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Research article

Backstepping-based delayed impulsive control for input-to-state stabilization of disturbed electrohydraulic servo systems

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Abstract: This paper investigates the input-to-state stability (ISS) of disturbed electrohydraulic servo systems (EHSSes) via a novel backstepping-based delayed impulsive correction control scheme, where the time delay in impulsive actions is fully considered and rigorously proven to facilitate system stabilization. First, a third-order nonlinear model of EHSS is established with explicit consideration of external disturbances. Then, a recursive backstepping method is developed to derive virtual control laws for the position velocity subsystem, rendering it ISS with respect to the acceleration error. Furthermore, by employing Lyapunov stability theory, impulsive delay inequalities, and ISS lemmas, sufficient conditions are derived to guarantee the ISS of the closed-loop error system, where the stabilizing contribution of delays involved in impulses is quantitatively analyzed within the established sufficient criteria. Finally, numerical simulations are conducted to validate that the proposed control strategy.

Keywords: impulsive control; input-to-state stability; electrohydraulic servo systems; backstepping-based control; time-delay
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1. Introduction

Electrohydraulic servo systems (EHSSes) are widely employed as critical actuation components in high-precision engineering applications, including aerospace actuators [1, 2], heavy-duty industrial machinery [3, 4], and robot joint drives [5, 6], due to their superior merits of high power density, fast dynamic response, and strong load-bearing capability . However, the inherent nonlinearities of EHSSes, such as flow pressure nonlinearity of proportional valves, oil compressibility, and mechanical load coupling together with ubiquitous external disturbances such as load fluctuations, parameter perturbations, and environmental interference pose severe challenges to the stability and trajectory tracking performance of such systems [7–9]. For nonlinear disturbed systems, input-to-state stability (ISS) has emerged as a fundamental stability metric, as it quantitatively characterizes the boundedness of system states with respect to disturbances and guarantees robust tracking performance even under perturbations [10, 11]. Therefore, developing an effective control strategy to achieve ISS for disturbed EHSSes is of great theoretical significance and practical engineering value [12, 13].

Among various nonlinear control approaches, backstepping control stands out for its recursive design framework, which is particularly suitable for high-order strict-feedback nonlinear systems [14–16]. It decomposes the complex high-order system into low-order subsystems and designs virtual control laws step by step, effectively handling nonlinear couplings and ensuring the asymptotic stability of each subsystem. Nevertheless, conventional continuous backstepping control consumes considerable control energy and may induce actuator saturation in practical EHSS applications. Impulsive control [17–19], as a discrete control, only updates the control signal at some discrete instants, which significantly reduces control energy consumption and improves the reliability of actuation systems [20–22]. This feature makes impulsive control highly compatible with the energy-saving and high-efficiency requirements of EHSSes. It is noteworthy that time delays in impulsive control are inevitable and indispensable in practical engineering implementations, which mainly originate from signal transmission latency, actuator response lag, sampling, and computation constraints in digital control systems. Such delayed impulses are an objective existence in real industrial applications. Unfortunately, the majority of existing studies on impulsive control ignore the existence of time delays in impulsive controllers or merely treat delayed impulses as a detrimental factor that impairs system stability and performance [23–25]. In recent years, by constructing a novel representation of time delays in impulses, [26] has accurately extracted the time-delay information embedded in impulses and demonstrated the positive effect of such time delays on the steady-state stability of the system. However, for electrohydraulic servo systems with external disturbances, the design of backstepping-based delayed impulsive correction control and the rigorous ISS stability analysis with explicit extract the information of time delays in impulses still remain open and unsolved.

Motivated by the above observations, this paper investigates the ISS problem of EHSSes with external disturbances by proposing a backstepping-based delayed impulsive correction control strategy. The main contributions of this paper are summarized as follows: First, a third-order nonlinear state-space model of EHSSes is established, which integrates proportional valve dynamics, cylinder pressure characteristics, and mechanical load dynamics, with explicit consideration of external disturbances. Second, this paper presents a backstepping method to design virtual control laws for the position velocity subsystem, rendering the subsystem ISS with respect to the acceleration error. Third, a novel delayed impulsive correction control law is proposed, where the time delay in impulsive actions is naturally introduced from engineering practice, and the information of the time delay in impulse actions is fully extracted, which shows that the time delay in the impulse has a positive effect on the steady state of the system. Based on Lyapunov stability theory, the closed-loop error system is demonstrated to be ISS. The remainder of this paper is organized as follows. Section 2 introduces some basic definitions and lemmas. Section 3 derives the simplified nonlinear mathematical model of the EHSS. The backstepping-based delayed impulsive correction control design and ISS stability analysis are presented in Section 4. Simulation results to validate the effectiveness of the proposed control strategy are given in Section 5. Finally, Section 6 concludes the paper.

Notations. Denote $\mathbb{R}$, $\mathbb{Z}_+$, and $\mathbb{R}_+$ as the classes of real numbers, positive integers, and non-negative real numbers, respectively. $\mathbb{R}^n$ represents the real space which is n-dimensional and furnished with the Euclidean norm $|\cdot|$. Let $\sup_{s\in J} g(s)$ denote the supremum norm of function $g(\cdot)$ over the interval J, and let $|g|_J = \sup_{s\in J} |g(s)|$. We say $\alpha(s) \in \mathcal{K}$ when $\alpha(s)$ is a continuous increasing function and satisfies $\alpha(0) = 0$. Additionally, let $\mathcal{K}_\infty$ be the subset of the set $\mathcal{K}$ containing the unbounded function. We say that a non-negative function $b(m, n)$ of two variables defined on $[0, \infty) \times [0, \infty)$ belongs to the set $\mathcal{KL}$ if, for every fixed second variable n, the function $b(\cdot, n) \in \mathcal{K}$ holds, and for every fixed first variable m, the function $b(m, n)$ decreases to 0 as $n \to 0$. For a constant $\rho > 0$, let $S_\rho = \{u \in \mathbb{R}, |u| \le \rho\}$.

2. Preliminaries

Consider the following inequality with external disturbance:

$$\dot{s}(t) = g(s(t) + \upsilon(t)),\ t \ge t_0, \tag{2.1}$$

where $s(t) \in \mathbb{R}^n$ denotes the state of System (2.1) with initial condition $s(t_0) = s_0$, $\upsilon(t) \in \mathbb{R}^m$ denotes the external disturbances, and $g(t)$ (from $\mathbb{R}^n \times \mathbb{R}^m$ to $\mathbb{R}^n$) is a continuous function.

Definition 1. *System* (2.1) *is said to be ISS if there exist a $\mathcal{KL}$ function $\tilde{\beta}$ and a $\mathcal{K}_\infty$ function $\tilde{\gamma}$ such that, for every initial time $t_0 \ge 0$, every initial state s_0, and every measurable input signal υ, the corresponding solution $s(t)$ satisfies the following inequality for all $t \ge t_0$:*

$$|s(t)| \le \tilde{\beta}(|s_0|, t - t_0) + \tilde{\gamma}(\sup_{h\in[t_0,t]} |\upsilon(h)|).$$

Definition 2. *A function F defined on $\mathbb{R}^n \to \mathbb{R}_+$ is said to be an ISS Lyapunov function if it satisfies the following conditions:*

(i) F is continuous and positive definite;

(ii) $\mu_1(|s|) \le F(s) \le \mu_2(|s|)$, where μ_1 and μ_2 belong to $\mathcal{K}_\infty$;

(iii) $\dot{F} \le -\mu_3(|s|) + \mu_4(|\upsilon|)$, where μ_3 and μ_4 belong to $\mathcal{K}_\infty$, and υ is the external disturbance.

Lemma 1. *[27] If System* (2.1) *admits an ISS Lyapunov function, then System* (2.1) *is ISS.*

Lemma 2. *[27] Consider the cascaded system given by*

$$\dot{s}_1(t) = g_1(s_1, s_2), \tag{2.2}$$

$$\dot{s}_2(t) = g_2(s_2, \upsilon), \tag{2.3}$$

where $s_1, s_2 \in \mathbb{R}^n$ are the system state; $\upsilon \in \mathbb{R}^m$ is the external disturbance; and g_1, g_2 are continuous functions. If both subsystems (2.2) *and* (2.3) *are ISS, then the cascaded system consisting of* (2.2) *and* (2.3) *is ISS.*

3. Mathematical model of electrohydraulic servo system

This section deduces the simplified mathematical model of the electrohydraulic servo system (EHSS) based on the physical structure and operational characteristics of the system in accordance with the basic laws of hydraulic transmission and mechanical dynamic equations. The EHSS is mainly composed of three core components: a proportional valve, hydraulic cylinder, and load actuator. The model construction integrates the flow pressure characteristics of the proportional valve-controlled hydraulic cylinder, the dynamic pressure characteristics of the cylinder chambers, and the dynamic characteristics of the mechanical load, and the coupling relationship among each part forms the complete dynamic model of the EHSS. The proportional valve of the EHSS can be described by a second-order dynamic system of mass-spring-dampers to characterize the spool motion characteristics [28, 29]. Its dynamic equation is expressed as

$$\ddot{o} + 2\xi\tilde{\omega}\dot{o} + \tilde{\omega}^2 o = ku\tilde{\omega}^2, \tag{3.1}$$

where o is the spool displacement of proportional valve, ξ is the damping ratio of spool motion, $\tilde{\omega}$ is the natural vibration frequency, and k and u denote the control gain and control input, respectively. For the sake of model simplification and in line with the practical engineering application scenarios, it is assumed that the spool displacement of the proportional valve can be ideally controlled; that is, the spool displacement is linearly proportional to the control input voltage [30, 31]. Thus, there exists a positive proportional constant σ that satisfies $o = \sigma u$. Based on the above linear relationship, the expression of the hydraulic oil flow into the hydraulic cylinder through the proportional valve can be derived. The flow of hydraulic oil through the valve orifice of the proportional valve follows the orifice flow formula. Combined with the linear control characteristic of the spool displacement, the output flow rates of the hydraulic oil entering the rodless chamber (denoted as R_1) and the rod chamber (denoted as R_2) are obtained as

$$R_1 = C_*\hat{\omega}\sigma u\sqrt{\frac{2}{\rho}L_l}, \quad R_2 = C_*\hat{\omega}\sigma u\sqrt{\frac{2}{\rho}L_r}, \tag{3.2}$$

where C_* is the flow coefficient, $\hat{\omega}$ is the valve orifice area gradient, L_l is the effective pressure difference at the proportional valve orifice, and its value is determined by the positive and negative of the control input voltage. This reflects reflecting the flow direction of hydraulic oil, which is defined as

$$\begin{cases} L_l = |l_s - l_1|, & u \geq 0, \\ L_l = |l_1 - l_a|, & u \leq 0. \end{cases}$$

Additionally, the oil return side effective pressure difference L_r is defined for the subsequent derivation of the chamber pressure characteristics, and its value is

$$\begin{cases} L_r = |l_2 - l_a|, & u \geq 0, \\ L_r = |l_s - l_2|, & u \leq 0, \end{cases}$$

where l_1, l_2 are the pressures and l_s and l_a are the supply pressure and tank pressure, respectively, which can be seen in Figure 1. Beacuse the influence of leakage on the main flow state of the system is negligible, in this paper, the influence of internal and external leakage of the hydraulic cylinder is neglected; we only consider the compressibility of hydraulic oil to deduce the dynamic pressure characteristics of the rodless chamber and rod chamber of the hydraulic cylinder. This is in line with the engineering simplified modeling principle. The compressibility of hydraulic oil is characterized by the bulk modulus ϖ. Combined with the relationship between the volume change of the chamber and the flow balance, the differential equations of the rodless chamber pressure $\dot{l}_1$ and the rod chamber pressure $\dot{l}_2$ are derived as

$$\begin{aligned} \dot{l}_1 &= \frac{\varpi}{V_0^* + A_1 s_1}(R_1 - A_1 \dot{s}_1), \\ \dot{l}_2 &= \frac{\varpi}{V_{*0} + A_2 s_1}(-R_2 + A_2 \dot{s}_1), \end{aligned} \tag{3.3}$$

where V_0^* and V_{*0} are the initial volume of rodless and rod chamber, respectively. A_1 and A_2 are annulus areas of position and rod side. Based on Newton's second law, the dynamic equation of the mechanical load is derived by considering the total mass of the piston and the load, viscous damping, load stiffness, and external uncertain disturbance:

$$\ddot{s} = \frac{1}{M}(l_1 A_1 - l_2 A_2 - c_1 \dot{s} - c_2 s - \dot{I}), \tag{3.4}$$

where M is the total mass of hydraulic cylinder piston and controlled load; c_1 and c_2 are the coefficients of the actuator and the external disturbance, respectively; and I is the external uncertain disturbance force on the system. To facilitate the subsequent control law design and stability analysis, the above partial models are integrated to construct a third-order state-space model of the EHSS. The system state variables are defined as follows:

$$s_1 = s,\ s_2 = \dot{s},\ s_3 = \ddot{s},$$

where s_1, s_2, and s_3 are the position state, velocity state, and the acceleration state, respectively. Then, by (3.2), (3.3), and (3.4), the state-space differential equation of the EHSS is obtained as

$$\begin{aligned} \dot{s}_1 &= s_2, \\ \dot{s}_2 &= s_3, \\ \dot{s}_3 &= \frac{1}{M}(\lambda_1 u - \lambda_2 s_2 - c_1 s_3 - \dot{I}). \end{aligned} \tag{3.5}$$

In (3.5), λ_1 is the control gain coefficient of the system, which is related to the structural parameters of the hydraulic system and the oil characteristics, and λ_2 is the comprehensive coefficient of damping and stiffness of the system. Their specific expressions are:

$$\begin{aligned} \lambda_1 &= \sigma \varpi C_* \tilde{\omega} \left(\frac{A_1 \sqrt{L_l}}{V_0^* + A_1 s_1} + \frac{A_2 \sqrt{L_r}}{V_{*0} - A_2 s_1} \right) \sqrt{\frac{2}{\rho}}; \\ \lambda_2 &= \varpi \left(\frac{A_1^2}{V_0^* + A_1 s_1} + \frac{A_2^2}{V_{*0} - A_2 s_1} \right) + c_2. \end{aligned}$$

In this model, it is assumed that the position, velocity, and acceleration states of the system can all be fed back through hardware detection. The control objective of the system is to make the piston position state s_1 track the given reference position signal z_1 by designing the control input. All signals involved in the model satisfy the characteristics of right continuity and existence of left limits.

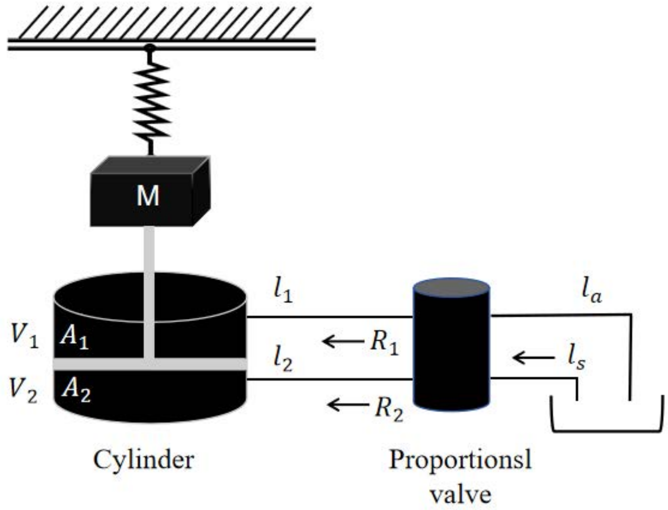

Figure 1. The mathematical model of the EHSS.

4. Main results

This section presents the control design and stability analysis for the position-tracking problem of the electrohydraulic servo system. First, a backstepping framework is constructed to derive virtual control laws. Then, an delayed impulsive correction mechanism is introduced to reduce control energy consumption. Finally, the ISS of the closed-loop error system is rigorously established via the Lyapunov method.

4.1. Backstepping control design

We start from the position and velocity subsystem given by

$$\begin{cases} \dot{s}_1 = s_2, \\ \dot{s}_2 = s_3. \end{cases} \tag{4.1}$$

The control goal is to steer s_1 to a smooth reference z_1. To this end, we use the backstepping technique to design a suitable virtual control signal z_3. The actual acceleration is decomposed as

$$s_3 = z_3 + sz_e,$$

where sz_e denotes the error. Then, we can rewrite System (4.1) as

$$\begin{cases} \dot{s}_1 = s_2, \\ \dot{s}_2 = z_3 + sz_e. \end{cases} \tag{4.2}$$

Define the candidate position error Lyapunov function

$$L_1(t) = \frac{1}{2}(s_1 - z_1)^2.$$

Differentiating L_1 yields

$$\dot{L}_1(t) = (s_1 - z_1)(\dot{s}_1 - \dot{z}_1) = (s_1 - z_1)(s_2 - \dot{z}_1).$$

We choose the virtual velocity control

$$z_2 = -\frac{1}{2}(s_1 - z_1) + \dot{z}_1.$$

Next, introduce the Lyapunov function

$$L_2(t) = \frac{1}{2}(s_1 - z_1)^2 + \frac{1}{2}(s_2 - z_2)^2.$$

Computing its time derivative gives

$$\begin{aligned}
\dot{L}_2(t) =&(s_1 - z_1)(\dot{s}_1 - \dot{z}_1) + (s_2 - z_2)(\dot{s}_2 - \dot{z}_2)\\
=&(s_1 - z_1)(s_2 - \dot{z}_1) + (s_2 - z_2)(s_3 + \frac{1}{2}(s_2 - \dot{z}_1) - \ddot{z}_1)\\
=&(s_1 - z_1)(s_2 - \dot{z}_1) + (s_2 - z_2)(z_3 + sz_e + \frac{1}{2}(s_2 - \dot{z}_1) - \ddot{z}_1).
\end{aligned}$$

From this, we derive the virtual acceleration control

$$z_3 = -\frac{1}{2}(s_2 - \dot{z}_1) + \ddot{z}_1 - (s_2 - z_2) - (s_1 - z_1). \tag{4.3}$$

By (4.3), we can obtain

$$\begin{aligned}
\dot{L}_2(t) &= (s_1 - z_1)(\dot{s}_1 - \dot{z}_1) + (s_2 - z_2)(-(s_2 - z_2) - (s_1 - z_1) + sz_e)\\
&= (s_1 - z_1)(s_2 - \dot{z}_1 - s_2 - \frac{1}{2}(s_1 - z_1) + \dot{z}_1) - (s_2 - z_2)^2 + sz_e(s_2 - z_2)\\
&= -\frac{1}{2}(s_1 - z_1)^2 - (s_2 - z_2)^2 + sz_e(s_2 - z_2)\\
&\le -\frac{1}{2}(s_1 - z_1)^2 - (s_2 - z_2)^2 + \frac{1}{2}sz_e^2 + \frac{1}{2}(s_2 - z_2)^2\\
&= -L_2(t) + \frac{1}{2}sz_e^2.
\end{aligned}$$

According to Definition 2, the function $L_2(t)$ serves as a valid ISS Lyapunov function for the (s_1, s_2) subsystem. Specifically, the term $-L_2(t)$ provides the required negative definite state dissipation (corresponding to the $\mathcal{K}_\infty$ function μ_3), and the term $sz_e^2/2$ characterizes the bounded influence of the external input sz_e (corresponding to the $\mathcal{K}_\infty$ function μ_4). Therefore, by Lemma 1, the cascaded subsystem (4.1) is ISS with respect to the acceleration error sz_e. It should be noted that the ultimate boundedness and ISS properties of the error state sz_e itself will be rigorously established in Section 4.2 via the proposed delayed impulsive correction control, which subsequently guarantees the ISS of the overall closed-loop system in Theorem 2.

4.2. Design of delayed impulsive correction control

In this section, let sz_e represent the error between the dynamics of s_3 and the virtual control z_3, that is, $sz_e = s_3 - z_3$. Throughout the ensuing impulsive stability analysis, it is assumed that all system state trajectories are right-continuous with left limits; specifically, at any impulsive instant t_n, the state satisfies $sz_e(t_n) = sz_e(t_n^+) = \lim_{t\to t_n^+} sz_e(t)$. By the dynamics of the system, the dynamics of the error system can be obtained as follows:

$$\begin{aligned} s\dot{z}_e &= \dot{s_3} - \dot{z_3} \\ &= \frac{1}{M}(\lambda_1 u - \lambda_2 s_2 - c_1 s_3 - \dot{I}) - \dot{z_3}. \end{aligned} \tag{4.4}$$

We propose a continuous virtual tracking control coupled with a delayed impulsive correction mechanism. For $t \neq t_n$, the continuous control input is designed to compensate for the known system dynamics:

$$u_1(t) = \frac{M}{\lambda_1}\left(\frac{1}{M}\lambda_2 s_2 + \frac{1}{M}c_1 s_3 + \dot{z}_3\right).$$

At the impulsive instants $t = t_n$, the acceleration error is corrected via the impulsive update. Under this control strategy, the dynamics of the error system (4.4) can be explicitly rewritten in the standard impulsive difference form as

$$\begin{cases} s\dot{z}_e(t) = \frac{1}{M}\dot{I}, t \neq t_n, n \in \mathbb{Z}_+ \\ sz_e(t_n^+) = d \cdot sz_e(r_n), \end{cases}$$

where $d \in (0, 1)$ is the impulsive gain. This paper assumes that the impulsive time sequence $\{t_n\}$ satisfies $t_0 < t_1 < t_2 < \cdots < t_l \to \infty$ as $l \to \infty$, and we let $\mathbb{G}$ denote such sequences. For any given $\{t_n\} \in \mathbb{G}$ and $\phi \in (0, 1]$, denote the corresponding $r_n = \phi t_n + (1 - \phi)t_{n-1}$, and then, $r_n \in (t_n, t_{n-1}]$, and let $\mathbb{G}^{\phi}_{\{t_n\}}$ denote such sequences. Because $\phi \in (0, 1]$, and $t_{n-1} < t_n$, it strictly follows that $r_n \leq t_n$. This mathematically guarantees that the delayed impulsive controller relies solely on historical or current state information, thereby strictly preserving physical causality without requiring any future knowledge. Additionally, if $\{t_n\} \in \mathbb{G}$ satisfies $t_n - t_{n-1} \leq \Delta$, $n \in \mathbb{Z}_+$, we say $\{t_n\} \in \mathbb{G}_\Delta$.

Theorem 1. *Under the delayed impulsive correction control, if there exist constants $\phi, \Delta > 0$ and $d \in (0, 1)$ such that*

$$\frac{\ln d}{\Delta} + \phi < 0, \tag{4.5}$$

then System (4.4) is ISS over the time sequence $\{t_n\} \in \mathbb{G}_\Delta$ and $\{r_n\} \in \mathbb{G}^{\phi}_{\{t_n\}}$.

Proof. Let $sz_e(t)$ be the solution of System (4.4) with respect to the initial condition $sz_{e0} = sz_e(t_0)$. Consider the Lyapunov function

$$L_3(t) = \frac{1}{2}sz_e^2.$$

It is obvious that $L_3(t)$ is a positive function and satisfies $h_1(|sz_e|) \leq L_3 \leq h_2(|sz_e|)$ for some functions $h_1, h_2 \in \mathcal{K}_\infty$. With the delayed impulsive control, it can be obtained that at the impulse time t_n,

$$L_3(t_n) = \frac{1}{2}d^2 sz_e^2(r_n) = d^2 L_3(r_n). \tag{4.6}$$

For $t \neq t_n$, it can be deduced that

$$\begin{aligned} D^+ L_3(t) &= sz_e s\dot{z}_e \\ &= sz_e \frac{1}{M}\dot{I} \\ &\leq |sz_e||\frac{1}{M}\dot{I}| \\ &\leq \frac{1}{2} sz_e^2(t) + \frac{1}{2}(\frac{1}{M}\dot{I})^2 \\ &= L_3(t) + \frac{1}{2}(\frac{1}{M}\dot{I})^2. \end{aligned}$$

In practical engineering applications, the rate of change of the external disturbance is physically bounded. Thus, it is reasonable to assume there exists a known positive constant $\bar{I}$ such that $|\dot{I}(t)| \leq \bar{I}$ for all $t \geq t_0$. Let us define the constant bound $\tilde{\epsilon} = \dot{I}/M$. First, consider the region where the error state satisfies $|sz_e| \geq |\tilde{\varepsilon}|$. In this region, we can obtain

$$D^+ L_3(t) \leq sz_e^2 = 2L_3, \ t \neq t_n. \tag{4.7}$$

It may be assumed without loss of generality that there exists an impulsive sequence on the time interval $[l^\star, l_\star]$ where $|sz_e| \leq |\tilde{\varepsilon}|$ holds such that it satisfies $l^\star = t_0 < t_1 < \cdots < t_n = l^\star$. In this case, we then prove when $t \in [t_l, t_{l+1})$

$$L_3(t) \leq \exp(2l \ln d + 2(t - t_0) + 2\sum_{i=1}^{l}(r_i - t_i))L_3(t_0). \tag{4.8}$$

Obviously, (4.8) holds when $t = t_0$. When $t \in [t_0, t_1)$, it follows from (4.7) that

$$L_3(t) \leq \exp(2(t - t_0))L_3(t_0);$$

thus, (4.8) holds on the time interval $[t_0, t_1)$. Then, we assume that (4.8) holds on $l \leq m - 1$, and thus, when $t \in [t_{m-1}, t_m)$,

$$L_3(t) \leq \exp(2(m-1)\ln d + 2(t - t_0) + 2\sum_{i=1}^{m-1}(r_i - t_i))L_3(t_0).$$

Next, consider the following time interval, namely, $t \in [t_m, t_{m+1})$. It follows from (4.7) that, we can derive

$$\begin{aligned} L_3(t) &\leq \exp(2(t - t_m))L_3(t_m) \\ &= d^2 \exp(2(t - t_m))L_3(r_m) \\ &\leq \exp(2m \ln d + 2(r_m - t_0) + 2(t - t_m) + 2\sum_{i=1}^{m-1}(r_i - t_i))L_3(t_0) \\ &= \exp(2m \ln d + 2(t - t_0) + 2\sum_{i=1}^{m}(r_i - t_i))L_3(t_0). \end{aligned}$$

This implies that (4.8) holds in the time interval $[t_m, t_{m+1})$. Thus, by induction, (4.8) holds on the interval $[l^\star, l_\star]$. Note that

$$\begin{aligned}\sum_{i=1}^{m}(r_i - t_i) &= \sum(\phi t_i + (1-\phi)t_{i-1} - t_i)\\ &= \sum_{i=1}^{m}(\phi - 1)(t_i - t_{i-1})\\ &= (\phi - 1)(t_m - t_0).\end{aligned}$$

Then, it can be further deduced that $L_3(t)$ satisfies

$$\begin{aligned}L_3(t) &\le \exp(2l\ln d + 2(t - t_0) + 2(\phi - 1)(t_l - t_0))L_3(t_0)\\ &\le \exp(-2\ln d + 2(1-\phi)\Delta + 2(\frac{\ln d}{\Delta} + \phi)(t - t_0))L_3(t_0).\end{aligned}$$

For later use, we denote $\Theta \doteq \exp(-2\ln d + 2(1-\phi)\Delta)$ and $\eta \doteq -2(\ln d/\Delta + \phi)$. From Condition (4.5), we have $\eta > 0$, and when $t \in [l_\star, l^\star)$,

$$L_3(t) \le \Theta L_3(t_0)\exp(-\eta(t - t_0)). \tag{4.9}$$

Combining the properties of the function L_3 and the Ineq (4.9), we can obtain that

$$\begin{aligned}|sz_e(t)| &\le h_1^{-1}(\Theta\exp(-\eta(t - t_0))h_2(|sz_{e0}|))\\ &= \varsigma(|sz_{e0}|, t - t_0),\end{aligned} \tag{4.10}$$

where $t \in [l_\star, l^\star]$, and $\varsigma \in \mathcal{KL}$. Additionally, if there exists the case of $|sz_e| < |\tilde{\varepsilon}|$, define $\check{q}_1 = \inf\{t \ge t_0, |sz_e| \le |\tilde{\varepsilon}|\}$. If $\check{q}_1 = +\infty$, then System (4.4) is ISS. If $\check{q}_1 < +\infty$, take $\hat{q}_1 = \inf\{t > \check{q}_1, |sz_e| > |\tilde{\varepsilon}|\}$. Then, in the case of $\hat{q}_1 = +\infty$, System (4.4) satisfies

$$|sz_e(t)| \le \varsigma(|sz_{e0}|, t - t_0) + |\tilde{\varepsilon}|_{[t_0,t]}, \tag{4.11}$$

where $t \ge t_0$, which implies that System (4.4) is ISS. In the case of $\hat{q}_1 < +\infty$, we can derive $|sz_e| \le |\tilde{\varepsilon}|$ in the time interval $[\check{q}_1, \hat{q}_1]$, and we define $\check{q}_2 = \inf\{t \ge \hat{q}_1, |sz_e| \le |\tilde{\varepsilon}|\}$. Then, $|sz_e| \ge |\tilde{\varepsilon}|$ in the time interval $[\hat{q}_1, \check{q}_2]$. It follows from (4.10) that

$$\begin{aligned}|sz_e(t)| &\le h_1^{-1}(\Theta\exp(-\eta(t - \hat{q}_1))h_2(|sz_e(\hat{q}_1)|))\\ &\le h_1^{-1}(\Theta\exp(-\eta(t - \hat{q}_1))h_2(|\tilde{\varepsilon}(\hat{q}_1)|))\\ &\le h_1^{-1}(\Theta h_2(|\tilde{\varepsilon}(\hat{q}_1)|))\\ &\le h_1^{-1}(\Theta h_2(|\tilde{\varepsilon}|_{[t_0,t]})),\ t \in [\hat{q}_1, \check{q}_2].\end{aligned} \tag{4.12}$$

Let $\tilde{\varsigma}(|\tilde{\varepsilon}|_{[t_0,t]}) \doteq \max\{|\tilde{\varepsilon}|_{[t_0,t]}, h_1^{-1}(\Theta h_2(|\tilde{\varepsilon}|_{[t_0,t]}))\}$. Then, it follows from (4.10), (4.11), and (4.12) that

$$|sz_e(t)| \le \varsigma(|sz_{e0}|, t - t_0) + \tilde{\varsigma}(|\tilde{\varepsilon}|_{[t_0,t]}),\ t \ge t_0.$$

Therefore, System (4.4) is ISS, which completes the proof. □

Theorem 2. *Under the delayed impulsive controller, if the condition in Theorem 1 holds, then the full tracking errors between the full EHSS system* (3.5) *and the reference signals* (z_1, z_2, z_3) *are ISS.*

Proof. The overall system is a cascade of the ISS (s_1, s_2) subsystem and the ISS sz_e subsystem. By Lemma 2, the cascade is ISS. Therefore, all tracking errors between the full EHSS system (3.5) and the reference signals (z_1, z_2, z_3) are ISS. The proof is completed. □

Remark 1. The delayed impulse sequence $\mathbb{G}^{\phi}_{\{t_n\}}$ is constructed to quantify the potential magnitude of impulse delays through the tuning parameter ϕ. Acting as a bridge between the continuous time evolution and discrete impulsive events in System (4.4), Condition (4.5) explicitly characterizes the interplay among impulse delays, impulsive control parameters, and the system's continuous dynamic behavior. Notably, it reveals a complementary relationship between ϕ and d when describing the overall effect of impulsive actions: that is, it reveals that appropriately introduced delays in impulsive control may actually contribute to the stability of system.

Remark 2. It is worth noting that although the mathematical formulation of the time delay in impulses has similarities with the delay in [26], our core research problem and control methodology are fundamentally distinct. The work in [26] primarily investigates the exponential stability of general nonlinear systems via pure impulsive control. In contrast, this study specifically tackles the ISS of strict-feedback electrohydraulic servo systems (EHSSes) subjected to continuous external disturbances. Furthermore, rather than relying on a standalone impulsive controller, we propose a novel hybrid backstepping-based delayed impulsive correction strategy. We utilize a recursive continuous backstepping method to handle the nonlinearities of the position velocity subsystem and then uniquely synthesize the delayed impulse formulation to correct the acceleration error.

Remark 3. Compared to recent advancements in impulsive control literature, the proposed hybrid architecture demonstrates distinct engineering and theoretical novelty. For instance, although [32] and [33] have excellently addressed complex stochastic delayed systems leveraging Lyapunov–Razumikhin techniques and finite-time frameworks, our work specifically targets a deterministic, strict-feedback EHSS to guarantee robust ISS performance against persistent, bounded mechanical load fluctuations. Additionally, compared to [34], our strategy extends beyond standalone impulsive inputs. By integrating continuous virtual control with delayed impulsive correction, our mathematical synthesis explicitly extracts the time delay information to prove its positive stabilizing contribution within the closed-loop ISS analysis.

5. Simulations

This section presents numerical simulation examples to verify the effectiveness and feasibility of the proposed impulsive control strategy for the EHSS. The simulation parameters are set in accordance with the EHSS mathematical model established in Section 2, where the total mass of the piston and load M equals 80, the proportional constant σ is 20, the bulk modulus ϖ equals 850, the flow coefficient C_* and the valve orifice area gradient $\hat{\omega}$ are set to 1, the density ρ is 900, the annulus areas of position A_1 equals 2.3 and rod side A_2 equals 2.2, the total length of the hydraulic cylinder is 1, the coefficient of actuator c_1 equals 0.2, the coefficient of external disturbance c_2 is set to 100, the supply pressure l_s is $35,000$, and the tank pressure l_a equals 30000.

In simulations, the position reference signal $z_1 = \cos(t)$ is selected, and the initial state of the EHSS is set as $s_1(0) = 0$, $s_2(0) = 0$, and $s_3(0) = 0$. The instants of impulses are $t_n = 0.6n$, which implies $\Delta = 0.6$. The parameter of delay is set to $\phi = 0.7$, and the impulsive gain is $d = 0.6$. Then, it can be verified that Condition (4.5) is satisfied. Take a disturbance $I(t) = 20t + 100$, which leads to $\dot{I} = 20$. By Theorem 2, the errors between the reference systems the the EHSS are ISS. It can be seen in Figure 2 that states s_1 and s_2 are driven to the desired signals z_1 and z_2, and owing to the delayed impulsive control, the state s_3 has instantaneous state jumps. Furthermore, their errors are bounded, which means it is ISS. Figure 3 shows the position state s_1, the reference state z_1, and their respective error. It also can be found that the error is bounded; that is, the position error is ISS. Thus, this simulation demonstrates the effectiveness of the proposed results.

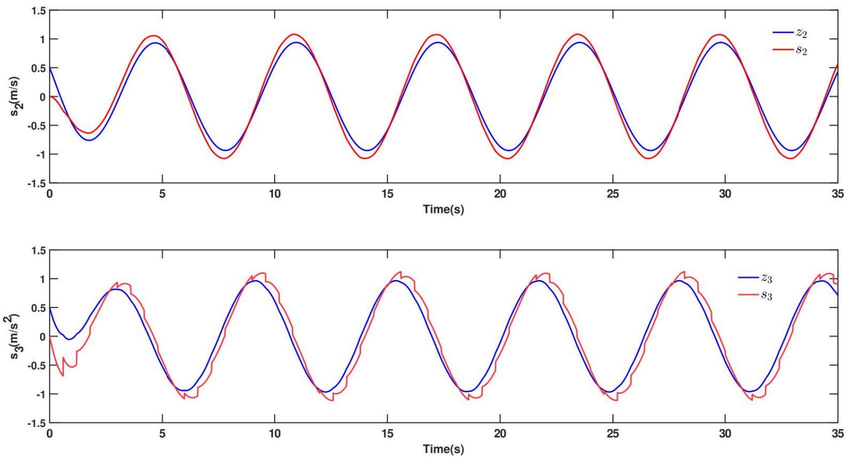

Figure 2. The trajectories of velocity and acceleration.

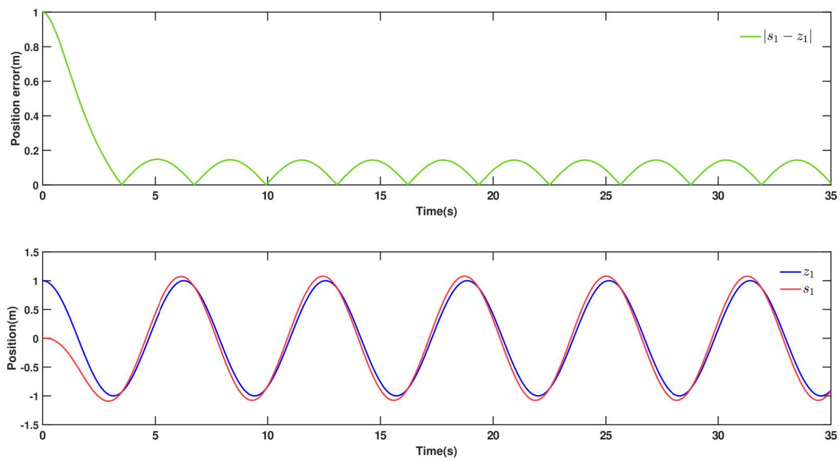

Figure 3. The trajectories of position and position error.

6. Conclusions

This paper addresses the ISS of the EHSS with external disturbances via a backstepping-based delayed impulsive correction control. A third-order nonlinear model of the EHSS is constructed with external disturbances considered. By a recursive backstepping design, virtual control laws are developed to render the position velocity subsystem ISS. With Lyapunov theory, sufficient conditions are derived to guarantee the ISS of the closed-loop system, and it is rigorously proved that time delays in impulsive actions can facilitate system stabilization. Numerical simulation verifies the proposed approach. This work extends the application of stabilizing delayed impulsive control to EHSSes and provides a practical control method for disturbed EHSSes. Future work will extend the results to systems with unbounded time-varying delays and finite-time ISS.

Author contributions

Mingzhong Li: Writing–original draft, Data curation; Yongming She: Visualization, Conceptualization; Wei Wang: Software; Shuai Liu: Supervision.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare there is no conflict of interest.

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