \ln\left[(\frac{\bar{k_1}}{r_1})(\bar{r}_1 - \bar{\sigma}_1)\right] = \ln b_2.$$

This together with (2.9), gives

$$u_1(t) \ge u_1(\eta_1) - \int_0^\omega |\dot{u}_1(t)| dt > \ln b_2 - 2\bar{\sigma}_1 \omega := B_{12}.$$
(2.18)

It follows from (2.17) and (2.18) that

$$\max_{t \in [0,\omega]} |u_1(t)| = \max_{t \in [0,\omega]} \{|B_{11}|, |B_{12}|\} := B_1.$$

It follows from that (2.7) that

$$\bar{\sigma}_2\omega \ge -a^{-1}\bar{\alpha}\omega e^{u_3(\eta_3)} + \bar{r}_2\omega - (\frac{\bar{r}_2}{k_2})\omega l_+,$$

thus

$$u_3(\eta_3) \ge \ln\{a\bar{\alpha}^{-1}[(\bar{r}_2 - \bar{\sigma}_2) - (\frac{\bar{r}_2}{k_2})l_+]\} = \ln b_3.$$

This combined with (2.11), give

$$u_{3}(t) \ge u_{3}(\eta_{3}) - \int_{0}^{\omega} |\dot{u}_{3}(t)| dt > \ln b_{3} - 2\bar{d}_{2}\omega := B_{31},$$
(2.19)

Similarly,

$$\bar{\sigma}_2\omega \leq \frac{-\bar{\alpha}\omega e^{u_3(\xi_3)}}{a+l_+} + \bar{\sigma}_1\omega l_-e^{u_1(\eta_1)} + \bar{r}_2\omega,$$

therefore

$$e^{u_3(\xi_3)} \le (a+l_+)[(\bar{r}_2 - \bar{\sigma}_2) + \bar{\sigma}_1 l_- e^{u_1(\eta_1)}],$$

or

$$u_3(\xi_3) < \ln \left\{ \bar{\alpha}^{-1}(a+l_+) [(\bar{r}_2 - \bar{\sigma}_2) + \bar{\sigma}_2 l_- e^{B_1}] \right\} = \ln b_4.$$

Combining with (2.11), give

$$u_3(t) < u_3(\xi_3) + \int_0^\omega |\dot{u}_3(t)| dt < \ln b_4 + 2\bar{d}_2\omega := B_{32}.$$
(2.20)

It follows from (2.19) and (2.20) that

$$\max_{t \in [0,\omega]} |u_3(t)| = \max_{t \in [0,\omega]} \{|B_{31}|, |B_{32}|\} := B_3.$$

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Next, let's consider QNu with $u = (u_1, u_2, u_3) \in \mathbb{R}^3$. Note that

$$QN(u_1, u_2, u_3) = [(\bar{r}_1 - \bar{\sigma}_1) + \bar{\sigma}_2 e^{u_2(t) - u_1(t)} - (\frac{\bar{r}_1}{k_1})e^{u_1(t)},$$
$$(\bar{r}_2 - \bar{\sigma}_2) + \bar{\sigma}_1 e^{u_1(t) - u_2(t)} - (\frac{\bar{r}_2}{k_2})e^{u_2(t)} - \bar{\alpha}\frac{e^{u_3(t)}}{a + e^{u_2(t)}}, -\bar{d}_2 + \frac{\bar{\beta}e^{u_2(t)}}{a + e^{u_2(t)}}].$$

In view of (H_1) , (H_2) , (H_3) , $QN(u_1, u_2, u_3) = 0$ has a solution $\tilde{u} = (\ln b_5, \ln u_0, \ln b_6)$. Take $B = \max\{B_1 + C, B_2 + C, B_3 + C\}$, where C > 0 is taken sufficiently large such that $\|(\ln b_5, \ln u_0, \ln b_6)\| < C$. Define $\Omega = \{u(t) = (u_1(t), u_2(t), u_3(t))^T \in U : \|u\| < B\}$. Then Ω is a bounded open subset of U, therefore Ω satisfies the requirement (a) in *Lemma* 1. Moreover, it's not difficult to verify $QNu \neq 0$ for $u \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^3$. A direct computation gives deg $\{JQN, \Omega \cap \ker L, 0\} \neq 0$. Therefore, system (2.1) has at least one ω periodic solution \tilde{u} .

Step 2: We prove that the third equation of system (1.2) has a unique ω -periodic solution associated with the obtained \tilde{u} . Letting

$$h(t) = \frac{\beta(t)y(t)z_2(t)}{a+y(t)} - \frac{\beta(t-\tau)y(t-\tau)z_2(t-\tau)}{a+y(t-\tau)}$$

then the third equation of (1.2) is

$$\frac{dz_1}{dt} = -d_1(t)z_1(t) + h(t).$$
(2.21)

Obviously,

$$d_1(t+\omega) = d_1(t)$$

and

$$h(t+\omega) = \frac{\beta(t+\omega)y(t+\omega)z_2(t+\omega)}{a+y(t+\omega)} - \frac{\beta(t+\omega-\tau)y(t+\omega-\tau)z_2(t+\omega-\tau)}{a+y(t+\omega-\tau)}$$
$$= \frac{\beta(t)y(t)z_2(t)}{a+y(t)} - \frac{\beta(t-\tau)y(t-\tau)z_2(t-\tau)}{a+y(t-\tau)}$$
$$= h(t).$$

Since $d_1(t)$ is nonnegative, $\bar{d}_1 > 0$, it follows that

$$\frac{dz_1}{dt} = -d_1(t)z_1(t),$$
(2.22)

admits exponential dichotomy. Therefore, we have

$$z_1(t) = \int_{-\infty}^t e^{-\int_s^t d_1(\sigma)d\sigma} h(s)ds.$$

Consequently, $(\ln x(t), \ln y(t), z_1(t), \ln z_2(t))$ is a ω -periodic solution of system (1.2). This completes the proof.

3. An example

As an example, corresponding to the model (1.2), we have the following stage-structured predatorprey model with Holling type II functional response incorporating prey refuge with actual biological parameters:

$$\begin{cases} \frac{dx}{dt} = (1.2 + \sin 20\pi t)x(t)(1 - \frac{x(t)}{10 + \sin 20\pi t}) - 0.2x(t) + 0.15y(t), \\ \frac{dy}{dt} = (1.5 + \sin 20\pi t)y(t)(1 - \frac{y(t)}{15 + \sin 20\pi t}) + 0.2x(t) - 0.15y(t) \\ - \frac{(2 + \sin 20\pi t)y(t)z_2(t)}{2 + y(t)}, \\ \frac{dz_1}{dt} = \frac{(1.5 + \sin 20\pi t)y(t)z_2(t)}{2 + y(t)} - \frac{(1.5 + \sin 20\pi (t - 1))y(t - 1)z_2(t - 1)}{2 + y(t - 1)} - 0.2z_1(t), \end{cases}$$
(3.1)

where $r_1(t) = 1.2 + \sin 20\pi t$ and $r_2(t) = 1.5 + \sin 20\pi t$ denote intrinsic growth rates, they are directly proportional to the the density of the prey *x* and *y*; $k_1(t) = 10 + \sin 20\pi t$ and $k_2(t) = 15 + \sin 20\pi t$ denote the environment carrying capacity for the prey *x* and *y*; $\sigma_1(t) = 0.2$ and $\sigma_2(t) = 0.15$ denote the per unit migration of the prey in the refuge to the predatory region and the opposite of it; $\alpha(t) = 2 + \sin 20\pi t$ denotes the maximal predator per capita consumption rate and $\beta(t) = 1.5 + \sin 20\pi t$ denotes the rate of the predator consumes prey; $\frac{(2+\sin 20\pi t)y(t)}{2+y(t)}$ denotes Holling type II functional response, which reflects the capture ability of the predator; $d_1(t) = 0.2$ and $d_2(t) = 0.1$ are the death rate of predator at any time *t*; $(1.5 + \sin 20\pi t)y(t - 1)z_2(t - 1)/(2 + y(t - 1))$ stands for the number of immature predator born at time (t - 1) which still survive at time *t* and become mature predator.

Since $\tau = 1$, a = 2, $d_2 = 0.1$, $\sigma_2 = 0.15$, $r_2(t) = 1.5 + \sin 20\pi t$, $k(t) = 15 + \sin 20\pi t$, $\alpha(t) = 2 + \sin 20\pi t$, $\beta(t) = 1.5 + \sin 20\pi t$, we have $\bar{d}_2 = d_2^M = d_2^L = 0.1$, $\bar{\sigma}_2 = 0.15$, $\bar{r}_2 = 1.5$, $(\frac{\bar{r}_2}{k_2}) = 0.1$, $\bar{\alpha} = 2$, $\beta^L = 0.5$, $\beta^M = 2.5$, $\omega = 0.1$. Simple computation shows

$$\beta^{M} e^{2\bar{\sigma}_{2}\omega} = 2.5 \times e^{2 \times 0.15 \times 0.1} = 2.5 \times e^{0.03} > 2.5 > 0.1 = d_{2}^{L},$$
$$(d_{2}^{L})^{-1} d_{2}^{M} \beta^{M} = 10 \times 0.1 \times 2.5 = 2.5 > 0.5 = \beta^{L}$$

and

$$\begin{split} \bar{\alpha}a^{-1}e^{B_3} + (\frac{\bar{r}_2}{k_2})l_+ + \bar{\sigma}_2 \\ &= \frac{1}{2} \times 2 \times e^{\max\{|B_{31}|, |B_{32}|\}} + \frac{1}{10} \times \frac{2 \times 0.1}{0.5 \times e^{-0.03} - 0.1} + 0.15 \\ &= e^{B_{32}} + \frac{2}{50e^{0.03} - 10} + 0.15 \\ &> (1.35 + (2.5e^{0.03} - 0.01)e^{\max\{|B_{11}|, |B_{12}|\}})e^{0.02} + \frac{2}{50e^{0.03} - 10} + 0.15 \\ &> (1.35 + (2.5e^{0.03} - 0.01)e^{0})e^{0.02} + \frac{2}{50e^{0.03} - 10} + 0.15 > 1.5 = \bar{r}_2. \end{split}$$

The above inequalities show that system (3.1) satisfies the hypothesis $(H_1), (H_2), (H_3)$ in *Theorem* 1. Therefore, system (3.1) has at least one positive periodic solution.

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4. Conclusion

In mathematical biology, dynamic relationship between predator and prey is always and will continue to be one of the main themes , many researchers have contributed to the study and improvement for the predator-prey model [17, 38–65]. The model in the present paper mimics the dynamic nature of the refuge. The populations varies due to the rates of emergence from and re-entry into refuge, which incorporates simultaneous effects of the refuge and migration of the population from the refuge area to the predatory area. For instance, it may happen in the birds migration. In fact, the dynamic nature of the refuge is an effective strategy for reducing predation as a prey population evolved. For this reason, a delayed stage-structured predator-prey model with a prey refuge is considered in this paper.

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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