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Research article

On q-differential operators acting on bounded turning functions associated with epicycloidal curves

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Abstract: The q-calculus has found numerous applications in engineering, particularly in mass and heat transfer problems, as well as in the study of nonlinear and fuzzy differential equations arising in science and technology. The q-difference operator $\mathfrak{D}_q$ and related q-calculus operators play an important role in the development of this theory. In this paper, we introduce and investigate a new class of analytic functions associated with epicycloidal domains by means of subordination theory and a q-differential operator. Our primary objective is to characterize this class and establish several of its geometric properties in the unit disk $\mathbb{U}$. In particular, we study coefficient estimates, the Zalcman conjecture and its generalized form, the second and third Hankel determinants, the Krushkal inequality, and the Fekete–Szegö functional.

Keywords: q-differential operator; Fekete–Szegö problem; Zalcman conjecture; Hankel determinant; Krushkal inequality; q-derivative; bounded turning function; epicycloidal domain; quantum calculus; analytic functions
Mathematics Subject Classification: 30C50, 30C80, 30C45, 47B38, 11B65

1. Introduction and definitions

Geometric function theory (GFT) is a main area of complex analysis that investigates and analyzes geometric properties of analytic and univalent functions defined on the open unit disk. Important directions in this field include coefficient estimates, Hankel determinants, Fekete–Szegö inequalities, Zalcman-type functionals, and subordination theory. These problems play a useful role in understanding the geometric behavior of analytic functions and continue to attract considerable attention in contemporary research.

The study of special functions has played an important role in the study of GFT and its applications. In particular, hypergeometric functions of a single complex variable have been extensively analyzed due to their rich analytic structure and their close connection with coefficient problems of univalent functions, including the Bieberbach conjecture proved by de Branges [1]. Motivated by this connection, many researchers have established the geometric properties of special functions and their normalized forms. In particular, generalized hypergeometric functions have been extensively investigated with respect to analyticity, univalence, starlikeness, convexity, and coefficient inequalities [2–4]. Moreover, important contributions concerning generalized Lommel functions, Lommel–Wright functions, and Fox–Wright functions have been obtained in the context of GFT and differential subordinations [5–7]. In addition, several studies related to Struve functions and Bessel functions have focused on close-to-convexity, coefficient estimates, and other geometric characteristics of their normalized forms [8] and the references cited therein. These investigations have significantly enriched the relation between special functions and GFT and continue to inspire the development of new subclasses of analytic and univalent functions.

Very recently, the methods of quantum calculus (or q-calculus) have been successfully studied under GFT. The introduction of the q-difference operator has led to a number of generalizations of classical function classes and geometric operators. Such developments have given a new approaches for studying coefficient problems, differential subordinations, and other geometric properties of analytic and univalent functions. For further details on q-calculus and its applications in GFT, see [9–11]. Recent developments in generalized fractional operators have also led to useful and significant advances in the numerical analysis of fractional differential equations. For example, Srivastava [12] studied univalent functions, fractional calculus, and associated generalized hypergeometric functions and Baroudi et al. [13] developed an efficient numerical scheme for integro-partial fractional diffusion heat equations involving the tempered ψ-Caputo fractional derivative and discussed its stability via Von Neumann analysis. Such investigations demonstrate the continuing impact of generalized calculus operators in both theoretical and applied mathematics.

Motivated by these developments, Shaba et al. [14] introduced and studied a q-differential operator which extends several previously known operators. This operator has proved useful in defining and investigating new subclasses of analytic and univalent functions related to various geometric regions. On the other hand, epicycloidal domains have recently gained considerable attention due to their rich geometric structure and their connections with Ma–Minda subordination theory. Gandhi et al. [15, 16] introduced a family of epicycloid associated functions that map the unit disk onto domains bounded by epicycloidal curves.

Inspired by the above works, in this paper, we introduce and investigate a new subclass of analytic functions, denoted by $MS^{m,b,c}_{\gamma,\omega}(q, g)$, defined through the q-differential operator of Shaba et al. [14] and the epicycloidal mapping function $\psi_{gL}(z)$. For this class, we obtain sharp coefficient bounds, investigate Zalcman-type inequalities, determine the second and third Hankel determinants, investigate estimates for inverse coefficients, derive Fekete–Szegö and prove a Krushkal-type inequality. Several known results are proved as special cases of our findings.

Let $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disk and if the condition $1 = f'(0) = 0 = f(0)$ holds. Then the class of analytic functions ($\mathcal{AF}s$), symbolized by $\mathcal{A}$, consists of functions written in the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

and let the class $\mathcal{S}$ contain univalent functions ($\mathcal{UF}s$) in $\mathbb{U}$.

A locally closed and bounded class is also known as a compact class, with the proof relying on distortion and growth theorems that regulate the distortion and derivative of any function in the class $\mathcal{S}$ with precise limits. The function introduced by Koebe [17] is given as follows:

$$K(z) = \sum_{k=1}^{\infty} kz^k.$$

The series expansion $\wp(z) = \sum_{k=1}^{\infty} \wp_k z^k$ remains valid assuming $\wp \in \mathcal{A}$ and $\wp_1 = 1$, and $(f \star \wp)(z) = \sum_{k=1}^{\infty} \wp_k f_k z^k$, $z \in \mathbb{U}$, represents the convolution of $\wp$ and f. It can be inferred from [17] that:

$$(f \star \wp)(\jmath^2 e^{i\Theta}) = (2\pi)^{-1} \int_0^{2\pi} f(\jmath e^{i(\Theta-b)})\wp(\jmath e^{ib})db, \quad \jmath < 1.$$

The present study concerns analytic functions possessing geometric properties such as starlikeness and convexity. A function is said to be starlike if its image domain is starlike with respect to the origin, whereas a function is convex if its image domain is a convex set. Let $\mathcal{S}^\star$ denote the class of functions that are starlike in the open unit disk $\mathbb{U}$. Recall that a domain $\Omega \subset \mathbb{C}$ is called starlike with respect to the origin if, for every point $w \in \Omega$, the line segment joining 0 and w lies entirely in Ω. In this case, Ω is referred to as a starlike domain.

If

$$f \in \mathcal{A} \quad \text{and} \quad \Re\left(\frac{zf'(z)}{f(z)}\right) > 0, \qquad z \in \mathbb{U},$$

then $f \in \mathcal{S}^\star$.

Mathematically, convex functions in the set $\mathcal{A}$ have the following criteria:

$$\Re\,(N+1) > 0, \quad z \in \mathbb{U},$$

where $N = [f'(z)]^{-1}[zf''(z)]$. Moreover, for $\alpha \in [0, 1]$, any function that satisfies

$$\Re\,(M) > \alpha, \quad z \in \mathbb{U}$$

and

$$\Re\,(N+1) > \alpha, \quad z \in \mathbb{U}$$

is classified as starlike and convex with order α, respectively, which is indicated as $\mathcal{S}^{\star}(\alpha)$.

Function theory is an area of mathematics whose origins can be traced back to the nineteenth century, when researchers first began to study analytic and univalent functions systematically. In 1916, Bieberbach's celebrated work [18] demonstrated the importance of this field and led to the formulation of the famous Bieberbach conjecture, which was eventually proved by de Branges [1] in 1985. To study the growth and geometric behavior of functions in various domains, it is important to understand both the theory of univalent functions ($\mathcal{UF}s$) and analytic functions ($\mathcal{AF}s$). This theory involves the investigation of Taylor coefficients and related functional inequalities. In 1933, Fekete and Szegö [19] introduced the celebrated Fekete–Szegö problem, which is closely related to the Bieberbach conjecture and the coefficient theory of univalent functions. The problem concerns the maximization of the nonlinear functional $|a_3 - \phi a_2^2|$ and has become one of the most important topics in geometric function theory. Since its introduction, numerous researchers have investigated the Fekete–Szegö problem for various subclasses of analytic and univalent functions and have obtained many significant results. Consequently, the study of the Fekete–Szegö problem for different classes of analytic functions remains an active and important area of research.

Noonan and Thomas, as reported in their 1976 work [20], brought forth the concept of the tth Hankel determinant for functions in $\mathcal{S}$, specifically formulated in Eq (1.1).

$$\mathcal{H}_{t,k}(z) = \begin{vmatrix} a_k & a_{k+1} & a_{k+2} & \cdots & a_{k+t-1} \\ a_{k+1} & a_{k+2} & a_{k+3} & \cdots & a_{k+t} \\ a_{k+2} & a_{k+3} & a_{k+4} & \cdots & a_{k+t+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{k+t-1} & a_{k+t} & a_{k+t+1} & \cdots & a_{k+2(t-1)} \end{vmatrix}, \qquad k \geq 1,\ t \geq 1,\ a_1 = 1. \tag{1.2}$$

Recent investigations have focused on obtaining bounds for Hankel determinants in various subclasses of analytic functions; see [21–23]. Guangadharan [24] investigated a class of bounded turning functions associated with three-leaf domains. The study of Hankel determinants for analytic functions has attracted considerable attention in recent years and continues to be an active area of research in geometric function theory.

Representing the $\mathcal{AF}s$ as τ and requiring a positive real part in $\mathbb{C}$, τ transforms $\mathbb{U}$ into a starlike domain centered at $\tau(0) = 1$, where $\tau'(0) > 0$ and is symmetric about the real axis. The class $S^{\star}(\tau)$ can be delineated.

$$S^{\star}(\tau) = \{f \in \mathcal{S} : M \prec \tau(z)\}. \tag{1.3}$$

Ma and Minda presented the class $S^{\star}(\tau)$ [25]. Different results are obtained when the function $\tau(z)$ is altered in Eq (1.3). Recently, numerous contributions in this direction have conducted thorough investigations into this subject and achieved various findings that we may mention:

(a) The expression $\tau(z) = 2[1 + e^{-z}]^{-1}$ is defined in [26].
(b) Extensive research on the function $\tau(z) = (1 + z)^{1/2}$ has been conducted in [27].
(c) The equation $\tau(z) = 1 + (4/3)z + (2/3)z^2$ was introduced in [28] and later analyzed in [29].
(d) The function $\tau(z) = e^z$ was introduced and studied in [30].
(e) The set $\mathcal{S}_l^{\star}$ refers to the group of functions $\tau(z) = (1 + z^2)^{1/2} + z$ and was analyzed in detail in [31].
(f) Kumar and Arora [32] examined the set $\mathcal{S}_p^{\star} = \mathcal{S}^{\star}(1 + \sinh^{-1} z)$ with the function $\tau(z) = 1 + \sinh^{-1} z$.
(g) Alotaibi et al. defined the set $\mathcal{S}_{\cosh}^{\star}$ as $\mathcal{S}^{\star}(\tau(z))$, where $\tau(z) = \cosh z$ [33].

Gandhi [16] introduced a particular group of starlike functions, labeled as $\mathfrak{S}^*_{3l}$, that is defined in the following manner:

$$\mathfrak{S}^*_{3l} = \left\{ f \in \mathcal{S} : M \prec 1 + \frac{4}{5}z + \frac{1}{5}z^4 \quad (z \in \mathbb{U}) \right\}.$$

Simply put, this class consists of functions where the ratio M is contained in a three-leaf-shaped area in the right half plane. Shi et al. [34] provided approximations for coefficients of functions in the class $\mathfrak{S}^*_{3l}$, specifically the third Hankel determinant coefficient. Arif et al. [35] created a type of bounded turning functions denoted by

$$\mathfrak{B}\mathfrak{T}^*_{3l} = \left\{ f \in \mathcal{S} : f'(z) \prec 1 + \frac{4}{5}z + \frac{1}{5}z^4 \quad (z \in \mathbb{U}) \right\}.$$

A geometric curve known as an epicycloid is generated by tracing a point on the circumference of a circle of radius h that rolls without slipping around the exterior of a fixed circle of radius y (see [36]). The curve is described by the parametric equations

$$x(l) = -\left[h\cos\left(\frac{kl}{h}\right) - k\cos l \right], \qquad y(l) = -\left[h\sin\left(\frac{kl}{h}\right) - k\sin l \right],$$

where $k = y + h$, and $-\pi \leq l \leq \pi$. The number of cusps of the epicycloid depends on the ratio k/h. In particular, when k/h is an integer, the curve possesses $(k/h) - 1$ cusps. Several special cases of epicycloids are well–known. For example, when $y = h$, the resulting curve is a cardioid with one cusp; when $y = 2h$, the curve is a nephroid with two cusps; and when $y = 5h$, it is called a ranunculoid, which has five cusps. These curves represent important examples within the broader family of epicycloids.

Gandhi [16] investigated a class of analytic functions whose image domain is bounded by a curve possessing three cusps, one located on the positive real axis and the other two situated at the angles $\pi/3$ and $5\pi/3$. Subsequently, Gandhi et al. [15] generalized this geometric construction by introducing a family of domains whose boundaries are parametrically represented by

$$x(l) = 1 + \frac{g}{g+1}\cos l + \frac{1}{g+1}\cos(gl), \tag{1.4}$$

$$y(l) = 1 + \frac{g}{g+1}\sin l + \frac{1}{g+1}\sin(gl), \tag{1.5}$$

where $g \in \mathbb{N}$. They further introduced the function

$$\psi_{g\mathcal{L}}(z) = 1 + \frac{gz}{g+1} + \frac{z^g}{g+1}, \qquad z \in \mathbb{U},$$

which maps the unit disk $\mathbb{U}$ onto a domain bounded by the curve defined in (1.4).

Definition 1.1. [15] Gandhi et al. defined $\mathfrak{S}^*_{g\mathcal{L}} = \mathfrak{S}^*(\psi_{g\mathcal{L}})$ to be the class of functions $f : \mathbb{U} \longrightarrow \mathbb{C}$ that satisfies

$$M \prec 1 + \frac{gz}{g+1} + \frac{z^g}{g+1}, \quad (z \in \mathbb{U}).$$

The condition is that g must be greater than or equal to 2, and the symbol $\prec$ is referred to as the subordination principle. For a deeper grasp of the subordination principle, refer to [37].

Quantum calculus, referred to as q-difference calculus, eliminates the need for derivatives when finding the limit of a ratio as the increment approaches zero. Instead, it depends on the q-difference operator, which is crucial for our subject matter. This calculus builds upon traditional mathematical analysis concepts by incorporating the parameter q, which belongs to the interval $(0, 1)$. The comprehensive research by Gasper and Rahman [9] offers detailed explanations and real-world uses of q-difference calculus in number theory, physics, and combinatorics. The q-calculus is commonly employed in engineering for mass and heat transfer, as well as for solving nonlinear and fuzzy differential equations in science and technology. In [10, 11], the q-difference operator $\mathfrak{D}_q$ and the q-calculus operator were groundbreaking advancements in the field. Ismail and colleagues were the first to employ the q-difference operator to expand the range of $\mathcal{S}^\star$ in quantum calculus. Srivastava [12] was a trailblazer in introducing basic (or q-) hypergeometric functions in GFT. Consequently, multiple mathematicians carried out significant studies that impacted the domain of GFT.

Jackson [10, 11] provided a definition for the q-difference operator in $\mathcal{UF}s$:

$$\mathfrak{D}_q f(z) = \frac{-f(z) + f(qz)}{z(-1+q)} = 1 + \sum_{k=2}^{\infty} [k]_q a_k z^{k-1}, \quad z \in \mathbb{U},$$

where

$$[k]_q = \frac{1-q^k}{1-q}, \quad k \in \mathbb{N}.$$

The deformation parameter q plays a essential role in the proposed framework. The coefficient estimates and functional inequalities obtained in this paper depend explicitly on the q-numbers $[k]_q$ and the operator coefficients $[\Phi_n(\gamma, \omega, b, c)]_q^m$. Therefore, the parameter q controls the transition between the quantum and classical settings. In particular, as $q \to 1^-$, the q-numbers converge to the corresponding integers, and the derived results reduce to their classical parts. Thus, the present study may be viewed as a continuous deformation of several known results in geometric function theory, while simultaneously providing new estimates in the q-calculus setting.

In Geometric function theory (GFT), Shaba et al. [14] introduced a new q-differential operator, denoted by $\mathcal{DO}_q$, in 2023. This operator provides a unified framework that extends several previously studied operators. In this paper, we use $\mathcal{DO}_q$ to define a new subclass of analytic functions and derive various coefficient estimates and functional inequalities. The operator is defined as follows:

Definition 1.2. [14] For $f \in \mathcal{A}$, $c \in \mathbb{N}$, $\omega \in [0, b]$, $\gamma \geq 0$, $m \in \mathbb{N}_0$, $q \in (0, 1)$, $z \in \mathbb{U}$, the $\mathcal{DO}q$ is defined by:

$$f(z) = \mathfrak{D}_q\left(\mathcal{S}_{\gamma,q}^{m-1,\omega,b,c} f(z)\right) = z + \sum_{r=2}^{\infty} [\Phi_r(\gamma, \omega, b, c)]_q^m d_r z^r,$$

where

$$\Phi_r(\gamma, \omega, b, c) = 1 + (r + b - \omega - 1)\mathcal{L}_s^c(\gamma),$$

and

$$\mathcal{L}_s^c(\gamma) = \sum_{s=1}^{c} \binom{c}{s} (-1)^{s+1} \gamma^r.$$

We can have the following:

$$D_q(f(z)) = 1 + \sum_{r=2}^{\infty}[\Phi_r(\gamma, \omega, b, c)]_q^m [r]_q d_r z^{r-1}. \tag{1.6}$$

$$zD_q^2(f(z)) = \sum_{r=2}^{\infty}[r-1]_q[\Phi_r(\gamma, \omega, b, c)]_q^m [r]_q d_r z^{r-1}. \tag{1.7}$$

The present study extends and substantially generalizes the bounded turning class introduced by Arif et al. [35]. Whereas the class studied in [35] is recovered as a special case when $m = 0$, $b = \omega$, $c = 1$, $\gamma = 1$, $g = 4$, and $q \to 1^-$, the proposed class $MS_{\gamma,\omega}^{m,b,c}(q, g)$ incorporates the recently introduced q-differential operator of Shaba et al. [14] along with the epicycloidal mapping function $\psi_{gL}(z)$. This additional structure introduces the parameter-dependent factors $[\Phi_r(\gamma, \omega, b, c)]_q^m$ into the coefficient relations, which are absent in the classical setting. Consequently, the coefficient bounds, Zalcman-type inequalities, Hankel determinant estimates, inverse coefficient bounds, Fekete–Szegö inequalities, and Krushkal-type inequalities obtained in this paper depend simultaneously on the q-deformation parameter and the operator parameters $(m, \gamma, \omega, b, c)$. Therefore, the present class is not merely a formal q-extension of [35]; rather, it provides a unified framework containing several previously studied subclasses as special cases while yielding new families of sharp coefficient estimates and functional inequalities in the setting of quantum calculus.

Definition 1.3. A function represented as f from a set $\mathcal{A}$ is considered to be in the class $\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q, g)$ if it satisfies the specified subordination condition

$$\partial_{\text{timilehin}f(z)} \prec \psi_{gL}(z), \quad z \in \mathcal{U}. \tag{1.8}$$

In simpler terms, the class $\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q, g)$ can be defined as:

$$\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q, g) = \left\{ f(z) \in \mathcal{S} : \partial_{\text{timilehin}f(z)} \prec \psi_{gL}(z) \right\}.$$

Here, $\psi_{gL}(z)$ is defined as $1 + (g/g + 1)z + z^g/g + 1$. The functions

$$f_1(z) = z + \frac{g}{[2]_q(g+1)[\Phi_2(\gamma, \omega, b, c)]_q^m} z^2 + \frac{1}{[5]_q(g+1)[\Phi_5(\gamma, \omega, b, c)]_q^m} z^5, \tag{1.9}$$

$$f_2(z) = z + \frac{g}{[3]_q(g+1)[\Phi_3(\gamma, \omega, b, c)]_q^m} z^3 + \frac{1}{[9]_q(g+1)[\Phi_9(\gamma, \omega, b, c)]_q^m} z^9, \tag{1.10}$$

$$f_3(z) = z + \frac{g}{[4]_q(g+1)[\Phi_4(\gamma, \omega, b, c)]_q^m} z^4 + \frac{1}{[13]_q(g+1)[\Phi_{13}(\gamma, \omega, b, c)]_q^m} z^{13}, \tag{1.11}$$

$$f_4(z) = z + \frac{g}{[5]_q(g+1)[\Phi_5(\gamma, \omega, b, c)]_q^m} z^5 + \frac{1}{[17]_q(g+1)[\Phi_{17}(\gamma, \omega, b, c)]_q^m} z^{17}, \tag{1.12}$$

belong to the category specified by $\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q, g)$ and play a vital role by showcasing exceptional characteristics in most of the issues discussed under $\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q, g)$ in the ongoing study.

Remark 1.1. *The extremal functions* $f_1, f_2, f_3,$ *and* f_4 *defined in* (1.9)–(1.12) *belong to the class* $MS_{\gamma,\omega}^{m,b,c}(q, g)$*. Indeed, for each of these functions, a direct computation shows that*

$$D_q\Big[S^{m,\omega,b,c}_{\gamma,q} f_i(z)\Big] = \psi_{gL}(z^r),$$

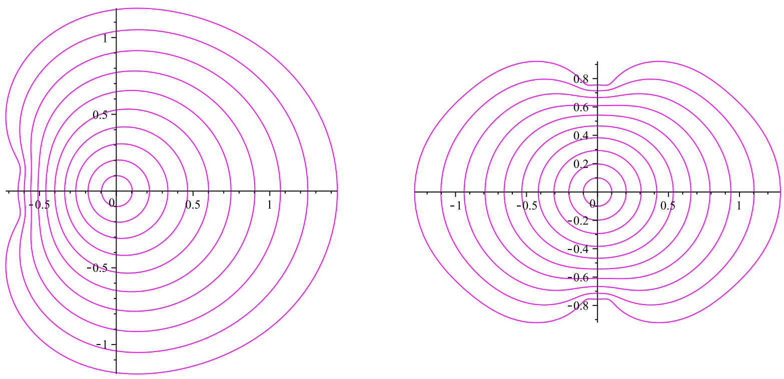
for a suitable positive integer r. Because z^r is a Schwarz function in $\mathbb{U}$, it follows that

$$D_q\Big[S^{m,\omega,b,c}_{\gamma,q} f_i(z)\Big] \prec \psi_{gL}(z),$$

and therefore, $f_i \in MS^{m,b,c}_{\gamma,\omega}(q, g)$, $i = 1, 2, 3, 4$.

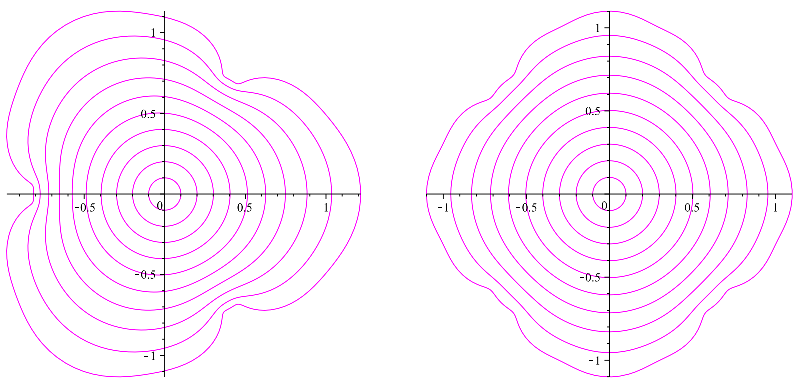
Furthermore, equality is attained in the corresponding coefficient estimates and functional inequalities established in this paper.

Geometric interpretation of figures: The image domains provided in Figures 1 and 2 correspond to the extremal functions f_1, f_2, f_3, and f_4. The figures analyzed the geometric effect of the dominant coefficients and show how the higher-order terms produce boundary deformations related with epicycloidal mappings. The symmetry and boundedness of the images are consistent with the defining subordination condition.



For $f_1(z)$ when $m = 0$, and $q \to 1-$. For $f_2(z)$ when $m = 0$, and $q \to 1-$.

Figure 1. The graphs of $f_1(z)$ and $f_2(z)$.



For $f_3(z)$ when $m = 0$, and $q \to 1-$. For $f_4(z)$ when $m = 0$, and $q \to 1-$.

Figure 2. The graphs of $f_3(z)$ and $f_4(z)$.

Remark 1.2. *When $m = 0, g = 4$, b is represented as ω, γ is one, c is one, and as q approaches the inverse of one, the operation $\mathcal{MS}^{m,b,c}_{\gamma,\omega}(q, g)$ simplifies to the class denoted as $\mathfrak{B}\mathfrak{T}^{*}_{3l}$ investigated by Arif* *et al.* in [35].

2. Relevant lemmas

For a deeper understanding of our findings, we emphasize the significance of the lemmas presented. Specifically, we introduce a set of functions denoted by $\mathcal{P}$, which can be vividly described as:

$$\wp(z) = 1 + \sum_{k=1}^{\infty} \wp_k z^k \quad (z \in \mathcal{U}, \quad \wp(0) = 1 \quad \text{and} \quad \Re\{\wp(z)\} > 0). \tag{2.1}$$

This specific family, labeled as $\mathcal{P}$, is associated with the well-known Carathéodory functions mentioned in [38]. The lemmas mentioned can be used for any function $\wp$ within the set $\mathcal{P}$.

Lemma 2.1. *[39] Suppose $\wp$ belongs to $\mathcal{P}$ and has the form* (2.1). *Then*

$$|\wp_k| \leq 2 \quad (k \in \mathbb{N}). \tag{2.2}$$

Additionally,

$$|\wp_k - \rho \wp_j \wp_{k-j}| \leq 2, \quad k > j, \quad \rho \in [0, 1], \tag{2.3}$$

which gives

$$f(z) = (1 + z)(1 - z)^{-1}.$$

Lemma 2.2. *[40] Suppose $\wp$ belongs to $\mathcal{P}$ and has the form* (2.1). *Afterward,*

$$|\wp_{k+j} - \rho \wp_k \wp_j| \leq \begin{cases} 2 & \text{for } \rho \in [0, 1], \\ 2|2\rho - 1| & \text{elsewhere}, \end{cases}$$

$$|\wp_k \wp_v - \wp_j \wp_\iota| < 4 \quad \text{for} \quad k + v = \iota + j,$$

$$|\wp_{k+2j} - \rho \wp_k \wp_j^2| \leq 2(1 + 2\rho) \quad \text{for} \quad \rho \in \mathbb{R},$$

and

$$\left| \wp_2 - \frac{\wp_1^2}{2} \right| \leq 2 - \frac{|\wp_1|^2}{2}.$$

Lemma 2.3. *[41] Suppose $\wp$ belongs to $\mathcal{P}$ and has the form* (2.1). *Afterward,*

$$|\wp_2 - \phi \wp_1^2| \leq 2 \max\{1, |2\phi - 1|\} \quad (\phi \in \mathbb{C}). \tag{2.4}$$

Lemma 2.4. *[42] Suppose $\wp$ belongs to $\mathcal{P}$ and has the form* (2.1). *Afterward,*

$$\begin{aligned}
\wp_2 &= \frac{1}{2}(4 - \wp_1^2)s + \frac{1}{2}\wp_1^2, \\
\wp_2^2 &= \frac{1}{4}\wp_1^4 + \frac{1}{2}(4 - \wp_1^2)\wp_1^2 s + (4 - \wp_1^2)^2 s^2, \\
\wp_3 &= \frac{1}{4}\wp_1^3 + \frac{1}{2}(4 - \wp_1^2)\wp_1 s - \frac{1}{4}(4 - \wp_1^2)\wp_1 s^2 + \frac{1}{2}(4 - \wp_1^2)(1 - |s|^2)z.
\end{aligned}$$

For s and z falling within the interval of -1 to 1, their absolute values must be less than or equal to 1.

Lemma 2.5. *[43] Suppose $\wp$ is in the set $\mathcal{P}$ and has the form* (2.1). *Afterward,*

$$|\wp_3 - 2Q\wp_1\wp_2 + C\wp_1^3| \leq 2$$

if

$$0 \leq Q \leq 1, \text{ and } Q(2Q-1) \leq C \leq Q.$$

Lemma 2.6. *[44] If $\sigma \in (0,1)$, $\varrho \in (0,1)$, and $\wp$ belongs to the class $\mathcal{P}$ and is of the form* (2.1)*, then*

$$\begin{aligned}&8\sigma(1-\sigma)[(\varrho\varphi - 2\nu)^2 + ((\varrho+\sigma)\varrho - \varphi)^2] + \varrho(1-\varrho)(\varphi - 2\sigma\varrho)^2\\&\leq 4\varrho^2\sigma(-\varrho+1)^2(-\sigma+1).\end{aligned}$$

Therefore,

$$\left|\nu\wp_1^4 + \sigma\wp_2^2 + 2\varrho\wp_1\wp_3 - \frac{3}{2}\varphi\wp_1^2\wp_2 - \wp_4\right| \leq 2. \tag{2.5}$$

The novelty of this paper lies in the introduction of a new subclass $MS_{\gamma,\omega}^{m,b,c}(q,g)$ of analytic functions defined via the recently introduced q-differential operator of Shaba et al. [14] and an epicycloidal mapping function. The proposed class unifies and extends several previously known subclasses as special cases. Within this framework, we derive sharp coefficient estimates, Zalcman-type inequalities, Hankel determinant bounds, inverse coefficient estimates, Fekete–Szegö inequalities, and Krushkal-type inequalities. The results explicitly depend on the deformation parameter q and the operator parameters, thereby providing new contributions to the theory of analytic functions in the setting of quantum calculus and epicycloidal image domains.

3. Coefficient bounds

Theorem 3.1. *If $g \geq 2$, and $f \in \mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$, then*

$$\begin{aligned}|a_2| &\leq \frac{g}{(1+g)[2]_q[\Phi_2(\gamma,\omega,b,c)]_q^m},\\|a_3| &\leq \frac{g}{(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m},\\|a_4| &\leq \frac{g}{(1+g)[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m},\\|a_5| &\leq \frac{g}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}.\end{aligned}$$

Moreover, the functions defined below attain their respective sharp bounds:

$$\begin{aligned}f_1(z) &= z + \frac{g}{[2]_q(g+1)[\Phi_2(\gamma,\omega,b,c)]_q^m}z^2 + \frac{1}{[5]_q(g+1)[\Phi_5(\gamma,\omega,b,c)]_q^m}z^5,\\f_2(z) &= z + \frac{g}{[3]_q(g+1)[\Phi_3(\gamma,\omega,b,c)]_q^m}z^3 + \frac{1}{[9]_q(g+1)[\Phi_9(\gamma,\omega,b,c)]_q^m}z^9,\\f_3(z) &= z + \frac{g}{[4]_q(g+1)[\Phi_4(\gamma,\omega,b,c)]_q^m}z^4 + \frac{1}{[13]_q(g+1)[\Phi_{13}(\gamma,\omega,b,c)]_q^m}z^{13},\\\text{and } f_4(z) &= z + \frac{g}{[5]_q(g+1)[\Phi_5(\gamma,\omega,b,c)]_q^m}z^5 + \frac{1}{[17]_q(g+1)[\Phi_{17}(\gamma,\omega,b,c)]_q^m}z^{17}.\end{aligned}$$

Proof. Let $f \in \mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$; then f satisfies (1.8), and we obtain that:

$$\partial_{\text{timilehin}f(z)} \prec 1 + \frac{gz}{g+1} + \frac{z^g}{g+1}.$$

By using subordination, it follows that

$$\partial_{\text{timilehin}f(z)} = 1 + \frac{g}{g+1}w(z) + \frac{1}{g+1}(w(z))^g. \tag{3.1}$$

Denote

$$\psi_{gL}(w(z)) = 1 + \frac{g}{g+1}w(z) + \frac{1}{g+1}(w(z))^g,$$

and

$$u(z) = 1 + \wp_1 z + \wp_2 z^2 + \cdots = \frac{1+w(z)}{1-w(z)}.$$

It is evident that the function $u(z)$ is a member of the set $\mathcal{P}$, and $w(z) = (u(z)-1)/(u(z)+1)$. This leads to

$$w(z) = \frac{u(z)-1}{u(z)+1} = \frac{\wp_1 z + \wp_2 z^2 + \wp_3 z^3 + \cdots}{2 + \wp_1 z + \wp_2 z^2 + \wp_3 z^3 + \cdots}$$

and

$$1 + \frac{g}{g+1}w(z) + \frac{1}{g+1}(w(z))^g$$

$$\begin{aligned} &= 1 + \frac{g\wp_1}{2(1+g)}z + \left[\frac{g\wp_2}{2(1+g)} - \frac{g\wp_1^2}{4(1+g)}\right]z^2 + \left[\frac{g\wp_1^3}{8(1+g)} - \frac{g\wp_2\wp_1}{2(1+g)} + \frac{g\wp_3}{2(1+g)}\right]z^3 \\ &+ \left[\frac{(1-g)\wp_1^4}{16(1+g)} + \frac{3g\wp_2\wp_1^2}{8(1+g)} - \frac{g\wp_3\wp_1}{2(1+g)} - \frac{g\wp_2^2}{4(1+g)} + \frac{g\wp_4}{2(1+g)}\right]z^4 + \cdots; \end{aligned} \tag{3.2}$$

also,

$$\begin{aligned} \partial_{\text{timilehin}f(z)} &= 1 + [2]_q[\Phi_2(\gamma,\omega,b,c)]_q^m a_2 z + [3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m a_3 z^2 \\ &+ [4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m a_4 z^3 + [5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m a_5 z^4 + \cdots. \end{aligned} \tag{3.3}$$

When we contrast (3.2) with (3.3), we note that

$$a_2 = \frac{g\wp_1}{2(1+g)[2]_q[\Phi_2(\gamma,\omega,b,c)]_q^m}, \tag{3.4}$$

$$a_3 = \frac{g}{2(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\left[\wp_2 - \frac{\wp_1^2}{2}\right], \tag{3.5}$$

$$a_4 = \frac{g}{2(1+g)[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}\left[\wp_3 - 2\left(\frac{1}{2}\right)\wp_2\wp_1 + \frac{1}{4}\wp_1^3\right], \tag{3.6}$$

$$a_5 = \frac{1}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}\left[\frac{-(g-1)}{16}\wp_1^4 + \frac{3g}{8}\wp_2\wp_1^2 - \frac{g}{2}\wp_3\wp_1 - \frac{g}{4}\wp_2^2 + \frac{g}{2}\wp_4\right]. \tag{3.7}$$

Applying Lemma 2.1 in relation to Eq (2.2) yields

$$|a_2| \leq \frac{g}{(1+g)[2]_q[\Phi_2(\gamma,\omega,b,c)]_q^m}.$$

By applying Lemma 2.3 to Eq (2.4), we get

$$|a_3| \leq \frac{g}{(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}.$$

After considering Eq (3.6), we get

$$a_4 = \frac{g}{2(1+g)[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}\left|\wp_3 - 2\left(\frac{1}{2}\right)\wp_2\wp_1 + \frac{1}{4}\wp_1^3\right|.$$

Given that $Q = 1/2$, and $C = 1/4$, which satisfy $Q(2Q-1) \leq C \leq Q$, when $g \geq 2$, and $q \in (0,1)$, Lemma 2.5 implies that

$$|a_4| \leq \frac{g}{(1+g)[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}.$$

Considering Eq (3.7), we conclude

$$a_5 = \frac{2g}{4(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}\left[\frac{g-1}{8g}\wp_1^4 - \frac{3}{2}\left(\frac{1}{2}\right)\wp_2\wp_1^2 + 2\left(\frac{1}{2}\right)\wp_3\wp_1 + \frac{1}{2}\wp_2^2 - \wp_4\right].$$

By using Lemma 2.6 with $\nu = (g-1)/8g$, $\sigma = 1/2$, $\varrho = 1/2$, and $\varphi = 1/2$ as parameters, we can verify that all conditions of Lemma 2.6 are satisfied, thereby confirming that

$$\begin{aligned}|a_5| &= \frac{-2g}{4(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}\left|\frac{g-1}{8g}\wp_1^4 - \frac{3}{2}\left(\frac{1}{2}\right)\wp_2\wp_1^2 + 2\left(\frac{1}{2}\right)\wp_3\wp_1 + \frac{1}{2}a_2^2 - \wp_4\right| \\ &\leq \frac{2g}{4(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m} \times 2 = \frac{g}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m},\end{aligned}$$

and hence,

$$|a_5| \leq \frac{g}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}.$$

□

Remark 3.1. *When m equals zero, b is represented as ω, $\gamma = 1$, and $c = 1$, $g = 4$, and $q \to 1^-$. The statement in Theorem 3.1 corresponds to the result shown earlier in [35].*

4. The Zalcman conjecture and its extended version

Zalcman came up with a result in 1960 regarding univalent functions, which suggests that any function $f(z)$ in class $\mathcal{S}$ and defined by Eq (1.1) must satisfy a certain inequality:

$$|a_k^2 - a_{2k-1}| < (k-1)^2, \quad k \geq 2. \tag{4.1}$$

In 1999, Ma expanded on the Zalcman conjecture, which was previously mentioned in [45], with additional details. He claimed that for any single-valued function belonging to the set $\mathcal{S}$, a specific inequality holds true:

$$|a_k a_j - a_{k-1+j}| \leq (k-1)(-1+j) \text{ for any } i, r \in \mathbb{N}, \quad k \geq 2, j \geq 2. \tag{4.2}$$

Setting k to be 2, Eq (4.1) transforms into another expression. The absolute difference between the square of a_2 and a_3 should be less than or equal to 1.

Theorem 4.1. *Assuming that $f \in \mathcal{MS}^{m,b,c}_{\gamma,\omega}(q, g)$, and it has the structure shown in Eq* (1.1)*, then*

$$|a_2^2 - a_3| \leq \frac{g}{(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}. \tag{4.3}$$

The maximum strictness of Inequality (4.3) *is reached when used with the function $f_2(z)$ as defined in* (1.10).

Proof. Examining Eqs (c1) and (c2) reveals

$$\begin{aligned}
|a_2^2 - a_3| &= \left| \frac{g}{2(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m} \left[\wp_2 - \frac{1}{2}\wp_1^2 \right] - \frac{g^2\wp_1^2}{4(1+g)^2[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2} \right| \\
&= \frac{g}{2(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m} \left| \wp_2 - \frac{1}{2}\left[1 + \frac{g[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{(1+g)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2}\right]\wp_1^2 \right| \\
&= \frac{g}{2(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m} |\wp_2 - \rho\wp_1^2|,
\end{aligned}$$

where

$$\rho = \frac{1}{2}\left[1 + \frac{g[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{(1+g)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2}\right].$$

Given that $0 < q < 1$, and $g \geq 2$, it can be inferred that ρ falls between the inclusive range of 0 and 1. Using Lemma 2.1 from Eq (2.3) with $k = 2$ and $j = 1$ leads to the derivation of Eq (4.3).

To validate the result, we have

$$f_2(z) = z + \frac{g}{[3]_q(g+1)[\Phi_3(\gamma,\omega,b,c)]_q^m} z^3 + \frac{1}{[9]_q(g+1)[\Phi_9(\gamma,\omega,b,c)]_q^m} z^9.$$

□

Remark 4.1. *When $m = 0$, b is represented as ω, $\gamma = 1, c = 1, g = 4$, and $q \to 1^-$, the statement in Theorem 4.1 coincides with the finding presented in [35].*

Setting k as 3 and j as 2 in Inequality (4.2) gives us the requirement that $|a_4 - a_2 a_3| \leq 2$. Next, we have:

Theorem 4.2. *Assuming that $f \in \mathcal{MS}^{m,b,c}_{\gamma,\omega}(q, g)$, and it has the structure shown in Eq* (1.1)*, then*

$$|a_4 - a_2 a_3| \leq \frac{g}{(1+g)[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}. \tag{4.4}$$

Inequality (4.4) *is most stringent when used with the function $f_3(z)$ as defined in* (1.11).

Proof. With Eqs (3.4)–(3.6) in mind, we find

$$|a_4 - a_2a_3| = \frac{g}{2(1+g)[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}\Bigg|\wp_3 - 2\Bigg[\frac{1}{2}+ \frac{g[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}{4(1+g)[2]_q[3]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_3(\gamma,\omega,b,c)]_q^m}\Bigg]\wp_1\wp_2 + \Bigg[\frac{1}{4}+ \frac{g[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}{4(1+g)[2]_q[3]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_3(\gamma,\omega,b,c)]_q^m}\Bigg]\wp_1^3\Bigg|.$$

Considering the value gives that

$$Q = \frac{1}{2} + \frac{g[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}{4(1+g)[2]_q[3]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_3(\gamma,\omega,b,c)]_q^m},$$

$$C = \frac{1}{4} + \frac{g[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}{4(1+g)[2]_q[3]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_3(\gamma,\omega,b,c)]_q^m}.$$

We observe that

$$Q(2Q-1) - C < 0, \text{when} \;\; g \geq 2,$$

which demonstrates that

$$Q(2Q-1) \leq C \leq Q.$$

As a result of using Lemma 2.5, we obtain the following outcome:

$$|a_4 - a_2a_3| \leq \frac{g}{(1+g)[4]_q[\Phi_4(\gamma,\omega,b,c)]_q^m}.$$

In order to attain clarity, consider the function $f_3(z)$ that meets the requirements:

$$f_3(z) = z + \frac{g}{[4]_q(g+1)[\Phi_4(\gamma,\omega,b,c)]_q^m}z^4 + \frac{1}{[13]_q(g+1)[\Phi_{13}(\gamma,\omega,b,c)]_q^m}z^{13}.$$

□

Remark 4.2. *When $m = 0$, b is represented as ω, $\gamma = 1, c = 1, g = 4$, and $q \to 1^-$, Theorem 4.2 corresponds to the result previously shown in [35].*

When the value of k is 3, Eq (4.1) is represented as follows:

$$|a_3^2 - a_5| \leq 4.$$

Theorem 4.3. *Suppose $f \in \mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$ and has the structure shown in Eq* (1.1)*; then*

$$|a_3^2 - a_5| \leq \frac{g}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}. \tag{4.5}$$

When applied to the function $f_4(z)$ as specified in (1.12)*, the strictest level of inequality* (4.5) *is attained.*

Proof. Using (3.5) and (3.7) yields

$$
\begin{aligned}
|a_3^2 - a_5| &= \Bigg| \frac{g^2}{4(1+g)^2[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\left[\wp_2^2 - \wp_1^2\wp_2 + \frac{\wp_1^4}{4}\right] - \\
&\quad \frac{g}{4(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}\left[\frac{g-1}{4g}\wp_1^4 - \frac{3}{2}\wp_2\wp_1^2 + 2\wp_3\wp_1 + \wp_2^2 - 2\wp_4\right]\Bigg| \\
&= \frac{g}{2[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m(1+g)}\Bigg|\left(\frac{[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{32(1+g)[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2} + \frac{g-1}{8g}\right)\wp_1^4 \\
&\quad + \left(\frac{g[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{2(g+1)[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2} + \frac{1}{2}\right)\wp_2^2 + 2\left(\frac{1}{2}\right)\wp_1\wp_3 \\
&\quad - \frac{3}{2}\left(\frac{1}{2} + \frac{g[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{3(1+g)[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\right)\wp_1^2\wp_2 - \wp_4\Bigg|.
\end{aligned}
$$

Applying Lemma 2.6 using the specified parameters implies that

$$
\nu = \frac{[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{32(1+g)[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2} + \frac{g-1}{8g}, \quad \sigma = \frac{g[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{2(g+1)[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2} + \frac{1}{2},
$$

$$
\varrho = \frac{1}{2}, \quad \varphi = \frac{1}{2} + \frac{g[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{3(1+g)[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2};
$$

we can verify that all the conditions required for Lemma 2.6 are satisfied. Therefore,

$$
|a_3^2 - a_5| = |a_5 - a_3^2| \leq \frac{g}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}.
$$

□

Remark 4.3. *When $m = 0$, b is represented as ω, $\gamma = 1, c = 1, g = 4$, and $q \to 1^-$. The result shown in [35] is consistent with Theorem 4.3.*

5. The Hankel determinant

Theorem 5.1. *Assuming $f(z) \in \mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$, then*

$$
|a_2a_4 - a_3^2| \leq \frac{g^2}{(g+1)^2[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}. \tag{5.1}
$$

Inequality (5.1) *is most strict when used with the function $f_2(z)$ as defined in* (1.10).

Proof. Through the use of the formulas denoted as (3.4)–(3.6), we obtain the following outcome:

$$
\begin{aligned}
a_2a_4 - a_3^2 &= \frac{g^2\wp_1^4}{16(1+g)^2[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m} \\
&\quad - \frac{g^2\wp_1^2\wp_2}{4(1+g)^2[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m} + \frac{g^2\wp_1^2\wp_2}{4(1+g)^2[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}
\end{aligned}
$$

$$+\frac{g^2\wp_1\wp_3}{4(1+g)^2[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}-\frac{g^2\wp_2^2}{4(1+g)^2[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}$$
$$-\frac{g^2\wp_1^4}{16(1+g)^2[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}.$$

After simplifying the process, we get

$$a_2a_4-a_3^2=\frac{g^2}{16(1+g)^2}\Bigg[\frac{4}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp_1\wp_3-\frac{4}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\wp_2^2$$
$$+\left(\frac{4}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}-\frac{4}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\right)\wp_1^2\wp_2$$
$$+\left(\frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}-\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\right)\wp_1^4\Bigg].$$

By substituting the values of $\wp_2$ and $\wp_3$ according to Lemma 2.4, and then simplifying, we get

$$a_2a_4-a_3^2=\frac{g^2}{16(1+g)^2}\Bigg[\left(\frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}-\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\right)\wp_1^4$$
$$+\frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp_1\Big[\wp_1^3+2\wp_1(4-\wp_1^2)s-\wp_1(4-\wp_1^2)s^2$$
$$+2(4-\wp_1^2)(1-|s|^2)z\Big]+\left(\frac{2}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}-\frac{2}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\right)\wp_1^2$$
$$[\wp_1^2+(4-\wp_1^2)s]-\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\left[\wp_1^2+(4-\wp_1^2)s\right]^2\Bigg].$$

By replacing $|s|$ with δ and $\wp_1$ with $\wp$ in the module, it follows that

$$|a_2a_4-a_3^2|\le\frac{g^2}{16(1+g)^2}\Bigg[\frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp^2(4-\wp^2)\delta^2$$
$$+\frac{2}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp(4-\wp^2)\delta^2+\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}(4-\wp^2)^2\delta^2$$
$$+\frac{2}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp(4-\wp^2)\Bigg]=\mathfrak{F}(\wp,\delta).$$

Hence, we have

$$\frac{\partial\mathfrak{F}(\wp,\delta)}{\partial\delta}=\frac{g^2}{16(1+g)^2}\Bigg[\Bigg(\frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp^2$$
$$+\frac{2}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp+\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}(4-\wp^2)\Bigg)2\delta(4-\wp^2)\Bigg].$$

It is clear that the partial derivative of $\mathfrak{F}(\wp,\delta)$ with respect to δ is positive. The data indicates that the function $\mathfrak{F}$ of the variables $\wp$ and δ gradually increases as δ moves within the interval from 0 to 1. It follows that the largest value of $\mathfrak{F}$ occurs when $\delta=1$. Consequently, we can conclude that

$$\max\{\mathfrak{F}(\wp,\delta)\}=\mathfrak{F}(\wp,1)=\mathcal{G}(\wp).$$

At this moment, we are observing that

$$\mathcal{G}(\wp) = \frac{g^2}{16(1+g)^2}\Bigg[\left(\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2} - \frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\right)\wp^4$$
$$- \frac{4}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp^3 + \frac{8}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp$$
$$+ \left(\frac{4}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m} - \frac{8}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\right)\wp^2$$
$$+ \frac{16}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\Bigg]. \tag{5.2}$$

Differentiating (5.2) with respect to $\wp$ results in

$$\mathcal{G}'(\wp) = \frac{g^2}{16(1+g)^2}\Bigg[4\left(\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2} - \frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\right)\wp^3$$
$$- \frac{12}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp^2 + \frac{8}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}$$
$$+ 2\left(\frac{4}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m} - \frac{8}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\right)\wp\Bigg].$$

By differentiating again with respect to $\wp$ the equation provided, we get

$$\mathcal{G}''(\wp) = \frac{g^2}{16(1+g)^2}\Bigg[12\left(\frac{1}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2} - \frac{1}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\right)\wp^2$$
$$+ 2\left(\frac{4}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m} - \frac{8}{[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}\right)$$
$$- \frac{24}{[2]_q[4]_q[\Phi_2(\gamma,\omega,b,c)]_q^m[\Phi_4(\gamma,\omega,b,c)]_q^m}\wp\Bigg].$$

When $\wp$ is zero, the highest value of $\mathcal{G}(\wp)$ within the range $[0,2]$ is found at $\wp = 0$. As a result, we can conclude that

$$|a_2a_4 - a_3^2| \leq \frac{g^2}{(g+1)^2[3]_q^2([\Phi_3(\gamma,\omega,b,c)]_q^m)^2}.$$

Therefore, the proof of Theorem 5.1 has been completed. □

Remark 5.1. *When $m = 0$, b is represented as ω, $\gamma = 1, c = 1, g = 4$, and $q \to 1^-$. The statement in Theorem 5.1 is consistent with the finding shown earlier in [35].*

Theorem 5.2. *If the function $f(z)$ satisfies Eq (1.1) and belongs to the class $\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$, then*

$$|\mathcal{H}_3(1)| \leq \frac{g^3}{(1+g)^3}\Bigg[\frac{1+g}{g[5]_q[3]_q[\Phi_5(\gamma,\omega,b,c)]_q^m[\Phi_3(\gamma,\omega,b,c)]_q^m} + \frac{1+g}{g[4]_q^2([\Phi_4(\gamma,\omega,b,c)]_q^m)^2}$$
$$+ \frac{1}{[3]_q^3([\Phi_3(\gamma,\omega,b,c)]_q^m)^3}\Bigg].$$

Proof. Using (1.2) and $a_1 = 1$, we obtain

$$H_3(1) = \begin{vmatrix} 1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \\ a_3 & a_4 & a_5 \end{vmatrix}. \tag{5.3}$$

Expanding the determinant of (5.3), we get

$$|\mathcal{H}_3(1)| \leq |a_3|\,|a_3^2 - a_2a_4| + |a_4|\,|a_2a_3 - a_4| + |a_5|\,|a_2^2 - a_3|. \tag{5.4}$$

Using the Theorems 3.1, 4.1, 4.2 and 5.1 in Inequality (5.4), we have

$$|\mathcal{H}_3(1)| \leq \frac{g^3}{(1+g)^3}\Bigg[\frac{1+g}{g[5]_q[3]_q[\Phi_5(\gamma,\omega,b,c)]_q^m[\Phi_3(\gamma,\omega,b,c)]_q^m} + \frac{1+g}{g[4]_q^2([\Phi_4(\gamma,\omega,b,c)]_q^m)^2}$$
$$+ \frac{1}{[3]_q^3([\Phi_3(\gamma,\omega,b,c)]_q^m)^3}\Bigg].$$

Therefore, the proof of Theorem 5.2 has been completed. □

6. Disparities in coefficients of the inverse function $f^{-1}(z)$

Theorem 6.1. *The functions $f(z)$ and its inverse $f^{-1}(\kappa) = \kappa + \sum_{r=2}^{\infty} e_r\kappa^r$, both defined according to $\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$, can be extended analytically to the region Δ, assuming that the absolute value of κ is bounded by a specific value,*

$$|e_2| \leq \frac{g}{(1+g)[2]_q[\Phi_2(\gamma,\omega,b,c)]_q^m}$$

$$|e_3| \leq \frac{2g^2}{(1+g)^2[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2},$$

and

$$|e_3 - \phi e_2^2| \leq \frac{g}{(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\max\left\{1, \frac{g(2-\phi)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{(1+g)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2}\right\}.$$

Proof. Let

$$f^{-1}(\kappa) = \kappa + \sum_{r=2}^{\infty} e_r\kappa^r$$

be the inverse of $f(z)$ and thus,

$$z = z + (a_2 + e_2)z^2 + (a_3 + 2a_2e_2 + e_3)z^3 + \cdots. \tag{6.1}$$

By equating the coefficients of z^2 and z^3 in Eq (6.1), we get the following result:

$$e_2 = -a_2 \tag{6.2}$$

and

$$e_3 = -a_3 - 2a_2e_2 = 2a_2^2 - a_3. \tag{6.3}$$

By utilizing Eqs (3.4), (3.5), (6.2), and (6.3), we can obtain the following results:

$$e_2 = -\frac{g\wp_1}{2(1+g)[2]_q[\Phi_2(\gamma,\omega,b,c)]_q^m}$$

and

$$e_3 = \frac{-g}{2(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\left[\wp_2 - \left(\frac{g[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{(g+1)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2} + \frac{1}{2}\right)\wp_1^2\right].$$

Furthermore, we can calculate for every complex number ϕ:

$$|e_3 - \phi e_2^2| = \frac{g}{2(g+1)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\left|\wp_2 - \left(\frac{g(2-\phi)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{2(g+1)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2} + \frac{1}{2}\right)\wp_1^2\right|. \tag{6.4}$$

Using a previously established result (Lemma (2.3)) and applying the modulus to both sides of the Eq (6.4), we can obtain

$$|e_3 - \phi e_2^2| \le \frac{g}{(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\max\left\{1, \frac{g(2-\phi)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{(1+g)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2}\right\}.$$

□

7. Functional coefficients related to the ratio of ξ divided by $f(\xi)$

We have the condition for the ratio as:

$$N(z) = z\cdot f^{-1}(z) = \sum_{r=1}^{\infty} p_r z^r + 1. \tag{7.1}$$

In this case, the function f is a member of the class represented by $\mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$.

Theorem 7.1. *We have $f \in \mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$ and $N(z)$ given in* (7.1)*; then the following is true:*

$$|p_2 - \phi p_1^2| \le \frac{g}{(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\max\left\{1, \left|\frac{g(1-\phi)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{(1+g)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2}\right|\right\}.$$

Proof. By simple calculation, it can be shown that

$$N(z) = 1 - a_2 z + (a_2^2 - a_3)z^2 + \cdots.$$

Hence, we get

$$p_1 = -a_2 \tag{7.2}$$

and

$$p_2 = a_2^2 - a_3. \tag{7.3}$$

By making use of (3.4) and (3.5) in Eqs (7.2) and (7.3), we derive the subsequent outcome:

$$|p_2 - \phi p_1^2| = \frac{g}{2(g+1)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\left|\wp_2 - \left(\frac{g(1-\phi)[3]_q[3]_q^m}{(g+1)[2]_q^2([2]_q^m)^2} + \frac{1}{2}\right)\wp_1^2\right|.$$

By utilizing Lemma 2.3, we obtain

$$|p_2 - \phi p_1^2| \le \frac{g}{(1+g)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}\max\left\{1, \left|\frac{2q(1-\phi)[3]_q[\Phi_3(\gamma,\omega,b,c)]_q^m}{(1+g)[2]_q^2([\Phi_2(\gamma,\omega,b,c)]_q^m)^2}\right|\right\}.$$

□

8. Krushkal inequality

Krushkal [46] initiated and demonstrated (8.1) for the set $\mathcal{S}$. Hence, we have

$$|a_k^{\varsigma} - a_2^{\varsigma(k-1)}| \leq 2^{\varsigma(k-1)} - k^{\varsigma}. \tag{8.1}$$

The following theorem shows that Inequality (8.1) is valid when $\varsigma = 1$, and $k = 5$ in the class.

Theorem 8.1. *If* $f \in \mathcal{MS}_{\gamma,\omega}^{m,b,c}(q,g)$. *Therefore*

$$|a_5 - a_2^4| \leq \frac{g}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}. \tag{8.2}$$

Inequality (8.2) *is most strictly enforced when applied to the function* $f_4(z)$ *outlined in* (1.12).

Proof. Using Eqs (3.4) and (3.7), it can be derived that

$$\begin{aligned}|a_5 - a_2^4| = &\Bigg|\frac{g^4\wp_1^4}{16(1+g)^4[2]_q^4([\Phi_2(\gamma,\omega,b,c)]_q^m)^4} + \\ &\frac{g}{4(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}\left[\frac{g-1}{4g}\wp_1^4 - \frac{3}{2}\wp_2\wp_1^2 + 2\wp_3\wp_1 + \wp_2^2 - 2\wp_4\right]\Bigg| \\ = &\frac{g}{2[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m(1+g)}\Bigg|\left(\frac{g^3[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{8(1+g)^3[2]_q^4([\Phi_2(\gamma,\omega,b,c)]_q^m)^4} + \frac{g-1}{8g}\right)\wp_1^4 \\ &+ \left(\frac{1}{2}\right)\wp_2^2 + 2\left(\frac{1}{2}\right)\wp_1\wp_3 - \frac{3}{2}\left(\frac{1}{2}\right)\wp_1^2\wp_2 - \wp_4\Bigg|.\end{aligned}$$

Applying Lemma 2.6 using the specified parameters gives

$$\nu = \frac{g^3[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}{8(1+g)^3[2]_q^4([\Phi_2(\gamma,\omega,b,c)]_q^m)^4} + \frac{g-1}{8g}, \quad \sigma = \frac{1}{2},\ \varrho = \frac{1}{2},\ \varphi = \frac{1}{2},$$

we can verify that all the conditions for Lemma 2.6 have been satisfied. Therefore

$$|a_5 - a_2^4| \leq \frac{g}{(1+g)[5]_q[\Phi_5(\gamma,\omega,b,c)]_q^m}.$$

□

9. Conclusions

In Section 1, we reviewed the necessary background material and introduced the fundamental concepts required throughout the paper. We also discussed number of subclasses of univalent functions related with domains in the right-half plane and presented the fundamental properties of the q-derivative along with the q-differential operator introduced by Shaba et al. [14]. Using this new operator and an epicycloidal mapping function, we introduced a new subclass of analytic and univalent functions. Section 2 provided the auxiliary lemmas that serve as the foundation for the proofs of our main results. In Section 3, sharp estimates for the first five Taylor coefficients were obtained.

Section 4 investigated Zalcman-type inequalities and their generalized forms, which extends several existing results. In Section 5, estimates for the second and third Hankel determinants were established. Section 6 was devoted to inverse coefficient estimates for the inverse function $f^{-1}(z)$, which generalize a number of previously known results. In Section 7, we established coefficient estimates related with the function $z/f(z)$ and derived corresponding Fekete–Szegö type inequalities. Finally, in Section 8, a Krushkal-type inequality was established for the proposed class. Several known special cases of our main findings were discussed.

There are several directions for future research. One may investigate the newly defined class by using subordination, alternative q-calculus operators, or newly developed differential and integral operators. The techniques employed in this research may also be extended to harmonic, multivalent, meromorphic, and bi-univalent function subclasses. Furthermore, the interaction between epicycloidal domains and quantum calculus remains a promising area for future investigation and may lead to the discovery of new classes.

Author contributions

Isra Al-Shbeil1: Conceptualization, methodology, software, formal analysis, investigation, writing-original draft preparation, writing-review and editing, supervision; Wael Mahmoud Mohammad Salameh: Conceptualization, software, validation, formal analysis, investigation, writing-review and editing; Maha Alammari: Methodology, software, validation, formal analysis, investigation, writing-review and editing; Alina Alb Lupas: Methodology, software, validation, formal analysis, investigation, writing-review and editing. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no competing interests.

References

1. L. D. Branges, A proof of the Bieberbach conjecture, *Acta Math.*, **154** (1985), 137–152. https://doi.org/10.1007/BF02392821
2. H. M. Zayed, T. Bulboacă, Normalized generalized Bessel function and its geometric properties, *J. Inequal. Appl.*, **2022** (2022), 158. https://doi.org/10.1186/s13660-022-02891-0
3. H. M. Zayed, T. Bulboacă, On some geometric properties for the combination of generalized Lommel–Wright function, *J. Inequal. Appl.*, **2021** (2021), 158. https://doi.org/10.1186/s13660-021-02690-z
4. I. Al-Shbeil, N. Khan, F. Tchier, Q. Xin, S. N. Malik, S. Khan, Coefficient bounds for a family of s-fold symmetric bi-univalent functions, *Axioms*, **12** (2023), 317. https://doi.org/10.3390/axioms12040317
5. K. Ullah, I. Al-Shbeil, M. I. Faisal, M. Arif, H. Alsaud, Results on second-order Hankel determinants for convex functions with symmetric points, *Symmetry*, **15** (2023), 939. https://doi.org/10.3390/sym15040939
6. A. O. Lasode, T. O. Opoola, I. Al-Shbeil, T. G. Shaba, H. Alsaud, Concerning a novel integral operator and a specific category of starlike functions, *Mathematics*, **11** (2023), 4519. https://doi.org/10.3390/math11214519
7. A. Saliu, K. Jabeen, I. Al-Shbeil, N. Aloraini, S. N. Malik, On q-limaçon functions, *Symmetry*, **14** (2022), 2422. https://doi.org/10.3390/sym14112422
8. M. F. Khan, I. Al-Shbeil, S. Khan, N. Khan, W. U. Haq, J. Gong, Applications of a q-differential operator to a class of harmonic mappings defined by q-Mittag–Leffler functions, *Symmetry*, **14** (2022), 1905. https://doi.org/10.3390/sym14091905
9. G. Gasper, M. Rahman, *Basic hypergeometric series*, Cambridge: Cambridge University Press, **96** (2011).
10. F. H. Jackson, On q-functions and a certain difference operator, *Earth Env. Sci. T. R. So.*, **46** (1909), 253–281. https://doi.org/10.1017/S0080456800002751
11. F. H. Jackson, On q-definite integrals, *Quart. J.*, **41** (1910), 193–203.
12. H. M. Srivastava, *Univalent functions, fractional calculus, and associated generalized hypergeometric functions*, In: Univalent Functions, Fractional Calculus, and Their Applications, John Wiley & Sons, New York: Halsted Press, 1989, 329–354.
13. S. Baroudi, M. Elomari, A. E. Mfadel, A. Kassidi, Numerical solutions of the integro-partial fractional diffusion heat equation involving tempered ψ-Caputo fractional derivative, *J. Math. Sci.*, **271** (2023), 555–567. https://doi.org/10.1007/s10958-023-06640-6
14. T. G. Shaba, S. Araci, B. O. Adebesin, F. Tchier, S. Zainab, B. Khan, Sharp bounds of the Fekete–Szegö problem and second Hankel determinant for certain bi-univalent functions defined by a novel q-differential operator associated with a q-limaçon domain, *Fractal Fract.*, **7** (2023), 506. https://doi.org/10.3390/fractalfract7070506
15. S. Gandhi, P. Gupta, S. Nagpal, V. Ravichandran, Starlike functions associated with an epicycloid, *Hacet. J. Math. Stat.*, **51** (2022), 1637–1660.

16. S. Gandhi, *Radius estimates for the three-leaf function and convex combinations of starlike functions*, In: Mathematical Analysis I: Approximation Theory, Singapore: Springer, 2018.

17. P. L. Duren, *Univalent functions*, New York: Springer, **259** (2001).

18. L. Bieberbach, Über die Koeffizienten derjenigen Potenzreihen, welche eine schlichte Abbildung des Einheitskreises vermitteln, *Sitzungsber. Königl. Preuss. Akad. Wiss.*, **138** (1916), 940–955.

19. M. Fekete, G. Szegö, Eine Bemerkung über ungerade schlichte Funktionen, *J. Lond. Math. Soc.*, **8** (1933), 85–89.

20. J. W. Noonan, D. K. Thomas, On the second Hankel determinant of areally mean p-valent functions, *T. Am. Math. Soc.*, **223** (1976), 337–346. https://doi.org/10.1090/S0002-9947-1976-0422607-9

21. M. Arif, K. I. Noor, M. Raza, Hankel determinant problem of a subclass of analytic functions, *J. Inequal. Appl.*, **2012** (2012), 22. https://doi.org/10.1186/1029-242X-2012-22

22. N. E. Cho, B. Kowalczyk, O. S. Kwon, A. Lecko, Y. J. Sim, Some coefficient inequalities related to the Hankel determinant for strongly starlike functions of order α, *J. Math. Inequal.*, **11** (2017), 429–439. https://doi.org/10.7153/jmi-2017-11-36

23. N. E. Cho, V. Kumar, Initial coefficients and fourth Hankel determinant for certain analytic functions, *Miskolc Math. Notes*, **21** (2020), 763–779. https://doi.org/10.18514/MMN.2020.3083

24. G. Murugusundaramoorthy, M. G. Khan, B. Ahmad, W. K. Mashwani, T. Abdeljawad, Z. Salleh, Coefficient functionals for a class of bounded turning functions connected to the three-leaf function, *J. Math. Comput. Sci.*, **28** (2022), 213–223. https://doi.org/10.22436/jmcs.028.03.01

25. W. Ma, C. Minda, *A unified treatment of some special classes of univalent functions*, In: Proceedings of the Conference on Complex Analysis, Boston: International Press, 1992.

26. P. Goel, S. S. Kumar, Certain classes of starlike functions associated with a modified sigmoid function, *B. Malays. Math. Sci. So.*, **43** (2020), 957–991. https://doi.org/10.1007/s40840-019-00784-y

27. J. Sokół, J. Stankiewicz, Radius of convexity of some subclasses of strongly starlike functions, *Zesz. Nauk. Politech. Rzesz. Mat.*, **19** (1996), 101–105.

28. K. Sharma, N. K. Jain, V. Ravichandran, Starlike functions associated with a cardioid, *Afr. Mat.*, **27** (2016), 923–939. https://doi.org/10.1007/s13370-015-0387-7

29. L. Shi, I. Ali, M. Arif, N. E. Cho, S. Hussain, H. Khan, A study of the third Hankel determinant problem for certain subfamilies of analytic functions involving a cardioid domain, *Mathematics*, **7** (2019), 418. https://doi.org/10.3390/math7050418

30. R. Mendiratta, S. Nagpal, V. Ravichandran, On a subclass of strongly starlike functions associated with an exponential function, *B. Malays. Math. Sci. So.*, **38** (2015), 365–386. https://doi.org/10.1007/s40840-014-0026-8

31. R. K. Raina, J. Sokół, On coefficient estimates for a certain class of starlike functions, *Hacet. J. Math. Stat.*, **44** (2015), 1427–1433.

32. S. S. Kumar, K. Arora, Starlike functions associated with a petal-shaped domain, *arXiv Preprint*, 2020, 10072. https://doi.org/10.48550/arXiv.2010.10072

33. A. Alotaibi, M. Arif, M. A. Alghamdi, S. Hussain, Starlikeness associated with the hyperbolic cosine function, *Mathematics*, **8** (2020), 1118. https://doi.org/10.3390/math8071118

34. L. Shi, M. G. Khan, B. Ahmad, W. K. Mashwani, P. Agarwal, S. Momani, Certain coefficient estimate problems for three-leaf-type starlike functions, *Fractal Fract.*, **5** (2021), 137. https://doi.org/10.3390/fractalfract5040137

35. M. Arif, O. M. Barukab, S. A. Khan, M. Abbas, The sharp bounds of Hankel determinants for the families of three-leaf-type analytic functions, *Fractal Fract.*, **6** (2022), 291. https://doi.org/10.3390/fractalfract6060291

36. J. D. Lawrence, *A catalog of special plane curves*, New York: Dover Publications, 1972.

37. P. L. Duren, *Univalent functions*, New York: Springer-Verlag, 1983.

38. O. A. F. Joseph, Y. O. Afolabi, B. O. Moses, Coefficient inequalities for a class of harmonic univalent functions, *Gulf J. Math.*, **3** (2015), 85–97. https://doi.org/10.56947/gjom.v3i4.33

39. A. W. Goodman, *Univalent functions*, Tampa, FL: Mariner Publishing Company, **1** (1983).

40. F. Keogh, E. Merkes, A coefficient inequality for certain subclasses of analytic functions, *P. Am. Math. Soc.*, **20** (1969), 8–12. https://doi.org/10.1090/S0002-9939-1969-0232926-9

41. H. M. Srivastava, D. Răducanu, P. Zaprawa, A certain subclass of analytic functions defined by means of differential subordination, *Filomat*, **30** (2016), 3743–3757. https://doi.org/10.2298/FIL1614743S

42. R. J. Libera, E. J. Złotkiewicz, Early coefficients of the inverse of a regular convex function, *P. Am. Math. Soc.*, **85** (1982), 225–230. https://doi.org/10.1090/S0002-9939-1982-0652447-5

43. M. Arif, M. Raza, H. Tang, S. Hussain, H. Khan, Hankel determinant of order three for familiar subsets of analytic functions related to the sine function, *Open Math.*, **17** (2019), 1615–1630. https://doi.org/10.1515/math-2019-0132

44. V. Ravichandran, S. Verma, Bound for the fifth coefficient of certain starlike functions, *C. R. Math. Acad. Sci. Paris*, **353** (2015), 505–510. https://doi.org/10.1016/j.crma.2015.03.003

45. W. Ma, Generalized Zalcman conjecture for starlike and typically real functions, *J. Math. Anal. Appl.*, **234** (1999), 328–339. https://doi.org/10.1006/jmaa.1999.6378

46. S. K. Krushkal, A short geometric proof of the Zalcman and Bieberbach conjectures, *arXiv Preprint*, 2014. https://doi.org/10.48550/arXiv.1408.1948