Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/MBE

MBE, 17(4): 2936–2949.
DOI: 10.3934/mbe.2020165
Received: 06 February 2020
Accepted: 22 March 2020
Published: 30 March 2020

Research article

Modelling and assessing the effects of medical resources on transmission of novel coronavirus (COVID-19) in Wuhan, China

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Abstract: The coronavirus disease 2019 (COVID-2019), a newly emerging disease in China, posed a public health emergency of China. Wuhan is the most serious affected city. Some measures have been taken to control the transmission of COVID-19. From Jan. 23rd, 2020, gradually increasing medical resources (such as health workforce, protective clothing, essential medicines) were sent to Wuhan from other provinces, and the government has established the hospitals to quarantine and treat infected individuals. Under the condition of sufficient medical resources in Wuhan, late-stage of epidemic showed a downward trend. Assessing the effectiveness of medical resources is of great significance for the future response to similar disease. Based on the transmission mechanisms of COVID-19 and epidemic characteristics of Wuhan, by using time-dependent rates for some parameters, we establish a dynamical model to reflect the changes of medical resources on transmission of COVID-19 in Wuhan. Our model is applied to simulate the reported data on cumulative and new confirmed cases in Wuhan from Jan. 23rd to Mar. 6th, 2020. We estimate the basic reproduction number $R_0 = 2.71$, which determines whether the disease will eventually die out or not under the absence of effective control measures. Moreover, we calculate the effective daily reproduction ratio $R_e(t)$, which is used to measure the 'daily reproduction number'. We obtain that $R_e(t)$ drops less than 1 since Feb. 8th. Our results show that delayed opening the 'Fire God Hill' hospital will greatly increase the magnitude of the outbreak. This shows that the government's timely establishment of hospitals and effective quarantine via quick detection prevent a larger outbreak.

Keywords: novel coronavirus; medical resources; mathematical modelling; reproduction number

1. Introduction

Wuhan, the capital city of the Hubei province in China, is the focus of global attention because of the outbreak of a febrile respiratory disease caused by novel coronavirus. On Dec. 12nd, 2019, a pneumonia case of unknown etiology was reported in Wuhan, associated with the Huanan Seafood Wholesale Market where there was also sale of live wild animals. On Dec. 31st, 2019, Chinese authorities alerted the World Health Organization (WHO) on the outbreak of pneumonia of unknown aetiology. Subsequently, Chinese authorities isolated a novel strain of coronavirus on Jan. 7th, 2020 [1]. On Feb. 11st, WHO named a novel coronavirus as COVID-19 [2]. In the early stages, little was known about the COVID-19. Due to an annual spring migration in China, COVID-19 spread rapidly from Wuhan to almost every province in China, to reach various countries including Japan, Thailand, South Korea, Philippines, Singapore, Mexico, Italy and the United States of America. As of Jan. 29th, cumulative 7711 confirmed cases with 170 deaths were reported in China. The size of the outbreak prompted the WHO to declare the outbreak of COVID-19 as a "Public Health Emergency of International Concern" on Jan. 30th, 2020 [3].

Based on transmission mechanisms and routes of COVID-19, some researchers try to use mathematical models to study rapid spread of COVID-19 [4–11]. However, there are very few models to investigate the effects of medical resources on transmission dynamics of COVID-19. For rapid spread of COVID-19 in China, Wuhan is the most serious affected city. In China, a regional university teaching hospital is usually equipped with 500-1000 beds, with only a small portion for isolation purpose [12]. When the infection started in districts of Wuhan, the available medical resources of the health system (health workforce, hospital beds, protective clothing, essential medicines, and the like) sufficed for diagnosing and treating the infected cases. As the number of infections keeps increasing rapidly, the medical resources proved incapable of treating the emerging cases in a timely way. In response to this situation, public health decision-and policy-makers took timely measures. After Wuhan was sealed off on Jan. 23rd, 2020, gradually increasing COVID-19 detecting kits and health workforces were sent to Wuhan from other provinces in succession. Government started setting up some hospitals for infected cases. On Feb. 4th, the 'Fire God Hill' hospital began to receive confirmed cases. Later, Feb. 8th, the 'Thunder God Hill' hospital also began to receive confirmed cases. Up to Feb. 28th, about sixteen the 'Mobile Field' hospitals had been gradually built to provide adequate hospital beds with infected individuals for diagnosis and treatment. Under the condition of sufficient medical resources in Wuhan, late-stage of epidemic showed a downward trend. Therefore, the medical resources can be considered as one of the main reasons for the transmission and control of COVID-19 in Wuhan.

The main purpose of this paper is to build a time-dependent model of COVID-19 to study the effects of medical resources on transmission of COVID-19 in Wuhan. In this paper, we first establish a model and use our model to fit the cumulative and new confirmed cases in Wuhan for the period from Jan. 23rd to Mar. 6th, 2020. Our model will allow us to estimate the basic reproduction number and effective daily reproduction ratio. Lastly, we assess the effects of medical resources on transmission of COVID-19 in Wuhan.

2. Materials and methods

2.1. Data

This study is based on daily reported cases extracted from the National Health Commission of the People's Republic of China [13] and the Health Commission of Hubei Province [14]. The data contain suspected cases, new confirmed cases, cumulative confirmed cases, hospitalized cases and death cases due to COVID-19 infection. Although the data do not include patient level information, they provide the best available data on the outbreak of COVID-19. On Jan. 23rd, 2020, Chinese government announced to seal off Wuhan from all outside contact to stop the spread of COVID-19. Hence, we choose Jan. 23rd, 2020 as the initial observation date and use the daily reported cumulative and new confirmed cases (see Figure 1) in Wuhan from Jan. 23rd to Mar. 6th, 2020 to do our study.

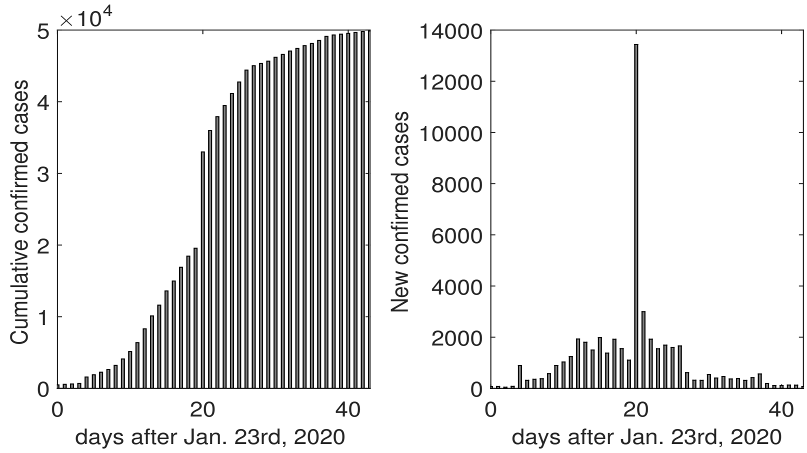

Figure 1. Reported cumulative and new confirmed cases from Jan. 23rd to Mar. 6th, 2020 in Wuhan, China.

2.2. Model formulation

The main purpose in this work is to investigate the effects of medical resources which include effective quarantine via quick detection of post-stage exposed individuals, contact rates and hospitalization rates of infected individuals. Up to Feb. 28th, about sixteen the 'Mobile Field' hospitals and two 'Hill' hospitals had been gradually built to provide adequate hospital beds with infected individuals for diagnosis and treatment. The first hospital to be built was 'Fire God Hill', which began to receive confirmed cases on Feb. 4th. Thus, we can treat the opening time of the 'Fire God Hill' hospital (Feb. 4th) as the initial time for hospital intervention.

Based on transmission mechanisms and routes of COVID-19, we draw a flow diagram shown in Figure 2. We divide the total number of individuals at time t, denoted by $N_h(t)$, into seven categories: susceptible $S(t)$, pre-stage exposed $E_1(t)$, post-stage exposed $E_2(t)$, infected with mild symptoms $I_1(t)$, infected with serious symptoms $I_2(t)$, hospitalized $H(t)$ and recovered $R(t)$ individuals. So, $N_h(t) = S(t) + E_1(t) + E_2(t) + I_1(t) + I_2(t) + H(t) + R(t)$. Assume that hospitalized individuals $H(t)$ are isolated and are unable to contact with susceptible individuals $S(t)$. We also capture the fact that post-stage exposed individuals have infectious ability. So, infectious individuals include $E_2(t)$, $I_1(t)$ and $I_2(t)$.

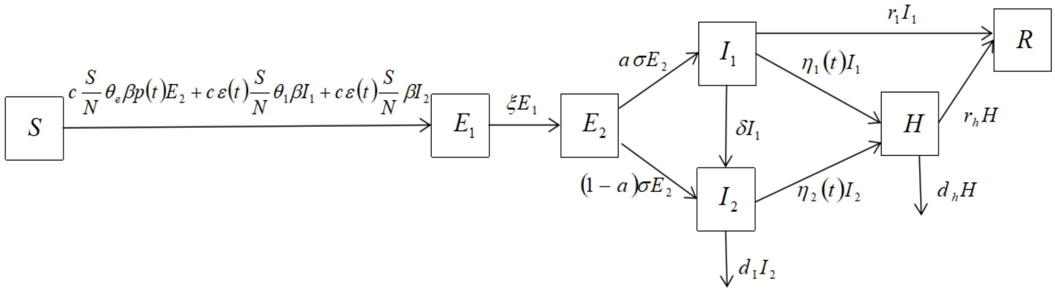

Figure 2. Flow diagram of COVID-19 transmission. The parameters are given in Table 1.

Susceptible individuals can be infected with COVID-19 via effective contact with post-stage exposed individuals E_2 at a rate $c\frac{S}{N}\theta_e\beta p(t)E_2$. c represents the mean number of an infectious individual contacting with others in unit time. $c\frac{S}{N}$ represents the mean number of an infectious individual contacting with susceptible individuals in unit time. β represents transmission probability from infectious individuals with serious symptoms I_2 to susceptible individuals per contact. θ_e $(0 < \theta_e < 1)$ accounts for reduction in transmissibility of post-stage exposed individuals compare to I_2. So, $c\frac{S}{N}\theta_e\beta$ represents the mean number of a post-stage exposed individual contacting and infecting susceptible individuals in unit time under the absence of effective control measures. Let $q(t)$ be a proportion, and it indicates that effective quarantine via quick detection of post-stage exposed individuals is adopted to keep the quarantined population from infecting others. So, the other unquarantined proportion $1 - q(t)$ of the post-stage exposed individuals is infectious. Here, we set $p(t) = 1 - q(t)$. Then $p(t)$ represents the unquarantined proportion of post-stage exposed individuals. From Jan. 23rd, 2020, gradually increasing COVID-19 detecting kits were sent to Wuhan from other provinces. It implies that the unquarantined proportion $p(t)$ would gradually decrease. According to the idea of modeling in [10, 11], in our study, we use the following form $p(t)$ to represent the exponential decay of the unquarantined proportion of the post-stage exposed individuals due to effective quarantine via quick detection,

$$p(t) = \begin{cases} p_0, & 0 \le t \le \tau_1, \\ (p_0 - p_1)e^{-m_0(t-\tau_1)} + p_1, & \tau_1 < t < \tau_2, \\ (p_0 - p_1)e^{-m_0(t-\tau_1)}e^{-m_1(t-\tau_2)} + p_1, & t \ge \tau_2. \end{cases} \tag{2.1}$$

Here, p_0 represents initial unquarantined proportion of post-stage exposed individuals. p_1 represents the minimum proportion under the current control strategies. Deservedly, we have $p(0) = p_0$ and $\lim_{t\to+\infty} p(t) = p_1$ with $p_1 < p_0$. τ_1 represents the time for sealing off the city. In this study, since Wuhan was sealed off on Jan. 23rd and we choose Jan. 23rd as the initial observation date, we can treat τ_1 as 0. τ_2 represents the time for opening the 'Fire God Hill' hospital. Since 'Fire God Hill' hospital was opened on Feb. 4th, we can treat τ_2 as 12. Obviously, $\tau_1 < \tau_2$. When $\tau_1 < t < \tau_2$, $p(t) = (p_0 - p_1)e^{-m_0(t-\tau_1)} + p_1$ represents the exponential decay at a rate m_0 of unquarantined proportion due to sealing off the city. When $t \ge \tau_2$, on the basis of $e^{-m_0(t-\tau_1)}$, we add the exponential decay at a rate m_1 of unquarantined proportion due to opening the 'Fire God Hill' hospital $e^{-m_1(t-\tau_2)}$.

Susceptible individuals can also be infected with COVID-19 via effective contact with infected individuals at a rate $c\epsilon(t)\frac{S}{N}\theta_1 I_1 + c\epsilon(t)\frac{S}{N}I_2$, in which θ_1 $(0 < \theta_1 < 1)$ accounts for reduction in transmissibility of infected individuals with mild symptoms compare to individuals with serious symptoms. After the 'Fire God Hill' hospital was opened on Feb. 4th, infected individuals would be arranged for hospitalization in succession. It implies that contact rate of infected individuals to susceptible individuals would gradually decrease. According to the idea of modeling in [10, 11], we use $\epsilon(t)$ to represent the exponential decay of contact rate of infected individuals to susceptible individuals due to opening the 'Fire God Hill' hospital with the following form

$$\epsilon(t) = \begin{cases} 1, & 0 \le t < \tau_2, \\ (1-\epsilon_1)e^{-m_2(t-\tau_2)} + \epsilon_1, & t \ge \tau_2. \end{cases} \tag{2.2}$$

Here, ϵ_1 represents the minimum proportion of contact rate of infected individuals to susceptible individuals under the measure of opening the 'Fire God Hill' hospital. Obviously, $\lim_{t\to+\infty} \epsilon(t) = \epsilon_1$. m_2 represents the exponential decreasing rate of contact rate of infected individuals to susceptible individuals due to opening the 'Fire God Hill' hospital.

Susceptible individuals are infected with COVID-19 and move into pre-stage exposed class E_1 with mean duration of $\frac{1}{\xi}$ days, later, enter into post-stage exposed class. Then individuals enter into infected class with mild symptoms I_1 at a proportion a, and into infected class with serious symptoms I_2 at a proportion $1-a$ over post-stage exposed period $\frac{1}{\sigma}$. The infected class with mild symptoms may recover through medication or self-immunity at a rate r_1, and may also be serious and enters to I_2 at a rate δ. If medical resources are adequate, then I_1 and I_2 can be admitted to hospital at rates $\eta_1(t)$ and $\eta_2(t)$, respectively. I_2 without recovery will die at a rate d_I. The hospitalized individual either recovers at a rate r_h or dies at a rate d_h.

After the 'Fire God Hill' hospital was opened on Feb. 4th, 2020, infected individuals would be arranged for hospitalization in succession. It implies that hospitalization rate $\eta_1(t)$ and $\eta_2(t)$ would gradually increase. According to the idea of modeling in [10], we use the following forms to represent the raises of hospitalization rate due to opening the 'Fire God Hill' hospital, respectively,

$$\eta_1(t) = \begin{cases} \eta_{1_0}, & 0 \le t < \tau_2, \\ \dfrac{1}{\left(\frac{1}{\eta_{1_0}} - \frac{1}{\eta_{1_1}}\right)e^{-m_3(t-\tau_2)} + \frac{1}{\eta_{1_1}}}, & t \ge \tau_2, \end{cases} \tag{2.3}$$

and

$$\eta_2(t) = \begin{cases} \eta_{2_0}, & 0 \le t < \tau_2, \\ \dfrac{1}{\left(\frac{1}{\eta_{2_0}} - \frac{1}{\eta_{2_1}}\right)e^{-m_3(t-\tau_2)} + \frac{1}{\eta_{2_1}}}, & t \ge \tau_2, \end{cases} \tag{2.4}$$

in which η_{1_0} and η_{2_0} represent the average hospitalization rates of infectious individuals with mild symptoms and serious symptoms between Jan. 23rd and Feb. 3rd, 2020. η_{1_1} and η_{2_1} represent maximum hospitalization rates with $\lim_{t\to+\infty} \eta_1(t) = \eta_{1_1}$ and $\lim_{t\to+\infty} \eta_2(t) = \eta_{2_1}$. m_3 represents the exponential decreasing rate of period of arranged for hospitalization of infected individuals.

In view of the above analysis, we formulate a COVID-19 model with medical resources as follows

$$\begin{cases} \dfrac{dS}{dt} = -c\dfrac{S}{N}\theta_e\beta p(t)E_2 - c\epsilon(t)\dfrac{S}{N}\theta_1\beta I_1 - c\epsilon(t)\dfrac{S}{N}\beta I_2, \\ \dfrac{dE_1}{dt} = c\dfrac{S}{N}\theta_e\beta p(t)E_2 + c\epsilon(t)\dfrac{S}{N}\theta_1\beta I_1 + c\epsilon(t)\dfrac{S}{N}\beta I_2 - \xi E_1, \\ \dfrac{dE_2}{dt} = \xi E_1 - \sigma E_2, \\ \dfrac{dI_1}{dt} = a\sigma E_2 - \delta I_1 - \eta_1(t)I_1 - r_1 I_1, \\ \dfrac{dI_2}{dt} = (1-a)\sigma E_2 + \delta I_1 - \eta_2(t)I_2 - d_I I_2, \\ \dfrac{dH}{dt} = \eta_1(t)I_1 + \eta_2(t)I_2 - r_h H - d_h H, \\ \dfrac{dR}{dt} = r_1 I_1 + r_h H. \end{cases} \tag{2.5}$$

3. Results

In this section, we first apply model (2.5) to fit the cumulative and new confirmed cases from Jan. 23rd to Mar. 6th, 2020 in Wuhan and estimate unknown parameters. Then, based on the parameters, the basic reproduction number and effective daily reproduction ratio are estimated. Later, we will assess the effects of medical resources on transmission of COVID-19.

3.1. Fitting

Let $X(t)$ be theoretic cumulative confirmed cases and its change with time is determined by $\frac{dX(t)}{dt} = \sigma E_2$, $Y(t)$ be theoretic new confirmed cases and its change with time is determined by $Y(t) = X(t) - X(t-1)$. Let $\bar{X}(t)$ and $\bar{Y}(t)$ is reported cumulative and new confirmed cases in day t, respectively. About 9 million are living in the city after sealing off the city. We can set the initial number of susceptible individuals $S(0) = 9,000,000$. The reported cumulative and new confirmed cases on Jan. 23rd are 495 and 70, respectively. So, $X(0) = 495$ and $Y(0) = 70$. Based on reported data from websites [14], we can set $I_1(0) = 495 \cdot a$ and $I_2(0) = 495 \cdot (1-a)$. Then, we set $H(0) = \eta_{1_0} I_1(0) + \eta_{2_0} I_2(0)$ and $R(0) = r_1 I_1(0)$. Because the numbers of the initial pre-stage exposed population $E_1(0)$ cannot be obtained directly, its value can be estimated as a parameter. Then, we derive $E_2(0) = \xi E_1(0)$. Next, the least-square estimation is used to calculate the parameter values to minimize the objective functions

$$J_1 = \frac{1}{n}\sum_{t=1}^{n}(X(t) - \bar{X}(t))^2,\ J_2 = \frac{1}{n}\sum_{t=1}^{n}(Y(t) - \bar{Y}(t))^2,$$

in which n is the number of reported data. In our model, take day as the unit time. So n=44. By applying method of random simulation and running 1000 simulations, the values of parameter are given in Table 1.

Table 1. Parameters description.

Parameter	Description	Value	Source
c	Contact rate (Day^{-1})	13.8	Assumption
β	Transmission probability of I_2 per contact (Dimensionless)	0.036	Estimation
$\frac{1}{\xi}$	Pre-stage exposed period (Day)	1.52	Estimation
$\frac{1}{\sigma}$	Post-stage exposed period (Day)	6.67	Estimation
a	Proportion of post-stage exposed class entering I_1 compartment (Dimensionless)	0.8	Calculation [14]
δ	Transition rate from infectious cases with mild symptoms to infectious cases with serious symptoms (Day^{-1})	0.2	Estimation
η_{1_0}	Average hospitalization rate of infectious cases with mild symptoms between Jan. 23rd and Feb. 3rd (Day^{-1})	0.31	Assumption
η_{1_1}	Maximum hospitalization rate of infectious cases with mild symptoms (Day^{-1})	0.93	Estimation
η_{2_0}	Average hospitalization rate of infectious cases with serious symptoms between Jan. 23rd and Feb. 3rd (Day^{-1})	0.7	Assumption
η_{2_1}	Maximum hospitalization rate of infectious cases with serious symptoms (Day^{-1})	0.98	Estimation
r_1	Recovery rate of infectious cases with mild symptoms (Day^{-1})	$\frac{1}{12.5}$	[12]
r_h	Recovery rate of hospitalized cases (Day^{-1})	0.05	Estimation
d_I	Disease-related death rate of infectious cases with serious symptoms (Day^{-1})	0.055	Assumption [13]
d_h	Disease-related death rate of hospitalized cases (Day^{-1})	0.049	Assumption [13]
p_0	The initial unquarantined proportion of post-stage exposed individuals (Dimensionless)	0.76	Estimation
p_1	The minimum unquarantined proportion of post-stage exposed individuals (Dimensionless)	0.00608	Estimation
θ_e	Modification parameter or infectious reduction factor (Dimensionless)	0.72	Estimation
θ_1	Modification parameter or infectious reduction factor (Dimensionless)	0.87	Estimation
m_0	The exponential decreasing rate of the unquarantined proportion of E_2 due to sealing off the city (Dimensionless)	0.02	Estimation
m_1	The exponential decreasing rate of the unquarantined proportion of E_2 due to opening the 'Fire God Hill' hospital (Dimensionless)	0.19	Estimation
m_2	The exponential decreasing rate of contact rate of I_1 and I_2 to S due to opening the 'Fire God Hill' hospital (Dimensionless)	0.22	Estimation
m_3	The exponential decreasing rate of period of arranged for hospitalization of I_1 and I_2 (Dimensionless)	0.3	Estimation
ϵ_1	Minimum proportion of contact rate of I_1 and I_2 to S due to opening the 'Fire God Hill' hospital (Dimensionless)	0.1	Estimation
τ_1	The time for sealing off the city (Dimensionless)	0	Calculation
τ_2	The time for opening the 'Fire God Hill' hospital (Dimensionless)	12	Calculation
$E_1(0)$	The initial number of pre-stage exposed individuals (Dimensionless)	1800	Estimation

Through some rational assumptions and parameter estimations, fitting curves of cumulative and new confirmed cases using model (2.5) are shown in Figures 3 and 4. Figures 3 and 4 indicate that our model (2.5) provides well fit to the reported data from Jan. 23rd to Mar. 6th, 2020 in Wuhan. From Figure 3, in the early stages, disease grows rapidly. The possible reason is the lack of medical resources. With the opening of the 'Fire God Hill' hospital and other the 'Mobile Field' hospitals in succession, later, new confirmed cases drop rapidly (Figure 4).

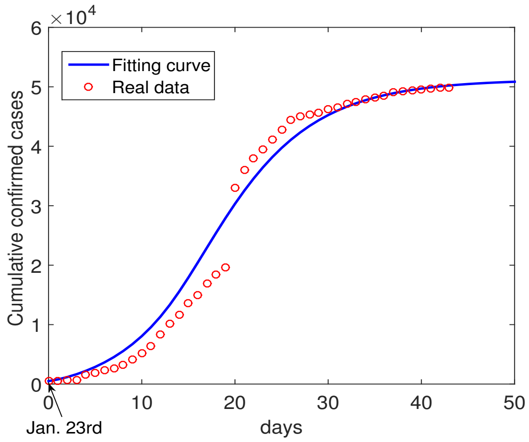

Figure 3. The fitting results of cumulative confirmed cases from Jan. 23rd to Mar. 6th, 2020 and prediction results for the following week. The blue curve denotes fitting curve of model (2.5). The star denotes the real data of cumulative confirmed cases.

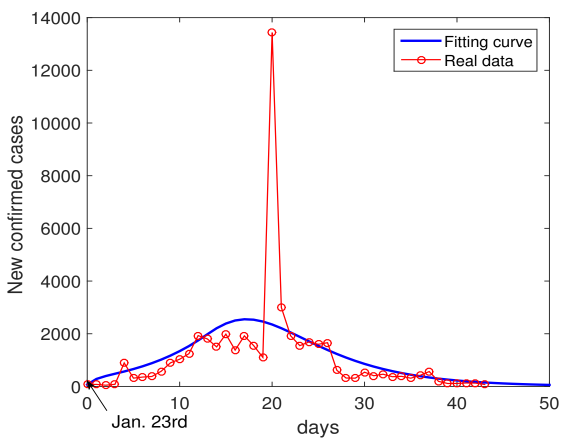

Figure 4. The fitting results of new confirmed cases from Jan. 23rd to Mar. 6th, 2020 and prediction results for the following week. The blue curve denotes fitting curve of model (2.5). The star denotes the real data of new confirmed cases.

3.2. *Estimation of the basic reproduction number and effective daily reproduction ratio*

3.2.1. Estimation of the basic reproduction number

Now, we give a risk index: the basic reproduction number R_0. The value of R_0 determines whether the disease will eventually die out or not under the absence of effective control measures. To be specific, if $R_0 > 1$, then the disease will break out. Conversely, if $R_0 < 1$, then the disease will die out. R_0 can be calculated under conditions: $p(t) \equiv p_0$, $\epsilon(t) \equiv 1$, $\eta_1(t) \equiv \eta_{1_0}$ and $\eta_2(t) \equiv \eta_{2_0}$. By using the next generation matrix, we can calculate R_0, the spectral radius of the next generation matrix [15, 16] given by

$$R_0 = R_E + R_{I_1} + R_{I_2}, \tag{3.1}$$

where

$$R_E = \frac{\beta c \theta_e p_0}{\sigma},\ R_{I_1} = \frac{\beta c \theta_1 a}{\delta + \eta_{1_0} + r_1},\ R_{I_2} = \beta c \left((1-a) + \frac{a\delta}{\delta + \eta_{1_0} + r_1} \right) \frac{1}{\eta_{2_0} + d_I}.$$

Biologically, R_0 represents the number of secondary infections produced by an infected individual during its average infectious lifetime in a completely susceptible population. Susceptible individuals acquire infection with COVID-19 by an effective contact with either post-stage exposed E_2, or infected with mild symptoms I_1, or infected with serious symptoms I_2 individuals. Thus, R_0 consists of three parts: R_E, R_{I_1} and R_{I_2}. Firstly, the number of new infectious individuals produced by a post-stage exposed individual E_2 via effective contact with susceptible individuals at a rate $\beta c \theta_e p_0$ during its average infectious lifetime $\frac{1}{\sigma}$ is $\frac{\beta c \theta_e p_0}{\sigma}$. Secondly, a indicates the proportion that an individual in the E_2 class will become I_1. $\frac{1}{\delta+\eta_{1_0}+r_1}$ represents the average infectious lifetime of I_1. So, the number of new infectious individuals generated by an infected individual with mild symptoms I_1 during its average infectious lifetime is given by $\theta_1 \beta c \cdot a \cdot \frac{1}{\delta+\eta_{1_0}+r_1}$. Lastly, the number of new infectious individuals generated by an infected individual with serious symptoms I_2 consists of two parts. The first path is given by the product of the infection rate βc, the proportion that E_2 becomes I_2 class $(1-a)$, and the average infectious duration of an individual in the I_2 class $\frac{1}{\eta_{2_0}+d_I}$. The second path is given by the product of the infection rate βc, the proportion that E_2 becomes I_1 class a, the proportion that I_1 becomes I_2 class $\frac{\delta}{\delta+\eta_{1_0}+r_1}$, and the average infectious duration of an individual in the I_2 class $\frac{1}{\eta_{2_0}+d_I}$.

We estimate that the value of R_0 is 2.71. It is contributed by post-stage exposed individuals R_E =1.81, contributed by infected individuals with mild symptoms R_{I_1} =0.59, contributed by infected individuals with serious symptoms $R_{I_2} = 0.31$. This suggests that transmission of post-stage exposed individuals is main factor of an outbreak of COVID-19. However, the effects of transmission of infected individuals on the spread of COVID-19 cannot be neglected.

3.2.2. Estimation of the effective daily reproduction ratio

Based on the definition of the effective daily reproduction ratio in [10, 11], which is used to measure the 'daily reproduction number', the number of new infectious individuals induced by a single infectious individual during its infectious period per day. In this paper, the effective daily reproduction ratio is calculated by

$$R_e(t) = \left(\frac{\beta c \theta_e p(t)}{\sigma} + \frac{\beta \theta_1 a c \epsilon(t)}{\delta + \eta_1(t) + r_1} + \beta c \epsilon(t) \left((1-a) + \frac{a\delta}{\delta + \eta_1(t) + r_1} \right) \frac{1}{\eta_2(t) + d_I} \right) \frac{S(t)}{N(t)}. \tag{3.2}$$

The effective daily reproduction ratio can be simulated and shown in Figure 5. $R_e(0)$ is a risk index of the initial transmissibility of COVID-19 on Jan. 23rd. We know that $R_e(0) = R_0$. From Figure 5, under the scenario sealing off the city, the effective daily reproduction ratio begins to decrease and drops to 2.33 as of Feb. 4th. After the 'Fire God Hill' hospital was opened on Feb. 4th, 2020, infected individuals would be arranged for hospitalization in succession. It makes infected individuals quarantine, thereby reduces contact rate. Under the combination of the restrictive measures, including sealing off the city and opening the 'Fire God Hill' hospital, the effective daily reproduction ratio declines rapidly and drops less than 1 since Feb. 8th. As of Mar. 6th, it drops to 0.06, and the epidemic will gradually die off.

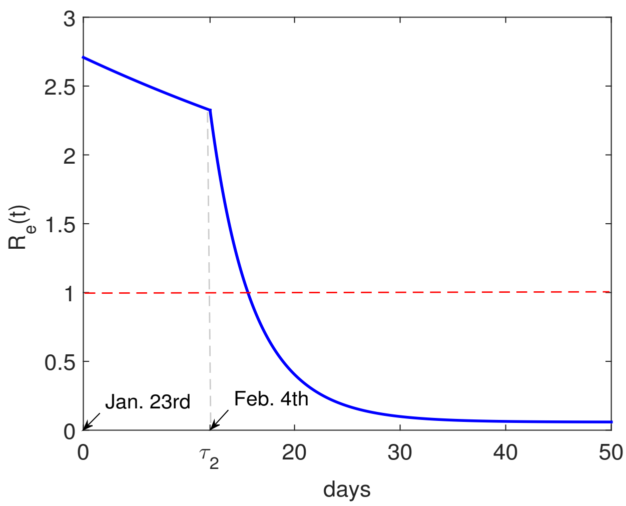

Figure 5. The effective daily reproduction ratio $R_e(t)$ with $R_e(0) = R_0$.

3.3. The effects of medical resources on transmission of COVID-19

In this part, we will examine the effects of opening the 'Fire God Hill' hospital later on cumulative and new confirmed cases, and the effects of the exponential decreasing rates of unquarantined proportion of E_2, the contact rate of I_1 and I_2 to S, and the period of arranged for hospitalization of infected individuals due to opening the 'Fire God Hill' hospital on cumulative confirmed cases.

Firstly, we consider different values of the time for opening the 'Fire God Hill' hospital: τ_2=12, 13, 14, 15, 16, 17. From Figure 6(A), if the 'Fire God Hill' hospital is opened 1 day delay (i.e., $\tau_2 = 13$), then the number of cumulative confirmed cases in Wuhan will be about 30% higher than the real reported cases (49,871) by Mar. 6th. If the 'Fire God Hill' hospital is opened 5 day delay (i.e., $\tau_2 = 17$), then the number of cumulative confirmed cases in Wuhan will be about 80.1% higher than the real data by Mar. 6th. It implies that delayed opening the 'Fire God Hill' hospital will greatly increase the magnitude of the outbreak. From Figure 6(B), if the 'Fire God Hill' hospital is opened later, the inflection point of the epidemic will appear later and the peak will be increased. This implies that the government's timely establishment of hospitals prevent a larger outbreak.

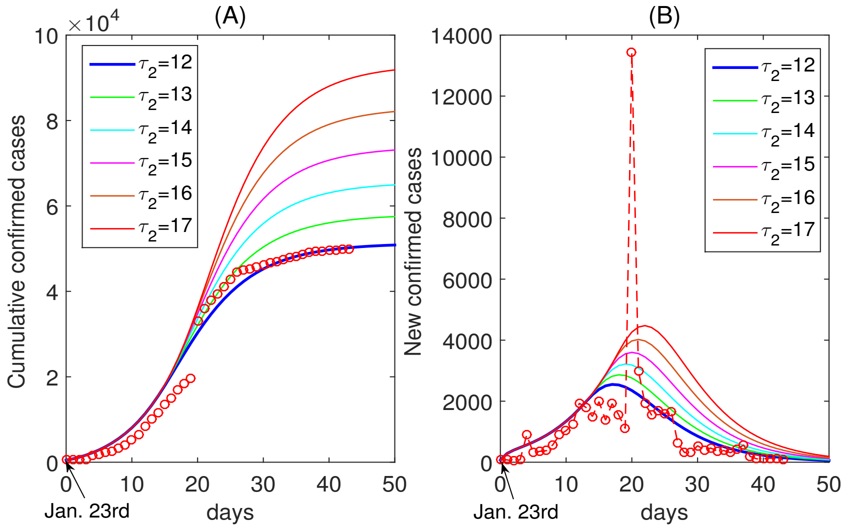

Figure 6. The effects of the time for opening the 'Fire God Hill' hospital τ_2 on (A) cumulative and (B) new confirmed cases.

Next, we examine the effects of the exponential decreasing rates of unquarantined proportion of E_2, the contact rate of I_1 and I_2 to S, and the period of arranged for hospitalization of infected individuals due to opening the 'Fire God Hill' hospital on cumulative confirmed cases. For this purpose, we choose different exponential decreasing rates m_1, m_2 and m_3, respectively. Figure 7 shows that the larger the strength of quarantine is, the smaller the number of cumulative confirmed cases is. The results shown in Figures 8 and 9 indicate the larger the strength of acceptance patients is, the smaller the number of cumulative confirmed cases is. It suggests that effective quarantine via quick detection and adequate number of available hospital beds will effectively reduce the number of cases of COVID-19.

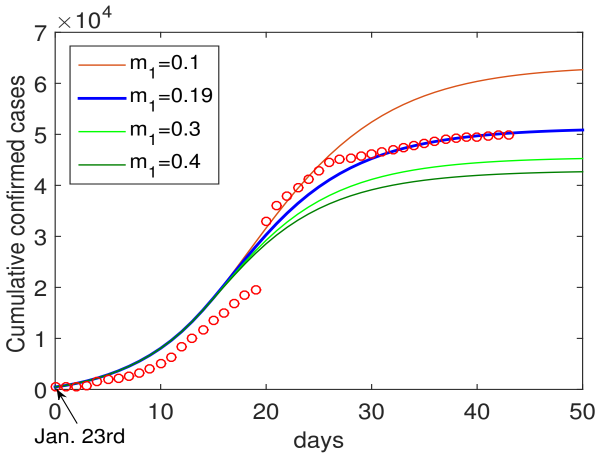

Figure 7. The effects of the exponential decreasing rate of unquarantined proportion of E_2 due to opening the 'Fire God Hill' hospital m_1 on cumulative confirmed cases.

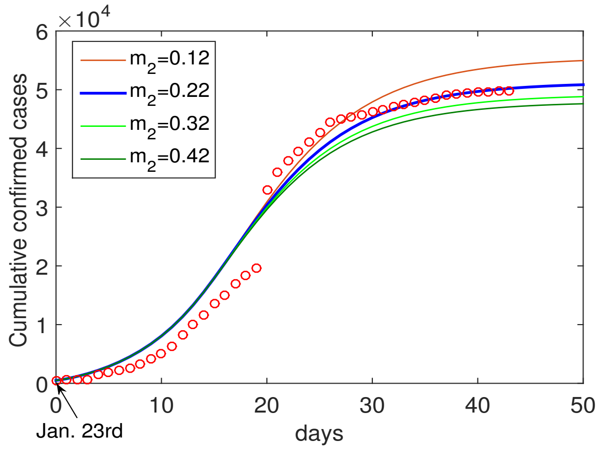

Figure 8. The effects of the exponential decreasing rate of contact rate of I_1 and I_2 to S due to opening the 'Fire God Hill' hospital m_2 on cumulative confirmed cases.

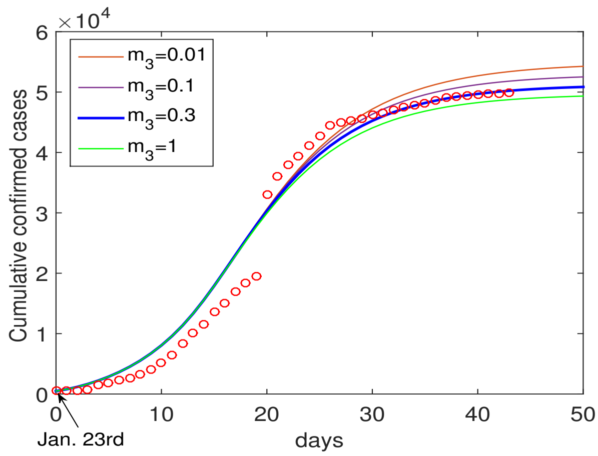

Figure 9. The effects of the exponential decreasing rate of period of arranged for hospitalization of infected individuals m_3 on cumulative confirmed cases.

4. Conclusion

Based on the transmission mechanisms of COVID-19 and epidemic characteristics of Wuhan, we build a time-dependent mathematical model of COVID-19 to focus on investigating the effects of medical resources on transmission of COVID-19. We apply our model to fit the cumulative and new confirmed cases in Wuhan from Jan. 23rd to Mar. 6th, 2020 and get reasonable match. The basic reproduction number is estimated $R_0 = 2.71$. Li et al. [17] estimated the basic reproduction number of COVID-19 to be 2.2 (95% CI, 1.4–3.9). Zhao et al. estimated range of the basic reproduction number R_0 from 3.30 (95% CI: 2.73–3.96) to 5.47 (95% CI: 4.16–7.10) based on the early outbreak data following the exponential growth in [19]. Later, in [18], they also estimated R_0 at 2.56 (95% CI: 2.49–2.63) through modelling the epidemic curve of COVID-19 cases, from Dec. 1st, 2019 to Jan. 24th, 2020. Other studies have shown that the range of the basic reproductive number is about 2–3. In this study, our estimate is reasonable.

Due to variation in the control strategies with the time, we calculate the effective daily reproduction ratio $R_e(t)$, which is used to measure the 'daily reproduction number'. From Jan. 23rd, the effective daily reproduction ratio begins to decrease from 2.71 to 2.33 as of Feb. 4th. After the 'Fire God Hill' hospital was opened on Feb. 4th, it declines rapidly and drops less than 1 since Feb. 8th. As of Mar. 6th, it drops to 0.06, the epidemic will gradually die off.

Our results show that opening the 'Fire God Hill' hospital delay will greatly increase the magnitude of the outbreak and disappear later the inflection point of the epidemic. Moreover, effective quarantine via quick detection and adequate number of available hospital beds will effectively reduce the number of cases of COVID-19. By establishing mathematical model, our results are basically consistent with the current government decision-making. We suggest that other countries may make the similar measures for this disease.

Acknowledgments

We are very grateful to anonymous referees and editor for careful reading and helpful suggestions which led to an improvement of our original manuscript. The work is supported by the National Natural Science Foundation of China (Nos 11971013, 11571170).

Conflict of interest

The authors declare there is no conflict of interest.

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