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*Research article*

# Stability analysis and $H_\infty$ control for rough singular systems with Pythagorean functions

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**Abstract:** This paper integrated Pythagorean fuzzy sets, intuitionistic $(\lambda, \eta)$-function rough sets, and singular systems to develop rough singular systems of Pythagorean functions (RSSPFs) and investigate its $H_\infty$ control. Focusing on uncertain information, RSSPFs first adopted Pythagorean fuzzy sets to accurately model uncertainties. Based on rough set theory, RSSPFs were decomposed into a lower-approximation set subsystem and a boundary set subsystem. By combining these two subsystems within the $H_\infty$ control framework for singular systems, a closed-loop control system was constructed to suppress disturbances. Applied to the path-tracking problem of autonomous vehicles, simulation experiments showed that the $H_\infty$ control for RSSPFs ensured the asymptotic stability of all subsystems, and effectively balanced the tracking accuracy and robustness.

**Keywords:** $H_\infty$ control for rough singular systems of Pythagorean functions; fuzzy rough set; admissibility
**Mathematics Subject Classification:** 93C05, 03E72, 93C42

## 1. Introduction

Within the research framework of uncertainty system analysis and intelligent information processing, fuzzy set theory stands as one of the foundational theories that first enabled quantitative descriptions of fuzziness and uncertainty. In 1965, American cybernetic expert Zadeh formally introduced the core concept of fuzzy sets based on classical set theory, establishing a comprehensive theoretical system [1]. Bulgarian scholar Atanassov made significant advancements in 1986 by extending Zadeh's framework and proposing intuitionistic fuzzy set theory [2]. In 2013, Yager further introduced the Pythagorean fuzzy set [3]. This innovation significantly broadened the representation space of fuzzy sets, enabling them to express more complex and extensive uncertainty relationships.

Rough set theory, as a pivotal mathematical framework, was formally introduced by Polish scientist Pawlak in 1982, establishing the foundational framework for uncertainty data processing and knowledge discovery [4]. Professor Kaiquan Shi later extended and refined rough sets by first proposing the concept of S-function rough sets [5]. Subsequently, Rui Guo and Feng Liu developed intuitionistic $(\lambda, \eta)$-function rough sets based on this framework, enabling more in-depth research in the field of rough set theory [6]. The complementary advantages of fuzzy sets and rough sets have been extensively studied, such as rough Pythagorean fuzzy sets [7], which have become a focal point in uncertainty theory research.

Compared with traditional regular systems, singular systems enable a more accurate representation of complex characteristics such as redundancy, coupling, and singularity prevalent in engineering, economics, and control domains.In the design of control system, it is not only required to make the system stable, but also to make the system reach a certain performance index. The $H_\infty$ norm characterizes system performance metrics, and the generated $H_\infty$ control is one of the effective approaches to enhance system robustness [8–10], playing an increasingly vital role in engineering design [11–14].

However, current research on control for singular systems still faces several limitations. On the one hand, most control systems designed by modern control-theory methods based on precise mathematical models often struggle to achieve expected performance, leading to severe degradation of both modeling accuracy and control effectiveness under actual operating conditions [15]. On the other hand, existing control strategies typically rely on fixed system partitions, in which even rules that require no computation are still processed, thereby increasing the computational load [16]. Therefore, given the aforementioned issues, this paper integrates intuitionistic $(\lambda, \eta)$-function rough sets, Pythagorean fuzzy sets, and singular systems. The $H_\infty$ control for RSSPFs enables precise modeling of fuzzy uncertain information through Pythagorean fuzzy sets. Meanwhile, leveraging the concepts of upper and lower approximation sets in rough sets, it divides complex control systems into deterministic subsystems and uncertain subsystems; dynamic adjustment of membership thresholds $\lambda$ and nonmembership thresholds $\eta$ enables accurate demarcation of uncertainty boundaries. This methodology effectively captures fuzziness and ambiguity in sensor data, facilitates the design of targeted control strategies, and enhances the overall system performance.

In autonomous vehicle path-following scenarios, the vehicle kinematic model incorporates nonholonomic constraints, while variables such as tire forces and road adhesion coefficients exhibit fast-changing dynamics. These constraints and fast-changing behaviors inherently manifest as a combination of algebraic equations and differential equations; employing a singular system model allows for the natural preservation of these constraints without requiring simplification or elimination [17]. This approach not only overcomes the limitations of traditional single-control methods in handling uncertainty and achieving robust control, but also provides a novel systematic solution for autonomous vehicle path following, as well as offering valuable insights for research on multi-source uncertainty information processing and multi-theory integrated control, thereby holding significant theoretical importance and engineering application value.

The structure of this paper is as follows: Section 2 introduces the necessary preliminary knowledge; Section 3 establishes the $H_\infty$ for RSSPF and models the autonomous driving path tracking problem; next, a proof of admissibility for this system is provided, demonstrating its theoretical feasibility; Section 4 demonstrates practical feasibility through an example of autonomous vehicle path tracking; and the final section summarizes the entire paper and outlines future research directions.

## 2. Prerequisite knowledge

This paper defines $D$ as the function domain, $Q = \{q_1, q_2 \cdots q_m\} \subset D$ as the function set, $R$ as the equivalence relation on $D$, and $[z]_R$ as the function equivalence class.

**Definition 2.1.** *[18–21] Consider two subsets:*

$$R_-(Q) = \cup \{z \in D : [z]_R \subseteq Q\},$$
$$R^-(Q) = \cup \{z \in D : [z]_R \cap Q \neq \emptyset\}.$$

*where $R_-(Q)$ is the $R$ lower approximation of $D$, $R^-(Q)$ is the $R$ upper approximation of $Q$, $BN_R(Q) = R^-(Q) - R_-(Q)$ is the boundary, and the set of them is called the rough set of function $Q \subset D$.*

**Definition 2.2.** *[22, 23] $\tilde{D} = \{(z, \mu_{\tilde{D}}(z), \nu_{\tilde{D}}(z)) | z \in D\}$ is the Pythagorean fuzzy set, where $\mu_{\tilde{D}}, \nu_{\tilde{D}} : D \to [0, 1]$, $\mu_{\tilde{D}}(z)$ and $\nu_{\tilde{D}}(z)$ are the membership degree and nonmembership degree of element $z$ with respect to $\tilde{D}$, $0 \leq \mu^2_{\tilde{D}}(z) + \nu^2_{\tilde{D}}(z) \leq 1$, and $\pi_{\tilde{D}}(z)$ is the hesitation of element $z$ with respect to $\tilde{D}$, where $\pi_{\tilde{D}}(z) = \sqrt{1 - \mu^2_{\tilde{D}}(z) - \nu^2_{\tilde{D}}(z)}$.*

Based on the above knowledge and the concept of intuitionistic $(\lambda, \eta)$-function rough set [6], a new set of the Pythagorean $(\lambda, \eta)$-function rough set is proposed.

**Definition 2.3.**

$$Q^1_\lambda = \{z \in D | \mu_{\tilde{D}}(z) \geq \lambda \text{ and } z \notin Q\},$$

$$Q^2_\eta = \{z \in D | \nu_{\tilde{D}}(z) \geq \eta \text{ and } z \in Q\},$$

$$Q^3 = \{z \in D | 0 < \mu_{\tilde{D}}(z) < \lambda \text{ and } 0 < \nu_{\tilde{D}}(z) < \eta \text{ and } z \notin Q\},$$

*are defined as Pythagorean $\lambda$-migration set, Pythagorean $\eta$-exogenous set, and Pythagorean swing set of $Q$ about the Pythagorean fuzzy set $\tilde{D}$. $(Q^1_\lambda, Q^2_\eta, Q^3)$ is the Pythagorean $(\lambda, \eta)$-migration set of $Q$ about the Pythagorean fuzzy set $\tilde{D}$, where $0 < \lambda \leq 1$, $0 < \eta \leq 1$, then $Q^\times = (Q - Q^2_\eta) \cup Q^1_\lambda \cup Q^3$ is called the Pythagorean $(\lambda, \eta)$-migration set of $Q$ about the Pythagorean fuzzy set $\tilde{D}$, respectively.*

**Definition 2.4.**

$$R_-(Q^\times) = \{z | z \in D, [z]_R \in Q^\times\},$$
$$R^-(Q^\times) = \{z | z \in D, [z]_R \cap Q^\times \neq \emptyset\}$$

*are called the Pythagorean $(\lambda, \eta)$-lower approximation and the Pythagorean $(\lambda, \eta)$-upper approximation of $Q$ about Pythagorean fuzzy set $\tilde{D}$, respectively, and $BN_R(Q^\times) = R^-(Q^\times) - R_-(Q^\times)$ is called the boundary of the Pythagorean $(\lambda, \eta)$- function rough set.*

**Definition 2.5.** *The set pair formed by $R_-(Q^\times)$ and $R^-(Q^\times)$ is termed the Pythagorean $(\lambda, \eta)$-function rough set of $Q \subset D$, denoted as $(R_-(Q^\times), R^-(Q^\times))$.*

## 3. $H_\infty$ control for rough singular systems of Pythagorean functions

### 3.1. $H_\infty$ control for rough singular systems of Pythagorean functions

According to the Pythagorean $(\lambda, \eta)$-function rough set, the $H_\infty$ control for RSSPF is established.

Given $n \times l$ finite function knowledge base $K_{ji} = (D_{ji}, R_{ji})$, (where: $i = 1, 2, 3 \cdots l$; $j = 1, 2, 3 \cdots n$), for each function subset $Q_{ji} \subset D_{ji}$ and an equivalence relation $R_{ji}$, which can be divided into sets $Q_{ji}$ according to the description of the basic set of $R_{ji}$; then, according to the actual situation, the Pythagorean fuzzy set $\tilde{D}_{ji}$ is established on $D_{ji}$, and then the values of $\lambda$ and $\eta$ are determined according to the needs of the specific problem,so as to determine the corresponding Pythagorean $(\lambda, \eta)$-the lower approximation, the upper approximation, and the boundary set of the fuzzy set of the function. Then, two functional spaces are formed in the lower approximation and the boundary set, respectively, and two sets of corresponding classical singular systems with the same dimension are established. After completing the construction of these two systems, this paper further introduces the $H_\infty$ control strategy to achieve robust stabilization and disturbance suppression for the RSSPFs.

Based on the concept from literature [24, 25], the present work adopts the Pythagorean $(\lambda, \eta)$-function rough set, defined in Definitions 2.3–2.5, as the rule antecedent. Specifically, the $i$ rule premise is characterized by the rough approximation structure $((R_-)(Q_{ji}^\times), (R^-)(Q_{ji}^\times))$, for the $j$ antecedent variable $z_j$, and the overall degree of similarity between it and the lower approximation set as well as the boundary set is

$$\rho_{ji}\left(z_j\right) = \alpha \frac{Card\left(\left(R_-(Q_{ji}^\times)\right) \cap \left[z_j\right]_{R_{ji}}\right)}{Card\left(\left(R_-(Q_{ji}^\times)\right)\right)} + (1-\alpha)\frac{Card\left(BN_R\left(Q_{ji}^\times\right) \cap \left[z_j\right]_{R_{ji}}\right)}{Card\left(BN_R\left(Q_{ji}^\times\right)\right)},$$

where $\alpha = \frac{\lambda}{\lambda+\eta}$, $\left[z_j\right]_{R_{ji}}$ is an equivalence class under the equivalence relation $R_{ji}$, which is the set of all elements indistinguishable from $z_j$. $Card\left(\left(R_-(Q_{ji}^\times)\right) \cap \left[z_j\right]_{R_{ji}}\right)$ denotes the number of elements belonging to $R_-\left(Q_{ji}^\times\right)$ in the equivalence class, and $Card\left(\left(R_-(Q_{ji}^\times)\right)\right)$ represents the total number of elements in $R_-\left(Q_{ji}^\times\right)$. $Card\left(BN_R\left(Q_{ji}^\times\right) \cap \left[z_j\right]_{R_{ji}}\right)$ denotes the number of elements belonging to $BN_R\left(Q_{ji}^\times\right)$ in the equivalence class, and $Card\left(BN_R\left(Q_{ji}^\times\right)\right)$ represents the total number of elements in $BN_R\left(Q_{ji}^\times\right)$. $\rho_i(z) = \prod_{j=1}^{n} \rho_{ji}\left(z_j\right) \geq 0$ denotes the matching degree of $z = (z_1, z_2 \cdots z_n)^T$ with rule $i$.

To accomplish this objective, the following system forms are presented:

If $z_j$ matches $(R_-(Q_{ji}^\times), R^-(Q_{ji}^\times))$, then establish a $H_\infty$ control for RSSPF global model

$$\sum_{i=1}^{l} h_i(z) E_i^1 \dot{x}_{S_1}(t) = \sum_{i=1}^{l} h_i(z)(A_i^1 x_{S_1}(t) + B_{1i}^1 u_{S_1}(t) + B_{2i}^1 \omega_1(t))$$

$$\sum_{i=1}^{l} h_i(z) E_i^2 \dot{x}_{S_2}(t) = \sum_{i=1}^{l} h_i(z)(A_i^2 x_{S_2}(t) + B_{1i}^2 u_{S_2}(t) + B_{2i}^2 \omega_2(t))$$

$$y(t) = \sum_{i=1}^{l} h_i(z)\left(C_i^1 x_{S_1}(t) + C_i^2 x_{S_2}(t)\right)$$

where $A_i^1, A_i^2 \in R^{n\times n}$, $B_i^1, B_i^2$, $C_i^1, C_i^2$ are the dimensionality parameters, $z_1, z_2 \cdots z_n$ are the premise variables, $x_{S_1}(t), x_{S_2}(t) \in R^n$ are the state vectors of the system, $u_{S_1}(t), u_{S_2}(t) \in R^m$ are the controlled inputs, $y_{S_1}(t), y_{S_2}(t) \in R^q$ are the controlled outputs, and $\omega_1(t), \omega_2(t)$ are the external disturbances. $h_i(z) = \frac{\rho_i(z)}{\sum\limits_{i=1}^{l}\rho_i(z)} \geq 0$, $\sum\limits_{i=1}^{l} h_i(z) = 1$.

If $x(t) = \begin{pmatrix} x_{S_1}(t) \\ \dot{x}_{S_1}(t) \\ x_{S_2}(t) \\ \dot{x}_{S_2}(t) \end{pmatrix}$, then the above system is transformed into

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{l} h_i(z)(A_i x(t) + B_{1i}u(t) + B_{2i}\omega(t)), \\ y(t) &= \sum_{i=1}^{l} h_i(z)C_i x(t). \end{aligned} \tag{3.1}$$

where $E = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $A_i = \begin{pmatrix} 0 & I & 0 & 0 \\ A_i^1 & -E_i^1 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & A_i^2 & -E_i^2 \end{pmatrix}$, $B_{1i} = \begin{pmatrix} 0 & 0 \\ B_{1i}^1 & 0 \\ 0 & 0 \\ 0 & B_{1i}^2 \end{pmatrix}$,
$B_{2i} = \begin{pmatrix} 0 \\ B_{2i}^1 \\ 0 \\ B_{2i}^2 \end{pmatrix}$, $C_i = \begin{pmatrix} 0 & C_i^1 & 0 & C_i^2 \end{pmatrix}$, $u(t) = \begin{pmatrix} u_{S_1}(t) \\ u_{S2}(t) \end{pmatrix}$.

This system dynamically adjusts the degree of uncertainty through parameters $\lambda$ and $\eta$, while leveraging the combination of Pythagorean fuzzy sets and functional rough sets to achieve a precise characterization of dual uncertainties. By integrating deterministic and uncertain information via a comprehensive matching function, independent singular subsystems are constructed for the lower approximation set and boundary set respectively. This ultimately merges both components into a unified global system while preserving information integrity and ensuring smooth transitions, achieving seamless integration from rough partitioning to robust control and significantly enhancing the modeling accuracy and control smoothness of the system in uncertain regions.

Below is a feasibility analysis of $H_\infty$ control for RSSPFs in practical scenarios.

Example 1: In autonomous vehicle path tracking, let $D_{ji}$ denote the functional domain of the path at a given moment. The Pythagorean fuzzy set $\tilde{D}_{ji}$ is defined as follows:

$$\tilde{D}_{ji} = \left\{ \left(z_j, \mu_{\tilde{D}_{ji}}\left(z_j\right), \nu_{\tilde{D}_{ji}}\left(z_j\right)\right) | z_j \in D_{ji} \right\},$$

where $\mu_{\tilde{D}}\left(z_j\right)$ denotes membership degree, indicating the degree of ideal tracking state, and $\nu_{\tilde{D}}\left(z_j\right)$ denotes nonmembership degree, indicating the degree to which it does not represent an ideal tracking state.

Select $m$ ideal state trajectories $Q_{ji} = \{q_{1i}, q_{2i} \cdots q_{mi}\} \subset D_{ji}$ from the function domain. In advance, we divide all states into several fixed "precision regions" based on the magnitude of the tracking error. States falling within the same region are considered indistinguishable by the system. This constitutes

the equivalence relation $R_{ji}$; these regions are known as equivalence classes $\left[z_j\right]_{R_{ji}}$.
So $R_{-}(Q_{ji}^{\times}) = \left\{z_j | z_j \in D_{ji}, \left[z_j\right]_{R_{ji}} \subseteq Q_{ji}^{\times}\right\}$, which indicates that it is determined to be in an ideal state;
$(R^{-})(Q_{ji}^{\times}) = \left\{z_j | z_j \in D_{ji}, \left[z_j\right]_{R_{ji}} \cap Q_{ji}^{\times} \neq \emptyset\right\}$;
$BN_{R_{ji}}(Q_{ji}{}^{\times}) = R_{-}(Q_{ji}^{\times}) - R^{-}(Q_{ji}^{\times})$, which indicates uncertainty about whether it is an ideal state.

According to the vehicle path tracking error model presented in the literature [26],

$x(t) = \begin{pmatrix} e_{d1}(t) & \dot{e}_{d1}(t) & e_{\varphi 1}(t) & \dot{e}_{\varphi 1}(t) & e_{d2}(t) & \dot{e}_{d2}(t) & e_{\varphi 2}(t) & \dot{e}_{\varphi 2}(t) \end{pmatrix}^T$,

where $e_{d1}(t)$ is denoted lateral error when the stiffness is equal to its nominal value, $\dot{e}_{d1}(t)$ is denoted lateral error rate when the stiffness is equal to its nominal value, $e_{\varphi 1}(t)$ is denoted heading angle error when the stiffness is equal to its nominal value, and $\dot{e}_{\varphi 1}(t)$ is denoted heading angle error rate when the stiffness is equal to its nominal value; $e_{d1}(t)$, $\dot{e}_{d1}(t)$, $e_{\varphi 1}(t)$, $\dot{e}_{\varphi 1}(t)$ are denoted the sum of the two errors and their rate of change under the stiffness condition of perturbation.

$u(t) = \begin{pmatrix} \delta_f(t) & \Delta M_z(t) \end{pmatrix}^T$,

where $\delta_f(t)$ is denoted the front wheel rotation angle, and $\Delta M_z(t)$ is denoted the additional yaw moment.

$A_i^1 = \begin{pmatrix} -\frac{C_{a1}+C_{a2}}{mv_i} & -v_i - \frac{aC_{a1}-bC_{a2}}{mv_i} \\ -\frac{aC_{a1}-bC_{a2}}{I_z v_i} & -\frac{a^2C_{a1}+b^2C_{a2}}{I_z v_i} \end{pmatrix}$, $A_i^2 = \begin{pmatrix} -\frac{C_{a1\Delta}+C_{a2\Delta}}{mv_i} & -v_i - \frac{aC_{a1\Delta}-bC_{a2\Delta}}{mv_i} \\ -\frac{aC_{a1\Delta}-bC_{a2\Delta}}{I_z v_i} & -\frac{a^2C_{a1\Delta}+b^2C_{a2\Delta}}{I_z v_i} \end{pmatrix}$,

$E^1 = \begin{pmatrix} 0 & \frac{C_{a1}+C_{a2}}{m} \\ 0 & \frac{aC_{a1}-bC_{a2}}{I_z} \end{pmatrix}$, $E^2 = \begin{pmatrix} 0 & \frac{C_{a1\Delta}+C_{a2\Delta}}{m} \\ 0 & \frac{aC_{a1\Delta}-bC_{a2\Delta}}{I_z} \end{pmatrix}$; $B^1 = \begin{pmatrix} \frac{2C_{a1}}{m} \\ \frac{2aC_{a1}}{I_z} \end{pmatrix}$, $B^2 = \begin{pmatrix} \frac{2C_{a1\Delta}}{m} \\ \frac{2aC_{a1\Delta}}{I_z} \end{pmatrix}$;

$B_{2i} = \begin{pmatrix} 0 & 0 & \frac{aC_{a1}-bC_{a2}}{mv_i} & \frac{a^2C_{a1}+b^2C_{a2}}{I_z v_i} & 0 & 0 & \frac{a^2C_{a1}+b^2C_{a2}}{I_z v_i} & \frac{a^2C_{a1\Delta}+b^2C_{a2\Delta}}{I_z v_i} \end{pmatrix}^T$, where $\dot{\theta}(t)$ is the path curvature.
(Symbol meanings are shown in Table 1).

**Table 1.** Vehicle parameters.

| Parameter name | Parameter values |
|---|---|
| Vehicle weight $m/kg$ | 1723 |
| Distance from vehicle center of mass to front axle $a/m$ | 1.232 |
| Distance from vehicle center of mass to rear axle $b/m$ | 1.468 |
| Lateral stiffness of front wheel $C_{a1}/\left(N \cdot m \cdot rad^{-1}\right)$ | 66900 |
| Lateral stiffness of rear wheel $C_{a2}/\left(N \cdot m \cdot rad^{-1}\right)$ | 62700 |
| Moment of inertia $I_z/kg \cdot m^2$ | 4175 |

The vehicle autonomous driving path tracking problem is abstracted into this model. By employing the Pythagorean fuzzy set's square sum constraint, the tolerance for conflicting information is relaxed, enabling more accurate quantification of sensor noise and cognitive uncertainty under complex operating conditions. Combined with dynamic threshold equivalence class partitioning from rough sets, the approach effectively prevents frequent control law switching and actuator jitter caused by minor disturbances. The singular system $H_\infty$ controller constructed on this foundation demonstrates theoretically guaranteed robust suppression capabilities against external disturbances such as lateral wind, ultimately achieving synergistic improvements in tracking accuracy and driving smoothness.

The admissibility of the system is derived with the help of the Lyapunov function and related inequalities. On this basis, the controller gain is solved by linear matrix inequality (LMI)[27], and finally the obtained results are applied to the actual scenario.

*3.2. The admissibility of $H_\infty$ control for rough singular systems of Pythagorean functions*

**Definition 3.1.** *[28, 29]*

*(1)* $E\dot{x}(t) = \sum_{i=1}^{l} h_i(z)A_i x(t)$ *is consistent and regular, if there is a constant s, so that* $\det(sE - \sum_{i=1}^{l} h_i(z)A_i) \neq 0$.

*(2) The above system is consistent impulse free, if* $\deg\det(sE - \sum_{i=1}^{l} h_i(z)A_i) = rankE$.

*(3) Consistent regular and impulse free self-consistent systems are asymptotically stable, if* $\frac{dV(x(t))}{dt} < 0$, *where* $V(x(t)) = x^T(t)E^T X x(t)$.

**Definition 3.2.** *[30] If the system is regular, impulse free and stable, then the system is said to be admissible.*

**Definition 3.3.** *(Schur complement lemma) [31, 32] For a given n-dimensional symmetric matrix*

$$S = \begin{pmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{pmatrix},$$

*where* $S_1 \in R^{r\times r}$, $S_3 \in R^{(n-r)\times(n-r)}$, $S_2 \in R^{r\times(n-r)}$, $(1 < r < n)$. *The following conditions are equivalent:*
*(1)* $S < 0$;
*(2)* $S_1 < 0,\ S_3 - S_2^T S_1^{-1} S_2 < 0$;
*(3)* $S_2 < 0,\ S_1 - S_2 S_3^{-1} S_2^T < 0$.

**Definition 3.4.** *[12] If* $\lambda_i\ (i = 1, 2, \cdots, rank(A))$ *is the positive eigenvalue of the matrix* $A^T A$, *then* $\sqrt{\lambda_i}$ *is called the nonzero singular value of matrix A, denoted as* $\sigma_i = \sqrt{\lambda_i}$.

**Definition 3.5.** *[12] For any matrix A, let* $rank(A) = r$, $\sigma_1, \sigma_2, \cdots \sigma_r$ *be the singular values of A, then there exist an n-dimensional orthogonal matrix U and an m-dimensional orthogonal matrix V, such that* $A = U \begin{pmatrix} \sum & 0 \\ 0 & 0 \end{pmatrix} V^T, \sum = diag(\sigma_1, \sigma_2, \cdots \sigma_r)$.

**Definition 3.6.** *[11] The* $H_\infty$ *stability of the system must satisfy the following two conditions:*

*(1) The system is admissible when* $\omega(t) = 0$.

*(2) For a given* $\gamma > 0$, *under zero initial conditions, the system satisfies* $\|y(t)\|_2 < \gamma\|\omega(t)\|_2$ *for all* $\omega(t) \in L_i[0, \infty)$.

**Theorem 1.** *For* $E\dot{x}(t) = \sum_{i=1}^{l} h_i(z)(A_i x(t) + B_{2i}\omega(t))$, *the system is admissible, if*

*(1) There are invertible real matrices* $P, Q \in R^{n\times n}$, *so that* $PE_iQ = \begin{pmatrix} I_{r_i} & 0 \\ 0 & 0 \end{pmatrix}$, *where* $r_i = rank(E_i)$, $PA_i^a Q = \begin{pmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{pmatrix}$, *and* $A_{i22}$ *is invertible;*

*(2) There exists $X \in R^{n\times n}$, satisfied $EX^T = XE^T \geq 0$,*

$$\begin{bmatrix} XA_i^T + A_iX^T & B_{2i} & XC_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, i = 1, 2, \cdots l. \tag{3.2}$$

*Proof.* Prove regularity and pulselessness first.

We do singular value decomposition of $E$,

$PEQ = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, where $P = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$, $Q = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$,

so $PA_iQ = \begin{pmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{pmatrix}$.

We can get

$\det\left(P\left(sE - \sum\limits_{i=1}^{l} h_i(z)A_i\right)Q\right) = s\sum\limits_{i=1}^{l} h_i(z)\det(-A_{i22})\det\left(sI_r - A_{i11} + A_{i12}(A_{i22})^{-1}A_{i21}\right)$. So when $s$ is not the eigenvalues of $sI_r - A_{i11} + A_{i12}(A_{i22})^{-1}A_{i21}$ and $A_{i22}$ is invertible, then $\det(sE - \sum\limits_{i=1}^{l} h_i(z)A_i) \neq 0$, hence the system is regular. It can also be seen from the above equation $\deg\det(sE - \sum\limits_{i=1}^{l} h_i(z)A_i) = rank(E)$, therefore, the system is impulse free.

Stability: Let $V = x(t)^T E^T P x(t)$.

When $\omega(t) = 0$,

$$\dot{V} = \sum_{i=1}^{l} h_i(z)\, x(t)^T \left(A_i^T P + P^T A_i\right) x(t).$$

Let $X = P^{-T}$, so $A_i^T P + P^T A_i = X^{-1}\left(XA_i^T + A_iX^T\right)X^{-T}$. As shown in (3.2), we can get $XA_i^T + A_iX^T < 0$, also $h_i(z) \geq 0$, $\sum\limits_{i=1}^{l} h_i(z) = 1$,, so $\dot{V} < 0$.

When $\omega(t) \neq 0$,

$$J = \sum_{i=1}^{l} h_i(z)\begin{pmatrix} x(t) \\ \omega(t) \end{pmatrix}^T \begin{pmatrix} X^{-1}\left(XA_i^T + A_iX^T\right)X^{-T} + C_i^T C_i & X^{-1}B_{2i} \\ * & -\gamma^2 I \end{pmatrix}\begin{pmatrix} x(t) \\ \omega(t) \end{pmatrix}.$$

By using the Schur complement lemma, (3.2) can be transformed into

$$\begin{pmatrix} XA_i^T + A_iX^T & B_{2i} \\ * & -\gamma^2 I \end{pmatrix} + \begin{pmatrix} XC_i^T C_i X^T & 0 \\ 0 & 0 \end{pmatrix} < 0,$$

so

$$\begin{pmatrix} X^{-1}\left(XA_i^T + A_iX^T\right)X^{-T} + C_i^T C_i & X^{-1}B_{2i} \\ * & -\gamma^2 I \end{pmatrix} < 0.$$

Also $h_i(z) \geq 0$, $\sum\limits_{i=1}^{l} h_i(z) = 1$, thus $J < 0$.

To $\forall T > 0$,

$$\int_0^T \left(y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t)\right) dt + V(x(T)) < 0,$$

so

$$V(x(T)) + \int_0^T y(t)^T y(t)\, dt < \gamma^2 \int_0^T \omega(t)^T \omega(t)\, dt.$$

When $T \to \infty$, we can get $\|y(t)\|_2^2 < \gamma^2 \|\omega(t)\|_2^2$, therefore $\|y(t)\|_2 < \gamma \|\omega(t)\|_2$.

In conclusion, the open-loop system is admissible.

□

The paralled distributed compensations (for short PDC), $u(t) = \sum_{i=1}^{l} h_i(z) L_i x(t)$, $L_i \in R^{n\times q}$, are designed to obtain a closed-loop system.

Let $x(t) = \begin{pmatrix} x_{S_1}(t) \\ \dot{x}_{S_1}(t) \\ x_{S_2}(t) \\ \dot{x}_{S_2}(t) \end{pmatrix}$, then

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{l} \sum_{j=1}^{l} h_i(z) h_j(z) [(A_i + B_{1i} L_j) x(t) + B_{2i} \omega(t)] \\ y(t) &= \sum_{i=1}^{l} h_i(z) C_i x(t). \end{aligned} \tag{3.3}$$

Next, the system is admissible to be certified.

**Theorem 2.** *The above system is admissible if*

*(1) There are invertible real matrices $P, Q \in R^{n\times n}$, so that $PE_iQ = \begin{pmatrix} I_{r_i} & 0 \\ 0 & 0 \end{pmatrix}$, where $r_i = rank(E_i)$, $PA_i^a Q = \begin{pmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{pmatrix}$, and $A_{i22}$ is invertible;*

*(2) There exists $X \in R^{n\times n}$, satisfied $EX^T = XE^T \geq 0$,*

$$\phi_{ii} = \begin{bmatrix} \varphi_{ii} & B_{2i} & XC_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, i = 1, 2, \cdots l \tag{3.4}$$

$$\phi_{ij} + \phi_{ji} = \begin{bmatrix} \varphi_{ij} + \varphi_{ji} & B_{2i} + B_{2j} & XC_i^T + XC_j^T \\ * & -2\gamma^2 I & 0 \\ * & * & -2I \end{bmatrix} < 0, i < j \leq l \tag{3.5}$$

*where*

$\varphi_{ii} = XA_i^T + A_i X^T + N_i^T B_{1i}^T + B_{1i} N_i$,

$\varphi_{ij} + \varphi_{ji} = XA_i^T + A_i X^T + N_j^T B_{1i}^T + B_{1i} N_j + XA_j^T + A_j X^T + N_i^T B_{1j}^T + B_{1j} N_i$

$N_i = L_i X^T$.

*Proof.* According to Theorem 1, the system is regular and impulse free.

Also, when $\omega(t) \neq 0$,

$$\dot{V} = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z)h_j(z)\, x(t)^T \left(\left(A_i + B_{1i}L_j\right)^T P + P^T\left(A_i + B_{1i}L_j\right)\right) x(t).$$

Let $X = P^{-T}$, $N_i = L_i X^T$, so

$$\left(A_i + B_{1i}L_j\right)^T P + P^T\left(A_i + B_{1i}L_j\right) = X^{-1}\varphi_{ij}X^{-T},$$

$$\dot{V} < \sum_{i=1}^{l} h_i^2(z)\varphi_{ii} + \sum_{i<j} h_i(z)\, h_j(z)\left(\varphi_{ij} + \varphi_{ji}\right).$$

As shown in (3.4)(3.5), $\varphi_{ii} < 0$, $\varphi_{ij} + \varphi_{ji} < 0$, thus $\dot{V} < 0$.

When $\omega(t) \neq 0$,

$$J = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z)h_j(z) \begin{pmatrix} x(t) \\ \omega(t) \end{pmatrix}^T \begin{pmatrix} X^{-1}\varphi_{ij}X^{-T} + C_i^T C_i & X^{-1}B_{2i} \\ * & -\gamma^2 I \end{pmatrix} \begin{pmatrix} x(t) \\ \omega(t) \end{pmatrix}.$$

Let $\phi_{ij} = \begin{bmatrix} \varphi_{ij} & B_{2i} & XC_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix}$,

$$\sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z)\, h_j(z)\, \phi_{ij} = \sum_{i=1}^{l} h_i^2(z)\phi_{ii} + \sum_{i<j} h_i(z)\, h_j(z)\left(\phi_{ij} + \phi_{ji}\right),$$

according to (3.4)(3.5), $\sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z)\, h_j(z)\, \phi_{ij} < 0$.

By using the Schur complement lemma for $\phi_{ij}$, we can get $J < 0$. Finally, $\|y(t)\|_2 < \gamma\|\omega(t)\|_2$.

In conclusion, the closed-loop system is admissible.

□

## 4. Path tracking of autonomous vehicle

### 4.1. Numerical simulation

With the rapid advancement of artificial intelligence and intelligent manufacturing technologies, autonomous driving has become a pivotal direction for the automotive industry's transformation and upgrading. Autonomous driving systems primarily encompass advanced technologies such as environmental perception, high-precision map positioning, decision-making, path planning, and path tracking [33]. In recent years, numerous scholars have conducted extensive research on path tracking control for autonomous vehicles, including feedforward nonlinear model predictive controllers [34], dual heuristic planning-based path tracking control methods using rolling optimization [35], autonomous drift-based path tracking control methods for autonomous vehicles [36], and weighted coefficient linear quadratic regulator (LQR) rear wheel steering angle control algorithms [37]. Compared to other methods, the LQR control algorithm utilizes dynamic models, enabling more precise system performance and stable path tracking [38, 39]. However, this approach demonstrates limited adaptability to varying driving conditions.

Set initial value to zero, $e_d = \frac{e_{d1}+e_{d2}}{2}$, $e_\varphi = \frac{e_{\varphi 1}+e_{\varphi 2}}{2}$, $z = \left(e_d, e_\varphi\right)^T$. Based on Table 1 in reference [26], equivalence classes for $e_d$ and $e_\varphi$ are defined, and the matching degree is defined as:

$$\rho_{11}\left(e_d\right) = \begin{cases} 1 & \left|e_d\right| \le 0.125 \\ 1.2 - 1.6\left|e_d\right| & 0.125 < \left|e_d\right| \le 0.25 \\ 0 & \left|e_d\right| > 0.25 \end{cases},$$

$$\rho_{12}\left(e_d\right) = 1 - \rho_{11}\left(e_d\right),$$

$$\rho_{21}\left(e_\varphi\right) = \begin{cases} 1 & \left|e_d\right| \le 0.125 \\ 1.2 - 2.29\left|e_d\right| & 0.0873 < \left|e_d\right| \le 0.1745 \\ 0 & \left|e_d\right| > 0.1745 \end{cases},$$

$$\rho_{22}\left(e_\varphi\right) = 1 - \rho_{21}\left(e_\varphi\right),$$

$\rho_1 = \rho_{11}\left(e_d\right)\rho_{12}\left(e_d\right)$, $\rho_2 = \rho_{21}\left(e_\varphi\right)\rho_{22}\left(e_\varphi\right)$, $h_1 = \frac{\rho_1}{\rho_1+\rho_2}$, $h_2 = 1 - h_1$.

According to Example 1 and Table 1, take the following control parameters:

$E = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$, where $E_1 = E_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$; $A_1 = \begin{pmatrix} A_{11} & 0 \\ 0 & A_{12} \end{pmatrix}$, where $A_{11} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -7.5218 & -9.4415 & 0 & 75.2176 \\ 0.2305 & -5.6686 & 0 & -2.3049 \end{pmatrix}$, $A_{12} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -7.8965 & -9.7894 & 0 & 75.2176 \\ 0.2792 & -5.8826 & 0 & -2.7698 \end{pmatrix}$; $A_2 = \begin{pmatrix} A_{21} & 0 \\ 0 & A_{22} \end{pmatrix}$, where $A_{21} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3.7609 & -19.7208 & 0 & 75.2176 \\ 0.1153 & -2.8343 & 0 & -2.3049 \end{pmatrix}$,

$A_{22} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3.9483 & -19.9842 & 0 & 75.7146 \\ 0.1396 & -2.9413 & 0 & -2.7698 \end{pmatrix}$;

$$B_{11} = B_{12} = \begin{pmatrix} 0 & 0 & 77.6553 & 39.4830 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 78.7792 & 40.5674 \end{pmatrix}^T;$$

$$B_{21} = \begin{pmatrix} 0 & 0 & -0.5294 & 0.5669 & 0 & 0 & -0.4769 & 0.4772 \end{pmatrix}^T,$$

$$B_{22} = \begin{pmatrix} 0 & 0 & -0.2647 & 0.2834 & 0 & 0 & -0.2384 & 0.2386 \end{pmatrix}^T,$$

$$C_1 = \begin{pmatrix} 0 & 0 & 0.7 & 0.4 & 0 & 0 & 0.5 & 1 \end{pmatrix},$$

$$C_2 = \begin{pmatrix} 0 & 0 & -1 & -0.8 & 0 & 0 & -0.4 & -0.9 \end{pmatrix}.$$

Let $\gamma = 0.1$, and using LMI, the controller gain can be obtained as:

$$L_1 = \begin{pmatrix} -3.4488 & -10.0425 & -3.4194 & -2.9117 & -2.8949 & -11.0365 & -2.8546 & -4.6752 \\ 9.4321 & 37.1831 & 4.9298 & 6.4506 & 3.7664 & 14.7176 & 1.8647 & 2.9988 \end{pmatrix},$$

$$L_2 = \begin{pmatrix} -2.9622 & -8.3712 & -3.0890 & -2.7393 & -2.4931 & -9.5034 & -2.5294 & -4.2026 \\ 8.0944 & 32.0233 & 4.3217 & 5.6286 & 3.1666 & 12.5976 & 1.6354 & 2.5348 \end{pmatrix}.$$

Figures 1 and 2 show the state response plots of both the open-loop system and the closed-loop system controlled by the controller.

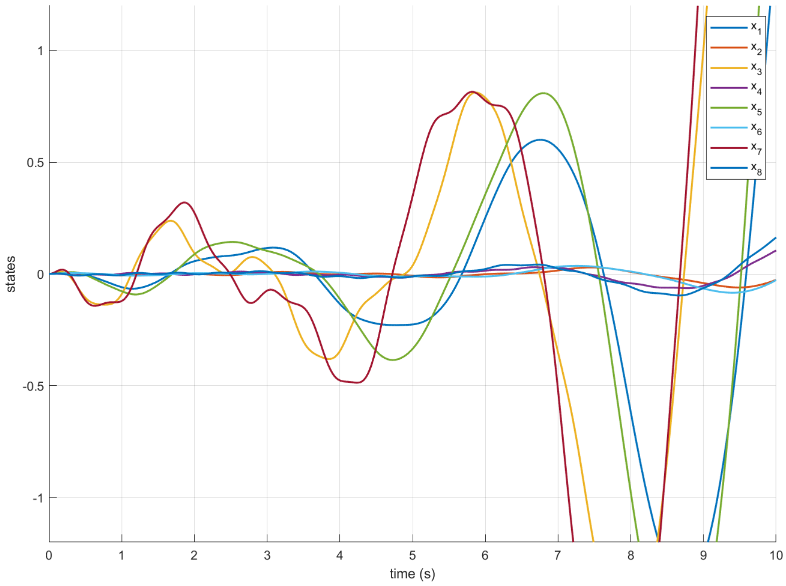


**Figure 1.** Open-loop system image.

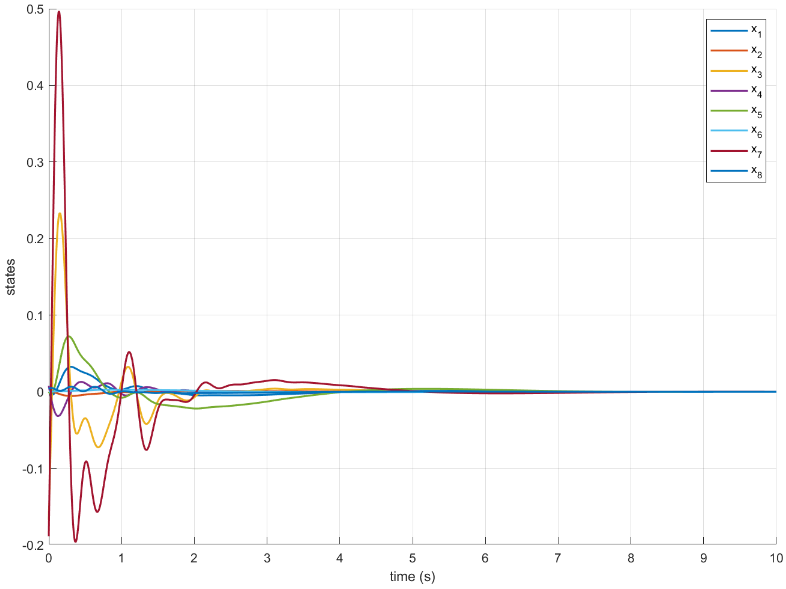


**Figure 2.** Closed-loop system image.

This section uses autonomous vehicle path tracking as a typical scenario to demonstrate the effectiveness of the proposed $H_\infty$ control for the RSSPFs method. By substituting real vehicle parameters into the model, a closed-loop controller was successfully derived, which demonstrates theoretical feasibility.

### *4.2. Dynamic adjustment comparison*

To investigate the influence of the Pythagorean rough set parameters, we set three representative cases: $\alpha = 0.2, 0.5, 0.8$, corresponding to different trade-offs between membership and non-membership degrees. The figures validate the active intervention capability of the proposed Pythagorean rough set rule antecedent.

Figures 3 and 4 respectively illustrate the time-domain response curves of the lateral error and heading angle error for the vehicle path-following system under different values of $\alpha$. In the absence of control input, both the lateral error and the heading angle error exhibit exponential divergence over time, indicating that the nominal vehicle dynamics model itself is inherently unstable. Without the application of active control, the vehicle will rapidly deviate from the desired trajectory, thereby validating the necessity of incorporating an $H_\infty$ robust controller. After incorporating the designed $H_\infty$ state feedback controller, the error curves under all three parameter configurations converge to zero within a finite time, thereby fully demonstrating the effectiveness of the proposed control strategy. This phenomenon validates that the Pythagorean rough set parameters $(\lambda, \eta)$ possess an active regulatory capability over the system's dynamic performance, providing a theoretical basis for flexibly configuring controller parameters in practical engineering applications based on the degree of uncertainty.

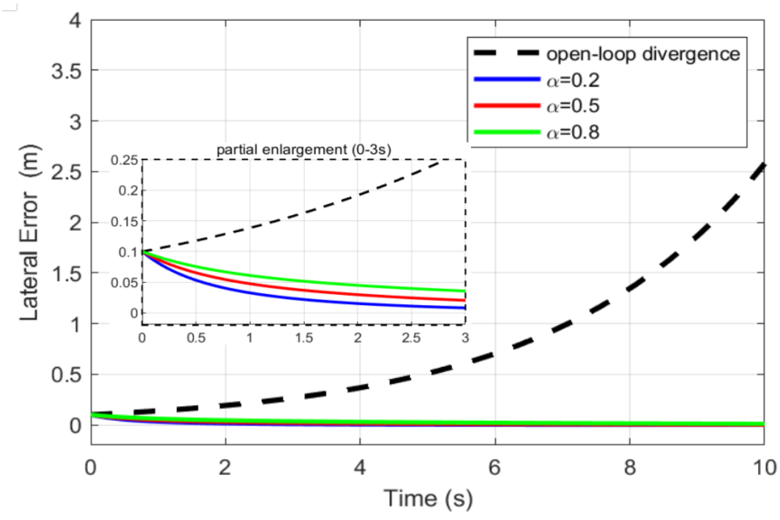


**Figure 3.** Lateral error.

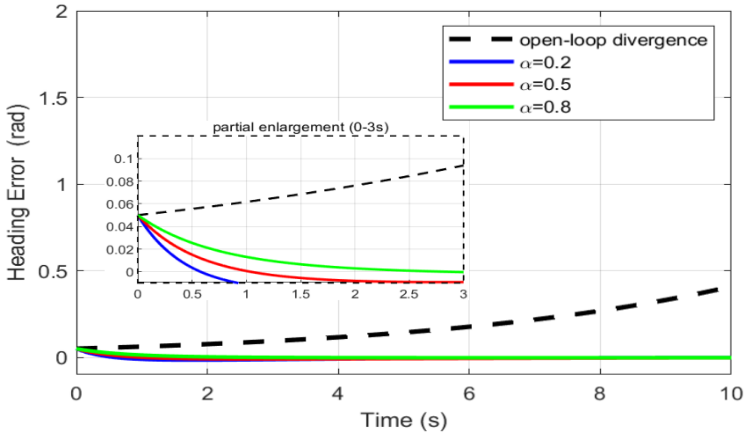


**Figure 4.** Heading angle error.

## 4.3. Theoretical comparison

While LQR control performs well in certain scenarios, it exhibits limitations in terms of control accuracy and robustness under complex road conditions and high-speed operating conditions, as well as poor performance in handling external disturbances [26, 39].

(1) The LQR controller is designed based on the nominal model and lacks an explicit mechanism for handling external disturbances. When the vehicle is subjected to interferences such as cross-wind, road unevenness, and sensor noise, its tracking accuracy degrades significantly. In contrast, the method proposed in this paper directly introduces $H_\infty$ performance constraints, which provide a quantitatively guaranteed attenuation level for all energy-bounded external disturbances, ensuring that the system maintains stable path-tracking performance even under harsh working conditions.

(2) The standard LQR adopts a fixed-gain matrix and can hardly adapt to variations in driving conditions including vehicle speed, road curvature, and road adhesion coefficient. This paper adopts a Takagi-Sugeno (T-S) fuzzy architecture, where membership functions are automatically generated via rough-set matching degrees to realize smooth switching among multiple rules. When the system operates in the high-precision tracking state, the nominal-stiffness control rule is fully activated. When errors increase and enter the boundary uncertainty region, the system smoothly switches to the perturbed-stiffness compensation rule. Such a mechanism enables the controller to automatically adjust control strategies in different error regions, and greatly improves its adaptability to varying driving conditions.

(3) Quantification and utilization of boundary uncertainty: LQR relies on the precise-model assumption and cannot cope with the boundary uncertainty of "whether a given state belongs to the ideal tracking region". In this paper, the lower approximation set and boundary set of Pythagorean rough sets are adopted, and the contribution ratios of the lower approximation and boundary are dynamically adjusted by parameters. The boundary uncertainty measure of rough sets is embedded into the membership functions of the controller, so as to achieve accurate quantification and smooth transition of control strategies for boundary regions. This feature has not been covered by existing LQR methods.

In future work, we can conduct a comprehensive comparative study with LQR and other methods through simulations and real-vehicle experiments under complex road conditions, thereby further validating the practical advantages of the proposed approach.

### 4.4. Engineering implementation considerations

(1) The $H_\infty$ control framework proposed in this paper exhibits robustness against energy-bounded disturbances, thereby demonstrating a certain ability to suppress high-frequency measurement noise with finite amplitudes.

(2) In the simulation, none of the control inputs exceeded the physical limits of the actuator. Potential saturation scenarios can be investigated within the framework of this system; further research in this area is warranted in future work.

## 5. Conclusions

This paper integrates pythagorean fuzzy sets, rough sets, intuitionistic$(\lambda, \eta)$-function rough sets, and $H_\infty$ control for singular system theory to develop a novel path-following control framework. Theoretical rigor is demonstrated through rigorous mathematical proofs of system admissibility, establishing the stability, feasibility, and theoretical soundness of the control system. The proposed hybrid control architecture effectively combines the core strengths of multiple uncertainty processing theories with generalized system control, overcoming the limitations of traditional single-control theories in complex system modeling and robust design. This advancement significantly enhances the theoretical framework of $H_\infty$ control for singular system path-following control.

This research provides a novel theoretical framework and technical implementation pathway for control problems in multi-source uncertain complex singular systems. It expands the application boundaries of fuzzy set and rough set theories in engineering control, enriching the theoretical research on singular system control. Moreover, it offers core technological support for high-precision and high-robustness control in fields such as intelligent vehicles, demonstrating significant theoretical innovation value and broad engineering application potential.

## Author contributions

Li Li is responsible for paper topic selection, model construction, manuscript drafting and initial manuscript review; Xiaolin Zhao undertakes theoretical proof within the model and simulation experiments. All authors have read and approved the final version of the manuscript for publication.

## Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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## Conflict of interest

The authors declare that they have no competing interests.

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