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Research article

Logarithmic decay of solutions to coupled cubic nonlinear Schrödinger systems with defocusing dissipative nonlinearities

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Abstract: We investigate the time-decay estimates for solutions to the Cauchy problem for a system of two coupled cubic nonlinear Schrödinger equations in one space dimension. Without any size restriction on the initial data, we prove the global existence and uniqueness of solutions in the defocusing dissipative case. The main result shows that the L^2-norm of the solution decays at the rate $O((\log(2+t))^{-1/3})$ as $t \to +\infty$.

Keywords: coupled cubic nonlinear Schrödinger equations; L^2-decay rate; without any size restriction on initial data; defocusing dissipative nonlinearity; Cauchy problem
Mathematics Subject Classification: 35B40, 35Q55

1. Introduction

In this paper, we consider the following Cauchy problem for a system of two coupled cubic nonlinear Schrödinger equations:

$$\begin{cases} i\partial_t u_1 + \dfrac{1}{2}\partial_x^2 u_1 = \lambda|u_1|^2 u_1 + bi|u_2|^2 u_1, \\ i\partial_t u_2 + \dfrac{1}{2}\partial_x^2 u_2 = \lambda|u_2|^2 u_2 + bi|u_1|^2 u_2, \end{cases} \qquad (t,x) \in (0,\infty)\times\mathbb{R}, \tag{1.1}$$

with the initial data

$$u_1(0,x) = u_{1,0}, \qquad u_2(0,x) = u_{2,0}, \qquad x \in \mathbb{R}, \tag{1.2}$$

where $i = \sqrt{-1}$, $u_j = u_j(t,x)$ is an unknown complex-valued function for $j = 1,2$, $\lambda = \lambda_1 + i\lambda_2$ is a complex constant satisfying $\lambda_1 \geq 0, \lambda_2 < 0$, and b is a real constant satisfying $\lambda_2 \leq b < 0$. The conditions $\lambda_1 \geq 0, \lambda_2 < 0$ and $\lambda_2 \leq b < 0$ correspond to the defocusing dissipative case (see e.g., [1]).

Our aim is to prove the following time-decay estimate for solutions to (1.1)–(1.2):

$$\|u_1\|_{L^2} + \|u_2\|_{L^2} \leq C_0(\log(2+t))^{-1/3}, \qquad t \geq 0$$

without any size restriction on initial data.

From the viewpoint of nonlinear optics, the system (1.1) can be regarded as a simplified effective model for the interaction of two orthogonal polarization components of a pulse in a birefringent fiber. The cubic nonlinear terms describe Kerr-type nonlinear effects and nonlinear interactions between the two polarization components, while the coupling structure the polarization interactions induced by the medium. The complex coefficients $\lambda = \lambda_1 + i\lambda_2$ and bi characterize the nonlinear response, where $\lambda_1 \geq 0$ corresponds to the defocusing effect, and $\lambda_2 < 0$ and $b < 0$ represent nonlinear dissipation. In randomly birefringent media, coupled nonlinear Schrödinger systems describe the slow evolution of polarized pulses and, more generally, provide a basic framework for the study of vector solitons and polarization-locked states under suitable physical conditions [2, 3]. This physical background has also motivated several studies on coherent structures in related vector Schrödinger-type models. For instance, soliton and rogue-wave dynamics have been investigated in nonautonomous and partially nonlocal settings [4], while traveling-wave solutions have been studied from the viewpoint of bifurcation analysis [5]. These related studies further illustrate the physical relevance of coupled Schrödinger-type systems and motivate analyses of the global existence and decay properties of solutions to the system (1.1).

The long-time behavior of dissipative nonlinear Schrödinger equations under small initial data has been extensively studied. In 2021, Li et al. [6] studied the two-component model in one space dimension

$$\begin{cases} i\partial_t u_1 + \frac{1}{2}\partial_x^2 u_1 = -i|u_2|^2 u_1, \\ i\partial_t u_2 + \frac{1}{2}\partial_x^2 u_2 = -i|u_1|^2 u_2, \end{cases} \qquad (t,x) \in (0,\infty) \times \mathbb{R}.$$

Under the assumption of small initial data in the Sobolev space, they showed that each component of the solution to the system is asymptotically free as $t \to +\infty$. Additionally, they verified that the scattering profiles satisfy a crucially restricted condition. In a subsequent work [7], for the same system and within the small-amplitude framework, they established criteria for large-time L^2 decay/non-decay in terms of the Fourier transforms of the initial data, yielding a refined description of the large-time dynamics. In 2025, Sagawa [8] investigated a general two-component cubic Schrödinger system in one dimension

$$\begin{cases} i\partial_t u_1 + \dfrac{1}{2}\partial_x^2 u_1 = ai|u_1|^2 u_1 + bi|u_2|^2 u_1, \\ i\partial_t u_2 + \dfrac{1}{2}\partial_x^2 u_2 = ci|u_1|^2 u_2 + di|u_2|^2 u_2, \end{cases} \qquad (t,x) \in (0,\infty) \times \mathbb{R}.$$

Under small initial data assumptions, the solution was shown to behave asymptotically like a free solution as $t \to +\infty$. Moreover, a non-decay result for the solution was obtained, which was non-trivial in terms of the long range scattering. Besides the long-time behavior of solutions, other theories for coupled Schrödinger equations have also been developed, including studies on low-regularity local well-posedness, the global existence versus finite-time blow-up dichotomy, the normalized ground states and mountain-pass solutions for the associated standing-wave problems, which can be seen in [9–11].

Early work of Kita et al. [12] obtained the L^∞-decay estimates of solutions to the nonlinear Schrödinger equation

$$i\partial_t u + \frac{1}{2}\Delta u = \lambda|u|^{p-1}u, \qquad (t,x) \in (0,\infty) \times \mathbb{R}^n \tag{1.3}$$

with strong dissipative nonlinearity ($\lambda = \lambda_1 + i\lambda_2$ is a complex constant satisfying $\lambda_2 < 0$ and $(p-1)|\lambda_1| \leq 2\sqrt{p}|\lambda_2|$) for arbitrarily large initial data when $n = 1$ and $1 < p \leq 3$. Later, in the case $n \geq 1$ and $1 < p \leq 1 + 2/n$, Hayashi et al. established L^2-decay estimates of the strong dissipative nonlinear Schrödinger equation (1.3) with arbitrarily large initial data in [13] . We now turn to the defocusing dissipative case $\lambda_1 \geq 0$ and $\lambda_2 < 0$, for which a natural question is whether the L^2-decay estimates of (1.3) for arbitrarily large initial data still hold without the strong dissipative condition. In [1], Gerelmaa et al. investigated the L^2-decay properties of solutions to the dissipative nonlinear Schrödinger equation (1.3) with $n = 1$ for large initial data, in the range $1 < p \leq 3$, where $\lambda = \lambda_1 + i\lambda_2$ satisfies $\lambda_1 \geq 0$ and $\lambda_2 < 0$. By combining energy methods with Fourier analysis, the authors established a decay rate for $\|u(t)\|_{L^2}$. In a subsequent work [14], they obtained L^2-decay estimates for solutions to the n-dimensional nonlinear Schrödinger equation (1.3) without any size restriction on the initial data in the case $\lambda_1 \geq 0, \lambda_2 < 0$ and $1 < p \leq 1 + 2/n$. More work on the related nonlinear Schrödinger equations can be found in [15–20].

We note that [6–8] mainly deal with the long-time asymptotic behavior of small solutions to coupled cubic Schrödinger equations in one space dimension. In contrast, the present paper is concerned with time-decay estimates of solutions to the Cauchy problem in (1.1)–(1.2) under the defocusing dissipative condition and does not impose any smallness assumption on the initial data. A more detailed study of the long-time asymptotic behavior in this setting is left for future work. Compared with the defocusing dissipative equations studied in [1], the main difficulty here is caused by the cross-interaction terms between the two components in the Cauchy problem in (1.1)–(1.2).

To overcome this difficulty, we mainly use the condition $\lambda_2 \leq b < 0$ to control both the self-dissipative terms and the coupled dissipative terms in the Cauchy problem in (1.1)–(1.2), and use the condition $\lambda_1 \geq 0$ to ensure the nonnegativity of the energy functionals. More precisely, the condition $\lambda_2 < 0$ controls the dissipative terms of the two components themselves, and $b < 0$ makes the cross-coupling term also provide a nonpositive contribution. The relation $\lambda_2 \leq b$ is mainly used to control some terms that appear in the derivative and weighted estimates for $\|\partial_x u_j(t)\|_{L^2}$ and $\|Ju_j(t)\|_{L^2}$.

The proof is based on energy estimates, derivative estimates, the factorization formula of the Schrödinger group, the standard generator of the Galilei transform, and interpolation inequalities. In this way, we extend the logarithmic decay argument to the coupled cubic system (1.1) by exploiting the defocusing dissipative coupling structure.

By using the techniques and insights developed in [1], we obtain the following result for the Cauchy problem in (1.1)–(1.2).

Theorem 1.1. *(Defocusing dissipative case) Assume that $\lambda_1 \geq 0$, $\lambda_2 < 0$ and $\lambda_2 \leq b < 0$. If $u_{j,0} \in H^1$ and $x\,u_{j,0} \in L^2$ for $j = 1, 2$, then the Cauchy problem in* (1.1) – (1.2) *admits a unique global solution $u = (u_1, u_2)$ satisfying*

$$u_j \in C([0,\infty); H^1) \cap C^1([0,\infty); H^{-1}), \qquad j = 1, 2,$$

$$x\,u_j \in C([0,\infty); L^2), \qquad j = 1, 2.$$

Furthermore, we have

$$\|u_1(t)\|_{L^2} + \|u_2(t)\|_{L^2} \leq C_0(\log(2+t))^{-1/3}, \qquad t \geq 0,$$

where $C_0 > 0$ is a constant independent of t.

In our result, Theorem 1.1 establishes global existence and uniqueness for the Cauchy problem in (1.1)–(1.2) in the defocusing dissipative case, without any size restriction on the initial data.

We now introduce some notation. The function space L^q ($1 \leq q < \infty$) for $\mathbb{C}$-valued functions is defined by $L^q = \{\, u(x) \mid \|u\|_{L^q} < \infty \,\}$, where

$$\|u\|_{L^q} = \left(\int_{\mathbb{R}} |u(x)|^q \, dx \right)^{1/q}, \qquad 1 \leq q < \infty.$$

The Sobolev space H^1 is equipped with the norm

$$\|u\|_{H^1} = \left(\|u\|_{L^2}^2 + \|\partial_x u\|_{L^2}^2 \right)^{1/2}.$$

The negative-index Sobolev space H^{-1} is defined as the dual space of H^1. For $t \neq 0$, we define

$$(M\varphi)(x) = e^{ix^2/(2t)}\varphi(x), \qquad (D\varphi)(x) = (it)^{-1/2}\varphi(x/t),$$

and the Fourier transform on $\mathbb{R}$ is given by

$$(\mathcal{F}\varphi)(\xi) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-ix\xi}\varphi(x)\, dx.$$

With this notation, the Schrödinger group $U(t) = \exp(it\partial_x^2/2)$ admits the factorization formula (see e.g., [21,22])

$$U(t) = MD\mathcal{F}M.$$

Since $U(-t)$ is the inverse of $U(t)$, it follows that $U(-t) = M^{-1}\mathcal{F}^{-1}D^{-1}M^{-1}$. The standard generator of the Galilei transform is given by $J = U(t)\, x\, U(-t) = x + it\partial_x$. It commutes with the Schrödinger operator $i\partial_t + \frac{1}{2}\partial_x^2$.

The remainder of the paper is organized as follows. In Section 2, we give some estimates for the norms $\|u_j\|_{L^2}$, $\|\partial_x u_j\|_{L^2}$, and $\|Ju_j\|_{L^2}$. In Section 3, we prove Theorem 1.1.

2. Preliminary estimates

The subsequent lemmas (Lemmas 2.1–2.3) give a priori estimates for $\|u_j\|_{L^2}$, $\|\partial_x u_j\|_{L^2}$, and $\|Ju_j\|_{L^2}$, for $j = 1, 2$. C denotes a positive constant, which may vary from line to line.

Lemma 2.1. *Assume that $\lambda_2 \leq 0, b \leq 0$, and that $u_{j,0} \in H^1, xu_{j,0} \in L^2$ for $j = 1, 2$. Let $T > 0$, and let (u_1, u_2) be a local solution of the Cauchy problem in* (1.1)–(1.2) *satisfying*

$$u_j \in C([0,T]; H^1) \cap C^1([0,T]; H^{-1}), \qquad j = 1, 2,$$

$$x\, u_j \in C([0,T]; L^2), \qquad j = 1, 2.$$

For any $t \in [0, T]$, we have

$$\|u_1(t)\|_{L^2}^2 + \|u_2(t)\|_{L^2}^2 \leq \|u_{1,0}\|_{L^2}^2 + \|u_{2,0}\|_{L^2}^2. \tag{2.1}$$

Proof. Multiplying both sides of the first equation of (1.1) by $2\overline{u_1}$, integrating over $\mathbb{R}$, and taking imaginary parts, we obtain

$$\operatorname{Im}\int_{\mathbb{R}}\Big(2i\,\partial_t u_1\,\overline{u_1}+\partial_x^2 u_1\,\overline{u_1}\Big)\,dx=\operatorname{Im}\int_{\mathbb{R}}\Big(2\lambda|u_1|^2u_1\,\overline{u_1}+2ib|u_2|^2u_1\,\overline{u_1}\Big)\,dx.$$

We then have

$$\frac{d}{dt}\|u_1(t)\|_{L^2}^2=2\lambda_2\int_{\mathbb{R}}|u_1|^4\,dx+2b\int_{\mathbb{R}}|u_1|^2|u_2|^2\,dx. \tag{2.2}$$

Similarly, we derive

$$\frac{d}{dt}\|u_2(t)\|_{L^2}^2=2\lambda_2\int_{\mathbb{R}}|u_2|^4\,dx+2b\int_{\mathbb{R}}|u_1|^2|u_2|^2\,dx. \tag{2.3}$$

By (2.2) and (2.3), we have

$$\frac{d}{dt}\Big(\|u_1(t)\|_{L^2}^2+\|u_2(t)\|_{L^2}^2\Big)=2\lambda_2\Big(\|u_1\|_{L^4}^4+\|u_2\|_{L^4}^4\Big)+4b\int_{\mathbb{R}}|u_1|^2|u_2|^2\,dx. \tag{2.4}$$

Under the assumptions $\lambda_2\le 0, b\le 0$ and $u_{1,0},u_{2,0}\in H^1$, we conclude that (2.1) holds. □

It also follows from (2.4) that the system admits no nontrivial localized traveling-wave solutions in the present energy class. Indeed, such a wave has a fixed profile up to translation and phase modulation, so its total L^2-norm is constant in time. By (2.4), under the assumptions $\lambda_2<0$ and $b<0$, this can happen only when the dissipative terms vanish, which implies $u_1\equiv 0$ and $u_2\equiv 0$.

Lemma 2.2. *Assume that $\lambda_1\ge 0$, $\lambda_2<0$, and $\lambda_2\le b<0$, and that $u_{j,0}\in H^1, xu_{j,0}\in L^2$ for $j=1,2$. Let $T>0$, and let (u_1,u_2) be a local solution of the Cauchy problem in (1.1)–(1.2) satisfying*

$$u_j\in C([0,T];H^1)\cap C^1([0,T];H^{-1}),\qquad j=1,2,$$

$$x\,u_j\in C([0,T];L^2),\qquad j=1,2.$$

For any $t\in[0,T]$, we then have

$$\|\partial_x u_1(t)\|_{L^2}^2+\|\partial_x u_2(t)\|_{L^2}^2\le C_0. \tag{2.5}$$

Proof. Multiplying both sides of the first equation of (1.1) by $2\overline{\partial_t u_1}$, integrating over $\mathbb{R}$, and taking real parts, we get

$$\operatorname{Re}\int_{\mathbb{R}}\partial_x^2 u_1\,\overline{\partial_t u_1}\,dx=2\operatorname{Re}\int_{\mathbb{R}}\Big(\lambda|u_1|^2u_1+bi|u_2|^2u_1\Big)\overline{\partial_t u_1}\,dx.$$

Hence, we obtain

$$\frac{1}{2}\frac{d}{dt}\|\partial_x u_1\|_{L^2}^2+\frac{\lambda_1}{2}\frac{d}{dt}\|u_1\|_{L^4}^4=-2\lambda_2\operatorname{Im}\int_{\mathbb{R}}|u_1|^2\,\overline{u_1}\,\partial_t u_1\,dx-2b\operatorname{Im}\int_{\mathbb{R}}|u_2|^2\,\overline{u_1}\,\partial_t u_1\,dx. \tag{2.6}$$

Applying the same argument to the second equation of (1.1), we derive

$$\frac{1}{2}\frac{d}{dt}\|\partial_x u_2\|_{L^2}^2+\frac{\lambda_1}{2}\frac{d}{dt}\|u_2\|_{L^4}^4=-2\lambda_2\operatorname{Im}\int_{\mathbb{R}}|u_2|^2\,\overline{u_2}\,\partial_t u_2\,dx-2b\operatorname{Im}\int_{\mathbb{R}}|u_1|^2\,\overline{u_2}\,\partial_t u_2\,dx. \tag{2.7}$$

Multiplying both sides of the first equation of (1.1) by $2\lambda_2|u_1|^2\overline{u_1}$, integrating over $\mathbb{R}$, and taking real parts, we obtain

$$-2\lambda_2 \operatorname{Im}\int_{\mathbb{R}} \partial_t u_1\, |u_1|^2\, \overline{u_1}\, dx + \lambda_2 \operatorname{Re}\int_{\mathbb{R}} \partial_x^2 u_1\, |u_1|^2\, \overline{u_1}\, dx = 2\lambda_1\lambda_2\|u_1\|_{L^6}^6. \tag{2.8}$$

Multiplying both sides of the first equation of (1.1) by $2b|u_2|^2\overline{u_1}$, integrating over $\mathbb{R}$, and taking real parts, we get

$$-2b \operatorname{Im}\int_{\mathbb{R}} \partial_t u_1\, |u_2|^2\, \overline{u_1}\, dx + b \operatorname{Re}\int_{\mathbb{R}} \partial_x^2 u_1\, |u_2|^2\, \overline{u_1}\, dx = 2b\lambda_1 \int_{\mathbb{R}} |u_2|^2|u_1|^4\, dx. \tag{2.9}$$

Similarly,

$$-2\lambda_2 \operatorname{Im}\int_{\mathbb{R}} \partial_t u_2\, |u_2|^2\, \overline{u_2}\, dx + \lambda_2 \operatorname{Re}\int_{\mathbb{R}} \partial_x^2 u_2\, |u_2|^2\, \overline{u_2}\, dx = 2\lambda_1\lambda_2\|u_2\|_{L^6}^6, \tag{2.10}$$

$$-2b \operatorname{Im}\int_{\mathbb{R}} \partial_t u_2\, |u_1|^2\, \overline{u_2}\, dx + b \operatorname{Re}\int_{\mathbb{R}} \partial_x^2 u_2\, |u_1|^2\, \overline{u_2}\, dx = 2b\lambda_1 \int_{\mathbb{R}} |u_1|^2|u_2|^4\, dx. \tag{2.11}$$

By (2.6)–(2.11) we derive

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\Big(\|\partial_x u_1\|_{L^2}^2 + \|\partial_x u_2\|_{L^2}^2\Big) + \frac{\lambda_1}{2}\frac{d}{dt}\Big(\|u_1\|_{L^4}^4 + \|u_2\|_{L^4}^4\Big)\\
&= -2\lambda_2 \operatorname{Im}\Big(\int_{\mathbb{R}} |u_2|^2\, \overline{u_2}\, \partial_t u_2\, dx + \int_{\mathbb{R}} |u_1|^2\, \overline{u_1}\, \partial_t u_1\, dx\Big) - 2b \operatorname{Im}\Big(\int_{\mathbb{R}} |u_1|^2\, \overline{u_2}\, \partial_t u_2\, dx + \int_{\mathbb{R}} |u_2|^2\, \overline{u_1}\, \partial_t u_1\, dx\Big)\\
&= \lambda_2 \operatorname{Re}\Big[\int_{\mathbb{R}} \partial_x u_1\, \partial_x(|u_1|^2\, \overline{u_1})\, dx + \int_{\mathbb{R}} \partial_x u_2\, \partial_x(|u_2|^2\, \overline{u_2})\, dx\Big] + 2\lambda_1\lambda_2\Big(\|u_1\|_{L^6}^6 + \|u_2\|_{L^6}^6\Big)\\
&\quad + b \operatorname{Re}\Big[\int_{\mathbb{R}} \partial_x u_1\, \partial_x(|u_2|^2\, \overline{u_1})\, dx + \int_{\mathbb{R}} \partial_x u_2\, \partial_x(|u_1|^2\, \overline{u_2})\, dx\Big] + 2\lambda_1 b\Big(\int_{\mathbb{R}} |u_2|^4\, |u_1|^2\, dx + \int_{\mathbb{R}} |u_1|^4\, |u_2|^2\, dx\Big)\\
&= \lambda_2 \operatorname{Re}\int_{\mathbb{R}}\Big((\partial_x u_1)^2\, \overline{u_1}^{\,2} + 2|\partial_x u_1|^2\, |u_1|^2 + (\partial_x u_2)^2\, \overline{u_2}^{\,2} + 2|\partial_x u_2|^2\, |u_2|^2\Big)\, dx + 2\lambda_1\lambda_2\Big(\|u_1\|_{L^6}^6 + \|u_2\|_{L^6}^6\Big)\\
&\quad + b\Big(\int_{\mathbb{R}} |u_1|^2\, |\partial_x u_2|^2\, dx + \int_{\mathbb{R}} |u_2|^2\, |\partial_x u_1|^2\, dx + 4\int_{\mathbb{R}} |u_1|\, |u_2|\, \partial_x|u_1|\, \partial_x|u_2|\, dx\Big)\\
&\quad + 2\lambda_1 b\Big(\int_{\mathbb{R}} |u_2|^4\, |u_1|^2\, dx + \int_{\mathbb{R}} |u_1|^4\, |u_2|^2\, dx\Big).
\end{aligned} \tag{2.12}$$

Since

$$\operatorname{Re}\int_{\mathbb{R}}\Big((\partial_x u_1)^2\, \overline{u_1}^{\,2} + |\partial_x u_1|^2|u_1|^2\Big)\, dx \geq 0, \qquad \operatorname{Re}\int_{\mathbb{R}}\Big((\partial_x u_2)^2\, \overline{u_2}^{\,2} + |\partial_x u_2|^2|u_2|^2\Big)\, dx \geq 0,$$

it follows that

$$\begin{aligned}
&\operatorname{Re}\int_{\mathbb{R}}\Big((\partial_x u_1)^2\, \overline{u_1}^{\,2} + 2|\partial_x u_1|^2\, |u_1|^2 + (\partial_x u_2)^2\, \overline{u_2}^{\,2} + 2|\partial_x u_2|^2\, |u_2|^2\Big)\, dx\\
&\geq \int_{\mathbb{R}} |\partial_x u_1|^2|u_1|^2\, dx + \int_{\mathbb{R}} |\partial_x u_2|^2|u_2|^2\, dx.
\end{aligned} \tag{2.13}$$

Substituting (2.13) into (2.12), we obtain

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\Big(\|\partial_x u_1(t)\|_{L^2}^2 + \|\partial_x u_2(t)\|_{L^2}^2\Big) + \frac{\lambda_1}{2}\frac{d}{dt}\Big(\|u_1(t)\|_{L^4}^4 + \|u_2(t)\|_{L^4}^4\Big)\\
&\leq \lambda_2 \int_{\mathbb{R}}\Big((|\partial_x u_1|^2\,|u_1|^2 + |\partial_x u_2|^2\,|u_2|^2\Big)\,dx + 2\lambda_1\lambda_2\Big(\|u_1\|_{L^6}^6 + \|u_2\|_{L^6}^6\Big)\\
&\quad + b\Big(\int_{\mathbb{R}} |u_1|^2\,|\partial_x u_2|^2\,dx + \int_{\mathbb{R}} |u_2|^2\,|\partial_x u_1|^2\,dx + 4\int_{\mathbb{R}} |u_1|\,|u_2|\,\partial_x|u_1|\,\partial_x|u_2|\,dx\Big)\\
&\quad + 2\lambda_1 b\Big(\int_{\mathbb{R}} |u_2|^4\,|u_1|^2\,dx + \int_{\mathbb{R}} |u_1|^4\,|u_2|^2\,dx\Big)\\
&\leq \lambda_2\Big(\int_{\mathbb{R}} |\partial_x u_1|^2|u_1|^2\,dx + \int_{\mathbb{R}} |\partial_x u_2|^2|u_2|^2\,dx\Big) - 2b\int_{\mathbb{R}} |u_1||u_2|\,|\partial_x u_1|\,|\partial_x u_2|\,dx\\
&\quad + 2\lambda_1\Big[\lambda_2(\|u_1\|_{L^6}^6 + \|u_2\|_{L^6}^6) + b\Big(\int_{\mathbb{R}} |u_1|^4|u_2|^2\,dx + \int_{\mathbb{R}} |u_2|^4|u_1|^2\,dx\Big)\Big]\\
&\leq (\lambda_2 - b)\Big(\int_{\mathbb{R}} |\partial_x u_1|^2|u_1|^2\,dx + \int_{\mathbb{R}} |\partial_x u_2|^2|u_2|^2\,dx\Big)\\
&\quad + 2\lambda_1\Big[\lambda_2(\|u_1\|_{L^6}^6 + \|u_2\|_{L^6}^6) + b\Big(\int_{\mathbb{R}} |u_1|^4|u_2|^2\,dx + \int_{\mathbb{R}} |u_2|^4|u_1|^2\,dx\Big)\Big]\\
&\leq 0.
\end{aligned}$$

Hence, for any $t \in [0, T]$ we get

$$\|\partial_x u_1(t)\|_{L^2}^2 + \|\partial_x u_2(t)\|_{L^2}^2 \leq C_0.$$

□

Lemma 2.3. *Assume that $\lambda_1 \geq 0$, $\lambda_2 < 0$, and $\lambda_2 \leq b < 0$, and that $u_{j,0} \in H^1, xu_{j,0} \in L^2$ for $j = 1, 2$. Let $T > 0$, and let (u_1, u_2) be a local solution of the Cauchy problem in* (1.1)–(1.2) *satisfying*

$$u_j \in C([0, T]; H^1) \cap C^1([0, T]; H^{-1}), \qquad j = 1, 2,$$

$$x\,u_j \in C([0, T]; L^2), \qquad j = 1, 2.$$

For any $t \in [0, T]$ we then have

$$\|Ju_1(t)\|_{L^2}^2 + \|Ju_2(t)\|_{L^2}^2 \leq C_0(1 + t). \tag{2.14}$$

Proof. Let $v_j(t) = U(-t)u_j(t)$ for $j = 1, 2$. We deduce that

$$i\partial_t u_j + \frac{1}{2}\,\partial_x^2 u_j = iU(t)\partial_t v_j.$$

Substituting this identity into the first equation of (1.1), we see that

$$i\partial_t v_1 = \lambda\, U(-t)|U(t)v_1|^2\, U(t)v_1 + ib\, U(-t)|U(t)v_2|^2\, U(t)v_1.$$

Using $U(t) = MD\mathcal{F}M$ and $U(-t) = M^{-1}\mathcal{F}^{-1}D^{-1}M^{-1}$, we find

$$
\begin{aligned}
i\partial_t v_1 &= \lambda\, U(-t)|U(t)v_1|^2\, U(t)v_1 + ib\, U(-t)|U(t)v_2|^2\, U(t)v_1 \\
&= \lambda\, M^{-1}\mathcal{F}^{-1}D^{-1}\Big(|D\mathcal{F}Mv_1|^2\, D\mathcal{F}Mv_1\Big) + ib\, M^{-1}\mathcal{F}^{-1}D^{-1}\Big(|D\mathcal{F}Mv_2|^2\, D\mathcal{F}Mv_1\Big) \\
&= \lambda\, t^{-1}M^{-1}\mathcal{F}^{-1}\Big(|\mathcal{F}Mv_1|^2\,\mathcal{F}Mv_1\Big) + ib\, t^{-1}M^{-1}\mathcal{F}^{-1}\Big(|\mathcal{F}Mv_2|^2\,\mathcal{F}Mv_1\Big).
\end{aligned}
$$

Hence

$$i\mathcal{F}M\partial_t v_1 = \lambda t^{-1}(|\mathcal{F}Mv_1|^2\,\mathcal{F}Mv_1) + ib\, t^{-1}(|\mathcal{F}Mv_2|^2\,\mathcal{F}Mv_1).$$

Similarly, we have

$$i\mathcal{F}M\partial_t v_2 = \lambda t^{-1}(|\mathcal{F}Mv_2|^2\,\mathcal{F}Mv_2) + ib\, t^{-1}(|\mathcal{F}Mv_1|^2\,\mathcal{F}Mv_2).$$

Set $w_j = \mathcal{F}Mv_j$, $j = 1, 2$. Since

$$\mathcal{F}M\partial_t v_j = \partial_t\mathcal{F}Mv_j + \mathcal{F}\left(\frac{ix^2}{2t^2}Mv_j\right) = \partial_t\mathcal{F}Mv_j - i\left(\frac{1}{2t^2}\right)\partial_\xi^2\mathcal{F}Mv_j,$$

the function $w_j(t,\xi) = \mathcal{F}Mv_j(t,\xi)$ satisfies the following system

$$
\begin{cases}
i\,\partial_t w_1 + \dfrac{1}{2t^2}\partial_\xi^2 w_1 = \lambda t^{-1}(|w_1|^2 w_1) + b\,i\,t^{-1}(|w_2|^2 w_1), \\
i\,\partial_t w_2 + \dfrac{1}{2t^2}\partial_\xi^2 w_2 = \lambda t^{-1}(|w_2|^2 w_2) + b\,i\,t^{-1}(|w_1|^2 w_2).
\end{cases}
\tag{2.15}
$$

We note that

$$\|\partial_\xi w_j\|_{L^2} = \Big\|\mathcal{F}(-ix\,MU(-t)u_j)\Big\|_{L^2} = \|U(t)xU(-t)u_j\|_{L^2} = \|Ju_j\|_{L^2}.$$

We now estimate $\|\partial_\xi w_1\|_{L^2} + \|\partial_\xi w_2\|_{L^2}$. Multiplying both sides of the first equation of (2.15) by $2\overline{\partial_t w_1}$, integrating over $\mathbb{R}$, and taking real parts, we get

$$
\begin{aligned}
&\frac{1}{t^2}\mathrm{Re}\int_{\mathbb{R}} \partial_\xi^2 w_1\,\overline{\partial_t w_1}\,d\xi \\
&= \frac{2\lambda_1}{t}\mathrm{Re}\int_{\mathbb{R}} |w_1|^2 w_1\,\overline{\partial_t w_1}\,d\xi - \frac{2b}{t}\mathrm{Im}\int_{\mathbb{R}} |w_2|^2 w_1\,\overline{\partial_t w_1}\,d\xi - \frac{2\lambda_2}{t}\mathrm{Im}\int_{\mathbb{R}} |w_1|^2 w_1\,\overline{\partial_t w_1}\,d\xi.
\end{aligned}
$$

From

$$\mathrm{Re}\int_{\mathbb{R}} \partial_\xi^2 w_j\,\overline{\partial_t w_j}\,d\xi = -\mathrm{Re}\int_{\mathbb{R}} \partial_\xi w_j\,\overline{\partial_t\partial_\xi w_j}\,d\xi = -\frac{1}{2}\frac{d}{dt}\|\partial_\xi w_j\|_{L^2}^2$$

and

$$\mathrm{Re}\int_{\mathbb{R}} |w_j|^2 w_j\,\overline{\partial_t w_j}\,d\xi = \frac{1}{4}\frac{d}{dt}\|w_j\|_{L^4}^4,$$

we obtain

$$\frac{d}{dt}\|\partial_\xi w_1\|_{L^2}^2 + \lambda_1 t\,\frac{d}{dt}\|w_1\|_{L^4}^4 = -4\lambda_2 t\,\mathrm{Im}\int_{\mathbb{R}} |w_1|^2\overline{w_1}\,\partial_t w_1\,d\xi - 4bt\,\mathrm{Im}\int_{\mathbb{R}} |w_2|^2\overline{w_1}\,\partial_t w_1\,d\xi. \tag{2.16}$$

Multiplying both sides of the first equation of (2.15) by $(\lambda_2|w_1|^2 + b|w_2|^2)\overline{w_1}$, integrating over $\mathbb{R}$, and taking real parts, we derive

$$\begin{aligned}
&-\operatorname{Im}\int_{\mathbb{R}} \partial_t w_1(\lambda_2|w_1|^2 + b|w_2|^2)\overline{w_1}\, d\xi \\
&= \frac{1}{2t^2}\operatorname{Re}\int_{\mathbb{R}} \partial_\xi w_1 \partial_\xi\Big[(\lambda_2|w_1|^2 + b|w_2|^2)\overline{w_1}\Big]\, d\xi + \frac{1}{t}\Big(\lambda_1\lambda_2\int_{\mathbb{R}} |w_1|^6\, d\xi + \lambda_1 b\int_{\mathbb{R}} |w_1|^4|w_2|^2\, d\xi\Big).
\end{aligned} \tag{2.17}$$

By (2.16) and (2.17), we have

$$\begin{aligned}
&\frac{d}{dt}\|\partial_\xi w_1\|_{L^2}^2 + \lambda_1 t\,\frac{d}{dt}\|w_1\|_{L^4}^4 \\
&= \frac{2}{t}\operatorname{Re}\int_{\mathbb{R}} \partial_\xi w_1\, \partial_\xi[(\lambda_2|w_1|^2 + b|w_2|^2)\overline{w_1}]\, d\xi + 4(\lambda_1\lambda_2\int_{\mathbb{R}} |w_1|^6\, d\xi + \lambda_1 b\int_{\mathbb{R}} |w_1|^4|w_2|^2\, d\xi).
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
&\frac{d}{dt}\|\partial_\xi w_2\|_{L^2}^2 + \lambda_1 t\,\frac{d}{dt}\|w_2\|_{L^4}^4 \\
&= \frac{2}{t}\operatorname{Re}\int_{\mathbb{R}} \partial_\xi w_2\, \partial_\xi[(\lambda_2|w_2|^2 + b|w_1|^2)\overline{w_2}]\, d\xi + 4(\lambda_1\lambda_2\int_{\mathbb{R}} |w_2|^6\, d\xi + \lambda_1 b\int_{\mathbb{R}} |w_2|^4|w_1|^2\, d\xi).
\end{aligned}$$

As in the case of (2.12), we have

$$\operatorname{Re}\int_{\mathbb{R}} \partial_\xi w_1 \partial_\xi(|w_1|^2\overline{w_1})\, d\xi + \operatorname{Re}\int_{\mathbb{R}} \partial_\xi w_2 \partial_\xi(|w_2|^2\overline{w_2})\, d\xi \geq \int_{\mathbb{R}} |w_1|^2|\partial_\xi w_1|^2\, d\xi + \int_{\mathbb{R}} |w_2|^2|\partial_\xi w_2|^2\, d\xi.$$

Therefore, we conclude that

$$\begin{aligned}
&\frac{d}{dt}\left(\|\partial_\xi w_1\|_{L^2}^2 + \|\partial_\xi w_2\|_{L^2}^2\right) + \lambda_1 t\frac{d}{dt}\left(\|w_1\|_{L^4}^4 + \|w_2\|_{L^4}^4\right) \\
&= \frac{2}{t}\operatorname{Re}\int_{\mathbb{R}} \partial_\xi w_2\, \partial_\xi[(\lambda_2|w_2|^2 + b|w_1|^2)\overline{w_2}] + \partial_\xi w_1\, \partial_\xi[(\lambda_2|w_1|^2 + b|w_2|^2)\overline{w_1}]d\xi \\
&\quad + 4\left(\lambda_1\lambda_2\int_{\mathbb{R}}(|w_1|^6 + |w_2|^6)d\xi + \lambda_1 b\int_{\mathbb{R}}(|w_2|^4|w_1|^2 + |w_1|^4|w_2|^2)d\xi\right) \\
&\leq \frac{2}{t}\left[\lambda_2\int_{\mathbb{R}}(|\partial_\xi w_1|^2|w_1|^2 + |\partial_\xi w_2|^2|w_2|^2)d\xi + b\int_{\mathbb{R}}(|w_1|^2|\partial_\xi w_2|^2 + |w_2|^2|\partial_\xi w_1|^2)d\xi\right] \\
&\quad - \frac{8b}{t}\int_{\mathbb{R}} |w_1||w_2||\partial_\xi w_1||\partial_\xi w_2|d\xi + 4\lambda_1\lambda_2\int_{\mathbb{R}}(|w_1|^6 + |w_2|^6)d\xi + 4\lambda_1 b\int_{\mathbb{R}}(|w_2|^4|w_1|^2 + |w_1|^4|w_2|^2)d\xi \\
&= \frac{2}{t}\left[\lambda_2\int_{\mathbb{R}}(|\partial_\xi w_1|^2|w_1|^2 + |\partial_\xi w_2|^2|w_2|^2)d\xi + b\int_{\mathbb{R}}(|w_1||\partial_\xi w_2| - |w_2||\partial_\xi w_1|)^2 d\xi\right] \\
&\quad - \frac{4b}{t}\int_{\mathbb{R}} |w_1||w_2||\partial_\xi w_1||\partial_\xi w_2|d\xi + 4\lambda_1\lambda_2\int_{\mathbb{R}}(|w_1|^6 + |w_2|^6)d\xi + 4\lambda_1 b\int_{\mathbb{R}}(|w_2|^4|w_1|^2 + |w_1|^4|w_2|^2)d\xi \\
&\leq \frac{2}{t}(\lambda_2 - b)\left(\int_{\mathbb{R}} |\partial_\xi w_1|^2|w_1|^2 d\xi + \int_{\mathbb{R}} |\partial_\xi w_2|^2|w_2|^2 d\xi\right) + 4\lambda_1\lambda_2(\|w_1\|_{L^6}^6 + \|w_2\|_{L^6}^6) \\
&\leq 4\lambda_1\lambda_2(\|w_1\|_{L^6}^6 + \|w_2\|_{L^6}^6).
\end{aligned} \tag{2.18}$$

Since

$$t\frac{d}{dt}(\|w_1\|_{L^4}^4+\|w_2\|_{L^4}^4)=\frac{d}{dt}(t(\|w_1\|_{L^4}^4+\|w_2\|_{L^4}^4))-(\|w_1\|_{L^4}^4+\|w_2\|_{L^4}^4),$$

it follows from Young's inequality, (2.18) and (2.1) that

$$\begin{aligned}
&\frac{d}{dt}(\|\partial_\xi w_1\|_{L^2}^2+\|\partial_\xi w_2\|_{L^2}^2)+\lambda_1\frac{d}{dt}(t(\|w_1\|_{L^4}^4+\|w_2\|_{L^4}^4))\\
\le\,& 4\lambda_1\lambda_2(\|w_1\|_{L^6}^6+\|w_2\|_{L^6}^6)+\lambda_1(\|w_1\|_{L^4}^4+\|w_2\|_{L^4}^4)\\
\le\,& (4\lambda_1\lambda_2\|w_1\|_{L^6}^6+\lambda_1\|w_1\|_{L^2}\|w_1\|_{L^6}^3)+(4\lambda_1\lambda_2\|w_2\|_{L^6}^6+\lambda_1\|w_2\|_{L^2}\|w_2\|_{L^6}^3)\\
\le\,& C(\|w_1\|_{L^2}^2+\|w_2\|_{L^2}^2)\\
\le\,& C_0.
\end{aligned}$$

Then, for any $t\in[0,T]$, we deduce that

$$\|Ju_1(t)\|_{L^2}^2+\|Ju_2(t)\|_{L^2}^2=\|\partial_\xi w_1(t)\|_{L^2}^2+\|\partial_\xi w_2(t)\|_{L^2}^2\le\ C_0(1+t).$$

□

3. Proof of Theorem 1.1

Now, we are in a position to prove Theorem 1.1.

Proof. For the initial data $u_{j,0}\in H^1$ and $xu_{j,0}\in L^2$, $j=1,2$, it follows from [23] that a time $T>0$ exists such that the Cauchy problem in (1.1)–(1.2) admits a unique local solution satisfying

$$u_j\in C([0,T];H^1)\cap C^1([0,T];H^{-1}),\qquad j=1,2,$$

$$x\,u_j\in C([0,T];L^2),\qquad j=1,2.$$

By Lemmas 2.1–2.3, we can extend the local solution globally in time. Since the solution u_j belongs to $C([0,\infty);H^1)\cap C^1([0,\infty);H^{-1})$, $\|u_j(t)\|_{L^2}^2$ is differentiable with respect to t for $j=1,2$. By (2.4), we obtain

$$\frac{d}{dt}(\|u_1(t)\|_{L^2}^2+\|u_2(t)\|_{L^2}^2)\le-2|\lambda_2|(\|u_1(t)\|_{L^4}^4+\|u_2(t)\|_{L^4}^4).\tag{3.1}$$

By Hölder's inequality, we have

$$\begin{aligned}
\|u_j\|_{L^2}^2&=\int_{\mathbb{R}}|u_j|^{\frac{2}{3}}|u_j|^{\frac{4}{3}}\,dx\\
&\le\left(\int_{\mathbb{R}}|u_j|\,dx\right)^{\frac{2}{3}}\left(\int_{\mathbb{R}}|u_j|^4\,dx\right)^{\frac{1}{3}}\\
&=\|u_j\|_{L^1}^{\frac{2}{3}}\|u_j\|_{L^4}^{\frac{4}{3}}.
\end{aligned}$$

Taking the square root on both sides, we have

$$\|u_j\|_{L^2}\le\|u_j\|_{L^1}^{1/3}\|u_j\|_{L^4}^{2/3}.\tag{3.2}$$

It follows from the Cauchy–Schwarz inequality that

$$\|u_j\|_{L^1} \le C\|(1+|x|)u_j\|_{L^2}.$$

Moreover, we define $u_{j,\eta}(x) = \eta u_j(\eta x)$ for $\eta > 0$. We then have

$$\|u_j\|_{L^1}^2 \le C(\eta\|u_j\|_{L^2}^2 + \eta^{-1}\|xu_j\|_{L^2}^2).$$

Taking $\eta = \|xu_j\|_{L^2}/\|u_j\|_{L^2}$, we obtain

$$\|u_j\|_{L^1} \le C\|u_j\|_{L^2}^{\frac{1}{2}}\|xu_j\|_{L^2}^{\frac{1}{2}}. \tag{3.3}$$

By (3.1)–(3.3), we have

$$\begin{aligned}\frac{d}{dt}\Big(\|u_1(t)\|_{L^2}^2 + \|u_2(t)\|_{L^2}^2\Big) &\le -2|\lambda_2|\left(\frac{\|u_1(t)\|_{L^2}^6}{\|u_1(t)\|_{L^1}^2} + \frac{\|u_2(t)\|_{L^2}^6}{\|u_2(t)\|_{L^1}^2}\right)\\ &\le -C\left(\frac{\|u_1(t)\|_{L^2}^6}{\|u_1(t)\|_{L^2}\,\|xu_1(t)\|_{L^2}} + \frac{\|u_2(t)\|_{L^2}^6}{\|u_2(t)\|_{L^2}\,\|xu_2(t)\|_{L^2}}\right)\\ &= -C\left(\frac{\|u_1(t)\|_{L^2}^5}{\|xu_1(t)\|_{L^2}} + \frac{\|u_2(t)\|_{L^2}^5}{\|xu_2(t)\|_{L^2}}\right).\end{aligned} \tag{3.4}$$

Since

$$\|xu_j\|_{L^2} = \|(x+it\partial_x)u_j - it\partial_x u_j\|_{L^2} \le \|Ju_j\|_{L^2} + t\|\partial_x u_j\|_{L^2},$$

it follows from Lemmas 2.2 and 2.3 that

$$\|xu_j\|_{L^2} \le C(1+t)^{1/2} + Ct \le C(1+t), \qquad t \ge 0.$$

Substituting this weighted estimate into (3.4), we obtain

$$\frac{d}{dt}(\|u_1(t)\|_{L^2}^2 + \|u_2(t)\|_{L^2}^2) \le -C(1+t)^{-1}(\|u_1(t)\|_{L^2}^2 + \|u_2(t)\|_{L^2}^2)^{\frac{5}{2}}, \qquad t \ge 0. \tag{3.5}$$

Integrating both sides of (3.5) from 0 to t, we obtain

$$(\|u_1(t)\|_{L^2}^2 + \|u_2(t)\|_{L^2}^2)^{-\frac{3}{2}} \ge (\|u_{1,0}\|_{L^2}^2 + \|u_{2,0}\|_{L^2}^2)^{-\frac{3}{2}} + C\int_0^t (1+\tau)^{-1}\,d\tau.$$

Hence

$$\|u_1(t)\|_{L^2}^2 + \|u_2(t)\|_{L^2}^2 \le [(\|u_{1,0}\|_{L^2}^2 + \|u_{2,0}\|_{L^2}^2)^{-\frac{3}{2}} + C\log(1+t)]^{-\frac{2}{3}} \le C_0(\log(2+t))^{-\frac{2}{3}}. \tag{3.6}$$

Taking the square root of (3.6) and using

$$\|u_1(t)\|_{L^2} + \|u_2(t)\|_{L^2} \le \sqrt{2}\left(\|u_1(t)\|_{L^2}^2 + \|u_2(t)\|_{L^2}^2\right)^{1/2},$$

we obtain

$$\|u_1(t)\|_{L^2} + \|u_2(t)\|_{L^2} \le C_0(\log(2+t))^{-1/3}, \quad t \ge 0.$$

This completes the proof of Theorem 1.1. □

4. Conclusions

In this paper, we studied the Cauchy problem in (1.1)–(1.2) for two coupled cubic nonlinear Schrödinger equations in one space dimension. In the defocusing dissipative case $\lambda_1 \geq 0$, $\lambda_2 < 0$, and $\lambda_2 \leq b < 0$, we proved the global existence and uniqueness of solutions without any size restriction on the initial data. We also obtained the logarithmic decay estimate

$$\|u_1(t)\|_{L^2} + \|u_2(t)\|_{L^2} \leq C_0(\log(2+t))^{-1/3}, \qquad t \geq 0.$$

The main point of the proof is to use the defocusing dissipative structure of the system to control the cross-interaction terms. This enables us to extend the large-data L^2-decay method for the nonlinear Schrödinger equation (1.3) to the coupled system (1.1). However, the present argument still depends on the conditions $\lambda_1 \geq 0$, $\lambda_2 < 0$, and $\lambda_2 \leq b < 0$. It would be interesting to consider more general coupling coefficients, higher-dimensional cases, or systems with derivative nonlinearities in future work.

Author contributions

Yuwen Sun: Conceptualization, methodology, validation, writing–original draft preparation; Chunhua Li: Writing–review and editing, supervision. All authors have read and agreed to the published version of the manuscript.

Use of generative-AI tools declaration

The authors declare they have not used artificial intelligence (AI) tools in the creation of this paper.

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Conflict of interest

The authors declare no conflicts of interest in this paper.

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