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Research article

Generalized analysis of the coupon collector problem with reset button

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Abstract: In this paper, we study a generalized version of the coupon collector problem that includes a reset button, which restarts the collection process when it is drawn. We derive an explicit formula for the expected waiting time to collect all distinct coupons in the general case where standard coupons are drawn with arbitrary probabilities. Using this result, we reprove the recent expected waiting time formula by Jocković and Todić in [8] for the case where standard coupons have equal probabilities, which is expressed in terms of the beta function.

Keywords: coupon collector problem; reset coupon; binomial inversion
Mathematics Subject Classification: 60C05, 60G50

1. Introduction

The coupon collector problem has a long history (see, e.g., [4, 6]) since its introduction. The goal is to determine the expected waiting time to collect all n distinct coupons from the set $\mathbb{N}_n = \{1, 2, \ldots, n\}$, assuming that coupons are drawn independently each time with a fixed probability distribution of $\mathbb{N}_n$.

The classical coupon collector problem is one of the fundamental models in probability theory and combinatorics. It has been widely studied because of its connections with occupancy problems, random processes, and various algorithmic applications. In particular, the analysis of the waiting time required to obtain a complete collection has attracted considerable attention, and many extensions of the original model have been proposed to capture more realistic stochastic mechanisms.

Recently, several generalizations of the coupon collector problem have been studied. One category introduces additional coupons or mechanisms designed for special purposes. For instance, the universal coupon is designed to speed up the collection process [7], whereas the null coupon [1] and the penalty coupon [17] are introduced to slow it down. Other variants incorporate bonus coupons [12], explore connections to hypergraph transversals [18], or introduce a reset button that restarts the process upon drawing [8]. Furthermore, analyzing the collection process when standard coupons are drawn with arbitrary probabilities remains an active area of research; recently, León-García et al. [9] addressed this by deriving explicit, computationally efficient recursive formulas for the higher-order moments of the waiting time in the general nonequiprobable case.

In [8], Jelena Jocković and Bojana Todić introduced a reset coupon which restarts the collection process when it is drawn (i.e., all previously collected coupons are lost and the count returns to zero). They derive the distribution of the waiting time until a full collection is completed. They apply Markov chain techniques to obtain an explicit formula for the expected waiting time for the case where standard coupons have equal probabilities, which involves the beta function. For other generating function theories related to the beta function, see [15, 16].

Generalized coupon collector problems have found applications in various fields, including cryptanalysis, engineering, and ecology (see, for example, [2, 13, 14, 20]). For instance, they have been used to distinguish the correct key guess from noisy data [13], and to compare the influence of the degree of heterogeneity among hosts on the speed of infection [20].

Building on the work in [8], this paper continues to explore the coupon collector problem with a reset button. We derive an explicit formula for the expected waiting time to complete a full collection in the general case and use it to deduce the expected waiting time formula presented in [8] for the case where all standard coupons have equal probabilities. To the best of our knowledge, an explicit formula for the expected waiting time in the general nonequiprobable case with a reset button has not previously appeared in the literature.

The main contribution of this work is an explicit closed-form expression for the expected waiting time in the nonequiprobable coupon collector setting with a reset mechanism. In contrast to the equiprobable case, the general case involves more complex combinatorial structures, which we treat using generating functions and binomial inversion. This result addresses a gap in the literature, as previous studies, such as [8], were restricted to uniform distributions. Furthermore, the analytical approach developed here may be adaptable to other stochastic processes that involve reset events.

From a practical standpoint, analyzing the coupon collector's problem with a reset mechanism may offer valuable perspectives in various domains. In computer science and network security, it could serve as a theoretical basis for evaluating system recovery times and vulnerability windows, where security keys or data packets might need to be re-collected after a failure or reset. In reliability engineering, our generalized formulation can provide useful guidance for optimizing maintenance cycles and for estimating the expected time to complete a sequence of operational milestones under intermittent power disruptions or mandatory reboots.

The structure of this paper is as follows. In Section 2, we present an explicit formula for the expected waiting time to collect all standard coupons in the general case. In Section 3, we present an alternative proof of Theorem 3 from [8]. Section 4 provides numerical results to validate our theoretical findings. Finally, Section 5 concludes the paper.

2. Expected waiting time

Throughout the paper, we always assume that the reset coupon is drawn with probability p_R, where $0 \le p_R < 1$, and the standard coupon i from the set $\mathbb{N}_n = \{1, 2, \dots, n\}$ is drawn with probability p_i such that $\sum_{i=1}^{n} p_i + p_R = 1$. Let W_n^* denote the waiting time to complete a full collection of standard coupons in the general case, and W_n denote the corresponding waiting time when $p_R = 0$. In other words, W_n can be equivalently viewed as the waiting time in the classical setting where the reset coupon is entirely absent from the pool, implying that $\sum_{i=1}^{n} p_i = 1$. We will also use the notation $p_J = \sum_{j\in J} p_j$ for $J \subset \mathbb{N}_n$.

The distribution of the waiting time W_n is well-known (see Theorem 1 in [1]):

$$P(W_n > k) = \sum_{i=0}^{n-1}(-1)^{n-1-i} \sum_{\substack{J\subset\mathbb{N}_n \\ |J|=i}} p_J^k, \text{ for } k \geq 0, \tag{2.1}$$

which can be used to evaluate the expected value $E(W_n)$ through a standard identity for nonnegative integer-valued random variables:

$$E(W_n) = \sum_{k=0}^{\infty} P(W_n > k). \tag{2.2}$$

Indeed, this relation can be verified directly as follows:

$$\begin{aligned} E(W_n) &= \sum_{j=1}^{\infty} jP(W_n = j) = \sum_{j=1}^{\infty}\sum_{k=0}^{j-1} P(W_n = j) \\ &= \sum_{k=0}^{\infty}\sum_{j=k+1}^{\infty} P(W_n = j) = \sum_{k=0}^{\infty} P(W_n > k), \end{aligned}$$

where the interchange of the summation order is justified by Tonelli's theorem since all terms are nonnegative. Substituting (2.1) into (2.2), one can get the standard result in the analysis of the coupon collector's problem (see, e.g., [3] or [11]):

$$E(W_n) = \sum_{i=0}^{n-1}(-1)^{n-1-i} \sum_{\substack{J\subset\mathbb{N}_n \\ |J|=i}} \frac{1}{1-p_J}. \tag{2.3}$$

To calculate $E(W_n^*)$ explicitly as in Theorem 2.3, we define the generating function of the tail probability of W_n as follows:

$$f_{W_n}(x) = \sum_{k=0}^{+\infty} P(W_n > k)x^k. \tag{2.4}$$

Note that $f_{W_n}(1) = \sum_{k=0}^{\infty} P(W_n > k) = E(W_n) < \infty$ by (2.3). Hence the power series $f_{W_n}(x)$ converges at $x = 1$, and its radius of convergence is at least 1. In fact, $f_{W_n}(x)$ can be calculated explicitly as follows.

Lemma 2.1. *The following identity of $f_{W_n}(x)$ holds:*

$$\sum_{k=0}^{+\infty} P(W_n > k)x^k = \sum_{i=0}^{n-1}(-1)^{n-1-i} \sum_{\substack{J\subset\mathbb{N}_n \\ |J|=i}} \frac{1}{1-p_Jx}, \tag{2.5}$$

provided that $|x| < (1 - \min_{1\leq i\leq n} p_i)^{-1}$.

Proof. The condition implies that $|p_J x| < 1$ for all $J \subsetneq \mathbb{N}_n$. By using (2.1), we obtain

$$\begin{aligned}\sum_{k=0}^{+\infty} P(W_n > k)x^k &= \sum_{k=0}^{+\infty}\sum_{i=0}^{n-1}(-1)^{n-1-i}\sum_{\substack{J\subset\mathbb{N}_n\\|J|=i}} p_J^k x^k\\ &= \sum_{i=0}^{n-1}(-1)^{n-1-i}\sum_{\substack{J\subset\mathbb{N}_n\\|J|=i}}\sum_{k=0}^{+\infty}(p_J x)^k \\ &= \sum_{i=0}^{n-1}(-1)^{n-1-i}\sum_{\substack{J\subset\mathbb{N}_n\\|J|=i}}\frac{1}{1-p_J x},\end{aligned} \tag{2.6}$$

where the interchange of the summation order is justified by the absolute convergence of the geometric series as $|p_J x| < 1$ for all $J \subsetneq \mathbb{N}_n$. □

Lemma 2.2. *For $\mathbf{x} \in (0, +\infty)^n$, the multivariable function*

$$g_n(\mathbf{x}) = \sum_{S\subset\mathbb{N}_n}(-1)^{|S|}\frac{1}{1+x_S} \tag{2.7}$$

is strictly positive and monotonically increasing with respect to each variable x_i, where $\mathbb{N}_n = \{1, 2, ..., n\}$ and $x_S = \sum_{i\in S} x_i$ for $S \subset \mathbb{N}_n$.

Proof. Before proceeding with the main proof, we establish the following auxiliary claim.
Claim. For positive integers k, n and a real number $a > 0$, the equation

$$\sum_{S\subset\mathbb{N}_n}(-1)^{|S|}\frac{1}{(a+x_S)^k} = k\int_0^{x_n}\sum_{S\subset\mathbb{N}_{n-1}}(-1)^{|S|}\frac{d\xi_n}{(a+\xi_n+x_S)^{k+1}}$$

holds for variables $x_1, x_2, \ldots, x_n > 0$.
Proof of the Claim For $n = 1$, the equation reduces to

$$\frac{1}{a^k} - \frac{1}{(a+x_1)^k} = k\int_0^{x_1}\frac{d\xi_1}{(a+\xi_1)^{k+1}},$$

since by definition $\mathbb{N}_0 = \emptyset$ and $x_\emptyset = 0$, which is true by the Newton-Leiznib formula.

For general $n \geq 2$, by partitioning the subsets of $\mathbb{N}_n$ into those not containing n (denoted as S) and those containing n (denoted as $S \cup \{n\}$) for $S \subset \mathbb{N}_{n-1}$, we have

$$\begin{aligned}\sum_{S\subset\mathbb{N}_n}(-1)^{|S|}\frac{1}{(a+x_S)^k} &= \sum_{S\subset\mathbb{N}_{n-1}}(-1)^{|S|}\left(\frac{1}{(a+x_S)^k} - \frac{1}{(a+x_n+x_S)^k}\right)\\ &= \sum_{S\subset\mathbb{N}_{n-1}}(-1)^{|S|}\int_0^{x_n}\frac{kd\xi_n}{(a+\xi_n+x_S)^{k+1}}\\ &= k\int_0^{x_n}\sum_{S\subset\mathbb{N}_{n-1}}(-1)^{|S|}\frac{d\xi_n}{(a+\xi_n+x_S)^{k+1}}.\end{aligned}$$

This completes the proof of the **Claim**.

Now, applying the **Claim** to the right-hand side of (2.7) (with $a = 1, k = 1$), we obtain:

$$g_n(\mathbf{x}) = 1! \int_0^{x_n} \sum_{S \subset \mathbb{N}_{n-1}} (-1)^{|S|} \frac{1}{(1 + \xi_n + x_S)^2} d\xi_n.$$

Applying the **Claim** to the integrand for the second time (with $a = 1 + \xi_n, k = 2$), we have:

$$g_n(\mathbf{x}) = 2! \int_0^{x_n} \int_0^{x_{n-1}} \sum_{S \subset \mathbb{N}_{n-2}} (-1)^{|S|} \frac{1}{(1 + \xi_n + \xi_{n-1} + x_S)^3} d\xi_{n-1} d\xi_n.$$

Repeating this process n times recursively yields the following n-fold integral representation:

$$g_n(\mathbf{x}) = n! \int_0^{x_n} \int_0^{x_{n-1}} \cdots \int_0^{x_1} \frac{1}{\left(1 + \sum_{i=1}^{n} \xi_i\right)^{n+1}} d\xi_1 \ldots d\xi_n = \int_{\Omega(\mathbf{x})} F(\boldsymbol{\xi}) d\boldsymbol{\xi},$$

where $\Omega(\mathbf{x}) = [0, x_1] \times [0, x_2] \times \cdots \times [0, x_n]$ and $F(\boldsymbol{\xi}) = \frac{n!}{\left(1+\sum_{i=1}^{n} \xi_i\right)^{n+1}}$.

Since the integrand $F(\boldsymbol{\xi})$ is strictly positive for all $\boldsymbol{\xi} \geq \mathbf{0}$, the integral $\int_{\Omega(\mathbf{x})} F(\boldsymbol{\xi}) d\boldsymbol{\xi}$ is strictly positive. Furthermore, as the domain $\Omega(\mathbf{x})$ expands (i.e., as any x_i increases), the integral monotonically increases by the mean value theorem for multiple integrals. This concludes the proof. $\square$

The waiting time W_n^* can be viewed through the lens of renewal theory. Each draw of a reset coupon acts as a renewal event that restarts the entire collection process. Let W_n' denote the waiting time in a rescaled process without resets, with probabilities $p_j' = p_j/(1 - p_R)$. The total waiting time W_n^* is composed of a geometric number of failed cycles followed by one successful cycle.

Although the distribution of the waiting time W_n^* was obtained in terms of the distribution of W_n' in [8] for the general case, an explicit formula for the expected waiting time $E(W_n^*)$ was not provided there. We establish this explicit expectation in the following theorem.

Theorem 2.3. *Let W_n' be the waiting time as defined above. Then, we obtain the following result:*

$$E(W_n^*) = f_{W_n'}(1 - p_R) \cdot \frac{1}{1 - p_R \cdot f_{W_n'}(1 - p_R)}, \tag{2.8}$$

where $f_{W_n'}(1 - p_R) = \sum_{i=0}^{n-1} (-1)^{n-1-i} \sum_{\substack{J \subset \mathbb{N}_n \\ |J|=i}} \frac{1}{1-p_J}$.

Proof. Rewriting part 1 of Theorem 1 of [8] as

$$P\{W_n^* > k\} = \sum_{i=0}^{k} p_R^i (1 - p_R)^{k-i} \sum_{\substack{k_1+\cdots+k_{i+1}=k-i \\ k_1,\cdots,k_{i+1} \geq 0}} \prod_{j=1}^{i+1} P(W_n' > k_j), \tag{2.9}$$

substituting it into $E(W_n^*) = \sum_{k=0}^{+\infty} P\{W_n^* > k\}$ and interchanging the order of summation, we obtain

$$\begin{aligned} E(W_n^*) &= \sum_{k=0}^{+\infty} \sum_{i=0}^{k} p_R^i (1 - p_R)^{k-i} \sum_{\substack{k_1+\cdots+k_{i+1}=k-i \\ k_1,\cdots,k_{i+1} \geq 0}} \prod_{j=1}^{i+1} P(W_n' > k_j) \\ &= \sum_{i=0}^{+\infty} p_R^i \sum_{k=i}^{+\infty} (1 - p_R)^{k-i} \sum_{\substack{k_1+\cdots+k_{i+1}=k-i \\ k_1,\cdots,k_{i+1} \geq 0}} \prod_{j=1}^{i+1} P(W_n' > k_j). \end{aligned} \tag{2.10}$$

The interchange of the summation order k and i is justified by Fubini's theorem for nonnegative series, as all terms in the summand are nonnegative.

Substituting $k - i$ with s, we get

$$E(W_n^*) = \sum_{i=0}^{+\infty} p_R^i \sum_{s=0}^{+\infty} (1 - p_R)^s \sum_{\substack{k_1+\cdots+k_{i+1}=s \\ k_1,\cdots,k_{i+1}\geq 0}} \prod_{j=1}^{i+1} P(W_n' > k_j). \tag{2.11}$$

Since all terms in the summation are nonnegative, the multidimensional sum in the inner loop can be decomposed into a product of multiple independent series under Tonelli's theorem for series. Therefore, we get

$$\begin{aligned} E(W_n^*) &= \sum_{i=0}^{+\infty} p_R^i \prod_{j=1}^{i+1} \sum_{k_j=0}^{+\infty} P(W_n' > k_j)(1 - p_R)^{k_j} \\ &= \sum_{i=0}^{+\infty} p_R^i \left(\sum_{k=0}^{+\infty} P(W_n' > k)(1 - p_R)^k \right)^{i+1}. \end{aligned} \tag{2.12}$$

The last equality is by changing the summation dummy indices k_j-s with k.

Substituting the definition (2.4) with W_n replaced by W_n' into (2.12), we have

$$E(W_n^*) = f_{W_n'}(1 - p_R) \sum_{i=0}^{+\infty} (p_R \cdot f_{W_n'}(1 - p_R))^i. \tag{2.13}$$

We now prove $p_R \cdot f_{W_n'}(1 - p_R) < 1$. Applying Lemma 2.1 with W_n replaced by W_n', we have

$$\begin{aligned} p_R \cdot f_{W_n'}(1 - p_R) &= \sum_{i=0}^{n-1} (-1)^{n-1-i} \sum_{\substack{J\subset \mathbb{N}_n \\ |J|=i}} \frac{p_R}{1 - p_J} \\ &= \sum_{i=0}^{n-1} (-1)^{n-1-i} \sum_{\substack{S\subset \mathbb{N}_n \\ |S|=n-i}} \frac{p_R}{p_R + p_S}. \end{aligned} \tag{2.14}$$

Setting $x_i = p_i/p_R$ and substituting it into (2.14), we have

$$p_R \cdot f_{W_n'}(1 - p_R) = \sum_{\substack{S\subset \mathbb{N}_n \\ |S|\neq 0}} (-1)^{|S|+1} \frac{1}{1 + x_S}. \tag{2.15}$$

Combining equations (2.7) and (2.15), we have

$$p_R \cdot f_{W_n'}(1 - p_R) = 1 - g_n(x_1, \ldots, x_n). \tag{2.16}$$

By Lemma 2.2, $g_n(x_1, \ldots, x_n) > 0$. Thus,

$$p_R \cdot f_{W_n'}(1 - p_R) < 1. \tag{2.17}$$

Since $p_R \cdot f_{W_n'}(1 - p_R) \geq 0$, the condition $p_R \cdot f_{W_n'}(1 - p_R) < 1$ ensures absolute convergence in (2.13).

Applying (2.17) to (2.13) yields the result. □

Remark. Note that while the explicit formula in Theorem 2.3 involves a summation over 2^n subsets, which is computationally expensive for large n, it provides an exact analytical benchmark.

3. A new proof of the expected waiting time formula

We now reprove the formula for the equal-probability case from [8].

Theorem 3.1. *Assume* $0 < p_R < 1$*. The expected waiting time to complete the collection when all standard coupons have equal probabilities* $p = (1 - p_R)/n$ *is:*

$$E(W_n^*) = \frac{1}{p_R}\left(\frac{1}{np_R B(n, \frac{np_R}{1-p_R})} - 1\right), \tag{3.1}$$

where $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ *denotes the Beta function for* $x, y > 0$*.*

Proof. Letting $\lambda := f_{W_n'}(1 - p_R)$, from Theorem 2.3, when all standard coupons are drawn with probability $p = \frac{1-p_R}{n}$, the expected waiting time is given by

$$E(W_n^*) = \frac{\lambda}{1 - p_R\lambda}, \quad \text{with} \quad \lambda = \sum_{i=0}^{n-1}(-1)^{n-1-i}\binom{n}{i}\frac{1}{1-ip}. \tag{3.2}$$

Using the relation $p_R + np = 1$ and changing the index i with $n - i$, we have

$$\frac{1}{p_R} - \lambda = \sum_{i=0}^{n}(-1)^{n-i}\binom{n}{i}\frac{1}{1-ip} = \sum_{i=0}^{n}(-1)^{i}\binom{n}{i}\frac{1}{p_R+ip}. \tag{3.3}$$

Using the integral representation of the Beta function, we have

$$B\left(n+1, \frac{p_R}{p}\right) = \int_0^1 (1-t)^n t^{\frac{p_R}{p}-1}dt = \sum_{i=0}^{n}(-1)^{i}\binom{n}{i}\frac{1}{\frac{p_R}{p}+i}. \tag{3.4}$$

Dividing (3.4) by p and then comparing it with (3.3), we obtain

$$\frac{1}{p_R} - \lambda = \frac{1}{p}B\left(n+1, \frac{p_R}{p}\right). \tag{3.5}$$

By applying the recurrence relation of the Beta function (e.g., see [10]), we get

$$B\left(n+1, \frac{p_R}{p}\right) = \frac{n}{n+\frac{p_R}{p}}B\left(n, \frac{p_R}{p}\right) = npB\left(n, \frac{p_R}{p}\right). \tag{3.6}$$

Thus,

$$\frac{1}{p_R} - \lambda = nB\left(n, \frac{p_R}{p}\right) \quad \text{and} \quad \lambda = \frac{1}{p_R} - nB\left(n, \frac{p_R}{p}\right). \tag{3.7}$$

Finally, substituting (3.7) into the expression for $E(W_n^*)$, we obtain

$$E(W_n^*) = \frac{\frac{1}{p_R} - nB\left(n, \frac{p_R}{p}\right)}{np_R B\left(n, \frac{p_R}{p}\right)} = \frac{1}{p_R}\left(\frac{1}{np_R B\left(n, \frac{p_R}{p}\right)} - 1\right). \tag{3.8}$$

Substituting $p = \frac{1-p_R}{n}$ into the above equation, we get the desired result. □

Remark. Note that the equation (3.4) can also be deduced from the following interesting identity of [19]:

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}B(i+1,s)=\frac{1}{n+s},\quad \text{for } s>0, \tag{3.9}$$

by taking $s = p_R/p$ in (3.9) and applying the binomial inversion formula:

$$a_n=\sum_{i=0}^{n}\binom{n}{i}b_i \iff b_n=\sum_{i=0}^{n}(-1)^{n-i}\binom{n}{i}a_i, \text{ where } a_n,b_n\in\mathbb{C} \text{ for } n=0,1,2,\ldots$$

(e.g., see [5]) to it.

4. Numerical results and simulations

To validate the formula in Theorem 2.3, we conducted Monte Carlo simulations. For $n = 3$, the base probabilities of the standard coupons were set according to ratio $p_1 : p_2 : p_3 = 2 : 3 : 4$. For each configuration, 10^6 trials were performed.

The results in Table 1 show that simulated averages are in excellent agreement with theoretical values, with errors below 0.04%. Beyond verifying the accuracy of our analytical formula, several meaningful physical insights can be extracted from the numerical trends shown in Table 1.

Table 1. Comparison of theoretical and simulated expected waiting times ($n = 3$).

Reset Prob. (p_R)	Theoretical $E(W_n^*)$	Simulated Average	Relative Error
0.00	6.1643	6.1630	0.020%
0.05	7.0506	7.0529	0.033%
0.10	8.1425	8.1437	0.014%
0.20	11.2163	11.2174	0.010%

First, as expected, the expected waiting time $E(W_n^*)$ increases monotonically with the reset probability p_R. This behavior is intuitively sound, as a higher reset probability increases the frequency of process restarts, repeatedly wiping out the collector's progress and forcing them to start anew.

Second, we observe that the expected waiting time grows nonlinearly with respect to p_R. Specifically, when p_R increases from 0.00 to 0.10, the expected waiting time rises by approximately 32.1% (from 6.1643 to 8.1425). However, when p_R is doubled further to 0.20, the expected waiting time surges to 11.2163, representing a much sharper increase of 81.9% compared to the baseline $p_R = 0.00$. This nonlinear acceleration reveals the compounding penalty of resets on the collection process: even small increments in the reset probability can significantly compound the expected completion time, especially in nonequiprobable coupon environments where collecting rare coupons is already challenging.

5. Conclusions

We have derived a generalized formula for the coupon collector problem with a reset button. While providing robust analytical insights, the current framework is limited to "full resets". Future work could explore partial resets or time-varying probabilities to enhance applicability in complex environments.

Author contributions

Y. Li and C. Dai developed the theoretical model and performed the numerical simulations. D. Kim supervised the project and verified the analytical proofs. All authors reviewed and approved the final manuscript. All authors contributed equally to this work.

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Conflict of interest

The authors declare no conflict of interest.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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