Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 11(9): 29312–29326.
DOI: 10.3934/math.20261165
Received: 14 March 2026
Revised: 03 June 2026
Accepted: 10 June 2026
Published: 11 September 2026

Research article

Some results on new subclasses of regular p-valent functions blended with Pascal distribution series defined with respect to other points

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Abstract: In this article, we consider an analytic p-valent function $f_p(\zeta)$ blended with a power series which has the probabilities of the Pascal distribution as its coefficients. Using the subordination principle, the subclasses $S_s^*(p, \alpha, \delta; I, J)$ and $S_{sc}^*(p, \alpha, \delta; I, J)$ are defined with respect to other points, and some properties (such as coefficient estimates, growth and distortion properties, radii of starlikeness, convexity, extreme points) of the function belonging to the aforementioned subclasses are investigated. Additionally, some consequences of the results obtained are discussed as corollaries.

Keywords: analytic; starlike; convex; symmetric; conjugate and pascal distribution
Mathematics Subject Classification: 30C45, 30C20

1. Introduction

Let us consider the regular function $f(\zeta)$ with the formation

$$f(\zeta) = \zeta + \sum_{k=2}^{\infty} a_k \zeta^k \qquad \zeta \in \mathbb{C} \tag{1.1}$$

meeting certain conditions in the disk $D = \{\zeta : |\zeta| < 1\}$. Let Ω constitute the class of all the functions of the form (1.1). Also, let Ω_p designate the group of analytic p-valent functions $h_p(\zeta)$ with the structure

$$f_p(\zeta) = \zeta^p + \sum_{k=1}^{\infty} a_{k+p} \zeta^{k+p} \qquad \zeta \in \mathbb{C}. \tag{1.2}$$

The function $f_p(\zeta)$ of the form (1.2) is said to be p-valent in D provided it is regular and does not presume any value more than p times for $|\zeta| < 1$. In recent times, the class Ω_p, which is invariant (symmetric) under rotations, has caught the attention of several researchers in geometric function theory, with several interesting results established diversely in the literature [1, 7, 9]. However, the study of said class Ω_p as related to the Poisson distribution, exponential distribution, binomial distribution, and Pascal distribution are not as well-known especially in geometric function theory. Consequently, in this exploration, the regular p-valent functions blended with Pascal distribution series (which is defined with respect to other points) are studied. In 2020, Serkan [10] examined $H(\mathcal{X} = k)$ such that

$$H(\mathcal{X} = k) = \binom{k+\alpha-1}{\alpha-1}\delta^k(1-\delta)^\alpha \quad (k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}), \tag{1.3}$$

where δ and α are parameters [2, 3, 8]. It is worthy to note that (1.3) is a discrete random variable $\mathcal{X}$ having a Pascal probability generating function.

In view of (1.3), the authors introduce and study a power series of the form

$$H_\delta^\alpha(\zeta) = \zeta^p + \sum_{k=1}^{\infty}\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha\zeta^{k+p} \tag{1.4}$$

$$(k \in \mathbb{N}_0,\ \alpha \geq 1,\ 0 \leq \delta < 1,\ p \in \mathbb{N},\ \zeta \in D).$$

Finding the Hadamard product of (1.2) and (1.4),

$$h_p(\zeta) = H_\delta^\alpha(\zeta) * f_p(\zeta).$$

That is,

$$h_p(\zeta) = \zeta^p + \sum_{k=1}^{\infty}\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha a_{k+p}\zeta^{k+p}. \tag{1.5}$$

In view of (1.5), we look at these definitions:

Definition 1.1. *Let $f_p(\zeta)$ be of the form (1.5). The function $h_p(\zeta)$ belongs to the subclass $S_s^*(p, \alpha, \delta; I, J)$, provided that*

$$\Re\left\{\frac{(1-(-1)^p)\zeta h_p'(\zeta)}{p\Big[h_p(\zeta) - h_p(-\zeta)\Big]}\right\} \prec \frac{1+I\zeta}{1+J\zeta}$$

and $S_s^*(p, \alpha, \delta; I, J)$ denote the class of starlike functions with respect to a symmetric point associated with Pascal distribution, where $-1 \leq J < I \leq 1$, $p = 2n+1$, and $n \in \mathbb{N}_0$, and $\prec$ denotes the subordination symbol.

Definition 1.2. *Let $f_p(\zeta)$ be of the form (1.5). The function $h_p(\zeta)$ belongs to the subclass $S_{sc}^*(p, \alpha, \delta; I, J)$, provided that*

$$\Re\left\{\frac{(1-(-1)^p)\zeta h_p'(\zeta)}{p\Big[h_p(\zeta) + \overline{h_p(\bar{\zeta})}\Big]}\right\} \prec \frac{1+I\zeta}{1+J\zeta}$$

and $S^*_{sc}(p,\alpha,\delta;I,J)$ *denote the class of starlike functions with respect to a conjugate point associated with Pascal distribution, where* $-1 \le J < I \le 1$, $p = 2n+1$, *and* $n \in \mathbb{N}_0$, *and* $\prec$ *denotes the subordination symbol.*

Geometrically, the subclass $S^*_{sc}(p,\alpha,\delta;I,J)$ represents a family of p-valent functions whose images are starlike with respect to conjugate points. The imposed Janowski subordination controls the radial expansion of the mapping and confines the associated logarithmic derivative to a prescribed Janowski region. As a result, the image domains retain a generalized starlike structure while exhibiting bounded angular variation and enhanced geometric regularity.

For recent studies in this direction, see [4–6], among others.

2. Coefficient estimates

At this point, we shall estimate the coefficient for $h_p(\zeta)$ in the subclasses $S^*_s(p,\alpha,\delta;I,J)$ and $S^*_{sc}(p,\alpha,\delta;I,J)$.

Theorem 2.1. *Let* $h_p(\zeta)$ *be defined by (1.5). Then* $f_p(\zeta) \in S^*_s(p,\alpha,\delta;I,J)$, $0 \le \delta < 1$, *provided that*

$$\sum_{k=1}^{\infty} \frac{\left[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\right]}{p(I-J)(1-(-1)^p)} \binom{k+p+\alpha-2}{\alpha-1} \delta^{k+p-1}(1-\delta)^{\alpha} |a_{k+p}| \le 1. \quad (2.1)$$

The result is sharp for the extremal function $h_p(\zeta)$ *given by*

$$h_p(\zeta) = \zeta^p + \sum_{k=1}^{\infty} \frac{p(I-J)(1-(-1)^p)}{\left[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\right]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}} \zeta^{k+p}.$$

Proof. By the principle of subordination, Definition 1.1 can be expressed as

$$\frac{(1-(-1)^p)\zeta h_p'(\zeta)}{p\left[h_p(\zeta) - h_p(-\zeta)\right]} = \frac{1+I\omega(\zeta)}{1+J\omega(\zeta)}. \quad (2.2)$$

Obviously, (2.2) can be written as

$$(1-(-1)^p)\zeta h_p'(\zeta) - p\left[h_p(\zeta) - h_p(-\zeta)\right] = \left\{Ip\left[h_p(\zeta) - h_p(-\zeta)\right] - J(1-(-1)^p)\zeta h_p'(\zeta)\right\}\omega(\zeta). \quad (2.3)$$

Simplifying (2.3) further, we obtain

$$\begin{aligned} &\sum_{k=1}^{\infty} M_1 \binom{k+p+\alpha-2}{\alpha-1} \delta^{k+p-1}(1-\delta)^{\alpha} a_{k+p}\zeta^{k+p} \\ &= \left\{p(I-J)(1-(-1)^p)\zeta^p - \sum_{k=1}^{\infty} M_2 \binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha} a_{k+p}\zeta^{k+p}\right\}\omega(\zeta), \end{aligned} \quad (2.4)$$

where

$$M_1 = \left[(1-(-1)^p)(k+p) - p(1-(-1)^{k+p})\right] \text{ and } M_2 = \left[J(1-(-1)^p)(k+p) - Ip(1-(-1)^{k+p})\right].$$

Using the fact that $|\omega(\zeta)| \leq 1$,

$$\begin{aligned}&\left|\sum_{k=1}^{\infty} M_1 \binom{k+p+\alpha-2}{\alpha-1} \delta^{k+p-1}(1-\delta)^{\alpha} a_{k+p}\zeta^{k+p}\right| \\ &\leq \left|p(I-J)(1-(-1)^p)\zeta^p - \sum_{k=1}^{\infty} M_2 \binom{k+p+\alpha-2}{\alpha-1} \delta^{k+p-1}(1-\delta)^{\alpha} a_{k+p}\zeta^{k+p}\right|,\end{aligned}$$

where M_1 and M_2 are as earlier defined. As $|\zeta| \to 1^{-1}$ through the real values, we have

$$\begin{aligned}&\sum_{k=1}^{\infty} \left[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\right] \binom{k+p+\alpha-2}{\alpha-1} \delta^{k+p-1}(1-\delta)^{\alpha} \left|a_{k+p}\right| \\ &\leq p(I-J)(1-(-1)^p).\end{aligned}$$

Therefore,

$$\sum_{k=1}^{\infty} \frac{\left[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\right]}{p(I-J)(1-(-1)^p)} \binom{k+p+\alpha-2}{\alpha-1} \delta^{k+p-1}(1-\delta)^{\alpha} |a_{k+p}| \leq 1,$$

which concludes the proof. □

Corollary 2.1. *Let $h_p(\zeta)$ be defined by (1.5). Then $h_1(\zeta) \in S_s^*(1, \alpha, \delta; I, J)$, $0 \leq \delta < 1$, provided that*

$$\sum_{k=1}^{\infty} \frac{\left[2(1-J)(k+1) + (I-1)(1-(-1)^{k+1})\right]}{2(I-J)} \binom{k+\alpha-1}{\alpha-1} \delta^{k}(1-\delta)^{\alpha} |a_{k+1}| \leq 1.$$

Corollary 2.2. *Let $h_p(\zeta)$ be defined by (1.5). Then $h_1(\zeta) \in S_s^*(1, \alpha, \delta; 1, -1)$, $0 \leq \delta < 1$, provided that*

$$\sum_{k=1}^{\infty} (k+1) \binom{k+\alpha-1}{\alpha-1} \delta^{k}(1-\delta)^{\alpha} |a_{k+1}| \leq 1.$$

Theorem 2.2. *Let $h_p(\zeta)$ be defined by (1.5). Then $h_p(\zeta) \in S_{sc}^*(p, \alpha, \delta; I, J)$, $0 \leq \delta < 1$, provided that*

$$\sum_{k=1}^{\infty} \frac{\left[k+Ip-J(k+p)\right]}{p(I-J)} \binom{k+p+\alpha-2}{\alpha-1} \delta^{k+p-1}(1-\delta)^{\alpha} |a_{k+p}| \leq 1. \tag{2.5}$$

Proof. Using the principle of subordination and Definition 1.2 and following the same steps as in Theorem 2.1. Inequality (2.5) is obtained. □

Corollary 2.3. *Let $h_p(\zeta)$ be defined by (1.5). Then $h_1(\zeta) \in S_{sc}^*(1, \alpha, \delta; I, J)$, $0 \leq \delta < 1$, provided that*

$$\sum_{k=1}^{\infty} \frac{\left[k(1-J) + (I-J)\right]}{(I-J)} \binom{k+\alpha-1}{\alpha-1} \delta^{k}(1-\delta)^{\alpha} |a_{k+1}| \leq 1. \tag{2.6}$$

Corollary 2.4. *Let $h_p(\zeta)$ be defined by (1.5). Then $h_1(\zeta) \in S_{sc}^*(1, \alpha, \delta; 1, -1)$, $0 \leq \delta < 1$, provided that*

$$\sum_{k=1}^{\infty} (k+1) \binom{k+\alpha-1}{\alpha-1} \delta^{k}(1-\delta)^{\alpha} |a_{k+1}| \leq 1. \tag{2.7}$$

3. Growth and distortion properties

Here, the growth and distortion results for the function $f_p(\zeta)$ in the subclasses $S_s^*(p,\alpha,\delta;I,J)$ and $S_{sc}^*(p,\alpha,\delta;I,J)$ are considered. When talking about the size of the image domain, $|f(\zeta)|$, we refer to the concept of the growth of function $f(\zeta)$, whereas when talking about the geometric analysis of $|f'(\zeta)|$ from the Jacobian $|f'(\zeta)|^2$ as the infinitesimal amplification factor of the area of the image domain (or as the nanoscopic amplification factor of the arc length under mapping/function f), we are refer to the distortion of $f(\zeta)$.

Theorem 3.1. *Assume the function* $f_p(\zeta) \in S_s^*(p,\alpha,\delta;I,J)$, $0 \le \delta < 1$. *Then*

$$r^p - \frac{p(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p) + p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^{p+1}$$
$$\le \left|f_p(\zeta)\right| \le r^p + \frac{p(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p) + p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^{p+1}.$$

Proof. From Theorem 2.1, we have

$$\left|a_{k+p}\right| \le \frac{p(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p) + p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha}. \tag{3.1}$$

Also, from (1.2), one can write that

$$\left|f_p(\zeta)\right| = \left|\zeta^p + \sum_{k=1}^{\infty} a_{k+p}\zeta^{k+p}\right| \tag{3.2}$$

or

$$\left|f_p(\zeta)\right| \le \left|\zeta\right|^p + \sum_{k=1}^{\infty} \left|a_{k+p}\right|\zeta^{k+p}$$

so that

$$\left|f_p(\zeta)\right| \le r^p + r^{p+1}\sum_{k=1}^{\infty} \left|a_{k+p}\right|. \tag{3.3}$$

In view of (2.1) and (3.3), we obtain

$$\left|f_p(\zeta)\right| \le r^p + \frac{p(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p) + p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^{p+1}.$$

Similarly,

$$\left|f_p(\zeta)\right| = \left|\zeta^p - \sum_{k=1}^{\infty} a_{k+p}\zeta^{k+p}\right|, \tag{3.4}$$

or

$$\left|f_p(\zeta)\right| \ge \left|\zeta\right|^p - \sum_{k=1}^{\infty} \left|a_{k+p}\right|\zeta^{k+p},$$

so that

$$\left|f_p(\zeta)\right| \geq r^p - r^{p+1}\sum_{k=1}^{\infty}\left|a_{k+p}\right|. \tag{3.5}$$

Then

$$\left|f_p(\zeta)\right| \geq r^p - \frac{p(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p)+p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^{p+1}.$$

This concludes the proof. □

Corollary 3.1. *Assume the function $f_1(\zeta) \in S_s^*(1, \alpha, \delta; I, J)$. Then*

$$r - \frac{(I-J)}{2(1-J)\alpha\delta(1-\delta)^\alpha} r^2 \leq \left|f_1(\zeta)\right| \leq r + \frac{(I-J)}{2(1-J)\alpha\delta(1-\delta)^\alpha} r^2.$$

Corollary 3.2. *Assume the function $f_1(\zeta) \in S_s^*(1, \alpha, \delta; 1, -1)$. Then*

$$r - \frac{r^2}{2\alpha\delta(1-\delta)^\alpha} \leq \left|f_1(\zeta)\right| \leq r + \frac{r^2}{2\alpha\delta(1-\delta)^\alpha}.$$

Theorem 3.2. *Assume the function $f_p(\zeta) \in S_s^*(p, \alpha, \delta; I, J)$, $0 \leq \delta < 1$. Then*

$$\begin{aligned} pr^{p-1} &- \frac{p(p+1)(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p)+p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^p \\ \leq \left|f_p'(\zeta)\right| &\leq pr^{p-1} + \frac{p(p+1)(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p)+p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^p. \end{aligned}$$

Proof. From (1.2), one can write that

$$f_p'(\zeta) = p\zeta^{p-1} + \sum_{k=1}^{\infty}(k+p)a_{k+p}\zeta^{k+p-1}, \tag{3.6}$$

$$\left|f_p'(\zeta)\right| = \left|p\zeta^{p-1} + \sum_{k=1}^{\infty}(k+p)a_{k+p}\zeta^{k+p-1}\right|,$$

or

$$\left|f_p'(\zeta)\right| \leq p\left|\zeta\right|^{p-1} + \sum_{k=1}^{\infty}(k+p)\left|a_{k+p}\right|\left|\zeta\right|^{k+p-1},$$

so that

$$\left|f_p'(\zeta)\right| \leq pr^p + r^p\sum_{k=1}^{\infty}(k+p)\left|a_{k+p}\right|. \tag{3.7}$$

Applying (3.1) in (3.7), we obtain

$$\left|f_p'(\zeta)\right| \leq pr^{p-1} + \frac{p(p+1)(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p)+p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^p.$$

Similarly,

$$\left|f_p'(\zeta)\right| \geq pr^{p-1} - \frac{p(p+1)(I-J)(1-(-1)^p)}{\left[(1-J)(p+1)(1-(-1)^p)+p(I-1)(1-(-1)^{p+1})\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^p,$$

which completes the proof. □

Corollary 3.3. *Assume the function* $f_1(\zeta) \in S_s^*(1,\alpha,\delta;I,J)$*. Then*

$$1 - \frac{(I-J)}{(1-J)\alpha\delta(1-\delta)^\alpha} r \leq \left|f_1'(\zeta)\right| \leq 1 - \frac{(I-J)}{(1-J)\alpha\delta(1-\delta)^\alpha} r.$$

Corollary 3.4. *Assume the function* $f_1(\zeta) \in S_s^*(1,\alpha,\delta;1,-1)$*. Then*

$$1 - \frac{r}{\alpha\delta(1-\delta)^\alpha} \leq \left|f_1'(\zeta)\right| \leq 1 + \frac{r}{\alpha\delta(1-\delta)^\alpha}.$$

Theorem 3.3. *Assume the function* $f_p(\zeta) \in S_{sc}^*(p,\alpha,\delta;I,J)$*,* $0 \leq \delta < 1$*. Then*

$$\begin{aligned} & r^p - \frac{p(I-J)}{\left[(1-J)+p(I-J)\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^{p+1} \\ & \leq \left|f_p(\zeta)\right| \leq r^p + \frac{p(I-J)}{\left[(1-J)+p(I-J)\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^{p+1}. \end{aligned} \tag{3.8}$$

Proof. Using the fact that

$$\left|a_{k+p}\right| \leq \frac{p(I-J)}{\left[k+Ip-J(k+p)\right]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}, \tag{3.9}$$

the proof is similar to that of Theorem 3.1. □

Corollary 3.5. *Let the function* $f_1(\zeta)$ *belong to the subclass* $S_{sc}^*(1,\alpha,\delta;I,J)$*. Then*

$$r - \frac{(I-J)}{\alpha\delta(1-2J+I)(1-\delta)^\alpha} r^2 \leq \left|f_1(\zeta)\right| \leq r + \frac{(I-J)}{\alpha\delta(1-2J+I)(1-\delta)^\alpha} r^2.$$

Corollary 3.6. *Let the function* $f_1(\zeta)$ *belong to the subclass* $S_{sc}^*(1,\alpha,\delta;1,-1)$*. Then*

$$r - \frac{r^2}{2\alpha\delta(1-\delta)^\alpha} \leq \left|f_1(\zeta)\right| \leq r + \frac{r^2}{2\alpha\delta(1-\delta)^\alpha}.$$

Theorem 3.4. *Assume the function* $f_p(\zeta) \in S_{sc}^*(p,\alpha,\delta;I,J)$*. Then*

$$\begin{aligned} & pr^{p-1} - \frac{p(p+1)(I-J)}{\left[(1-J)+p(I-J)\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^p \\ & \leq \left|f_p'(\zeta)\right| \leq pr^p + \frac{p(p+1)(I-J)}{\left[(1-J)+p(I-J)\right]\binom{p+\alpha-1}{\alpha-1}\delta^p(1-\delta)^\alpha} r^p. \end{aligned} \tag{3.10}$$

Corollary 3.7. *Assume the function* $f_1(\zeta) \in S_{sc}^*(1,\alpha,\delta;I,J)$*. Then*

$$1 - \frac{2(I-J)}{\alpha\delta(1-2J+I)(1-\delta)^\alpha} r \leq \left|f_1'(\zeta)\right| \leq 1 + \frac{2(I-J)}{\alpha\delta(1-2J+I)(1-\delta)^\alpha} r.$$

Corollary 3.8. *Assume the function* $f_1(\zeta) \in S_{sc}^*(1,\alpha,\delta;1,-1)$*. Then*

$$1 - \frac{r}{\alpha\delta(1-\delta)^\alpha} \leq \left|f_1'(\zeta)\right| \leq 1 + \frac{r}{\alpha\delta(1-\delta)^\alpha}.$$

4. Radii of starlikeness and convexity

Before proceeding to the results in this section, we note that the following classes of analytic p-valent functions in Eq (1.2) are well-known.

The analytic p-valent function defined in (1.2) is said to be starlike of order μ, provided the following geometric condition is satisfied:

$$\operatorname{Re}\left\{\frac{zf_p'(z)}{f_p(z)}\right\} > \mu, \quad 0 \le \mu < p, \quad p \in \mathbb{N}.$$

Also, the analytic p-valent function defined in (1.2) is said to be convex of order μ provided the following geometric condition is satisfied:

$$\operatorname{Re}\left\{1 + \frac{zf_p''(z)}{f_p'(z)}\right\} > \mu, \quad 0 \le \mu < p, \quad p \in \mathbb{N}.$$

Theorem 4.1. *Let $f_p(\zeta)$ be given by (1.2). If $f_p(\zeta) \in S_s^*(p, \alpha, \delta; I, J)$ (starlike class associated with a symmetric point), $0 \le \mu < 1$, then $f_p(\zeta)$ is called starlike of order $\mu(0 \le \mu < 1)$ with respect to a symmetric point in $|\zeta| < r_1$, where*

$$\begin{aligned} r_1 &= r_1(p, \alpha, \delta, \mu; I, J) \\ &= \inf_k \left\{ \frac{(p-\mu)\Big[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{p(k+p-\mu)(I-J)(1-(-1)^p)} \right\}^{\frac{1}{k}}. \end{aligned} \tag{4.1}$$

Proof. It is sufficient to prove that

$$\left|\frac{\zeta f_p'(\zeta)}{f_p(\zeta)} - p\right| \le p - \mu \qquad (0 \le \mu < 1). \tag{4.2}$$

This implies that

$$\left|\frac{\zeta f_p'(\zeta)}{f_p(\zeta)} - p\right| = \left|\frac{\sum_{k=1}^\infty k a_{k+p}\zeta^{k+p}}{\zeta^p + \sum_{k=1}^\infty a_{k+p}\zeta^{k+p}}\right| \le \frac{\sum_{k=1}^\infty k\left|a_{k+p}\right|\left|\zeta\right|^k}{1 - \sum_{k=1}^\infty \left|a_{k+p}\right|\left|\zeta\right|^k}. \tag{4.3}$$

Equation (4.3) holds true, provided that

$$\sum_{k=1}^\infty (k-\mu+p)\left|a_{k+p}\right|\left|\zeta\right|^k \le (p-\mu). \tag{4.4}$$

In view of (4.4) and (3.1), we have

$$\begin{aligned} &\frac{(k-\mu+p)\left|a_{k+p}\right|\left|\zeta\right|^k}{p-\mu} \\ &\le \frac{\Big[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{p(I-J)(1-(-1)^p)}. \end{aligned} \tag{4.5}$$

Solving for $\left|\zeta\right|$ in (4.5), we obtain

$$\left|\zeta\right| < \left\{\frac{(p-\mu)\Big[(1-J)(1-(-1)^p)(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{p(k+p-\mu)(I-J)(1-(-1)^p)}\right\}^{\frac{1}{k}},$$

and this concludes the proof Theorem 4.1. □

Corollary 4.1. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta) \in S_s^*(1,\alpha,\delta;I,J)$, then $f_1(\zeta)$ is called starlike of order $\mu(0 \le \mu < 1)$ with respect to a symmetric point in $|\zeta| < r_1$, where*

$$\begin{aligned} r_1 &= r_1(1,\alpha,\delta,\mu;I,J) \\ &= \inf_k \left\{\frac{(1-\mu)\Big[2(1-J)(k+1)+(I-1)(1-(-1)^{k+1})\Big]\binom{k+\alpha-1}{\alpha-1}\delta^k(1-\delta)^\alpha}{2(k+1-\mu)(I-J)}\right\}^{\frac{1}{k}}. \end{aligned}$$

Corollary 4.2. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta) \in S_s^*(1,\alpha,\delta;1,-1)$, then $f_1(\zeta)$ is called starlike of order $\mu(0 \le \mu < 1)$ with respect to a symmetric point in $|\zeta| < r_1$, where*

$$r_1 = r_1(1,\alpha,\delta,\mu;1,-1) = \inf_k \left\{\frac{(1-\mu)(k+1)\binom{k+\alpha-1}{\alpha-1}\delta^k(1-\delta)^\alpha}{(k+1-\mu)}\right\}^{\frac{1}{k}}.$$

Theorem 4.2. *Let $f_p(\zeta)$ be given by (1.2). If $f_p(\zeta) \in S_s^c(p,\alpha,\delta;A,B)$ (convex class associated with a symmetric point), $0 \le \delta < 1$, then $f_p(\zeta)$ is said to be convex of order $\mu(0 \le \mu < 1)$ with respect to a symmetric point in $|\zeta| < r_2$, where*

$$\begin{aligned} r_2 &= r_2(p,\alpha,\delta,\mu;I,J) \\ &= \inf_k \left\{\frac{(p-\mu)\Big[(1-J)(1-(-1)^p)(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{k(k+p-\mu)(k+p)(I-J)(1-(-1)^p)}\right\}^{\frac{1}{k}}. \end{aligned} \tag{4.6}$$

Proof. We shall show that

$$\left|1+\frac{\zeta f_p''(\zeta)}{f_p'(\zeta)}-p\right| \le p-\mu. \tag{4.7}$$

This implies that

$$\left|1+\frac{\zeta f_p''(\zeta)}{f_p'(\zeta)}-p\right| = \left|\frac{\sum_{k=1}^\infty k(k+p)a_{k+p}\zeta^k}{p+\sum_{k=1}^\infty (k+p)a_{k+p}\zeta^k}\right| \le \left|\frac{\sum_{k=1}^\infty k(k+p)a_{k+p}\zeta^k}{p-\sum_{k=1}^\infty (k+p)a_{k+p}\zeta^k}\right|. \tag{4.8}$$

Equation (4.8) holds true, provided that

$$\sum_{k=1}^\infty k(k+p-\mu)(k+p)\left|a_{k+p}\right|\left|\zeta\right|^k \le p(p-\mu). \tag{4.9}$$

Using (3.1) and (4.9), we obtain

$$\frac{(k+p-\mu)(k+p)\left|\zeta\right|^k}{p(p-\mu)} \le \frac{\Big[(1-J)(1-(-1)^p)(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{p(I-J)(1-(-1)^p)},$$

so that

$$\left|\zeta\right|^k < \frac{(p-\mu)\Big[(1-J)(1-(-1)^p)(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{k(k+p-\mu)(k+p)(I-J)(1-(-1)^p)},$$

from which we obtain the required result. □

Corollary 4.3. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta) \in S_s^c(1,\alpha,\delta;I,J)$, then $f_1(\zeta)$ is said to be convex of order $\mu(0 \le \mu < 1)$ with respect to a symmetric point in $|\zeta| < r_2$, where*

$$r_2 = r_2(1,\alpha,\delta,\mu;I,J) = \inf_k \left\{ \frac{(1-\mu)\Big[2(1-J)(k+1)+(I-1)(1-(-1)^{k+1})\Big]\binom{k+\alpha-1}{\alpha-1}\delta^k(1-\delta)^\alpha}{2(k+1-\mu)(k+1)(I-J)} \right\}^{\frac{1}{k}}.$$

Corollary 4.4. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta)$ belongs to the class $S_s^c(1,\alpha,\delta;1,-1)$, then $f_1(\zeta)$ is said to be convex of order $\mu(0 \le \mu < 1)$ with respect to a symmetric point in $|\zeta| < r_2$, where*

$$r_2 = r_2(1,\alpha,\delta,\mu;1,-1) = \inf_k \left\{ \frac{(1-\mu)(k+1)\binom{k+\alpha-1}{\alpha-1}\delta^k(1-\delta)^\alpha}{(k+1-\mu)(k+1)} \right\}^{\frac{1}{k}}.$$

Theorem 4.3. *Let $f_p(\zeta)$ be given by (1.2). If $f_p(\zeta) \in S_{sc}^*(p,\alpha,\delta;I,J)$ (starlike class associated with a symmetric conjugate point), then $f_p(\zeta)$ is called starlike of order $\mu(0 \le \mu < 1)$ with respect to a conjugate point in $|\zeta| < r_3$, where*

$$r_3 = r_3(p,\alpha,\delta,\mu;I,J) = \inf_k \left\{ \frac{(p-\mu)\Big[k+Ip-J(k+p)\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{p(k+p-\mu)(I-J)} \right\}^{\frac{1}{k}}. \tag{4.10}$$

Proof. It suffices to show that

$$\left|\frac{\zeta f_p'(\zeta)}{f_p(\zeta)} - p\right| \le p-\mu \qquad (0 \le \mu < 1). \tag{4.11}$$

This implies that

$$\left|\frac{\zeta f_p'(\zeta)}{f_p(\zeta)} - p\right| = \left|\frac{\sum_{k=1}^\infty k a_{k+p}\zeta^{k+p}}{\zeta^p + \sum_{k=1}^\infty a_{k+p}\zeta^{k+p}}\right| \le \frac{\sum_{k=1}^\infty k\left|a_{k+p}\right|\left|\zeta\right|^k}{1-\sum_{k=1}^\infty \left|a_{k+p}\right|\left|\zeta\right|^k}. \tag{4.12}$$

Equation (4.12) holds true if

$$\sum_{k=1}^\infty (k-\mu+p)\left|a_{k+p}\right|\left|\zeta\right|^k \le (p-\mu). \tag{4.13}$$

In view of (4.13) and (3.9), we have

$$\frac{(k-\mu+p)\left|a_{k+p}\right|\left|\zeta\right|^k}{p-\mu} \le \frac{\Big[k+Ip-J(k+p)\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}{p(I-J)}. \tag{4.14}$$

Solving for $\left|\zeta\right|$ in (4.14), we obtain

$$\left|\zeta\right| < \left\{\frac{(p-\mu)\Big[k+Ip-J(k+p)\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}{p(k+p-\mu)(I-J)}\right\}^{\frac{1}{k}},$$

which concludes the proof of Theorem 4.3. □

Corollary 4.5. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta) \in S^*_{sc}(1,\alpha,\delta;I,J)$, then $f_1(\zeta)$ is called starlike of order $\mu(0 \leq \mu < 1)$ with respect to a conjugate point in $|\zeta| < r_3$, where*

$$r_3 = r_3(1,\alpha,\delta,\mu;I,J) = \inf_k \left\{\frac{(1-\mu)\Big[k+I-J(k+1)\Big]\binom{k+\alpha-1}{\alpha-1}\delta^{k}(1-\delta)^{\alpha}}{(k+1-\mu)(I-J)}\right\}^{\frac{1}{k}}.$$

Corollary 4.6. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta) \in S^*_{sc}(1,\alpha,\delta;1,-1)$, then $f_1(\zeta)$ is called starlike of order $\mu(0 \leq \mu < 1)$ with respect to a conjugate point in $|\zeta| < r_3$, where*

$$r_3 = r_3(1,\alpha,\delta,\mu;1,-1) = \inf_k \left\{\frac{(1-\mu)(k+1)\binom{k+\alpha-1}{\alpha-1}\delta^{k}(1-\delta)^{\alpha}}{(k+1-\mu)}\right\}^{\frac{1}{k}}.$$

Theorem 4.4. *Let $f_p(\zeta)$ be given by (1.2). If $f_p(\zeta) \in S^c_{sc}(p,\alpha,\delta;I,J)$ (covex class associated with a symmetric conjugate point), then $f_p(\zeta)$ is said to be convex of order $\mu(0 \leq \mu < 1)$ with respect to a conjugate point in $|\zeta| < r_4$, where*

$$r_4 = r_4(p,\alpha,\delta,\mu;I,J) = \inf_k \left\{\frac{(p-\mu)\Big[k+Ip-J(k+p)\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}{k(k+p-\mu)(k+p)(I-J)}\right\}^{\frac{1}{k}}. \tag{4.15}$$

Proof. Using (3.9), the proof is similar to that of Theorem 4.3. □

Corollary 4.7. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta) \in S^c_{sc}(1,\alpha,\delta;I,J)$, then $f_1(\zeta)$ is said to be convex of order $\mu(0 \leq \mu < 1)$ with respect to a conjugate point in $|\zeta| < r_4$, where*

$$r_4 = r_4(1,\alpha,\delta,\mu;I,J) = \inf_k \left\{\frac{(1-\mu)\Big[k+I-J(k+1)\Big]\binom{k+\alpha-1}{\alpha-1}\delta^{k}(1-\delta)^{\alpha}}{k(k+1-\mu)(k+1)(I-J)}\right\}^{\frac{1}{k}}.$$

Corollary 4.8. *Let $f_p(\zeta)$ be given by (1.2). If $f_1(\zeta) \in S^c_{sc}(1,\alpha,\delta;1,-1)$, then $f_1(\zeta)$ is said to be convex of order $\mu(0 \leq \mu < 1)$ with respect to a conjugate point in $|\zeta| < r_4$, where*

$$r_4 = r_4(1,\alpha,\delta,\mu;1,-1) = \inf_k \left\{\frac{(1-\mu)(k+1)\binom{k+\alpha-1}{\alpha-1}\delta^{k}(1-\delta)^{\alpha}}{k(k+1-\mu)(k+1)}\right\}^{\frac{1}{k}}.$$

5. Extreme points

Theorem 5.1. *Let*

$$f_p(\zeta) = \zeta^p \tag{5.1}$$

and

$$f_{k+p}(\zeta) = \zeta^p + \frac{p(I-J)(1-(-1)^p)}{\Big[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}\zeta^{k+p}, \tag{5.2}$$

$$0 \le \delta < 1, \quad -1 \le J < I \le 1, \quad p = 2n+1, n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}.$$

Then, $f_p(\zeta) \in S_s^*(p, \alpha, \delta; I, J)$ *if and only if it can be written as*

$$f_p(\zeta) = \sum_{k=0}^{\infty} \psi_{k+p}^s f_{k+p}(\zeta), \tag{5.3}$$

where

$$\psi_{k+p}^s \ge 0, \quad \textit{and} \quad \sum_{k=0}^{\infty} \psi_{k+p}^s = 1. \tag{5.4}$$

Proof. Suppose that

$$f_p(\zeta) = \sum_{k=0}^{\infty} \psi_{k+p}^s f_{k+p}(\zeta),$$

as stated in (5.3). Then

$$\begin{aligned} f_p(\zeta) = &\sum_{k=0}^{\infty} \psi_{k+p}^s \zeta^p \\ &+ \sum_{k=1}^{\infty} \psi_{k+p}^s \frac{p(I-J)(1-(-1)^p)}{\Big[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}\zeta^{k+p}. \end{aligned}$$

Because (5.4) is given, it follows that

$$f_p(\zeta) = \zeta^p + \sum_{k=1}^{\infty} \psi_{k+p}^s \frac{p(I-J)(1-(-1)^p)}{\Big[(1-J)(1-(-1)^p)(k+p) + p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}\zeta^{k+p}.$$

Also, because

$$\frac{\sum_{k=1}^{\infty} \psi_{k+p}^s \frac{p(I-J)\left(1-(-1)^p\right)}{\left[(1-J)\left(1-(-1)^p\right)(k+p)+p(I-1)\left(1-(-1)^{k+p}\right)\right]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}\zeta^{k+p}}{\frac{p(I-J)\left(1-(-1)^p\right)}{\left[(1-J)\left(1-(-1)^p\right)(k+p)+p(I-1)\left(1-(-1)^{k+p}\right)\right]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^\alpha}\zeta^{k+p}} = 1,$$

then

$$\sum_{k=1}^{\infty} \psi_{k+p}^s = 1. \tag{5.5}$$

From (5.4), one can say that

$$\sum_{k=0}^{\infty}\psi_{k+p}^{s}=\sum_{k=1}^{\infty}\psi_{k+p}^{s}+\psi_{p}.$$

Therefore, (5.5) can be written as

$$1-\psi_{p}\leq 1$$

so that

$$\psi_{p}\geq 0.$$

Hence, the function $f_p(\zeta)$ belongs to the subclass $S_s^*(p,\alpha,\delta;I,J)$.

Conversely, let $f_p(\zeta)\in S_s^*(p,\alpha,\delta;I,J)$. Then, from (3.1),

$$\left|a_{k+p}\right|\leq\frac{p(I-J)(1-(-1)^{p})}{\Big[(1-J)(1-(-1)^{p})(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}.$$

Also, let

$$\psi_{k+p}=\frac{\Big[(1-J)(1-(-1)^{p})(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}{p(I-J)(1-(-1)^{p})}\left|a_{k+p}\right|$$

and

$$\psi_{p}=1-\sum_{k=1}^{\infty}\psi_{k+p},\quad\Big(\sum_{k=1}^{\infty}\psi_{k+p}=\sum_{k=0}^{\infty}\psi_{k+p}-\psi_{p}\ \ and\ \ \sum_{k=0}^{\infty}\psi_{k+p}=1\Big).$$

From (1.2), we have

$$f_p(\zeta)=\zeta^{p}+\sum_{k=1}^{\infty}a_{k+p}\zeta^{k+p};$$

then,

$$\begin{aligned}
f_p(\zeta)&=\zeta^{p}+\sum_{k=1}^{\infty}\psi_{k+p}\frac{p(I-J)(1-(-1)^{p})}{\Big[(1-J)(1-(-1)^{p})(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}\zeta^{k+p}\\
&=\Big(\psi_{p}+\sum_{k=1}^{\infty}\psi_{k+p}\Big)\zeta^{p}\\
&+\sum_{k=1}^{\infty}\psi_{k+p}\frac{p(I-J)(1-(-1)^{p})}{\Big[(1-J)(1-(-1)^{p})(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}\zeta^{k+p}\\
&=\psi_{p}\zeta^{p}+\sum_{k=1}^{\infty}\psi_{k+p}\Big[\zeta^{p}+\frac{p(I-J)(1-(-1)^{p})}{\Big[(1-J)(1-(-1)^{p})(k+p)+p(I-1)(1-(-1)^{k+p})\Big]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}\Big]\\
&=\sum_{k=1}^{\infty}\psi_{k+p}f_{k+p}(\zeta),
\end{aligned}$$

which shows that the function $f_p(\zeta)$ can be represented by (5.3). This concludes the proof. □

Corollary 5.1. *The extreme points of the subclass $S_s^*(p,\alpha,\delta;I,J)$ are the functions $f_p(\zeta)$ and $f_{k+p}(\zeta)$ given by (5.1) and (5.2), respectively.*

Theorem 5.2. *Let*

$$f_p(\zeta) = \zeta^p \tag{5.6}$$

and

$$f_{k+p}(\zeta) = \zeta^p + \frac{p(I-J)}{\left[k + Ip - J(k+p)\right]\binom{k+p+\alpha-2}{\alpha-1}\delta^{k+p-1}(1-\delta)^{\alpha}}\zeta^{k+p}. \tag{5.7}$$

Then, $f_p(\zeta) \in S^*_{sc}(p, \alpha, \delta; I, J)$ if and only if it can be written as

$$f_p(\zeta) = \sum_{k=0}^{\infty} \psi^{sc}_{k+p} f_{k+p}(\zeta),$$

where

$$\psi^{sc}_{k+p} \geq 0, \quad \textit{and} \quad \sum_{k=0}^{\infty} \psi^{sc}_{k+p} = 1.$$

Proof. The proof is similar to that of Theorem 5.1. □

Corollary 5.2. *The extreme points of the subclass $S^*_{sc}(p, \alpha, \delta; I, J)$ are the functions $f_p(\zeta)$ and $f_{k+p}(\zeta)$ given by (5.6) and (5.7), respectively.*

6. Conclusions

The research in this article is primarily concerned with the analytic function $h_p(\zeta)$ blended with a power series having Pascal distribution as its coefficients using the Hadamard product. Having defined two subclasses $S^*_s(p, \alpha, \delta; I, J)$ and $S^*_{sc}(p, \alpha, \delta; I, J)$, certain properties, including the extreme points for the function $f_p(\zeta)$ belonging to the aforementioned subclasses, are investigated using a succinct mathematical approach, and several corollaries follow as simple consequences.

Author contributions

Hamzat Jamiu Olusegun: Conceptualization, Methodology, Software, Formal analysis, Visualization, Investigation, Data curation, Writing–original draft; Abbas Kareem Wanas: Conceptualization, Methodology, Software, Formal analysis, Visualization, Project administration, Data curation, Supervision, Writing–review and editing; Daniel Breaz: Validation, Resources, Methodology, Investigation, Project administration, Data curation, Funding acquisition. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no conflicts of interest.

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