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*Research article*

# Stabilization and controller design of switched Boolean networks with minimum dwell time

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**Abstract:** This paper investigates the stabilization problem and the feedback controller design of switched Boolean control networks (SBCNs) consisting of two subnetworks, where minimum dwell time constraints are taken into consideration. First, the semi-tensor product method is utilized to convert the SBCNs with minimum dwell time constraints into an algebraic state-space model. Second, based on the relation between dwell time and the stabilization time of individual subsystems, some sufficient conditions are obtained to guarantee the global stabilization of the system, and feedback controllers which are dependent on system states and switching signals are designed. Finally, two practical examples are presented to validate the theoretical results of this paper.

**Keywords:** switched Boolean networks; minimum dwell time; stability; controller design; semi-tensor product
**Mathematics Subject Classification:** 93DXX; 93C05, 93D15, 93D23

## 1. Introduction

Switched Boolean control networks (SBCNs) are discrete-time hybrid systems composed of finite Boolean subnets and switching signals that carry out switching among control subsystems [1]. Since their proposal, SBCNs have been widely recognized as a powerful modeling tool for characterizing complex logical dynamic behaviors in multiple fields, including gene regulatory networks, neural networks, networked evolutionary games, and economic systems [2–4]. For example, in gene regulatory networks, SBCNs can accurately describe the switching mechanisms between different functional states of genes, such as the alternation between the lytic and lysogenic states of bacteriophage $\lambda$, as well as the four different phases of eukaryotic cell cycles [5].

The semi-tensor product (STP) of matrices has brought revolutionary progress to the research of SBCNs [6]. By converting logical operations into algebraic expressions, this method can transform

SBCNs from original logical forms into equivalent algebraic state-space representations. Based on this approach, numerous fundamental problems of SBCNs have been systematically studied, such as controllability [7, 8], observability [9, 10], decoupling problems [11, 12], and output tracing [13–15]. Furthermore, the STP method is also applied to other finite-valued systems, including Boolean networks and probabilistic Boolean networks [16–18], networked evolutionary games [19–22], multiagent systems defined over finite fields [23], and finite automata [24, 25]. Besides the analysis and control of finite-valued dynamical systems, the STP can also be applied to image processing [26] and solving quaternion matrix equations [27, 28].

Stability analysis explores whether a system can converge to equilibrium points spontaneously without external control, wheres stabilization focuses on driving unstable or multistable systems to expected equilibrium points via external control inputs [29, 30]. Although extensive studies have been carried out on the stabilization of SBCNs, most existing results mainly focus on arbitrary switching [31], random switching [32, 33], and periodic switching [34, 35], ignoring the practical minimum dwell time constraints. In practical engineering and biological applications, SBCNs are usually subject to various constraints, among which the minimum dwell time constraint is one of the most common and essential. A representative example is the lytic–lysogenic cycle of bacteriophage $\lambda$, which can be naturally modeled as an SBCN. Because mode transitions depend on the gradual accumulation of regulatory proteins, the system must remain in its current subsystem for a minimum duration before any switch occurs, which inherently forms a minimum dwell time constraint [36]. When modeling networked evolutionary games under different government policies, policy changes are described by switching signals, and the implementation duration of each policy generally needs to exceed a minimum threshold [37], which also conforms to the minimum dwell time constraint studied in this paper. In fact, there have been many interesting results on minimum dwell times for both classical linear and nonlinear switching systems [38, 39]. However, these systems are investigated using conventional control theory tools, which cannot be directly applied to discrete-valued Boolean networks.

In previous studies, Lu et al. [40] first converted aperiodic sampled Boolean control networks into switched systems with dwell time constraints and established sufficient stabilization conditions via Lyapunov methods. Nevertheless, their conclusions are only existential results without strict limitation on minimum dwell time. Sun et al. [41] further investigated the scenario with all unstable subsystems and designed switching strategies via upper and lower dwell time bounds to achieve stabilization; however, their work is only applicable to uncontrolled Boolean networks (SBNs) without external control access. Nie et al. [42] initially established necessary and sufficient conditions for global stability of uncontrolled SBNs with minimum dwell time, improving the theoretical framework of uncontrolled systems. Additionally, [33, 43, 44] studied asynchronous control for SBCNs, in which the controller switches with a time-delay lag behind subsystem mode transitions. Though dwell time related constraints are utilized in these papers, such constraints are placed on the controller to cope with asynchronous switching rather than serving as minimum dwell time limits for the actual subsystem modes.

In view of the above research gaps and practical demands, this paper concentrates on the stabilization problem of SBCNs with minimum dwell time constraints. On one hand, the two-subsystem framework serves as a fundamental building block for multimode switched system analysis. It facilitates clear characterization of the intrinsic interplay between stabilization time and minimum dwell time and supports future multisubsystem extensions. On the other hand, two-subsystem SBCNs are widely encountered in practical systems. Typical instances include cell cycle networks switching between $G1/S$ and $G2/M$ phases with phase-dependent Boolean update rules as well as digital sequential logic circuits featuring two discrete operating modes. The main contributions are summarized as follows. First, this paper is the first to systematically study the global stabilization and feedback controller design problem for SBCNs under minimum dwell time constraints. The STP method is adopted to convert this SBCN model into its algebraic form, which provides a unified computable basis for subsequent stability analysis and controller construction. Second, feedback controllers that rely on both system states and switching signals are constructed. The proposed controllers assign separate gain matrices to different subsystems based on the active switching mode. This design increases the flexibility of controller tuning, reduces theoretical conservatism, and yields improved transient performance and system robustness.

The rest of this paper is arranged as follows. Section 2 presents the necessary preliminaries, including the definition, related properties, and lemmas of STP. Section 3 analyzes the stability and controller design of SBCNs with minimum dwell time constraints under arbitrary switching signals. Combining the relationship between subsystem stabilization time and dwell time, global stabilization conditions are given in three cases, and state feedback controllers are constructed correspondingly. Section 4 provides a numerical example based on the Escherichia coli lactose operon. Finally, conclusions are drawn in Section 5.

## 2. Preliminaries

We first present Table 1 to show some necessary notations.

**Definition 2.1.** *[6] Let $M \in \mathbb{R}_{m\times a}$, $N \in \mathbb{R}_{b\times n}$, and $\alpha$ be the least common multiple of a and b. Then*

$$M \ltimes N = (M \otimes I_{\alpha/a})(N \otimes I_{\alpha/b}).$$

Using the vector representation of logical values via the equivalence relation

$$1 \sim \delta_2^1, \quad 0 \sim \delta_2^2,$$

a Boolean function $f : \mathcal{D}^n \to \mathcal{D}$ can be equivalently expressed as a mapping

$$f : \Delta_2 \times \Delta_2 \times \cdots \times \Delta_2 \to \Delta_2,$$

where $\Delta_2 = \{\delta_2^1, \delta_2^2\}$. We call $\delta_2^1$ and $\delta_2^2$ the vector forms of Boolean values 0 and 1, respectively.

**Lemma 2.2.** *[6] Let $f : \Delta^n \to \Delta$; then, there exists a unique $M_f \in \mathcal{L}_{2\times 2^n}$ such that*

$$f(x_1, x_2, \ldots, x_n) = M_f \ltimes_{i=1}^n x_i,$$

*where $\ltimes_{i=1}^n x_i = x_1 \ltimes x_2 \ltimes \cdots \ltimes x_n$.*

**Table 1.** Notations.

| Notation | Definitions |
|---|---|
| $\mathbb{N}$ | Set of natural numbers |
| $\mathbb{Z}_+$ | Set of positive integers |
| $[a : b]$ | Defined as $[a : b] := \{a, a+1, a+2, \cdots, b\}$ |
| $\mathbb{R}_{p\times q}$ | Set of $p \times q$-dimensional real matrices |
| $A^T$ | Transpose of matrix $A$ |
| $A_{i,j}$ | The $(i, j)$-th element of matrix $A$ |
| $\mathrm{Col}_i(A)$ | The $i$th column of matrix $A$ |
| $\mathrm{Col}(A)$ | Set of all columns of matrix $A$ |
| $\delta_n^i$ | Defined as $\delta_n^i = \mathrm{Col}_i(I_n)$ |
| $\Delta_n$ | Defined as $\Delta_n := \mathrm{Col}(I_n)$ |
| $\Delta$ | Defined as $\Delta := \mathrm{Col}(I_2)$ |
| $\mathcal{D}$ | Binary set $\{0, 1\}$ |
| $\mathcal{D}^n$ | $n$-dimensional binary vector set: $\mathcal{D}^n := \underbrace{\mathcal{D} \times \cdots \times \mathcal{D}}_{n}$ |
| Boolean matrix | A matrix with all elements taking values 0 or 1 |
| $\mathcal{B}_{p\times q}$ | Set of $p \times q$-dimensional Boolean matrices |
| Logical matrix | A matrix $L \in \mathbb{R}_{p\times q}$ satisfying $\mathrm{Col}(L) \subseteq \Delta_p$ |
| $\mathcal{L}_{p\times q}$ | Set of $p \times q$-dimensional logical matrices |
| $\delta_p[i_1, i_2, \cdots, i_q]$ | Abbreviation for $[\delta_p^{i_1}, \delta_p^{i_2}, \cdots, \delta_p^{i_q}]$ |
| $\otimes$ | Kronecker product operator |
| $\mathcal{L}(X)$ | Set defined as $\{x \in \Delta_m \mid x \leq X,\ X \in \mathcal{B}_{m\times l}\}$ |
| $J_s$ | Index vector for set $S$: $J_s := \delta_n^{i_1} + \delta_n^{i_2} + \cdots + \delta_n^{i_t}$ where $S = \{\delta_n^{i_1}, \ldots, \delta_n^{i_t}\}$ |

## 3. Main results

### 3.1. Problem formulation

To facilitate comparative analysis of the relationship between the global stabilization time of subsystems and system minimum dwell time, this subsection focuses on an SBCN comprising two subnetworks. A system model under minimum dwell time.

Consider an SBCN with $n$ nodes, $m$ control inputs, and two subnetworks as follows:

$$X_i(t+1) = f_i^{\omega(t)}(X(t); U(t)), \quad i \in [1 : n], \tag{3.1}$$

where $X(t) = (X_1(t), X_2(t), \cdots X_n(t)) \in \mathcal{D}^n$ is the state, $U(t) = (U_1(t), U_2(t), \cdots, U_m(t)) \in \mathcal{D}^m$ is the control input, and $\omega : \mathbb{N} \mapsto [1, 2]$ is the switching signal.

Let $x_i = \delta_2^{2-X_i}$, and $u_j = \delta_2^{2-U_j}$. Using the STP method, SBCN (3.1) can be converted into algebraic form as

$$x(t+1) = L_\omega(t)u(t)x(t), \tag{3.2}$$

where $x(t) = \ltimes_{i=1}^n x_i(t) \in \Delta_{2^n}$, $u(t) = \ltimes_{i=1}^m u_i(t) \in \Delta_{2^m}$, and $L \in \mathcal{L}_{2^n\times 2^{n+m}}$.

Denote $L := [L_1, L_2]$. Similar to Boolean variables, we also convert switching signals into an algebraic form. Let $\omega(t) = 1 \sim \delta_2^1$, and $\omega(t) = 2 \sim \delta_2^2$. Then, System (3.2) can be rewritten as

$$x(t+1) = [L_1, L_2]\sigma(t)u(t)x(t), \tag{3.3}$$

where $\sigma(t)$ stands for the vectorized switching signal satisfying $\sigma(t) \in \Delta_2$.

In this paper, the switching signal satisfies a predefined minimum dwell time on each subsystem. It should be noted that there is no maximum dwell time limit for any subsystem. Suppose that the minimum dwell time is the same for all subsystems, denoted as $\tau$. Let $t_l$ denote the $l$th switching instant; then,

$$\tau \leq t_{l+1} - t_l, \quad \forall l \in \mathbb{N}.$$

A switching signal sequence $(\sigma(0), \sigma(1), \ldots, \sigma(s-1))$ is simply denoted as $\sigma^s$, and the switching signal sequence $\sigma^s$ satisfying the minimum dwell time condition is denoted as $\overline{\sigma}^s$. It should be noted that for the switching signal sequence $\bar{\sigma}^s$, $s$ does not necessarily need to precisely correspond to the switching moment.

The state at time $t$ under the switching signal $\sigma^t$ and control input sequence $\{u(t), t = 0, 1, \ldots, t-1\}$, starting from the initial state $x_0 \in \Delta_{2^n}$, is denoted as $x(t; \sigma^t, u(t), x_0)$.

We consider the feedback controller dependent on both state and switching signal as

$$U_j(t) = h_j\left(X(t), \omega(t)\right), \quad j \in [1:m],$$

where $h_j : \{1, 2\} \times \mathcal{D}^n \to \mathcal{D}^m$ are logical functions whose algebraic form can be expressed as

$$u(t) = H\sigma(t)x(t). \tag{3.4}$$

**Remark 3.1.** *This paper designs a feedback controller dependent on both system states and switching signals. Traditional state feedback controllers use one gain matrix for all subsystems, which results in strict stability conditions and high conservatism. Our controllers apply different gain matrices to each subsystem according to switching modes. It improves design flexibility, lowers conservatism, and delivers better dynamic performance and robustness.*

Next, we define the global stabilization of SBCN with minimal dwell time.

**Definition 3.2.** *The SBCN* (3.3) *with the minimum dwell time is said to be globally stabilizable to a given state $x^* \in \Delta_{2^n}$ if there exists a controller $u(t) = H\sigma(t)x(t)$ and an integer $T \geq 0$ such that for any initial state $x(0) \in \Delta_{2^n}$ and all admissible switching signals, $x(t) = x^*$ holds for all $t \geq T$.*

**Definition 3.3.** *A state $x^* = \delta_{2^n}^{\gamma}$ is a common control fixed point (CCFP) of System* (3.3) *if for every subsystem $x(t+1) = L_i u(t)x(t)$, there exists a control $u_i \in \Delta_{2^m}$ such that $L_i u_i x^* = x^*$.*

### *3.2. Stability analysis and feedback controller design*

In this subsection, we proceed to analyze the stability of SBCNs with minimum dwell time and design the corresponding feedback controller. We begin with a key proposition on the relationship between system-level stability and individual subsystem stability.

**Proposition 3.4.** *If the SBCN* (3.3) *is globally stabilizable to $x^*$ for all switching signals satisfying the minimum dwell time constraint, then each subsystem is globally stabilizable to $x^*$.*

*Proof.* We prove it by contradiction. Without loss of generality, assume that the SBCN (3.3) can be globally stablizable to $x^*$, but Subsystem 2 is not globally stabilizable to $x^*$.

Because there is no upper bound on the dwell time, the constant switching signal $\sigma(t) \equiv \delta_2^2$ is admissible. Under this signal, the system dynamics reduce to $x(t+1) = L_2 u(t) x(t)$.

However, because Subsystem 2 is not globally stabilizable to $x^*$, there exists an initial state for which the trajectory of the closed-loop system fails to converge to $x^*$ under this constant control and switching law. This contradicts the assumption that the SBCN (3.3) is globally stabilizable to $x^*$ for all admissible switching signals. The proof is complete. □

**Proposition 3.5.** *If the SBCN* (3.3) *with minimum dwell time can be globally stabilizable to $x^*$ under all admissible switching signals, then $x^*$ is the CCFP of System* (3.3).

*Proof.* First, recall that if the SBCN (3.3) with the minimum dwell time is globally stabilizable to $x^*$, then there exist $u(t) = H\sigma(t)x(t)$ and $T \in \mathbb{Z}_+$ such that for all $x_0 \in \Delta_{2^n}$, any admissible switching signal sequence $\bar{\sigma}^t$, and all $t \geq T$, it holds that $x(t; \bar{\sigma}^t, u(t), x_0) = x^*$.

We proceed by contradiction. Assume that $x^*$ is not a control fixed point of Subsystem 2, that is, $L_2 u x^* \neq x^*$, for $\forall u \in \Delta_{2^m}$.

**Case 1:** At time $T$, we have $x(T) = x^*$ by the definition of global stabilization, and consider the case where $\sigma(T) = \delta_2^2$.

- If $T$ is not a switching instant, the system evolves under Subsystem 2 at $t = T$. Thus, $x(T+1) = L_2 u(T) x(T) = L_2 u(T) x^* \neq x^*$.
- If $T$ is a switching instant, we construct an admissible switching signal satisfying the dwell time constraint as

$$\bar{\sigma}(t) = (\bar{\sigma}(T), \underbrace{\delta_2^1, \delta_2^1, \cdots, \delta_2^1}_{\tau}, \delta_2^2).$$

  This leads to $x(t) = x(T+\tau+1) = L_2 u(t) x^* \neq x^*$.

Both subcases contradict the global stability of (3.3) under any switching signal satisfying the dwell time constraint.

**Case 2:** Consider the case where $\sigma(T) = \delta_2^1$ at time $T$.

- If $T$ is a switching moment, the system switches to Subsystem 2 immediately, leading to $x(T+1) = L_2 u(T) x(T) = L_2 u(T) x^* \neq x^*$.
- If $T$ is not a switching instant, we construct an admissible switching signal that remains in Subsystem 1 for exactly $\tau$ steps before switching to Subsystem 2:

$$\bar{\sigma}(t) = (\bar{\sigma}(T), \underbrace{\delta_2^1, \delta_2^1, \cdots, \delta_2^1}_{\tau}, \delta_2^2).$$

  At $t = T + \tau + 1$, the system switches to Subsystem 2, and because $x(T+\tau) = x^*$, we have $x(T+\tau+1) = L_2 u(t) x(T+\tau) = L_2 u(t) x^* \neq x^*$.

Both subcases also contradict the stabilizability of (3.3) under any switching signal which satisfies the dwell time constraint.

In conclusion, if the SBCN (3.3) with the minimum dwell time can be globally stabilizable to $x^*$ under all admissible switching signals, then $x^*$ must be the CCFP of System (3.3). □

From the preceding analysis, both the existence of a CCFP and the stabilizability of each subsystem are necessary conditions for global stabilization. Accordingly, we make the following definition and assumptions.

**Assumption 3.6.** *$x^* = \delta_{2^n}^{\gamma}$ is the CCFP of the SBCN* (3.3).

**Definition 3.7.** *[29] A Boolean control network $x(t+1) = Lu(t)x(t)$ is said to be globally stabilizable to $x^* \in \Delta_{2^n}$ if there exists a state feedback controller $u(t) = Hx(t)$ and an integer $T \geq 0$ such that for any initial state $x(0) \in \Delta_{2^n}$, $x(t) = x^*$ holds for all $t \geq T$.*

**Assumption 3.8.** *Both subsystems can be globally stabilizable to $x^*$.*

With these assumptions, we now design state feedback controllers for each subsystem to achieve stabilization to $x^*$. Because both subsystems can be stabilized to $x^*$, we first design separate controllers $u^1(t) = H_1x(t)$ and $u^2(t) = H_2x(t)$ for Subsystem 1 and Subsystem 2, respectively. For more details on the calculation of $H_1$, $H_2$, and the stabilization time $h_1$, $h_2$, please refer to [29]. The truth matrix method is adopted to construct these controllers explicitly.

Divide the matrix $L_1$ into $2^m$ equal blocks as

$$L_1 = [L_1^1, L_1^2, \cdots, L_1^{2^m}],$$

where $L_1^i$ represent the $i$th block. We first design the state feedback controller $u^1(t) = H_1x(t)$ such that all states in Subsystem 1 converge to $x^* = \delta_{2^n}^{\gamma}$. Define

$$T_1^1(x^*)_{i,j} = \begin{cases} 1, & \text{if } (\delta_{2^n}^{\gamma})^T L_1^i \delta_{2^n}^j = 1,\ \delta_{2^n}^j \in \Delta_{2^n}, \\ 0, & \text{otherwise.} \end{cases}$$

We can see that $T_1^1(x^*)_{i,j} = 1$ if and only if $L_1\delta_{2^m}^i\delta_{2^n}^j = \delta_{2^n}^{\gamma}$. That is, the state $\delta_{2^n}^j$ can reach $x^*$ in one step under $u = \delta_{2^m}^i$. Let $\Omega_1(x^*) = \{\delta_{2^n}^j \mid \mathrm{Col}_j(T_1^1(x^*)) \neq 0_{2^n}\}$. Then, $\Omega_1(x^*)$ contains all states that can reach $x^*$ in one step.

Subsequently, we construct a sequence of truth matrices $T_t^1(x^*)$ for $t \geq 2$ as follows:

$$(T_t^1(x^*))_{i,j} = \begin{cases} 1, & \text{if } J_{\Omega_{t-1}(x^*)}^T L_1^i \delta_{2^n}^j = 1,\ \delta_{2^n}^j \in \Delta_{2^n} \setminus \bigcup_{v=1}^{t-1} \Omega_v(x^*), \\ 0, & \text{otherwise.} \end{cases}$$

Define $\Omega_t(x^*) = \{\delta_{2^n}^j \mid \mathrm{Col}_j(T_2(x^*)) \neq 0_{2^n}\}$. For any state $\delta_{2^n}^j \in \Omega_t(x^*)$, there exists a control input $\delta_{2^m}^i$ such that $L_1\delta_{2^m}^i\delta_{2^n}^j \in \Omega_{t-1}(x^*)$. By induction, all states in $\Omega_t(x^*)$ can reach $x^*$ at step $t$.

Because $\Delta_{2^n}$ is a finite set, and the sets $\Omega_i(x^*)$ are pairwise disjoint, there must exist an integer $h_1 \in [0 : 2^n]$ such that

$$\begin{cases} \Omega_{h_1}(x^*) \neq \varnothing, \\ \Omega_{h_1+1}(x^*) = \varnothing. \end{cases}$$

Then, Subsystem 1 can achieve global stabilization to the point $x^*$ at time $h_1$.

By summing the previously constructed truth matrices, we obtain a combined truth matrix

$$\Lambda_1 = \sum_{i=1}^{h_1} T_i^1(x^*).$$

We now construct the state feedback controller $u^1(t) = H_1x(t)$ that globally stabilizes Subsystem 1 to $x^*$. The feedback matrix $H_1 \in \mathcal{L}_{2^m\times 2^n}$ can be designed as follows:

$$H_1\delta_{2^n}^i \in \mathcal{L}(\Lambda_1\delta_{2^n}^i), \quad \forall \delta_{2^n}^i \in \Delta_{2^n}.$$

For Subsystem 2, the truth matrix and corresponding sets can be constructed using the same recursive method as for Subsystem 1. Let $h_2$ denote the stabilization time of Subsystem 2 to $x^*$. We define the combined truth matrix for Subsystem 2 as $\Lambda_2 = \sum_{i=1}^{h_2} T_i^2(x^*)$, where each $T_i^2(x^*)$ is the truth matrix constructed for the $i$th step reachability analysis of Subsystem 2. With this, we design the state feedback controller $u^2(t) = H_2x(t)$, $H_2 \in \mathcal{L}_{2^m\times 2^n}$ for Subsystem 2, where

$$H_2\delta_{2^n}^i \in \mathcal{L}(\Lambda_2\delta_{2^n}^i), \quad \forall \delta_{2^n}^i \in \Delta_{2^n}.$$

We first classify System (3.3) into three scenarios and discuss whether it can be globally stabilizable to $x^*$ while providing corresponding controller design schemes for the stabilizable cases.

**Theorem 3.9.** *The SBCN* (3.3) *can be globally stabilizable at $x^*$ if the minimum dwell time $\tau > max\{h_1, h_2\}$. Moreover, the feedback controller is given by*

$$u(t) = [H_1, H_2]\sigma(t)x(t).$$

*Proof.* Let $\sigma(0) = \delta_2^i$ and $t_1$ be the first switching instant. By the minimum dwell time constraint, $\sigma(t) = \delta_2^i$, $\forall t \in [0, t_1)$. Substituting $\sigma(t) = \delta_2^i$ into (3.3), the system dynamics reduce to

$$x(t+1) = L\delta_2^i u^i(t)x(t) = L_iH_ix(t)x(t) = L_iu^i(t)x(t), \quad \forall t \in [0, t_1).$$

By Assumption 3.8, Subsystem $i$ stabilizes to $x^*$ at step $h_i$, and the condition $\tau > \max\{h_1, h_2\}$ implies that $t_1 \geq \tau > h_i$. For any $x_0 \in \Delta_{2^n}$, we can apply the state feedback controller $u^i(t) = H_ix(t)$ to ensure that

$$x(t, u^i(t), x_0) = x^*, \quad h_i \leq t \leq t_1.$$

Because $x^*$ is the CCFP of two subsystems, for each $i \in \{1, 2\}$, the controller $u^i(t) = H_ix(t)$ ensures that the state remains at $x^*$ permanently once it reaches this equilibrium, that is,

$$x(t, u^i(t), x^*) = x^*, \quad \forall t \geq t_1.$$

The composite controller $u(t) = [H_1, H_2]\sigma(t)x(t)$ guarantees

$$x(t, u(t), x_0) = x^*, \quad \forall t \geq t_1.$$

Therefore, if $\tau > \max\{h_1, h_2\}$, SBCN (3.3) is globally stabilizable to CCFP $x^*$ via the feedback controller

$$u(t) = [H_1, H_2]\sigma(t)x(t).$$

□

**Remark 3.10.** *Although Theorem 3.9 only considers two-subsystem SBCNs, the proposed results can be generalized to SBCNs with multiple subsystems. For an SBCN containing g subsystems, if the minimum dwell time $\tau > \max\{h_1, h_2, \ldots, h_g\}$, the controller can be designed as $u(t) = [H_1, \ldots, H_g]\sigma(t)x(t)$, where $H_i$ is the feedback matrix of the ith subsystem.*

Next, we analyze the case where one subsystem's stabilization time is less than the minimum dwell time, whereas the other's is greater. This yields the following theorem.

**Theorem 3.11.** *The SBCN* (3.3) *can be globally stabilizable to $x^*$ if the minimum dwell time satisfies* $\min\{h_1, h_2\} \le \tau \le \max\{h_1, h_2\}$. *Moreover, the feedback controller is given by $u(t) = [H_1, H_2]\sigma(t)x(t)$.*

*Proof.* Without loss of generality, assume $h_1 \le h_2$, so the condition becomes $h_1 \le \tau \le h_2$.

Let $\sigma(0) = \delta_2^1$. Because $h_1 \le \tau$, there exists a controller $u^1(t) = H_1x(t)$ such that

$$x(\tau; u^1, x_0) = x^*, \quad \forall\, x_0 \in \Delta_{2^n}.$$

Because $x^*$ is a CCFP, once the state reaches $x^*$, it remains there under any valid control. Hence, for all $t \ge \tau$, $x(t; u, x_0) = x^*$.

Now, consider $\sigma(0) = \delta_2^2$. Let $t_1$ be the first switching instant. If $t_1 \ge h_2$, the system is trivially stabilized to $x^*$ under valid controller $u^2(t) = H_2x(t)$. If $t_1 < h_2$, for any initial state $x_0 \in \Delta_{2^n}$, there exists a controller $u^2(t) = H_2x(t)$ such that

$$x(t_1; u^2, x_0) \in \Delta_{2^n}, \quad x(1; u^2, x^*) = x^*.$$

Because $\sigma(t_1) = \delta_2^1$, the system dynamics for $t \in [t_1, t_2)$ are given by $x(t+1) = L_1u^1(t)x(t)$. Given $t_2 - t_1 \ge \tau \ge h_1$, there exists a controller $u^1(t) = H_1x(t)$ such that

$$x(t_2; u^1, x(t_1)) = x^*, \quad \forall x(t_1) \in \Delta_{2^n}.$$

This shows that when $h_1 \le \tau \le h_2$, the SBCN (3.3) is globally stabilizable to $x^*$ by the controller $u(t) = [H_1, H_2]\sigma(t)x(t)$.

By symmetry, the same result holds when $h_2 \le \tau \le h_1$. □

**Remark 3.12.** *It is noted that this result cannot be directly generalized to the multisubsystem case. The reason is that the stabilization times of multiple subsystems may be longer than the minimum dwell time $\tau$. When the system switches among such subsystems under the proposed controller, cyclic state trajectories may arise, which prevents the system from converging to the target fixed point $x^*$.*

Finally, we consider the case where the stabilization time of both subsystems to the fixed point exceeds the minimum dwell time, that is, $\tau < \min\{h_1, h_2\}$.

Denote $Q = L_1 + L_2$, and divide $Q$ into $2^m$ equal blocks as

$$Q = [Q_1, Q_2, \cdots, Q_{2^m}].$$

We define the matrix $T_1(x^*)$ as

$$T_1(x^*)_{j,l} = \begin{cases} 1, & \text{if } (\delta_{2^n}^{\gamma})^T Q_j \delta_{2^n}^l = 2,\ \forall \delta_{2^n}^l \in \Delta_{2^n} \backslash \{x^*\}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.5}$$

Define $\Theta_1(x^*) = \{\delta_{2^n}^l \mid \mathrm{Col}_l(T_1(x^*)) \ne 0_{2^n}\}$. We obtain that for any state $\delta_{2^n}^l \in \Theta_1(x^*)$, there exists at least one control input $\delta_{2^m}^j$ such that $(\delta_{2^n}^{\gamma})^T Q_j \delta_{2^n}^l = 2$; that is, $L\delta_2^i\delta_{2^m}^j\delta_{2^n}^l = \delta_{2^n}^{\gamma}$ for all $i \in [1:2]$. This indicates that the state $\delta_{2^n}^j$ can reach $x^*$ under any switching signal. Therefore, if $T_1(x^*)_{j,l} = 1$, the control input corresponding to the state $\delta_{2^n}^l$ can be selected as $u = \delta_{2^m}^j$.

Subsequently, we compute the states that can reach $x^*$ in two steps. Denote $\Gamma_1 = \cup_{i=0}^1 \Theta_i(x^*)$, where $\Theta_0(x^*) := x^*$. Define the matrix $T_2(x^*)$ as

$$T_2(x^*)_{j,l} = \begin{cases} 1, & \text{if} J_{\Gamma_1}^T Q_j \delta_{2^n}^l = 2,\ \delta_{2^n}^l \in \Delta_{2^n} \setminus \Gamma_1, \\ 0, & \text{otherwise.} \end{cases} \tag{3.6}$$

Define $\Theta_2(x^*) = \{\delta_{2^n}^l \mid \mathrm{Col}_l(T_2(x^*)) \neq 0_{2^n}\}$. We obtain that for any state $\delta_{2^n}^l \in \Theta_2(x^*)$, there exists at least one control input $\delta_{2^m}^j$ such that $L\delta_2^i\delta_{2^m}^j\delta_{2^n}^l \in \Gamma_1$, $\forall\, i \in [1:2]$. Because all states in $\Gamma_1$ can reach $x^*$ in one step, then the states in $\Theta_2(x^*)$ require two steps to reach $x^*$. By constructing the truth matrices $T_1(x^*)$ and $T_2(x^*)$, we can obtain all states that can reach $x^*$ within two steps. The set of these states is $\Theta_1(x^*) \cup \Theta_2(x^*)$.

Denote $\Gamma_t = \cup_{i=0}^t \Theta_i(x^*)$. Following the same procedure, for $t \geq 2$, we define a sequence of truth matrices $T_t(x^*)$ by

$$T_t(x^*)_{j,l} = \begin{cases} 1, & \text{if} J_{\Gamma_{t}-1}^T Q_j \delta_{2^n}^l = 2,\ \delta_{2^n}^l \in \Delta_{2^n} \setminus \Gamma_{t-1}, \\ 0, & \text{otherwise.} \end{cases}$$

Define

$$\Theta_t(x^*) = \{\delta_{2^n}^l \mid \mathrm{Col}_l(T_2(x^*)) \neq 0_{2^n}\}.$$

We obtain that all states in $\Theta_t(x^*)$ can reach $x^*$ at step $t$.

Because $\Delta_{2^n}$ is finite, and $\Theta_p(x^*) \cap \Theta_q(x^*) = \emptyset$ for all $p \neq q$, there exists an integer $k \in [0, 2^n]$ such that

$$\begin{cases} \Theta_k(x^*) \neq \varnothing, \\ \Theta_{k+1}(x^*) = \varnothing. \end{cases}$$

Here, $k$ is the global stabilization time of the System (3.3) to $x^*$. Summing the constructed truth matrices yields the combined truth matrix

$$\Lambda = \sum_{i=1}^{k} T_i(x^*).$$

**Algorithm 1** Design of the state-dependent switching matrix $H$

**Input:** $Q = L_1 + L_2$, $x^* = \delta_{2^n}^{\gamma}$
**Output:** $H$

Initialize $t = 1$
**while** $t \leq 2^n - 1$ **do**
  Construct matrix:
  $T_t(x^*)_{j,l} = \begin{cases} 1, & \text{if } J_{\Gamma_{t-1}}^\top Q_j \delta_{2^n}^l = 2,\ \delta_{2^n}^l \in \Delta_{2^n} \setminus \Gamma_{t-1}, \\ 0, & \text{otherwise.} \end{cases}$
  Compute $\Theta_t(x^*) = \left\{\delta_{2^n}^l \mid \mathrm{Col}_l(T_t(x^*)) \neq 0_{2^n}\right\}$
  **if** $\Theta_t(x^*) \neq \emptyset,\ \Theta_{t+1}(x^*) = \emptyset$ **then**
    Denote $k := t$
    Compute $\Lambda = \sum_{i=1}^k T_i(x^*)$.
    Construct the state-dependent switching matrix $H$ as follows:
    $H\delta_{2^n}^l \in \mathcal{L}(\Lambda\delta_{2^n}^l)$, $\forall\, l \in [1:2^n]$
    **stop**
  **end if**
  $t := t + 1$
**end while**

**Theorem 3.13.** *The SBCN* (3.3) *with minimum dwell time $\tau < min\{h_1, h_2\}$ can be globally stabilizable to $x^*$, if* $\mathrm{Col}_l(\Lambda) \neq 0$ *for all $l \in [1 : 2^n]$. Moreover, the stabilizing controller can be designed as $u(t) = Hx(t)$, where $H$ satisfies $H\delta_{2^n}^l \in \mathcal{L}(\Lambda\delta_{2^n}^l)$, $\forall\, l \in [1 : 2^n]$.*

*Proof.* Recall that $\Lambda = \sum_{i=1}^{k} T_i(x^*)$, where each $T_i(x^*)$ is constructed to identify states that reach $x^*$ in exactly $i$ steps. The condition $\mathrm{Col}_l(\Lambda) \neq 0$ for all $l \in [1 : 2^n]$ implies that for every state $\delta_{2^n}^l \in \Delta_{2^n}$, there exists some $i \in [1 : k]$ such that $\delta_{2^n}^l \in \Theta_i$. In other words, $\bigcup_{i=1}^{k} \Theta_i = \Delta_{2^n}$. By construction, for any $\delta_{2^n}^l \in \Theta_i$, there exists a control input $\delta_{2^m}^j$ such that the state transitions to $\Gamma_{i-1}$ in one step. Repeating this process, any initial state $x_0 = \delta_{2^n}^l$ will reach $x^*$ in at most $k$ steps under the controller $u(t) = Hx(t)$, no matter what switching signal $\sigma(t)$ is applied. Let $T = \max_{i\in[1,2^n]} T_i$, where $T_i$ is the stabilization time for the initial state $\delta_{2^n}^i$. Then, for any $x_0 \in \Delta_{2^n}$ and any switching signal $\sigma(t)$, $x(t; x_0, \sigma, u) = x^*, \quad \forall t \geq T$. This shows that the system is globally stabilizable to $x^*$ even without a minimum dwell time constraint. Therefore, the conclusion also holds when the switching signal satisfies $\tau < \min\{h_1, h_2\}$. □

**Remark 3.14.** *The result of Theorem 3.13 can be generalized to multisubsystem SBCNs with arbitrary switching signals. Because its conclusion holds for any switching signal, the controller $u(t) = Hx(t)$ is switching independent, and the unified gain matrix $H$ applies to all subsystems. Stabilization is thus guaranteed for multisubsystem cases and the scenarios in Theorems 3.9 and 3.11.*

This result completes our dwell time-dependent characterization of global stabilization conditions for SBCNs. By combining the preceding analysis with the truth matrix-based reachability condition in Theorem 3.13, we have obtained a unified framework that covers all cases of minimum dwell time constraints.

**Remark 3.15.** *We mainly use the truth matrix method to design the feedback controller, and the complexity to calculate each element in the truth matrix is $O(2^n)$. Each truth matrix contains $2^{m+n}$ elements, so the time complexity for building one truth matrix is $O(2^{n+m})$. Such matrix construction is performed at most $k$ times, where $k$ denotes the global stabilization time given in Algorithm 1 for driving System* (3.3) *to the target state $x^*$, with $k < 2^n$.* The computational complexity is $O(k \cdot 2^{n+m})$.

### *3.3. Key distinctions from asynchronous control and aperiodic sampled data control*

As stated in reference [40], the aperiodic sampled data stabilization problem of Boolean control networks can be transformed into the stabilization problem of SBCNs with dwell time. Moreover, asynchronous control is widely adopted for the analysis and synthesis of SBCNs subject to dwell time restrictions. Nevertheless, there exist essential distinctions between our developed controller and the aforementioned two approaches. The detailed analysis is provided as follows.

- **Nonperiodic sampled data control**: In nonperiodic sampled data control, state measurements and control inputs are updated solely at aperiodic discrete sampling moments. The control signal is held constant during each intersampling interval. Its core challenge stems from intermittent control updates induced by irregular sampling. In our work, subsystems are required to satisfy the dwell time condition, whereas the control inputs can be updated arbitrarily at any time instant. This feature clearly distinguishes our framework from nonperiodic sampled data control.
- **Asynchronous control:** Research on asynchronous control mainly aims to address the problem of system mode–controller mode mismatch. Representative works [33, 43, 44] all focus on this mismatch phenomenon. In [33], mode mismatch arises from random communication constraints

such as transmission delays and data packet loss, which are characterized by a conditional probability model. In [43], mismatch is induced by fixed signal transmission delay: Once the system switches to a new subsystem, the controller retains the previous mode gain for a predetermined number of steps. In [44], mismatch originates from the dwell time restriction imposed on the controller: The controller cannot update its gain immediately even if the system mode has switched. Obviously, in all these references, the dwell time constraints are imposed on controller updates rather than on subsystems. If system–controller mode mismatch is absent, asynchronous control is no longer required.

- **Mode-dependent state feedback:** Our work investigates SBCNs subject to strict minimum dwell time constraints on switching signals. The controller is perfectly synchronized with the currently activated subsystem at every time instant, so plant–controller mode mismatch does not exist. Our controller is a flexible control scheme, where the control input at each time step is determined by the current mode and system state. Even when subsystems must obey the dwell time condition for switching, we can still update the control input at every time instant using real-time state information.

From the above discussions, one can conclude that our control scheme is neither a special case nor a direct extension of asynchronous control or nonperiodic sampled data control.

## 4. Examples

**Example 4.1.** *The simplified model of the Escherichia coli lactose operon is given by*

$$\begin{cases} X_1(t+1) = \neg U_1(t) \wedge (X_2(t) \vee X_3(t)), \\ X_2(t+1) = \neg U_1(t) \wedge U_2(t) \wedge X_1(t), \\ X_3(t+1) = \neg U_1(t) \wedge (U_2(t) \vee (U_3(t) \wedge X_1(t))), \end{cases} \tag{4.1}$$

*where $X_1, X_2$, and $X_3$ represent high-concentration lactose, medium-concentration lactose, and lactose mRNA, respectively, and $U_1, U_2$, and $U_3$ represent the control groups for high-concentration extracellular glucose, high-concentration extracellular lactose, and medium-concentration extracellular lactose, respectively. To simplify the analysis, we set $U_1(t) \equiv 0$ and introduce new control variables $\mu_1(t) = U_2(t)$, $\mu_2(t) = U_3(t)$. System (4.1) then reduces to*

$$\begin{cases} X_1(t+1) = X_2(t) \vee X_3(t), \\ X_2(t+1) = \mu_1(t) \wedge X_1(t), \\ X_3(t+1) = \mu_1(t) \vee (\mu_2(t) \wedge X_1(t)). \end{cases} \tag{4.2}$$

*Next, we consider a second operating mode where the dynamics of $X_3$ are simplified. Motivated by the fact that $X_3$ is primarily regulated by the intracellular lactose levels $X_1$ and $X_2$, and under conditions where the direct control effect of $V_2$ is negligible [45], we adopt the following simplified model for this mode:*

$$\begin{cases} X_1(t+1) = X_2(t) \vee X_3(t), \\ X_2(t+1) = \mu_1(t) \wedge X_1(t), \\ X_3(t+1) = X_1(t) \vee \neg X_2(t). \end{cases} \tag{4.3}$$

*We treat* (4.2) *and* (4.3) *as two subsystems of an SBCN, where* (4.2) *is the first subsystem, and* (4.3) *is the second subsystem.*

We assume that the system switches with a minimum dwell time of $\tau = 2$ for each subsystem. Using the vector representation of logical variables, we transform the SBCN into the following algebraic form:

$$x(t+1) = L\sigma(t)u(t)x(t), \tag{4.4}$$

where $x(t) \in \Delta_8$, $u(t) \in \Delta_4$, $\sigma(t) \in \Delta_2$, $L = [L_1, L_2]$, and

$$L_1 = \delta_8[1,1,1,5,3,3,3,7,1,1,1,5,3,3,3,7,3,3,3,7,4,4,4,8,3,3,3,7,4,4,4,8],$$
$$L_2 = \delta_8[1,1,1,5,4,4,3,7,1,1,1,5,4,4,3,7,3,3,3,7,4,4,3,7,3,3,3,7,4,4,3,7].$$

In SBCN (4.4), state $(1,0,1)$ indicates that the lactose operon is in the open state. Using the vector representation, we have $(1,0,1) \sim \delta_8^3$. A simple verification shows that

$$L_1\delta_4^1\delta_8^3 = \delta_8^3, L_2\delta_4^1\delta_8^3 = \delta_8^3,$$

so $\delta_8^3$ is the CCFP of both subsystems.

Next, we verify whether both subsystems are globally stabilizable to $\delta_8^3$.

For Subsystem 1, construct the truth matrix

$$T_1^1(x^*) = \begin{pmatrix} 0&0&0&0&1&1&1&0\\ 0&0&0&0&1&1&1&0\\ 1&1&0&0&0&0&0&0\\ 1&1&0&0&0&0&0&0 \end{pmatrix}.$$

We can obtain $\Omega_1 = \{\delta_8^1, \delta_8^2, \delta_8^5, \delta_8^6, \delta_8^7\}$. Construct the truth matrix

$$T_2^1(x^*) = \begin{pmatrix} 0&0&0&1&0&0&0&1\\ 0&0&0&1&0&0&0&1\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{pmatrix}.$$

We can obtain that $\Omega_2 = \left\{\delta_8^4, \delta_8^8\right\}$. Obviously, we can observe that

$$\Omega_0 \cup \Omega_1 \cup \Omega_2 = \Delta_8,$$

where $\Omega_0 = \{\delta_8^3\}$. Therefore, Subsystem 1 can be globally stabilizable to $\delta_8^3$, and its maximum stabilization time is $h_1 = 2$. Moreover, we have

$$\Lambda_1(x^*) = \begin{pmatrix} 0&0&0&1&1&1&1&1\\ 0&0&0&1&1&1&1&1\\ 1&1&1&1&0&0&0&0\\ 1&1&1&1&0&0&0&0 \end{pmatrix};$$

therefore, the state feedback gain matrix $H_1$ of Subsystem 1 can be designed as

$$H_1\delta_{2^n}^i \in \mathcal{L}(\Lambda_1(x^*)\delta_{2^n}^i), \quad \forall \delta_{2^n}^i \in \Delta_8.$$

Using the same approach, we can construct the truth matrix to identify states in Subsystem 2 that can reach the equilibrium point $x^* = \delta_8^3$. We obtain that

$$\Omega_1^2 = \{\delta_8^1, \delta_8^2, \delta_8^7\}, \Omega_2^2 = \{\delta_8^4, \delta_8^8\}, \Omega_3^2 = \{\delta_8^5, \delta_8^6\},$$

and

$$\Lambda_2(x^*) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix},$$

where $\Omega_i^2$ is the $i$th step reachable set of $x^*$ in Subsystem 2.

Because $\cup_{i=1}^{3} \Omega_i^2 \cup \delta_8^3 = \Delta_8$, Subsystem 2 can achieve global stabilization to $\delta_8^3$, and its maximum stabilization time is $h_2 = 3$. The state feedback control matrix $H_2$ of Subsystem 2 can be designed as

$$H_2 \delta_{2^n}^i \in \mathcal{L}(\Lambda_2(x^*)\delta_{2^n}^i), \quad \forall \delta_{2^n}^i \in \Delta_8.$$

Because the minimum dwell time is set to $\tau = 2$, which satisfies $\min\{h_1, h_2\} \leq \tau \leq \max\{h_1, h_2\}$, it follows from Theorem 3.11 that the SBCN (4.4) is globally stabilizable to the equilibrium $x^*$. Based on the preceding reachability analysis, we can design a controller

$$u(t) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \sigma(t)x(t). \tag{4.5}$$

The state transition diagram of the SBCN is presented in Figure 1. We can see that all states will converge to the common control fixed point $x^* = \delta_8^3$ within finite steps under the designed switching rules.

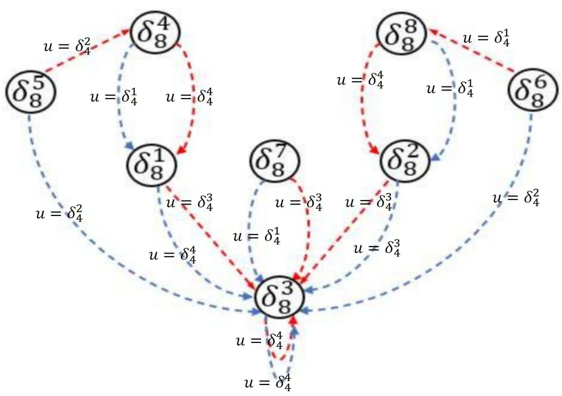


**Figure 1.** State transitions of the SBCN (4.4). Dashed blue lines denote transitions of Subsystem 1, and dashed red lines denote transitions of Subsystem 2.

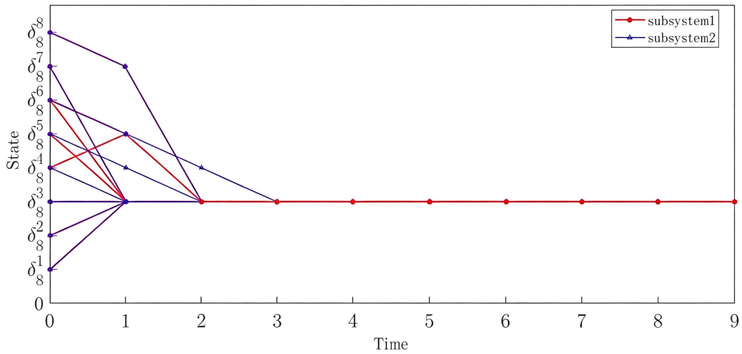

**Figure 2.** State trajectories of the SBCN (4.4) subject to the minimum dwell time constraint under control (4.5).

**Example 4.2.** *Consider the bacteriophage $\lambda$ Boolean network from [36]. The full regulatory dynamics are given by*

$$\begin{cases} N(t+1) = \neg cI(t) \wedge \neg cro(t), \\ cI(t+1) = \neg cro(t) \wedge \neg u_1(t) \wedge (cI(t) \vee cII(t)), \\ cII(t+1) = \neg cI(t) \wedge \neg cro(t) \wedge u_2(t) \wedge N(t), \\ cro(t+1) = \neg cI(t) \wedge \neg cII(t), \end{cases} \tag{4.6}$$

*where $N$, $cI$, $cII$, and $cro$ denote the binary states of the antiterminator gene $N$, lysogenic repressor gene $cI$, lysogenic establishment gene $cII$, and lytic regulator gene $cro$, respectively. $u_1$ is the control input that represses $cI$ to trigger the lytic pathway, and $u_2$ represents environmental conditions that modulate $cII$ expression.*

*The external environmental variable $u_2(t)$ cannot be adjusted manually, so it is not treated as a control input. Accordingly, the system exhibits two distinct operating modes corresponding to different environmental states.*

*When $u_2(t) \equiv 1$, the fist subsystem is given by*

$$\begin{cases} N(t+1) = \neg cI(t) \wedge \neg cro(t), \\ cI(t+1) = \neg cro(t) \wedge \neg u(t) \wedge (cI(t) \vee cII(t)), \\ cII(t+1) = \neg cI(t) \wedge \neg cro(t) \wedge N(t), \\ cro(t+1) = \neg cI(t) \wedge \neg cII(t). \end{cases} \tag{4.7}$$

*When $u_2(t) \equiv 0$, the second subsystem is formulated as*

$$\begin{cases} N(t+1) = \neg cI(t) \wedge \neg cro(t), \\ cI(t+1) = \neg cro(t) \wedge \neg u(t) \wedge (cI(t) \vee cII(t)), \\ cII(t+1) = 0, \\ cro(t+1) = \neg cI(t) \wedge \neg cII(t). \end{cases} \tag{4.8}$$

Treating (4.7) and (4.8) as two subsystems of an SBCN, we assume that the system switches with a minimum dwell time of $\tau = 2$ for each subsystem. By constructing the state transition matrices of the two subsystems, we obtain

$$\begin{aligned} L_1 = \delta_{16}[&16,16,16,16,16,6,15,5,16,16,16,16,16,8,15,7,\\ &16,12,16,12,16,2,15,5,16,12,16,12,16,4,15,7],\\ L_2 = \delta_{16}[&16,16,16,16,16,8,15,7,16,16,16,16,16,8,15,7,\\ &16,12,16,12,16,4,15,7,16,12,16,12,16,4,15,7]. \end{aligned}$$

Here, we denote $(0,0,0,1)$ as the lytic state. Using the vector representation, we have $(0,0,0,1) \sim \delta_{16}^{15}$. A simple verification shows that $L_1\delta_2^1\delta_{16}^{15} = \delta_{16}^{15}$, $L_2\delta_2^2\delta_{16}^{15} = \delta_{16}^{15}$, so $\delta_{16}^{15}$ is the CCFP of both subsystems. Next, we verify whether both subsystems are globally stabilizable to $\delta_{16}^{15}$.

Following the same truth matrix method in Example 4.1, we derive that Subsystem 1 can be globally stabilized to $x^* = \delta_{16}^{15}$ in $h_1 = 4$ steps, and Subsystem 2 can be globally stabilized to the same equilibrium in $h_2 = 3$ steps.

Because the minimum dwell time $\tau = 2$ satisfies $\tau < \min\{h_1, h_2\}$, we apply the conditions of Theorem 3.13 to verify whether the system can be stabilized. Define $Q = L_1 + L_2$, and partition it into two equal blocks as follows:

$$\begin{aligned} Q_1 = [&2\delta_{16}^{16}, 2\delta_{16}^{16}, 2\delta_{16}^{16}, 2\delta_{16}^{16}, 2\delta_{16}^{16}, \delta_{16}^{6}+\delta_{16}^{8}, 2\delta_{16}^{15}, \delta_{16}^{5}+\delta_{16}^{7},\\ &2\delta_{16}^{16}, 2\delta_{16}^{16}, 2\delta_{16}^{16}, 2\delta_{16}^{16}, 2\delta_{16}^{16}, 2\delta_{16}^{8}, 2\delta_{16}^{16}, 2\delta_{16}^{7}].\\ Q_2 = [&2\delta_{16}^{16}, 2\delta_{16}^{12}, 2\delta_{16}^{16}, 2\delta_{16}^{12}, 2\delta_{16}^{16}, \delta_{16}^{2}+\delta_{16}^{4}, 2\delta_{16}^{15}, \delta_{16}^{5}+\delta_{16}^{7},\\ &2\delta_{16}^{16}, 2\delta_{16}^{12}, 2\delta_{16}^{16}, 2\delta_{16}^{12}, 2\delta_{16}^{16}, 2\delta_{16}^{4}, 2\delta_{16}^{15}, 2\delta_{16}^{7}]. \end{aligned}$$

Based on Algorithm 1, we can construct the truth matrix $T_1(x^*)$ as

$$T_1(x^*) = \begin{bmatrix} 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \end{bmatrix}.$$

This matrix allows us to find the one-step reachable set $\Theta_1 = \{\delta_{16}^7\}$. Further computation yields

$$T_2(x^*) = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}.$$

Accordingly, all states that can reach $x^*$ in two steps form the set $\Theta_2 = \{\delta_{16}^{16}\}$. We then establish $T_3(x^*)$, where

$$T_3(x^*) = \begin{bmatrix} 1&1&1&1&1&0&0&0&1&1&1&1&1&0&0&0\\ 1&1&1&1&1&0&0&0&1&0&1&0&1&0&0&0 \end{bmatrix}.$$

With this matrix, we get the three-step reachable set: $\Theta_3 = \{\delta_{16}^1, \delta_{16}^2, \delta_{16}^3, \delta_{16}^4, \delta_{16}^5, \delta_{16}^9, \delta_{16}^{10}, \delta_{16}^{11}, \delta_{16}^{12}, \delta_{16}^{13}\}$. Moreover, we have

$$T_4(x^*) = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \end{bmatrix}.$$

From this result, we have $\Theta_4 = \{\delta_{16}^{14}\}$. Note that $T_5(x^*)$ is a zero matrix, which means $\Theta_5 = \emptyset$. The iteration terminates under the condition

$$\begin{cases} \Theta_4 \neq \emptyset, \\ \Theta_5 = \emptyset. \end{cases}$$

Adding up all previously derived truth matrices following Algorithm 1, we obtain the combined matrix $\Lambda$ as

$$\Lambda = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

We can see that $\Lambda$ contains zero columns corresponding to the states $\delta_{16}^6$ and $\delta_{16}^8$. This violates Theorem 3.13, so the considered SBCN (4.6) cannot be globally stabilized to equilibrium $x^*$.

We can design a state feedback controller as

$$u(t) = \delta_2[2, 1, 1, 2, 1, *, 2, *, 2, 1, 2, 1, 2, 2, 2, 1]x(t),$$

where each symbol $*$ can take any value from $\{1, 2\}$ freely. Under this controller, every state belonging to set $\Delta_{16} \setminus \{\delta_{16}^6, \delta_{16}^8\}$ converges to $x^*$.

## 5. Conclusions

This paper has investigated the stabilization and controller design for SBCNs with minimum dwell time. First, the STP method has been applied to transform the logical dynamic model of such networks into an algebraic form. Subsequently, by analyzing the relationship between the minimum dwell time and the stabilization time of each subsystem, three typical scenarios are studied with corresponding sufficient stabilization conditions, and a feedback controller dependent on both states and switching signal is constructed via the truth matrix method. Finally, the effectiveness and feasibility of the proposed conclusions are validated by the classical Escherichia coli lactose operon example.

Nevertheless, this work has two main limitations. First, all results are derived for two-subnetwork SBCNs with constant minimum dwell time, and only sufficient stabilization conditions are obtained. Second, the proposed method only applies to small-scale SBCNs and cannot be directly generalized to large-scale networks. For future work, we will extend the framework to multi-subnetwork scenarios to derive necessary and sufficient conditions, and we willintegrate mature BN complexity-reduction methods [30, 46, 47] to study large-scale SBCN stabilization.

## Author contributions

Xinling Li and Lei Deng: Methodology, Formal analysis, Investigation, Resources, Writing–original draft preparation; Shilin Wang: Conceptualization, Software, Validation; Xueying Nie and Tingmin Liu: Writing–review and editing, Supervision. All authors have read and approved the final version of the manuscript for publication.

## Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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**Conflict of interest**

All authors declare no conflicts of interest in this paper.

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