



*Research article***Synchronization of the memristive recurrent neural networks via fuzzy adaptive strategy and its application****Lixia Ye^{1,2}, Qiling Zhao^{1,2} and Hang Zheng^{1,2,*}**¹ Department of Mathematics and Computer, Wuyi University, Wuyishan, 354300, China² Fujian Key Laboratory of Big Data Application and Intellectualization for Tea Industry (Wuyi University), Digital Fujian Institute of Tourism Big Data (Wuyi University)*** Correspondence:** Email: zhenghang513@163.com.

Abstract: This study examines the synchronization issue for memristive recurrent neural networks (MRNNs) subject to random disturbances. Asymptotic synchronization and mean-square synchronization can be realized via the fuzzy adaptive strategy, whose adaptive rules are designed for the controller of the slave system, allowing iterative updates of its connection weights. The established synchronization criteria are applicable to various disturbance scenarios. Finally, the validity of the synchronization results are verified via multiple examples, further demonstrating that the proposed model with fuzzy adaptive strategy achieve higher SMD2 than 3-GMM and CNN in image enhancement. For the denoising of color images corrupted with noise intensities of 0.02 and 0.05, the proposed model yields higher EME and PSNR (over 30dB) values than BM3D, and its SMD2 values are closer to the original-image counterparts.

Keywords: synchronization; random disturbances; MRNNs; fuzzy adaptive strategy; image processing

Mathematics Subject Classification: 34E13, 34K35, 91B55

1. Introduction

The memristor is a new type of circuit element, dubbed a resistor with memory, because its resistance is not fixed but determined by the cumulative charge that it has accumulated (see [1]). Owing to its distinctive properties, such as nanoscale dimensions and memory capability, the memristor has found widespread application. For instance, it can act as a synaptic weight that relates voltage to its feedback function in artificial neural networks [2]. As a result, the memristor can be constructed in various architectures, including Hopfield networks [3, 4], cellular neural networks [5], BAM networks [6–8], Duplex networks [9], and others, which are collectively referred to

as memristive neural networks (MNNs). The complex structure of MNNs gives rise to rich dynamical behaviors, which facilitate the realistic simulation of human brain activities [5]. Moreover, the authors have revealed that improved mechanisms of MNNs can generate diverse chaotic phenomena including multi-butterfly attractors, which exhibit promising prospects for engineering hardware implementation [4]. To effectively emulate synaptic activity or track the transmitted signal, it is essential to study the synchronization behavior of MNNs.

In terms of dynamic differential characteristics, MNNs can be broadly categorized into discrete impulsive MNNs and discontinuous switched MNNs, which exhibit distinct differences in dynamic characteristics and control complexity. For discrete impulsive systems, researchers have focused on sudden emergencies arising in biological population evolution, disease prevention, and signal transmission [10–18]. For example, Liu et al. [13] addressed new stability analysis for nonlinear hybrid systems featuring both switched impulsive dynamics and control inputs. The authors in [14] further investigated a hybrid impulsive system characterized by abstract nonlinear mappings and established some ISS results. In addition, based on the theory of differential inclusions, impulsive effects have also been incorporated into neural network models [15, 19–22]. Nevertheless, discrete impulsive-driven neural networks require complicated discrete control strategies to suppress impulsive disturbances, which greatly increase the difficulty of dynamic modeling and synchronization control for such systems. By comparison, these switched network models allowed for continuous signal transmission and supported event-triggered communication [23]. For a class of stochastic MNNs, the authors in [24, 25] designed two different control strategies to synchronization results. In recent years, researchers have studied finite-time and fixed-time synchronization of NNs to satisfy the requirements of deadline-constrained control systems. Zhou et al. adopted stochastic MNNs with coupling terms to realize fixed-time synchronization [26, 27]. The authors derived fixed-time synchronization conclusions for Kuramoto-Oscillator networks in [28]. They further explored event-triggered fixed-time synchronization and energy consumption properties of convolutional neural networks with stochastic disturbances in [29]. For discontinuous NNs, Zhang et al. obtained tighter bounds on the settling time in [30]. Furthermore, such systems are investigated in the complex domain to analyze the finite-time synchronization [31–34]. Meanwhile, a characteristic approach is introduced in MNNs to investigate their stability [7, 8, 35].

Among the above investigations on synchronization, the main control strategies adopted include impulsive control [10, 12, 20], finite-time control [11, 36, 37], sliding mode control [31], event-triggered control [29, 34, 38], and adaptive control [22, 24, 25, 27, 39, 40]. Their main distinction lies in control actuation manners and handling capability. For instance, finite-time control prioritizes settling time. Meanwhile, event-triggered control prioritizes responding to unexpected events. In turn, adaptive control is mainly utilized to handle the uncertainties or unknown nonlinear terms of the master system. In general, controllers for these strategies mostly are adaptive. The researchers in [29] designed adaptive controller with fixed gains to achieve fixed-time synchronization via event-triggered mechanism. While the researchers in [25] investigated differences between other feedback controller with fixed gain and adaptive controller with adaptive rules in settling time. It turns out that the adaptive strategy enables the synchronization errors to converge more rapidly. Since the gains of fuzzy adaptive strategy can be updating automatically. Moreover, its adaptive rules enable adaptive controller to possess favorable convergence performance. Consequently, T-S fuzzy adaptive strategies have obtained more opportunities to be designed to solve the synchronization issue for MNNs [41, 42].

At present, for memristive recurrent neural networks (MRNNs), since their weights are state dependent and exhibit discontinuous switching behaviors, the theory of differential inclusions is generally adopted to establish the corresponding synchronization conclusions [37, 42, 45, 46]. Moreover, the weights of their slave systems are fixed or rest on the structural information of weights in the master system. Synchronization results derived in such a way necessitate extra constraints on the activation functions, such as the activation functions satisfying $f_j(0) = 0, g_j = 0$ [30, 38], $f_i(\pm\Lambda_i) = 0, g_i(\pm\Lambda_i) = 0$ [46], and the boundedness condition [40]. While uncertain slave systems with synaptic weights updated by adaptive control rules have the capacity to handle the activation function only satisfying the local Lipschitz condition, the adopted controller is a T-S adaptive strategy with adaptive rules [39]. The T-S fuzzy strategy is able to handle the synchronization problem effectively because of its insensitivity to parameters variation, which possesses a better robustness. However, synchronization results derived via the T-S fuzzy strategy may impose extra assumptions concerning the prior knowledge of external disturbances, for instance, boundedness constraints or feasibility only under specific scenarios. It is still a challenge for T-S fuzzy adaptive strategies to derive synchronization criteria where slave-system weights exhibit weaker reliance on master system information, especially under incomplete prior information of external perturbations, stochastic noise in particular. In future investigations, it is important and necessary to design an excellent adaptive controller to resolve synchronization problems with lower conservatism.

Inspired by the above discussions and referring to the method in existing literature, the delayed MRNNs with random disturbances are established to realize two different types of synchronization via the T-S fuzzy adaptive strategy. The central thesis of this paper is supported by several contributions.

- Compared with [24, 27, 39, 47], the applicable disturbance scenarios are generalized in this paper.
- Two adaptive controllers are designed for the delayed MRNNs subject to random disturbances, achieving both asymptotic synchronization and mean square synchronization. The control gains and the weights of slave system can be explicitly computed. Adjustment schemes for control parameter k_i and for control parameter h_i are developed, whereby certain robustness of the designed adaptive strategy is demonstrated against estimation errors in the disturbance bounds.
- This paper shows that the proposed MRNNs with fuzzy adaptive strategy can be utilized for image-processing applications. Compared with several other existing algorithms, the proposed model exhibits some favorable denoising-fidelity performance relative to BM3D.

2. Problem description

2.1. MRNNs with arbitrary deterministic disturbance

In this section, we consider the delayed MRNNs with deterministic disturbance. The disturbance is arbitrarily selected.

$$\begin{aligned} \dot{s}_i(t) = & -c_i(s_i(t))s_i(t) + \sum_{j=1}^n p_{ij}(s_j(t))\Phi_j(s_j(t)) + \sum_{j=1}^n q_{ij}(s_j(t))\Psi_j(s_j(t - \vartheta_j(t))) \\ & + I_i + \delta_i(t), i \in \mathbb{Z}^+, \end{aligned} \quad (2.1)$$

where $s_i(t) \in R$ is the state of neuron i , $c_i(s_i(t)) = c_{1i}$ if $s_i(t) \leq h$, otherwise $c_i(s_i(t)) = c_{2i}$, $\vartheta_j(t)$ satisfies $0 \leq \vartheta_j(t) \leq \tau$, and $\dot{\vartheta}_j(t) \leq \tau_0 < 1$ (τ_0 is a known constant). I_i is an external input, and $\delta_i(t)$ is an

arbitrary disturbance. The connection memristive weights $p_{ij}(s_j(t))$ and $q_{ij}(s_j(t))$ satisfy

$$p_{ij}(s_j(t)) = \begin{cases} \hat{p}_{ij}, & |s_j(t)| \leq \Lambda_j, \\ \check{p}_{ij}, & |s_j(t)| > \Lambda_j, \end{cases} \quad q_{ij}(s_j(t)) = \begin{cases} \hat{q}_{ij}, & |s_j(t)| \leq \Lambda_j, \\ \check{q}_{ij}, & |s_j(t)| > \Lambda_j, \end{cases}$$

where the switching jump $\Lambda_j > 0$, $\hat{p}_{ij}, \check{p}_{ij}, \hat{q}_{ij}, \check{q}_{ij}$ are all known constants, and $s_i(\theta) = \varphi_i(\theta) \in C^1([-\tau, 0], \mathbb{R}^n)$ is the initial condition. The activation functions $\Phi_j, \Psi_j (j \in \mathbb{Z}^+)$ satisfy the following assumption.

Assumption 1. $\forall x, y \in \mathbb{R}, x \neq y$, there exist $M_j^{(1)} \geq 0, M_j^{(2)} \geq 0, \xi_j \geq 0, l_j \geq 0$ such that $|\Phi_j| \leq M_j^{(1)}, |\Psi_j| \leq M_j^{(2)}$ and

$$\frac{|\Phi_j(x) - \Phi_j(y)|}{|x - y|} \leq \xi_j, \quad \frac{|\Psi_j(x) - \Psi_j(y)|}{|x - y|} \leq l_j.$$

Assumption 2. Assume that $\delta_i(t), \sigma_i(t)$ satisfy $|\delta_i(t)| \leq \bar{\delta}_i, |\sigma_i(t)| \leq \bar{\sigma}_i$, where $\bar{\delta}_i$ and $\bar{\sigma}_i$ are positive constants.

For the purpose of managing nonlinear control systems, the memristive weight $c_i(s_i(t))$ is taken into account to describe some fuzzy piecewise functions. Following the approach in [39], system (2.1) is transformed using fuzzy IF-THEN rules into the following form

$$\begin{aligned} \dot{s}_i(t) = & -\sum_{l=1}^2 \chi_{li}(t) c_{li} s_i(t) + \sum_{j=1}^n p_{ij}(s_j(t)) \Phi_j(s_j(t)) + \sum_{j=1}^n q_{ij}(s_j(t)) \Psi_j(s_j(t - \vartheta_j(t))) \\ & + I_i + \delta_i(t), \end{aligned} \quad (2.2)$$

where

$$\chi_{li}(t) = \begin{cases} 0, & s_i(t) \leq h, \\ 1, & s_i(t) > h. \end{cases}$$

Definition 1. [48] For the discontinuous differential system

$$\frac{dx(t)}{dt} = f(t, x), \quad (2.3)$$

the associated Filippov set-valued maps $F : \mathbb{R} \times \mathbb{H}^n \longrightarrow \mathbb{H}^n$,

$$F(t, x) = \bigcap_{\delta > 0} \bigcap_{\mu(N)=0} co[f(B(X, \delta) \setminus N)], \quad (2.4)$$

where $B(x, \delta)$ is a punctured ball. A function $x(t)$ qualifies as a Filippov solution on the interval $[t_0, T] (T \in [t_0, +\infty))$ if it is absolutely continuous and satisfies $x(0) = x_0, \frac{dx(t)}{dt} \in F(t, x)$, for a.a. $t \in [0, T]$.

As shown in Definition 1, we obtain set-valued maps in this part

$$co[p_{ij}(s_j(t))] = \begin{cases} \hat{p}_{ij}, & |s_j(t)| < \Lambda_j, \\ co[\check{p}_{ij}, \hat{p}_{ij}], & |s_j(t)| = \Lambda_j, \\ \check{p}_{ij}, & |s_j(t)| > \Lambda_j, \end{cases} \quad co[q_{ij}(s_j(t))] = \begin{cases} \hat{q}_{ij}, & |s_j(t)| \leq \Lambda_j, \\ co[\check{q}_{ij}, \hat{q}_{ij}], & |s_j(t)| = \Lambda_j, \\ \check{q}_{ij}, & |s_j(t)| > \Lambda_j. \end{cases}$$

Then, a reformulation of system (2.2) leads to

$$\begin{aligned} \dot{s}_i(t) \in & -\sum_{l=1}^2 \chi_{li}(t) c_{li} s_i(t) + \sum_{j=1}^n co[p_{ij}(s_j(t))] \Phi_j(s_j(t)) \\ & + \sum_{j=1}^n co[q_{ij}(s_j(t))] \Psi_j(s_j(t - \vartheta_j(t))) + I_i + \delta_i(t). \end{aligned} \quad (2.5)$$

Based on differential inclusion theory, the master system (2.2) belonging to the closure of differential sets can be effectively tracked by the following slave system

$$\begin{aligned} \dot{\tilde{s}}_i(t) = & -\sum_{l=1}^2 \chi_{li}(t) c_{li} \tilde{s}_i(t) + \sum_{j=1}^n p_{ij}(t) \Phi_j(\tilde{s}_j(t)) \\ & + \sum_{j=1}^n q_{ij}(t) \Psi_j(\tilde{s}_j(t - \vartheta_j(t))) + I_i + u_i(t). \end{aligned} \quad (2.6)$$

Let $r_i(t) = \tilde{s}_i(t) - s_i(t)$, $r_i(t - \vartheta_i(t)) = \tilde{s}_i(t - \vartheta_i(t)) - s_i(t - \vartheta_i(t))$. The error system between the MRNNs (2.2) and (2.6) can be written as

$$\begin{aligned} \dot{r}_i(t) = & -\sum_{l=1}^2 \chi_{li}(t) c_{li} r_i(t) + \sum_{j=1}^n (p_{ij}(t) - p_{ij}(s_j(t))) \Phi_j(\tilde{s}_j(t)) \\ & + \sum_{j=1}^n p_{ij}(s_j(t)) \tilde{\Phi}_j(\tilde{s}_j(t), s_j(t)) + \sum_{j=1}^n q_{ij}(s_j(t)) \tilde{\Psi}_j(\tilde{s}_j(t - \vartheta_j(t)), s_j(t - \vartheta_j(t))) \\ & + \sum_{j=1}^n (q_{ij}(t) - q_{ij}(s_j(t))) \Psi_j(\tilde{s}_j(t - \vartheta_j(t))) + u_i(t) - \delta_i(t), \end{aligned} \quad (2.7)$$

where

$$\begin{aligned} \tilde{\Phi}_j(\tilde{s}_j(t), s_j(t)) &= \Phi_j(\tilde{s}_j(t)) - \Phi_j(s_j(t)), \\ \tilde{\Psi}_j(\tilde{s}_j(t - \vartheta_j(t)), s_j(t - \vartheta_j(t))) &= \Psi_j(\tilde{s}_j(t - \vartheta_j(t))) - \Psi_j(s_j(t - \vartheta_j(t))). \end{aligned}$$

2.2. MRNNs with white noise disturbance

We extend the above model to the white noise case and propose a stochastic memristive neural network

$$\begin{aligned} ds_i(t) = & \left(-c_i(s_i(t)) s_i(t) + \sum_{j=1}^n p_{ij}(s_j(t)) \Phi_j(s_j(t)) + \sum_{j=1}^n q_{ij}(s_j(t)) \right. \\ & \left. \times \Psi_j(s_j(t - \vartheta_j(t))) + I_i \right) dt + \sigma_i(t) d\omega_i(t), i \in \mathbb{Z}^+, \end{aligned} \quad (2.8)$$

where the Wiener process $\omega_i(t)$ satisfies $\mathbb{E}\omega_i(t) = 0, \mathbb{E}\omega_i^2(t) = 1, \mathbb{E}\omega_i(t)\omega_j(t) = 0 (i \neq j)$ and $\sigma_i(t)$ denotes the noise intensity. Based on Definition 1, system (2.8) can be transformed into

$$ds_i(t) \in \left(-\sum_{l=1}^2 \chi_{li}(t) c_{li} s_i(t) + \sum_{j=1}^n co[p_{ij}(s_j(t))\Phi_j(s_j(t))] \right. \\ \left. + \sum_{j=1}^n co[q_{ij}(s_j(t))]\Psi_j(s_j(t - \vartheta_j(t))) + I_i \right) dt + \sigma_i(t) d\omega_i(t). \quad (2.9)$$

The slave system (2.6) is adopted to realize the tracking of the master system (2.8). The error system between the stochastic MRNNs (2.8) and (2.6) can be described as

$$dr_i(t) = \left(-\sum_{l=1}^2 \chi_{li}(t) c_{li} r_i(t) + \sum_{j=1}^n (p_{ij}(t) - p_{ij}(s_j(t))) \Phi_j(\tilde{s}_j(t)) \right. \\ \left. + \sum_{j=1}^n p_{ij}(s_j(t)) \tilde{\Phi}_j(\tilde{s}_j(t), s_j(t)) + \sum_{j=1}^n q_{ij}(s_j(t)) \tilde{\Psi}_j(\tilde{s}_j(t - \vartheta_j(t)), s_j(t - \vartheta_j(t))) \right. \\ \left. + \sum_{j=1}^n (q_{ij}(t) - q_{ij}(s_j(t))) \Psi_j(\tilde{s}_j(t - \vartheta_j(t))) + u_i(t) \right) dt - \sigma_i(t, s_i(t)) d\omega_i(t). \quad (2.10)$$

For the purpose of proving synchronization of the MRNNs, Proposition 1 is stated below.

Proposition 1. [49] For an autonomous system $\dot{x} = f(x)$ with continuous f and a C^1 scalar function $V(x) : \mathbb{R}^n \mapsto \mathbb{R}$, assume that

- (1) $V(x)$ is radially unbounded,
- (2) $\forall x(t) \in \mathbb{R}^n, \dot{V}(x(t)) \leq 0$.

Therefore, all trajectories $x(t)$ ultimately approach the set M as $t \rightarrow \infty$, where M is the maximal invariant set contained in $S = \{x \in \mathbb{R}^n \mid \dot{V}(x(t)) = 0\}$.

3. Main results

In this section, an adaptive controller is designed to ensure the MRNNs with arbitrarily deterministic disturbance asymptotic synchronization. Specially, an updated adaptive strategy is provided for the stochastic MRNNs.

3.1. Synchronization under arbitrarily deterministic disturbance

The following criteria can be established if the adaptive controller $u_i(t)$ is given as

$$u_i(t) = \eta_i(t)r_i(t) - k_i \text{sign}(r_i(t)), \quad (3.1)$$

where $k_i > 0$ is a tunable parameter and $\eta_i(t)$ satisfies

$$\dot{\eta}_i(t) = -\mu_i |r_i(t)|,$$

the connection weights $p_{ij}(t), q_{ij}(t)$ are updated by

$$\dot{p}_{ij}(t) = -\Phi_j(\tilde{s}_j(t)) \text{sign}(r_i(t)), \quad (3.2)$$

$$\dot{q}_{ij}(t) = -\Psi_j(\tilde{s}_j(t - \vartheta_j(t))) \text{sign}(r_i(t)). \quad (3.3)$$

Remark 1. For the controller (3.1) with adaptive rules (3.2) and (3.3), the parameters $\eta_i(t)$, $\text{sign}(r_i(t))$, and $\Phi_j(\tilde{s}_j(t))$, $\Psi_j(\tilde{s}_j(t) - \vartheta_j(t))$ can be computed easily, and k_i, μ_i are known. The automatic control gain $\eta_i(t)$ can be adjusted by $\dot{\eta}_i(t) = -\mu_i|r_i(t)|$. In addition, the connection weights $p_{ij}(t), q_{ij}(t)$ of the slave system (2.6) can be updated by adaptive rules (3.2) and (3.3). Similar to [42], the fuzzy adaptive strategy with tunable parameter k_i is designed for MRNNs (2.1), which differs from [39]. Moreover, the lower bound of k_i can be increased according to $\bar{\delta}_i + \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij})M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij})M_j^{(2)} + \|r\|_2^2$ in the presence of estimation error for the disturbance bound $\bar{\delta}_i$. This is of great importance for synchronization considering bounded disturbance.

Theorem 1. Suppose that Assumptions 1 and 2 hold. The global asymptotic synchronization of the MRNNs between (2.2) and (2.6) under controller (3.1) with adaptive rules (3.2) and (3.3) can be realized, where the control parameter $k_i \geq \bar{\delta}_i + \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij})M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij})M_j^{(2)}$.

Proof: A Lyapunov-Krasovskii functional is constructed as follows

$$V(t) = \sum_{i=1}^n \left\{ |r_i(t)| + \frac{1}{2\mu_i} (\eta_i(t) + \Theta_i^{(1)})^2 + \sum_{j=1}^n \frac{1}{2} (p_{ij}(t) - \bar{p}_{ij})^2 + \sum_{j=1}^n \frac{1}{2} (q_{ij}(t) - \bar{q}_{ij})^2 \right\} + \frac{1}{1-\tau_0} \sum_{i=1}^n \varphi_i^{(1)} \int_{t-\vartheta_i(t)}^t |r_i(\theta)| d\theta, \quad (3.4)$$

where $\bar{p}_{ij} = \max\{|\hat{p}_{ij}|, |\check{p}_{ij}|\}$, $\bar{q}_{ij} = \max\{|\hat{q}_{ij}|, |\check{q}_{ij}|\}$, and $\Theta_i^{(1)} > 0, \varphi_i^{(1)} > 0, i, j = 1, 2, \dots, n$.

By calculating $D^+V(t)$, under the controller (3.1) with adaptive rules (3.2) and (3.3) and Assumption 1, we obtain

$$\begin{aligned} D^+V(t) &= \sum_{i=1}^n \left\{ \dot{r}_i(t) \text{sign}(r_i(t)) + \frac{1}{\mu_i} (\eta_i(t) + \Theta_i^{(1)}) \dot{\eta}_i(t) + \sum_{j=1}^n (p_{ij}(t) - \bar{p}_{ij}) \dot{p}_{ij}(t) \right. \\ &\quad \left. + \sum_{j=1}^n (q_{ij}(t) - \bar{q}_{ij}) \dot{q}_{ij}(t) \right\} + \frac{1}{1-\tau_0} \varphi_i^{(1)} (|r_i(t)| - |r_i(t - \vartheta_i(t))| (1 - \dot{\vartheta}_i(t))) \\ &\leq \sum_{i=1}^n \left\{ -\sum_{l=1}^2 \chi_{li}(t) c_{li} |r_i(t)| + \sum_{j=1}^n \bar{p}_{ij} \xi_j |r_j(t)| + \sum_{j=1}^n \bar{q}_{ij} l_j |r_j(t - \vartheta_j(t))| \right. \\ &\quad \left. + \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)} - k_i + |\delta_i(t)| - \Theta_i^{(1)} |r_i(t)| \right. \\ &\quad \left. + \frac{1}{1-\tau_0} \varphi_i^{(1)} (|r_i(t)| - |r_i(t - \vartheta_i(t))| (1 - \dot{\vartheta}_i(t))) \right\}. \end{aligned} \quad (3.5)$$

Under Assumption 2, we have $|\delta_i(t)| \leq \bar{\delta}_i$, so we select the parameter k_i in the controller (3.1) such that $k_i \geq \bar{\delta}_i + \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)}$. Because of $\dot{\vartheta}_i(t) \leq \tau_0 < 1$, one has

$$\begin{aligned} D^+V(t) &\leq \sum_{i=1}^n \left\{ \left(\frac{1}{1-\tau_0} \varphi_i^{(1)} + \sum_{j=1}^n \bar{p}_{ij} \xi_j - c_i - \Theta_i^{(1)} \right) |r_i(t)| \right. \\ &\quad \left. + \left(\sum_{j=1}^n \bar{q}_{ij} l_j - \varphi_i^{(1)} \right) |r_i(t - \vartheta_i(t))| \right\}, \end{aligned} \quad (3.6)$$

where $\underline{p}_{ij} = \min \{|\hat{p}_{ij}|, |\check{p}_{ij}|\}$, $\underline{q}_{ij} = \min \{|\hat{q}_{ij}|, |\check{q}_{ij}|\}$, and $\underline{c}_i = \min \{|c_{1i}|, |c_{2i}|\}$.

The selected $\varphi_i^{(1)}$ satisfies

$$\varphi_i^{(1)} > \sum_{j=1}^n \bar{q}_{ji} l_i. \quad (3.7)$$

With $\varphi_i^{(1)}$ chosen according to inequality (3.7), one can select $\Theta_i^{(1)}$ that satisfies

$$\Theta_i^{(1)} > \sum_{j=1}^n \bar{p}_{ji} \xi_i + \frac{1}{1 - \tau_0} \varphi_i^{(1)} - \underline{c}_i. \quad (3.8)$$

It is ensured that $D^+V(t) \leq 0$, and the equality holding is equivalent to $r_i(t) = 0$.

In accordance with Proposition 1, $r(t) = (r_1(t), r_2(t), \dots, r_n(t))^T$ converges to zero as $t \rightarrow \infty$. This result demonstrates that the MRNN systems described by (2.2) and (2.6) achieve global asymptotic synchronization.

Remark 2. Here, $\varphi_i^{(1)}$ and $\Theta_i^{(1)}$ are arbitrary constants of the Lyapunov-Krasovskii functional (3.4). In Theorem 1, $\varphi_i^{(1)}$ and $\Theta_i^{(1)}$ satisfying inequalities (3.7) and (3.8) can be realized easily. In contrast to [40, 50] that assume bounded activation functions, the designed fuzzy adaptive strategy can ensure that the weights of the slave system are updated automatically, so the synchronization result is free of constraints on the parameter values of the master system.

When the disturbance $\delta_i(t)$ is unbounded or grows linearly, it is difficult to guarantee the synchronization of the master-slave system if the control parameter k_i is selected according to the scheme given in Theorem 1. For this case, the following assumption is introduced to achieve synchronization result similar to Theorem 1. Considering $\delta_i(t)$ depends on time t and $s_i(t)$, we use the notation $\delta_i(t, s_i(t))$ instead.

Assumption 3. $\forall x, y \in R, x \neq y$, there exists $\rho_i > 0$ such that the disturbance $\delta_i(t, \cdot)$ with initial value $\delta_i(0, s_i(0)) = 0$ satisfies

$$\frac{|\delta_i(t, x) - \delta_i(t, y)|}{|x - y|} \leq \rho_i.$$

Hence, k_i needs to be updated and depend on the state of the slave system. We set

$$k_i = \rho_i |\tilde{s}_i(t) - s_i(0)| + \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)}. \quad (3.9)$$

Theorem 2. Suppose that Assumptions 1 and 3 hold. The global asymptotic synchronization of the MRNNs between (2.2) and (2.6) under controller (3.1) with adaptive rules (3.2), (3.3), and (3.9) can be realized.

Proof: Similar to Theorem 1, we choose the Lyapunov function $V(t)$ from Theorem 1. From Assumption 3, we have

$$|\delta_i(t, s_i(t))| = |\delta_i(t, s_i(t)) - \delta_i(0, s_i(0))| \leq \rho_i |s_i(t) - s_i(0)|.$$

By virtue of inequality (3.5), it suffices to derive

$$-k_i + |\delta_i(t, s_i(t))| = -\rho_i |\tilde{s}_i(t) - s_i(0)| + |\delta_i(t, s_i(t))| - \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} - \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)}$$

$$\leq \rho_i |r_i(t)| - \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} - \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)}.$$

Hence, we only need to reselect $\Theta_i^{(1)} > \sum_{j=1}^n \bar{p}_{ji} \xi_i + \frac{1}{1-\tau_0} \varphi_i^{(1)} + \rho_i - \underline{c}_i$ to ensure $D^+V(t) \leq 0$.

Remark 3. Here, $s_i(0), \tilde{s}_i(t)$ can be obtained from the master-slave MRNNs (2.2) and (2.6), so the fuzzy adaptive controller (3.2) with (3.9) can be implemented based on [42]. In Theorem 2, we extend the deterministic disturbance $\delta_i(t, s_i(t))$ to satisfy $\delta_i(0, s_i(0))$ and the local Lipschitz condition, which implies that $\delta_i(t, s_i(t))$ can be unbounded. This reveals that the incorporation of adaptive rule (3.9) enables controller (3.2) to cope well with the robustness induced by disturbance. Compared with [39], the established synchronization results possess wider applicability.

3.2. Synchronization under white noise disturbance

In this part, the updated controller for the stochastic MRNNs is designed.

$$u_i(t) = \frac{1}{m} \eta_i(t) (|r_i(t)| + \varepsilon) \text{sign}(r_i(t)) - \frac{1}{2} h_i^2 (m-1) \frac{\text{sign}(r_i(t))}{|r_i(t)| + \varepsilon} - d_i \text{sign}(r_i(t)), \quad (3.10)$$

where $\varepsilon > 0, h_i > 0, d_i > 0$ are tunable parameters and $\eta_i(t)$ satisfies

$$\dot{\eta}_i(t) = -\mu_i (|r_i(t)| + \varepsilon)^m,$$

and $p_{ij}(t), q_{ij}(t)$ are updated by

$$\dot{p}_{ij}(t) = -\Phi_j(\tilde{s}_j(t)) (|r_i(t)| + \varepsilon)^{(m-1)} \text{sign}(r_i(t)), \quad (3.11)$$

$$\dot{q}_{ij}(t) = -\Psi_j(\tilde{s}_j(t - \vartheta_j(t))) (|r_i(t)| + \varepsilon)^{(m-1)} \text{sign}(r_i(t)). \quad (3.12)$$

Theorem 3. Suppose that the Assumptions 1 and 2 hold. The stochastic MRNNs between (2.8) and (2.6) under the updated controller (3.10) and adaptive rules (3.11) and (3.12) are synchronized in mean square, where the control parameters $h_i \geq \bar{\sigma}_i, d_i \geq \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)}$.

Proof: A Lyapunov-Krasovskii functional that differs from the previous candidate should be established for the considered stochastic system

$$\begin{aligned} V(t) = & \sum_{i=1}^n \left\{ (|r_i(t)| + \varepsilon)^m + \frac{1}{2\mu_i} (\eta_i(t) + \Theta_i^{(2)})^2 + \sum_{j=1}^n \frac{m}{2} (p_{ij}(t) - \bar{p}_{ij})^2 \right. \\ & \left. + \sum_{j=1}^n \frac{m}{2} (q_{ij}(t) - \bar{q}_{ij})^2 \right\} + \frac{1}{1-\tau_0} \sum_{i=1}^n \varphi_i^{(2)} \int_{t-\vartheta_i(t)}^t (|r_i(\theta)| + \varepsilon)^m d\theta. \end{aligned} \quad (3.13)$$

Applying Itô isometry yields

$$\begin{aligned}
 \mathcal{L}V(t) = & \sum_{i=1}^n \left\{ m(|r_i(t)| + \varepsilon)^{(m-1)} \text{sign}(r_i(t)) \left(-\sum_{l=1}^2 \chi_{li}(t) c_{li} r_i(t) \right. \right. \\
 & + \sum_{j=1}^n p_{ij}(s_j(t)) \tilde{\Phi}_j(\tilde{s}_j(t), s_j(t)) + \sum_{j=1}^n (p_{ij}(t) - p_{ij}(s_j(t))) \Phi_j(\tilde{s}_j(t)) \\
 & + \sum_{j=1}^n q_{ij}(s_j(t)) \tilde{\Psi}_j(\tilde{s}_j(t - \vartheta_j(t)), s_j(t - \vartheta_j(t))) + \sum_{j=1}^n (q_{ij}(t) - q_{ij}(s_j(t))) \\
 & \times \Psi_j(\tilde{s}_j(t - \vartheta_j(t))) + u_i(t) \Big) + \frac{1}{\mu_i} (\eta_i(t) + \Theta_i^{(2)}) \dot{\eta}_i(t) + \sum_{j=1}^n m(p_{ij}(t) - \bar{p}_{ij}) \dot{p}_{ij}(t) \\
 & + \sum_{j=1}^n m(q_{ij}(t) - \bar{q}_{ij}) \dot{q}_{ij}(t) + \frac{1}{2} m(m-1) \sigma_i(t)^2 (|r_i(t)| + \varepsilon)^{(m-2)} \\
 & \left. + \frac{1}{1 - \tau_0} \sum_{i=1}^n \varphi_i^{(2)} \left((|r_i(t)| + \varepsilon)^m - (|r_i(t - \vartheta_i(t))| + \varepsilon)^m (1 - \dot{\vartheta}_i(t)) \right) \right\}. \tag{3.14}
 \end{aligned}$$

Under the controller (3.10) with adaptive rules (3.11) and (3.12), and Assumption 1, one has

$$\begin{aligned}
 \mathcal{L}V(t) \leq & \sum_{i=1}^n \left\{ m(|r_i(t)| + \varepsilon)^{(m-1)} \left(\sum_{j=1}^n \bar{p}_{ij} \xi_j |r_j(t)| + \sum_{j=1}^n \bar{q}_{ij} l_j |r_j(t - \vartheta_j(t))| - \underline{c}_i |r_i(t)| \right. \right. \\
 & + \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)} - d_i \Big) - \Theta_i^{(2)} (|r_i(t)| + \varepsilon)^m + \frac{1}{2} m(m-1) \\
 & \times (\sigma_i(t)^2 - h_i^2) (|r_i(t)| + \varepsilon)^{(m-2)} + \frac{1}{1 - \tau_0} \varphi_i^{(2)} \left((|r_i(t)| + \varepsilon)^m - (|r_i(t - \vartheta_i(t))| + \varepsilon)^m (1 - \dot{\vartheta}_i(t)) \right) \Big\}. \tag{3.15}
 \end{aligned}$$

According to Assumption 2, we select the parameters d_i, h_i in the controller (3.10) such that $d_i \geq \sum_{j=1}^n (\bar{p}_{ij} - \underline{p}_{ij}) M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{ij} - \underline{q}_{ij}) M_j^{(2)}, h_i \geq \bar{\sigma}_i$. As $\dot{\vartheta}_i(t) \leq \tau_0 < 1$ and $\varepsilon > 0$, one has

$$\begin{aligned}
 \mathcal{L}V(t) \leq & \sum_{i=1}^n \left\{ -m \underline{c}_i |r_i(t)|^m + \left(\frac{1}{1 - \tau_0} \varphi_i^{(2)} - \Theta_i^{(2)} \right) (|r_i(t)| + \varepsilon)^m \right. \\
 & - \varphi_i^{(2)} |r_i(t - \vartheta_i(t))|^m + \sum_{j=1}^n m \bar{p}_{ij} \xi_j (|r_i(t)| + \varepsilon)^{(m-1)} |r_j(t)| \\
 & \left. + \sum_{j=1}^n m \bar{q}_{ij} l_j (|r_i(t)| + \varepsilon)^{(m-1)} |r_j(t - \vartheta_j(t))| \right\}. \tag{3.16}
 \end{aligned}$$

In fact, it is readily apparent that the inequality below is invariably valid

$$m a_1 a_2 \dots a_m \leq a_1^m + a_2^m + \dots + a_m^m, \quad a_i \geq 0, \quad i = 1, 2, \dots, m. \tag{3.17}$$

We can get

$$\begin{aligned} & \sum_{j=1}^n m \bar{p}_{ij} \xi_j (|r_i(t)| + \varepsilon)^{m-1} |r_j(t)| \\ &= \sum_{j=1}^n m \left(\bar{p}_{ij}^{\frac{\alpha_{ij}}{m-1}} \xi_j^{\frac{\beta_{ij}}{m-1}} (|r_i(t)| + \varepsilon) \right)^{m-1} \bar{p}_{ij}^{(1-\alpha_{ij})} \xi_j^{(1-\beta_{ij})} |r_j(t)| \end{aligned} \quad (3.18)$$

$$\leq (m-1) \sum_{j=1}^n \bar{p}_{ij}^{\frac{m\alpha_{ij}}{m-1}} \xi_j^{\frac{m\beta_{ij}}{m-1}} (|r_i(t)| + \varepsilon)^m + \sum_{j=1}^n \bar{p}_{ij}^{m(1-\alpha_{ij})} \xi_j^{m(1-\beta_{ij})} |r_j(t)|^m,$$

$$\begin{aligned} & \sum_{j=1}^n m \bar{q}_{ij} l_j (|r_i(t)| + \varepsilon)^{m-1} |r_j(t - \vartheta_j(t))| \\ &= \sum_{j=1}^n m \left(\bar{q}_{ij}^{\frac{\lambda_{ij}}{m-1}} l_j^{\frac{\gamma_{ij}}{m-1}} (|r_i(t)| + \varepsilon) \right)^{m-1} \bar{q}_{ij}^{(1-\lambda_{ij})} l_j^{(1-\gamma_{ij})} |r_j(t - \vartheta_j(t))| \end{aligned} \quad (3.19)$$

$$\begin{aligned} & \leq (m-1) \sum_{j=1}^n \bar{q}_{ij}^{\frac{m\lambda_{ij}}{m-1}} l_j^{\frac{m\gamma_{ij}}{m-1}} (|r_i(t)| + \varepsilon)^m + \sum_{j=1}^n \bar{q}_{ij}^{m(1-\lambda_{ij})} l_j^{m(1-\gamma_{ij})} \\ & \quad \times |r_j(t - \vartheta_j(t))|^m. \end{aligned}$$

Substituting Eqs (3.18) and (3.19) into inequality (3.16), we get

$$\begin{aligned} \mathcal{L}V(t) & \leq \sum_{i=1}^n \left\{ \left(\sum_{j=1}^n \bar{p}_{ji}^{m(1-\alpha_{ji})} \xi_i^{m(1-\beta_{ji})} - m \underline{c}_i \right) |r_i(t)|^m + \left(\frac{1}{1-\tau_0} \varphi_i^{(2)} \right. \right. \\ & \quad + (m-1) \sum_{j=1}^n \left(\bar{p}_{ij}^{\frac{m\alpha_{ij}}{m-1}} \xi_j^{\frac{m\beta_{ij}}{m-1}} + \bar{q}_{ij}^{\frac{m\lambda_{ij}}{m-1}} l_j^{\frac{m\gamma_{ij}}{m-1}} \right) - \Theta_i^{(2)} \Big) (|r_i(t)| + \varepsilon)^m \\ & \quad \left. + \left(\sum_{j=1}^n \bar{q}_{ji}^{m(1-\lambda_{ji})} l_i^{m(1-\gamma_{ji})} - \varphi_i^{(2)} \right) |r_i(t - \vartheta_i(t))|^m \right\}. \end{aligned} \quad (3.20)$$

The selected $\varphi_i^{(2)}$ satisfies

$$\varphi_i^{(2)} > \sum_{j=1}^n \bar{q}_{ji}^{m(1-\lambda_{ji})} l_i^{m(1-\gamma_{ji})}. \quad (3.21)$$

With $\varphi_i^{(2)}$ chosen according to inequality (3.21), one can select $\Theta_i^{(2)}$ satisfies

$$\begin{aligned} \Theta_i^{(2)} & > \sum_{j=1}^n (m-1) \left(\bar{p}_{ij}^{\frac{m\alpha_{ij}}{m-1}} \xi_j^{\frac{m\beta_{ij}}{m-1}} + \bar{q}_{ij}^{\frac{m\lambda_{ij}}{m-1}} l_j^{\frac{m\gamma_{ij}}{m-1}} \right) \\ & \quad + \frac{1}{1-\tau_0} \varphi_i^{(2)} + \left| \bar{p}_{ji}^{m(1-\alpha_{ji})} \xi_i^{m(1-\beta_{ji})} - m \underline{c}_i \right|. \end{aligned} \quad (3.22)$$

From inequality (3.22), one has

$$\sum_{j=1}^n (m-1) \left(\bar{p}_{ij}^{\frac{m\alpha_{ij}}{m-1}} \xi_j^{\frac{m\beta_{ij}}{m-1}} + \bar{q}_{ij}^{\frac{m\lambda_{ij}}{m-1}} l_j^{\frac{m\gamma_{ij}}{m-1}} \right) + \frac{1}{1-\tau_0} \varphi_i^{(2)} - \Theta_i^{(2)} < 0,$$

so inequality (3.20) can be reduced to

$$\begin{aligned} \mathcal{L}V(t) \leq & \sum_{i=1}^n \left\{ \left(\sum_{j=1}^n \bar{p}_{ji}^{m(1-\alpha_{ji})} \xi_i^{m(1-\beta_{ji})} - m\bar{c}_i + \frac{1}{1-\tau_0} \varphi_i^{(2)} \right. \right. \\ & + (m-1) \sum_{j=1}^n \left(\bar{p}_{ij}^{\frac{m\alpha_{ij}}{m-1}} \xi_j^{\frac{m\beta_{ij}}{m-1}} + \bar{q}_{ij}^{\frac{m\lambda_{ij}}{m-1}} l_j^{\frac{m\gamma_{ij}}{m-1}} \right) - \Theta_i^{(2)} \Big) |r_i(t)|^m \\ & \left. + \left(\sum_{j=1}^n \bar{q}_{ji}^{m(1-\lambda_{ji})} l_i^{m(1-\gamma_{ji})} - \varphi_i^{(2)} \right) |r_i(t - \vartheta_i(t))|^m \right\}. \end{aligned} \quad (3.23)$$

Combined with inequalities (3.21) and (3.22), it follows that $\mathcal{L}V(t) < 0$. Obviously, we obtain $E[\mathcal{L}V(t)] < 0$. Then, we have $E[dV(t)] < 0$. Hence, the stochastic MRNNs between (2.8) and (2.6) under the updated controller (3.10) and adaptive rules (3.11) and (3.12) are globally synchronized in mean square.

Remark 4. In Theorem 3, $\varphi_i^{(2)}$ and $\Theta_i^{(2)}$ satisfying the conditions of (3.21) and (3.22) can be easily obtained. Compared with the previous stochastic MNNs [24, 27, 29, 47], the boundedness condition of noise may be more conservative than the local Lipschitz condition. However, the main advantage of this paper lies in that the weights of the slave system are independent of the master-system weights and noise model structure, and can be updated automatically. Moreover, by statistical estimation based on real-time system operation data and sensor sampling data, the upper bound of noise density can be obtained in scenarios where disturbance amplitude is constrained by hardware devices, output-power limits, and operating ranges. The lower bound of the controller parameter h_i can be increased according to $\bar{\sigma}_i + \|\mathbf{r}\|_2^2$, which implies that the bound of noise intensity is allowed to exist collection error. The proposed fuzzy adaptive strategy may enjoy favorable performance.

Remark 5. Here, it is difficult to handle noise intensity $\sigma_i(t)$ satisfying the local Lipschitz assumption, which brings considerable energy loss of the controller. Currently, if we use the stochastic slave system and $\sigma_i(t, \cdot)$ satisfies the Lipschitz condition, synchronization can be achieved. In future work, we will focus on weakening the assumptions imposed on noise and designing effective controllers, so that the synchronization of stochastic systems can adapt to a broader range of scenarios.

4. Simulation results and analysis

4.1. Synchronization simulation

To better illustrate the synchronization results of the MRNNs, two examples are provided.

Example 1. Consider the MRNNs (2.2) and (2.6) with the following parameters:

$$\begin{aligned} \Phi_i(s_i(t)) &= \sin(s_i(t)), \quad \Psi_i(s_i(t)) = \cos(s_i(t)), \quad \vartheta_i(t) = \frac{e^t}{e^t + 1}, \quad \tau = 1, \quad \dot{\vartheta}_i(t) \leq \tau_0 = \frac{1}{4} < 1, \\ I_i &= 0.2, \quad i = 1, 2, \quad \delta_1(t) = 1.25 \cos(1.6t) \sin(s_2(t)), \quad \delta_2(t) = 3.25 \cos(1.6t) \sin(s_1(t)), \\ c_1(s_1(t)) &= \begin{cases} 1.3, & s_1(t) \leq 0, \\ 0.9, & s_1(t) > 0, \end{cases} \quad c_2(s_2(t)) = \begin{cases} 0.009, & s_2(t) \leq 0, \\ 0.013, & s_2(t) > 0, \end{cases} \end{aligned}$$

$$\begin{aligned}
p_{11}(s_1(t)) &= \begin{cases} -1.8, & |s_1(t)| \leq 1 \\ -1.5, & |s_1(t)| > 1, \end{cases} & p_{12}(s_2(t)) &= \begin{cases} 2.8, & |s_2(t)| \leq 1, \\ 2.5, & |s_2(t)| > 1, \end{cases} \\
p_{21}(s_1(t)) &= \begin{cases} 1, & |s_1(t)| \leq 1, \\ 0.8, & |s_1(t)| > 1, \end{cases} & p_{22}(s_2(t)) &= \begin{cases} 1, & |s_2(t)| \leq 1, \\ -0.8, & |s_2(t)| > 1, \end{cases} \\
q_{11}(s_1(t)) &= \begin{cases} 1.5, & |s_1(t)| \leq 1, \\ 0.8, & |s_1(t)| > 1, \end{cases} & q_{12}(s_2(t)) &= \begin{cases} 2.8, & |s_2(t)| \leq 1, \\ 2.5, & |s_2(t)| > 1, \end{cases} \\
q_{21}(s_1(t)) &= \begin{cases} 0.6, & |s_1(t)| \leq 1, \\ 0.4, & |s_1(t)| > 1, \end{cases} & q_{22}(s_2(t)) &= \begin{cases} 1, & |s_2(t)| \leq 1, \\ -0.8, & |s_2(t)| > 1. \end{cases}
\end{aligned}$$

Obviously, we can get $\bar{p}_{11} = 1.8, \bar{p}_{12} = 2.8, \bar{p}_{21} = \bar{p}_{22} = \bar{q}_{22} = 1, \bar{q}_{11} = 1.5, \bar{q}_{12} = 2.8, \bar{q}_{21} = 0.6, \underline{p}_{11} = 1.5, \underline{p}_{12} = \underline{q}_{12} = 2.5, \underline{p}_{21} = \underline{p}_{22} = \underline{q}_{11} = \underline{q}_{22} = 0.8, \underline{q}_{21} = 0.4, \underline{c}_1 = 0.9, \underline{c}_2 = 0.009$. The selected $\xi_i = l_i = 1, M_i^{(1)} = M_i^{(2)} = 1, \bar{\delta}_1 = 1.25, \bar{\delta}_2 = 3.25$ are easy to verify Assumptions 1 and 2 are satisfied.

From inequalities (3.7) and (3.8) in Theorem 1, we calculate $\bar{q}_{11} + \bar{q}_{21} = 2.1$ and $\bar{q}_{12} + \bar{q}_{22} = 3.8$ yield $\varphi_1^{(1)} \in (2.1, +\infty), \varphi_2^{(1)} \in (3.8, +\infty)$. For convenience, $\varphi_1^{(1)} = 2.4, \varphi_2^{(1)} = 3.9$, we derive $\Theta_1^{(1)} > \bar{p}_{11}\xi_1 + \bar{p}_{21}\xi_1 + \frac{1}{1-\tau_0}\varphi_1^{(1)} - \underline{c}_1 = 5.1, \Theta_2^{(1)} > \bar{p}_{12}\xi_2 + \bar{p}_{22}\xi_2 + \frac{1}{1-\tau_0}\varphi_2^{(1)} - \underline{c}_2 = 8.991$. Therefore, the conditions of Theorem 1 hold.

For the simulations, let $s_1(0) = 4, s_2(0) = 6, \tilde{s}_1(0) = \text{rand}, \tilde{s}_2(0) = \text{rand}$ be the initial states of the MRNNs (2.2) and the slave system (2.6). Set $p_{ij}(0) = 0.01$ and $q_{ij}(0) = 0.01$ as the initial states of the adaptive rules (3.2)–(3.3), and $k_1 = \bar{\delta}_1 + \sum_{j=1}^2 (\bar{p}_{1j} - \underline{p}_{1j})M_j^{(1)} + \sum_{j=1}^n (\bar{q}_{1j} - \underline{q}_{1j})M_j^{(2)} = 2.85, k_2 = \bar{\delta}_2 + \sum_{j=1}^2 (\bar{p}_{2j} - \underline{p}_{2j})M_j^{(1)} + \sum_{j=1}^2 (\bar{q}_{2j} - \underline{q}_{2j})M_j^{(2)} = 4.05, \eta_i(0) = \text{rand}, \mu_i = 0.005$ as the initial parameters of the controller (3.1). The simulation results are depicted in Figures 1, 2, 5(a), and 5(b).

From Figure 1, it is observed that the phase trajectories of the MRNNs (2.2) are chaotic and the state variables are unstable. Meanwhile, it also shows that the synchronization errors $r_i(t)$ converge to zero. Figure 2(a) shows that the parameter $\eta_i(t)$ tends to be stable, which ensures that the controller (3.1) operates in a stable state (see Figures 5(a) and 5(b)). Besides, Figures 2(b) and 2(c) show that the states of adaptive rules (3.2)–(3.3) vary continuously. This illustrates that the weights of the slave system (2.6) are updated automatically. The above results demonstrate that the MRNNs (2.2) and the slave system (2.6) achieve favorable asymptotic synchronization under the designed controller without excessive energy loss.

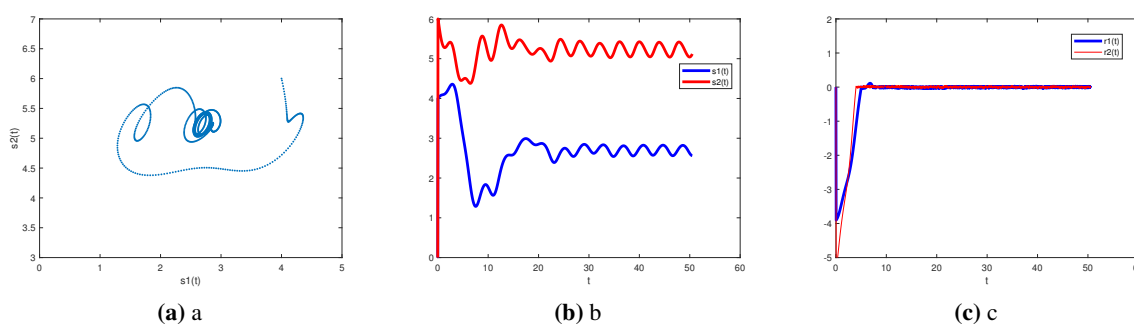


Figure 1. (a) The phase trajectories of MRNNs (2.2); (b) The states of MRNNs (2.2); (c) The errors of MRNNs (2.2) and (2.6).

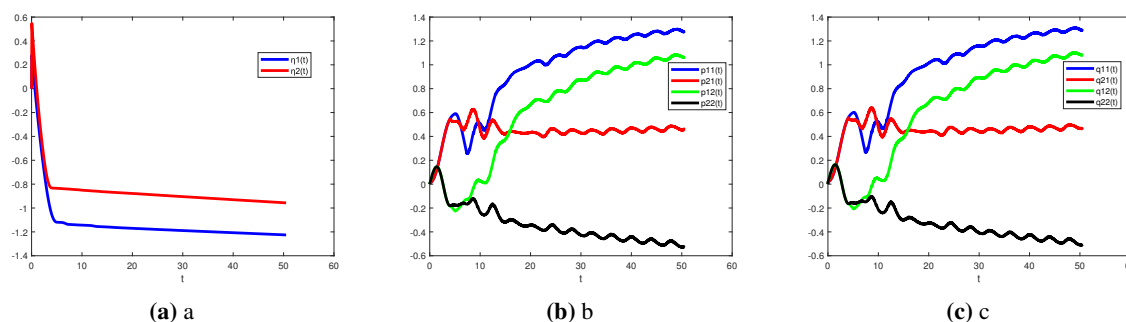


Figure 2. Trajectories of the adaptive parameters: (a) $\eta_i(t)$, (b) $p_{ij}(t)$, (c) $q_{ij}(t)$, $i, j = 1, 2$ in Example 1.

Example 2. Consider the stochastic MRNNs (2.8) with following parameters:

$$\begin{aligned}\sigma_1(t) &= 0.8 \cos(1.6t) \sin(s_2(t)), & \sigma_2(t) &= 1.2 \cos(1.6t) \sin(s_1(t)), \\ p_{12}(s_2(t)) &= \begin{cases} 1, & |s_2(t)| \leq 1, \\ 0.9, & |s_2(t)| > 1, \end{cases} & p_{22}(s_2(t)) &= \begin{cases} -1, & |s_2(t)| \leq 1, \\ -0.8, & |s_2(t)| > 1, \end{cases} \\ q_{12}(s_2(t)) &= \begin{cases} 2.5, & |s_2(t)| \leq 1, \\ 2.2, & |s_2(t)| > 1, \end{cases} & q_{21}(s_1(t)) &= \begin{cases} 0.3, & |s_1(t)| \leq 1, \\ 0.2, & |s_1(t)| > 1. \end{cases}\end{aligned}$$

and all other system parameters stay consistent with Example 1 without any adjustments.

Similarly, we can easily calculate $\bar{p}_{11} = 1.8, \bar{p}_{12} = 1, \bar{p}_{21} = 1, \bar{p}_{22} = 1, \bar{q}_{11} = 1.5, \bar{q}_{12} = 2.5, \bar{q}_{21} = 0.3, \bar{q}_{22} = 1, \underline{c}_1 = 0.9, \underline{c}_2 = 0.009, \xi_i = l_i = 1, \bar{\sigma}_1 = 0.8, \sigma_2 = 1.2$. Now, let $m = 2, \lambda_{ij} = \gamma_{ij} = \alpha_{ij} = \beta_{ij} = 0.5$, and set $\mu_i = 0.006, \varepsilon = 0.2, h_1 = 0.8, h_2 = 1.2, d_1 = 2.3, d_2 = 0.7$.

From inequalities (3.21) and (3.22) in Theorem 3, we calculate $\sum_{j=1}^2 \bar{q}_{j1}^{m(1-\lambda_{j1})} l_i^{m(1-\gamma_{j1})} = \bar{q}_{11} + \bar{q}_{21} = 1.8, \sum_{j=1}^2 \bar{q}_{j2}^{m(1-\lambda_{j2})} l_2^{m(1-\gamma_{j2})} = \bar{q}_{12} + \bar{q}_{22} = 3.5$, yielding $\varphi_1^{(2)} \in (1.8, +\infty), \varphi_2^{(2)} \in (3.5, +\infty)$. For convenience, $\varphi_1^{(2)} = 2.1, \varphi_2^{(2)} = 3.6$, we derive $\Theta_1^{(2)} > \bar{p}_{11} + \bar{q}_{11} + \bar{p}_{12} + \bar{q}_{12} + \frac{1}{1-\tau_0} \varphi_1^{(2)} + |\bar{p}_{11} + \bar{p}_{21} - m \underline{c}_1| = 10.6, \Theta_2^{(2)} > \bar{p}_{21} + \bar{q}_{21} + \bar{p}_{22} + \bar{q}_{22} + \frac{1}{1-\tau_0} \varphi_2^{(2)} + |\bar{p}_{12} + \bar{p}_{22} - m \underline{c}_2| = 10.082$. The sufficient conditions of Theorem 3 hold for any parameter chosen within the above interval.

The controller (3.10) can be reduced to $u_i(t) = 0.5\eta_i(t)(|r_i(t)| + 0.2)\text{sign}(r_i(t)) - \frac{h_i^2 \text{sign}(r_i(t))}{2(|r_i(t)| + 0.2)} - d_i \text{sign}(r_i(t))$, and adaptive rules $\dot{p}_{ij}(t) = -\Phi_j(\tilde{s}_j(t))(|r_i(t)| + 0.2)\text{sign}(r_i(t)), \dot{q}_{ij}(t) = -\Psi_j(\tilde{s}_j(t) - \vartheta_j(t))(|r_i(t)| + 0.2)\text{sign}(r_i(t))$, where $\eta_i(t)$ satisfies $\dot{\eta}_i(t) = -\mu_i(|r_i(t)| + 0.2)^2$. The simulation results are depicted in Figures 3–4 and 5(b)–5(c).

As shown in Figures 3(a) and 3(b), the trajectories of the stochastic MRNNs (2.8) are chaotic and the state variables are unstable. Figure 3(c) shows that the errors $r_1(t), r_2(t)$ nearly converge to zero. Figure 4 depicts states of the control gains $\eta_i(t), p_{ij}(t), q_{ij}(t)$. This indicates that the controller (3.10) may generate small-amplitude fluctuations. Furthermore, Figures 5(c) and 5(d) show that the states of the controller (3.10). Compared with the controller (3.1), the controller (3.10) stays within a reasonable fluctuation range. The above results further support the conclusion that stochastic MRNNs (2.8) and slave system (2.6) achieve mean square synchronization under controller (3.10) with the adaptive rules (3.11) and (3.12).

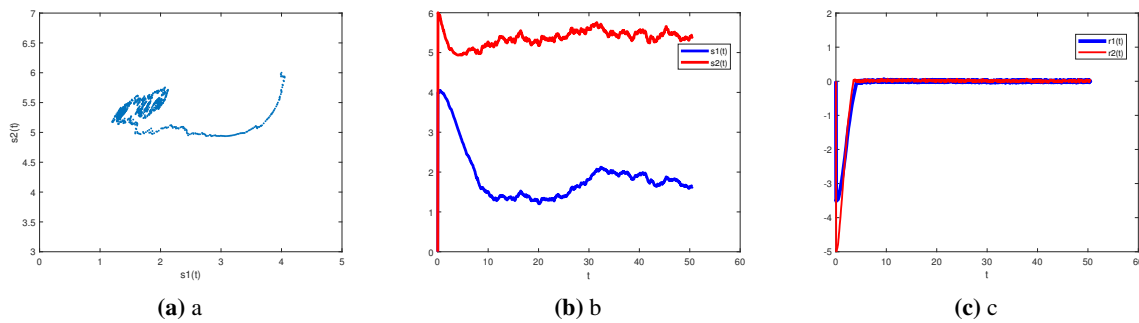


Figure 3. (a) The phase trajectories of MRNNs (2.8); (b) The states of MRNNs (2.8); (c) The errors of MRNNs (2.8) and (2.6).

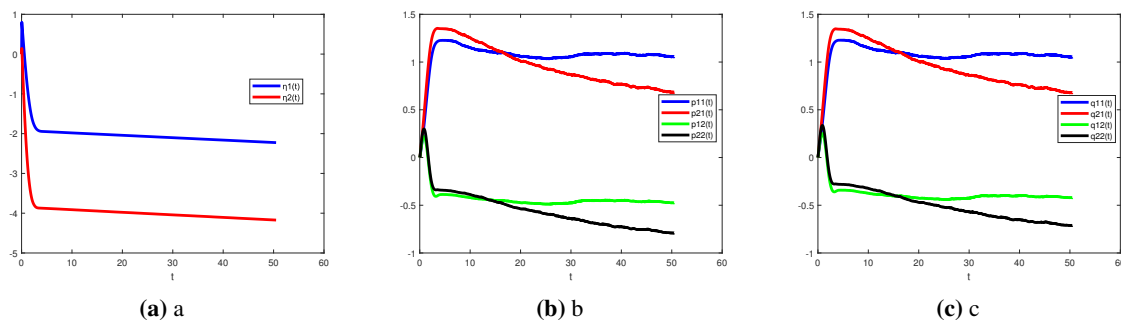


Figure 4. Trajectories of the adaptive parameters: (a) $\eta_i(t)$; (b) $p_{ij}(t)$; (c) $q_{ij}(t)$, $i, j = 1, 2$ in Example 2.

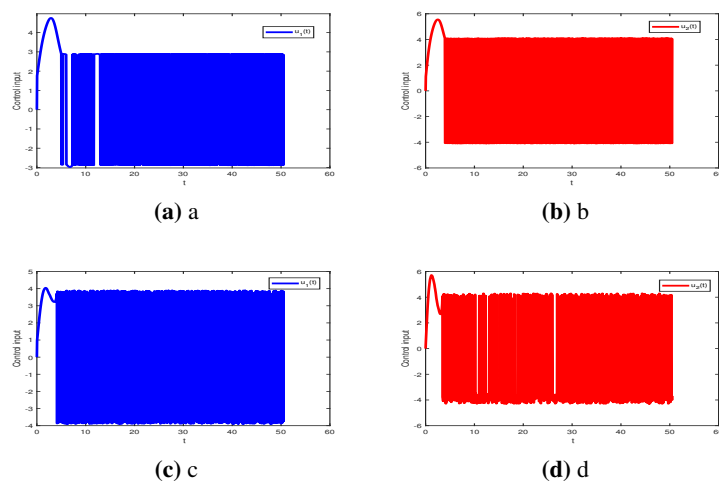


Figure 5. Trajectories of controller: (a) $u_1(t)$ in controller (3.1); (b) $u_2(t)$ in controller (3.1); (c) $u_1(t)$ in controller (3.10); (d) $u_2(t)$ in controller (3.10).

4.2. Image enhancement based on MRNNs

In this section, we implement image enhancement and denoising based on the MRNNs (2.8) and the corresponding slave system (2.6). To better apply the circuit design of MRNNs (2.8) to image processing tasks, we denote $\Phi_{ij}(s_{ij}(t))$ instead of $\Phi_j(s_j(t))$, which signifies the output of the neuron at pixel (i, j) . Set $\Phi_{ij}(s_{ij}(t)) = s_{ij}(t)$, and let $p_{i,j,k,l}$ replace $p_{ij}(s_j(t))$, which denotes the connection weight between the neuron at pixel (i, j) and the neighboring neuron at (k, l) . Set $\Psi_j(s_j(t - \vartheta_j(t))) = 0, i, j = 1, 2, 3$. The parameters of controller (3.10) and adaptive rules (3.11) and (3.12) take the same values as those in Example 2. Here, the standard deviation of the external white noise corresponds to the noise intensity of the master system (2.8). Subsequently, pixel-intensity features extracted from the image are adopted as the initial states of the master system (2.8). We compute the coupling weights between the neuron at pixel (i, j) and adjacent neuron at (k, l) and input them into the master system for iteration. The iterative outputs serve as the initial states of the slave system (2.6), and the slave system under controller (3.10) and adaptive rules (3.11) and (3.12) performs iterative evolution. Upon the completion of iteration, the pixel-intensity features output are used to reconstruct the image.

We evaluate the enhancement and denoising effects of four algorithms, namely the three-Gaussian mixture model (3-GMM), CNN, BM3D, and our MRNNs. The 3-GMM, CNN, and MRNNs are first applied to enhance two batches of original images. After adding additive Gaussian white noise to these original images, a denoising comparison between BM3D and MRNNs is conducted. The EME, SMD2 and PNSR indices from [51] are adopted to evaluate the image enhancement and denoising performance. All evaluation indices of the four methods are provided subsequently. As indicated by the overall results, our MRNNs yields weaker contrast enhancement than 3-GMM and CNN, but gains advantages in sharpness and brightness. Besides, it achieves superior denoising performance over BM3D on noisy color images.

Example 3. Consider the MRNNs (2.8) with disturbance $\sigma_{ij}(t) = 0$, and

$$c_{ij}(s_{ij}(t)) = \begin{cases} 1.06, & s_{ij}(t) \leq 0, \\ 1, & s_{ij}(t) > 0, \end{cases}, i, j = 1, 2, 3.$$

We randomly select one color image (Figure 6(a)) and one grayscale image (Figure 7(a)). The 3-GMM, CNN, and our MRNNs model are employed to process these images. As shown in Figure 6, the processed image by MRNNs is brighter than the original from a visual perspective. Compared with 3-GMM and CNN, the results obtained MRNNs have lower EME and higher SMD2 values. From Figure 7, our MRNNs yields a slightly higher EME but much higher SMD2 than other methods, which may arise from fine high-frequency artifacts generated during processing. Therefore, our model exhibits limited advantages for image-enhancement tasks. The corresponding evaluation metrics are provided in Table 1.



Figure 6. Color image bionic enhancement: (a) Origin; (b) 3-GMM; (c) CNN; (d) MRNNs.



Figure 7. Grayscale image bionic enhancement: (a) Origin; (b) 3-GMM; (c) CNN; (d) MRNNs.

Example 4. In the MRNNs (2.8), let

$$c_{ij}(s_{ij}(t)) = \begin{cases} 1.1, & s_{ij}(t) \leq 0, \\ 0.9, & s_{ij}(t) > 0, \end{cases}, i, j = 1, 2, 3,$$

and σ be the standard deviation of Gaussian white noise.

We feed two original images (Figures 6(a) and 7(a)) into MRNNs (2.8) to obtain images corrupted by Gaussian white noise with $\sigma = 0.02$ (Figures 8(a) and 9(a)). Select controller (3.10) with adaptive rules (3.11) and (3.12) to denoise the noisy images. Let $\mu = 0.005$, $h_i = 0.05$, $\varepsilon = 0.2$. Comparative denoising experiments are conducted between the classical BM3D algorithm and the proposed MRNNs model with control strategy (3.10). Combining Figure 8 with the data in Table 1, our model achieves higher EME and PSNR (over 30dB) values than BM3D, and its SMD2 value is closer to original image. From Figure 9 and Table 1, our model achieves a higher EME value than BM3D. Meanwhile, BM3D yields a considerably higher SMD2 than the original image, and its PSNR is below 30dB. This implies that BM3D may suffer from distortion for grayscale images.

To demonstrate the denoising effect of proposed MRNNs equipped with control strategy (3.10), we obtain images corrupted by Gaussian white noise with $\sigma = 0.05$ (Figures 10(a) and 11(a)). By comparing the two groups of images in Figures 10(b)–10(c), and 11(b)–11(c), it can be observed that the images processed by MRNNs contain fewer background textures and granular artifacts than that obtained by BM3D. From Table 1, our model yields higher EME and PSNR (over 30 dB) for denoising the noisy color image, whereas for the grayscale image, only PSNR is improved and the

remaining metrics are closer to the original image. Experimental results indicate that our approach achieves certain advantages over BM3D in terms of denoising and fidelity for image-processing tasks.

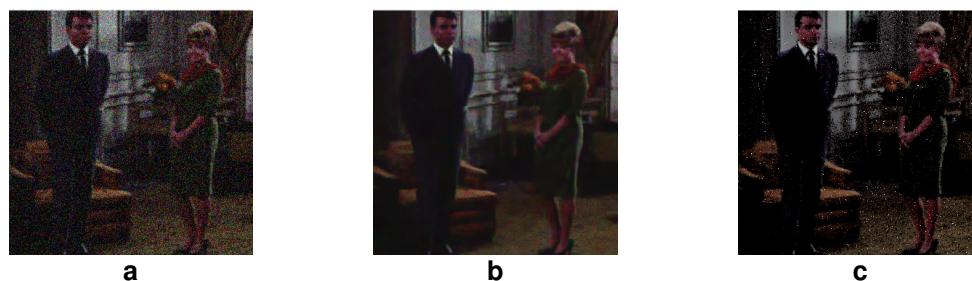


Figure 8. Color image denoising enhancement: (a) Gaussian noisy ($\sigma = 0.02$); (b) BM3D; (c) MRNNs with control strategy (3.10).



Figure 9. Grayscale image denoising enhancement: (a) Gaussian noisy ($\sigma = 0.02$); (b) BM3D; (c) MRNNs with control strategy (3.10).

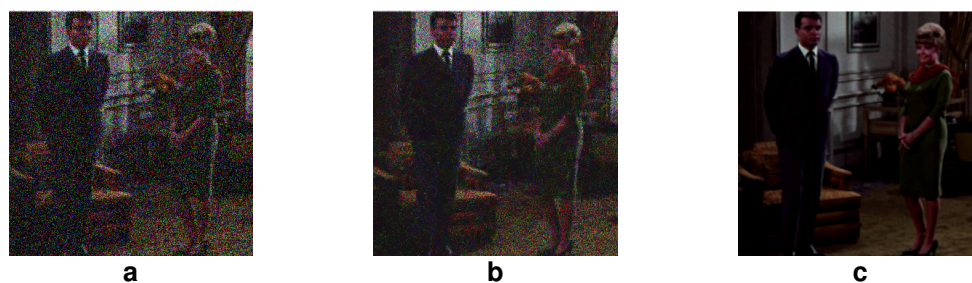


Figure 10. Color image denoising enhancement: (a) Gaussian noisy ($\sigma = 0.05$); (b) BM3D; (c) MRNNs with control strategy (3.10).

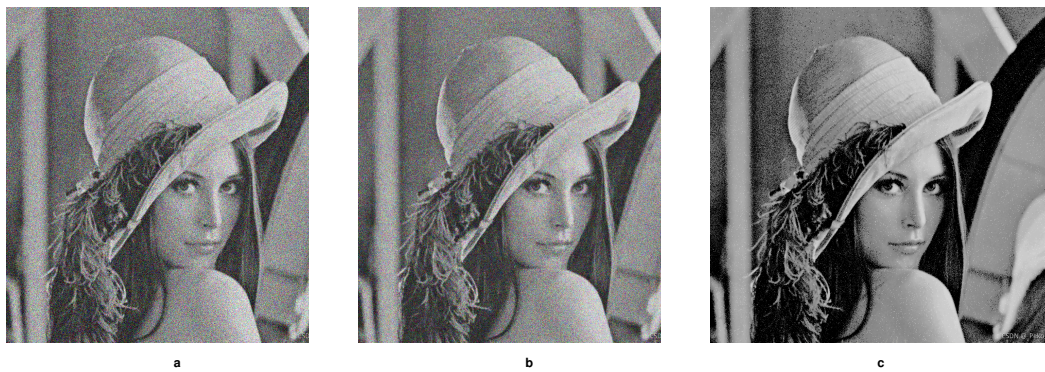


Figure 11. Grayscale image denoising enhancement: (a) Gaussian noisy ($\sigma = 0.05$); (b) BM3D; (c) MRNNs with control strategy (3.10).

Table 1. Comparison of the effects of the four enhancement models

No.	Index	Origin	Gaussian	3-GMM	CNN	BM3D	MRNNs
1	EME	14.15	—	17.8	16.67	—	14.81
	SMD2	1385039	—	1425163	1229519	—	5703553
	PSNR(db)	99	—	35.21	40.77	—	16.10
2	EME	3.62	—	6.14	5.63	—	6.181
	SMD2	12578931	—	41643945	15102663	—	129866538
	PSNR(db)	99	—	20.99	25.59	—	9.78
3	EME	14.15	101.39	—	—	10.01	53.23
	SMD2	1385039	59215804	—	—	5526064	1692851
	PSNR(db)	99	21.93	—	—	27.80	31.42
4	EME	3.62	17.40	—	—	7.16	9.84
	SMD2	12578931	818207001	—	—	164611846	24445980
	PSNR(db)	99	20.63	—	—	26.52	23.34
5	EME	14.15	111.03	—	—	34.72	53.33
	SMD2	1385039	125563009	—	—	96064573	1778532
	PSNR(db)	99	18.19	—	—	19.02	31.39
6	EME	3.62	35.58	—	—	28.78	9.89
	SMD2	12578931	1833530179	—	—	2736415944	26206290
	PSNR(db)	99	17.05	—	—	15.23	23.39

5. Conclusions

We have investigated the asymptotic synchronization issue for the delayed MRNNs with random disturbances using the fuzzy adaptive control method. Two controllers with adaptive rules are designed to achieve better synchronization of MRNNs. This approach allows the connection weights of the slave MRNNs to be updated automatically through adaptive rules. The synchronization criteria in Theorems 1–3 rest on a novel approach grounded in differential inclusions, set-valued maps, and

the Lyapunov-Krasovskii functional method. This framework sets our results apart from existing literature. First, the boundedness requirement on disturbance term is extended to the conditions of zero initial value and local Lipschitz property, and corresponding synchronization results are established for MRNNs with arbitrary deterministic disturbance. Second, mean square synchronization for stochastic systems is derived under the bounded white noise condition. Finally, multiple simulation examples are implemented to check and analyze the synchronization results, thereby improving the applicability and robustness of the MRNNs and the related control strategy. Quantitative assessment confirms the model's substantial effectiveness in tasks of image enhancement and denoising, with full experimental data and analysis presented in Section 4. Our future research will prioritize fixed-time synchronization of MNNs with discontinuous switching, including complex dynamic networks with semi-Markov jumps. In parallel, we will develop effective control strategies for these systems under Event-Triggered Communication Scheme. We evaluate the controller performance using the evaluation criteria proposed in [30].

Author contributions

Lixia Ye: Writing-original draft, Writing-review & editing; Qiling Zhao: Investigation; Hang Zheng: Validation, Supervision. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgements

The authors express gratitude to the reviewers and editors for their helpful comments and suggestions, as well as to the Natural Science Foundation of Fujian Province, China (No. 2024J01926), Scientific Research Startup Fund for High-level Talents of Wuyi University (No. YJ202317).

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. L. O. Chua, Memristor-the missing circuit element, *IEEE Transactions on Circuit Theory*, **18** (1971), 507–519. <https://doi.org/10.1109/TCT.1971.1083337>
2. J. Hu, J. Wang, Global uniform asymptotic stability of memristor-based recurrent neural networks with time delays, *The 2010 International Joint Conference on Neural Networks (IJCNN)*, Barcelona, Spain, 2010, 1–8. <https://doi.org/10.1109/IJCNN.2010.5596359>

3. C. Xu, M. L. Liao, C. H. Wang, J. R. Sun, H. R. Lin, Memristive competitive hopfield neural network for image segmentation application, *Cogn. Neurodyn.*, **17** (2023), 1061–1077. <https://doi.org/10.1007/s11571-022-09891-2>
4. H. R. Lin, X. H. Deng, Y. Zhang, G. Y. Min, A second-order memristor method to construct memristive neural networks with multi-butterfly and multi-scroll dynamics, *IEEE T. Circuits-I*, **73** (2026), 4840–4852. <https://doi.org/10.1109/TCSI.2026.3663432>
5. H. R. Lin, Z. T. Li, Y. Zhang, X. H. Deng, X. C. Chen, G. Y. Min, Memristive cnn with multi-butterfly attractors: Mathematical modeling, dynamics analysis and application in secure communication, *Chaos Soliton. Fract.*, **206** (2026), 117905. <https://doi.org/10.1016/j.chaos.2026.117905>
6. L. C. Li, R. Xu, Q. T. Gan, J. Z. Lin, A switching control for finite-time synchronization of memristor-based bam neural networks with stochastic disturbances, *Nonlinear Anal.-Model.*, **25** (2020), 958–979. <https://doi.org/10.15388/namc.2020.25.20557>
7. W. T. Wang, W. Chen, New study on neutral-type inertial bam neural networks via the characteristic method, *J. Math. Anal. Appl.*, **557** (2026), 130325. <https://doi.org/10.1016/j.jmaa.2025.130325>
8. W. T. Wang, W. Chen, Exponential input-to-state stability for delayed inertial bam neural networks, *Electron. Res. Arch.*, **34** (2026), 5842–5860. <https://doi.org/10.3934/era.2026259>
9. J. S. Zhuang, Y. Zhou, Y. H. Xia, Intra-layer synchronization in duplex networks with time-varying delays and stochastic perturbations under impulsive control, *Neural Process. Lett.*, **52** (2020), 785–804. <https://doi.org/10.1007/s11063-020-10281-0>
10. X. D. Li, J. D. Cao, D. W. C. Ho, Impulsive control of nonlinear systems with time-varying delay and applications, *IEEE T. Cybernetics*, **50** (2020), 2661–2673. <https://doi.org/10.1109/TCYB.2019.2896340>
11. D. S. Wang, L. K. Li, Fixed-time stability analysis of discontinuous impulsive systems, *Commun. Nonlinear Sci.*, **120** (2023), 107153. <https://doi.org/10.1016/j.cnsns.2023.107153>
12. Y. K. Xie, Z. Wang, A ratio-dependent impulsive control of an siqs epidemic model with non-linear incidence, *Appl. Math. Comput.*, **423** (2022), 127018. <https://doi.org/10.1016/j.amc.2022.127018>
13. Y. Liu, J. Q. Lu, B. Wu, Stability and l2-gain performance for non-linear switched impulsive systems, *IET Control Theory A.*, **9** (2015), 300–307. <https://doi.org/10.1049/iet-cta.2014.0015>
14. F. Shi, Y. Liu, Y. Y. Li, J. L. Qiu, Input-to-state stability of nonlinear systems with hybrid inputs and delayed impulses, *Nonlinear Anal.-Hybri.*, **44** (2022), 101145. <https://doi.org/10.1016/j.nahs.2021.101145>
15. Z. Y. Wang, J. D. Cao, Z. W. Cai, M. Abdel-Aty, A novel lyapunov theorem on finite/fixed-time stability of discontinuous impulsive systems, *Chaos*, **30** (2020), 013139. <https://doi.org/10.1063/1.5121246>
16. Y. Liu, X. Y. Chen, J. Q. Lu, W. H. Gui, Stability of time-delay systems with delayed impulses: average impulsive estimation approach, *SIAM J. Control Optim.*, **61** (2023), 620–646. <https://doi.org/10.1137/22M1476332>

17. J. Q. Lu, B. X. Jiang, W. X. Zheng, Potential impacts of delay on stability of impulsive control systems, *IEEE T. Automat. Contr.*, **67** (2022), 5179–5190. <https://doi.org/10.1109/TAC.2021.3120672>
18. Y. B. Li, D. S. Wang, Z. W. Cai, On asymptotic fixed-time controller design for uncertain nonlinear systems with pure state constraints, *AIMS Math.*, **8** (2023), 27151–27174. <https://doi.org/10.3934/math.20231389>
19. X. S. Yang, X. D. Li, J. Q. Lu, Z. S. Cheng, Synchronization of time-delayed complex networks with switching topology via hybrid actuator fault and impulsive effects control, *IEEE T. Cybernetics*, **50** (2020), 4043–4052. <https://doi.org/10.1109/TCYB.2019.2938217>
20. X. Y. Zhang, C. D. Li, Z. L. He, Cluster synchronization of delayed coupled neural networks: Delay-dependent distributed impulsive control, *Neural Networks*, **14** (2021), 34–43. <https://doi.org/10.1016/j.neunet.2021.04.026>
21. X. X. Lv, J. D. Cao, X. D. Li, M. Abdel-Aty, U. A. Al-Juboori, Synchronization analysis for complex dynamical networks with coupling delay via event-triggered delayed impulsive control, *IEEE T. Cybernetics*, **51** (2021), 5269–5278. <https://doi.org/10.1109/TCYB.2020.2974315>
22. N. N. Zhao, Y. H. Qiao, C. Q. Xu, New results on synchronization control of memristor-based quaternion-valued fuzzy neural networks with delayed impulses, *Fuzzy Set. Syst.*, **484** (2024), 108940. <https://doi.org/10.1016/j.fss.2024.108940>
23. H. Sang, C. Y. Cui, P. Wang, Y. Liu, C. X. Huang, J. D. Cao, Further on pinning quasi-synchronization of switched directed networks with event-triggered communication, *IEEE T. Automat. Contr.*, **70** (2025), 6127–6134. <https://doi.org/10.1109/TAC.2025.3552027>
24. J. Gao, P. Y. Zhu, W. J. Xiong, J. D. Cao, L. Zhang, Asymptotic synchronization for stochastic memristor-based neural networks with noise disturbance, *J. Franklin I.*, **353** (2016), 3271–3289. <https://doi.org/10.1016/j.jfranklin.2016.06.002>
25. H. W. Ren, Z. P. Peng, Y. Gu, Fixed-time synchronization of stochastic memristor-based neural networks with adaptive control, *Neural Networks*, **130** (2020), 165–175. <https://doi.org/10.1016/j.neunet.2020.07.002>
26. Y. Zhou, X. X. Wan, C. X. Huang, X. S. Yang, Finite-time stochastic synchronization of dynamic networks with nonlinear coupling strength via quantized intermittent control, *Appl. Math. Comput.*, **376** (2020), 125157. <https://doi.org/10.1016/j.amc.2020.125157>
27. L. L. Zhou, H. Lin, F. Tan, Fixed/predefined-time synchronization of coupled memristor-based neural networks with stochastic disturbance, *Chaos Soliton. Fract.*, **173** (2023), 113643. <https://doi.org/10.1016/j.chaos.2023.113643>
28. Z. F. Ma, D. B. Tong, Q. Y. Chen, W. N. Zhou, Fixed/prescribed-time synchronization and energy consumption for kuramoto-oscillator networks, *IEEE T. Cybernetics*, **55** (2025), 3379–3389. <https://doi.org/10.1109/TCYB.2025.3556103>
29. Q. J. Jiang, D. B. Tong, Q. Y. Chen, F. Hu, W. N. Zhou, Event-triggered fixed-time synchronization and energy consumption prediction for coupled neural networks with stochastic disturbances, *Eng. Appl. Artif. Intel.*, **166** (2026), 113531. <https://doi.org/10.1016/j.engappai.2025.113531>

30. G. D. Zhang, J. H. Hua, S. P. Wen, New results on fixed/preassigned-time stabilization of the discontinuous neural networks with mixed time-varying delays, *Neurocomputing*, **682** (2026), 133464. <https://doi.org/10.1016/j.neucom.2026.133464>
31. X. Li, H. Q. Wu, J. D. Cao, Synchronization in finite time for variable-order fractional complex dynamic networks with multi-weights and discontinuous nodes based on sliding mode control strategy, *Neural Networks*, **139** (2021), 335–347. <https://doi.org/10.1016/j.neunet.2021.03.033>
32. L. Feng, C. Hu, J. Yu, H. J. Jiang, S. P. Wen, Fixed-time synchronization of coupled memristive complex-valued neural networks, *Chaos Soliton. Fract.*, **148** (2021), 110993. <https://doi.org/10.1016/j.chaos.2021.110993>
33. M. Iswarya, R. Raja, J. D. Cao, M. Niezabitowski, J. Alzabut, C. Maharajan, New results on exponential input-to-state stability analysis of memristor based complex-valued inertial neural networks with proportional and distributed delays, *Math. Comput. Simulat.*, **201** (2022), 440–461. <https://doi.org/10.1016/j.matcom.2021.01.020>
34. Y. F. Liu, B. Shen, P. Zhang, Synchronization and state estimation for discrete-time coupled delayed complex-valued neural networks with random system parameters, *Neural Networks*, **150** (2022), 181–193. <https://doi.org/10.1016/j.neunet.2022.02.028>
35. Q. Wang, L. Duan, L. H. Huang, Z. H. Su, Global exponential stability of a periodic inertial memristive neural networks with time delays: characteristic approach, *Neurocomputing*, **668** (2026), 132408. <https://doi.org/10.1016/j.neucom.2025.132408>
36. X. Y. Liu, H. S. Su, M. Z. Q. Chen, A switching approach to designing finite-time synchronization controllers of coupled neural networks, *IEEE T. Neur. Net. Lear.*, **27** (2016), 471–482. <https://doi.org/10.1109/TNNLS.2015.2448549>
37. Z. Y. Wang, J. D. Cao, Z. W. Cai, L. Rutkowski, Anti-synchronization in fixed time for discontinuous reaction-diffusion neural networks with time-varying coefficients and time delay, *IEEE T. Cybernetics*, **50** (2020), 2758–2769. <https://doi.org/10.1109/TCYB.2019.2913200>
38. Y. Fan, X. Huang, H. Shen, J. D. Cao, Switching event-triggered control for global stabilization of delayed memristive neural networks: an exponential attenuation scheme, *Neural Networks*, **117** (2019), 216–224. <https://doi.org/10.1016/j.neunet.2019.05.014>
39. A. Alsaedi, J. D. Cao, B. Ahmad, A. Alshehri, X. G. Tan, Synchronization of master-slave neural networks via fuzzy output-based adaptive strategy, *Chaos Soliton. Fract.*, **158** (2022), 112095. <https://doi.org/10.1016/j.chaos.2022.112095>
40. Z. Y. Lu, Q. B. Ge, Y. Li, J. H. Hu, Finite-time synchronization of memristor-based recurrent neural networks with inertial items and mixed delays, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, **51** (2021), 2701–2711. <https://doi.org/10.1109/TSMC.2019.2916073>
41. R. X. Li, J. D. Cao, Passivity and dissipativity of fractional-order quaternion-valued fuzzy memristive neural networks: nonlinear scalarization approach, *IEEE T. Cybernetics*, **52** (2019), 2821–2832. <https://doi.org/10.1109/TCYB.2020.3025439>
42. Q. W. Liu, H. C. Yan, H. Zhang, C. Y. Chen, Y. F. Chang, Exponential synchronization of second-order fuzzy memristor-based neural networks with mixed time delays via fuzzy adaptive control, *IEEE T. Fuzzy Syst.*, **31** (2023), 1953–1965. <https://doi.org/10.1109/TFUZZ.2022.3216777>

43. S. P. Wen, Z. G. Zeng, T. W. Huang, Dynamic behaviors of memristor-based delayed recurrent networks, *Neural Comput. Appl.*, **23** (2013), 815–821. <https://doi.org/10.1007/s00521-012-0998-y>
44. S. P. Wen, G. Bao, Z. G. Zeng, Y. R. Chen, T. W. Huang, Global exponential synchronization of memristor-based recurrent neural networks with time-varying delays, *Neural Networks*, **48** (2013), 195–203. <https://doi.org/10.1016/j.neunet.2013.10.001>
45. L. M. Wang, H. Y. Li, C. Hu, J. H. Hu, Q. Y. Wang, Synchronization and settling-time estimation of fuzzy memristive neural networks with time-varying delays: Fixed-time and preassigned-time control, *Fuzzy Set. Syst.*, **470** (2023), 108654. <https://doi.org/10.1016/j.fss.2023.108654>
46. J. Gao, P. Y. Zhua, A. Alsaedi, F. E. Alsaadi, T. Hayat, A new switching control for finite-time synchronization of memristor-based recurrent neural networks, *Neural Networks*, **86** (2017), 1–9. <https://doi.org/10.1016/j.neunet.2016.10.008>
47. F. Wang, Y. L. Chen, Mean square exponential stability for stochastic memristor-based neural networks with leakage delay, *Chaos Soliton. Fract.*, **146** (2021), 110811. <https://doi.org/10.1016/j.chaos.2021.110811>
48. A. F. Filippov, *Differential equations with discontinuous righthand sides*, Dordrecht: Springer, 1988. <https://doi.org/10.1007/978-94-015-7793-9>
49. J.-J. E. Slotine, W. P. Li, *Applied nonlinear control*, Englewood Cliffs: Prentice-Hall, 1991.
50. C. Yang, Y. C. Liu, F. M. Li, Y. F. Li, Finite-time synchronization of a class of coupled memristor-based recurrent neural networks: Static state control and dynamic control approach, *Int. J. Control Autom. Syst.*, **19** (2021), 426–438. <https://doi.org/10.1007/s12555-019-0616-1>
51. S. S. Agaian, B. Silver, K. A. Panetta, Transform coefficient histogram-based image enhancement algorithms using contrast entropy, *IEEE T. Image Process.*, **16** (2007), 741–758. <https://doi.org/10.1109/TIP.2006.888338>



AIMS Press

© 2026 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (<https://creativecommons.org/licenses/by/4.0>)