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Research article

A stochastic differential game approach to multi-stakeholder collaborative governance of quality–reputation risks in livestream e-commerce scenarios: a "quality–reputation–performance" symbiosis perspective

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Abstract: In this study, we constructed a stochastic differential game (SDG) model for multi-stakeholder collaborative governance of quality–reputation risks in livestream e-commerce scenarios driven by quality complaints, and proposed an adaptive multi-agent collaborative response model based on dynamic event-triggering and Q-learning. We further analyzed the effectiveness conditions for collaborative governance and derived corresponding multi-stakeholder collaborative strategies under relevant factors. The major findings were threefold: (1) From the perspective of strategy adaptation, collaborative strategies were applicable to manufacturers only when the commission rate was relatively high, whereas for streamers and livestreaming platforms, such strategies were applicable only when the commission rate was relatively low. When the intensity of quality complaints was at a moderate threshold, the benefits of reputational collaborative governance were highest, prompting the system to favor collaborative mechanisms under the "quality–reputation–performance" symbiosis; once the complaint intensity deviated from this threshold, the system begin to deviate from collaboration. (2) From the perspective of quality complaints, the theoretical condition for maintaining an upward trajectory of system reputation was that complaint intensity remained relatively low in the absence of government penalties. After introducing government penalties, the external conditions for achieving Pareto optimality in the reputation trajectory were either that system traffic was weak with complaint intensity at a low or high level, or that system traffic was strong with complaint intensity at a moderate threshold. (3) Collaborative decision-making always yielded higher total system profit than decentralized decision-making, and enabled the expected value and variance of the reputation trajectory to achieve Pareto optimality while slowing its declining trend, subject to the condition that the platform's revenue coefficient (take rate) varies in the same direction as its reputation-restoration cost coefficient; with stronger synchronicity producing greater slowing effects.

Keywords: stochastic differential game; collaborative governance; quality–reputation risks; Q-learning multi-agent collaborative response model; "quality-reputation- performance" symbiosis
Mathematics Subject Classification: 91A15, 91A80, 91B06

1. Introduction

In recent years, the livestream e-commerce industry has experienced rapid growth; however, product quality issues have also emerged frequently. Incidents involving counterfeit or substandard products sold by popular streamers have occurred repeatedly (e.g., typical cases such as the sale of "Meicai Braised Pork" by "Crazy Xiao Yangge" and "sweet potato vermicelli" by "Northeast Yujie"), severely undermining consumer rights and market trust. Correspondingly, behind the frequent occurrence of product quality incidents in livestreaming, challenges such as difficulties in returns and exchanges and high rates of consumer quality complaints have become increasingly prominent. According to statistics from the State Administration for Market Regulation, in 2024, market regulatory authorities nationwide received a total of 402,000 complaints and reports related to livestream sales, representing a year-on-year increase of 19.3%[1]. High return rates and frequent quality disputes not only increase social transaction costs but also reveal a latent "low-quality" crisis underlying the "high-growth" of the livestream e-commerce industry, thereby significantly triggering systemic reputational risks within the sector. However, the governance and remediation of such systemic reputational risks induced by quality complaints cannot be addressed by a single agent alone; rather, they are the result of the joint actions of multiple stakeholders, including manufacturers, streamers, livestreaming platforms, and government regulatory authorities. For instance, manufacturers' fulfillment of responsibilities and after-sales responses, streamers' communication and trust repair, platforms' complaint coordination and rule enforcement, and government authorities' complaint handling and supervision all play indispensable roles in addressing reputational risks triggered by consumer quality complaints. The absence or improper response of any single party may amplify overall risks through reputational spillover effects, leading to systemic consequences. Therefore, in response to systemic reputational risks in livestreaming triggered by consumer quality complaints, how to break the governance boundaries among stakeholders and construct a multi-agent collaborative governance mechanism, led by regulatory authorities and involving "manufacturers–streamers–livestreaming platforms–regulatory authorities", has become an urgent and important practical issue. On this basis, characterizing the collaborative behavioral logic and response pathways of multiple agents in the post-event governance stage is of significant theoretical

[1] https://news.qq.com/rain/a/20250415A07P5L00.

and practical value for developing multi-agent collaborative governance strategies for livestreaming reputational risk.

Relevant research mainly unfolds from the following aspects: First, research on the governance of livestream product quality problems. Aiming at product quality problems in livestream e-commerce, scholars mainly use evolutionary game theory to explore governance strategies and supervision paths from the perspective of multi-agent interaction. Luo and Xie [1] constructed a multi-participant supervision game model among the government, the public and livestream platforms, and pointed out that the government's one-sided punishment intensity is not sufficient to affect the platform's final decision, while public participation has a significant impact on platform supervision. Hu et al. [2] focused on the livestream supply chain with multiple participants such as platforms, operators, and consumers, and proposed that the absence of government supervision is likely to lead to difficulties in consumers' rights protection, and strengthening in-process supervision and post-event punishment intensity helps to improve the integrity level of operators. He et al. [3] also adopted the evolutionary game method to construct a livestream supply chain model with three participants: Brand owners, MCN institutions, and platforms, and found that the return rate is a key factor promoting the evolution of the system to an ideal equilibrium. In terms of livestream reputation governance, Xu et al. [4] designed a multimodal analysis framework based on streamers' reputation for product sales prediction in livestream e-commerce, and innovatively incorporated historical reputation and real-time reputation signals to deeply explore the impact of streamers' reputation on product sales volume. Yang and Yin [5] explored the collaborative governance behavior between government departments and social platforms during rumor spread from the perspective of cooperative game. Fan and Guo [6] aimed at the governance dilemma of livestream platforms under the threat of credibility crisis, constructed a tripartite differential game model, including the government, product suppliers, and livestream platforms, and explored the collaborative governance strategy of livestream reputation under multi-agent collaboration.

Second, research on multi-agent collaborative governance mainly unfolds from four dimensions: conceptual definition, application fields, participation mechanisms, and research methods. In terms of conceptual definition, multi-agent collaborative governance is regarded as a governance model evolving from a single agent to multiple agents, which realizes information sharing, resource integration, and synergy effects through cooperative forms such as partnership and intergovernmental agreements (see [7,8]). In terms of application, research on multi-agent collaborative governance has been extended to many public affairs fields. For example, in the field of food safety, Yang et al. [9] constructed an evolutionary game model with multiple participants such as the government, enterprises, and consumers, and found that introducing social forces and strengthening supervision can effectively regulate the behavior of food enterprises. In the field of internet governance, Chen [10] emphasized the need for the joint participation of national guidance, platform technology improvement and users' safety awareness. In the field of e-commerce quality risk control, Huang et al. [11] constructed a tripartite game model, including fresh e-commerce platforms, logistics enterprises, and supervision institutions, introduced a dynamic supervision mechanism on the basis of the traditional bilateral game model, and proposed a collaborative governance strategy for quality risk management in the fresh e-commerce ecosystem. In the field of water resource management, Yuan et al. [12] developed a differential game model of trans-jurisdictional water pollution governance involving upstream governments, downstream governments, and sewage enterprises, and proposed cooperation strategies as well as reward and punishment mechanisms, which provide a solid theoretical reference for the construction of collaborative decision-making and penalty mechanisms in this study.

In terms of participation mechanisms, scholars not only affirm the positive significance of multi-agent participation, but also emphasize that the government should assume the core guiding role. Lu and He [13,14] pointed out that although multi-agent participation is necessary, it is not advisable to blindly pursue the participation of industry associations and the public; rather, the government should guide all parties and coordinate the relations between agents. Wang et al. [15] showed through research on the government, enterprises, social organizations, and investment and financing institutions that linkage incentives and strengthened implementation can improve the degree of collaborative participation. In addition to government guidance, it is necessary to optimize the multi-agent supervision structure to strengthen participation and control risks [16]. Beyond these mechanisms, Yuan et al. [17] built an allocation mechanism combining Bankruptcy Theory with Bargaining and Concession games (BCBGM) to ensure multi-agent participation, multi-stage negotiation, and optimal allocation.

In terms of research methods, evolutionary game models are widely adopted. Scholars analyze the strategic evolution of multiple agents by introducing relevant factors: Wang et al. [18] constructed a tripartite evolutionary game model to analyze the interaction between enterprises, the public, and the government; Song et al. [19] used a stochastic evolutionary game to analyze the strategies of industry-university-research alliances; Xu and Yang [20] considered the uncertainty of the external environment and established a stochastic game model to study collaborative governance in environmental supervision. Yang et al. [21] constructed a tripartite game model involving producers, sales and operation enterprises, and local governments, considering the impact of the evolutionary stability of complex system behavior on the collaborative promotion of brand building and the strategic choices of all parties. Furthermore, Yuan et al. [22] combined evolutionary game theory with system dynamics to simulate the dynamic evolution of strategic decision-making in transboundary resource sharing problems, providing a methodological reference for analyzing how stakeholder strategies evolve over time under different initial conditions and payoff structures.

In summary, the above literature provides a solid theoretical foundation for this study; however, several limitations remain: (1) Researchers have mainly focused on single governance perspectives, such as livestream product quality or livestreaming reputation, and have rarely revealed the symbiotic relationship among livestream product quality, system reputation, and system profit. However, the chain reaction of reputational risks in livestream systems triggered by product quality issues can further affect total system profit, which in turn influences the quality investment decisions of all participants. These three aspects are highly interdependent, requiring coordinated participation from all stakeholders, as well as guidance from regulatory authorities. (2) Studies on multi-agent co-governance have primarily entailed the mechanisms of collaboration from the perspective of equal participation among agents, with relatively limited attention to the role of government (or regulatory authorities) as a leading agent. In particular, insufficient analysis has been conducted on the elastic effects and external constraints of governance strategies and collaborative mechanisms following the introduction of government regulation. (3) Game-theoretic studies on livestream governance predominantly employ static or evolutionary game approaches, which cannot capture the random and dynamic nature of reputation evolution influenced by unpredictable factors such as consumer sentiment and public opinion. Stochastic differential game methods are better suited to modeling such uncertain and dynamic processes; yet, such approaches have seldom been applied to livestream reputation governance. Furthermore, the integration of analytical stochastic differential game solutions with adaptive simulation approaches enables the simulation of reputation dynamics under various shock scenarios, making the evolution of system reputation more intuitive and significantly enhancing the practical applicability of theoretical findings; an aspect that has received little attention in previous livestream quality or reputation studies. The comparison of related studies is shown in Table 1:

Table 1. Comparison between this study and the literature.

Dimension	Representative Studies	Existing Literature	This Study
Research Focus	[1], [2], [3], [4], [5], [6]	Focuses separately on livestream product quality governance, livestream reputation prediction, rumor governance	Unifies quality, reputation, and performance into a single "quality–reputation–performance" symbiosis framework, revealing their interdependencies and co-evolutionary dynamics
Stakeholder Coverage	[1], [2], [3], [6], [12]	Typically involves two or three stakeholders: government-public-platform, platform-operator-consumer, brand owner-MCN-platform, government-supplier-platform, or upstream/downstream governments and enterprises	Involves four distinct stakeholders: manufacturers, streamers, livestreaming platforms, and regulatory authorities, with government explicitly modeled as a leading agent
Theoretical Approach	[1], [2], [3], [9], [11], [12], [17], [22], [4]	Primarily adopts evolutionary game theory, or differential game, evolutionary game with system dynamics, or bankruptcy -concession -bargaining game model, or a multimodal analysis framework	Integrates stochastic differential game theory with Q-learning-based adaptive multi-agent simulation, combining analytical equilibrium solutions with computational reinforcement learning
Governance Mechanisms	[1], [3], [4], [12]	Focuses on punishment intensity, return rates, streamer reputation signals, or cost-sharing and reward-punishment mechanisms	Incorporates government penalty mechanisms with effort-based penalty reduction structures, and dynamic event-triggered communication among agents
Dynamic Analysis	[1], [2], [3]	Mostly employs static equilibrium analysis or evolutionary game simulations, which do not capture the continuous and stochastic evolution of system reputation over time	Analyzes expected value, variance, and trend of reputation trajectories; examines Pareto optimality conditions of collaborative governance; integrates Q-learning for adaptive dynamic simulation

Accordingly, the contributions of this study are as follows: (1) A four-dimensional collaborative stochastic differential game model is constructed, driven by consumer quality complaints and involving "manufacturers–streamers–livestreaming platforms–regulatory authorities", which captures the dynamic and stochastic nature of reputation evolution and derives explicit equilibrium conditions for collaborative governance; (2) an innovative Q-learning-based adaptive multi-agent collaborative response simulation model is developed under consumer quality complaint-triggered livestream scenarios, enabling the simulation of dynamic reputation adjustment processes and bridging the gap between theoretical analysis and operational strategies; (3) from the novel perspective of "quality–reputation–performance" symbiosis, we explore collaborative governance strategies for livestream system reputational risk, analyze the applicability regions of multi-agent collaboration, and further examine multi-agent decision-making and the evolution of system reputation differential trajectories, along with their Pareto conditions. Therefore, this study provides important theoretical value and practical implications for improving the governance effectiveness of consumer quality complaints, optimizing livestream system reputation, and promoting multi-agent collaborative governance.

2. Model

In this study, we consider constructing a government supervision department-led four-dimensional collaborative governance model of "manufacturer-streamer-livestream platform-regulatory authority", focusing on post-governance of consumers' product quality complaints in livestream scenarios to avoid or reduce systemic reputational risks. The selection of these four stakeholders is grounded in the institutional reality of China's livestream e-commerce governance. Documented practices show that regulatory authorities play a leading role in issuing guidelines and imposing penalties; for example, in 2024, the State Administration for Market Regulation processed over 400,000 livestream-related complaints and imposed administrative penalties on non-compliant agents, with the Chengdu Kuaigou Technology case resulting in a penalty of 26.6929 million yuan[2]; livestreaming platforms (e.g., Douyin, Kuaishou) coordinate complaint handling and rule enforcement, with platforms now required by the newly implemented "Supervision and Management Measures for Live e-commerce" to establish complaint mechanisms in prominent locations on live pages and provide convenient channels for consumer complaints[3,4]; streamers serve as the direct interface for consumer trust repair, yet frequently face reputation crises—in the "Northeast Yujie" sweet potato vermicelli incident, the streamer was fined 1.65 million yuan and ordered to suspend operations for making false or misleading commercial representations[5,6]; and manufacturers bear the ultimate responsibility for product quality and recall actions: In the "Meicai Braised Pork" incident, the manufacturer was required to issue public apologies, initiate product recalls, and provide consumer compensation[7]. These four parties consistently appear in documented governance practices following major quality incidents. The four-dimensional collaborative governance system led by the regulatory authorities is shown in Figure 1.

[2] https://www.sohu.com/a/982035444_362042?scm=10001.1554_13-100000-0_8000.0-0.0-1-0-0-0.0_1020.
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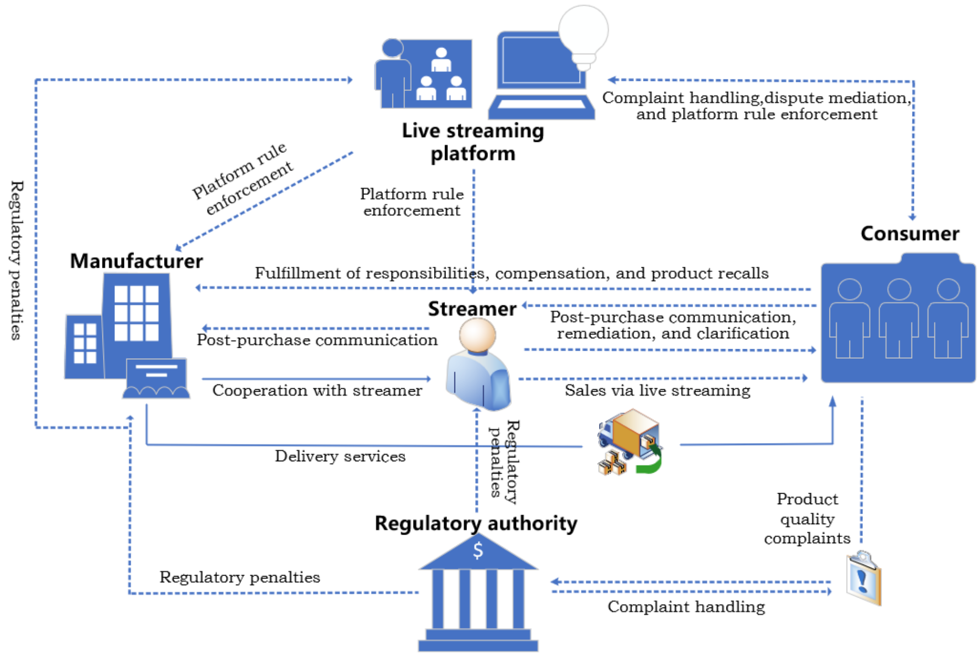

Figure 1. Collaborative governance model of livestream e-commerce based on consumer quality complaints.

In the above four-dimensional collaborative governance model, the supervision department leads all participants to actively handle consumers' quality complaints, thus inhibiting the spread of individual complaints to systemic reputational risks. The relevant basic assumptions are as follows:

Assumption 1: In the collaborative governance process, the manufacturer's reputation repair effort is B_m, realized through product liability fulfillment, compensation, and recall; the streamer's effort is B_c, realized through after-sales communication, corrective explanation, and risk clarification; the platform's effort is B_p, realized through complaint acceptance, dispute coordination, and rule enforcement. The regulatory authority dynamically intervenes according to complaint intensity and imposes penalties on non-compliant behaviors, forming external constraints that promote collaborative investment among all agents in the post-governance stage.

Assumption 2: The change process of the livestreaming system reputation under consumer quality complaints conforms to the Itô process, as follows:

$$dX(t)=\left(B_m+B_c+B_p-\Theta RX(t)-\xi X(t)\right)dt+\sigma\left(X(t)\right)dz(t) \tag{1}$$

Among them, $X(t)$ represents the livestream system reputation level at time t, and the initial system reputation level is $X(0)=X_0\geq 0$; Θ is the intensity of consumers' quality complaints, and R is the system traffic level. When traffic volume is higher and the intensity of consumers' quality complaints is more severe, these two factors generate a 'reverse multiplication effect', which will cause great damage to the reputation of the livestream system. The reputation repair efforts of each agent B_m, B_c, and B_p, reflecting the reality that manufacturers, streamers, and platforms jointly contribute to reputation restoration from different dimensions, through product recalls, public apologies, and complaint coordination, and that multi-party collaborative efforts can effectively offset the reputational damage caused by quality complaints and slow the declining trend of system reputation. ξ represents the natural attenuation coefficient of reputation, $\sigma(X(t))$ represents the random interference coefficient, $z(t)$ is a one-dimensional standard Brownian motion, and the random interference coefficient is proportional to the square root of the livestream system reputation, that is, $\sigma(X(t))dz(t) = \sigma\sqrt{X}dz(t)$. Additionally, $\sigma(X(t))dz(t)$ reflects the inherent uncertainty in consumer sentiment and public opinion dynamics. Furthermore, the dynamic change process of livestream system reputation can be rewritten as:

$$dX(t) = \left(B_m + B_c + B_p - \Theta RX(t) - \xi X(t)\right)dt + \sigma\sqrt{X}dz(t) \tag{2}$$

The Itô process is adopted because livestream system reputation evolves through a combination of predictable forces, such as stakeholders' repair efforts, complaint intensity, and natural decay, and unpredictable shocks from consumer sentiment and public opinion. In practice, consumer quality complaints typically follow an outbreak-peak-subsidence pattern, as observed in the "Northeast Yujie" and "Meicai Braised Pork" incidents, while the diffusion term captures the inherent randomness of viral social media events and shifting consumer perceptions, making the Itô process a natural and well-established framework for modeling such dynamics (see [6,23]).

Assumption 3: In the livestream system, its total revenue is closely related to the livestream system reputation and livestream traffic, and the livestream traffic and the livestream system reputation produce a "positive multiplication effect". Therefore, it is assumed that the total system revenue is:

$$W = RX \tag{3}$$

The linear specification $W = RX$ draws on the standard Nerlove-Arrow goodwill framework, which models the dynamic evolution of reputation or goodwill as a state variable influenced by investment and decay [24]. In the livestream e-commerce context, this formulation aligns with the revenue structure adopted in studies, where total system revenue is expressed as a linear function of streamer traffic and product goodwill [6]. This reflects the empirical reality that revenue is generated only when traffic and trust are present; traffic without conversion yields low revenue, while trust without scale limits market reach.

Furthermore, in the aforementioned equations, the traffic level R plays a dual role: On the one hand, as shown in Eq (1), traffic amplifies the negative reputational impact of consumer quality complaints through the "reverse multiplication effect", meaning that higher traffic leads to more rapid and widespread reputation damage during incidents; on the other hand, as shown in Eq (3), traffic also amplifies the positive effect of reputation recovery through the "positive multiplication effect", meaning that restored reputation generates greater revenue returns in high-traffic scenarios. In reality, this dual amplification effect is clearly observed in livestream e-commerce; during the "Meicai Braised Pork" incident, affected livestreaming rooms with high viewership experienced sharper reputation declines and revenue losses, while streamers who successfully rebuilt consumer trust subsequently benefited from faster sales recovery due to their large audience base. The above equation for the change process of livestream system reputation and the system revenue function fully reflect the mathematical relationship of "quality-reputation-performance" symbiosis in the livestream system.

Assumption 4: Referring to general assumptions [25,26], in the process of livestream system revenue distribution, the streamer will charge a commission of α proportion based on the total livestream revenue, and the livestream platform will charge a platform service fee of β proportion on the basis of the streamer's commission. In practice, platforms such as Douyin and Kuaishou typically charge take rates ranging from 5% to 20%, while streamers' commission rates vary according to their popularity and contractual arrangements, directly affecting the revenue shares of all stakeholders.

Assumption 5: Each participant will incur corresponding repair costs in the process of system reputation repair. Assuming that the repair cost is positively correlated with the repair effort cost of each agent, and drawing on the convex characteristic of general costs [27], the cost function expressions of manufacturers, streamers, and livestream platforms are respectively:

$$C_m(t)=\frac{1}{2}\mu_m B_m^{\ 2},\quad C_c(t)=\frac{1}{2}\mu_c B_c^{\ 2},\quad C_p(t)=\frac{1}{2}\mu_p B_p^{\ 2} \tag{4}$$

In practice, the convex quadratic form captures the reality that reputation repair requires substantial upfront investment to establish mechanisms and protocols, while marginal costs gradually decrease as procedures mature and experience accumulates.

Assumption 6: The regulatory authority imposes penalties at level $(\omega_{i1}-\varphi B_i)\Theta$ on all relevant agents according to the intensity of consumer quality complaints, where ω_{i1} denotes the initial responsibility that the regulatory authority assigns to agent $i=m,c,p$ for the complaint incident, and φB_i is the degree to which the authority reduces penalties based on each agent's reputation repair effort. The penalty intensity is treated as an exogenous variable in this study.

Therefore, based on the aforementioned basic assumptions, the revenue function of each agent can be obtained as:

$$\max_{B_m(t)}\Pi_m=\int_0^\infty e^{-\rho t}\left[(1-\alpha)RX-(\omega_{m1}-\varphi B_m)\Theta-\frac{1}{2}\mu_m B_m^{\ 2}\right]dt \tag{5}$$

$$\max_{B_c(t)}\Pi_c=\int_0^\infty e^{-\rho t}\left[(1-\beta)\alpha RX-(\omega_{c1}-\varphi B_c)\Theta-\frac{1}{2}\mu_c B_c^{\ 2}\right]dt \tag{6}$$

$$\max_{B_p(t)}\Pi_p=\int_0^\infty e^{-\rho t}\left[\beta\alpha RX-(\omega_{p1}-\varphi B_p)\Theta-\frac{1}{2}\mu_c B_c^{\ 2}\right]dt \tag{7}$$

where ρ denotes the discount factor, which is assumed to be constant [28]. For notational convenience, the time index t is omitted in the subsequent analysis. The parameters and their meanings are shown in Table 2.

Table 2. List of parameters and their meanings.

Parameter	Meaning	practical significance
$X(t)$	Livestream system reputation state at time t	Represents the overall trust and credibility level of the livestream ecosystem at a given time. In practice, this can be proxied by consumer satisfaction indices, platform credit scores, or brand equity metrics.
B_m	Manufacturer's reputation repair effort due to systemic reputational risks caused by consumers' quality complaints	Refers to the intensity of manufacturer's post-incident actions such as product recalls, quality rectifications, and consumer compensation. For example, in the "Meicai Braised Pork" incident, the manufacturer's recall scope and compensation scale directly reflected this effort level.

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Parameter	Meaning	practical significance
B_c	Streamer's reputation repair effort due to systemic reputational risks caused by consumers' quality complaints	Refers to the streamer's post-incident remedial actions, including public apologies, live-streamed explanations, and after-sales coordination. In the "Northeast Yujie" sweet potato vermicelli incident, the streamer's apology and subsequent communication efforts exemplified this parameter.
B_p	Livestream platform's reputation repair effort due to systemic reputational risks caused by consumers' quality complaints	Refers to the platform's actions in coordinating complaint resolution, enforcing rules, and mediating disputes. Platforms such as Douyin and Kuaishou have dedicated teams that activate complaint-handling protocols following major quality incidents, reflecting this effort dimension.
Θ	Intensity of consumer complaints about quality problems	Captures the severity of consumer dissatisfaction, typically measured by complaint volume, negative reviews, and social media sentiment.
R	Livestream system traffic level, including private domain traffic such as manufacturer's brand effect and streamer's influence, and public domain traffic allocated by livestream platforms	Encompasses both private-domain traffic (e.g., manufacturer brand equity, streamer influence) and public-domain traffic allocated by platforms.
ξ	Natural attenuation coefficient of system reputation	Reflects the inherent tendency of reputation to decay over time in the absence of active maintenance. In practice, even without new negative incidents, consumer trust naturally erodes due to competing information and changing preferences.
$\sigma(G(t))$	Random interference coefficient	Captures the volatility and unpredictability of public opinion and consumer sentiment.
$z(t)$	One-dimensional standard Brownian motion	Represents the random shocks to system reputation, including unforeseen public opinion swings, viral social media events, and other stochastic disturbances affecting consumer perception.
Π_m	Manufacturer's revenue	The proportion of total system revenue allocated to the manufacturer after commission and platform fees.
Π_c	Streamer's revenue	The streamer's share of total system revenue after platform fees.
Π_p	Livestream platform's revenue	The revenue the platform earns by charging a service fee based on the streamer's commission.

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Parameter	Meaning	practical significance
α	Streamer's commission rate	The percentage of total revenue paid to the streamer. In practice, this varies based on the streamer's popularity and contractual arrangements. For example, top-tier streamers may command commission rates of 20-30%, while emerging streamers receive lower rates.
β	Livestream platform's take rate	The platform service fee charged on the basis of the streamer's commission. In practice, platforms such as Douyin and Kuaishou typically charge take rates ranging from 5% to 20%.
ω_{i1}	$i \in \{m,c,p\}$ denotes the initial responsibility that the regulatory authority assigns to agent i for the systemic reputational risk triggered by consumer quality complaints	The baseline liability percentage assigned to each agent based on their role and culpability in the quality complaint incident. In practice, this is determined by regulatory investigation and enforcement guidelines
φ	φ denotes the extent to which agent i's reputation repair efforts reduce the initial responsibility it bears	The degree to which proactive repair actions mitigate penalties. In practice, this reflects regulatory discretion where voluntary compliance and remedial actions are considered mitigating factors, as observed in actual enforcement cases.
$C_i(t)$	$i \in \{m,c,p\}$ denotes the effort cost of each agent	Represents the actual cost incurred by each agent in undertaking reputation repair activities. This includes financial costs (e.g., recall expenses, compensation payouts) and non-financial costs (e.g., reputational damage, operational disruption).
μ_i	$i \in \{m,c,p\}$ denotes the effort cost coefficient of each agent	The marginal cost parameter in the convex cost function, reflecting the efficiency of each agent in converting effort into reputation repair outcomes.

3. Equilibrium results

3.1. Scenario without government penalties (N)

In this scenario, the model assumes an absence of government regulation and punishment, and self-governance is implemented within the livestream system, which indicates that the penalty function satisfies $(\omega_{i1} - \varphi B_i)\Theta = 0$. All parties make decisions to maximize their own interests. For the convenience of distinction, *N* is used to represent the scenario without government penalties.

3.1.1. Decentralized decision-making without government penalties (*NF*)

According to the Bellman continuous dynamic programming theory, Theorem 1-1 can be obtained.

Theorem 1-1. Under the decentralized decision-making scenario without government Penalties, for the system reputation risks caused by consumers' quality complaints, the optimal reputation repair efforts of manufacturers, streamers, and livestream platforms are respectively: $B_m^{NF}=\frac{(1-\alpha)R}{(R\Theta+\rho+\xi)\mu_m}$, $B_c^{NF}=\frac{(1-\beta)\alpha R}{(R\Theta+\rho+\xi)\mu_c}$, $B_p^{NF}=\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}$.

Proof. To find the equilibrium solution, the optimal value functions $V_m(X)$、$V_c(X)$, and $V_p(X)$ of manufacturers, streamers, and livestream platforms must be continuous and bounded, have first and second derivatives, and satisfy the Hamilton-Jacobi-Bellman (HJB) equation for any $X(t)\geq 0$, that is:

$$\begin{cases}\rho V_m=\max\limits_{B_m}\left\{(1-\alpha)RX-\frac{1}{2}\mu_m B_m^2+V_{mx1}\left(-R\Theta X-\xi X+B_c+B_m+B_p\right)+\frac{1}{2}\sigma^2 V_{mx2}\right\}\\ \rho V_c=\max\limits_{B_c}\left\{(1-\beta)\alpha RX-\frac{1}{2}\mu_c B_c^2+V_{cx1}\left(-R\Theta X-\xi X+B_c+B_m+B_p\right)+\frac{1}{2}\sigma^2 V_{cx2}\right\}\\ \rho V_p=\max\limits_{B_p}\left\{\beta\alpha RX-\frac{1}{2}\mu_p B_p^2+V_{px1}\left(-R\Theta X-\xi X+B_c+B_m+B_p\right)+\frac{1}{2}\sigma^2 V_{px2}\right\}\end{cases} \tag{a1}$$

where V_{mx1} denotes the first-order partial derivative of value function V_m with respect to X, V_{cx1} denotes the first-order partial derivative of value function V_c with respect to X, and V_{px1} denotes the first-order partial derivative of value function V_p with respect to X, which represents the marginal effect of growth in system reputation level on growth in payoffs of all parties. Taking the first partial derivatives of equation (a1) with respect to B_m、B_c, and B_p, respectively, and setting them to zero, we can obtain: $B_m=\frac{V_{mX1}}{\mu_m}$、$B_c=\frac{V_{cX1}}{\mu_c}$、$B_p=\frac{V_{pX1}}{\mu_p}$. Substituting the above results into equation (a1) and simplifying and sorting out, we can obtain:

$$\rho V_m(X)=\left[(1-\alpha)R+V_{mx1}(-R\Theta-\xi)\right]X-\frac{V_{mx1}^2}{2\mu_m}+V_{mx1}\left(\frac{V_{cx1}}{\mu_c}+\frac{V_{mx1}}{\mu_m}+\frac{V_{px1}}{\mu_p}\right)+\frac{1}{2}\sigma^2 V_{mx2} \tag{a2}$$

$$\rho V_c(X)=\left[(1-\beta)\alpha R+V_{cx1}(-R\Theta-\xi)\right]X-\frac{V_{cx1}^2}{2\mu_c}+V_{cX1}\left(\frac{V_{cX1}}{\mu_c}+\frac{V_{mX1}}{\mu_m}+\frac{V_{pX1}}{\mu_p}\right)+\frac{1}{2}\sigma^2 V_{cX2} \tag{a3}$$

$$\rho V_p(X)=\left[\beta\alpha R+V_{px1}(-R\Theta-\xi)\right]X-\frac{V_{pX1}^2}{2\mu_p}+V_{pX1}\left(\frac{V_{cX1}}{\mu_c}+\frac{V_{mX1}}{\mu_m}+\frac{V_{pX1}}{\mu_p}\right)+\frac{1}{2}\sigma^2 V_{pX2} \tag{a4}$$

From the structural characteristics of the above Eqs (a2)–(a4), it can be seen that the linear optimal revenue function about $X(t)$ is the solution of the *HJB* equation. Let:

$$V_m(X)=J_{mx1}X+J_{mx2}\,,\quad V_c(X)=J_{cx1}X+J_{cx2}\,,\quad V_m(X)=J_{px1}X+J_{px2} \tag{a5}$$

where the above coefficients J_{ix1} and J_{ix2}（$i\in m,c,p$）are all constants, which are calculated by the method of undetermined coefficients:

$$J_{mx1}=\frac{(1-\alpha)R}{R\Theta+\rho+\xi},\quad J_{cx1}=\frac{(1-\beta)\alpha R}{R\Theta+\rho+\xi},\quad J_{px1}=\frac{\beta\alpha R}{R\Theta+\rho+\xi} \tag{a6}$$

$$J_{mx2}=\frac{-\frac{1}{2}\frac{(-1+\alpha)^2R^2}{(R\Theta+\rho+\xi)^2\mu_m}-\frac{(-1+\alpha)R\left(\frac{V_{cX1}}{\mu_c}-\frac{(-1+\alpha)R}{(R\Theta+\rho+\xi)\mu_m}+\frac{V_{pX1}}{\mu_p}\right)}{R\Theta+\rho+\xi}}{\rho} \tag{a7}$$

$$J_{cx2}=\frac{-\frac{1}{2}\frac{(\beta-1)^2\alpha^2R^2}{(R\Theta+\rho+\xi)^2\mu_c}-\frac{(\beta-1)\alpha R\left(-\frac{(\beta-1)\alpha R}{(R\Theta+\rho+\xi)\mu_c}+\frac{V_{mx1}}{\mu_m}+\frac{V_{px1}}{\mu_p}\right)}{R\Theta+\rho+\xi}}{\rho} \tag{a8}$$

$$J_{px2}=\frac{-\frac{1}{2}\frac{\beta^2\alpha^2R^2}{(R\Theta+\rho+\xi)^2\mu_p}+\frac{\beta\alpha R\left(\frac{J_{cx1}}{\mu_c}+\frac{V_{mx1}}{\mu_m}+\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}\right)}{R\Theta+\rho+\xi}}{\rho} \tag{a9}$$

Substituting (a6) into the relevant equilibrium decisions, the equilibrium strategies of each livestream agent under the decentralized decision-making scenario without government penalties can be obtained.

Since the dynamic change process of livestream system reputation is affected by random factors, it is necessary to explore whether the stochastic evolution characteristics of livestream system reputation can be grasped and what evolutionary characteristics it has. In this section, we will continue to discuss the expectation, variance, and stability of livestream system reputation under consumer quality complaints in the decentralized decision-making scenario without government penalties.

Substituting the equilibrium strategies of manufacturers, streamers, and livestream platforms in Theorem 1-1 into Eq (1), we can obtain:

$$\begin{aligned}
&dX(t)=\left(\frac{(1-\alpha)R}{(R\Theta+\rho+\xi)\mu_m}+\frac{(1-\beta)\alpha R}{(R\Theta+\rho+\xi)\mu_c}+\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}-(\Theta R+\xi)X(t)\right)dt+\sigma\sqrt{X}dz(t)\\
&X(0)=X_0>0
\end{aligned} \tag{8}$$

where letting $\Omega^{NF}=\frac{(1-\alpha)R}{(R\Theta+\rho+\xi)\mu_m}+\frac{(1-\beta)\alpha R}{(R\Theta+\rho+\xi)\mu_c}+\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}$, $\delta=\Theta R+\xi$, the expectation and variance of the dynamic trajectory for reputation in the livestream system can be further derived as follows:

Theorem 1-2. Under the decentralized decision-making scenario without government penalties, for consumer quality complaints:

i) The expected value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively:

$$EX(t)=\left(X_0-\frac{\frac{\left(1-\alpha\right)R}{(R\Theta+\rho+\xi)\mu_m}-\frac{(\beta-1)\alpha R}{(R\Theta+\rho+\xi)\mu_c}+\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}}{(\Theta R+\xi)}\right)e^{-(\Theta R+\xi)t}$$

$$+\frac{\frac{(1-\alpha)R}{(R\Theta+\rho+\xi)\mu_m}-\frac{(\beta-1)\alpha R}{(R\Theta+\rho+\xi)\mu_c}+\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}}{(\Theta R+\xi)}$$

$$\lim_{t\to\infty}EX(t)=\frac{\Omega^{NF}}{\delta}=\frac{-\frac{(-1+\alpha)R}{(R\Theta+\rho+\xi)\mu_m}-\frac{(\beta-1)\alpha R}{(R\Theta+\rho+\xi)\mu_c}+\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}}{(\Theta R+\xi)}$$

ii) The variance value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively:

$$DX(t)=\frac{\sigma^2\left[\Omega^{NF}-2\left(\Omega^{NF}-\delta X_0\right)\mathrm{e}^{-\delta t}+\left(\Omega^{NF}-2\delta X_0\right)\mathrm{e}^{-2\delta t}\right]}{2\delta^2}$$

$$\lim_{t\to\infty}DX(t)=\frac{\sigma^2\Omega^{NF}}{2\delta^2}=\frac{\sigma^2\left(-\frac{(-1+\alpha)R}{(R\Theta+\rho+\xi)\mu_m}-\frac{(\beta-1)\alpha R}{(R\Theta+\rho+\xi)\mu_c}+\frac{\beta\alpha R}{(R\Theta+\rho+\xi)\mu_p}\right)}{2(\Theta R+\xi)^2}$$

Proof. Rewrite Eq (8) into the stochastic integral form:

$$X(t)=X_0+\int_0^t\left(\Omega^{NF}-\delta X(s)\right)\mathrm{d}s+\int_0^t\sigma(\sqrt{X(s)})\mathrm{d}z(s) \tag{b1}$$

Taking the expectation of Eq (b1) and using the zero expectation property of the Wiener process, we can obtain:

$$E[X(t)]=X_0+\int_0^t\left[\Omega^{NF}-\delta E\left(X\left(s\right)\right)\right]\mathrm{d}s \tag{b2}$$

Taking the derivative of equation (b2) further, and $E[X(0)]=X_0$, we can obtain:

$$E[X(t)]=\mathrm{e}^{-\delta t}\left(X_0-\frac{\Omega^{NF}}{\delta}\right)+\frac{\Omega^{NF}}{\delta} \tag{b3}$$

Taking the limit of Eq (b3), we can obtain $\lim_{t\to\infty}E[G(t)]=\frac{\Omega^{NF}}{\delta}$.

To find the variance, it is only necessary to obtain $E[X^2(t)]$. Applying the Ito formula to Eq (8), we can obtain:

$$dX^2(t)=\left[2X\left(\Omega^{NF}-\delta X\right)+\sigma^2X\right]\mathrm{d}t+2X\sigma\sqrt{X}\mathrm{d}z\left(s\right) \tag{b4}$$

Rewrite Eq (b4) into the stochastic integral form:

$$X^2(t)=X_0^2+\int_0^t\left[2X(s)\left(\Omega^{NF}-\delta X(s)\right)+\sigma^2X(s)\right]\mathrm{d}s+\int_0^t2X(s)\sigma\sqrt{X}\mathrm{d}z\left(s\right) \tag{b5}$$

Taking the expectation of Eq (b5), we can obtain:

$$E\left[X^2(t)\right] = X_0^2 + \int_0^t \left[2E[X(s)]\left(\Omega^{NF} - \delta E[X(s)]\right) + \sigma^2 E[X(s)]\right]\mathrm{d}s \tag{b6}$$

Taking the derivative of Eq (b6) further, and $E[X^2(0)] = X_0^2$, we can obtain:

$$\mathrm{d}E\left[X^2(t)\right] = \left(2\Omega^{NF} + \sigma^2\right)E[X(t)] - 2\delta E\left[X^2(t)\right] \tag{b7}$$

Solving Eq (b7), we can obtain:

$$E\left[X^2(t)\right] = X_0^2 \mathrm{e}^{-2\delta t} + \frac{\left(2\Omega^{NF} + \sigma^2\right)\left(\Omega^{NF}\mathrm{e}^{2\delta t} + 2\delta X_0 \mathrm{e}^{\delta t} - 2\Omega^{NF}\mathrm{e}^{\delta t} - 2\delta X_0 + \Omega^{NF}\right)}{2\delta^2 \mathrm{e}^{2\delta t}} \tag{b8}$$

Taking the limit of Eq (b8), we can obtain $\lim\limits_{t\to\infty} E\left[X^2(t)\right] = \dfrac{\Omega^{NF}\left(2\Omega^{NF} + \sigma^2\right)}{2\delta^2}$

From $D[X(t)] = E\left[X^2(t)\right] - \left(E\left(X(t)\right)\right)^2$, we can obtain:

$$D[X(t)] = \frac{\sigma^2\left[\Omega^{NF} - 2\left(\Omega^{NF} - \delta X_0\right)\mathrm{e}^{-\delta t} + \left(\Omega^{NF} - 2\delta X_0\right)\mathrm{e}^{-2\delta t}\right]}{2\delta^2}$$

Furthermore, taking the limit of it, we can obtain $\lim\limits_{t\to\infty} D[X(t)] = \dfrac{\sigma^2\Omega^{NF}}{2\delta^2}$.

3.1.2. Collaborative decision-making without government penalties (*NC*)

Theorem 2-1. Under the collaborative decision-making scenario without government penalties, for the system reputation risks caused by consumers' quality complaints, the optimal reputation repair efforts of manufacturers, streamers, and livestream platforms are respectively:

$$B_m^{NC} = \frac{R}{\left(R\Theta + \rho + \xi\right)\mu_m},\quad B_c^{NC} = \frac{R}{\left(R\Theta + \rho + \xi\right)\mu_c},\quad B_p^{NC} = \frac{R}{\left(R\Theta + \rho + \xi\right)\mu_p}.$$

Proof. Similarly, to find the equilibrium solution, the overall optimal value function $V_z\left(X\right)$ of the livestream system must be continuous and bounded, have first and second derivatives, and satisfy the *Hamilton-Jacobi-Bellman (HJB)* equation for any $X\left(t\right) \geq 0$, that is:

$$\rho V_z = XR - \frac{1}{2}\mu_m B_m^2 - \frac{1}{2}\mu_c B_c^2 - \frac{1}{2}\mu_p B_p^2 + V_{zx1}\left(-R\Theta X - X\xi + B_c + B_m + B_p\right) + \frac{1}{2}\sigma^2 V_{zx2} \tag{c1}$$

where V_{zx1} denotes the first-order partial derivative of value function V_z with respect to X, and V_{zx2} denotes the second-order partial derivative of value function V_z with respect to X. Taking the first-order partial derivatives of Eq (c1) with respect to B_m、B_c, and B_p, respectively, and setting them equal to zero yields the following: $B_m = \dfrac{V_{zx1}}{\mu_m}$, $B_c = \dfrac{V_{zx1}}{\mu_c}$, and $B_p = \dfrac{V_{zx1}}{\mu_p}$. Substituting the above results into Eq (c1) and simplifying and sorting out, we can obtain:

$$\rho V_z = \left(R - V_{zx1}\left(R\Theta + \xi\right)\right)X - \frac{V_{zx1}^2}{2\mu_m} - \frac{V_{zx1}^2}{2\mu_c} - \frac{V_{zx1}^2}{2\mu_p} + V_{zx1}\left(\frac{V_{zx1}}{\mu_c} + \frac{V_{zx1}}{\mu_m} + \frac{V_{zx1}}{\mu_p}\right) + \frac{1}{2}\sigma^2 V_{zx2} \tag{c2}$$

Referring to the proof process of Theorem 1-1, it can be seen from the structural characteristics of the above Eq (c2) that its linear optimal revenue function about $X(t)$ is the solution of the *HJB* equation. Let $V_z(X)=J_{zx1}X+J_{zx2}$. Among them, the above coefficients J_{zx1} and J_{zx2} are all constants, which are calculated by the method of undetermined coefficients:

$$J_{z1}=\frac{R}{R\Theta+\rho+\xi} \tag{c3}$$

$$J_{z2}=\frac{-\frac{R^2}{2\left(R\Theta+\rho+\xi\right)^2\mu_m}-\frac{R^2}{2\left(R\Theta+\rho+\xi\right)^2\mu_c}-\frac{R^2}{2\left(R\Theta+\rho+\xi\right)^2\mu_p}}{\rho}+\frac{R\left(\frac{R}{(R\Theta+\rho+\xi)\mu_c}+\frac{R}{(R\Theta+\rho+\xi)\mu_m}+\frac{R}{(R\Theta+\rho+\xi)\mu_p}\right)}{\rho\left(R\Theta+\rho+\xi\right)} \tag{c4}$$

Substituting Eq (c3) into the relevant equilibrium decisions yields equilibrium strategies for all livestreaming agents under the scenario of collaborative decision-making without government penalties.

Substituting the equilibrium strategies of manufacturers, streamers, and livestream platforms in Theorem 2-1 into Eq (1), we can obtain:

$$dX(t)=\left(\frac{R}{(R\Theta+\rho+\xi)\mu_m}+\frac{R}{(R\Theta+\rho+\xi)\mu_c}+\frac{R}{(R\Theta+\rho+\xi)\mu_p}-(\Theta R+\xi)X(t)\right)dt+\sigma\sqrt{X}dz(t)\ ,$$

$$X(0)=X_0>0 \tag{9}$$

where, let $\Omega^{NC}=\frac{R}{(R\Theta+\rho+\xi)\mu_m}+\frac{R}{(R\Theta+\rho+\xi)\mu_c}+\frac{R}{(R\Theta+\rho+\xi)\mu_p}$, $\delta=\Theta R+\xi$.

Furthermore, the expectation and variance of the dynamic change trajectory of livestream system reputation can be obtained, as shown in Theorem 2-2 below:

Theorem 2-2. Under the collaborative decision-making scenario without government penalties, for consumer quality complaints:

i) The expected value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively:

$$EX(t)=\left(X_0-\frac{\frac{R}{(R\Theta+\rho+\xi)\mu_m}+\frac{R}{(R\Theta+\rho+\xi)\mu_c}+\frac{R}{(R\Theta+\rho+\xi)\mu_p}}{(\Theta R+\xi)}\right)e^{-(\Theta R+\xi)t}+\frac{\frac{R}{(R\Theta+\rho+\xi)\mu_m}+\frac{R}{(R\Theta+\rho+\xi)\mu_c}+\frac{R}{(R\Theta+\rho+\xi)\mu_p}}{(\Theta R+\xi)}$$

$$\lim_{t\to\infty}EX(t)=\frac{\Omega^{NC}}{\delta}=\frac{\frac{R}{(R\Theta+\rho+\xi)\mu_m}+\frac{R}{(R\Theta+\rho+\xi)\mu_c}+\frac{R}{(R\Theta+\rho+\xi)\mu_p}}{(\Theta R+\xi)}$$

ii) The variance value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively:

$$DX(t)=\frac{\sigma^2\left[\Omega^{NC}-2\left(\Omega^{NC}-\delta X_0\right)\mathrm{e}^{-\delta t}+\left(\Omega^{NC}-2\delta X_0\right)\mathrm{e}^{-2\delta t}\right]}{2\delta^2}$$

$$\lim_{t\to\infty}DX(t)=\frac{\sigma^2\Omega^{NC}}{2\delta^2}=\frac{\sigma^2\left(\dfrac{R}{(R\Theta+\rho+\xi)\mu_m}+\dfrac{R}{(R\Theta+\rho+\xi)\mu_c}+\dfrac{R}{(R\Theta+\rho+\xi)\mu_p}\right)}{2((\Theta R+\xi))^2}$$

Proof. Same as Theorem 1-2.

3.2. Scenario with government penalties (G)

3.2.1. Decentralized decision-making with government penalties (*GF*)

Theorem 3-1. Under the decentralized decision-making scenario with government penalties, for the system reputation risks caused by consumers' quality complaints, the optimal reputation repair efforts of manufacturers, streamers, and livestream platforms are respectively; $B_m^{GF}=\dfrac{1}{\mu_m}\left(\Theta\varphi-\dfrac{(-1+\alpha)R}{R\Theta+\rho+\xi}\right)$, $B_c^{GF}=\dfrac{1}{\mu_c}\left(\Theta\varphi-\dfrac{(\beta-1)\alpha R}{R\Theta+\rho+\xi}\right)$, $B_p^{GF}=\dfrac{1}{\mu_p}\left(\Theta\varphi+\dfrac{\beta\alpha R}{R\Theta+\rho+\xi}\right)$.

Proof. The proof process here is the same as Theorem 1-1.

Substituting the equilibrium strategies of manufacturers, streamers, and livestream platforms in Theorem 3-1 into Eq (1), we can obtain;

$$dX(t)=\left(\frac{\Theta\varphi-\dfrac{(-1+\alpha)R}{R\Theta+\rho+\xi}}{\mu_m}+\frac{\Theta\varphi-\dfrac{(\beta-1)\alpha R}{R\Theta+\rho+\xi}}{\mu_c}+\frac{\Theta\varphi+\dfrac{\beta\alpha R}{R\Theta+\rho+\xi}}{\mu_p}-(\Theta R+\xi)X(t)\right)dt+\sigma\sqrt{X}dz(t),$$

$$X(0)=X_0>0 \tag{10}$$

where let $\Omega^{GF}=\dfrac{\Theta\varphi-\dfrac{(-1+\alpha)R}{R\Theta+\rho+\xi}}{\mu_m}+\dfrac{\Theta\varphi-\dfrac{(\beta-1)\alpha R}{R\Theta+\rho+\xi}}{\mu_c}+\dfrac{\Theta\varphi+\dfrac{\beta\alpha R}{R\Theta+\rho+\xi}}{\mu_p}$, $\delta=\Theta R+\xi$

Furthermore, the expectation and variance of the dynamic change trajectory of livestream system reputation can be obtained as follows:

Theorem 3-2. Under the decentralized decision-making scenario with government penalties, for consumer quality complaints:

i) The expected value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively;

$$EX(t)=\left(X_0-\frac{\dfrac{\Theta\varphi-\dfrac{(-1+\alpha)R}{R\Theta+\rho+\xi}}{\mu_m}+\dfrac{\Theta\varphi-\dfrac{(\beta-1)\alpha R}{R\Theta+\rho+\xi}}{\mu_c}+\dfrac{\Theta\varphi+\dfrac{\beta\alpha R}{R\Theta+\rho+\xi}}{\mu_p}}{\Theta R+\xi}\right)e^{-(\Theta R+\xi)t}$$

$$+\frac{\dfrac{\Theta\varphi-\dfrac{(-1+\alpha)R}{R\Theta+\rho+\xi}}{\mu_m}+\dfrac{\Theta\varphi-\dfrac{(\beta-1)\alpha R}{R\Theta+\rho+\xi}}{\mu_c}+\dfrac{\Theta\varphi+\dfrac{\beta\alpha R}{R\Theta+\rho+\xi}}{\mu_p}}{\Theta R+\xi}$$

$$\lim_{t\to\infty}EX(t)=\frac{\Omega^{GF}}{\delta}=\frac{\dfrac{\Theta\varphi-\dfrac{(-1+\alpha)R}{R\Theta+\rho+\xi}}{\mu_m}+\dfrac{\Theta\varphi-\dfrac{(\beta-1)\alpha R}{R\Theta+\rho+\xi}}{\mu_c}+\dfrac{\Theta\varphi+\dfrac{\beta\alpha R}{R\Theta+\rho+\xi}}{\mu_p}}{\Theta R+\xi}$$

ii) The variance value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively:

$$DX(t)=\frac{\sigma^2\left[\Omega^{GF}-2\left(\Omega^{GF}-\delta X_0\right)e^{-\delta t}+\left(\Omega^{GF}-2\delta X_0\right)e^{-2\delta t}\right]}{2\delta^2}$$

$$\lim_{t\to\infty}DX(t)=\frac{\sigma^2\Omega^{GF}}{2\delta^2}=\frac{\sigma^2\left(\dfrac{\Theta\varphi-\dfrac{(-1+\alpha)R}{R\Theta+\rho+\xi}}{\mu_m}+\dfrac{\Theta\varphi-\dfrac{(\beta-1)\alpha R}{R\Theta+\rho+\xi}}{\mu_c}+\dfrac{\Theta\varphi+\dfrac{\beta\alpha R}{R\Theta+\rho+\xi}}{\mu_p}\right)}{2(\Theta R+\xi)^2}$$

Proof. The proof process here is the same as Theorem 1-2.

3.2.2. Collaborative decision-making with government penalties (*GC*)

Theorem 4-1. Under the collaborative decision-making scenario with government penalties, for the system reputation risks caused by consumers' quality complaints, the optimal reputation repair efforts of manufacturers, streamers, and livestream platforms are respectively; $B_m^{GC}=\dfrac{1}{\mu_m}\left(\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}\right)$, $B_c^{GC}=\dfrac{1}{\mu_c}\left(\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}\right)$, $B_p^{GC}=\dfrac{1}{\mu_p}\left(\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}\right)$.

Proof. The proof process here is the same as Theorem 1-1.

Substituting the equilibrium strategies of manufacturers, streamers, and livestream platforms in Theorem 4-1 into Eq (1), we can obtain:

$$dX(t)=\left(\frac{\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}}{\mu_m}+\frac{\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}}{\mu_c}+\frac{\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}}{\mu_p}-(\Theta R+\xi)X(t)\right)dt+\sigma\sqrt{X}dz(t),$$

$$X(0)=X_0>0 \tag{11}$$

we let $\Omega^{GC}=\dfrac{\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}}{\mu_m}+\dfrac{\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}}{\mu_c}+\dfrac{\Theta\varphi+\dfrac{R}{R\Theta+\rho+\xi}}{\mu_p}$, $\delta=\Theta R+\xi$.

Furthermore, the expectation and variance of the dynamic change trajectory of livestream system reputation can be obtained, as follows:

Theorem 4-2. Under the collaborative decision-making scenario with government penalties, for consumer quality complaints:

i) The expected value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively:

$$EX(t) = \left(X_0 - \frac{\dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_m} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_c} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_p}}{\Theta R + \xi} \right) e^{-(\Theta R+\xi)t}$$

$$+ \frac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_m} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_c}}{\Theta R + \xi}$$

$$\lim_{t\to\infty} EX(t) = \frac{\Omega^{GC}}{\delta} = \frac{\dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_m} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_c} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_p}}{\Theta R + \xi}$$

ii) The variance value and steady-state value of the dynamic change trajectory of livestream system reputation are respectively:

$$DX(t) = \frac{\sigma^2\left[\Omega^{GC} - 2\left(\Omega^{GC} - \delta X_0\right)\mathrm{e}^{-\delta t} + \left(\Omega^{GC} - 2\delta X_0\right)\mathrm{e}^{-2\delta t}\right]}{2\delta^2}$$

$$\lim_{t\to\infty} DX(t) = \frac{\sigma^2\Omega^{GC}}{2\delta^2} = \frac{\sigma^2\left(\dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_m} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_c} + \dfrac{\Theta\varphi + \dfrac{R}{R\Theta+\rho+\xi}}{\mu_p}\right)}{2(\Theta R + \xi)^2}$$

Proof. The proof process here is the same as Theorem 1-2.

4. Discussion

4.1. Results analysis

Proposition 1. Comparative analysis between decision variables

1-(1)

i) There exists a threshold $\alpha' = \dfrac{(R\Theta+\rho+\xi)\varphi\Theta}{R}$, such that when $\alpha < \alpha'$, $B_m^{NF} < B_m^{NC} < B_m^{GF} < B_m^{GC}$, and when $\alpha > \alpha'$, $B_m^{NF} < B_m^{GF} < B_m^{NC} < B_m^{GC}$;

ii) There exist thresholds $\alpha'' = \dfrac{1}{1-\beta}$, $\bar{\alpha} = \dfrac{\Theta^2\varphi - 1}{\beta - 1}$, and $\bar{\bar{\alpha}} = -\dfrac{\Theta^2\varphi + 1}{\beta - 1}$, and $\alpha'' > \bar{\alpha}$, $\alpha'' < \bar{\bar{\alpha}}$, such that when $\alpha < \alpha''$ is satisfied, if $\alpha < \bar{\alpha}$, $B_c^{NF} < B_c^{GF} < B_c^{NC} < B_c^{GC}$, and if $\bar{\alpha} < \alpha < \alpha''$, $B_c^{NF} < B_c^{NC} < B_c^{GF} < B_c^{GC}$; When $\alpha > \alpha''$ is satisfied, if $\alpha'' < \alpha < \bar{\bar{\alpha}}$, $B_c^{NC} < B_c^{NF} < B_c^{GC} < B_c^{GF}$, and if $\alpha > \bar{\bar{\alpha}}$, $B_c^{NC} < B_c^{GC} < B_c^{NF} < B_c^{GF}$.

iii) There exists a threshold $\alpha''' = \dfrac{R-(R\Theta+\rho+\xi)\varphi\Theta}{R\beta}$, such that when $\alpha < \alpha'''$, $B_p^{NF} < B_p^{GF} < B_p^{NC} < B_p^{GC}$, and when $\alpha > \alpha'''$, $B_p^{NF} < B_p^{NC} < B_p^{GF} < B_p^{GC}$.

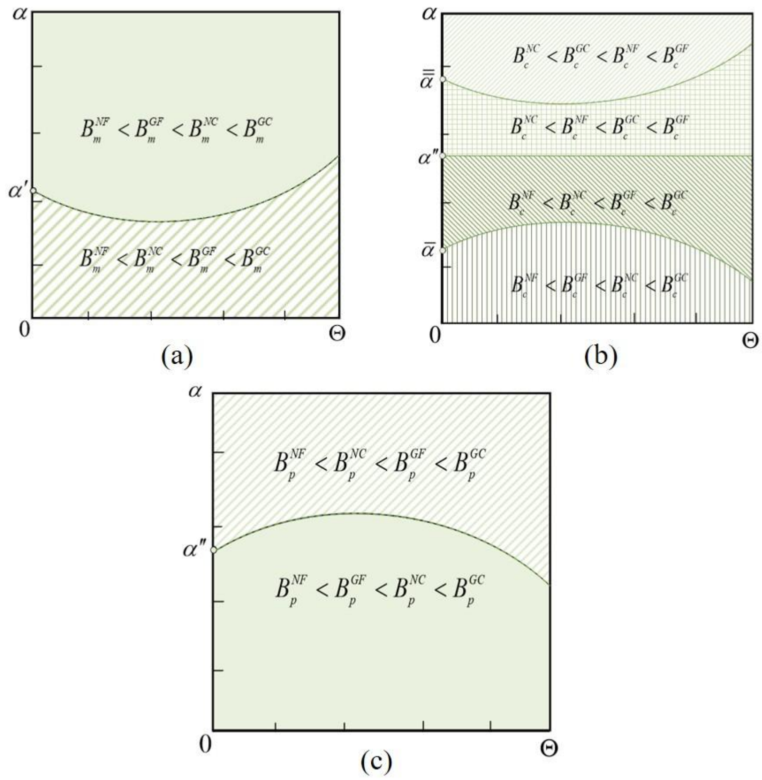


Figure 2. Comparison of decision variables under different scenarios.

Proposition 1-(1) and Figure 2 jointly reflect several key insights. (1) Under all conditions, the levels of system reputation repair efforts by manufacturers and livestreaming platforms are always highest under collaborative decision-making with government penalties, and lowest under decentralized decision-making without government penalties. (2) Collaborative decision-making yields an optimal level of reputation repair effort for manufacturers when the streamer commission rate is relatively high, whereas for livestreaming platforms, the optimal level is achieved when the commission rate is relatively low. Moreover, the advantage range of collaborative decision-making reaches its maximum when the intensity of consumer quality complaints is at a moderate level. Once the complaint intensity either increases or decreases, this advantage range shrinks. The underlying mechanism can be explained as follows: When the complaint intensity is either low or high, the collaborative mechanism tends to fail; Specifically, when complaint intensity is low, collaborative decision-making is unnecessary, as each agent can independently restore system reputation. When complaint intensity is high, the situation tends to become uncontrollable, requiring timely government intervention. In contrast, when complaint intensity is moderate, collaborative decision-making achieves its maximum effectiveness. (3) For streamers, their system reputation repair effort reaches its highest level under collaborative decision-making with government penalties only when the commission rate is low. Conversely, when the commission rate is high, the streamer's effort level is highest under decentralized decision-making with government penalties. This occurs because an increase in commission rates reduces the share of benefits allocated to streamers under collaborative decision-making, thereby weakening their incentives for reputation repair. The condition under which collaborative decision-making yields an optimal level of reputation repair effort for streamers is that the commission rate is low, and the intensity of consumer quality complaints is at a moderate level. These findings further indicate that: (1) The introduction of government penalty mechanisms consistently enhances the incentives of all participants in livestreaming system to engage in reputation repair; (2) collaborative decision-making is more suitable for manufacturers in high-commission scenarios, whereas for streamers and livestreaming platforms, it is more suitable in low-commission scenarios; (3) when quality complaint intensity is at a moderate threshold, the benefits of collaborative governance of system reputation are maximized, encouraging the system to prefer collaborative mechanisms within the co-evolutionary framework of "quality–reputation–performance". Once complaint intensity deviates from this threshold (either increasing or decreasing), the applicability of collaborative decision-making diminishes, and the system begins to deviate from collaboration; (4) with increasing streamer commission rates, collaborative decision-making becomes less effective in enhancing the reputation repair incentives of streamers and livestreaming platforms.

The above findings can be observed in practice. When complaint intensity is low, as seen in routine consumer feedback scenarios, individual agents can independently restore reputation through standard after-sales procedures without costly cross-agent collaboration. Conversely, when complaint intensity is extremely high, as in the "Northeast Yujie" sweet potato vermicelli incident, where the streamer was fined 1.65 million yuan and ordered to suspend operations, and the "Meicai Braised Pork" incident, where the manufacturer was required to issue public apologies, initiate product recalls, and provide consumer compensation, the situation becomes uncontrollable through self-governance alone, requiring timely government intervention. In such high-intensity cases, regulatory authorities step in with administrative penalties, public warnings, and operational restrictions to contain the reputational damage. These observations support our theoretical finding that collaborative governance achieves maximum effectiveness only when complaint intensity is at a moderate level; severe enough to necessitate cooperation but not so severe that it exceeds the capacity of multi-agent coordination.

1-(2) There exist $s_m = 1-\alpha$, $s_c = (1-\beta)\alpha$, and $s_p = \beta\alpha$, which are the final revenue distribution ratios of manufacturers, streamers, and livestream platforms, respectively. The correlation between the equilibrium decision level and relevant factors under each scenario is as follows:

Table 3. Sensitivity analysis of equilibrium decision level under each scenario.

Decision Variable\ Parameter		s_i （$i \in m,c,p$）	R	μ_i （$i \in m,c,p$）	Θ
NF Scenario	B_m^{NF}	↗	↗	↘	↘
	B_c^{NF}	↗	↗	↘	↘
	B_p^{NF}	↗	↗	↘	↘
NC Scenario	B_m^{NC}	—	↗	↘	↘
	B_c^{NC}	—	↗	↘	↘
	B_p^{NC}	—	↗	↘	↘

Continued on next page

Decision Variable\ Parameter		s_i ($i \in m,c,p$)	R	μ_i ($i \in m,c,p$)	Θ	
GF Scenario	B_m^{GF}	↗	↗	↘	$\varphi < \varphi'$	↘
					$\varphi > \varphi'$	↗
	B_c^{GF}	↗	↗	↘	$\varphi < \varphi'$	↘
					$\varphi > \varphi'$	↗
	B_p^{GF}	↗	↗	↘	$\varphi < \varphi'$	↘
					$\varphi > \varphi'$	↗
GC Scenario	B_m^{GC}	—	↗	↘	$\varphi < \varphi''$	↘
					$\varphi > \varphi''$	↗
	B_c^{GC}	—	↗	↘	$\varphi < \varphi''$	↘
					$\varphi > \varphi''$	↗
	B_c^{GC}	—	↗	↘	$\varphi < \varphi''$	↘
					$\varphi > \varphi''$	↗

Proposition 1-(2) and Table 3 jointly indicate that, under the four decision-making scenarios, the effort levels of all participants in the livestream system are positively correlated with the system traffic level and negatively with their own effort cost coefficients. Under decentralized decision-making (i.e., the NF and GF scenarios), the effort levels of all participants are positively correlated with their respective revenue-sharing proportions. In the absence of government penalties, the effort levels of all participants are negatively correlated with the intensity of consumer complaints. However, under government penalties, when the effort-based penalty reduction coefficient φ is relatively small, the effort levels of all participants remain negatively correlated with complaint intensity; conversely, when φ is relatively large, the effort levels become positively correlated with complaint intensity. This finding further indicates that, in the absence of government penalties, higher consumer complaint intensity leads to lower incentives for reputation repair efforts among all participants. When government penalties are introduced, if the degree of effort-based penalty reduction is small (i.e., although participants exert effort in reputation repair, the reduction in penalties is limited), the incentives for reputation repair remain limited even under high complaint intensity. In contrast, when the degree of effort-based penalty reduction is large (i.e., if all parties make efforts to restore their reputations, the government will reduce the penalties based on the extent of the efforts made), the incentives for reputation repair increase significantly. These findings further imply that the theoretical condition under which the introduction of government penalty mechanisms enhances the incentives for reputation repair is that the degree of effort-based penalty reduction must exceed certain thresholds, denoted as φ' (under decentralized decision-making) and φ'' (under collaborative decision-making). Therefore, when consumer quality complaint intensity increases, a single government penalty mechanism is insufficient to stimulate reputation repair incentives among participants. The establishment of an effort-based penalty reduction mechanism can significantly enhance such incentives and improve the overall system reputation, thereby leading the system, within the co-evolutionary framework of “quality–reputation–performance”, to prefer a governance structure that combines government penalties with effort-based penalty reduction mechanisms.

Proposition 2. Limit comparative analysis of the expected value and variance of livestream system reputation differential trajectory

2-(1) There exists a threshold $\beta' = \dfrac{\alpha\mu_c\mu_p - \alpha\mu_m\mu_p + \mu_c\mu_m + \mu_m\mu_p}{\alpha\left(\mu_c - \mu_p\right)\mu_m}$, such that

$$\mu_p < \mu_c \Rightarrow \begin{cases} \beta < \beta' \Rightarrow \lim_{t\to\infty} E\left[X^{GC}(t)\right] > \begin{cases} \lim_{t\to\infty} E\left[X^{NC}(t)\right] \\ \lim_{t\to\infty} E\left[X^{GF}(t)\right] \end{cases} > \lim_{t\to\infty} E\left[X^{NF}(t)\right] \\ \beta > \beta' \Rightarrow \lim_{t\to\infty} E\left[X^{GF}(t)\right] > \begin{cases} \lim_{t\to\infty} E\left[X^{NF}(t)\right] \\ \lim_{t\to\infty} E\left[X^{GC}(t)\right] \end{cases} > \lim_{t\to\infty} E\left[X^{NC}(t)\right] \end{cases},$$

$$\mu_p > \mu_c \Rightarrow \begin{cases} \beta < \beta' \Rightarrow \lim_{t\to\infty} E\left[X^{GF}(t)\right] > \begin{cases} \lim_{t\to\infty} E\left[X^{NF}(t)\right] \\ \lim_{t\to\infty} E\left[X^{GC}(t)\right] \end{cases} > \lim_{t\to\infty} E\left[X^{NC}(t)\right] \\ \beta > \beta' \Rightarrow \lim_{t\to\infty} E\left[X^{GC}(t)\right] > \begin{cases} \lim_{t\to\infty} E\left[X^{NC}(t)\right] \\ \lim_{t\to\infty} E\left[X^{GF}(t)\right] \end{cases} > \lim_{t\to\infty} E\left[X^{NF}(t)\right] \end{cases}$$

When $\mu_p < \mu_c$, $\dfrac{\partial\beta'}{\partial\alpha} < 0$, when $\mu_p > \mu_c$, $\dfrac{\partial\beta'}{\partial\alpha} > 0$.

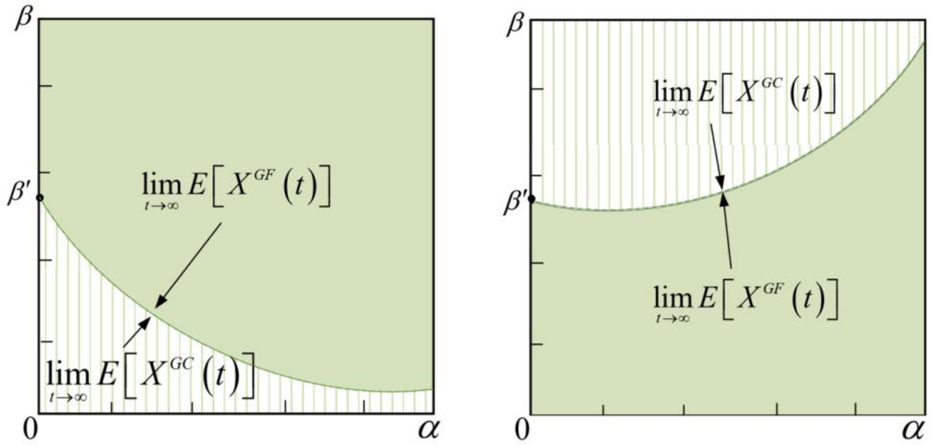


Figure 3. Comparison of the maximum expected values of livestream system reputation trajectory under the influence of relevant factors.

Proposition 2-(1) and Figure 3 jointly reveal that, under all conditions, the expected value of the system reputation trajectory is always highest in the presence of government penalties and lowest in the absence of government penalties. This finding further indicates that government penalty mechanisms can effectively enhance the expected system reputation trajectory. In addition, when the unit cost coefficient of reputation repair effort for the livestreaming platform is lower than that of the streamer, and if the platform's take rate is relatively low, the expected value of the reputation trajectory is highest under collaborative decision-making with government penalties; however, if the platform's take rate is relatively high, the expected value is highest under decentralized decision-making with government penalties. Conversely, when the unit cost coefficient of reputation repair effort for the livestreaming platform is higher than that of the streamer, the opposite pattern emerges: When the platform's take rate is low, the expected value is highest under decentralized decision-making with government penalties, whereas when the platform's take rate is high, it is highest under collaborative decision-making with government penalties. The underlying mechanism can be explained as follows: The revenue of the livestreaming platform must be sufficient to compensate for its reputation repair costs. When the cost of reputation repair effort is relatively high, the platform needs to increase its service fee to ensure operational revenue, thereby providing sufficient profit margins to support reputation repair efforts. Moreover, under both types of conditions described above, as the streamer commission rate increases, the dominant region (in which the expected value of the reputation trajectory is higher) under decentralized decision-making with government penalties continuously expands, whereas the dominant region under collaborative decision-making with government penalties continuously shrinks. These findings further indicate that: (1) The introduction of government penalty mechanisms always leads to the highest expected system reputation trajectory; (2) the theoretical condition under which collaborative decision-making achieves Pareto optimality in terms of the expected reputation trajectory is that the platform's revenue coefficient (take rate) must move in the same direction as its reputation-repair cost coefficient; and (3) an increase in streamer commission rates consistently reduces the dominant region of collaborative decision-making in terms of the expected reputation trajectory (i.e., the scope of collaborative decision-making).

The above findings can be observed in practice. First, the effectiveness of government penalties in enhancing reputation governance is evidenced by regulatory actions. In 2024, the State Administration for Market Regulation processed over 400,000 livestream-related complaints nationwide, with the Chengdu Kuaigou Technology case resulting in a penalty of 26.69 million yuan, demonstrating that regulatory penalty mechanisms are an effective means of improving system reputation. Second, the condition that the platform's take rate must co-move with its cost coefficient is reflected in practice: When platforms face high reputation repair costs, they require sufficient take-rate revenue to cover these expenses; platforms unable to generate adequate revenue from take rates are less capable of supporting effective reputation restoration. Third, the finding that higher streamer commission rates reduce the applicability of collaborative decision-making is consistent with observed practices, as higher commissions increase the financial burden on platforms and manufacturers, thereby weakening their willingness to engage in coordinated reputation repair efforts.

2-(2) There exists a threshold $\beta'' = \dfrac{\left((1-\alpha)\mu_p + \mu_c\right)\mu_m + \alpha\mu_c\mu_p}{\alpha\left(\mu_c - \mu_p\right)\mu_m}$, such that

$$\mu_p < \mu_c \Rightarrow \begin{cases} \beta < \beta'' \Rightarrow \lim\limits_{t\to\infty} D\left[X^{GF}(t)\right] > \begin{cases} \lim\limits_{t\to\infty} D\left[X^{NF}(t)\right] \\ \lim\limits_{t\to\infty} D\left[X^{GC}(t)\right] \end{cases} > \lim\limits_{t\to\infty} D\left[X^{NC}(t)\right] \\ \beta > \beta'' \Rightarrow \lim\limits_{t\to\infty} D\left[X^{GC}(t)\right] > \begin{cases} \lim\limits_{t\to\infty} D\left[X^{NC}(t)\right] \\ \lim\limits_{t\to\infty} D\left[X^{GF}(t)\right] \end{cases} > \lim\limits_{t\to\infty} D\left[X^{NF}(t)\right] \end{cases},$$

$$\mu_p > \mu_c \Rightarrow \begin{cases} \beta < \beta'' \Rightarrow \lim\limits_{t\to\infty} D\left[X^{GC}(t)\right] > \begin{cases} \lim\limits_{t\to\infty} D\left[X^{NC}(t)\right] \\ \lim\limits_{t\to\infty} D\left[X^{GF}(t)\right] \end{cases} > \lim\limits_{t\to\infty} D\left[X^{NF}(t)\right] \\ \beta > \beta'' \Rightarrow \lim\limits_{t\to\infty} D\left[X^{GF}(t)\right] > \begin{cases} \lim\limits_{t\to\infty} D\left[X^{NF}(t)\right] \\ \lim\limits_{t\to\infty} D\left[X^{GC}(t)\right] \end{cases} > \lim\limits_{t\to\infty} D\left[X^{NC}(t)\right] \end{cases}$$

When $\mu_p < \mu_c$, $\frac{\partial \beta''}{\partial \alpha} < 0$, when $\mu_p > \mu_c$, $\frac{\partial \beta''}{\partial \alpha} > 0$.

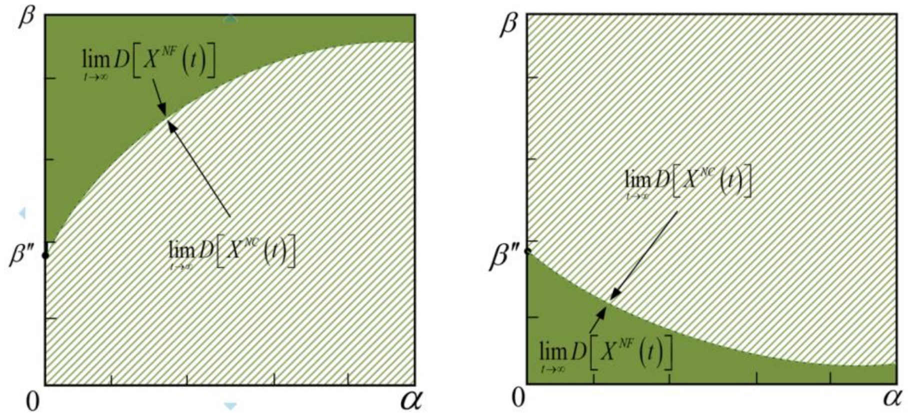


Figure 4. Comparison of the minimum variance values of livestream system reputation trajectory under the influence of relevant factors.

Proposition 2-(2) and Figure 4 jointly reveal that, under all conditions, the variance of the system reputation trajectory is always highest in the presence of government penalties and lowest in the absence of government penalties. In addition, when the unit cost coefficient of reputation repair effort for the livestreaming platform is lower than that of the streamer, if the platform's take rate is relatively low, the variance of the reputation trajectory is minimized under collaborative decision-making without government penalties; however, if the platform's take rate is relatively high, the variance is minimized under decentralized decision-making without government penalties. Conversely, when the unit cost coefficient of reputation repair effort for the livestreaming platform is higher than that of the streamer, the opposite pattern emerges: When the platform's take rate is low, the variance is minimized under decentralized decision-making without government penalties, whereas when the take rate is high, it is minimized under collaborative decision-making without government penalties. Furthermore, under both types of conditions described above, as the streamer commission rate increases, the dominant region (in which the variance of the reputation trajectory is lower) under collaborative decision-making without government penalties continuously expands. These findings further indicate that: (1) Although the introduction of government penalty mechanisms leads to the highest expected value of the system reputation trajectory, it also increases the volatility of the trajectory; (2) the theoretical condition under which collaborative decision-making achieves Pareto optimality in terms of the variance of the reputation trajectory (i.e., lower volatility) is that the platform's revenue coefficient (take rate) must move in the same direction as its reputation-repair cost coefficient; and (3) an increase in streamer commission rates consistently expands the dominant region of collaborative decision-making with respect to the variance of the reputation trajectory (i.e., the scope of collaborative decision-making), indicating that collaborative decision-making can make the evolution of the system reputation trajectory more stable as commission rates increase.

The above findings can be observed in practice. First, while government penalty mechanisms effectively improve expected reputation levels, they also introduce volatility, as evidenced by the varying enforcement outcomes across livestream platforms. Public naming of major violators such as the "San Zhi Yang" and "Northeast Yujie" cases by regulatory authorities demonstrates that regulatory interventions, while necessary for raising overall reputation standards, also create fluctuations and uncertainty in reputation evolution. Second, the condition that the platform's take rate must co-move with its reputation-repair cost coefficient is reflected in practice: Platforms with take rates aligned with their reputation repair costs tend to experience more stable reputation trajectories, whereas platforms where take rates do not adequately cover repair costs face greater reputation fluctuations. Third, the finding that collaborative decision-making can mitigate the negative impact of rising streamer commission rates on reputation trajectory volatility is consistent with observed practices. As commission rates rise, streamers gain more revenue while manufacturers' profits decline, creating a "double marginalization" effect that destabilizes the system; however, coordinated governance mechanisms can offset this effect and help stabilize reputation evolution.

Proposition 3. Trend analysis of livestream system reputation differential trajectory.

3-(1) Judging the change trend of differential trajectory under four scenarios from the perspective of initial value.

There exist thresholds β_1 、 β_2 and β_3 , When $\mu_c < \mu_p$, $\beta_3 < \beta_1 < \beta_2$, if $\beta < \beta_3$, ① $E_{NC} > E_{GC} > E_{NF} > E_{GF}$,if $\beta_3 < \beta < \beta_1$, ② $E_{NC} > E_{NF} > E_{GC} > E_{GF}$, if $\beta_1 < \beta < \beta_2$, ③ $E_{NF} > E_{NC} > E_{GF} > E_{GC}$, if $\beta > \beta_2$, ④ $E_{NF} > E_{GF} > E_{NC} > E_{GC}$; When $\mu_c > \mu_p$, $\beta_2 < \beta_1 < \beta_3$, if $\beta < \beta_2$, ⑤ $E_{NF} > E_{GF} > E_{NC} > E_{GC}$, if $\beta_2 < \beta < \beta_1$, ⑥ $E_{NF} > E_{NC} > E_{GF} > E_{GC}$, if $\beta_1 < \beta < \beta_3$, ⑦ $E_{NC} > E_{NF} > E_{GC} > E_{GF}$, if $\beta > \beta_3$, ⑧ $E_{NC} > E_{GC} > E_{NF} > E_{GF}$.

Among them, the thresholds

$$\beta_1 = \frac{\alpha\mu_c\mu_p - \alpha\mu_m\mu_p + \mu_c\mu_m + \mu_m\mu_p}{\alpha\left(\mu_c - \mu_p\right)\mu_m}$$

$$\beta_2 = \frac{\left[\left(-\Theta\left(R\Theta+\rho+\xi\right)\varphi + R(1-\alpha)\right)\mu_p - \left(\Theta\left(R\Theta+\rho+\xi\right)\varphi - R\right)\mu_c\right]\mu_m - \left(\Theta\left(R\Theta+\rho+\xi\right)\varphi - R\alpha\right)\mu_c\mu_p}{\alpha\left(\mu_c - \mu_p\right)R\mu_m}$$

$$\beta_3 = \frac{\left[\left(\Theta\varphi\left(R\Theta+\rho+\xi\right) + R(1-\alpha)\right)\mu_p + (\Theta\varphi(R\Theta+\rho+\xi)+R)\mu_c\right]\mu_m + \mu_c\left(\Theta\varphi\left(R\Theta+\rho+\xi\right) + R\alpha\right)\mu_p}{\alpha\left(\mu_c - \mu_p\right)R\mu_m}.$$

The regional conditions in different scenarios are shown in Figure 5:

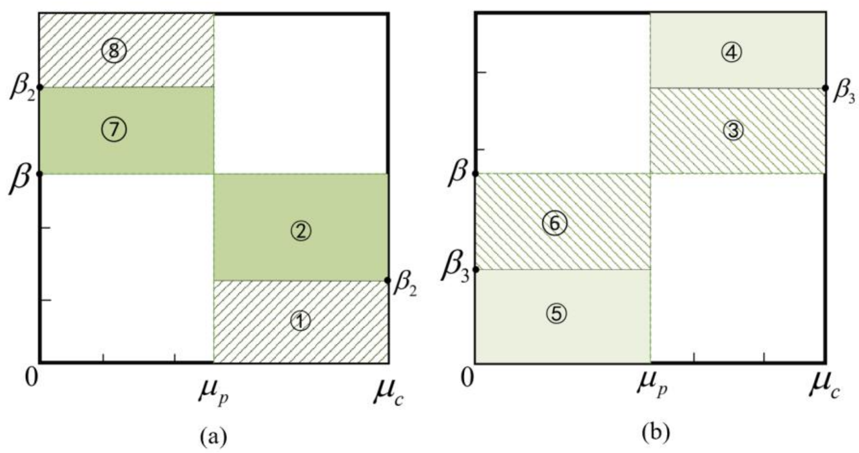


Figure 5. Existence interval of system reputation differential trajectory state.

Table 4. Evolution state of system reputation differential trajectory under different conditions.

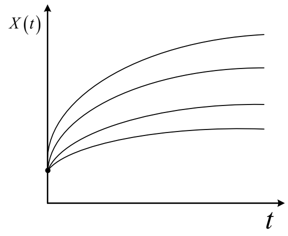
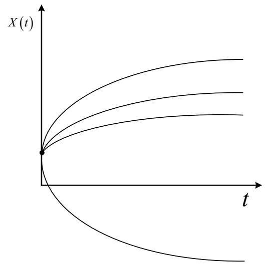
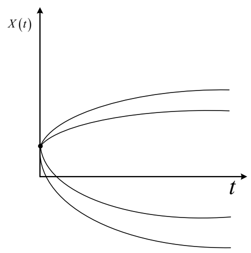
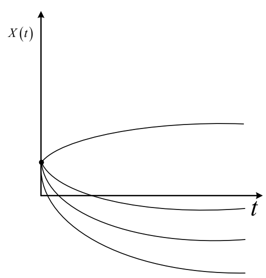
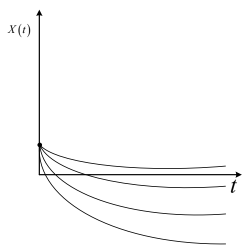
Theoretical Conditions \ Reputation Differential Trajectory	$\mu_c < \mu_p$ and $\beta > \beta_2$ or $\mu_c > \mu_p$ and $\beta < \beta_2$	$\mu_c < \mu_p$ and $\beta_1 < \beta < \beta_2$ or $\mu_c > \mu_p$ and $\beta_2 < \beta < \beta_1$	$\mu_c < \mu_p$ and $\beta_3 < \beta < \beta_1$ or $\mu_c > \mu_p$ and $\beta_1 < \beta < \beta_3$	$\mu_c < \mu_p$ and $\beta < \beta_3$ or $\mu_c > \mu_p$ and $\beta > \beta_3$
	$m_1 < m_3 < m_2 < m_4$	$m_1 < m_2 < m_3 < m_4$	$m_2 < m_1 < m_4 < m_3$	$m_2 < m_4 < m_1 < m_3$
	$X_0 < m_1$ $\begin{cases} E_{NF} < 0 \\ E_{GF} < 0 \\ E_{NC} < 0 \\ E_{GC} < 0 \end{cases}$	$X_0 < m_1$ $\begin{cases} E_{NF} < 0 \\ E_{NC} < 0 \\ E_{GF} < 0 \\ E_{GC} < 0 \end{cases}$	$X_0 < m_2$ $\begin{cases} E_{NC} < 0 \\ E_{NF} < 0 \\ E_{GC} < 0 \\ E_{GF} < 0 \end{cases}$	$X_0 < m_2$ $\begin{cases} E_{NC} < 0 \\ E_{GC} < 0 \\ E_{NF} < 0 \\ E_{GF} < 0 \end{cases}$
	$m_1 < X_0 < m_3$ $\begin{cases} E_{NF} > 0 \\ E_{GF} < 0 \\ E_{NC} < 0 \\ E_{GC} < 0 \end{cases}$	$m_1 < X_0 < m_2$ $\begin{cases} E_{NF} > 0 \\ E_{NC} < 0 \\ E_{GF} < 0 \\ E_{GC} < 0 \end{cases}$	$m_2 < X_0 < m_1$ $\begin{cases} E_{NC} > 0 \\ E_{NF} < 0 \\ E_{GC} < 0 \\ E_{GF} < 0 \end{cases}$	$m_2 < X_0 < m_4$ $\begin{cases} E_{NC} > 0 \\ E_{GC} < 0 \\ E_{NF} < 0 \\ E_{GF} < 0 \end{cases}$
	$m_3 < X_0 < m_2$ $\begin{cases} E_{NF} > 0 \\ E_{GF} > 0 \\ E_{NC} < 0 \\ E_{GC} < 0 \end{cases}$	$m_2 < X_0 < m_3$ $\begin{cases} E_{NF} > 0 \\ E_{NC} > 0 \\ E_{GF} < 0 \\ E_{GC} < 0 \end{cases}$	$m_1 < X_0 < m_4$ $\begin{cases} E_{NC} > 0 \\ E_{NF} > 0 \\ E_{GC} < 0 \\ E_{GF} < 0 \end{cases}$	$m_4 < X_0 < m_1$ $\begin{cases} E_{NC} > 0 \\ E_{GC} > 0 \\ E_{NF} < 0 \\ E_{GF} < 0 \end{cases}$
	$m_2 < X_0 < m_4$ $\begin{cases} E_{NF} > 0 \\ E_{GF} > 0 \\ E_{NC} > 0 \\ E_{GC} < 0 \end{cases}$	$m_3 < X_0 < m_4$ $\begin{cases} E_{NF} > 0 \\ E_{NC} > 0 \\ E_{GF} > 0 \\ E_{GC} < 0 \end{cases}$	$m_4 < X_0 < m_3$ $\begin{cases} E_{NC} > 0 \\ E_{NF} > 0 \\ E_{GC} > 0 \\ E_{GF} < 0 \end{cases}$	$m_1 < X_0 < m_3$ $\begin{cases} E_{NC} > 0 \\ E_{GC} > 0 \\ E_{NF} > 0 \\ E_{GF} < 0 \end{cases}$
	$X_0 > m_4$ $\begin{cases} E_{NF} > 0 \\ E_{GF} > 0 \\ E_{NC} > 0 \\ E_{GC} > 0 \end{cases}$	$X_0 > m_4$ $\begin{cases} E_{NF} > 0 \\ E_{NC} > 0 \\ E_{GF} > 0 \\ E_{GC} > 0 \end{cases}$	$X_0 > m_3$ $\begin{cases} E_{NC} > 0 \\ E_{NF} > 0 \\ E_{GC} > 0 \\ E_{GF} > 0 \end{cases}$	$X_0 > m_3$ $\begin{cases} E_{NC} > 0 \\ E_{GC} > 0 \\ E_{NF} > 0 \\ E_{GF} > 0 \end{cases}$

Table 4 presents the evolution states of the system reputation differential trajectories under four decision-making scenarios (NF, NC, GF, GC) based on the theoretical conditions derived from Proposition 3-(1). The left column displays the evolutionary states of the system reputation differential trajectories. The upper-right section presents four theoretical conditions, which describe how the relationship between the platform's cost efficiency and its commission rate influences: First, the magnitude of changes in the system reputation differential trajectories under the four scenarios, where different orderings of thresholds determine which scenario's reputation trajectory begins to decline earlier or later relative to the others; and second, the threshold value of the initial system reputation m, where four ordering patterns of the thresholds are examined: $m_1 < m_3 < m_2 < m_4$, $m_1 < m_2 < m_3 < m_4$, $m_2 < m_1 < m_4 < m_3$, and $m_2 < m_4 < m_1 < m_3$. When the initial system reputation X_0 exceeds its corresponding threshold m, the evolution state of the system reputation differential trajectory can be determined. Specifically, $E > 0$ indicates that the reputation differential trajectory under that scenario exhibits a declining trend, while $E < 0$ indicates an increasing trend.

Proposition 3-(1) yields several key insights. (1) From a longitudinal perspective, as the initial level of system reputation increases, the system reputation trajectory gradually exhibits a declining trend. Once the initial value exceeds a certain threshold, the reputation trajectories under all four decision-making scenarios exhibit a downward trend. (2) When the unit cost coefficient of reputation repair effort for the livestreaming platform is lower than that of the streamer and the platform take rate is extremely low, or when the platform's cost coefficient is higher than that of the streamer and the take rate is extremely high, Pattern One tends to emerge (the differential reputation trajectories of the livestream system under all four decision-making scenarios initially show an increasing trend → the reputation trajectory under the NF scenario begins to decline → the reputation trajectory under the GF scenario begins to decline → the reputation trajectory under the NC scenario begins to decline → the reputation trajectory under the GC scenario begins to decline → the differential reputation trajectories under all four scenarios exhibit a declining trend). When the unit cost coefficient of reputation repair effort for the livestreaming platform is lower than that of the streamer and the take rate is relatively low, or when the platform's cost coefficient is higher than that of the streamer and the take rate is relatively high, Pattern Two tends to emerge (the differential reputation trajectories under all four scenarios initially show an increasing trend → the reputation trajectory under the NF scenario begins to decline → the reputation trajectory under the NC scenario begins to decline → the reputation trajectory under the GF scenario begins to decline → the reputation trajectory under the GC scenario begins to decline → the differential reputation trajectories under all four scenarios exhibit a declining trend). Conversely, when the unit cost coefficient of reputation repair effort for the livestreaming platform is lower than that of the streamer and the take rate is relatively high, or when the platform's cost coefficient is higher than that of the streamer and the take rate is relatively low, Pattern Three tends to emerge (the differential reputation trajectories under all four scenarios initially show an increasing trend → the reputation trajectory under the NC scenario begins to decline → the reputation trajectory under the NF scenario begins to decline → the reputation trajectory under the GC scenario begins to decline → the reputation trajectory under the GF scenario begins to decline → the differential reputation trajectories under all four scenarios exhibit a declining trend). When the unit cost coefficient of reputation repair effort for the livestreaming platform is lower than that of the streamer and the take rate is extremely high, or when the platform's cost coefficient is higher than that of the streamer and the take rate is extremely low, Pattern Four tends to emerge (the differential reputation trajectories under all four scenarios initially show an increasing trend → the reputation trajectory under the NC scenario begins to decline → the reputation trajectory under the GC scenario begins to decline → the reputation trajectory under the

NF scenario begins to decline → the reputation trajectory under the GF scenario begins to decline → the differential reputation trajectories under all four scenarios exhibit a declining trend). These findings further indicate that: (1) An increase in the initial level of system reputation leads to a gradual decline in the reputation trajectory; (2) under all scenarios, the reputation trajectories ultimately exhibit a downward trend as the initial reputation increases, but the introduction of government penalty mechanisms can delay this decline. Moreover, collaborative decision-making can also delay the decline, subject to a theoretical condition that the platform's revenue coefficient (take rate) must move in the same direction as its reputation-repair cost coefficient. The higher the degree of such alignment, the stronger the delay effect. Conversely, when the platform's revenue coefficient moves in the opposite direction to its cost coefficient, the declining trend of the system reputation trajectory is intensified, and a higher degree of divergence further accelerates the decline.

The above findings can be observed in practice. First, the result that higher initial reputation leads to decline is explained by the fact that when consumers hold higher expectations of a brand or platform, their "loss aversion" becomes more pronounced once quality issues emerge, making reputational damage more severe. The "Neimeng Laogou" incident, where a streamer with over 10 million fans generated significant sales discrepancies, illustrates that even high initial reputation cannot guarantee sustained consumer trust without proper governance mechanisms in place. Second, the finding that government penalty mechanisms and collaborative decision-making can delay the decline of reputation trajectories, subject to the condition that the platform's take rate co-moves with its cost coefficient, is consistent with observed practices. Platforms that align their revenue structures with reputation repair investments are better able to sustain reputation over time.

3-(2) Analyzing the change trend of differential trajectory from the perspective of consumer quality complaint intensity

i) When $\Theta < \Theta_1$, $E_{NF} < 0$; when $\Theta > \Theta_1$, $E_{NF} > 0$;

ii) When $\Theta < \Theta_2$, $E_{NC} < 0$; when $\Theta > \Theta_2$, $E_{NC} > 0$;

iii) There exists a threshold $R' = \dfrac{\varphi\left(\mu_c\mu_m + \mu_c\mu_p + \mu_m\mu_p\right)}{X_0\mu_m\mu_c\mu_p}$, if $R < R'$ is satisfied, when $\Theta < \Theta_{31}$ or $\Theta > \Theta_{32}$, $E_{GF} < 0$, and when $\Theta_{31} < \Theta < \Theta_{32}$, $E_{GF} > 0$. If $R > R'$ is satisfied, when $\Theta < \Theta_{31}$ or $\Theta > \Theta_{32}$, $E_{GF} > 0$, when $\Theta_{31} < \Theta < \Theta_{32}$, $E_{GF} < 0$; iv) There exists a threshold $R' = \dfrac{\varphi\left(\mu_c\mu_m + \mu_c\mu_p + \mu_m\mu_p\right)}{X_0\mu_m\mu_c\mu_p}$, if $R < R'$ is satisfied, when $\Theta < \Theta_{41}$ or $\Theta > \Theta_{42}$, $E_{GC} < 0$, when $\Theta_{41} < \Theta < \Theta_{42}$, $E_{GC} > 0$. If $R > R'$ is satisfied, when $\Theta < \Theta_{41}$ or $\Theta > \Theta_{42}$, $E_{GC} > 0$, when $\Theta_{41} < \Theta < \Theta_{42}$, $E_{GC} < 0$.

Among them, Θ_1 is the zero point of function $f_1(\Theta)$ with respect to Θ, and Θ possesses non-negativity.

$$f_1(\Theta) = R^2 X_0\mu_c\mu_m\mu_p\Theta^2 + (2\xi+\rho)X_0\mu_m\mu_c R\mu_p\Theta + \left((-1+\alpha)\mu_c + \alpha\mu_m(\beta-1)\right)R\mu_p - \beta\alpha R\mu_m\mu_c$$
$$+\xi X_0\mu_c\mu_m(\xi+\rho)\mu_p$$

Among them, Θ_2 denotes the zero point of function $f_2(\Theta)$ with respect to Θ, and Θ possesses non-negativity.

$$f_2(\Theta) = X_0R^2\mu_p\mu_m\mu_c\Theta^2 + \left(X_0\xi R + X_0R(\rho+\xi)\right)\mu_p\mu_m\mu_c\Theta + \left[\left(X_0\xi(\rho+\xi)\mu_p - R\right)\mu_m - R\mu_p\right]\mu_c - R\mu_m\mu_p$$

Among them, Θ_{31} and Θ_{32} are zero points of function $f_3(\Theta)$ with respect to Θ, and Θ possesses non-negativity.

$$f_3(\Theta)=\left[\left(\left(R^2X_0\mu_m-R\varphi\right)\mu_c-\mu_mR\varphi\right)\mu_p-\mu_m\mu_cR\varphi\right]\Theta^2+\left(\left(X_0\xi R+X_0R(\xi+\rho)\right)\mu_m-\varphi(\xi+\rho)\right)\mu_c\mu_p\Theta$$
$$-\mu_m\varphi(\xi+\rho)\mu_p\Theta-\mu_m\mu_c\varphi(\xi+\rho)\Theta+\left[\left(X_0\xi(\xi+\rho)\mu_m-(1-\alpha)R\right)\mu_c+\mu_m(\beta-1)\alpha R\right]\mu_p-\mu_m\mu_c\beta\alpha R$$

Among them, Θ_{41} and Θ_{42} are zero points of function $f_4(\Theta)$ with respect to Θ, and Θ possesses non-negativity.

$$f_4(\Theta)=R\left(\left(\left(RX_0\mu_p-\varphi\right)\mu_m-\mu_p\varphi\right)\mu_c-\mu_m\mu_p\varphi\right)\Theta^2+\left[\left(\left((2\xi+\rho)RX_0\mu_p-\varphi(\xi+\rho)\right)\mu_m-\mu_p\varphi(\xi+\rho)\right)\mu_c\right]\Theta$$
$$-\mu_m\mu_p\varphi(\xi+\rho)\Theta+\left(\left(\xi X_0(\xi+\rho)\mu_p\right)-R\right)\mu_m-R\mu_p\big)\mu_c-R\mu_m\mu_p$$

Proposition 3-(2) indicates that, in the absence of government penalties, when the intensity of consumer quality complaints is low, the system reputation trajectory exhibits an upward trend, whereas when it is high, the trajectory exhibits a downward trend. In the presence of government penalties, when the system traffic level is relatively low, the system reputation trajectory exhibits an upward trend when the quality complaint intensity is either low or high, but exhibits a downward trend when the quality complaint intensity is at a moderate level. When the system traffic level is relatively high, the system reputation trajectory exhibits a downward trend when the quality complaint intensity is either low or high, but exhibits an upward trend when the quality complaint intensity is at a moderate level. These mathematical results further indicate that: (1) The theoretical condition for maintaining an increasing system reputation trajectory is that the intensity of consumer quality complaints remains at a low level in the absence of government penalties. After introducing government penalties, the external conditions for achieving Pareto-optimal reputation trajectories are either that the overall system traffic level is low with complaint intensity at a low or high level, or that the system traffic level is high with complaint intensity at a moderate threshold. (2) Under the co-evolution of "quality–reputation–performance," there exists a linear relationship between total system profit, traffic level, and reputation level. Therefore, when the traffic level is low and complaint intensity is either weak or strong, or when the traffic level is high and complaint intensity is at a moderate level, the system should introduce government penalty mechanisms in a timely manner to enhance system reputation.

The above findings can be observed in practice. The result that an upward reputation trajectory can be maintained under two conditions. either when complaint intensity remains low, or when government penalties are introduced under specific traffic-complaint intensity combinations, is consistent with observed regulatory practices. Under low complaint intensity, routine after-sales procedures suffice to maintain consumer trust without external intervention. However, when complaint intensity is high and traffic is large, problems rapidly generate amplification or systemic effects that exceed the capacity of market entities to self-correct, requiring timely government intervention to stabilize and restore market trust. The "Hema Taotao" case, where a private-domain livestream APP using low-price promotions attracted large audiences while making exaggerated claims, illustrates how high traffic creates a multiplier effect once problems emerge, necessitating timely regulatory intervention to stabilize and restore market trust.

Furthermore, the "moderate threshold" of complaint intensity, which appears in Proposition 1-(1) and Proposition 3-(2), refers to an intermediate range where collaborative governance achieves its maximum effectiveness. When complaint intensity is too low, individual agents can resolve issues through routine procedures without coordination; when it is too high, the situation escalates beyond the system's self-repair capacity and requires government intervention. In the moderate range, complaints are significant enough to motivate stakeholders to cooperate, yet not so severe as to exceed the capacity of multi-agent coordination. In practice, this threshold can be identified by monitoring complaint growth rates, negative sentiment intensity, and the effectiveness of platform-led remediation efforts.

3-(3) Change trend of system profit trajectory and differential trajectory under collaborative decision-making scenario.

Under the collaborative decision-making scenario, the impact of different initial reputation values on the livestream system reputation trajectory and manufacturer profit trajectory is generally divided into three trends: i) When $X_0 < m_2 < m_4$ is satisfied, $E_{NC} < 0$, $E_{GC} < 0$ and $E_{GC} < E_{NC}$, $T_{NC} < 0$, $T_{GC} < 0$ and $T_{GC} < T_{NC}$; ii) When $m_2 < X_0 < m_4$ is satisfied, $E_{NC} > 0$ and $E_{GC} < 0$, $T_{NC} > 0$ and $T_{GC} < 0$; iii) When $m_2 < m_4 < X_0$ is satisfied, $E_{NC} > 0$, $E_{GC} > 0$ and $E_{GC} < E_{NC}$, $T_{NC} > 0$, $T_{GC} > 0$ and $T_{GC} < T_{NC}$.

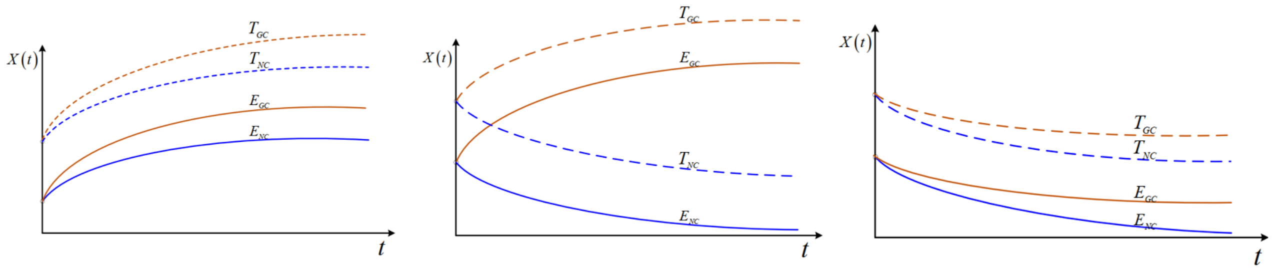


Figure 6. Change trend of reputation trajectory and manufacturer profit trajectory under collaborative decision-making scenario.

Proposition 3-(3) and Figure 6 jointly reveal that when the initial level of system reputation is low, over time, the manufacturer's profit trajectory and the system reputation trajectory exhibit an increasing trend under both collaborative decision-making scenarios (with and without government penalties). As the initial reputation level continues to increase and reaches a moderate threshold, the manufacturer's profit trajectory and the system reputation trajectory under the scenario without government penalties begin to decline, whereas with government penalties, both trajectories continue to increase. As the initial reputation level further increases to a relatively high level, both trajectories exhibit a declining trend under both collaborative decision-making scenarios. These results further indicate that the initial reputation level and the government penalty mechanism influence the evolution of the system reputation and manufacturer profit trajectories under collaborative decision-making. A continuous increase in the initial reputation level leads to a persistent decline in both trajectories under collaboration; however, the introduction of government penalty mechanisms can delay this trend. Moreover, the effectiveness of this delay effect is subject to an external condition, namely that the initial level of the system reputation trajectory lies within a moderate threshold range.

The above findings can be observed in practice. Even under collaborative decision-making, once the market has established high expectations and trust, any quality incident can still trigger a decline in the reputation trajectory. This is consistent with cases where well-known brands with strong market recognition nevertheless experienced reputation and profit declines following quality incidents. The introduction of government penalty mechanisms can delay this adverse trend by imposing external constraints that incentivize sustained quality investment and accountability, thereby mitigating the negative impact of high consumer expectations on system stability once problems emerge.

The parameter values used to generate Figure 7 are set with reference to multiple sources. Specifically, $\mu_m = 1.5$, $\mu_c = 1.2$, and $\mu_P = 1$ are chosen following the typical ranges in [26]; $\alpha = 0.2$ [8] and $\beta = 0.05$ [9] are calibrated based on publicly available data from online sources documenting platform revenue-sharing structures; $R = 1.5$, $\xi = 1.1$, and $\rho = 0.7$ are set in alignment with [28]; and $\omega_{m1} = 0.5$, $\omega_{c1} = 0.35$, and $\omega_{p1} = 0.15$ are adopted with reference to [1]. For computational tractability and normalization across variables, certain parameters have been appropriately adjusted while preserving the fundamental theoretical relationships. Further observation of Figures 7–9 indicates that, under all collaborative decision-making scenarios, the expected trajectory of total system profit is always at the highest level, exceeding that under decentralized decision-making. When the values of Θ and R are relatively low, the total system profit under collaborative decision-making without government penalties is higher than that under collaborative decision-making with government penalties. This indicates that, although quality complaints exist, their intensity is relatively low, and the system traffic level is also low. The negative impact of quality complaints has a weak diffusion effect, enabling the system to achieve self-governance. In such cases, the introduction of government regulation and penalties instead increases the cost of system reputation governance. In contrast, when the values of Θ and R are relatively high, the intensity of quality complaints and the system traffic level are high. The negative reputational impact caused by quality complaints can easily spread throughout the livestream system, making self-governance and system adjustment ineffective. Therefore, timely government regulation and penalties are required to guide and discipline the system, thereby facilitating effective governance.

[8] https://www.toutiao.com/article/7295161215762842138/.
[9] https://www.sohu.com/a/843913751_122004016.

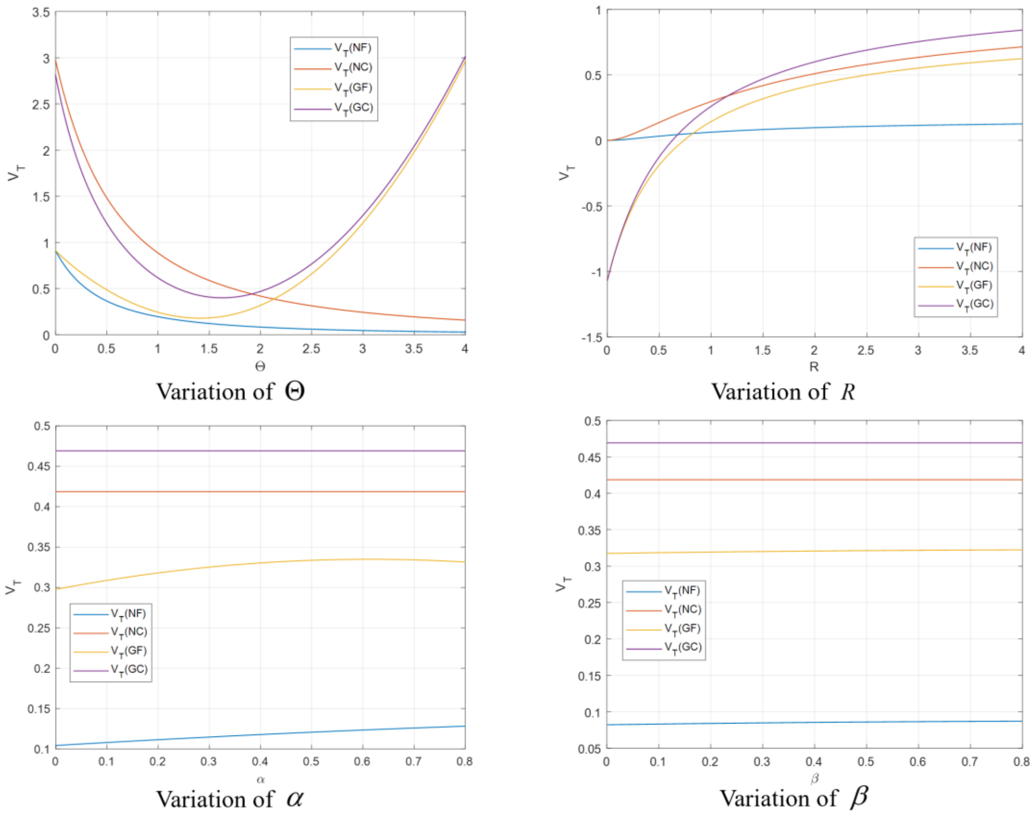


Figure 7. Comparison of the expected trajectories of total system profits under four decision scenarios under different factors ($\Theta = 2$, $\mu_m = 1.5$, $\mu_c = 1.2$, $\mu_p = 1$, $\alpha = 0.2$, $\beta = 0.05$, $R = 1.5$, $\xi = 1.1$, $\rho = 0.7$, $\omega_{m1} = 0.5$, $\omega_{c1} = 0.35$, $\omega_{p1} = 0.15$, and $\varphi = 0.5$).

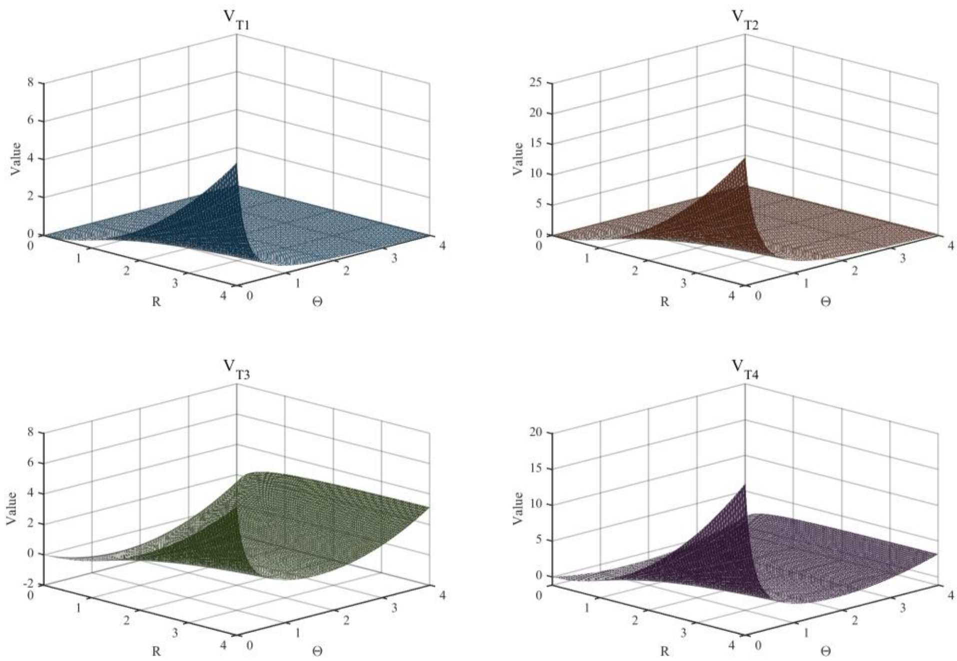


Figure 8. The expected trajectories of system profits under the influence of parameters R and Θ.

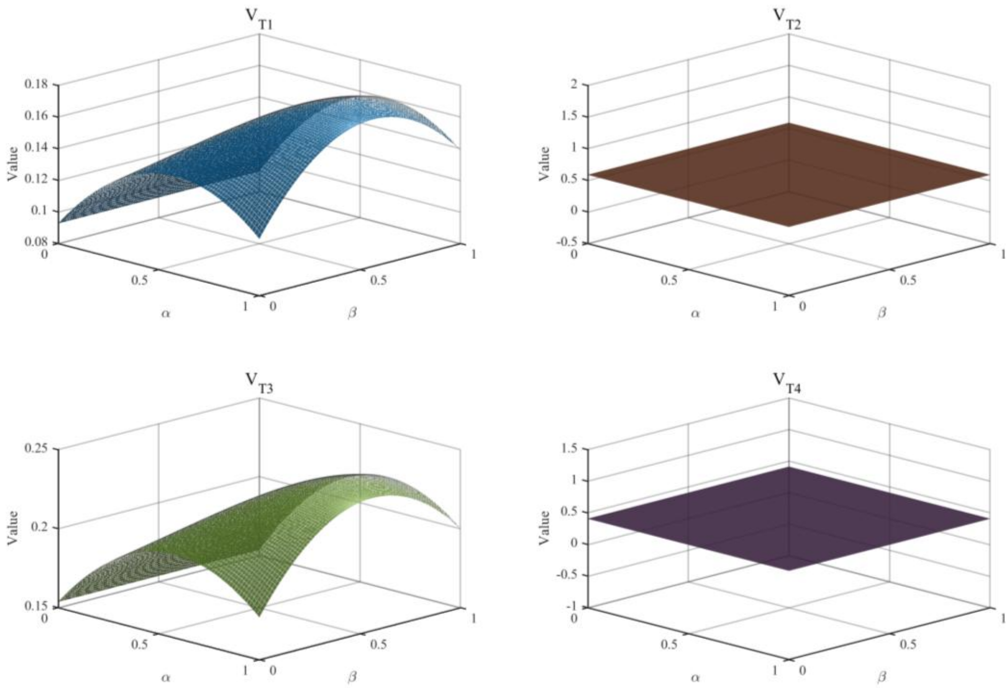


Figure 9. The expected trajectories of system profits under the influence of parameters α and β.

4.2. Multi-agent collaborative simulation analysis based on dynamic event triggering

In this section, we investigate how an adaptive multi-agent collaborative governance model based on dynamic event-triggering and Q-learning can effectively address reputational risks of the livestream system arising from consumer quality complaints, with numerical simulations conducted via MATLAB. The simulation serves as an operational complement to the analytical model developed in Sections 2–4: While the theoretical model provides equilibrium solutions under four decision-making scenarios, the simulation explores how agents can adaptively coordinate in a more realistic setting with limited communication, adopting the same stakeholder structure and reputation dynamics. The simulation settings include four agents: The manufacturer (M), the streamer (C), the livestream platform (P), and the government supervision department (G), with the supervision department acting as the leader and the others as followers. System states include reputation $X(t)$ of the livestream system and intensity $\Theta(t)$ of consumer quality complaints. Each agent maximizes long-term payoffs through adjustment of reputation repair efforts $B_i(t)$ and conducts distributed communication based on the event-triggered mechanism. The supervision department dynamically adjusts the platform's trigger threshold by means of Q-learning to reduce the complaint intensity while reducing communication costs.

4.2.1. Communication topology and graph-theoretic representation

Information interaction among agents is described by directed graph $G=(V,E,\mathcal{A})$, in which vertex set $V=\{m,c,p,g\}$ represents manufacturer, streamer, livestream platform, and government regulatory authority, respectively. Edge set E indicates the direction of information transmission: A directed edge (j,i) exists if agent j can receive information from agent i. Here, we set the regulatory authority as the leader that can send information to all followers; mutual communication among followers forms a fully connected topology. Accordingly, adjacency matrix $\mathcal{A} = \left[a_{ij}\right] \in \Re^{4\times 4}$ is defined as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Among them, $a_{ij} = 1$ means agent i can receive information from agent j. Since the graph is symmetric, the topology is an undirected graph. The corresponding degree matrix $D = diag\{3,3,3,3\}$, and the Laplacian matrix $L = D - \mathcal{A}$ is:

$$L = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

The connection relationship of the leader is represented by a diagonal matrix $B = diag\left\{b_m, b_c, b_p, b_g\right\}$, where $b_i = 1$ means agent i can receive information from the leader. In this section, the supervision department is the leader, and all followers can receive supervision information, so $b_m = b_c = b_p = 1$, $b_g = 0$.

4.2.2. Repair effort update protocol

Each agent dynamically adjusts its repair efforts based on its own instantaneous optimal response and the average effort of neighbors. The local optimal response of an agent is derived from the first-order condition for instantaneous revenue maximization:

$$B_i^{\mathrm{opt}}(t) = \frac{\varphi\Theta(t)}{\mu_i},$$

Among them, μ_i is the effort cost coefficient, and φ is the liability mitigation coefficient for reputation repair efforts. The average effort of neighbors is calculated using the cached value $\hat{B}_j$ obtained via event-triggered communication:

$$\bar{B}_i(t) = \frac{1}{|N_i|}\sum_{j\in N_i} \hat{B}_j(t),$$

N_i is the neighbor set of agent i. The effort update adopts a consensus protocol:

$$\dot{B}_i = \alpha_c\left(\bar{B}_i - B_i\right) + \beta_c\left(B_i^{\mathrm{opt}} - B_i\right),$$

where α_c and β_c are gain coefficients. The discretized form is:

$$B_i(t+\Delta t)=B_i(t)+\Delta t\left[\alpha_c\left(\bar{B}_i(t)-B_i(t)\right)+\beta_c\left(B_i^{\text{opt}}(t)-B_i(t)\right)\right]$$

4.2.3. Event-triggered mechanism

To reduce the communication burden, each agent broadcasts its current effort value only when the event-triggered condition is satisfied. Define the trigger error of agent i as $e_i(t)=\hat{B}_i(t)-B_i(t)$. Among them, $\hat{B}_i(t)$ is the value broadcast from last time. The trigger condition is:

$$\|e_i(t)\|>\delta_i \quad \text{or} \quad t-t_{\text{last},i}>T_{\max,i}$$

where δ_i is the trigger threshold, and $T_{\max,i}$ is the maximum allowable interval. When triggered, the agent broadcasts current B_i and updates cache $\hat{B}_i$; when not triggered, the controller uses the cached value to calculate the average of neighbors' efforts. During a malicious complaint attack, the communication of non-supervision subjects is blocked (the cache is not updated), while the supervision department maintains communication capability at all times.

4.2.4. Q-Learning for the supervision department

As the leader, the regulatory authority adjusts the trigger threshold δ_i of the platform (agent p) through learning to optimize system performance. The Q-learning is designed as follows: (1) State: The complaint intensity $\Theta(t)$ is discretized into three levels, low, medium, and high, with intervals $[0,3]$, $(3,6]$, and $(6,10]$, respectively. This discretization is chosen to capture the different regimes of complaint intensity observed in practice: Low intensity corresponds to routine complaints that can be handled through standard procedures, medium intensity reflects situations where collaborative efforts are most effective, and high intensity indicates systemic crises requiring urgent intervention. (2) Action: The optional values of the platform threshold are $\delta_p\in\{0.2,0.5,0.8\}$. A lower threshold triggers more frequent communication, enabling faster coordination but incurring higher communication costs, while a higher threshold reduces communication frequency at the cost of slower response. (3) Reward: The reward function is formulated as $r(t)=10[\Theta(t-1)-\Theta(t)]-0.1C_p(t)$, where $C_p(t)$ denotes the cumulative number of communication times of the platform. The first term incentivizes the reduction of complaint intensity, while the second term penalizes excessive communication to encourage efficient information exchange. (4) Update: Standard Q-learning is adopted, with a learning rate $\alpha_q=0.1$, a discount factor $\gamma_q=0.1$, and an exploration rate $\varepsilon=0.2$. The relatively low discount factor reflects the emphasis on short-term performance, as complaint dynamics evolve rapidly in practice. Convergence is assessed by monitoring the stability of the Q-table; the algorithm is considered to have reached a stable policy when the optimal action selections remain unchanged across consecutive episodes.

The main simulation parameters are set as follows: The initial reputation $X(0)=100$, the initial complaint intensity $\Theta(0)=3$, and the initial efforts of all agents are set as $B_m(0)=2$, $B_c(0)=2$, $B_p(0)=2$, $B_g(0)=0$. The initial values of the event-triggered thresholds are $\delta_m=0.5$, $\delta_c=0.5$, $\delta_p=0.3$, and $\delta_g=0.2$, respectively, with the maximum trigger intervals $T_{\max}=[8,6,4,3]s$. The intensity of random noise is $\sigma = 0.1$, the simulation time is $T = 60s$, and the time step is $dt = 0.01s$. The simulation results are presented as follows:

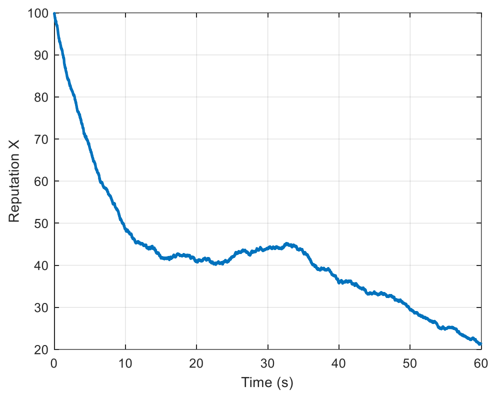


Figure 10. Evolution of livestreaming system reputation.

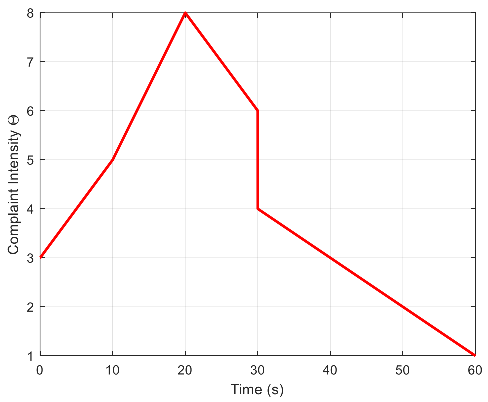


Figure 11. Evolution of consumer complaint intensity.

Figure 10 shows the time-varying curve of the livestream system reputation. The initial reputation value is 100, and as complaint intensity rises rapidly (see Figure 11), the reputation drops sharply to around 50 in the first 10 seconds, and then the decline rate slows down, falling to around 40 at approximately 20 seconds and fluctuating at a low level between 20 and 30 seconds. It continues to decline slowly after 30 seconds, finally reaching 27.5 at 60 seconds. The rapid decline phase of the reputation curve completely coincides with the peak period of complaint intensity. The slowing decline in the later period indicates that the repair efforts of each agent begin to offset part of the damage, but the overall reputation fails to fully recover, reflecting the long-term negative impact of complaint incidents on reputation.

Figure 11 shows the evolution of consumer complaint intensity over time. The complaint intensity rises rapidly from 3.0 at time 0, reaching a peak of 8 at 20 seconds, then declines rapidly to 4.0 at 30 seconds and 2.8 at 40 seconds, and continues to decline to 1.0 at 60 seconds thereafter. This dynamic fully simulates the typical process of a quality complaint incident from outbreak to diffusion to subsidence: 0–20 seconds is the outbreak period with a sharp increase in complaints; 20–30 seconds is the high-level oscillation period; and after 30 seconds is the subsidence period with complaints gradually calming down. The peak of complaint intensity appears at around 20 seconds, corresponding to the period of the fastest reputation decline in Figure 10.

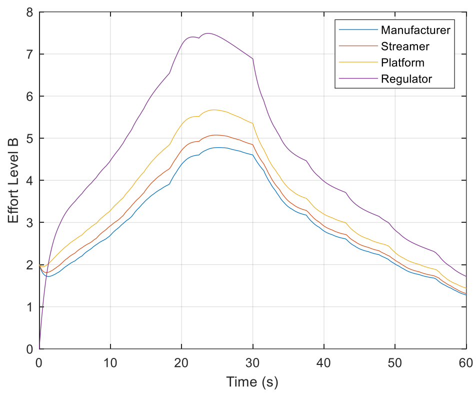


Figure 12. Evolution of reputation repair efforts of each agent.

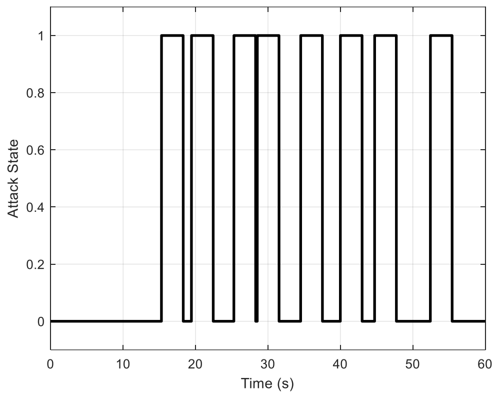


Figure 13. Time series attack state of malicious complaints.

Figure 12 presents the reputation repair effort curves of the manufacturer, streamer, platform, and supervision department. The effort levels of all agents increase continuously with time during the first 20 seconds, remain stable from 20 to 30 seconds, and start to decline at about 30 seconds. This indicates that the reputation repair efforts of each agent change with the intensity of consumers' quality complaints, but the growth rate and final level are different: The supervision department exhibits the highest effort, followed by the platform, while the streamer and manufacturer show relatively low efforts. This reflects the different responsibilities of each agent in reputation repair: The supervision department, as the leader, bears the greatest repair responsibility; the platform, as an intermediary hub, takes the second place; and the streamer and manufacturer are relatively passive. All effort curves show nonlinear growth, with rapid growth in the early stage (corresponding to the complaint outbreak period) and a slowing growth or a decline in the later stage. Figure 13 marks the occurrence period of malicious complaint attacks. A malicious complaint attack with a duration of 1 second is set in the simulation, occurring at the 15th second, with the attack state value set to 1 during the attack. The purpose of the attack is to test the robustness of the system in the case of communication interruption, where the communication of non-supervision subjects is blocked but the supervision department can still communicate normally. From Figure 11, the complaint intensity near the 15th second is approximately 5.5–6.5, with no obvious abnormal peak, indicating that the attack duration is short or its effect is not significantly reflected in the data.

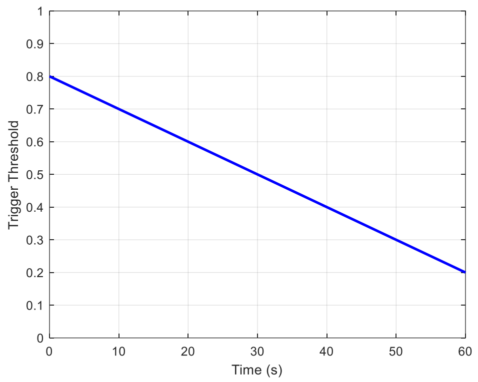


Figure 14. Event-triggered threshold of the platform.

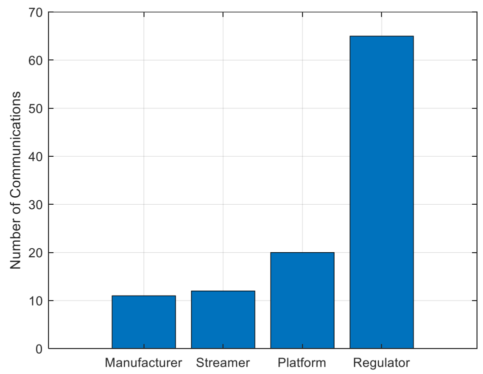


Figure 15. Communication frequency of each agent.

Figure 14 illustrates the dynamic adjustment process of the platform's event-triggered threshold under the effect of supervision Q-learning. The threshold shows a linear downward trend from the initial value of 0.8, finally dropping to 0.2. This indicates that, through learning, the supervision department finds that reducing the platform threshold (i.e., increasing the communication frequency) after the occurrence of a complaint incident can coordinate the reputation repair efforts of each agent in a more timely manner, thus obtaining a higher cumulative reward. The threshold continues to drop to the minimum value, which shows that a lower threshold is always conducive to improving the effect of collaborative governance during the simulation cycle. Figure 15 presents the communication frequency of each agent during the simulation process: The manufacturer communicates 10 times, the streamer 12 times, the platform 28 times, and the supervision department 65 times. The supervision department, as the leader, has the highest communication frequency, far exceeding other agents, reflecting its core role of continuous monitoring and coordination; the platform, as an intermediary hub, has the second most communication times; and the streamer and manufacturer communicate less, mostly receiving instructions and reporting status. Under the event-triggered mechanism, the actual communication frequency is much lower than that of periodic triggering, verifying its effectiveness in reducing communication costs.

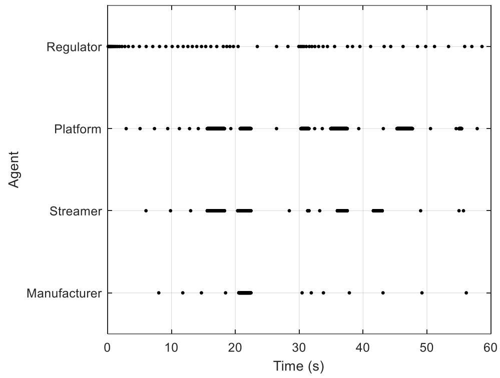


Figure 16. Distribution of event-triggered moments.

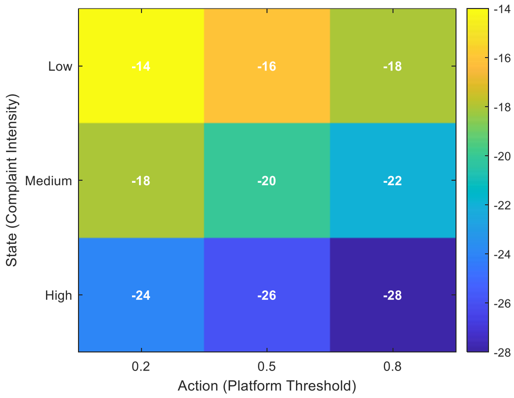


Figure 17. Q-table of the supervision.

Figure 16 presents the moments of event triggering of the four agents during the simulation process in the form of a scatter plot. The horizontal axis represents time (seconds), the vertical axis represents agents, and each black dot indicates that the agent triggers event communication at a specific moment. It can be observed that: The trigger points of the manufacturer (Agent 1) are mainly concentrated around 20 seconds and sparsely distributed, indicating that it is more active in the complaint outbreak period and the triggering decreases in the later stage. The streamer (Agent 2) has very few trigger points, with only sporadic triggers around 20 seconds, showing its low communication demand and reliance on receiving broadcasts from other agents. The platform (Agent 3) has evenly and densely distributed trigger points, with frequent triggering especially during 10–50 seconds, reflecting its core role as an intermediary hub in event coordination. The supervision department (Agent 4) has trigger points covering almost the entire simulation process with the highest density, verifying its role as a leader in continuous monitoring and coordination. This distribution is highly correlated with the complaint intensity curve in Figure 11 and the effort curve in Figure 12: Each agent triggers communication relatively densely during the complaint intensity peak (around 20 seconds); the triggering frequency gradually decreases as complaints ease, while the supervision department maintains high-frequency triggering to keep the system stable. The event-triggered mechanism effectively enables on-demand communication, avoiding unnecessary resource consumption while ensuring the timeliness of collaborative governance.

Figure 17 presents the Q-table values obtained by the supervision department through

Q-learning. The horizontal axis represents the three optional actions of the platform threshold (0.2, 0.5, 0.8), and the vertical axis represents the three discrete states of complaint intensity (low, medium, high). The value in each cell represents the expected cumulative reward of choosing the corresponding action in that state. All Q-values are negative; at the same complaint intensity state, the lower the threshold (0.2), the higher the Q-value (the smaller the negative value), indicating that a lower threshold yields a greater reward. With the increase of complaint intensity, the Q-values of all actions decrease (e.g., the highest is −14 in the low state and the lowest is −28 in the high state), which means the overall system reward is lower, and the governance difficulty is greater under high complaint intensity. Under high complaint intensity, the Q-value (−24) of choosing the low threshold (0.2) is still significantly better than that of the high threshold (0.8) (−28), reinforcing the decision logic of "more frequent communication is needed when complaints are high". The Q-table results verify that the supervision department successfully captures the optimal threshold strategy through learning; namely that a lower threshold is more conducive to collaborative governance in any complaint state.

Therefore, the proposed multi-agent collaborative governance model based on dynamic event triggering and Q-learning can effectively respond to complaint incidents in livestream reputation management, adaptively adjusting communication frequency to conserve network resources while ensuring rapid reputation recovery. The robustness under malicious attacks and the learning ability of the supervision leader have been verified.

4.3. Comparative discussion and research limitations

Our study differs from other research in several important respects. First, in terms of analytical dimensions, while researchers have separately examined livestream product quality governance [1–3] or livestream reputation prediction [4], we integrate quality, reputation, and performance into a unified "quality–reputation–performance" symbiosis framework, revealing their interdependencies and co-evolutionary dynamics. Second, regarding stakeholder coverage, whereas most studies involve two or three stakeholders [1,2,6], our model incorporates four distinct stakeholders, manufacturers, streamers, livestreaming platforms, and regulatory authorities, with the government explicitly modeled as a leading agent. Third, in terms of theoretical approach and methodology, unlike the static or evolutionary game methods predominantly employed in prior livestream governance research, we adopt a stochastic differential game framework that captures the random and dynamic nature of reputation evolution influenced by unpredictable factors such as consumer sentiment and public opinion; moreover, the integration of analytical stochastic differential game solutions with Q-learning-based adaptive simulation enables the simulation of reputation dynamics under representative scenarios, making the evolution of system reputation more intuitive and enhancing the practical applicability of theoretical findings. Fourth, concerning our key findings, we find that there exists a "moderate threshold" of complaint intensity, where collaborative governance achieves maximum effectiveness, and a theoretical condition that the platform's take rate must co-move with its reputation-repair cost coefficient to achieve Pareto optimality, which provides new insights into when and how collaborative governance is most effective. These conditions have not been systematically explored in previous livestream reputation studies. These distinctions collectively highlight the academic value of our work in offering a more comprehensive analytical framework and strategic inspiration for multi-stakeholder collaborative governance of quality–reputation risks in livestream e-commerce.

Nevertheless, this study has several limitations that merit consideration. First, the model considers only four core stakeholders, manufacturers, streamers, livestreaming platforms, and regulatory authorities, and does not include other important participants such as consumers, industry associations, or media oversight bodies, whose roles in reputation governance may also be significant. Second, the collaborative governance relationships among agents are modeled in a relatively simplified manner, without fully capturing the complexity and diversity of real-world interactions, such as information asymmetry, dynamic coalition formation, or conflicting interests among stakeholders. In future research, we will extend this framework by incorporating a broader set of agents and more realistic governance structures, thereby advancing the study of emergency collaborative governance strategies for livestreaming reputation under quality complaint scenarios.

5. Conclusions

5.1. Conclusions

In this study, we examine the multi-agent collaborative governance of livestreaming reputation in the context of quality complaint incidents. A stochastic differential game model is constructed under government regulation, incorporating manufacturers, streamers, livestreaming platforms, and regulatory authorities into a multi-agent collaborative framework. An adaptive multi-agent collaborative response model based on dynamic event triggering and Q-learning is then proposed. The governance mechanisms and effective intervals of multi-agent collaboration are analyzed, leading to collaborative governance strategies for livestreaming reputation from the perspective of quality complaints. The main findings are as follows:

(1) The introduction of government penalty mechanisms consistently motivates all participants in the livestream system to engage in reputation repair efforts and leads to the highest expected value of the system reputation trajectory while increasing the volatility of the reputation trajectory. However, under continuously increasing consumer quality complaint intensity, the theoretical condition under which government penalties enhance reputation repair incentives is that the degree of effort-based penalty reduction must exceed the corresponding thresholds φ' (under decentralized decision-making) and φ'' (under collaborative decision-making).

(2) For manufacturers, collaborative decision-making is more suitable under high commission rates, whereas for streamers and livestreaming platforms, it is more suitable under low commission rates. When the intensity of quality complaints is at a moderate threshold, the benefits of collaborative governance are maximized, prompting the system, under the co-evolution of "quality–reputation–performance," to prefer collaborative mechanisms. Once the complaint intensity deviates from the moderate threshold (either increasing or decreasing), the applicability of collaborative decision-making shrinks, and the system begins to deviate from collaborative mechanisms. However, as streamer commission rates increase, collaborative decision-making becomes less effective in enhancing the reputation repair incentives of streamers and livestreaming platforms.

(3) The theoretical condition under which collaborative decision-making achieves Pareto optimality in terms of the expected value and variance of the system reputation trajectory is that the platform's revenue coefficient (take rate) must move in the same direction as its reputation-repair cost coefficient. Moreover, an increase in streamer commission rates consistently reduces the dominant region of collaborative decision-making in terms of the expected reputation trajectory while expanding it in terms of variance.

(4) Collaborative decision-making can delay the declining trend of the system reputation trajectory caused by increasing initial reputation levels, but this effect is subject to a theoretical condition that the platform's revenue coefficient (take rate) must move in the same direction as its reputation-repair cost coefficient, and the higher the degree of alignment, the stronger the delay effect. Additionally, from the quality complaint perspective, the theoretical condition for maintaining an increasing system reputation trajectory is that, in the absence of government penalties, the intensity of consumer quality complaints remains low. After introducing government penalties, the external conditions for achieving Pareto-optimal reputation trajectories are that the system traffic level is low with complaint intensity at either a low or high level, or that the system traffic level is high with complaint intensity at a moderate threshold. Furthermore, under the co-evolution of "quality–reputation–performance", where total system profit is linearly related to traffic and reputation levels, the system tends to adopt government penalty mechanisms to achieve Pareto-optimal reputation trajectories.

5.2. Managerial insights

Based on the above findings, we derive the following managerial insights for multi-stakeholder collaborative governance of systemic reputational risks in livestream e-commerce.

First, regulatory authorities should adopt differentiated penalty mechanisms with effort-based reduction clauses. Our analysis shows that government penalties effectively enhance reputation recovery incentives only when the penalty reduction coefficient exceeds certain thresholds. In practice, this implies that regulators should design penalty frameworks that reward proactive reputation repair efforts, such as manufacturers' product recalls, streamers' public apologies, and platforms' complaint coordination, by reducing fines proportionally to the demonstrated effort. This approach encourages stakeholders to engage in timely remediation rather than passively awaiting regulatory intervention.

Second, platforms should align their revenue structures with reputation restoration costs. Our findings reveal that collaborative decision-making achieves Pareto optimality in reputation trajectory only when the platform's take rate moves in the same direction as its reputation-repair cost coefficient. This suggests that platforms with higher repair costs should appropriately adjust their take rates to ensure sufficient financial capacity for reputation governance, while platforms with lower costs can maintain more flexible pricing strategies. Platforms should also be mindful of the commission rates streamers charge manufacturers, as higher rates may discourage manufacturers from participating in collaborative governance by reducing their profit margins.

Third, stakeholders should adopt collaborative governance mechanisms when complaint intensity reaches a moderate level. Our results indicate that collaborative decision-making is most effective when complaint intensity is neither too low or too high. When complaints are mild, decentralized self-governance suffices; when complaints escalate to systemic levels, government intervention becomes necessary. Therefore, platforms and regulators should establish early warning systems to monitor complaint growth rates and negative sentiment intensity, enabling timely activation of collaborative governance protocols before systemic risks materialize.

Fourth, the "quality–reputation–performance" symbiosis should be explicitly integrated into governance design. Our analysis demonstrates that product quality, system reputation, and total profit are highly interdependent. Investments in quality improvement and reputation repair generate positive feedback loops that enhance overall system performance. Platforms should therefore incentivize manufacturers to maintain product quality and streamers to uphold trustworthiness, as these efforts collectively contribute to long-term profitability and systemic stability.

Author contributions

Yuexiang Yang: Conceptualization, Supervision, Funding acquisition; Zhenwu Chen: Methodology, Writing–original draft, Writing–review & editing; Ketao Huang: Validation; Xinyi Li: Investigation. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declares that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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