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Research article

An application of stochastic optimization through barter exchange

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Abstract: In an increasingly complex supply chain network, market demand integrates structural and nonlinear dynamics to reshape the current landscape. Due to these advancements, the concept of barter exchange has emerged as an innovative approach for exchanging products/services without capital investment in the smart production system. It recommends a variety of advantages to facilitate commercialism. The proposed study demonstrates information-based smart supply chain business models and transforms physical activities into digital challenges by highlighting the analytical complexity of verifying retail profitability. Despite the challenges, the study initiates a novel approach to establishing the barter mechanism with promotional efficiency under a stochastic demand function. To facilitate digital implementation, the methodology derives several differential equations with random nonlinearity and proposes complex intelligent optimization algorithms to determine the optimal parameters in the nonlinear mathematical system. The numerical experiment yields a mathematically grounded closed-form solution for an enhanced profit of 42.47% and reduces carbon emissions by 33.33% on a dual digital platform using a stochastic optimization technique. The sensitivity analysis identifies the key elements that emphasize the precision of a flexible alternative decision. It offers significant managerial insights to guide suitable pricing policies and diverse retail interests, and manage complex retail dynamics. Indeed, the barter mechanism supports nonlinear transformation to retail businesses by easing financing and reveals practical applicability in mitigating the complexities of the circular economy.

Keywords: barter exchange; closed-form solution; retail management; nonlinear mathematical system; random nonlinearity; stochastic optimization
Mathematics Subject Classification: 90B06, 90B50, 90B90

1. Introduction

Despite the rapid advancement of sustainable development, several issues persist in accurately measuring the potential impact of supply chain practices. Collecting information involves complexities in a global supply chain framework. In the transition towards a circular economy, the paradigm reduces resource extraction and waste through numerous retail market failures. The approach not only preserves natural resources but also minimizes the ecological impact. However, this research focuses on how the circular economy shapes consumers' purchasing behavior in retail. The findings are especially relevant for ecology-conscious consumers and are aligned with circular economy policies in retail. In contrast, the circular economy intensifies consumers' intention in the retailing context, including concerns about pricing, trading, and greening products. Better communication about pro-green activities and a better understanding of the circular economy reinforce the consumer's intention to purchase the product. In contrast, the distinction highlights the beneficiary of self-driven and financially driven achievements, as opposed to environmentally driven selections. The variability indicates the critical function of price sensitivity in the complex retail landscape. The wider acceptance of the circular economy in retail businesses reshapes the e-commerce systems. Utilizing circular economy principles, a contextual barter exchange framework delineates the retail system, including sustainable and socio-economic aspects. Fostering dual benefits, a barter system emphasizes complementary currencies that are closely aligned with the principles of sustainability and recognized as an alternative avenue for exchanging products and serving as a unit of price-setting policy. The development provides consumers' satisfaction and regional identity with the values of the products in the barter system. Indeed, the innovative instrument enhances qualitative understanding in the context of sustainable barter growth.

In this context, new digital methodologies offer a significant opportunity to enhance industrial performance by maintaining a high-level consumer service [10]. In these challenging contexts, the supply chain digital twin represents a promising concept for implementing a cyclical and holistic mechanism in the supply chain paradigm [2]. It aims to improve the sustainability and resiliency of the chain under crucial scenarios [35]. The trend intensifies to protect consumer satisfaction and guarantees of continuous retail activities [23]. Despite the pricing competency in an uncertain environment, barter exchange is a unique alternative policy to recover expenses in a counter-trade procedure [50]. In general, bartering deals with a trading exchange to complement the digital economy. It recommends pricing initiatives to add value to products/services. To ensure competency in the commercial market, the supplier often selects the best strategy to capture the retail market. They offer various alternatives to consumers that enhance the trading volume and satisfaction [45]. In the current era, online purchasing pays more attention to the supply chain due to the uncertain situation caused by trading congestion [42]. As a result, the retailer concentrates more on online trading to maximize retail profit. In practice, retailers may obtain incomplete information about the online market size and demand for the product. In this context, it generates a gap between growing research and real-world applications. Information resources, a fundamental parameter in digital financing, play a crucial role in virtual retail businesses, aiming to enrich an exclusive competitive dimension. The data resources of an enterprise influence the decisions of products and services uniquely. The dual-channel integrates offline-to-online services that can perform consumer services at the post-retail stage of products. To deal with the pricing conflict in dual-channel retailing, e-reviews with in-trade service are prime factors in consumers' shopping decisions, and it explores the potential contribution of digital twins in the retail environment. Thus, this study discusses the optimal pricing decisions to analyze retail services in the digital platform [67].

In recent eras, the global supply chain has become more complex in servitization and faces many challenges digitally [52]. The challenges encourage retail managers to seek for innovative solutions to resolve the operations in the supply chain network. The digital enhancement promises to improve performance-based visibility in the supply chain environment. In this digital context, pricing and profit management are crucial for retail management. The variable selling price in a dual-channel leads to the utility values of the products. The utility values of the products depend on the trust value of the brand's reputation. Thus, the consumer accepts online products by investigating their price and quality. The retail manager adopts the dual-channel retailing strategy to allow consumers to engage according to their purchasing satisfaction, and it depends on the primary preference of the consumer. The price management mechanism incorporates retailing flexibility and enhances the commercial advantages. It enables consumers with customization schemes to influence the products through online channels. In the customization context, the quality of the products extensively reduces the probability of imperfect customization. It enhances cost-effective retailing with consumer satisfaction. The impact of dual-channel retailing optimizes dynamic pricing to balance inventory levels between both channels. The inventory management strategy reduces excess inventory by highlighting the brand's visibility to satisfy more consumers in the retailing system [40].

Varying channel selection distinguishes collaborative methods between online retail platforms and manufacturers. The online retail platform provides strategic interaction between brand and channel selection under various pricing policies [32]. It influences competitive relationships and the distribution of interest in retailing. Thus, online trading alleviates dual marginalization and determines comprehensive retail research [33]. Consequently, the retailer trades the products through an online retail platform. The distribution channel strategy cooperates with e-tail platforms to capitalize on the channel's expertise in retail promotion. The mechanism guides the retail manager in operating the dual market retail system. Very recently, retailing has emphasized store-brand products. It encompasses a value-added service, including retailing efforts to enhance consumer loyalty. The service highlights the consumer's product-specific purchasing and changes the diversification of product distribution in retail outlets. In this dual-channel context, store brand's introduction and service investment are embedded in the supply chain to enhance the service levels in retailing. It highlights the conflict situation under dual-channel strategic retailing and implements consistent contextual retail information to stabilize brand images in the retail market. However, the study incorporates additional measures to better account for the retailer's perception by streamlining the barter condition in a dual-channel preference approach that establishes a well-suggested benchmark to distinguish the brand level in smart retail competition [73] (Figure 1a). It investigates the capability of promotional growth to aggregate the measure of consumers' loyalty to retail practices [56].

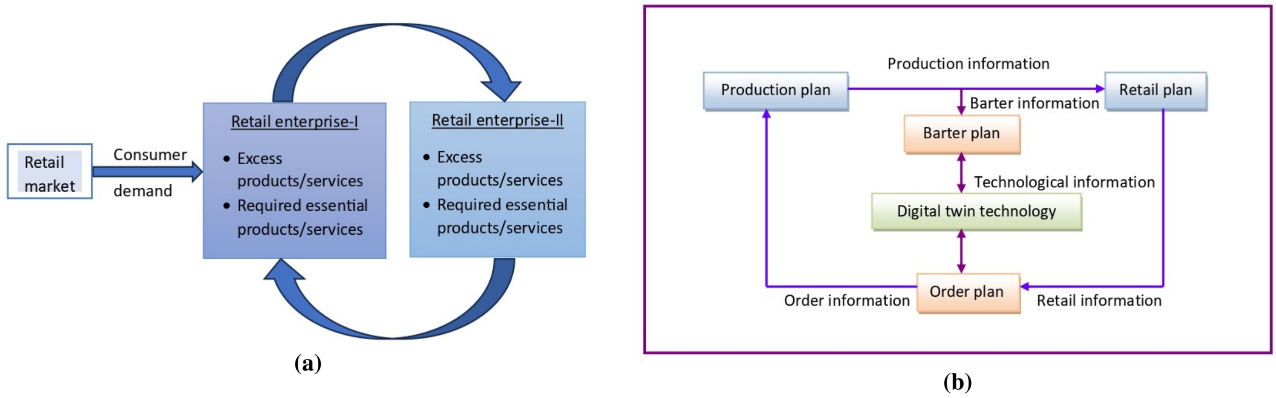


Figure 1. Retail barter exchange mechanism in the digital platform: **(a)** Schematic structure between two enterprises in retailing under a barter policy; **(b)** Digital twin performance in the supply chain paradigm.

Additionally, the rapid proliferation of e-purchasing brings several challenges caused by the advancement of digital methodologies (Figure 1b). It shifts consumers' behavior in a dynamic retail platform towards sustainability. Online retailing reshapes commercial engagement with consumers and conducts trading in the digital realm [4]. Consequently, it redefines e-marketing strategies to explore the risks of consumers' engagement digitally with expanding interactive retailing experiences [69]. The digital revolution investigates consumer perception in channel switching intention. Thus, it becomes more challenging for the retail manager to retain consumer engagement in retailing with the brand image during channel shifting. Digitalization transforms the behavioral approach to purchasing products and sharing information [88]. During the operational activities under the dual-channel supply chain, the internal cooperation provides economic support that encourages advanced payment and quality investment schemes [6]. The outcome reveals improved product quality through supportive financing under a cooperative retail strategy [77, 89]. During market turbulence, retailing agility and competitive benefits reinforce supply chain management. It utilizes the digital mechanism to revolutionize operational management.

However, this dynamic capability fills the conceptual and empirical gaps by undergoing a significant retail transformation. The theoretical identification of e-retailing platforms empirically verifies a rigorous scale enhancement process. It measures e-reliability and e-satisfaction by identifying the prime elements of retail inter-relations, e-business quality, and consumer satisfaction to integrate e-retailing perceptions. Consumers' responses predict their purchasing intention via sentiment analysis in reviewing products to intensify promotional investment in retailing. Thus, retailing and e-retailing enterprises reshape their trading strategies with the concern of growing consumer engagement on online platforms [57]. An online review comprises a product's evaluation, services, and brand reputation. It explores the brand image and its associations to analyze these at an aggregate level that extracts significant insight.

1.1. Research gaps and contributions of the study

A comprehensive overview of channel efficacy brings significant guidance on competitive advantage with expected profits. The contribution to the literature demonstrates that pricing competition prompts retail managers to set lower retail prices [18]. It mitigates double marginalization in the retail chain context. Thus, the study addresses a supply chain structure to illustrate the retail mechanism and understand the implications of the retail barter system towards the digital paradigm. It summarizes a list of contributions that helps to eliminate research gaps.

- The research mainly emphasizes the digital twin technology in the supply chain framework. It addresses more reliable and intelligible retailing to achieve a potential outcome. The implementation integrates physical and virtual systems using real-time information. It guides practitioners to enhance strategic and sustainable development through the digital supply chain twin mechanism [103]. Indeed, the intelligent methodology enables the effective control of retail services and generates a cyberphysical environment [106]. It offers an explicit interaction and synchronization between physical and virtual performances [75].
- The rapid expansion of digital retail businesses has led to the emergence of e-commerce as a dominant platform that integrates real-time engagement with trading and instant purchasing techniques. It revolutionizes the traditional supply chain by enabling swifter interactions among consumers. The transition analyzes the structural impact on consumer perception and supply chain dynamics. It supports real-time visibility and interactivity, enhancing reliability by minimizing purchase uncertainty. The dynamism highlights a significant challenge for retail management with demand volatility and the need for real-time inventory. The operational complexities require flexible strategies that align retailing momentum and hinder coordinated decisions on capital investment and marketing efforts. It enables the trade-off of retail optimization and strategic planning. The framework assesses pricing efficiency, service quality, and demand responsiveness in an e-commerce environment. It prioritizes the consumer and business goals under fluctuating market conditions. This research aims to bridge gaps in developing an integrated framework to analyze decisions of dynamic retailing under various commercial strategies. It provides valuable insights into the interaction of pricing, quality, trading effort, and consumer involvement, which increasingly influence the real-time context.
- The retail channel integrates with digital channels to provide the e-shopping experience [47]. The application of channel integration facilitates consumers and provides extensive success in retail management. It recommends the purchase experience for retail development. The phenomenon indicates the increasing reliability of the retail platform through e-personalized recommendations.
- Consequently, price promotion supports the digital platform and offers guidance on the manufacturer-retailer relationship. It stimulates category-based trading through promotional interaction. The promotion of pricing enhances brand retailing through cross-brand shifting. It assesses promotional activities on e-platforms to support effective retailing [20].
- The development of barter actions supports an alternative communal economy that provides a unique characteristic. It promotes local production to stimulate financial growth and activates the labor market. The policy increases the circulation of money and prevents unnecessary

aggregation. Complex transactions encourage numerous collaborative participants in retail businesses and foster reliable relationships for exchanging products and services without the need for conventional capitalism in the global retail market. However, the central goal of the barter policy is to promote production and consumption through a sustainable approach in a smart production system. Highlighting the critical global challenge, the research intends to design a sustainable framework to manage the complexities of swapping green products and services into an e-bartering platform to intensify ecological awareness and eco-conscious consumption. Thus, the e-barter mechanism is a user-friendly digital instrument that promotes sustainable commercial practices. It demonstrates a potential e-commerce platform that combines digital exchange with eco-conscious decisions and the well-being outcomes of smart production. Barter interactions facilitates communal initiatives in response to the implications of modernization and industrialization. Indeed, the implementation of the barter exchange mechanism in production enterprises reveals valuable industrial circularity under sustainable production.

- Retail barter practice is an organized barter system in which products or services are sold or bought via a barter agreement. The transaction is successful without paying money to other members of the barter system. In this regard, the performance of retail barter operations offers several alternatives to demand and supply internationalization in supply chain management [1]. It makes a trade-off between supply and demand, which provides a competitive advantage. Barter recommends promotional efficacy that enables effective retailing and analyzes the economic structuring by utilizing the overstock inventory [22].
- The in-depth knowledge of loyalty programs extends to consumer enjoyment. It drives the measured degree to the consumers' association with the purchase. The happiness of consumers tends to incorporate the potential value of the brand image. The brand-equity-based reputation encourages retailing functions to integrate human resources. It drives the value of distinct retailers to generate better consumer experiences in retailing the product. It delivers personalized service by demonstrating the dual sales role of consumer-employee integration.
- The existing research exhibits incongruities in promotional activity with green consumption across various conditions of purchase. The research offers practitioners a precise methodology for designing a retail mechanism that aims at augmenting green belief by integrating trading promotion. Customizing promotional programs based on green performance emphasizes the experimental perspective toward promotional instruments.
- Indeed, the retail strategy illuminates psychological and empirical experiences in the digital age. It intensifies cybersecurity, cost efficiency, productivity, and knowledge management in the retail industry [9]. The concept promotes healthier digital practice and enhances the propensity of digitization.

Thus, the digital twin is an interesting research topic regarding the supply chain paradigm. It expects resiliency from every retail business that connects to digital methodologies. In particular, the proposed problem offers a digital twin-based supply chain network to elaborate on an extension of decision support management and delineate how the intelligent mechanism contributes to supply chain knowledge.

1.2. Orientation of the study

The remainder of the proposed study is structured as follows: A brief literature review is presented in Section 2 based on the technology within the supply chain domain. Section 3 describes the assumptions and notation of the proposed framework with a problem statement. Section 4 represents the mathematical model formulation. Section 5 illustrates the experimental solutions, including a sensitivity analysis in distinct cases. Managerial insights are suggested in Section 6. Finally, the research concludes with future extensions in Section 7.

2. Literature review

The study integrates the dual-channel product distribution and retailing streams with application of the barter mechanism in a digital supply chain framework. The section elaborates on how the proposed framework relates to the existing literature in the context of retail businesses.

2.1. Barter exchange in the digital retail businesses

Barter exchange is an alternative approach to transferring distressed inventory, which opens a new dimension in retail businesses digitally. The inventory managers retail their unsold products with a barter exchange option. The strategy assists managers in enhancing their retail profit by managing the uncertainty of demand [105]. The modern barter trade is more advantageous in lowering the barter commission by estimating the higher trade value it needs, which becomes close to the retail economy, relatively [60]. The application of barter policy emerges due to cashless retailing [29]. It drives the exchange of excess materials by the face value of the products. Thus, the collaborative swapping policy engages many retail enterprises in exchanging the solution of product shipments. In a conventional retail system, the credit facility leads to the possibility of risks [14]. In these circumstances, flexible production manages the investment on risks and diminishes all other expenses by enhancing the product quality of the inventory system [51, 72]. Indeed, the retail manager selects a proportional pricing markup to achieve the expected profit. The target-setting approach serves as a pervasive policy to categorize the utility of the newsvendor problem [109]. It assists the coordination of the supply chain members to maximize the channel's profit [28]. Initially, the decision-maker generates an advanced ordering decision by predicting the expert's opinion. The scheme derives conceptual guarantees for all regions of the planning horizon. It transfers the learned information of demand knowledge in each stage. Inventory sets decisions on the level of retail advertisement by maintaining commercialization. It aims to reduce complexities at the inventory level to indicate coordination between attitudinal ordering decision within the realm of retail operational research. In addition, risk-neutral newsvendor problem minimizes the value-at-risk concerning significant conditions.

In real-world retail operations, the target attention and decision navigation influenced by dual referencing points emphasize the newsvendor to boost the expected profit. It provides retail-oriented insights, including planning and control of inventory, that strike a balance between the lower and extreme ordering costs [81]. In this regard, retailers exhibit risk aversion for profits and a risk-seeking attitude to overcome complexities [16]. Developing a dynamic pricing strategy in the newsvendor problem clears the trading of leftover inventory and determines how to earn salvage revenue [84]. Interestingly, the classical newsvendor problem illustrates the financing edges by reducing the malicious ecological components of unrecyclable surplus products. In practice, the newsvendor problem deals with stochastic demand-based product marketing by shedding green responsibility [53]. The production, recyclability, and disposal motivate consumers to purchase green products. Producing green premium products under quality and durability integrates greener marketing performances. Consequently, in the newsvendor problem, retailers predict the demand to analyze the cost-benefit inventory decision. It encourages consumers to make other optimal decisions towards the retail economy. The findings set the pricing decisions and inventories for seasonal materials by the promotional investment of pre-purchase perceptions. However, the stocking and retailing of substitute products satisfy the diversity of consumers in the joint optimization problem for multiple products under budgetary constraints [58]. It is a prominent benefit asserted by manufacturers [112].

Financial risk management measures risks with a more realistic and in-depth understanding. It considers risk aversion pricing competency under the degree of risk tolerance [19, 79]. Market demand captures consumers' attention, which results in analytic impacts within pricing variability. However, markup pricing policies and price-dependent demand impose a restriction on the combined effect of trading advantages. Price-sensitive demand function optimizes the decisions and retail economic benefits to the newsvendor under upstream competition [74]. A decision-contextual approach characterizes strategic market conditions and investigates consumers' expectation of the retail outcome. The retail market reconfigures the consumption pattern by promoting brand image, a novel financing growth point. The findings offer an advanced premium trading adaptation that assists retailers in eliminating consumers' regret proactively. From an economic perspective, newsvendor experiments encourage the potential effect of uncertain demand on the level of inventory empirically. The policy manages fluctuations in demand, leading the inventory system to prevent distressed products. Indeed, the decision-maker needs to pay more attention when investigating diverse-level cooperation in the retail channels to obtain more profitability. It offers practical guidance in the evolving domain of retailing with an adverse impact on retailers' profits.

2.2. *Digital twin-based smart production*

Digital twin is an evolving concept in the industry and academia. It integrates real and cyber environments to simulate real-world attributes. Compared with conventional models, the digital twin has significant advantages with realistic information and mutual conversations between virtual and physical systems [15]. Unlike traditional problems, the digital twin has functioned over static and isolated environments with updated real-time information, enabling predictive decision-making capabilities. Owing to its benefits, the technology has emerged in several regions other than production in the multi-modal supply chain system [27]. It expands retail businesses to collaborate on the best retail decisions across distinct stages in retailing, optimizing, and analyzing the data [82]. Interactive digital models emphasize flexibility in dynamic environments. The industry earns more profit by visualizing inventory and mitigating risks with flexible demand. A real-time risk simulation demonstrates the scope of empirical validation but does not address the logical flows.

Despite the technological advancements, the application faces challenges in achieving the scalability features of the digital platform. The digital twin-based cyber-physical processes have promising solutions in an uncertain environment and shows a scientific interest in supporting market dynamics and disruptions in supply chain operations [90]. The necessary discussions on digital twin-based approaches have been a topic in the scientific panorama. A retail business can ensure consumer satisfaction, risk mitigation, and cost-effective strategic alignment. By streamlining the procedures, controlling risks, and internal cooperation, it adapts to the transition of retail market conditions and meets consumer demands, which drives success under retail competency. The revolution of existing methodologies offers a new opportunity that brings countless benefits to retailing. In this regard, Table 1 indicates the divergence of the proposed framework from the existing studies. It creates a virtual replica to analyze and optimize the activity of the supply chain and facilitates advanced decisions in the context of a digital supply chain regime. The application contributes qualitative insight to understanding integration mechanisms in the retail management sector. The complexities highlight the digital twin-based platform for ensuring flexible and scalable supply chain management. Thus, the prime objective of the proposed framework leverages the digital twin concept within an integrated supply chain environment and demonstrates its extensive platform effectiveness.

Table 1. Contribution of prior research.

Author(s)	Platform preference	Pricing promotion	Intelligent application	Green practice	Reflective constructs
Hua et al. [60]	Retail			✓	Retail barter system
Zhang et al. [112]	Retail	✓			Constrained financing
Lim et al. [78]	Omni		✓		Channel integration
Wang et al. [110]	Retail		✓	✓	Smart supply chain
Chen et al. [30]	Dual	✓			E-retailer bundling policy
Bose et al. [25]	Dual	✓			Consumer service
Cimino et al. [34]	Digital		✓		Simulated production
Chu et al. [33]	Dual	✓			Dual-channel selection
Datta et al. [40]	Retail	✓			Dynamic retailing
Sarkar et al. [98]	Retail	✓		✓	Complex retail strategy
Jin et al. [68]	Digital		✓	✓	Digital green supply chain
Bar et al. [17]	Dual		✓	✓	Carbon emission mitigation
Li & Jiang [77]	Dual	✓	✓	✓	E-commerce marketing
This research	Dual	✓	✓	✓	Customized retail e-bartering

2.2.1. Dual-channel customizable servitization

Due to the evolving retail landscape, consumer preferences in the dual-channel supply chain provide contemporary advantages [30]. This study investigates an intricate interplay among variable pricing strategies and services within the dual-channel under multiple elements-dependent demand. The mechanism explores the channel preference of consumers, pricing competency, and the retail market's changing effect with a strategic solution. The retail manager generates trading plans by predicting the trend of the subsequent retail market. In addition, the retailer's concerns about the demand-increment investment affect the selection of the channel structure. The investment policy expands the product delivery services to enhance retail market potential for long-term retail advancement [62]. The retail industry often selects a dual-channel trading policy to investigate the promotional demand of the product. It assists a flexible production system to play a potential role in an e-commerce platform. The strategic interactions among dual platform concerns and the relevant competitive advantages enhance retail intensification to alleviate some adverse effects of the steady growth of platform retailing. The platform achieves dual-purpose objectives and recommends digital services to increase consumers' interests. The platform-dependent policy enhances consumers' loyalty via the product's brand image [61]. Compared with profit sharing, the strategy combines the cooperative mechanism with a service to form a novel policy with a competitive advantage in the offline-to-online structure. Consumer relationship management determines the consumer services that emphasize all retailing efforts.

In reality, many retail enterprises initiate digital channels for trading to enhance internationalized retailing. The channel composition offers a beneficial channel pricing structure to provide retail services. The offering boosts the propensity of consumers to lower purchasing uncertainty [48]. Because of the massive transaction volume, e-retailers possess advantageous information for conducting in-depth analyses. They establish a strategic relationship with consumers and commit to sharing information that supports operational retail services. The service investigates the power structure preferences of supply chain members concerning the subsequent decisions [59]. In practice, buying online and picking up from store addresses are strategic values of supply chain performance to implement channel coordination at the maximum level. It initiates a cost-sharing policy to complement the retail industry. Thus, the retail operation is widely enriched in commercialization [91]. It implements multi-channel mechanisms with the advancement of the retail economy. The cross-brand attitudes of consumers on dynamic pricing determine the survival of the supply chain in a fiercely competitive marketplace. It integrates a physical store with an online marketing channel, which enables a new retail opportunity. Omnichannel retailing offers product information and pickup of online orders. The prime dimension of an omnichannel service is to fulfill the online orders of the product [46]. The optimum pricing under promotional effort generates revenue under green innovation. It is a unique challenge in the retail research to coordinate under channel conflicts. Indeed, the advertisement policy is a key feature of flexible commercialism [83]. It contributes to product greenness in both channels, which benefits retail managers.

Surprisingly, incentivization offers consumers with green consciousness under the expansion of greening cost components. In this context, the e-tailer demonstrates the diverse channel competitive coefficients on pricing and greenness of the product. The sensitivity of the retailing problem under a two-channel system leads to integrating online-to-offline trading concerning the consumer's changing purchasing mode [63]. Most consumers prefer to receive the desired products at their doorstep, thereby avoiding a physical visit to a nearby retail store. The home delivery policy always provides the best retail service to attract consumers. In reality, a digital e-commerce service implements the concept of home delivery to meet the basic requirements of human beings. Consequently, the research gaps contrast with the potential interests in understanding consumers' inspiration in the dual-channel environment that enhances purchasing intention and loyalty of consumers. The concept perceives an interesting research opportunity for a deep understanding of consumer attitudes in advanced retailing. The retail managers foster consumer inspiration across both channels and highlight sustainable advantages [104]. In this context, perception provides a novel stimulus in the channel structure. It demonstrates the benefits of constituting a novel retail environment that integrates the expansion of a dual-channel strategy. Thus, emphasizing a channel perspective on consumer perception allows practitioners to define the concept of retailing policy and realize an alternative marketing-related vision.

2.3. Dynamic challenges during e-purchasing

Digital retail cooperation is a key extension of dual-channel retail management, reflecting the degree of channel coordination. It interacts with diverse strategic orientations to formulate decisions regarding channel integration. The concentrated strategy combines seamless marketing experiences that attract enhanced attention to the retail industry. The planning of coordination induces an appropriate trading strategy to increase product exposure and attract more consumers to purchase them. Retail managers incorporate various criteria into the distribution channel, including pricing promotion, product delivery, and brand advertisement, to shape consumer preferences. The opportunity aims to enhance consumer retention through promoting brand experiences. It attracts a more convenient and intuitive approach to marketing by offering suitable customized services [43]. The findings provide empirical insights to researchers. In recent eras, the retail manager has attracted more new consumers in the supply chain framework to ensure good product quality. Pricing is one of the strategic components that influences profitability, along with demand value. The determination defines a deep understanding of the digital economy, which focuses on the value of consumers rather than competency [86]. The concept compels a holistic approach that melds the benefits of uninterrupted purchasing intentions irrespective of the retail channel system. Combining multiple channels reveals trading statistics to cope with business advancement through the highest level of coordination efforts. The concern indicates a better plan for omnichannel retail managers to enhance the patronage intention of consumers across distinct retail outlets [78].

With the wide application of digital coordination, the participants generate a level of digital methodology. Digitization promotes a prime project for the retail industry in upgrading the speed of retail responses. In this regard, transparency and consistency in digital services predominantly emphasize channel optimization to prioritize retail business strategyies [38]. It perceives privacy risks in the relationship between consumer loyalty and service integration. The retail operation strategy drives service efforts to provide consumers with the convenience of conversion and platform experiences. The omnichannel service prints a transparent introduction of the products' characteristics, promoting expert opinions. A comfortable environment facilitates the conversion of channels and enhances the degree of integration from offline to online [76]. In addition, the e-tailer contributes to expanding digital retailing and increasing revenue for retail services. With the advent of online retailing, selecting a suitable platform becomes a challenge for retail managers. Many retail sectors offer a return mechanism in the web store channel, alongside their revenue in each channel.

Besides retail services, the e-tailing platform provides promotional services to increase retail and traffic. The outcome-based shipment strategy derives the conditions that collaborate with the upstream retail enterprises to trade products on a dual platform [37]. The platform-based dual-channel supply chain demonstrates an intensity of retailing competition with incentives. It stimulates a remarkable growth in e-retailing and strengthens the retail businesses. The policy leverages a prime distinction in a retail marketplace [110]. In digital retailing, the encroaching decisions in an e-retailing marketplace shed light on the supply chain platform. It determines the associative retail operations to manage the challenging condition of channel encroachment. Consequently, the return strategies from the online channel increase online trading under profit margins. The e-commerce enterprises consider the amount of product returns to explore operational decisions under pricing competition in retailing. In reality, retail managers acknowledge the necessity of omnichannel policies. It identifies the essential components of channel integration to enjoy more personalized retail services. The conceptual foundation investigates the promotional reliability and consumer empowerment with patronage perception in retailing. It indicates the enhancement of retailing efforts and amplification of their advertising endeavors. In response to the challenging issue, the cooperative effort between downstream and upstream retailers in the retail chain is affected during the industrial transition [111].

2.3.1. Retail strategies for e-commerce platform competitiveness

In recent eras, supply chain management has led to the paramount significance of combining cooperative operations and services due to competitive industries. The challenge is to generate a digital methodology to enable virtual replicas of products and procedures with well-defined semantics. The application recommends a holistic insight into digital twins to overcome such challenges significantly. It identifies critical components that hinder retail profit. Moreover, the pricing policy is more challenging for the channel. The analysis claims to provide reliable outcomes by introducing the store brand's product images. It enables consumers to gain a better knowledge of the brand information of the product. The phenomenon develops novel profit growth in the retail market and generates an unprecedented opportunity for retailers to evaluate easily. A smart investigation manages dual-channel retailing to produce greener products. It earns consumer trust through promotional policy that secures the profit of retail businesses. The advertisement-driven demand controls a flexible production system and achieves a better environment by investing in green methodology in the retail management system [87]. Trading effort achieves revenue for the retail businesses [44]. It has profound implications for dynamic retail management. The decision provides consumer loyalty and competitive market advantages, which align with retail strategies [97]. Channel policies for dual-channel enterprises consider selecting predetermined pricing instead of dynamic pricing [36]. Strategic consumers seek to generate inter-temporal purchasing attitudes. However, practitioners avoid the negative impact of dual-channel retailing by demonstrating variable coordination contracts [54]. Indeed, it accelerates the development of digital e-commerce under the condition of competition [92].

The digital transition emphasizes a successful implementation of the evaluation of system processes to prioritize retail demonstration in this area [95]. The advanced changes in supply chain management yield better results in the reliable delivery of products and consumer satisfaction. Indeed, the product-specific dual-channel mechanism supports providing an essential consumer service. Consumers prefers to select the necessary materials according to the quality and pricing of products. Indeed, the consumer-centric retailing reshapes the retail industries in addressing consumer preferences through cost-effective retail contexts. It provides consumers with customization provision that requires extensive quality inspections. Automated investigation identifies defective products to intensify the retail sector. Trading the products through dual-channel-based retailing with digital information reassesses the retail businesses. Due to digitalization, dual-channel retailing attracts enhanced consideration for an imperfect manufacturing procedure [93]. It blends offline familiarity with online convenience. The consumer-based expansion simplifies the logistics expenses and enhances the order fulfillment efficacy [100]. Indeed, online-to-offline retailing bridges the gap between convenience service and shopping experiences. The strategic attempt at pricing relies on the shipments of the products and delivers consumer satisfaction by promoting demand [25]. The approach incorporates price-based communications between both channels. The challenge addresses the diverse purchasing interests of consumers and provides vital research-related insights to explore.

In this context, dual-channel delivery service provides satisfaction and assists the supply chain system to run smoothly toward commercialism. Consumers always prefer better retail service, which often opens several retail opportunities [99]. Many retailers coordinate with online retailers to minimize the burden of operating websites. They widely implement the price packaged retail practice of bundling to stimulate the consumer's consumption. This shows a significant understanding of the supply chain members to establish a cooperative relationship between the producer and retailer. As the degree of overconfidence on product demand incorporates the channel preference of the consumer, it derives a closed-form solution to the pricing decision. Additionally, pricing policy provides a warranty replacement service to consumers to achieve competitive advantages. The hybrid profitable tariff contract meets consumers' expectations and ensures confidence in the product's quality when purchasing online. Thus, intelligent production management has to take advantage of the pricing in both channels. Consumers get the desired products on their doorstep through an e-retailing strategy. The policy detects imperfect materials through a variable rate of production. Surprisingly, the digital retailing concept determines the product demand and attempts to fill retail research objectives. It motivates retailers to implement a dual-channel mechanism by overcoming all the retail complexities. The consideration should be prioritized for any production industry to illustrate the economic feasibility of retailing.

Indeed, the distribution channel has become diverse and competitive, with new opportunities. It emerges with challenges in the digital retail system under the complexities of channel conflict. Thus, from a dynamic perspective, research practitioners delve into long-run profitable attitudes to coordinate with retail efficacy, which meets consumer demand and demonstrates a widespread application. To address the issue, the proposed problem frames the supply chain digital twin framework and investigates the methodology that enables the enhancement of supply chain modelling. It detects the proactive risks under an uncertain environment and facilitates effective supply chain collaboration to achieve a more productive benefit.

3. Assumptions and notation

The research emphasizes retail practitioners who are digital natives and have a distinct perception of utilizing the supply chain's economic services [8]. Based on the cognitive virtual strategy, the study has listed the basic assumptions and notations to solve the proposed mathematical problem of generating maximum retail profit.

3.1. Assumptions

The study assumes the observations in the price-setting newsvendor problem to simplify the analysis procedure.

- A critical factor of the supply chain framework is demand forecasting. It fluctuates according to numerous factors: Market trends, seasonal changes, and economic conditions. In the supply chain, from demand to resource allocation, the capability of the proposed mathematical models is evaluated dynamically. Predictive analytics provides a proactive approach to address the critical issues of demand variability and supply chain disruptions. It enhances demand forecasting, optimizes inventory levels, and improves the operational efficiency of the supply chain. However, the proposed study provides a comprehensive overview of the benefits and

applications of linear demand in supply chain management. The application demonstrates how integrated information leads to significant cost-effective optimization and enhanced responsiveness to market demands. Indeed, this provides a holistic understanding of the current situation of retailing in the dynamic realm of supply chain management.
- In the dynamic e-commerce landscape, consumer perception intensifies the critical and sustained success of online retail businesses. The advent of intelligent strategies has highlighted consumer preferences. Their expectations expand the capabilities of e-commerce platforms. The relationship between consumer intention and e-commerce sheds light on transformative intelligent-powered personalization that influences retail market trends. In these circumstances, an increase in consumer demand adversely affects both retailing and distribution. To prevent the spread of conflict issues, uniform distribution related to performance is equally likely to occur in randomized conditions, with the possible outcomes within a specific range. It provides opportunities for growth and innovation towards a consumer-centric digital marketplace.
- The evolution of the payment system reflects communal and technological advancement. It provides a preliminary barter strategy for the digitalization paradigm. The investigation explores an innovative retail development that highlights the evolutionary trajectory of the retail industry.
- The retail practice determines the dynamic pricing strategy by considering a crucial challenge in consumer perception. Dual-channel pricing investigates the selection of a trading platform in a complex retail environment [98]. The price-based interaction between retail and digital channels allows diverse consumer reactions. Thus, the study integrates the channels to ease retail operations.
- Cross-channel integration influences consumer purchasing intention through the skepticism of consumers, ecological knowledge, and pricing sensitivity. Green consumption encourages consumers to participate in addressing ecological issues. It combines the retailer-consumer relationship with green reliability. In this context, the online experiment conducts an effective promotion to improve consumer green purchase perception digitally [68]. Thus, retail promotion and green consumption have empirical impacts on retailing strategy.

3.2. Notation

This subsection defines the notations of the proposed problem. The index, parameters, and variables are utilized in the proposed problem as shown in Table 2.

Table 2. *Notation.*

Index	
i	Model i; i = *1, 2, 3*
Input parameters	
w_s, w_d, w_e	Unit wholesale price of product for Model i ($/unit)
s_s, s_d, s_e	Unit shortage cost of product for Model i ($/unit)
h_s, h_d, h_e	Scaling parameter for switching platform in Model i
p_s, p_d, p_e	Unit retail price of product for Model i ($/unit)
v_s, v_d, v_e	Unit holding cost of product for Model i ($/unit)
c_s, c_d, c_e	Price sensitivity parameter for Model i
l_s, l_d, l_e	Product sensitivity parameter for Model i
b_s, b_d, b_e	Cross-price sensitivity of the product for Model i
a	Scaling parameter of product compatibility for Model i
g_s, g_d, g_e	Scaling parameter for reducing carbon emissions for Model i
E_0	Initial level of carbon emissions due to the product (unit)
E	Target level of carbon emissions set by the retailer (unit)
r_s, r_d, r_e	Retail commission paid for per unit product for Model i (%)
Q_0	Bartered quantity of product for Model i (unit)
A, B	Interval of uniform distribution for Model i
Random variable	
D_s, D_d, D_e, z	Random market demand for Model i (unit)
Decision variables	
Q_s^*, Q_d^*, Q_e^*	Optimal ordered quantity for Model i (unit)
Others	
$E[\cdot]$	Expected value of $(\cdot)$
$\Pi^S(Q_s), \Pi^D(Q_d)$	Maximum profits for Models 1 and 2, respectively ($)
$\Pi^E(Q_e)$	Maximum profit for Model 3 ($)

3.3. Problem statement

Developing an innovative methodological solution for intelligent barter transactions is a novel approach in the digital economy. The core function of barter policy is to predict and match market demand for the transparency of transactions. It ensures the advantages of successful transactions and security. The architecture of the barter platform evaluates the feasibility and efficiency of operational performance in managing retail complexity. It provides conceptual support and practical suggestions for the development of barter trade. Financial strategy expands information sources to assist in digital transition, maximize profit, and identify risks. In the barter platform, retail demand is matched with digital services through the exchange of products. The platform utilizes an intelligent risk control system to regulate strategic assessment during the decision-making stages, provides reliable information storage in the barter pool, and ensures smart agreement in the e-barter system. It enhances matching efficiency to maintain balance during operational interventions in forming a complex financing system. To analyze the strategic barter revolution, the retail manager selects appropriate commissions during the barter transactions to maximize profit and achieve long-term stability in the retail industry. In practice, the policy reshapes the dynamic market to explore the initial impact on the retail barter system. A reasonable commission measures outcomes for promoting international trade agreements through the exchange of products. Indeed, the research explores a barter-aligned economic strategy for achieving complete equivalent product exchange. The application enables complementary barter advantages and forms a dynamic retail management system under the constant flow of information. Thus, the research provides an interesting region for investigating the interactions between participants in expanding the retail scope and attracting more consumers in the retailing system.

In this context, the research proposes a digital pricing strategic problem based on the dual-channel purpose under a barter exchange environment. The barter strategy refers to an alternative mechanism in commercialism that represents a variety of opportunities in financing and the public sector. It indicates a well-accepted economic instrument for trading products or services that ensures perfect retailing [64]. Given recent market conditions, several technologies have transformed into a digital paradigm to generate a digital replication of real objects. The digital twin is a virtual depiction of a material to deal with the two realms [65]. The application supports effective pricing and scalable digital transition. Despite the transformative expansion of digital twin in supply chain management, the proposed study provides a significant insight into the motivational application. It stimulates the platform-based selection of consumers with cross-channel demand interaction. The research maps the real entity in the digital twin supply chain network [66]. It enhances connectivity, coordination, and optimization across multiple scenarios. In this regard, the implementation applies three distinct mathematical barter models under single- and dual-channel digital twin platforms with successful promotional performances [34]. It is the first attempt to quantify the cross-channel demand interaction, which depicts the non-negative realization of demand. The mathematical complexity of all the derivations of the objective function generates the concavity property in the retail region [13]. Indeed, the novel concept sheds light on retailer-consumer interaction and investigates retailer's reliability under cost-saving opportunities. In a digital economic context, the pricing policy in dual-channel retailing maximizes the retailer's profit, and the empirical analysis evaluates the digital performance by mitigating the existing gaps in barter retail businesses [3].

4. Mathematical model formulation

Interestingly, supply chain management's most critical research topics have faced complex challenges: Diverse market requirements, logistical risks, and data uncertainty, which bring them significantly closer to industrial realities. Due to increasing competition in global markets, the crucial task in designing and optimizing supply chains is to effectively perform under uncertain conditions. It has been recognized as a prime approach to balance the conflicting objectives of cost minimization, service-level development, and escalated flexibility. In this context, a dual-channel structure enhances the complexity in decision-making stages related to production, distribution, and shipment. Managing uncertainty in the supply chain has become a fundamental challenge in expanding the concept of reliability to a suitable barter platform. In this context, the expansion of digital methodologies highlights supply chain management based on a virtual version of the real system, in turbulent situations. In these circumstances, a digital twin provides real-time prediction, investigation, and optimization at strategic and operational levels. The integration of intelligent strategies into supply chain systems plays a powerful role in enhancing flexibility and mitigating uncertain situations. The concept of the digital twin mechanism is elevated by connecting data analytics. The research emphasizes the role of digital twins as a smart supply chain framework allowing accurate modeling and optimization of real-world scenarios. Implementing a digital twin in the supply chain represents an integrated strategy for mitigating risks between policy and processes during retailing. It recalibrates the decision-making framework and adapts to fluctuations in demand, which manages operational activities. However, digital twins operate as intelligent dynamic systems to receive and analyze information during the research optimization stages. Additionally, methodological advancements in the proposed framework contribute to operational and ecological sustainability by enabling continuous promotion of ecological consciousness and re-optimization of efficiency. Thus, the proposed framework explains the three models under the condition of bartering, which are as follows: Model 1 represents the problem in a single online channel, Model 2 represents the dual retail channel, and Model 3 represents the dual online channel.

4.1. Model 1. A single online channel environment with barter exchange

In this subsection, consumers choose the direct online channel rather than the conventional retail approach. According to product compatibility, a few consumers select the innovative technology through the online channel, and the remaining choose the other. Thus, the market demand (D_s) can be expressed for the single e-tail platform as

$$D_s = z + (1 - h_s)a - c_s p_s + g_s(E_0 - E).$$

Random and base market demands are defined in the first and second terms, respectively. Similarly, the price sensitivity demand of the e-tail channel is expressed by the third term, and the carbon preferences are presented by the fourth term. In the end, the retail profits can be obtained for a single bartered platform, which is given below.

- When market demand D_s is greater than the retailer's ordered quantity Q_s, then a shortage arises. In this phenomenon, the non-participation of retail manager in the barter platform allows them to purchase the required quantity of the product from the supply chain market to retail. Hence, the profit function of the retail manager is structured as follows:

$$\Pi^S(Q_s) = (p_s - w_s)Q_s - s_s(D_s - Q_s) - p_s Q_0 + l_s Q_s.$$

- In the situation where the ordered amount Q_s is higher than the demand D_s, the retail manager has to hold a certain quantity of products. The holding products are exchanged with the bartered quantity Q_0 in the barter exchange platform. If $D_s < Q_s$, then r_s is the commission paid over the retail price of those items held on the barter platform and rest of the bartered product ($Q_0 - (Q_s - D_s)$) is purchased separately. Thus, the retailer's profit becomes

$$\Pi^S(Q_s) = p_s D_s - r_s p_s(Q_s - D_s) - p_s\{Q_0 - (Q_s - D_s)\} - w_s Q_s + l_s Q_s.$$

- Finally, when the $Q_s > Q_0 + D_s$ scenario exists, i.e., the bartered amount is not greater than the ordered product, and all bartered products have been purchased with a commission by exchanging the holding products at the barter platform. The remaining products are held at a holding cost v_s. Then the profit function of the retailer becomes

$$\Pi^S(Q_s) = p_s D_s - r_s p_s Q_0 - v_s\{(Q_s - Q_0) - D_s\} - w_s Q_s + l_s Q_s.$$

Thus, the following expressions summarize the retail profit functions.

$$\Pi^S(Q_s) = \begin{cases} (p_s - w_s)Q_s - s_s(D_s - Q_s) - p_sQ_0 + l_sQ_s; & Q_s \leq D_s, \\ p_sD_s - r_sp_s(Q_s - D_s) - p_s\{Q_0 - (Q_s - D_s)\} - w_sQ_s + l_sQ_s; & D_s < Q_s \leq Q_0 + D_s, \\ p_sD_s - r_sp_sQ_0 - v_s\{(Q_s - Q_0) - D_s\} - w_sQ_s + l_sQ_s; & Q_s > Q_0 + D_s. \end{cases} \tag{4.1}$$

Equation (4.1) shows the corresponding expected total profits in the supply chain e-tail channel. In this context, considering the expected value of Π^S provides the expected profit of the retail manager.

$$E[\Pi^S] = [(p_s - w_s + s_s + l_s)Q_s - p_sQ_0 - s_s\{(1-h_s)a - c_sp_s + g_s(E_0 - E)\}]\int_{Q_s}^{\infty} f(z)dz - s_s\int_{Q_s}^{\infty} zf(z)dz$$

$$-[(r_sp_s - p_s + w_s - l_s)Q_s + p_sQ_0 + r_sp_s\{(1-h_s)a - c_sp_s + g_s(E_0 - E)\}]\int_{Q_s-Q_0}^{Q_s} f(z)dz + r_sp_s\int_{Q_s-Q_0}^{Q_s} zf(z)dz$$

$$-[(v_s + w_s - l_s)Q_s + (r_sp_s - v_s)Q_0 + (p_s + v_s)\{(1-h_s)a - c_sp_s + g_s(E_0 - E)\}]\int_0^{Q_s-Q_0} f(z)dz + (p_s + v_s)\int_0^{Q_s-Q_0} zf(z)dz \tag{4.2}$$

Take the first-order derivative of Eq (4.2) with respect to Q_s, which is formulated as

$$E'[\Pi^S] = (s_s + r_sp_s)F[Q_s] + (p_s + v_s - r_sp_s)F[Q_s - Q_0] - (p_s - w_s + s_s + l_s) - (v_s + w_s - l_s).$$

Similarly, the first-order derivative of Eq (4.2) with respect to Q_s is formulated as

$$E''[\Pi^S] = (s_s + r_sp_s)f(Q_s) + (p_s + v_s - r_sp_s)f(Q_s - Q_0).$$

Theorem 1. The optimality test of the order quantity Q_s^* of the retailer is satisfied by the first-order derivative of Eq (4.2).

$$(s_s + r_sp_s)F[Q_s^*] + (p_s + v_s - r_sp_s)F[Q_s^* - Q_0] = (p_s - w_s + s_s + l_s) + (v_s + w_s - l_s) \tag{4.3}$$

Proof. From Theorem 1, the optimal order quantity Q_s^* can be derived by following the uniform distribution of market demand $D_s \sim U[A, B]$, which is expressed by the following

$$Q_s^* = \frac{[p_s - w_s + s_s + l_s]B + [v_s + w_s - l_s]A}{(p_s + v_s + s_s)} + \frac{\{p_s(1 - r_s) + v_s\}}{(p_s + v_s + s_s)}Q_0. \tag{4.4}$$

Equation (4.4) is positive when $1 - r_s > 0$. This is a unique and closed-form of Eq (4.2). Hence, the theorem is proved.

Theorem 2. The expected profit of the retailer reaches a global maximal point when the second-order derivative of Eq (4.2) becomes ≤ 0.

Proof. The second-order derivative with respect to Q_s of Eq (2) provides

$$E''[\Pi^S] = -[\{p_s(1 - r_s) + v_s\}f(Q_0) - \{p_s + v_s + s_s\}f(Q_s)] < 0. \tag{4.5}$$

It implies that the nature of the expected profit function $E[\Pi^S]$ is concave. Thus, the optimum value of order quantity Q_s is obtained from the first-order derivative of Eq (4.2) under the necessary condition of $E''[\Pi^S]$, which is expressed by Eq (4.4).

4.2. Dual-channel environment with barter exchange

Due to advances in society and progressive digital communications, a large number of consumers have changed their purchasing preference from offline to online. However, a few consumers prefer a retail platform to examine their products offline before purchasing. To capture an enormous consumer base in the digital trading sectors, retail managers keep both retail and online approaches as a form of commercialism in the retail industry. In this context, trading is accelerated with digital twin technology in two scenarios: Retail and e-tail platforms under a digital dual-channel environment in retailing. Indeed, channel conflict between the supply chain members minimizes the potential effect on the expected total profit of the system. Thus, equivalent coordination among the components of the supply chain framework assists in achieving the retail benefit under a barter system.

4.2.1. Model 2. Dual-retail channel

In the retailing context, the market demand can be expressed in the following manner to determine the pricing and cross-pricing of the product under a digital dual-channel environment [11]:

$$D_d = z + h_d a - c_d p_d + b_d(p_e - p_d) + g_d(E_0 - E).$$

Random and base market demands are formulated in the first and second terms, respectively. Similarly, the price and cross-price sensitivity demands of dual-channel retailing are expressed by the third and fourth expressions, respectively. The final term represents carbon preferences. Thus, the profit functions of retailers can be obtained in the dual-channel retail environment under the barter condition as follows.

- When the market demand D_d is greater than the retailer's ordered quantity Q_d, then a shortage arises. In these circumstances, the non-participation policy of the retailer in the barter platform allows them to purchase the required quantity of product from the supply chain market to retail. Hence, the retail profit is structured in the following manner:

$$\Pi^D(Q_d) = (p_d - w_d)Q_d - s_d(D_d - Q_d) - p_d Q_0 + l_d Q_d.$$

- At the moment where the ordered amount Q_d is higher than the demand D_d, the retail manager holds a certain quantity of products. The holding products are exchanged with the bartered products Q_0 via the barter exchange platform. If $D_d < Q_d$, then a r_d commission is paid over the retail price of the held materials on the barter platform and rest of the bartered product $(Q_0 - (Q_d - D_d))$ is purchased separately. Thus, the retailer's profit becomes

$$\Pi^D(Q_d) = p_d D_d - r_d p_d(Q_d - D_d) - p_d\{Q_0 - (Q_d - D_d)\} - w_d Q_d + l_d Q_d.$$

- Finally, when the $Q_d > Q_0 + D_d$ scenario occurs, i.e., the bartered quantity is not more than the ordered quantity, then all bartered quantities are purchased with a commission by exchanging the holding products via the barter platform. The remaining products are held at a holding cost v_d. Then the profit in retailing becomes

$$\Pi^D(Q_d) = p_d D_d - r_d p_d Q_0 - v_d\{(Q_d - Q_0) - D_d\} - w_d Q_d + l_d Q_d.$$

Thus, the retail profits in the digital dual-channel framework are summarized in the following manner:

$$\Pi^D(Q_d) = \begin{cases} (p_d - w_d)Q_d - s_d(D_d - Q_d) - p_d Q_0 + l_d Q_d; & Q_d \le D_d, \\ p_d D_d - r_d p_d(Q_d - D_d) - p_d\{Q_0 - (Q_d - D_d)\} - w_d Q_d + l_d Q_d; & D_d < Q_d \le Q_0 + D_d, \\ p_d D_d - r_d p_d Q_0 - v_d\{(Q_d - Q_0) - D_d\} - w_d Q_d + l_d Q_d; & Q_d > Q_0 + D_d. \end{cases} \tag{4.6}$$

Equation (4.6) indicates the corresponding profit functions in the digital dual-retail channel under barter conditions. In this context, considering the expected value of Π^D provides the expected profit of the retail manager.

$$\begin{aligned} E[\Pi^D] &= [(p_d - w_d + s_d + l_d)Q_d - p_d Q_0 - s_d\{h_d a - c_d p_d + b_d(p_e - p_d) + g_d(E_0 - E)\}] \int_{Q_d}^{\infty} f(z)dz - s_d \int_{Q_d}^{\infty} z f(z)dz \\ &-[(r_d p_d - p_d + w_d - l_d)Q_d + p_d Q_0 + r_d p_d\{h_d a - c_d p_d + b_d(p_e - p_d) + g_d(E_0 - E)\}] \int_{Q_d - Q_0}^{Q_d} f(z)dz + r_d p_d \int_{Q_d - Q_0}^{Q_d} z f(z)dz \\ &-[(v_d + w_d - l_d)Q_d + (r_d p_d - v_d)Q_0 + (p_d + v_d)\{h_d a - c_d p_d + b_d(p_e - p_d) + g_d(E_0 - E)\}] \int_0^{Q_d - Q_0} f(z)dz \\ &+(p_d + v_d) \int_0^{Q_d - Q_0} z f(z)dz \end{aligned} \tag{4.7}$$

Take the first-order derivative of Eq (4.7) with respect to Q_d, which is formulated as

$$E'[\Pi^D] = (s_d + r_d p_d)F[Q_d] + (p_d + v_d - r_d p_d)F[Q_d - Q_0] - (p_d - w_d + s_d + l_d) - (v_d + w_d - l_d).$$

Similarly, the first-order derivative of Eq (4.7) with respect to Q_d is formulated as

$$E''[\Pi^D] = (s_d + r_d p_d)f(Q_d) + (p_d + v_d - r_d p_d)f(Q_d - Q_0).$$

Theorem 3. The optimality test of the order quantity Q_d^* of the retailer is satisfied by the first-order derivative of Eq (4.7).

$$(s_d + r_d p_d)F[Q_d^*] + (p_d + v_d - r_d p_d)F[Q_d^* - Q_0] = (p_d - w_d + s_d + l_d) + (v_d + w_d - l_d) \tag{4.8}$$

Proof. From Theorem 3, the optimal order quantity Q_d^* can be derived by following the uniform distribution of market demand $D_d \sim U[A, B]$, which is as follows:

$$Q_d^* = \frac{[p_d - w_d + s_d + l_d]B + [v_d + w_d - l_d]A}{(p_d + v_d + s_d)} + \frac{\{p_d(1 - r_d) + v_d\}}{(p_d + v_d + s_d)} Q_0. \tag{4.9}$$

Equation (4.9) is positive under the condition of $1 - r_d > 0$. However, the expression is a unique and closed-form of Eq (4.7). Hence, the theorem is proved.

Theorem 4. The expected profit of the retailer reaches a global maximal point when the second-order derivative of Eq (4.7) becomes ≤ 0.

Proof. The second-order derivative with respect to Q_d of Eq (4.7) provides

$$E''[\Pi^D] = -[\{p_d(1 - r_d) + v_d\}f(Q_0) - \{p_d + v_d + s_d\}f(Q_d)] < 0. \tag{4.10}$$

This implies that the nature of the expected profit function $E[\Pi^D]$ is concave. Thus, the optimum value of the order quantity Q_d is obtained from the first-order derivative of Eq (4.7) under the necessary condition of $E''[\Pi^D]$, which is expressed by Eq (4.9).

4.2.2. Model 3. Dual online channel

Despite the digital twin mechanism, the dual-channel retailer uses the equilibrium strategy for transforming the retail framework to the digital platform under the supply chain environment. The retailer operates dual-channel businesses to increase e-retailing and switch consumers' conventional purchasing behavior from retail to a digital network [108]. In the digital retailing context, the following market demand (D_e) determines the pricing and cross-pricing of the product under an e-tail channel environment with dual-channel retailing [41]:

$$D_d = z + (1 - h_e)a - c_e p_e + b_e(p_d - p_e) + g_e(E_0 - E).$$

Random and base market demands are defined in the first and second terms. Similarly, the price and cross-price sensitivity demands of dual-channel retailing are expressed by the third and fourth terms, respectively. The final term represents carbon preferences. Thus, the retail profits can be obtained for the e-tail channel under the barter condition as follows.

- When the market demand D_e is greater than the retailer's ordered quantity Q_e, then a shortage arises. In these circumstances, the non-participation policy of the retailer in the barter platform can purchase the required quantity of product from the supply chain market to retail. Hence, the formation of the profit function is as follows:

$$\Pi^E(Q_e) = (p_e - w_e)Q_e - s_e(D_e - Q_e) - p_e Q_0 + l_e Q_e.$$

- In the scenario where the ordered amount Q_e is higher than the demand D_e, the retail manager holds a certain quantity of products. The holding products are exchanged with the bartered products Q_0 via the barter exchange platform. If $D_e < Q_e$, then r_e is the commission paid over the retail price of the held items in the barter platform and rest of the bartered product $(Q_0 - (Q_e - D_e))$ is purchased separately. Thus, the retailer's profit becomes

$$\Pi^E(Q_e) = p_e D_e - r_e p_e(Q_e - D_e) - p_e\{Q_0 - (Q_e - D_e)\} - w_e Q_e + l_e Q_e.$$

- Finally, when the $Q_e > Q_0 + D_e$ scenario occurs, i.e., bartered quantity is not more than the ordered quantity, all bartered quantities are purchased with a commission by exchanging the holding products via the barter platform. The remaining products are held at a holding cost v_e. Then the retail profit function becomes

$$\Pi^E(Q_e) = p_e D_e - r_e p_e Q_0 - v_e\{(Q_e - Q_0) - D_e\} - w_e Q_e + l_e Q_e.$$

Thus, the expected retail profit functions in the digital dual-e-tail channel framework can be expressed as follows:

$$\Pi^E(Q_e) = \begin{cases} (p_e - w_e)Q_e - s_e(D_e - Q_e) - p_e Q_0 + l_e Q_e; & Q_e \le D_e, \\ p_e D_e - r_e p_e(Q_e - D_e) - p_e\{Q_0 - (Q_e - D_e)\} - w_e Q_e + l_e Q_e; & D_e < Q_e \le Q_0 + D_e, \\ p_e D_e - r_e p_e Q_0 - v_e\{(Q_e - Q_0) - D_e\} - w_e Q_e + l_e Q_e; & Q_e > Q_0 + D_e. \end{cases} \tag{4.11}$$

Equation (4.11) provides the corresponding profit functions of the e-tail channel under the digital dual-channel framework with a barter application. In this context, considering the expected value of Π^E provides the expected profit of the retail manager.

$$E[\Pi^E] = [(p_e - w_e + s_e + l_e)Q_e - p_eQ_0 - s_e\{(1-h_e)a - c_ep_e + b_e(p_d - p_e) + g_e(E_0 - E)\}]\int_{Q_e}^{\infty} f(z)dz - s_e\int_{Q_e}^{\infty} zf(z)dz$$
$$-[(r_ep_e - p_e + w_e - l_e)Q_e + p_eQ_0 + r_ep_e\{(1-h_e)a - c_ep_e + b_e(p_d - p_e) + g_e(E_0 - E)\}]\int_{Q_e-Q_0}^{Q_e} f(z)dz + r_ep_e\int_{Q_e-Q_0}^{Q_e} zf(z)dz$$
$$-[(v_e + w_e - l_e)Q_e + (r_ep_e - v_e)Q_0 + (p_e + v_e)\{(1-h_e)a - c_ep_e + b_e(p_d - p_e) + g_e(E_0 - E)\}]\int_0^{Q_e-Q_0} f(z)dz$$
$$+(p_e + v_e)\int_0^{Q_e-Q_0} zf(z)dz \tag{4.12}$$

Take the first-order derivative of Eq (4.12) with respect to Q_e, which is formulated as follows:

$$E'[\Pi^E] = (s_e + r_ep_e)F[Q_e] + (p_e + v_e - r_ep_e)F[Q_e - Q_0] - (p_e - w_e + s_e + l_e) - (v_e + w_e - l_e).$$

Similarly, the first-order derivative of Eq (4.12) with respect to Q_e is formulated as:

$$E''[\Pi^E] = (s_e + r_ep_e)f(Q_e) + (p_e + v_e - r_ep_e)f(Q_e - Q_0).$$

Theorem 5. The optimality test of the order quantity Q_e^* of the retailer is satisfied by the first-order derivative of Eq (4.12).

$$(s_e + r_ep_e)F[Q_e^*] + (p_e + v_e - r_ep_e)F[Q_e^* - Q_0] = (p_e - w_e + s_e + l_e) + (v_e + w_e - l_e). \tag{4.13}$$

Proof. From Theorem 5, the optimal order quantity Q_e^* can be derived by the uniform distribution of market demand $D_e \sim U[A, B]$, which is expressed as

$$Q_e^* = \frac{[p_e - w_e + s_e + l_e]B + [v_e + w_e - l_e]A}{(p_e + v_e + s_e)} + \frac{\{p_e(1-r_e) + v_e\}}{(p_e + v_e + s_e)}Q_0. \tag{4.14}$$

Equation (4.14) is positive under the condition of $1 - r_e > 0$. However, the expression is a unique and closed-form of Eq (4.12). Hence, the theorem is proved.

Theorem 6. The expected profit of the retailer reaches a global maximal point when the second order derivative of Eq (4.12) becomes ≤ 0.

Proof. The second-order derivative with respect to Q_e of Eq (4.12) provides

$$E''[\Pi^E] = -[\{p_e(1-r_e) + v_e\}f(Q_0) - \{p_e + v_e + s_e\}f(Q_e)] < 0. \tag{4.15}$$

This implies that the nature of the expected profit function $E[\Pi^E]$ is concave. Thus, the optimum value of order quantity Q_e is determined by the first order derivative of Eq (4.12) under the necessary condition of $E''[\Pi^E]$, which is expressed by Eq (4.14).

5. Experimental study

The mathematical problem explores experimental analyses of three consecutive models by using the LINGO optimization package 21. It uses a strategic solution to assess an analytic approach and offers the input parameters to initiate the process. The analysis provides a scientific observation regarding individual investigations compatible with extant studies by keeping the proposed problem's assumptions [49].

5.1. *Procedural approach*

The proposed framework explores potential trading channel configurations with maximum benefit. It investigates the bartering supply chain for the single and dual-channel environment under three different schemas. However, the following approaches explore the quantitative evaluations through online channels to achieve profitable solutions under the application of the barter strategy. In this context, Algorithm I is utilized for Model 1 and Algorithm II for Models 2 and 3.

5.1.1. Algorithm I for Model 1

The approach is used to seek for the optimum solutions of the Model 1.

- ***Step 1.*** Assign all the input parameters with their respective values for Model 1 to initiate the mathematical problem.
- ***Step 2.*** Evaluate the closed-formation of Model 1 from Eq (4.2).
- ***Step 3.*** Determine the optimal values for Model 1 from Eq (4.4).
- ***Step 4.*** Estimate the maximum sensitivity in the expected profit function of Model 1 by changing the initial values of the parameters.
- ***Step 5.*** Examine the maximality condition of Eq (4.5) globally by applying Theorem 1. If all the determinations are satisfied, then move to the next step; otherwise returns to the initial stage with revised valuations of the input parameters.
- ***Step 6.*** Terminate the program.

5.1.2. Algorithm II for Models 2 and 3

The following algorithmic process is applied to seek for the optimal solutions of Model 2 and 3.

- ***Step 1.*** Assign all the input parameters with their respective values for Models 2 and 3 to initiate the mathematical problems.
- ***Step 2.*** Evaluate the closed-formation of Model 2 from Eq (4.7) and Model 3 from Eq (4.12).
- ***Step 3.*** Determine the optimum values from Eqs (4.9) and (4.14) for Models 2 and 3, respectively.
- ***Step 4.*** Estimate the maximum sensitivity in the profit functions of two consecutive models, Models 2 and 3, by changing the initial values of the parameters.
- ***Step 5.*** Investigate the maximality condition of Eqs (4.10) and (4.15) globally by applying Theorems 4 and 6, for Models 2 and 3, respectively. If all the determinations are fulfilled, then move to the next step, or else return to the initial stage with revised valuations of input parameters.
- ***Step 6.*** Terminate the program.

5.2. *Result analysis*

The section explains the optimal solutions of three models, along with their profits from the retail barter system, based on numerical experiments. In this context, the random market demand z follows a uniform distribution within the given interval $[A, B]$ for Models 1, 2, and 3. However, the analysis of the results is discussed elaborately in detail below.

5.2.1. Model 1

Example I. The parameters are initialized to obtain the optimal solutions with the following considerations: A=30, B=100, the price sensitivity parameter c_s=0.25, the low carbon preferences sensitivity parameter g_s=0.40, the compatibility of product a=0.1, the sensitivity parameter of consumers' preference for platform transition h_s=0.30. In addition, the barter quantity of product Q_0=85 units, the holding cost v_s=\$17, the wholesale price w_s=\$15, the shortage cost s_s=\$27, the production cost l_s=\$30, the commission r_s=0.27, the initial carbon emissions E_0=90 unit, the retail price p_s=\$60, and the reduced emissions E=45 units. The optimum value of ordered amount (Q_s^*)=148.34 units and the expected total profit ($E[\Pi^S]$)=\$23,718.90. The market demand (D_s) becomes 3.07 units, i.e., optimal ordered amount exceeds the retail market demand ($Q_s^* > D_s$). Thus, the retail manager accepts the bartering policy to receive the bartered amount of products by swapping the excess products. Consequently, $Q_s^* > Q_0 + D_s$, and the holding cost requires the excess products to be held even after exchanging the barter quantity of product.

5.2.2. Model 2

Example II. The parameters are initialized to obtain the optimal solutions with the following considerations: A=30, B=110, the price sensitivity parameter c_d=0.27, the cross-price sensitivity parameter b_d=0.22, the low carbon preferences sensitivity parameter g_d=0.45, the compatibility of product a=0.1, the sensitivity parameter of consumers' preference for platform transition h_d=0.33, the barter quantity of product Q_0=90 units, the holding cost v_d=\$20, the wholesale price w_d=\$17, the shortage cost s_d=\$30, the production cost l_d=\$35, the commission r_d=0.28, the initial carbon emissions E_0=90 units, the retail price p_d=\$55, and the reduced emissions E=40 units. The optimal value of order quantity demand (Q_d^*)=159.56 units and the expected total profit ($E[\Pi^D]$)=\$27,807.14. The market demand (D_d) becomes 10 units, which implies that the market demand is less than the optimal order quantity. Thus, the retail manager applies the barter mechanism by receiving a bartered quantity of product in exchange for the excess products. Consequently, $Q_d^* > Q_0 + D_d$, and the holding cost requires the excess products to be held even after the barter exchange retail strategy. Indeed, the carbon emissions have reduced by 11.11% in Example II and increased in profit by 14.7% compared with Model 1 for retailing.

5.2.3. Model 3

Example III. The parameters are initialized to obtain the optimal solutions with the following considerations: A=15, B=190, the price sensitivity parameter c_e=0.30, the cross-price sensitivity parameter b_e=0.25, the low carbon preferences sensitivity parameter g_e=0.48, the compatibility of product a=0.1, the sensitivity parameter of consumers' behavior for platform transition h_e=0.35, the barter quantity of product Q_0=95 units, the holding cost v_e=\$25, the wholesale price w_e=\$20, the shortage cost s_e=\$38, the production cost l_e=\$36, the commission r_e=0.30, the initial carbon emissions E_0=90 units, the retail price p_e=\$40, and the reduced emissions E=30 unit. The optimal value of order quantity demand (Q_e^*)=223.59 units and the expected total profit ($E[\Pi^E]$)=\$41,228.08. The market demand (D_e) becomes 19 unit, which implies that the market demand is less than the optimal order quantity. Thus, the retail manager recommends the barter mechanism for exchanging the excess products. Consequently, $Q_e^* > Q_0 + D_e$, and the holding cost requires the excess products to be held even after the retail barter strategy. Noticeably, the carbon emissions have reduced by 33.33% and 25% in Example III compared with Models 1 and 2, respectively. Similarly, the retail profit has increased by 32.55% and 42.47% compared with Model 2 and Model 1, respectively.

5.3. Discussion

Figure 2 depicts an optimal profit of Models 1, 2, and 3. However, a descriptive illustration is provided below.

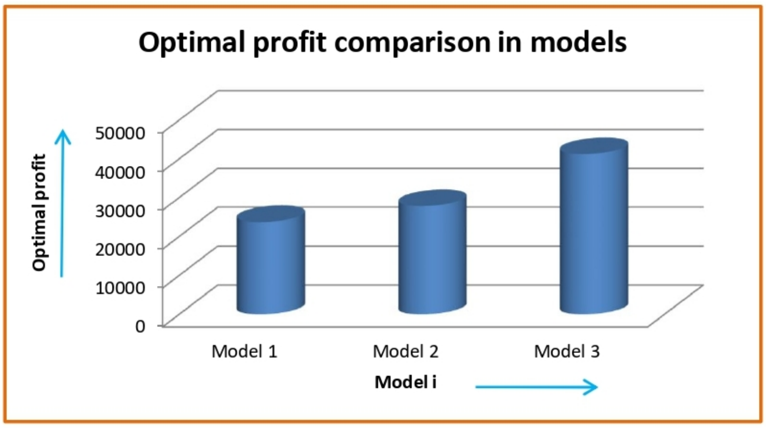


Figure 2. Solutions of optimal profits: Comparison of Models 1, 2, and 3.

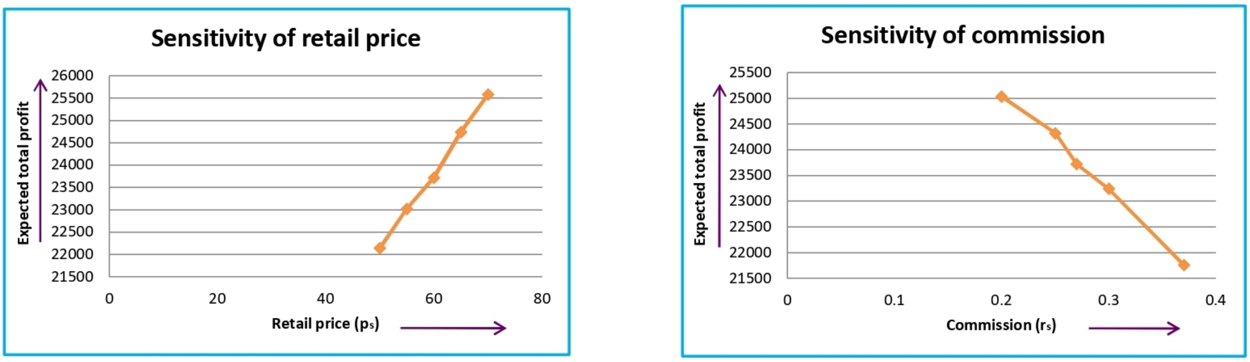
- It shows that the optimal profit of the third model is comparatively higher than that of the other models. Regarding demand, the function formulates the combined form of retail and e-retail prices. The production is completely dependent on the market demand. It is observed that the order quantity has increased by 28.6% and 33.6% in Model 3 compared with Models 2 and 1, respectively, which is relatively high. Thus, the optimal profit increases in Model 3 compared with Models 1 and 2.
- Besides, the optimal profit increases by 14.7% in Model 2 from compared with Model 1 and increases by 32.55% in Model 3 from compared with Model 2. Similarly, the profit increases by 42.47% in Model 3 compared with Model 1, which suggests that the retail manager achieves more profit in the dual-online channel comparatively. The research suggests incorporating a market-sensing strategy into the business model to maintain retailing agility. This illustrates the advantages of the commercial environment to capitalize on retailing abilities.
- Carbon emissions are reduced by 11.11% and 33.33% in Model 2 and Model 3 compared with Model 1. Similarly, emissions are reduced by 25% in Model 3 compared with Model 2. It implies that a more sustainable improvement is achieved by Model 3 by minimizing emissions. The findings reveal an empirical ramification that assists the policymaker in deriving an effective retail plan.
- Due to the increasing utilization of dual-purpose barter applications, creative uniformity addresses the long-term presence in the retail framework. The study reveals that a retailer can achieve more earnings in the digital dual-channel by applying a digital twin retail mechanism, and the barter strategy is more appropriate in Model 3 due to the expensive financing. It provides an alternative tool where the demand has decreased with increasing retail competition.

- Indeed, the experimental solutions evaluate the choice of retailers to transform their plan from a single to a dual platform digitally because of the huge number of consumers who make their purchases via the e-tail channel, irrespective of the retail platform. In addition, the retail demand is expressed in linear form consisting of the self-price, the cross price, and the reduction of carbon. The cross-price sensitivity parameter of the products evaluates the channel conflict according to the price distinction among distinct platforms. Thus, the trading behavior of the retailer has upgraded according to the market demand to the digital retail framework.

5.4. Sensitivity analysis and discussion

The sensitivity in the decision-making stage is often assessed using scientific methodology under uncertain situations. It examines the degree of retail satisfaction when changing the parameters used in the estimation. In this context, Figures 3–5 provide a sensitivity evaluation of the models to achieve a profit under distinct scenarios for retail management.

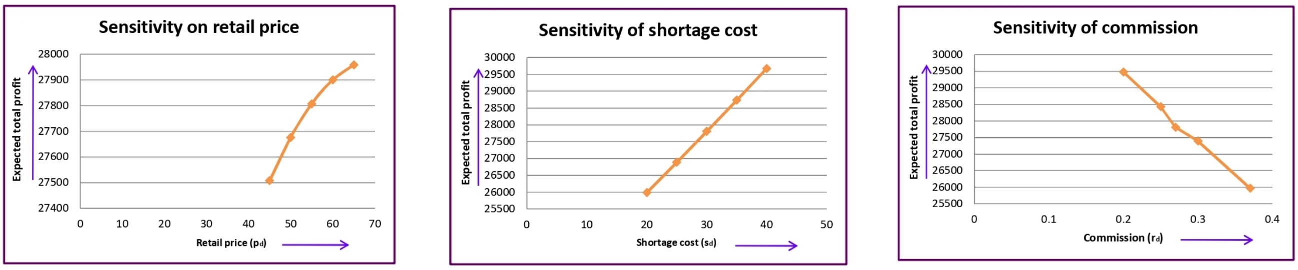
- Model 1 is sensitive to the retail price. A 7.3% increment in total profit increases the retail price by 14% and a 6.6% reduction in total profit reduces the retail price by 17% in Model 1 (Figure 3(a)). It emphasizes the relative importance of coordinated the pricing strategy with the product's brand image. The study shows that retail competitiveness depends on the elasticity of pricing. On the other hand, commission is more sensitive in Model 1 , as a 27% increase in the commission decreases the total profit by 8.3%, and a 26% reduction in commission enhances the total profit by 5.25% in Model 1 (Figure 3(b)). Thus, the research shows that all pricing parameters that directly affect the expected total profit (Π^S) in the single online channel of the supply chain framework.



(a) **(b)**

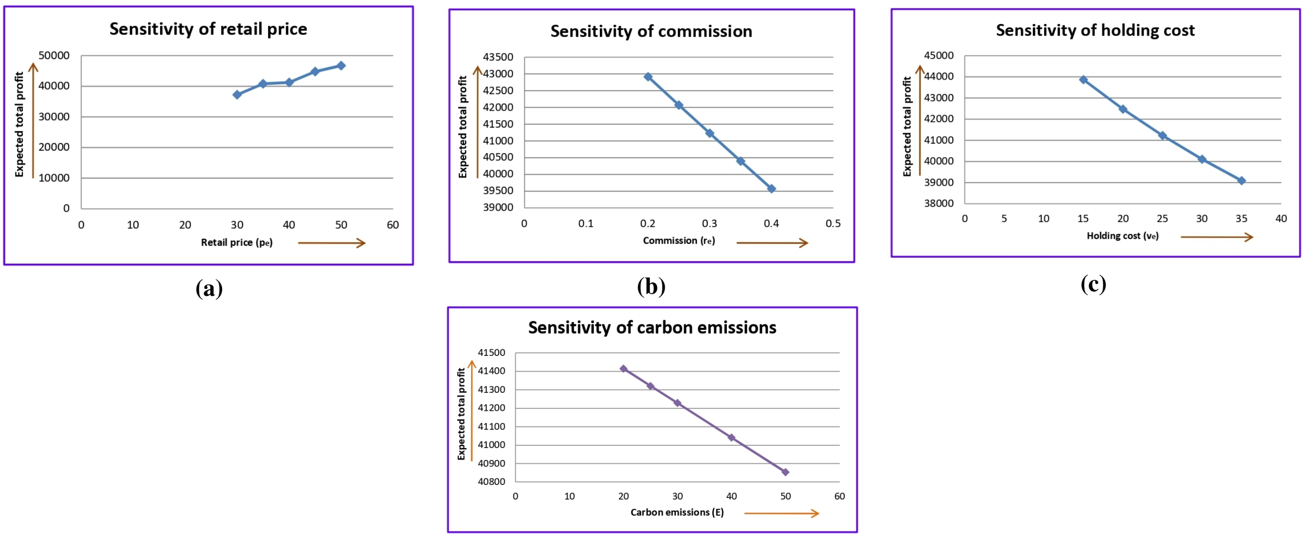
Figure 3. Sensitivity analysis on the single online platform of Model 1: **(a)** Evaluation of expected total profit with respect to the retail price (p_s); **(b)** evaluation of expected total profit with respect to commission (r_s).

- In Model 2, a 15% enhancement of the retail price the total profit by 0.5% and an 18% reduction of retail price reduces the total profit by 1% (Figure 4(a)). The retail price is a more sensitive parameter in the retail environment. Thus, the impact intensifies the retail profits and encourages consumers to purchase. Similarly, a 25% increment of the shortage cost increases the total profit of Model 2 by 6.3% (Figure 4(b)) and a 6.5% reduction of total profit reduces the shortage cost by 33%. Therefore, the shortage cost is directly proportional to the expected revenue, which implies that if product deficiency does not occur, then the retail manager can disregard the shortage-related costs. Besides these, due to the 4% decrease in commission, the total profit is 6.6%, and a 26% reduction in commission enhances the total profit by 5.7% in Model 2 (Figure 4(c)). Thus, the changes in all pricing parameters directly affect the estimated profit (Π^D) in the dual-retail channel.



(a) (b) (c)

Figure 4. Sensitivity analysis on the dual-retail platform of Model 2: **(a)** Evaluation of expected total profit with respect to the retail price (p_d); **(b)** evaluation of expected total profit with respect to shortage cost (s_d); **(c)** evaluation of expected total profit with respect to the commission (r_d).



(a) (b) (c)

(d)

Figure 5. Sensitivity analysis on the dual-online platform of Model 3: **(a)** Evaluation of expected total profit with respect to the retail price (p_e); **(b)** evaluation of expected total profit with respect to the commission (r_e); **(c)** evaluation of expected total profit with respect to the holding cost (v_e); **(d)** evaluation of expected total profit with respect to the emissions (E).

- Similarly, in Model 3, a 20% increment in the retail price increases the total profit by 11.8% and a 25% decrease in the retail price reduces the total profit by 9.8% (Figure 5(a)). On the other hand, the total profit increases by 6% when reducing the holding cost by 40% and the profit reduces by 5.2% by enhancing the holding cost by 28% (Figure 5(c)). Noticeably, commission is more

sensitive in Model 3 due to the enhancement of commission, 33% decreases the total profit by 4%, and a 10% reduction in commission enhances the total profit by 4% in Model 3 (Figure 5(b)). Thus, the changes in all pricing parameters directly affect the expected total profit (Π^E) in the dual-e-tail channel.
- In addition, the total profit increases by reducing the carbon emissions and profit reduces by enhancing the emissions (Figure 5(d)). As the goal of the trading is to improve the profit in retailing, the retailer does not emphasize increasing carbon emissions in the proposed framework.

In summary, the digital twin methodology changes the retail and supply chain landscapes by generating digital replicas of physical systems in the realm. The scientific analysis may help the retail manager to find the best opportunity for retailing the product. It identifies the pricing drivers to generate a profitable retail environment. Indeed, the scalability and flexibility of the solutions evolve with the trading platform economically. The approach significantly reduces the time and number of late orders in delivery. It indicates a proactive decision-making environment that maximizes the total profit of the retail system under pricing and cross-pricing elasticities.

6. Managerial implications on e-bartering

The barter mechanism is a unified avenue in the digital landscape. It integrates an authenticated e-commerce platform with a barter exchange policy, the prime sustainable challenges of accessibility, and communal integration. A community-based digital resource can be used in evaluating the single versus dual shipment strategy, which tends to provide flexible retailing under sustainable effort. But the crucial problem is how, in dual digital intervention in the green supply chain, each participant can achieve their individual profit. The critical issue is pricing decisions between production and retailing on a dual digital platform. The optimal decisions of the dual digital platform between the retailer and manufacturer promote cooperation among investors and facilitate their interest in improving overall digital activity. Enhancing the supply chain's operational system, a revised incentivization approach encourages manufacturers to lower wholesale prices and even encourages retail managers to contribute to lower expenses in minimizing carbon emissions. This motivational investment spreads the financial accountability towards sustainable development. Sustainable low-carbon capitalism intensifies the issue of double-profit marginalization by boosting operational efficiency in the dual-channel green supply chain framework. Indeed, the strategic mechanism determines the comparative advantages of profitable low-carbon incentivized barter practices that influence the adoption of more sustainable products by a dual digital retail platform. The advanced methodology escalates the dual-channel dynamism in implementing delivery planning policy and explores an alternative application for the green production manager, recommending commutable products through an e-commerce platform that enriches product financing and retail development.

In these circumstances, retail firms utilize the crisis conversation to understand consumers' concerns more accurately and use current methodology in shifting consumer sentiment. It grasps consumer's requirements and assists the retailer in the market to sell with comprehensive information. Due to the rapid development of the retail landscape, consumer preference encompasses dynamic coordination within supply chain members under multi-component dependent demand.

- Retail digitalization offers an opportunity for commercial paradigms to achieve enhanced e-commerce efficiency by promising sustainable development by combining human and information methodologies [96]. A comprehensive depiction of the system understands the activity parameters to increase the virtual value-added performance by the digital twin policy. It is a digital counterpart based on a simulation that deals with optimization in the retail system. The methodology creates a novel avenue to coordinate the platform-based physical and virtual world by meeting consumer requirements with purchase expectations.

- Retailing policy improves dual-channel retailing with a sustainable dimension. It offers price and quality-dependent demand, which enhances consumer service-based production. The change from retail to e-tail platforms, rather than establishing pure e-bartering retailing is a practical application for business-to-business (B2B) e-commerce platform [31].
- Diversity of products or services reduces the complexities in commercial markets. It explores an independent marketplace to collaborate with sophisticated and comprehensive retailing. In reality, generating an e-bartering system with a primary transaction procedure can solve many complex problems. It establishes an online small-sized or medium-sized barter market aiming at the marketing objective of attracting ample consumers digitally.
- The brand quality and power framework demonstrates the incentive and profitability of each retail enterprise. It increases bundling attention to optimize the incentive rate and customize the retailing power towards trade-ins. It prioritizes and observes the market during emerging issues.
- Finally, for retaining the loyalty of prime consumers [24], the market offers other services in the region of commercialism. The e-community provides value-added services for consumers. It enhances the ability to increase interactivity. The analysis lens is the theoretical and practical assessment framework. Indeed, the research guides how retailing can better counter the crisis in different dimensions, such as reliability, physical aspects, communal interactions, etc.

Indeed, socio-responsible and eco-conscious consumers have expressed their intention to purchase low-carbon products, which is a significant component of the supply chain market. Global enterprises are increasing the use of eco-friendly products to protect the environment by incorporating green elements into their supply chain mechanisms. As a result, prioritizing low-carbon practices can achieve a larger market to meet consumer expectations. In this regard, designing a low-carbon preference supply chain model is crucial for sustainable development. The manager proactively invests in reduction plans when several supply chain industries compete horizontally. The proposed research contributes to practical insights in formulating an effective carbon emissions policy that paves the way for more sustainable and informed retail business practices in a competitive environment. It indicates greater retail benefits and explores cost-effective management by achieving sustainability. The retail framework addresses stochastic consumer demand to strengthen collaboration within supply chain management. It leads to enhance performance for carbon neutrality by yielding higher profit in the system. Indeed, the vital task of policy-driven investment is promoting environmentally friendly practices. Policymakers can achieve mutual profit from financial aspects by investing in green decisions and pricing strategies within the supply chain system. Thus, the research reveals that effective cooperation and competition can expand retail demand, enhance the degree of carbon emissions reduction, and maximize the profit of the supply chain industry.

However, establishing tighter collaboration with manufacturers and developing flexible acquisition mechanisms can combat false information. It provides better knowledge by communicating with consumers online and offline. It strengthens in-store management and educates the participants via safety awareness training. The essence of retail service is made more active by communicating and understanding with consumers. Thus, the study is widely applicable in the management science domain, preferably for seeking an experimental solution to the mathematical or simulation models in the supply chain network, in which logistics is a challenging factor. However, to succeed in the commercial environment, barter managers should take the necessary actions towards the prospective marketing plan for the foreseeable future.

6.1. Theoretical implications

Our findings create conceptual inferences to suggest a sustainable effort to give valuable insights for any production-oriented retail enterprises. Moreover, it signifies that the financial benefits influence sustainable consumption through digital e-commerce platforms [26].

- The study offers a novel theoretical approach by constructing the relationship between sustainable consumption and economic interests. It reduces the negative panic during e-purchasing choices in risky and uncertain scenarios [101]. Migrating e-resources can use e-commerce platforms to serve consumers by facilitating interactive, valuable communication between the manufacturer and retailer. However, the synergy between integrity and reliability maintains the consumer-retailer relationship.
- Research needs to investigate the retail opportunities that digital transactions can secure for commercial administration to reinforce the expansion of methodological implementation with retailers and loyal organizations towards a strategic marketing intention. They construct reliable platforms to distribute the products and services by promoting e-transactions with privacy protection towards consumer savings. The ability to evaluate the experimental solutions of the mathematical model is an empirical implementation for the barter merchandising industry.
- The value of the barter currency is one of the prime elements in establishing an attractive commercial exchange in trades [80]. In general, the trade pool is associated with the barter currency, and its exposure has high value only if the pool has an abundant heterogeneity of products and services. The problem becomes complex when the number of trading members and the diversification of the products are sufficient in the trading pool. In these circumstances, enough evidence emerges for the consumer-focused management issues that ensure the components generate revenue for commercial exchange in the barter system.

6.2. Empirical implications

The study promotes several experimental impressions and bridges the gap between scientific and managerial outlooks. It offers three major key points for manufacturers, retailers, policymakers, and marketers, which are discussed briefly.

- Ecological awareness stimulates consumers' behavior and trust to translate to green apparel purchasing behavior. It suggests a pro-green attitude aimed at enhancing the eco-knowledge for societal benefit. The policymakers reduce the attitude-behavior gap by increasing the environmental understanding of consumers. Sharing public messages from the government upskills the retailers by enhancing their environmental concern [107].
- An agenda of digital transition determines the appropriate corrective action to support optimization in the supply chain framework. Despite the emerging technologies, manufacturers develop several belief-building measures to purchase their products. It reveals a positive

influence on the purchasing attitude of consumers. Endorsement from the experts confirms that the enterprise follows only an eco-friendly production process.
- Retailers establish purchasing efforts towards environmental accounting by providing several biodegradable initiatives. Specifically, the effective conversation mechanism can enhance consumers' perception and ensure the preservation of green apparel for future generations. The tagging satisfaction based on information about products with green characteristics highlights a significant prediction. It establishes the superiority of green apparel over traditional apparel and promotes increased purchase satisfaction [39].
- Due to global socio-ecological challenges, barter exchange has been referred to as an alternative financial practice that integrates environmental, communal, and economic aspects with nature-human interactions. The study strengthens communal well-being through sustainability and socio-ecological resilience. Conceptually, barter trade is considered to be a non-financial transaction that combats capitalism. The ethical advantage of social justice, interdependence, and ecological respect aims to identify solid strategies for potential communal impact, especially in minimizing inequalities, promoting consumption, and protecting the ecological system. To strengthen the practice and maximize the positive implications, a barter agreement promotes a collaborative approach among all participants in retail management. It enhances retail image and reinforces economic and social connections. However, promoting multidisciplinary research develops social awareness within the community and highlights the cultural importance of sustainable and communal cohesion.

7. Conclusions with future extensions

The pricing mechanism influences external perceptions of consumer loyalty and highlights critical digital challenges in retailing [55]. It offers equilibrium outcomes from each trading format and assists e-retailers in better dynamic optimization to control the time lag in inventory. However, price promotion affects green investment due to pricing differentiation. Thus, the research has offered novel insights by mitigating restrictions in retailing and provided commercial opportunities for future perspectives.

- The conceptual model has revealed that consumers vary in their perceptions due to sustainable perspectives. It was examined with online and offline consumers, and the respondents viewed it from the commercial platforms digitally. Hence, the empirical model should be cross-validated in the subsequent study by collecting the information from diverse economies to make more comprehensive findings in the future.
- The intrinsic variables in industry, such as differences in economy and market, should be incorporated in the proposed study for future extension to gain better knowledge of consumers' purchasing attitudes. Another constraint of this study is the target association. As per the expert reviews, the sub-civilization with communal networking sites incorporates financial benefits [94]. Several complications can exacerbate the countable issues in the dual trading platform. In these circumstances, a time-series longitudinal foundation investigates the boundary relationship between financial benefits and sustainable consumption to enhance the strategic effectiveness.
- In the foreseeable research dimension, the study may emphasize risk management by using

stochastic optimization and ensure robustness by integrating a robust manufacturer-retailer stochastic optimization framework. Robust stochastic optimization maximizes profit functions by satisfying the consumers' demands. In the context of uncertain complex performance, design robustness interprets the scientific significance and industrial impact in an advanced retail system. It sheds light on more efficient and accurate demand distribution models to estimate the degree of uncertainty in the retail environment.

The research proposes the barter supply chain problem in a dual-channel environment by considering all effective sensitive parameters. In addition, the channel preference of the consumers and product compatibility may also illustrate the problem more logically. The framework enriches analytic and experimental applications with significant resource methodologies to obtain more retailing-related perceptions [7]. Profitable production with relatively lesser carbon emissions can explore joint impact towards social welfare and investigate the strategic selection during network complexities [17, 70]. Sustainable development extends the cost-benefit methodological enhancement and promotes ecological balance in the dynamic environment.

The above observations and the analytic computations can help to promote the brand reputation with a more sustainable process. The proposed problem has multiple decision-making applications in production and utilities. It addresses a significant implication of time-specific financial uncertainty. The study assumes that the demand function is linear in decision variables, but it can extend to the nonlinear demand pattern in future analysis [71]. The flexible production under risk management in the bartering system enhances the system's profit. In recent studies, retailers have always selected the direct channel under a supply chain traceable environment. Thus, one can investigate the problem regarding the competition by taking multiple retailers. The empirical analysis and numerous computations demonstrate that barter is one of the beneficial trading approaches in the case of a superior rate of productive supply and demand from commercial perspectives. It derives an explicit decision that leads to an interaction mathematically. It promotes the degree of retail price observability to extend a new avenue in retail opportunities to practitioners [12]. Hence, the proposed research encourages optimizing the product's order quantity and retailing price to influence the exchange strategy [85].

The nonlinear and complex dynamics play a vital role in industrial processes, where higher uncertainty and critical interactions can significantly impact production networks. Managing and understanding dynamism can develop adaptive, robust, and resilient industrialization. It highlights the convergence of machine learning, robust control, IoT, digital twin, and Industry 4.0 strategies for managing industrial challenges. Digital twin methodology integrates stochastic parameters to optimize e-communication in the retail industry. Production optimization, risk elimination, and real-time monitoring are strategic priorities for demonstrating marketing demands. Multi-objective evolutionary algorithms could be used; for example, the genetic algorithm uses financial modeling to identify optimal trade-offs in distinct criteria. Enhancing nonlinear control and retail intelligence contributes to the supply chain framework for emerging trends of information-driven predictive control and hybrid artificial intelligence-based mathematical models that address the difficult tasks posed by demand uncertainty and industrial complexity. The primary objective of the research is to identify future research directions for significant industrial adoption by highlighting cybersecurity and sustainability-related issues. The contribution addresses a holistic approach that integrates nonlinearity, cybersecurity, and artificial intelligence. The novel innovation articulates a systematic vision where global competency coexists harmoniously. The implementation of cybersecure and intelligent networks requires appropriate socioeconomic transitional programs from conceptual investigation to industrial applications. Stochastic-based demand function and cybersecurity-based nonlinearity highlight the relevance of synergic intelligent modeling, control theory, and artificial intelligence in addressing the challenges of an evolving smart retail industry. Thus, the convergence of nonlinear dynamics and advanced optimization extends a wider range of opportunities for research practitioners aiming to contribute to sustainable development.

Indeed, the digital twin-based supply chain demonstrates the superiority of retail planning to practitioners. The service pricing policy evolves e-retailers' personalization to promote consumer-centric promotional strategies, and affects category management and dependence theory directly. Varying with an increasing commission recommends a joint promotion to consumer attitudes towards retail profit. The outcomes illustrate the potential strategy for achieving higher retail performance [5]. A shortage of products can delay distribution in service, which may lead to substituting the findings of the consumer behavior-attitude assessment from retail to e-commerce. However, the supply chain processes are more reciprocal with sustainable development to adopt the emerging digital twin methodology [102]. The study should emphasize the diversification of cultures and internet communities to compare several digital perspectives that may provide more insightful commercialism. It might emphasize the consumer-specific economic approaches to estimate the consumers' demand by using deep neural network methodology, which may guide a better illustration of the purchasing attitude of consumers under e-tail consumption [21]. Furthermore, the research recommends considering certain external variables (e.g., price sensitivity, consumer motivation, brand in retailing.) that may provide a potential digital benefit to the perception-behavior gap of consumers.

Author contributions

S. Bhattacharya: Data collection, Methodology, Software, Writing-original draft, Writing-review & editing; M. Sarkar: Formal analysis, Writing-review & editing; B. Sarkar: Conceptualization, Funding acquisition, Methodology, Investigation, Software, Supervision, Validation, Resources, Project administration; L. E. Cárdenas-Barrón: Formal analysis, Investigation, Visualization; A. Gunasekaran: Methodology, Resources, Visualization, Writing-review & editing.

Use of Generative-AI tools declaration

The premium version of Grammarly software has been used for grammar check and necessary corrections.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that they could have appeared to influence the work reported in this research.

References

1. H. G. Abdelwahed, M. A. E. Abdelrahman, The novel stochastic solutions for the Maccari system driven by Wiener perturbations, *AIMS Mathematics*, **11** (2026), 5365–5378. https://doi.org/10.3934/math.2026221
2. M. Abdullahi, A. S. Halilu, M. A. Saleh, A. Z. Almaymuni, Accelerated double step-length method for solving monotone nonlinear equations with convex-constraint and application, *AIMS Mathematics*, **11** (2026), 10908–10935. https://doi.org/10.3934/math.2026448
3. K. Abuasbeh, S. Trabelsi, A unified fractional-stochastic energy balance model: Theoretical analysis, numerical implementation, and implications for sustainable energy systems, *AIMS Mathematics*, **11** (2026), 8546–8583. https://doi.org/10.3934/math.2026352
4. W. Ahmed, M. Moazzam, B. Sarkar, S. Ur Rehman, Synergic effect of reworking for imperfect quality items with the integration of multi-period delay-in-payment and partial backordering in global supply chains, *Engineering*, **7** (2021), 260–271. https://doi.org/10.1016/j.eng.2020.07.022
5. K. A. Alattas, Correction: Advancing artificial intelligence-enabled cybersecurity framework using ensemble deep representation learning for intelligent cybersecurity in cloud-edge-IoT environments, *AIMS Mathematics*, **10** (2025), 30637–30638. https://doi.org/10.3934/math.20251343
6. S. A. Alharbi, M. A. A. El-Hadidy, A multivariate discrete Wiener range distribution with truncation: Theory, reliability properties, and applications to constrained financial markets, *AIMS Mathematics*, **11** (2026), 3563–3593. https://doi.org/10.3934/math.2026146
7. M. Alhazmi, M. O. Sidi, Bilinear optimal control problem with minimum energy for a fourth-order reaction diffusion system, *AIMS Mathematics*, **11** (2026), 10274–10310. https://doi.org/10.3934/math.2026425
8. H. K. Alkahtani, M. Baihan, M. B. Rehman, R. Allafi, S. Almutairi, I. Zalah, et al., Mitigating DDoS attacks through ensemble learning models integrated with two-tier heuristic optimisation techniques for effective cyber defence, *AIMS Mathematics*, **10** (2025), 30068–30108. https://doi.org/10.3934/math.20251322
9. R. Allafi, An attention mechanism based recurrent neural network with dimensionality reduction model for cyber threat detection in IoT environment, *AIMS Mathematics*, **10** (2025), 11998–12031. https://doi.org/10.3934/math.2025544
10. K. A. C. Amankou, R. Guchhait, B. Sarkar, H. Dem, Product-specified dual-channel retail management with significant consumer service, *J. Retail. Consum. Serv.*, **79** (2024), 103788. https://doi.org/10.1016/j.jretconser.2024.103788
11. A. A. Almoneef, A. Hyder, M. A. Barakat, A. M. Aly, Stochastic solutions of the geophysical KdV equation: Numerical simulations and white noise impact, *AIMS Mathematics*, **10** (2025), 5859–5879. https://doi.org/10.3934/math.2025269
12. Y. Alnafisah, M. El-Shahed, Stochastic dynamics of an elk–wolf system with inter-regional movement, *AIMS Mathematics*, **11** (2026), 6400–6419. https://doi.org/10.3934/math.2026264
13. H. Althagafi, Dynamics of difference systems: A mathematical study with applications to neural systems, *AIMS Mathematics*, **10** (2025), 2869–2890. https://doi.org/10.3934/math.2025134

14. M. Arab, A. Zaman, A. U. K. Niazi, M. Balti, Impulsive fault-tolerant control for multi-agent systems with stochastic disturbances, *AIMS Mathematics*, **10** (2025), 7414–7429. https://doi.org/10.3934/math.2025340
15. E. Badakhshan, P. Ball, Applying digital twins for inventory and cash management in supply chains under physical and financial disruptions, *Int. J. Prod. Res.*, **61** (2023), 5094–5116. https://doi.org/10.1080/00207543.2022.2093682
16. T. Bai, G. Feng, M. Wu, S. X. Zhu, A newsvendor model with multiple reference points: Target-setting for aspirational newsvendors, *Eur. J. Oper. Res.*, **320** (2024), 655–669. https://doi.org/10.1016/j.ejor.2024.09.015
17. N. Bar, I. Sangal, B. Sarkar, Impact of smart energy on a three-echelon closed-loop supply chain management with reduction of carbon emissions, *Sci. Rep.*, **16** (2026), 2245. https://doi.org/10.1038/s41598-025-32009-7
18. A. Barrera, A. D. L. P. Cuao, J. J. Serrano-Pérez, F. Torres-Ruiz, Estimating the consumer price index using the lognormal diffusion process with exogenous factors: The Colombian case, *AIMS Mathematics*, **10** (2025), 3334–3380. https://doi.org/10.3934/math.2025155
19. B. Basu, D. Chakraborty, A. K. Kar, How can we improve buyer experiences for the operations of e-grocery platforms? Towards a unified framework for digital service operations, *Int. J. Prod. Econ.*, **278** (2024), 109417. https://doi.org/10.1016/j.ijpe.2024.109417
20. A. Belhadi, M. Venkatesh, S. S. Kamble, M. Z. Abedin, Data-driven digital transformation for supply chain carbon neutrality: Insights from cross-sector supply chain, *Int. J. Prod. Econ.*, **270** (2024), 109178. https://doi.org/10.1016/j.ijpe.2024.109178
21. M. Benaissa, E. H. A. Al-Sabri, M. Iqbal, W. Abdelfattah, G. A. Alsawah, A. U. K. Niazi, Output formation containment of time-delayed heterogeneous singular multi-agent systems with stochastic impulsive effects, *AIMS Mathematics*, **11** (2026), 11706–11730. https://doi.org/10.3934/math.2026482
22. B. Benedek, O. Csiki, K. Demeter, D. Losonci, L. Szász, Financial impact of digitalization–A time-lagged analysis, *Int. J. Prod. Econ.*, **288** (2025), 109699. https://doi.org/10.1016/j.ijpe.2025.109699
23. G. S. Bharathi, S. Thirumalai, S. Elango, B. Unyong, Modeling and simulation of tax evasion dynamics: Optimal control and its cost-effectivenesss, *AIMS Mathematics*, **11** (2026), 9398–9438. https://doi.org/10.3934/math.2026390
24. S. Bhattacharya, B. Sarkar, M. Sarkar, A. Mukherjee, Hospitality for prime consumers and others under the retail management, *J. Retail. Consum. Serv.*, **80** (2024), 103849. https://doi.org/10.1016/j.jretconser.2024.103849
25. B. K. Dey, S. Pareek, M. Tayyab, B. Sarkar, Autonomation policy to control work-in-process inventory in a smart production system, *Int. J. Prod. Res.*, **59** (2021), 1258–1280. https://doi.org/10.1080/00207543.2020.1722325
26. O. Buberkoku, Bayesian analysis of a novel sophisticated stochastic volatility model: A comprehensive empirical application to stock and crude oil markets price returns, *AIMS Mathematics*, **11** (2026), 7400–7435. https://doi.org/10.3934/math.2026304

27. A. Busse, B. Gerlach, J. C. Lengeling, P. Poschmann, J. Werner, S. Zarnitz, Towards digital twins of multimodal supply chains, *Logistics*, **5** (2021), 25. https://doi.org/10.3390/logistics5020025

28. E. Cao, J. Tang, Managing regretful newsvendors and supply chain performance through target-setting, *Transport. Res. E: Log.*, **183** (2024), 103444. https://doi.org/10.1016/j.tre.2024.103444

29. U. Chaudhari, A. Bhadoriya, M. Y. Jani, B. Sarkar, A generalized payment policy for deteriorating items when demand depends on price, stock, and advertisement under carbon tax regulations, *Math. Comput. Simulat.*, **207** (2023), 556–574. https://doi.org/10.1016/j.matcom.2022.12.015

30. T. Chen, F. Shan, F. Yang, F. Xu, Online retailer bundling strategy in a dual-channel supply chain, *Int. J. Prod. Econ.*, **259** (2023), 108821. https://doi.org/10.1016/j.ijpe.2023.108821

31. X. Chen, Y. He, G. H. Pakdel, C. H. Yeh, Intelligent forecasting and distribution in cross-border e-commerce import trade: A deep-learning-based iterative optimization approach, *Omega*, **133** (2025), 103277. https://doi.org/10.1016/j.omega.2025.103277

32. Y. Chen, S. Cao, How customer digital orientation drives supplier green and low-carbon efforts: The roles of supplier dependence and common ownership, *Int. J. Prod. Econ.*, **287** (2025), 109680. https://doi.org/10.1016/j.ijpe.2025.109680

33. Q. Chu, Z. Zhang, T. J. Wu, Z. Zhang, Interaction between online retail platforms' private label brand introduction and manufacturers' channel selection, *J. Retail. Consum. Serv.*, **84** (2025), 104208. https://doi.org/10.1016/j.jretconser.2024.104208

34. A. Cimino, F. Longo, G. Mirabelli, V. Solina, P. Veltri, Enhancing internal supply chain management in manufacturing through a simulation-based digital twin platform, *Comput. Ind. Eng.*, **198** (2024), 110670. https://doi.org/10.1016/j.cie.2024.110670

35. A. Cimino, F. Longo, G. Mirabelli, V. Solina, A cyclic and holistic methodology to exploit the Supply Chain Digital Twin concept towards a more resilient and sustainable future, *Clean. Logist. Supply C.*, **11** (2024), 100154. https://doi.org/10.1016/j.clscn.2024.100154

36. J. Colombo, A. Boffelli, M. Kalchschmidt, H. Legenvre, Navigating the socio-technical impacts of purchasing digitalisation: A multiple-case study, *J. Purch. Supply Manag.*, **29** (2023), 100849. https://doi.org/10.1016/j.pursup.2023.100849

37. B. Dai, Y. Du, Q. Yang, X. Xie, Implications of unobservable promotion on distribution channel strategies in a retail platform, *Transport. Res. E: Log.*, **185** (2024), 103502. https://doi.org/10.1016/j.tre.2024.103502

38. Y. Dang, Intelligent optimization algorithm for strategic planning in economics with multi-factors assessment: A MEREC-driven Heronian mean framework, *AIMS Mathematics*, **10** (2025), 10866–10897. https://doi.org/10.3934/math.2025494

39. A. Sarkar, R. Guchhait, B. Sarkar, Application of the artificial neural network with multithreading within an inventory model under uncertainty and inflation, *Int. J. Fuzzy Syst.*, **24** (2022), 2318–2332. https://doi.org/10.1007/s40815-022-01276-1

40. A. Datta, B. K. Dey, S. Bhuniya, I. Sangal, B. Mandal, M. Sarkar, et al., Adaptation of e-commerce retailing to enhance customer satisfaction within a dynamical system under transfer of risk, *J. Retail. Consum. Serv.*, **84** (2025), 104129. https://doi.org/10.1016/j.jretconser.2024.104129

41. A. Debnath, B. Sarkar, Effect of circular economy for waste nullification under a sustainable supply chain management, *J. Clean. Prod.*, **385** (2023), 135477. https://doi.org/10.1016/j.jclepro.2022.135477

42. B. K. Dey, A. Datta, B. Sarkar, Effectiveness of carbon policies and multi-period delay in payments in a global supply chain under remanufacturing consideration, *J. Clean. Prod.*, **402** (2023), 136539. https://doi.org/10.1016/j.jclepro.2023.136539

43. M. Fang, Y. Yu, K. Park, F. Liu, S. Xiao, Y. Shi, Supply chain relationship dependencies and circular economy performance: The contingency role of digitalization capability, *J. Purch. Supply Manag.*, **30** (2024), 100902. https://doi.org/10.1016/j.pursup.2024.100902

44. M. Frasquet, M. Ieva, A. Mollá-Descals, Customer inspiration in retailing: The role of perceived novelty and customer loyalty across offline and online channels, *J. Retail. Consum. Serv.*, **76** (2024), 103592. https://doi.org/10.1016/j.jretconser.2023.103592

45. B. Sarkar, B. Mridha, S. Pareek, M. Sarkar, L. Thangavelu, A flexible biofuel and bioenergy production system with transportation disruption under a sustainable supply chain network, *J. Clean. Prod.*, **317** (2021), 128079. https://doi.org/10.1016/j.jclepro.2021.128079

46. S. K. Ghosh, B. Sarkar, P. Goswami, M. Sarkar, R. Guchhait, H. Dem, Quality control with autonomation for market segmentation in a service-based smart complex production system, *Ain Shams Eng. J.*, **17** (2026), 104229. https://doi.org/10.1016/j.asej.2026.104229

47. D. Grewal, D. Herhausen, S. Ludwig, F. V. Ordenes, The future of digital communication research: Considering dynamics and multimodality, *J. Retailing*, **98** (2022), 224–239. https://doi.org/10.1016/j.jretai.2021.01.007

48. S. Q. Gu, Y. Liu, G. Zhao, Pricing coordination of a dual-channel supply chain considering offline in-sale service, *J. Retail. Consum. Serv.*, **75** (2023), 103483. https://doi.org/10.1016/j.jretconser.2023.103483

49. R. Guchhait, S. Bhattacharya, B. Sarkar, A. Gunasekaran, A Pricing strategy based on a stochastic problem with barter exchange under variable promotional effort for a retail channel, *J. Retail. Consum. Serv.*, **81** (2024), 103954. https://doi.org/10.1016/j.jretconser.2024.103954

50. R. Guchhait, B. Sarkar, M. Sarkar, L. Yang, A. AlArjani, B. Mandal, Extended material requirement planning (MRP) within a hybrid energy-enabled smart production system, *J. Ind. Inf. Integr.*, **42** (2024), 100717. https://doi.org/10.1016/j.jii.2024.100717

51. R. Guchhait, B. Sarkar, Economic evaluation of an outsourced fourth-party logistics (4PL) under a flexible production system, *Int. J. Prod. Econ.*, **279** (2025), 109440. https://doi.org/10.1016/j.ijpe.2024.109440

52. M. S. Habib, M. Omair, M. B. Ramzan, T. N. Chaudhary, M. Farooq, B. Sarkar, A robust possibilistic flexible programming approach toward a resilient and cost-efficient biodiesel supply chain network, *J. Clean. Prod.*, **366** (2022), 132752. https://doi.org/10.1016/j.jclepro.2022.132752

53. S. H. R. Hajiagha, S. Alaei, H. A. Mahdiraji, Multi-objective multi-product sustainable newsvendor management in an emerging economy: Evidence and applications, *J. Clean. Prod.*, **310** (2021), 127466. https://doi.org/10.1016/j.jclepro.2021.127466

54. P. He, G. Zhang, T. Y. Wang, Y. Si, Optimal two-period pricing strategies in a dual-channel supply chain considering market change, *Comput. Ind. Eng.*, **179** (2023), 109193. https://doi.org/10.1016/j.cie.2023.109193
55. X. J. He, S. Lin, Analytically pricing European options under a two-factor Heston-Vasicek model with regime switching and stochastic interest rate, *AIMS Mathematics*, **11** (2026), 3986–4007. https://doi.org/10.3934/math.2026160
56. S. Herold, J. Heller, F. Rozemeijer, D. Mahr, Brave new procurement deals: An experimental study of how generative artificial intelligence reshapes buyer-supplier negotiations, *J. Purch. Supply Manag.*, **31** (2025), 101012. https://doi.org/10.1016/j.pursup.2025.101012
57. F. Hooshmand, M. Nikoomanesh, Integration of machine constraint learning methods within optimization models, *Appl. Math. Model.*, **146** (2025), 116184. https://doi.org/10.1016/j.apm.2025.116184
58. C. Hou, B. Ma, Finite volume element discretization of optimal control of the parabolic equation using the discretize-then-optimize approach, *AIMS Mathematics*, **11** (2026), 444–461. https://doi.org/10.3934/math.2026019
59. X. Hu, J. Cai, X. Yue, Power structure preferences in a dual-channel supply chain: Demand information symmetry vs. asymmetry, *Eur. J. Oper. Res.*, **314** (2024), 920–934. https://doi.org/10.1016/j.ejor.2023.10.041
60. G. Hua, Y. Zhang, T. C. E. Cheng, S. Wang, J. Zhang, The newsvendor problem with barter exchange, *Omega*, **92** (2020), 102149. https://doi.org/10.1016/j.omega.2019.102149
61. S. Huang, Y. Chen, Manufacturer encroachment with competing dual-purpose online retail platforms', *J. Retail. Consum. Serv.*, **73** (2023), 103370. https://doi.org/10.1016/j.jretconser.2023.103370
62. S. Huang, J. Xu, Y. Gao, Y. Chen, Manufacturer selling channel structure choice under retailer demand-enhancement investment with dual-purpose firms, *Expert Syst. Appl.*, **242** (2024), 122726. https://doi.org/10.1016/j.eswa.2023.122726
63. J. Huang, J. Du, H. Liang, Dynamics and stochastic sensitivity technique of a stochastic SIRS epidemic model, *AIMS Mathematics*, **11** (2026), 6720–6743. https://doi.org/10.3934/math.2026278
64. R. Issaoui, M. Elloumi, I. Bouzida, O. Naifar, Hierarchical neural identification approach for Hammerstein large-scale stochastic systems: A simulation study of hydraulic process, *AIMS Mathematics*, **11** (2026), 12132–12154. https://doi.org/10.3934/math.2026498
65. D. Ivanov, Intelligent digital twin (iDT) for supply chain stress-testing, resilience, and viability, *Int. J. Prod. Econ.*, **263** (2023), 108939. https://doi.org/10.1016/j.ijpe.2023.108938
66. D. Ivanov, Conceptual and formal models for design, adaptation, and control of digital twins in supply chain ecosystems, *Omega*, **137** (2025), 103356. https://doi.org/10.1016/j.omega.2025.103356
67. Y. Jia, L. Cui, J. Su, L. Wu, S. Akter, A. Kumar, Digital servitization in digital enterprise: Leveraging digital platform capabilities to unlock data value, *Int. J. Prod. Econ.*, **278** (2024), 109434. https://doi.org/10.1016/j.ijpe.2024.109434

68. H. Jin, N. Jiang, W. Su, S. Dalia, How does customer enterprise digitalization improve the green total factor productivity of state-owned suppliers: From the supply chain perspective, *Omega*, **133** (2025), 103248. https://doi.org/10.1016/j.omega.2024.103248

69. A. Kashyap, O. J. Shukla, R. Kumar, M. M. Alam, S. S. Oberoi, Online retailing and the metaverse: Addressing stakeholder impediments in e-commerce, *J. Retail. Consum. Serv.*, **84** (2025), 104171. https://doi.org/10.1016/j.jretconser.2024.104171

70. M. A. A. Khan, L. E. Cárdenas-Barrón, G. Treviño-Garza, A. Céspedes-Mota, Optimizing emissions reduction investments in livestock-producing farms under non-linear holding cost and power demand: A comparative analysis of carbon tax and cap-and-trade environmental regulations, *Appl. Math. Model.*, **138** (2025), 115821. https://doi.org/10.1016/j.apm.2024.115821

71. S. Kong, C. Wang, Y. Sun, A recursive filter for a class of two-dimensional nonlinear stochastic systems, *AIMS Mathematics*, **10** (2025), 1741–1756. https://doi.org/10.3934/math.2025079

72. A. S. H. Kugele, B. Sarkar, Reducing carbon emissions of a multi-stage smart production for biofuel towards sustainable development, *Alex. Eng. J.*, **70** (2023), 93–113. https://doi.org/10.1016/j.aej.2023.01.003

73. A. S. H. Kugele, Y. Oh, S. Kar, Y. Kwon, J. Jeon, Y. Kim, et al., A generative artificial intelligence-driven smart factory using generative adversarial networks, *Int. J. Prod. Res.*, 2026, 1–29. https://doi.org/10.1080/00207543.2026.2683883

74. E. Lee, S. Minner, How power structure and markup schemes impact supply chain channel efficiency under price-dependent stochastic demand, *Eur. J. Oper. Res.*, **318** (2024), 297–309. https://doi.org/10.1016/j.ejor.2024.05.019

75. J. Lee, P. C. Chua, B. Liu, S. K. Moon, M. Lopez, A hybrid data-driven optimization and decision-making approach for a digital twin environment: Towards customizing production platforms, *Int. J. Prod. Econ.*, **279** (2025), 109447. https://doi.org/10.1016/j.ijpe.2024.109447

76. J. Li, H. Wang, V. Shi, Q. Sun, Manufacturer's choice of online selling format in a dual-channel supply chain with green products, *Eur. J. Oper. Res.*, **318** (2024), 131–142. https://doi.org/10.1016/j.ejor.2024.04.038

77. L. Li, Y. Jiang, Data-driven neural network dynamics for customer behavior modeling and personalized E-commerce marketing, *AIMS Mathematics*, **11** (2026), 3221–3242. https://doi.org/10.3934/math.2026130

78. X. J. Lim, J. H. Cheah, Y. K. Dwivedi, J. E. Richard, Does retail type matter? Consumer responses to channel integration in omni-channel retailing, *J. Retail. Consum. Serv.*, **67** (2022), 102992. https://doi.org/10.1016/j.jretconser.2022.102992

79. C. Liu, W. Zhu, Newsvendor conditional value-at-risk minimisation: A feature-based approach under adaptive data selection, *Eur. J. Oper. Res.*, **313** (2024), 548–564. https://doi.org/10.1016/j.ejor.2023.08.043

80. H. Liu, T. Kan, Optimal investment strategy for an investor with partial information under exchange rate risk based on Malliavin calculus, *AIMS Mathematics*, **10** (2025), 30528–30543. https://doi.org/10.3934/math.20251339

81. M. Lüders, M. Klarmann, M. Wouters, A. Gerlach, How online information search behavior and the role of tacit knowledge differ across clusters of purchase situations, *J. Purch. Supply Manag.*, **29** (2023), 100862. https://doi.org/10.1016/j.pursup.2023.100862

82. A. S. Mahapatra, S. Sengupta, A. Dasgupta, B. Sarkar, R. T. Goswami, What is the impact of demand patterns on integrated online-offline and buy-online-pickup in-store (BOPS) retail in a smart supply chain management?, *J. Retail. Consum. Serv.*, **82** (2025), 104093. https://doi.org/10.1016/j.jretconser.2024.104093

83. B. Sarkar, R. Guchhait, M. Sarkar, A sustainable electric vehicle smart production with work-in-process inventory of outsourced spare parts, *J. Indust. Inform. Integ.*, **49** (2026), 100998. https://doi.org/10.1016/j.jii.2025.100998

84. S. Mitra, Newsvendor problem with clearance pricing, *Eur. J. Oper. Res.*, **268** (2018), 193–202. https://doi.org/10.1016/j.ejor.2018.01.023

85. M. Mittal, B. Sarkar, Stochastic behavior of exchange rate on an international supply chain under random energy price, *Math. Comput. Simulat.*, **205** (2023), 232–250. https://doi.org/10.1016/j.matcom.2022.09.007

86. B. Mridha, S. Pareek, A. Goswami, B. Sarkar, Joint effects of production quality improvement of biofuel and carbon emissions towards a smart sustainable supply chain management, *J. Clean. Prod.*, **386** (2023), 135629. https://doi.org/10.1016/j.jclepro.2022.135629

87. B. Mridha, B. Sarkar, L. E. Cárdenas-Barrón, G. V. Ramana, L. Yang, Is the advertisement policy for dual-channel profitable for retailing and consumer service of a retail management system under emissions-controlled flexible production system? *J. Retail. Consum. Serv.*, **78** (2024), 103662. https://doi.org/10.1016/j.jretconser.2023.103662

88. A. T. Van Nguyen, R. McClelland, N. H. Thuan, Exploring customer experience during channel switching in omnichannel retailing context: A qualitative assessment, *J. Retail. Consum. Serv.*, **64** (2022), 102803. https://doi.org/10.1016/j.jretconser.2021.102803

89. W. Niu, X. Chao, L. Liu, L. Zhang, M. Luo, Financial support to a supplier for quality improvement in a dual-channel supply chain, *Comput. Ind. Eng.*, **189** (2024), 109975. https://doi.org/10.1016/j.cie.2024.109975

90. K. T. Park, Y. H. Son, S. D. Noh, The architectural framework of a cyber physical logistics system for digital-twin-based supply chain control, *Int. J. Prod. Res.*, **59** (2021), 5721–5742. https://doi.org/10.1080/00207543.2020.1788738

91. Z. Pei, S. Ghose, R. Yan, S. B. Zhou, A. Yan, The strategic value of buy online and pick up from store service to the dual-channel coordination, *J. Retail. Consum. Serv.*, **77** (2024), 103648. https://doi.org/10.1016/j.jretconser.2023.103648

92. R. Qiu, C. Li, M. Sun, Impacts of consumer virtual showrooming behavior on manufacturer and retailer strategic decisions in a dual-channel supply chain, *Eur. J. Oper. Res.*, **313** (2024), 325–342. https://doi.org/10.1016/j.ejor.2023.08.025

93. S. Rahmanzadeh, M. S. Pishvaee, K. Govindan, Emergence of open supply chain management: the role of open innovation in the future smart industry using digital twin network, *Ann. Oper. Res.*, **329** (2023), 979–1007. https://doi.org/10.1007/s10479-021-04254-2

94. S. Samanta, V. K. Dubey, B. Sarkar, Measure of influences in social networks, *Appl. Soft Comput.*, **99** (2021), 106858. https://doi.org/10.1016/j.asoc.2020.106858

95. N. Saporiti, V.G. Cannas, R. Pozzi, T. Rossi, Challenges and countermeasures for digital twin implementation in manufacturing plants: A Delphi study, *Int. J. Prod. Econ.*, **261** (2023), 108888. https://doi.org/10.1016/j.ijpe.2023.108888

96. B. Sarkar, B. Mridha, S. Pareek, A sustainable smart multi-type biofuel manufacturing with the optimum energy utilization under flexible production, *J. Clean. Prod.*, **332** (2022), 132869. https://doi.org/10.1016/j.jclepro.2021.129869

97. B. Sarkar, R. Guchhait, Ramification of information asymmetry on a green supply chain management with the cap-trade, service, and vendor-managed inventory strategies, *Electron. Commer. Res. Appl.*, **60** (2023), 101274. https://doi.org/10.1016/j.elerap.2023.101274

98. B. Sarkar, S. Bhattacharya, M. Sarkar, Integrating smart production and multi-objective reverse logistics for the optimum consumer-centric complex retail strategy towards a smart factory's solution, *J. Ind. Inf. Integr.*, **46** (2025), 100856. https://doi.org/10.1016/j.jii.2025.100856

99. B. Sarkar, A. S. H. Kugele, M. Sarkar, Two non-linear programming models for the multi-stage multi-cycle smart production system with autonomation and remanufacturing in same and different cycles to reduce wastes, *J. Ind. Inf. Integr.*, **44** (2025), 100749. https://doi.org/10.1016/j.jii.2024.100749

100. B. Sarkar, M. Sarkar, B. Mridha, Mitigating the bullwhip effect and improving sustainable dual-channel retail management profitability, *Comput. Ind. Eng.*, **216** (2026), 111968. https://doi.org/10.1016/j.cie.2026.111968

101. R. Guchhait, B. Sarkar, A decision-making problem for product outsourcing with flexible production under a global supply chain management, *Int. J. Prod. Econ.*, **272** (2024), 109230. https://doi.org/10.1016/j.ijpe.2024.109230

102. S. K. Singh, A. Chauhan, B. Sarkar, Sustainable biodiesel supply chain model based on waste animal fat with subsidy and advertisement, *J. Clean. Prod.*, **382** (2023), 134806. 10.1016/j.jclepro.2022.134806

103. B. Sarkar, S. Kar, A. Pal, M. Sarkar, An investigation of retailing techniques within a supply chain model under a leader–follower game strategy, *Ain Shams Eng. J.*, **17** (2026), 103878. https://doi.org/10.1016/j.asej.2025.103878

104. S. K. Singh, A. Chauhan, B. Sarkar, Resilience of sustainability for a smart production system to produce biodiesel from waste animal fat, *J. Clean. Prod.*, **452** (2024), 142047. https://doi.org/10.1016/j.spc.2023.10.012

105. S. Sohani, T. Barman, B. Sarkar, A. Gunasekaran, S. Pareek, Retail management policy through firefly algorithm under uncertainty using Dempster-Shafer theory for production firm, *J. Retail. Consum. Serv.*, **79** (2024), 103760. https://doi.org/10.1016/j.jretconser.2024.103760

106. P. Supakar, M. Sarkar, B. Sarkar, Forecasting grocery item sales using gradient boosting models: A study of GridSearchCV, RandomizedSearchCV, and optuna optimization approaches, *Ain Shams Eng. J.*, **17** (2026), 104012. https://doi.org/10.1016/j.asej.2026.104012

107. A. S. Mahapatra, M. S. Mahapatra, B. Sarkar, S. K. Majumder, Benefit of preservation technology with promotion and time-dependent deterioration under fuzzy learning, *Exp. Syst. App.*, **201** (2022), 117169. https://doi.org/10.1016/j.eswa.2022.117169

108. M. Tayyab, B. Sarkar, An interactive fuzzy programming approach for a sustainable supplier selection under textile supply chain management, *Comput. Ind. Eng.*, **155** (2021), 107164. https://doi.org/10.1016/j.cie.2021.107164

109. M. Vishnoi, D. Deepi, S. R. Singh, B. Sarkar, Sustainable closed-loop supply chain management for a two-warehouse system with trade-credit and emissions constraints under dynamic demand, *Sci. Rep.*, **16** (2026), 18438. https://doi.org/10.1038/s41598-026-43484-x

110. T. Y. Wang, Z. S. Chen, K. Govindan, K. S. Chin, Manufacturer's selling mode choice in a platform-oriented dual-channel supply chain, *Expert Syst. Appl.*, **198** (2022), 116842. https://doi.org/10.1016/j.eswa.2022.116842

111. D. Yadav, R. Singh, A. Kumar, B. Sarkar, Reduction of pollution through sustainable and flexible production by controlling by-products, *J. Environ. Inform.*, **40** (2022), 106. https://doi.org/10.3808/jei.202200476

112. L. Zhang, G. Zhang, Z. Yao, Analysis of two substitute products newsvendor problem with a budget constraint, *Comput. Ind. Eng.*, **140** (2020), 106235. https://doi.org/10.1016/j.cie.2019.106235

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