



Editorial

Special issue “Advances in analysis and applied mathematics”

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Abstract: This special issue spans the interface of analysis and applied mathematics collecting recent advances in the following topics: Boundedness of integral operators on generalized function spaces, normality in several complex variables, fractional stochastic dynamics, functional inequalities and fixed-point methods, nonlinear waves, generalized thermoelasticity, and models in the life and earth sciences. We identify the four recurring themes and summarize the main results. We also collect some open problems emerging from these contributions.

Keywords: analysis and applied mathematics; integral operators; fractional differential equations; nonlinear dynamics; thermoelasticity; mathematical modeling

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Introduction

The contributions collected in this special issue can be framed by two questions. The first one is *where* analysis takes place: The classical Lebesgue and Sobolev spaces do not always suffice for equations with nonstandard growth or for operators whose order varies from point to point. Variable-exponent, Herz, Herz–Morrey, Triebel–Lizorkin, Besov, and Orlicz spaces provide the flexible settings required by such phenomena. The second one concerns *how* memory and nonlocality are represented. The presented results rely on the Caputo, Caputo–Katugampola, Caputo-tempered, Caputo–Hadamard, and conformable derivatives. Some papers address both questions at once, and there the operator and the ambient space are not independent technical choices: together, they determine which forms of regularity, stability, and dynamics can occur.

We present a discussion of the main results grouped by different topics: (1) integral operators and function spaces, (2) complex analysis and special functions, (3) fractional stochastic dynamics and control, (4) inequalities, convexity and fixed-point methods, (5) nonlinear waves, solitons, and chaotic dynamics, (6) thermoelasticity, and (7) models in the life and earth sciences. We also list some open

problems to be considered in the future. We refer the reader to the corresponding papers for clarification of the statements and notation describing each contribution.

1. Integral operators and function spaces

The most frequent topic addressed the question of how the boundedness, continuity, and evolution properties of integral operators survive when classical regularity, symmetry, or ambient geometry is weakened. Khan et al. [1] develop a variational framework for the $p(\cdot)$ -Laplace equation and prove a pointwise estimate for weak solutions in terms of truncated Havin–Maz’ya–Wolff potentials on variable Herz spaces, recovering classical potential estimates as limiting cases. AlNemer, Basendwah, Sultan, and Popa [2] prove the boundedness of multilinear Calderón–Zygmund operators on grand variable Herz–Morrey spaces, a framework containing several previously studied Herz-type scales. Nasir and Ghobber [3] introduce a variable-order Bessel–Riesz operator and establish boundedness on variable Lebesgue spaces in diagonal and non-diagonal regimes, and Liu, Liu, and Zhang [4] obtain boundedness and continuity of local bilinear maximal commutators on Triebel–Lizorkin and Besov spaces over a subdomain of $\mathbb{R}^n$. The last two papers present solutions to the loss of translation invariance: Young’s convolution inequality is unavailable in the variable-exponent setting, while translations do not preserve a proper subdomain. Both therefore replace the missing symmetry with pointwise and dyadic estimates adapted to the local or variable structure of the problem.

Two papers weaken the kernel instead. On the one hand, Lv and Tao [5] obtain quantitative two-weight A_1 – A_∞ estimates for the commutator $[b, T_\Omega]$ of a BMO symbol with a homogeneous singular integral, relaxing the customary assumption $\Omega \in L^\infty(S^{n-1})$ to $\Omega \in L^q(S^{n-1})$ for some $q \in (2, \infty)$. On the other hand, Shen and Yang [6] prove uniform $L^p(\mathbb{R}^n)$ bounds, $1 < p < \infty$, for oscillatory singular integrals whose kernel combines a polynomial phase, a rough homogeneous factor, and the first-order Taylor remainder of a function A with $\nabla A \in \text{BMO}$; the bound is independent of the coefficients of the phase polynomial, and a new oscillatory estimate replaces the van der Corput lemma of earlier work.

Lastly, Torresblanca-Badillo, Bolaño-Benitez, and Gutiérrez-García [7] introduce two families of p -adic pseudo-differential operators of Bochner–Riesz type on $\mathbb{Q}_p^n$. The first generates a convolution semigroup with explicit kernels and converges, as the smoothness parameter tends to zero, to the Fourier projection onto a compact frequency ball; notably, the kernel is non-positive away from the origin despite the non-negative symbol, in contrast with the classical setting. The second family is self-adjoint and m -dissipative on $L^2(\mathbb{Q}_p^n)$, so Hille–Yosida–Phillips theory yields well-posedness of the associated Cauchy problem.

2. Complex analysis and special functions

Two works extend classical one-variable structures to multivariable or q -deformed settings while preserving concrete criteria and calculational tools. Li and Lin [8] prove that a family $\mathcal{F} \subset \mathcal{O}(\mathbb{D}^n)$ on the unit polydisc is normal if and only if every associated one-dimensional coordinate-slice family is normal on the unit disc, reducing normality in several variables to the classical one-variable theory; they then define normal functions on $\mathbb{D}^n$ through invariance under its automorphism group and characterize them by the growth estimate $f^\sharp(z) \leq C \max_{1 \leq j \leq n} (1 - |z_j|^2)^{-1}$. Zayed, Khan, Ryoo, and Duran [9] introduce bivariate extended q -Laguerre-based Appell polynomials via zeroth-

order q -Bessel–Tricomi functions and, within the framework of quasi-monomiality, derive operational identities, q -differential and q -integro-differential equations, summation formulas, and a determinant representation, with Bernoulli, Euler, and Genocchi families as special cases.

3. Fractional stochastic dynamics and control

Some contributions study memory-dependent dynamics, seeking conditions for well-posedness, controllability, and stabilization under stochastic forcing or impulsive intervention. Liaqat, Akgül, and Conejero [10] establish existence and uniqueness of mild solutions, and approximate controllability, for fractional stochastic equations driven by fractional Brownian motion with Hurst parameter $H \in (\frac{1}{2}, 1)$. The governing Caputo–Katugampola derivative gives its second parameter β a genuine modeling role: $\beta = 1$ recovers power-law (Caputo) memory, the limit $\beta \rightarrow 0^+$ the logarithmic (Caputo–Hadamard) regime, and intermediate values interpolate between the two. Dlala, Zaidi, and Alharbi [11] address the complementary problem of stabilization: After deriving exponential-stability criteria for impulsive fractional equations by Lyapunov methods, they design two event-triggered impulsive-control strategies that guarantee exponential convergence, exclude Zeno behavior, and are illustrated on a chaotic fractional financial system.

4. Inequalities, convexity, and fixed-point methods

Some authors develop inequalities, iterative schemes, and generalized convexity as tools for proving bounds, convergence, existence, and stability in nonlinear or fractional problems. Etemad, Akgül, and Conejero [12] prove coupled and extended Gronwall–Bellman inequalities in the Hadamard fractional setting, applied respectively to the Ulam–Hyers stability of a delayed Caputo–Hadamard coupled system and to existence for a damped initial value problem, both motivated by population dynamics. Filali, Akram, and Dilshad [13] construct a four-step iterative scheme with strong convergence and weak ω^2 -stability for contractive-like mappings, together with convergence results for generalized α -Reich–Suzuki nonexpansive mappings; numerical experiments indicate faster convergence than several established schemes, with applications to a Caputo-type fractional Volterra–Fredholm integro-differential equation and a fractional diffusion equation. Guo, Li, and Zeng [14] extend the affine fractional L^p Sobolev inequalities of Haddad and Ludwig from $s \in (0, 1)$ to the full range $s \in (0, n) \setminus \mathbb{N}$ with $1 < p < n/s$, introducing higher-order fractional L^p Sobolev polar projection bodies; the resulting affine inequalities are stronger than their Euclidean counterparts, and the proofs rest on symmetric decreasing rearrangement, a Pólya–Szegő-type inequality, and anisotropic fractional Sobolev norms. We conclude this topic with Ali, Samraiz, Trabelsi, and Zaway [15], who introduce harmonically trigonometric p -convex functions and the corresponding coordinated class, deriving Hermite–Hadamard and Fejér-type inequalities whose bounds involve hypergeometric functions, with further notions of convexity arising in limiting cases.

5. Nonlinear waves, solitons, and chaotic dynamics

Fractional or deformed structures, from soliton families and bifurcations to stability and chaos, can reshape nonlinear dynamics. Naseem and Hussain [16] analyze the generalized $(3 + 1)$ -dimensional

fractional q -deformed tanh–Gordon model, in which a quantum deformation parameter and a fractional derivative act simultaneously: The modified Sardar sub-equation method yields dark, bright, periodic, and singular solitons, and a Galilean transformation leads to a planar system whose bifurcations mark transitions among periodic, quasi-periodic, and chaotic regimes. Elkhalloufy, Menchih, Hilal, and Kajouni [17] study the space–time conformable Von Foerster–Lasota equation of order $\kappa \in (0, 1)$ in Orlicz spaces $L^\Psi(0, 1)$. Writing $p_\Psi \leq q_\Psi$ for the Matuszewska–Orlicz indices of Ψ , $\aleph$ for the growth coefficient, and $\Upsilon > 0$ for the coefficient of the conformable transport term, the associated κ -semigroup is Devaney chaotic when $\aleph > -\Upsilon/q_\Psi$ and strongly stable when $\aleph \leq -\Upsilon/p_\Psi$. The unresolved interval $\aleph \in (-\Upsilon/p_\Psi, -\Upsilon/q_\Psi]$ disappears on the Lebesgue scale, where the two indices coincide: Enlarging the ambient space reveals a dynamical question absent from the classical setting.

6. Generalized thermoelasticity

Two studies examine how nonclassical heat conduction and material heterogeneity modify coupled thermal–mechanical waves and transient attenuation. Ismail, Ahmed, El-Bary, Youssef, and Samir [18] derive exact wave solutions for the Lord–Shulman theory with temperature-dependent material properties, a coupling that renders the governing system nonlinear; the improved simple equation method yields solution families with free parameters describing temperature, stress, and displacement fields in regimes of rapid thermal loading, where the finite propagation speed of the model is essential. Alanazi and Abouelregal [19] propose a fractional Caputo-tempered two-phase-lag model with spatial and temporal nonlocality for thermoelastic vibrations in porous media containing spherical cavities and voids: Lower fractional order with higher stiffness accelerates attenuation of the response, higher order with lower stiffness prolongs it, so the fractional order functions as a potential design variable rather than a mere fitting parameter.

7. Models in the life and earth sciences

Lastly, we have two applications that use PDE models to connect spatial heterogeneity with temporal evolution, focusing on stability, pattern formation, and parameter sensitivity. Ali and Islam [20] analyze a two-dimensional Gray–Scott reaction–diffusion system with a Caputo derivative in time, establishing local-stability criteria by linearization and global asymptotic stability of the discrete model by Lyapunov functionals; simulations combining a Chebyshev spectral discretization with the $L1$ scheme reproduce Turing patterns and self-replicating spots, so the fractional formulation retains the pattern-forming mechanisms of the classical model. Wu and Hsieh [21] derive a two-dimensional analytical solution, via the generalized integral transform technique, for groundwater flow in an unconfined two-region aquifer with piecewise hydraulic properties and spatially variable recharge, benchmarked against finite-difference simulations of the nonlinear Boussinesq equation. The series converges with roughly forty eigenvalues in each coordinate direction, and recharge variability dominates the sensitivity analysis. It changes mound height by up to 90% and the sensitivity measure by 33%, against 19% for specific yield and 11% for hydraulic conductivity. So that, the spatial distribution of recharge can outweigh geological asymmetry in determining groundwater levels.

8. Recurring themes and open problems

Four themes recur across the collection. First, *generalization along two axes*: Several papers broaden the operator and the ambient framework simultaneously: A variable-order operator on a variable-exponent space [3], a conformable derivative on an Orlicz space [17], nonlinear potential estimates on variable Herz spaces [1], a tempered model with space–time nonlocality [19]. Such double generalizations matter most when they reveal behavior absent from the classical setting, as with the Orlicz transition regime [17] and the non-positive p -adic kernel [7]. Second, *the memory kernel as a modeling choice*: The parameter β interpolates between power-law and logarithmic memory [10], tempering controls its decay [19], the Hadamard setting is logarithmically structured [12], and the conformable derivative preserves a local differential structure [17]; the question is not whether to use a fractional operator, but which one reflects the mechanism under study. Third, *instability and chaos in contrasting roles*: Chaotic dynamics are characterized in [16, 17] and suppressed through control in [11], whereas instability drives pattern formation in [20]. Fourth, *closed-form solutions as benchmarks*: Exact solutions validate numerical schemes, quantify convergence, and expose parameter dependence directly [16, 18, 21]—roles that simulation alone cannot fill.

Finally, we indicate some problems that remain unresolved in these works: The transitional Orlicz regime and density-dependent rates [17]; precise L^p – L^q mapping properties of p -adic Bochner–Riesz operators and non-Archimedean analogues of classical summability thresholds [7]; robustness of event-triggered impulsive control under uncertain delays and data loss [11]; endpoint cases and alternative ambient spaces for variable-order operators [3]; extensions of truncated Havin–Maz’ya–Wolff potentials to variable-exponent Morrey and Besov spaces [1]; and Gronwall–Bellman inequalities for non-singular kernels and (p, q) -calculus on time scales [12]. These problems provide natural starting points for further development of the methods represented in this special issue. We encourage authors and readers to pursue advances in their solutions.

Guest Editor

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Conflict of interest

JAC is a co-author of [10] and [12]. He was not involved in the editorial review of, or the decision to publish, either article. JAC declares no other conflict of interest.

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